Domain Decomposition

$\Omega = (0, L_x) \times (0, L_y)$

$$\begin{cases} (-\Delta - k^2)u = f & \text{in } \Omega \\ (\partial_n - ik)u = g & \text{in } \partial\Omega = \Gamma \end{cases}$$

SOURCE TERM $\quad f(\underline{x}) = \sum_{i=1}^{N_s} w_i e^{-\frac{10}{\lambda^2}|\underline{x} - s_i|^2}$

1) LOOKING FOR $u_h(x) = \sum_j u_j \phi_j(x)$

code.py: SOLVER FEM P1 FOR HELMHOLTZ EQUATION

$A = K - k^2 M - ik M_b$, THEN SOLVE $Ax = b$ — REFERRED TO f

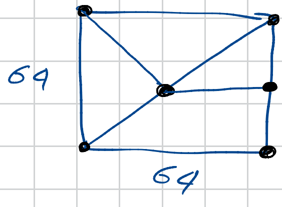
MESH REFINING: $\Delta x = \frac{L_x}{n_x - 1} \qquad \Delta y = \frac{L_y}{n_y - 1}$

(WITHOUT PRECOND.)

↑ DoFs ⇒ ↑ COST PER ITERATION (BIGGER A)
↳ BETTER REPRESENTATION OF OSCILLATIONS

maxiter = 200, BUT HERE WE USE RESTART! — EVERYTIME YOU USE RESTART, maxiter RE-SETS

GRAPH?



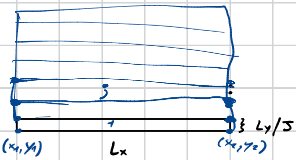
MESH

2) BUILDING A NON-OVERLAPPING DOMAIN DECOMPOSITION METHOD

SOLVING EVERYTHING ON $\Omega \mapsto \Omega_j$ SUB-DIVIDE IN J SUBDOMAINS $\quad j = \{1, \dots, J\}$

DEFINING BOOLEAN MATRICES OF RESTRICTION:

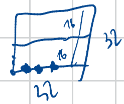
R_j (VOLUME), B_j (TRACE ON INTERFACE), C_j (SELECTING INTERFACE'S DOF IN GLOBAL SKELETON $V(S)$).



DECOMPOSITION IN THE y DIRECTION.

WE GET J HORIZONTAL SLABS $(0, L_x) \times (0, L_y/J)$

ASSUMPTION: $N_y - 1$ MULTIPLE OF J.



CHARACTERISTICS OF THE MESH:

vtx = COORD. OF VERTICES $|V(\Omega)| \times 2$ (x, y) [$V(\Omega)$: COORDINATES OF VERTEXES OF THE MESH]

elt = GIVES THE LIST OF VERTECES: $elt[t] = [i_0, i_1, i_2, i_3]$

$vtx[i_0]$ $vtx[i_1]$ $vtx[i_2]$ COORDINATE OF THAT VERTEX

1) local_mesh $(L_x, L_y, N_x, N_y, j, J)$

THAT RETURNS vtx_j, elt_j = MESH OF SUB-DOMAIN Ω_j

EACH SLUB IS A RECTANGLE $(0, L_x) \times (y_{j,min}, y_{j,max})$

vtx_j IS LIKE A MESH WITH N_x POINTS IN x AND $\frac{(N_y - 1)}{J} + 1$ POINTS ON y, BUT WITH y COORDINATES TRASLATED TO BE IN THE CORRECT POSITION IN THE GLOBAL DOMAIN.

2) local_boundary (N_x, N_y, j, J)

BUILD TWO EDGE ARRAYS OF THE j-TH SUBDOMAIN

beltj_phys: BORDERS THAT STAY ALSO IN THE PHYSICAL BOUNDARY $\partial\Omega$.

- INTERNAL SLABS: LEFT/RIGHT $(x = 0, x = L_x)$
- BOTTOM SLAB $(j = 0)$: $y = 0$, LEFT/RIGHT
- UPPER SLAB $(j = J - 1)$: $y = L_y$, LEFT/RIGHT

beltj_artf: ARTIFICIAL BORDERS, I.E. INTERFACES WITH NEIGHBOURS.

INTERNAL SLABS: BOTTOM + UP = ARTIF

BOTTOM SLAB: UP

UPPER SLAB: BOTTOM

LOCAL RESTRICTION MATRICES

1) R_j_matrix : $V(\Omega) \longrightarrow V(\Omega)$

TAKES A GLOBAL VECTOR OF DoF ON Ω AND OUTPUTS

JUST THOSE OF SUB-DOMAIN Ω_j

$R_j \in \{0,1\}^{|V(\Omega)| \times |V(\Omega)|}$ $\qquad u_j = R_j u$

2) B_j_matrix : $V(\Omega_j) \longrightarrow V(\Sigma_j)$

$u\_j \mapsto$ GIVES JUST VALUES AT NODES' INTERFACE $\to$ USE belt_j_phys

$B_j \in \{0,1\}^{|V(\Sigma_j)| \times |V(\Omega_j)|}$

$B_j u_1 = u_1$

3) C_j_matrix : $\overbrace{V(S)}^{\Sigma|V(\Sigma_j)|} \longrightarrow \overbrace{V(\Sigma_j)}^{|V(\Sigma_j)|}$, $\Sigma_j = \partial\Omega_j \setminus \partial\Omega$

$V(S) = \prod_{j=1}^{J} V(\Sigma_j) = x = (x_1, x_2, \dots, x_J)$ BLOCK OF THE DoFs IN INTERFACES OF SUBDOMAIN j $\to$ DoF VALUES

$x_j = C_j x$

$C_j \in \{0,1\}^{|V(\Sigma_j)| \times |V(S)|}$

$V(\Omega)$: GLOBAL FEM SPACE $\qquad V(\omega) = \text{SPAN}\{\varphi_i|_\omega\}_{i \in I(\omega)}$, $I(\omega)$ = INDEX OF GLOBAL VERTECES IN ω

LOCAL PROBLEMS

1) $A \mapsto A_j$ LOCAL PROBLEM

TAKING vtx_j, elt_j, belt_j_phys

2) $T_j = KM_j$ — ASSOCIATED TO INTERFACES Σ_j

3) $S_j := (A_j - B_j^* T_j B_j)$, PERFORM LU FACTORIZATION

4) b_j RHS

GLOBAL OPERATORS

1) S_operator : COMPUTES THE ACTION OF S ON A GLOBAL VECTOR INTERFACE x

2) Pi_operator : ACTION OF π ON A GLOBAL VECTOR INTERFACE

3) g_vector : GLOBAL RHS g OF INTERFACE PROBLEM