Macro patterns in the evolution of human aging

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Extended abstract

Typically, demographers summarize the distribution of remaining lifetimes with the mean, $e(a)$, i.e. remaining life expectancy. Life expectancy at birth, $e(0)$ is one of the most widely used measures to summarize population health. Its significant and consistent increase witnessed during the last two centuries is one of the most remarkable achievements of modern societies (Oeppen and Vaupel 2002). Nevertheless, life expectancy is not an omnibus descriptor of time to death. There are other useful measures of longevity that refer to the pace of aging, such as the modal or median ages at death (Canudas-Romo 2010). These measures also summarize the distribution of lifespans in the lifetable stationary population. These indicators, however, conceal variation in lifetimes and other aspects of the age at death distribution.

Variation in lifespans has recently arisen as an important dimension in demography and aging research. It expresses a fundamental inequality among individuals and addresses a growing interest in health inequality and its linkage with social behavior (Mackenbach 2012). Inequality measures have been found to be negatively associated with life expectancy levels in several countries and over millions of years of primate evolution (Vaupel et al. 2011, Colchero et al. 2016). As a result, demographers have responded to the need of accurate measures and have developed a battery of lifespan variability indicators (van Raalte and Caswell 2013), which refer to particular aspects of the shape of the distribution of mortality or the shape of aging (Wrycza et al. 2015).

In this paper, we extend previous research by expressing a set of formulas to measure other aspects of the age at death distribution from lifetables conditioned on surviving to any age, such as skewness and kurtosis, from a moment generator function. Further, we explore linkages between such indicators and the pace of aging from a macro shape framework, and we interpret results in terms of the evolution of aging.

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Definitions

Remaining life expectancy conditional on survival to age a is defined as

$$e(a) = \frac{1}{l(a)} \int_0^\infty l(a+y)\ \mathrm{d}y \qquad , \tag{1}$$

where $l(a)$ is lifetable survivorship to age a. Define the conditional deaths distribution

$$f(y|a) = \frac{1}{l(a)} \mu(a+y) l(a+y) \qquad , \tag{2}$$

where $\mu(a)$ is the force of mortality at age a. $f(y|a)$ is interpreted as the probability of surviving to and dying at exact age $a+y$ given survival to age a.

The conditional deaths distribution can be described empirically using quantiles, or other central measures such as the median or the mode, or perhaps more parsimoniously using its moments. The n^{th} central moment about the conditional mean of $f(y|a)$, $\eta_n(y|a)$ is defined as:

$$\eta_n(y|a) = \int_{y=0}^\infty (y - e(a))^n f(y|a)\ \mathrm{d}y \qquad , \tag{3}$$

where $\eta_2(y|a)$ gives the variance of remaining lifespan about $e(a)$, $\sigma^2(y|a)$.[1] Survival-conditioned variance is useful information, but it can be deceptive because lifespan variation is not symmetric around $e(a)$. The conditional skewness function, $Skew(y|a)$ captures most such variation and can be roughly interpreted in this way. It is defined as

$$Skew(y|a) = \frac{\eta_3(y|a)}{\sigma(y|a)^3} \qquad , \tag{4}$$

the third standardized moment. The conditional excess kurtosis of $f(y|a)$, $Kurt(y|a)$, can be defined as

$$Kurt(y|a) = \frac{\eta_4(y|a)}{\sigma(y|a)^4} - 3 \qquad . \tag{5}$$

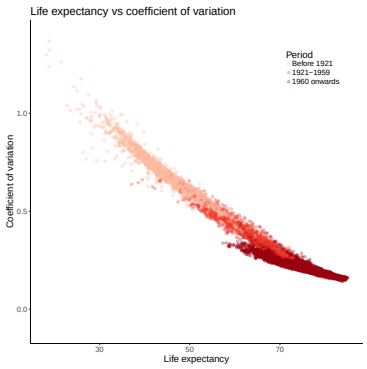
The age pattern of kurtosis describes how the peakedness, or the fatness of the tails of the remaining distribution change over age.

Data and Methods

All formulas are discretized to single ages using standard demographic approximations, and these are implemented in the `R` programming language (R Core Team 2016). We then calculate the above measures for each year, population, and sex in the Human Mortality Database (HMD). To examine macro patterns in the central and shape measures, we show a series of bivariate relationships.

[1] Compare with Chiang (1984), Chapter 10, Equation 6.10, where the author denotes $f(y|a)$ with Y_α.

Figure 1: Coefficient of variation by average length of life



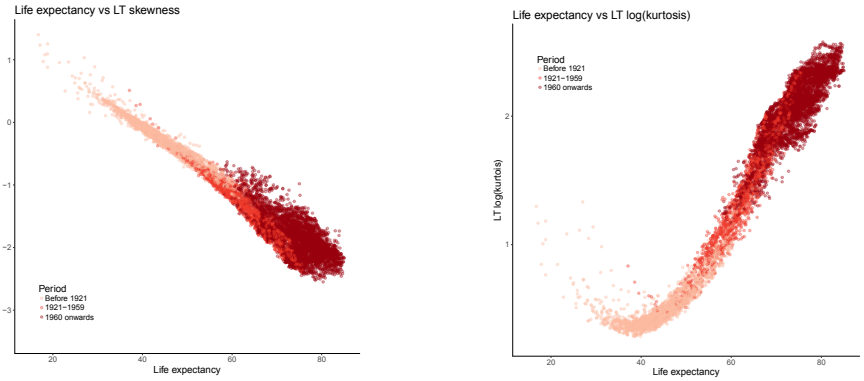
Preliminary Results

Figure 1 shows the relationship between life expectancy at birth and variation in lifespans, as measured by the lifetable coefficient of variation for 45 countries. Lighter colors refer to data before 1920, darker reds to years between 1920 and 1959, and darkest hues to the most recent period from 1960 onward. The results suggest that as populations tend to live longer, they also experience less uncertainty surrounding their eventual time of death. After 1960, there is a change in the slope and a level-off on the speed of reducing variability, as the coefficient of variation asymptotically reaches zero, the unrealistic scenario in which everybody dies at the same age.

Figure 2 shows the skewness of the age at death distribution and its relationship with the trend in life expectancy (panel 2a) and its corresponding kurtosis value (panel 2b). Before 1960, the macro pattern of life expectancy with skewness suggest that as the average length of life increases over time, the distribution of mortality gets a longer left tail. However, after 1960, populations have experienced a shift in the value of skewness, giving the overall pattern the shape of a Prince Rupert's drop. This might be a result of countries changing from a bimodal to a unimodal distribution. In addition, as mortality reductions are increasingly difficult at young ages, improvements are gradually concentrating in older ages, which makes the right tail of the distribution longer. Similarly, as life expectancy increases, the value of kurtosis also increases. However, after 1960 similar values of life expectancy correspond to different values in kurtosis. These results suggest that over time, the tails of the distribution are getting "fatter", paralleling the rise in life expectancy.

Future work: we will further compare linkages between different moments conditioned on surviving to higher ages. For example, what are the macro associations between these indicators conditioned on surviving to age 15 (or 50), leaving out all infant mortality and capturing just late-life trends.

Figure 2: Skewness and kurtosis at birth by average length of life



(a) Skewness vs $e(0)$

(b) Kurtosis vs $e(0)$

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