# Training Workshop

## Kinship Dynamics: Concept, Modelling, and Applications

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MAX PLANCK INSTITUTE FOR DEMOGRAPHIC RESEARCH
MAX-PLANCK-INSTITUT FÜR DEMOGRAFISCHE FORSCHUNG

# Introductions

Find someone you don't know and ask:

1. Their name
2. Where they study/work
3. Favorite food
4. What is the demography of kinship?

# Our plan for today

| | |
|---|---|
| 09:00 - 09:20 | Introductions |
| 09:20 - 10:00 | Kinship demographic theory |
| 10:00 - 10:30 | Technical setup |
| *10:30 - 11:00* | *Coffee break* |
| 11:00 - 12:00 | The simple kinship model |
| *12:00 - 13:30* | *Lunch* |
| 13:30 - 14:30 | Two-sex time-variant kinship models |
| *14:30 - 15:00* | *Coffee break* |
| 15:00 - 15:30 | Projections of kin by education for Singapore |
| 15:30 - 16:00 | Conclusions and closing |

# This presentation

1. Introduction to kinship demography
2. Demographic models of kinship
3. Example: projections of kinship

# Introduction to kinship demography

# Consider a baby born in Singapore in 2020...

1. How old were her grandparents when she was born, on average?
2. How many living children will she have on her 70th birthday?
3. How many grandchildren?

# Kinship structure is a questions of societal interest[1]



[1] 'The age of the grandparent has arrived.' (Jan 2023). The Economist. https://www.economist.com/international/2023/01/12/the-age-of-the-grandparent-has-arrived

# Definitions (1)[2]

## Kinship

Social relationships that bind individuals together through culturally shared definitions of relatedness on biological, legal, or normative grounds, ultimately constituting family systems.

## Family

More narrow group of kin given special privilege which, among other things, organize the provision of support, socialization, and social placement of its members.

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[2] Alburez-Gutierrez, D., Barban, N., Caswell, H., Kolk, M., Margolis, R., Smith-Greenaway, E., Song, X., Verdery, A., & Zagheni, E. (2022). Kinship, Demography, and Inequality: Review and Key Areas for Future Development. *SocArXiv*. https://doi.org/10.31235/osf.io/fk7x9

# Definitions (2)[3]

**Kinship demography**

The study of family networks, their structures and dynamics from a demographic perspective and using demographic methods.

[3] Alburez-Gutierrez, D., Barban, N., Caswell, H., Kolk, M., Margolis, R., Smith-Greenaway, E., Song, X., Verdery, A., & Zagheni, E. (2022). Kinship, Demography, and Inequality: Review and Key Areas for Future Development. *SocArXiv*. https://doi.org/10.31235/osf.io/fk7x9

# Kinship as a demographic human universal

1. All humans are born
2. All humans die
3. All humans are embedded in kinship structures[4]
4. No particular family configuration is universal or stable

[4] Caswell, H. (2019). The formal demography of kinship: A matrix formulation. *Demographic Research*, *41*, 679–712

# Demographic models of kinship

# What are kinship models?

1. Kinship is an *emergent property* of demographic systems
2. Simplified representation of interaction between reproduction, survival (and more)
3. Can be formal (mathematical) or simulation-based (computational)

# Formal models of kinship

Given a set of:

- age-specific fertility rates
- survival probabilities
- simplifying assumptions

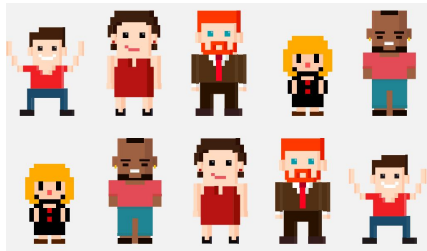
The models produce:

1. Number of (living/dead) kin
2. Age distribution of relatives
3. From the point of view of an average member of the population ('Focal')

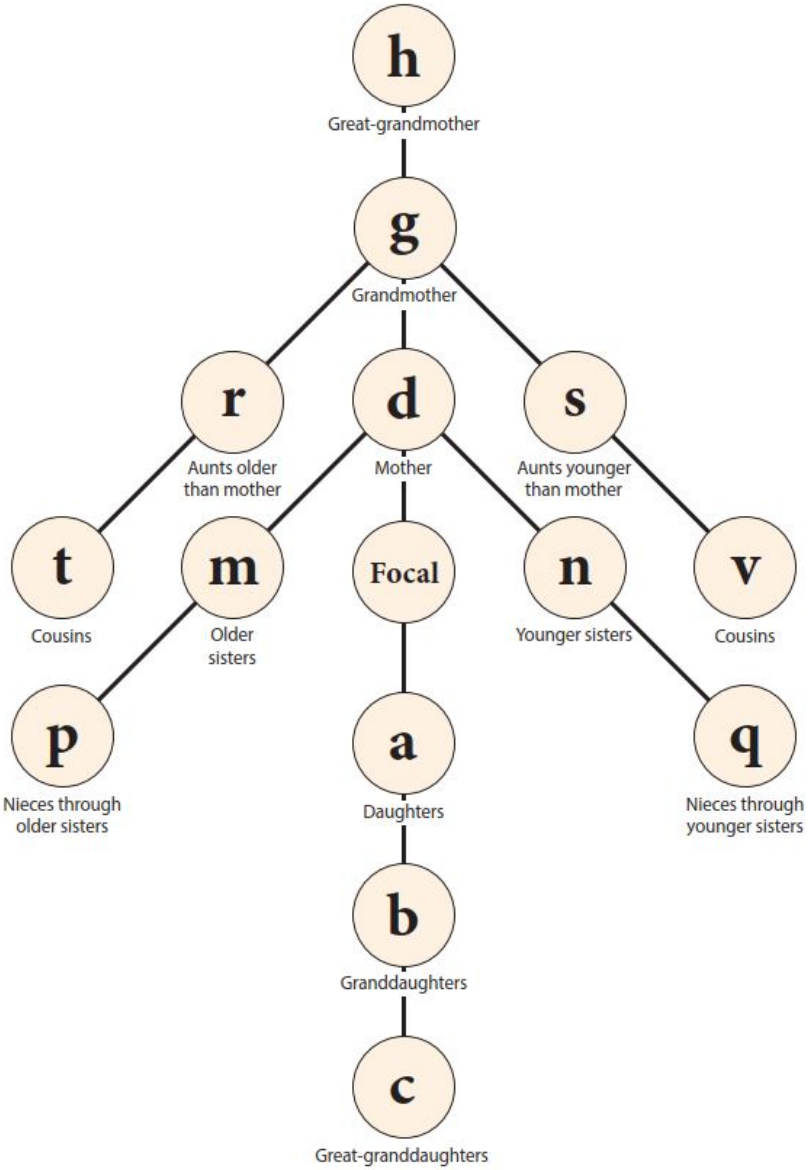
# Focal: an average member of the population

# Matrix kinship models

1. The relatives of Focal constitute a population
2. They can be modelled using traditional projection methods
3. Matrix operations provide an efficient implementation



# Kinship structure



# Implementation: time-invariant, one-sex models[5]

The models are of the general form:

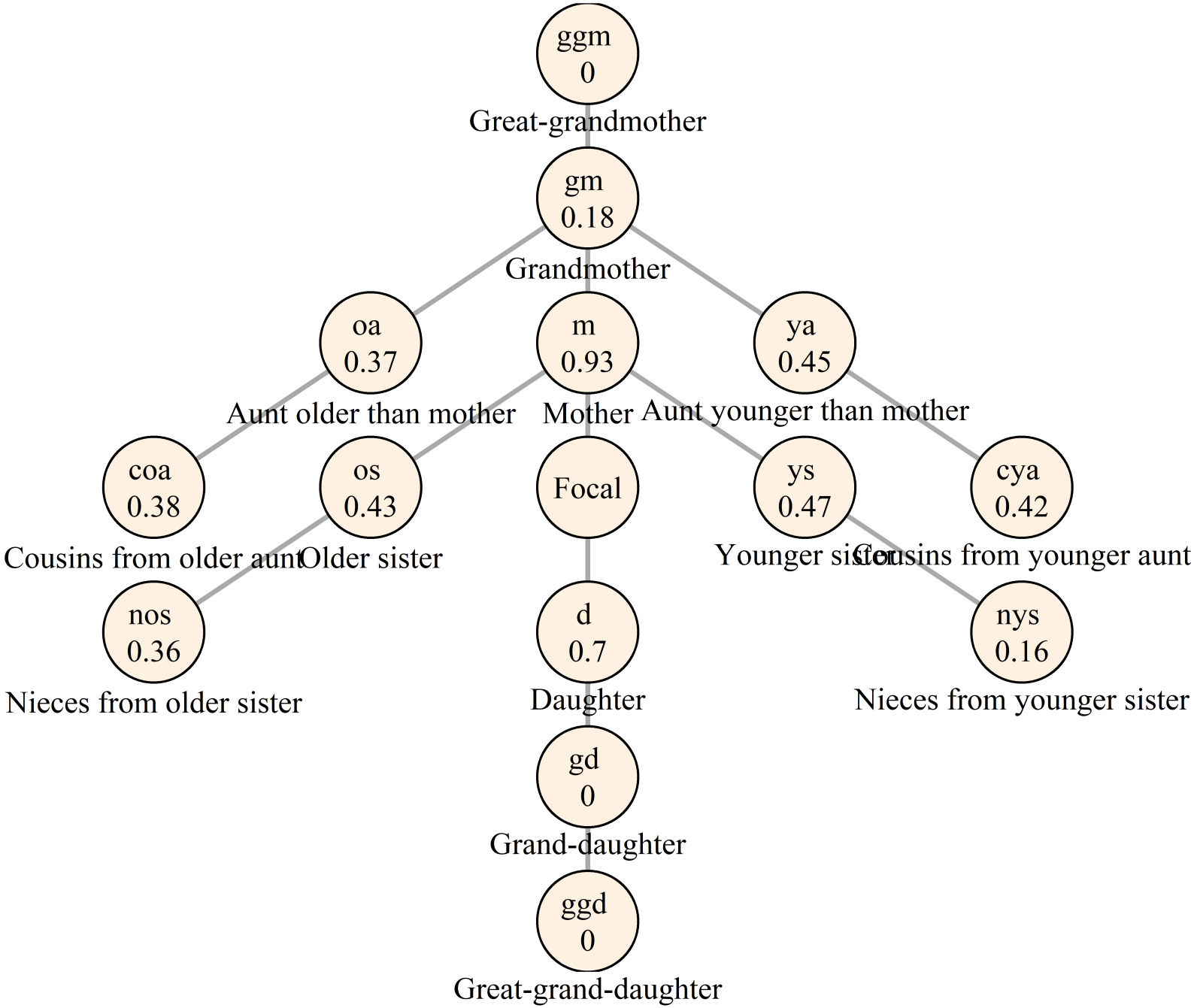
$$\underbrace{\mathbf{k}(x+1)}_{\substack{\text{age structure of kin} \\ \text{at Focal's age } x+1}} = \underbrace{\mathbf{U}\,\mathbf{k}(x)}_{\substack{\text{ageing and survival} \\ \text{of existing kin}}} + \underbrace{\begin{cases} \mathbf{0} \\ \mathbf{F}\,\mathbf{k}^*(x) \end{cases}}_{\substack{\text{new kin members} \\ \text{added to the population}}} .$$

where:

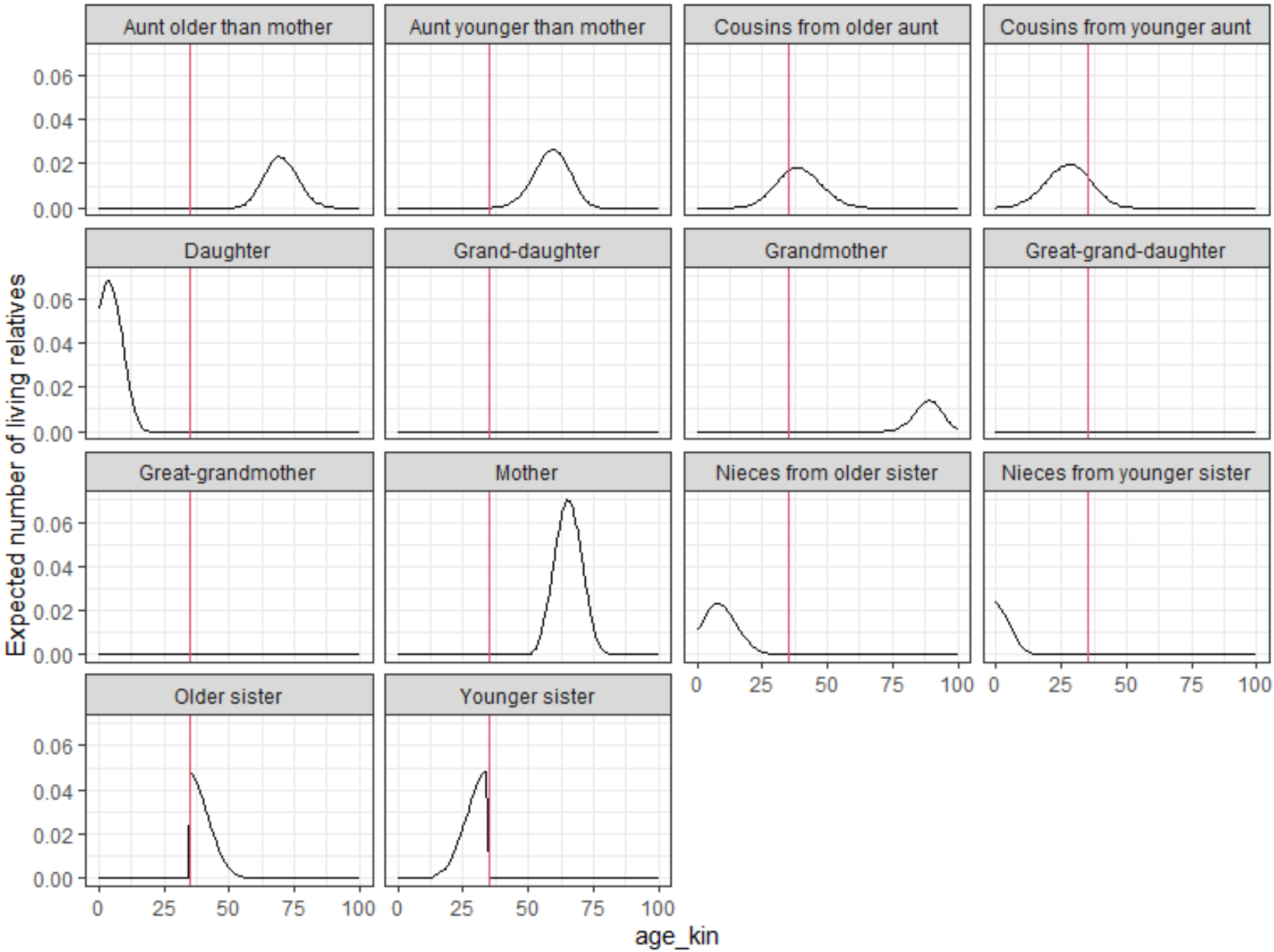
- **U** a matrix with survival probabilities in the subdiagonal
- **F** a matrix with fertility rates in the first row

[5] Caswell, H. (2019). The formal demography of kinship: A matrix formulation. *Demographic Research*, *41*, 679–712

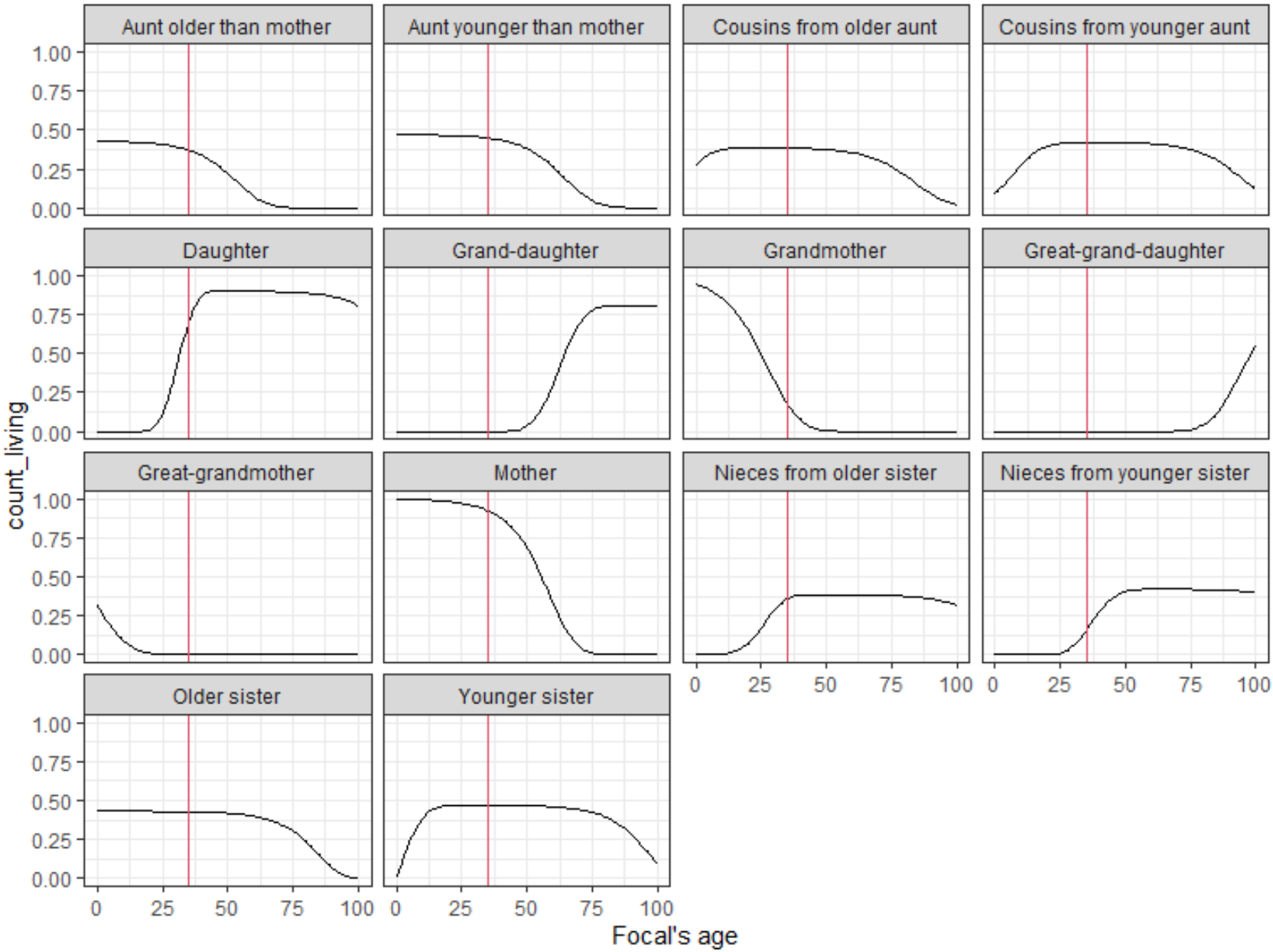
# Kinship structure



# Age distributions of kin



# Expected number of kin



# Daughters

Daughters ($\mathbf{a}$) are the result of the reproduction of Focal:

$$\underbrace{\mathbf{a}(x+1)}_{\substack{\text{age structure of daughters} \\ \text{at Focal's age } x+1}} = \underbrace{\mathbf{U}\,\mathbf{a}(x)}_{\substack{\text{ageing and survival} \\ \text{of existing daughters}}} + \underbrace{\mathbf{F}\,\mathbf{e}_x.}_{\substack{\text{new daughters} \\ \text{(subsidy)}}} \qquad (1)$$

$$a(0) = \mathbf{0}.$$

where:

- $\mathbf{U}$ is a matrix with survival probabilities in the subdiagonal
- $\mathbf{F}$ is a matrix with fertility rates in the first row
- $\mathbf{F}\,\mathbf{e}_x$ is the subsidy vector
- $e_x$ is the unit vector for age $x$
- $a(0)$ is the distribution of daughters at Focal's birth

# Mothers

The population of mothers ($\mathbf{d}$) of Focal consists of at most a single individual:

$$\underbrace{\mathbf{d}(x+1)}_{\substack{\text{age structure of mothers} \\ \text{at Focal's age } x+1}} = \underbrace{\mathbf{U}\,\mathbf{d}(x)}_{\substack{\text{ageing and survival} \\ \text{of existing mothers}}} + \underbrace{\mathbf{0}.}_{\substack{\text{new mothers} \\ \text{(subsidy)}}} \tag{2}$$

$$d(0) = \pi.$$

where:

- $b(0)$ is the distribution of mothers at Focal's birth
- $\pi$ is the distribution of ages of mothers in the population

# Typology of formal kinship models

| No | time | sex | state | reference |
|---|---|---|---|---|
| **1** | **invariant** | **female** | **age** | [6] |
| 2 | variant | female | age | [7] |
| 3 | invariant | two | age | [8] |
| 4 | invariant | female | multiple | [9] |
| 5 | variant | two | multiple | [10] |

[6] Caswell, H. (2019). The formal demography of kinship: A matrix formulation. *Demographic Research*, *41*, 679–712

[7] Caswell, H., & Song, X. (2021). The formal demography of kinship. III. kinship dynamics with time-varying demographic rates. *Demographic Research*, *45*, 517–546

[8] Caswell, H. (2022). The formal demography of kinship IV: Two-sex models and their approximations. *Demographic Research*, *47*, 359–396

[9] Caswell, H. (2020). The formal demography of kinship II: Multistate models, parity, and sibship. *Demographic Research*, *42*, 1097–1146

[10] Williams, I., Alburez-Gutierrez, D., Caswell, H., & Song, X. (2023). *DemoKin: 1.0.3*. https://CRAN.R-project.org/package=DemoKin

# Consider a baby born in Singapore in 2020...

1. How old were her grandparents when she was born, on average?
2. How many living children will she have on her 70th birthday?
3. How many grandchildren?

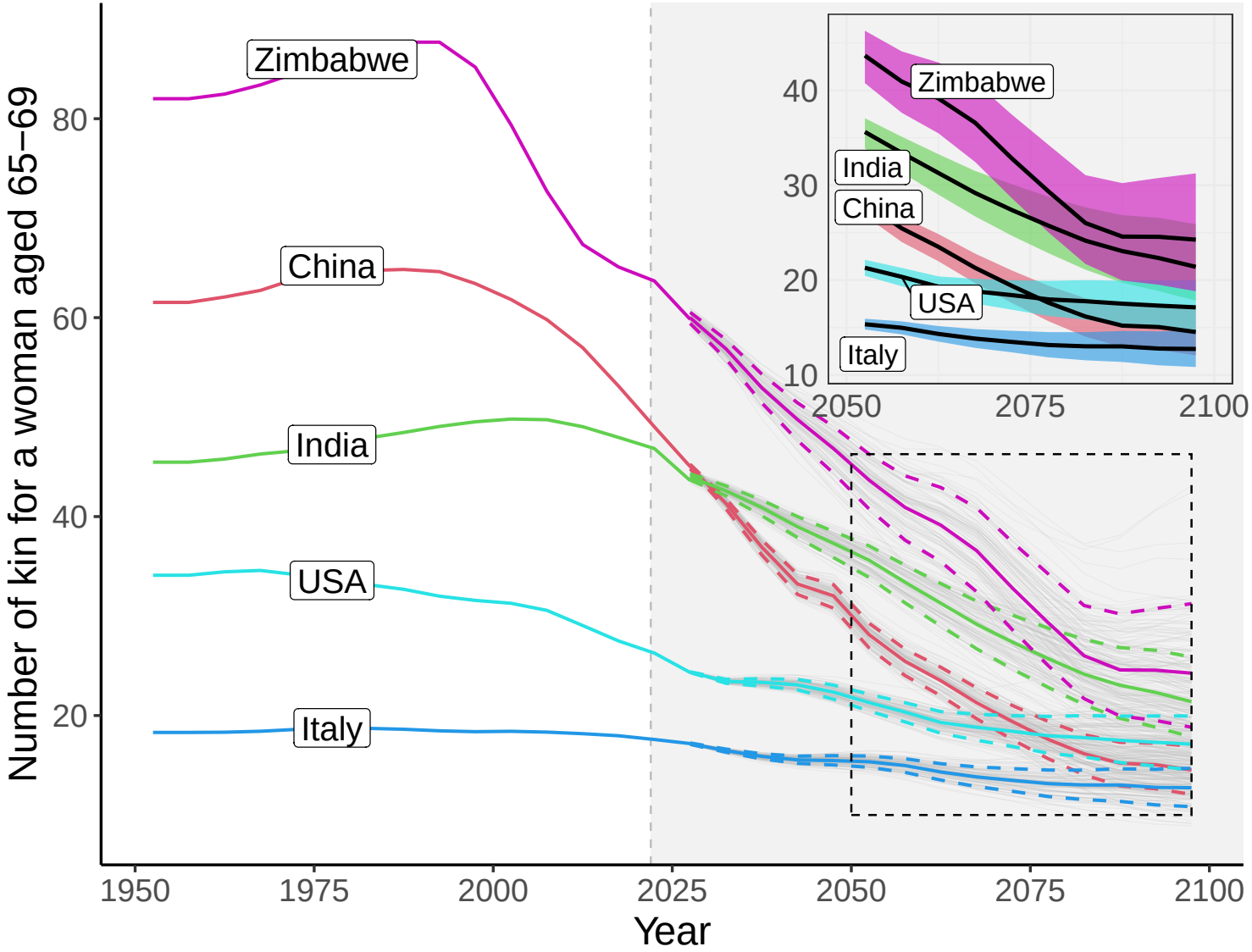
# DemoKin: matrix kinship models in R[11]

- Time-(in)variant models
- One/two-sex models
- Multistate models
- Kin loss by cause of death
- More in the lab session...


[11] Williams, I., Alburez-Gutierrez, D., Caswell, H., & Song, X. (2023). *DemoKin: 1.0.3*. https://CRAN.R-project.org/package=DemoKin

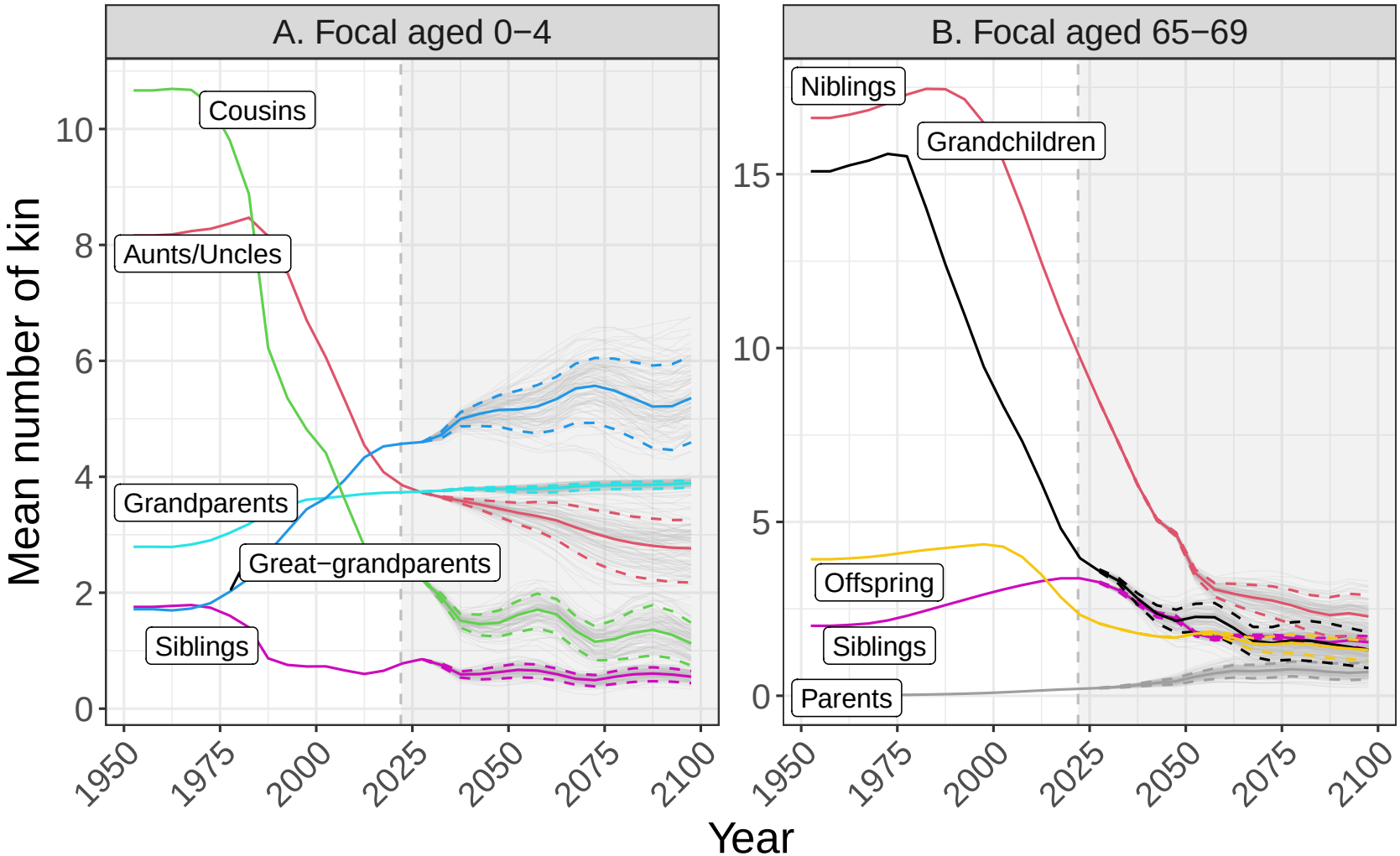
# Example: projections of kinship

# Total number of kin (all kin combined) for a 5yo woman[12]

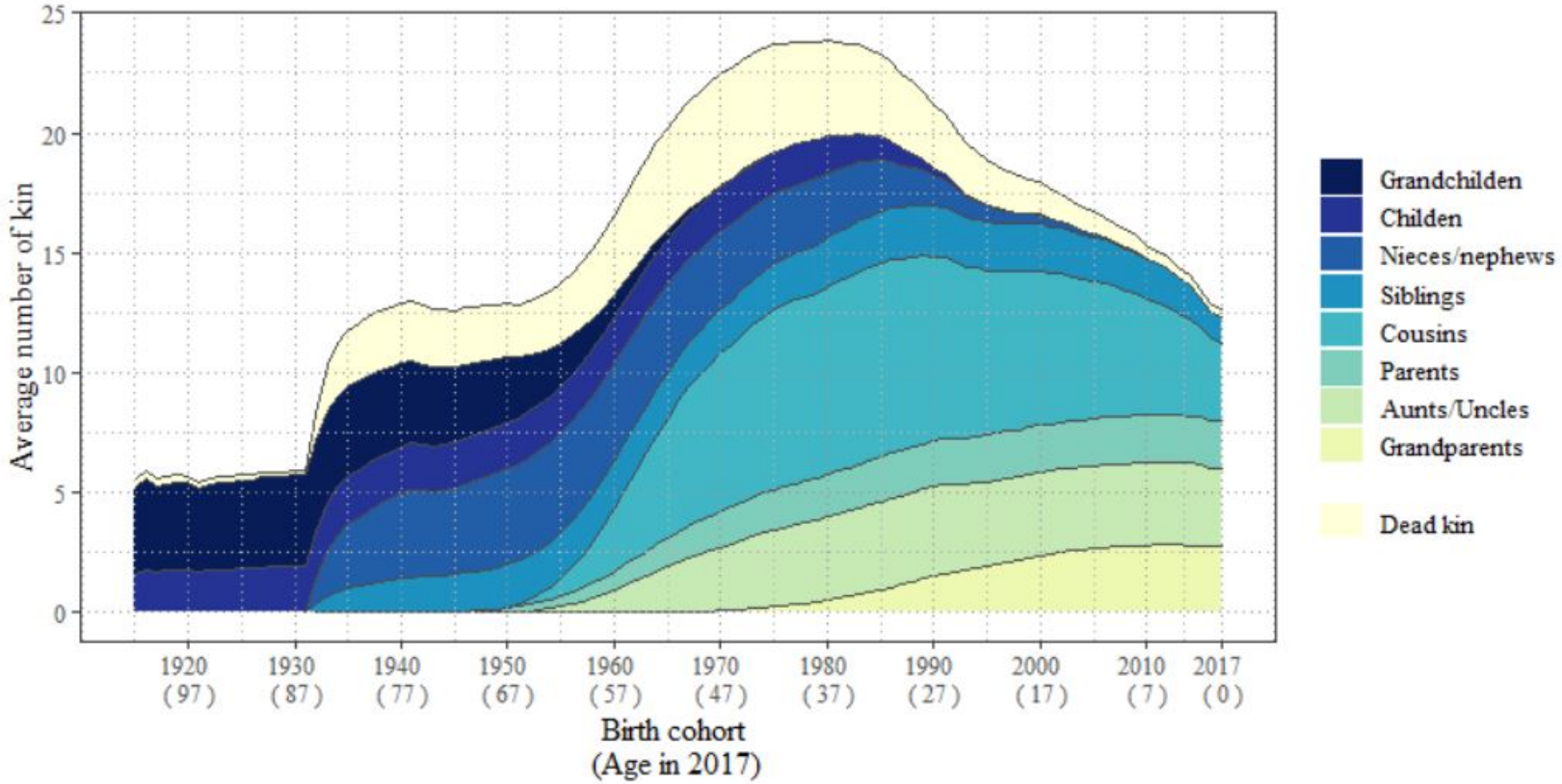



[12] Alburez-Gutierrez, D., Williams, I., & Caswell, H. (2023). Projections of human kinship for all countries. *Proceedings of the National Academy of Sciences, 120*(52), e2315722120. https://doi.org/10.1073/pnas.2315722120

# Number of living kin in China



# Note: Kinship structure also can be studied empirically[13]




[13] Kolk, M., Andersson, L., Pettersson, E., & Drefahl, S. (2021). The Swedish Kinship Universe – A demographic account of the number of children, parents, siblings, grandchildren, grandparents, aunts/uncles, nieces/nephews, and cousins using national population registers. *Stockholm Research Reports in Demography, 28*. https://doi.org/10.17045/sthlmuni.17704988.v1

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