

Displaying changes in bivariate relationships over age and time

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Abstract

Background Lexis surfaces are an established visualization technique to show how a given value changes over age and time. Vector fields are a two-dimensional representation of two variables: direction and speed (or force). Phase planes represent limits and thresholds of differential equations, such as the relationship between first and second derivatives.

Objective We aim to increase the dimensionality of patterns shown on the Lexis surface by placing a vector field on the Lexis surface. We also relate the phase planes to the Lexis surface by way of the Lexis field.

Results We show Lexis fields using different combinations of visual encodings, such as color, contour layering, and angle, length, and curvature of field elements. These instruments enable information layering that is not otherwise possible on standard Lexis surfaces.

Conclusions Lexis fields extend the analytic power of the Lexis surface, and these can be rendered to display information at higher densities than commonly made Lexis surfaces.

Abstract

Lexis surfaces are a graphical form used to display data on the Lexis coordinate plane, a Cartesian plane that is also a simplex relationship between age, period, and cohort. Surfaces are often displayed as heat maps, contours maps, perspective plots, or variants of these things. Various kinds of quantities, such as raw magnitudes, differences, ratios, intensities, proportions, derivatives, and even compositions (Schöley and Willekens 2017)) can be displayed on Lexis surfaces in order to put age, period, cohort, or other patterns in relief. Vector fields are a graphical form used to display variation in speed, direction, or force over a plane. Point estimates of variation on the plane are often represented with arrows where length is proportional to a function of magnitude (force, speed) and where angle indicates direction. We propose a fusion of these two graphical instruments, *Lexis fields*, as a method to display variation in relationships between variables over age and time. We give two different use examples of Lexis fields. The first example shows patterns in the relationship between average remaining lifespan and the standard deviation of remaining lifespan over age and time based on all available populations in the Human Mortality Database from 1950 onward. The second example shows changes in the relationship between the first and second derivative of period fertility rates over age and time, essentially a Lexis *phase plane*, but represented as a Lexis field.

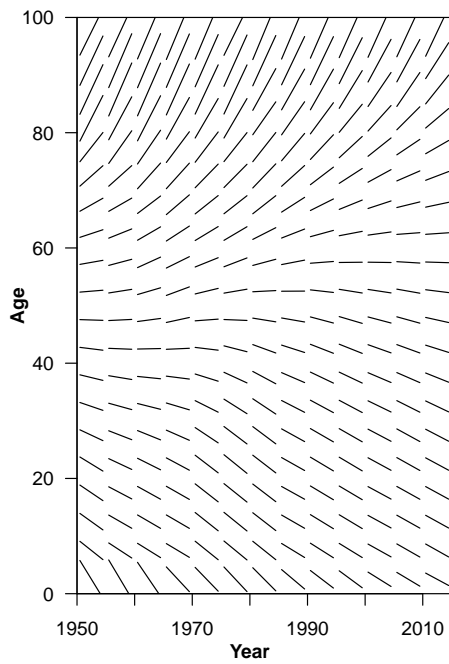
We offer several experimental variants of Lexis field design for our two examples. The first of these, Figure 1a, is a bare-bones Lexis field that serves to illustrate the underlying concept. This display differs from a standard vector field in two notable ways. 1) Segments are drawn rather than arrows, because each slope can be interpreted as a subplot, where the convention is to always read a relationship from left to right. 2) The left and right extremes are fixed for each segment according to the year-width rendered,

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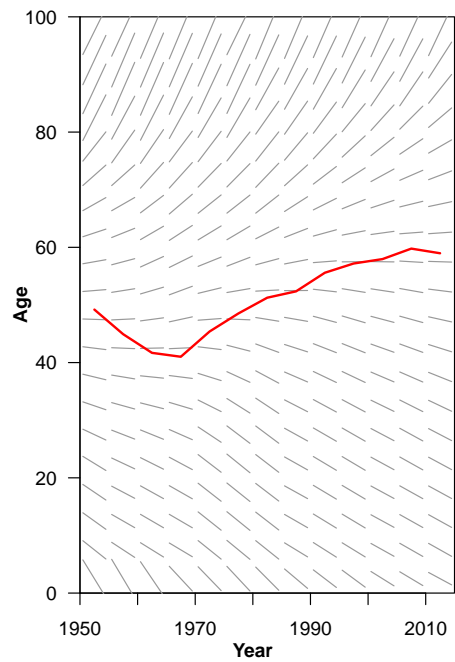
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in this case 5-year groupings. This means that relationships farther from zero also result in longer line segments, but the length of the line segment is not strictly proportional to the magnitude of the slope coefficient or any other quantity. For each subplot, the x range represents the same hypothetical range of variation in remaining life expectancy or 2 years. The y-axis is arbitrarily, but identically scaled for each subplot. Other criteria for slope length would also be possible and will be considered.

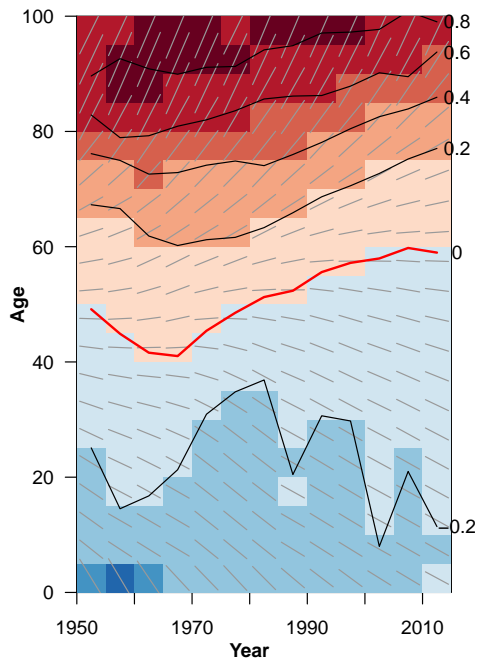
Figure 1b is identical to the previous, but adds a red contour line at the turning point in slopes. In this case there is only one sign change in slope coefficients over the age pattern. It is easier to locate and detect change in this (novel) threshold with the line markup than from the bare Lexis field. Indeed one could add further isoclines at slope break points, as in Figure 1c or 1d, and insodoing we begin a transition into the more standard graphical form of contour maps. Figure 1c blends the Lexis field with a heatmap, where hue indicates both direction and magnitude of slope coefficients. In this case, slope coefficients are double-coded, and this display is likely the most legible, but it comes at a cost of clutter. Figure 1d removes the slope segments, but is otherwise identical. We wish to explore variants of the first three of these Lexis field proposals, and may possibly display more example relationships between other variables in a full manuscript, data permitting.



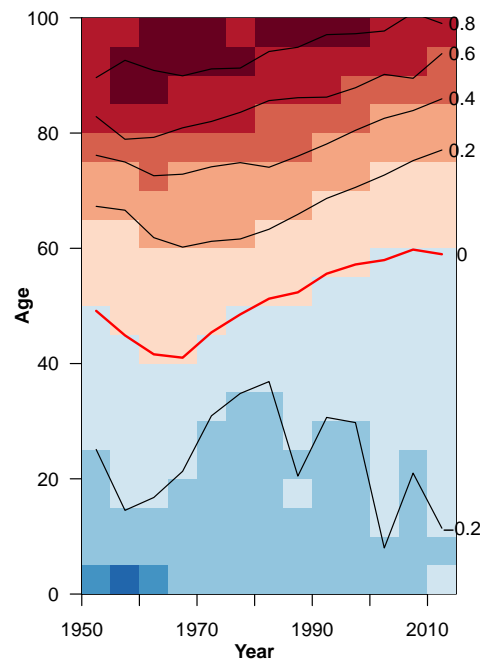
(a) A simple field.



(b) A simple field, with the zero-slope isocline drawn (red).



(c) Slopes doubly coded with field segments and discrete color, further slope isoclines drawn.



(d) Discrete color heatmap with selected isoclines drawn.

Figure 1: Four versions of Lexis fields displaying the linear relationship between the standard deviation and mean of remaining lifespan, males (HMD)

References

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- Jonas Schöley and Frans Willekens. Visualizing compositional data on the lexis surface. *Demographic Research*, 36:627–658, 2017.
- James W Vaupel, Bradley A Gambill, and Anatoli I Yashin. Thousands of data at a glance: shaded contour maps of demographic surfaces. 1987.