

Please write clearly in block capitals		
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INTERNATIONAL AS FURTHER MATHEMATICS

(9665/FM02) Unit FPSM1 - Pure Maths, Statistics and Mechanics

Friday 31 May 2019

07:00 GMT

Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables (enclosed).
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use		
Mark		

Section A

Pure Maths

Answer all questions in the spaces provided.

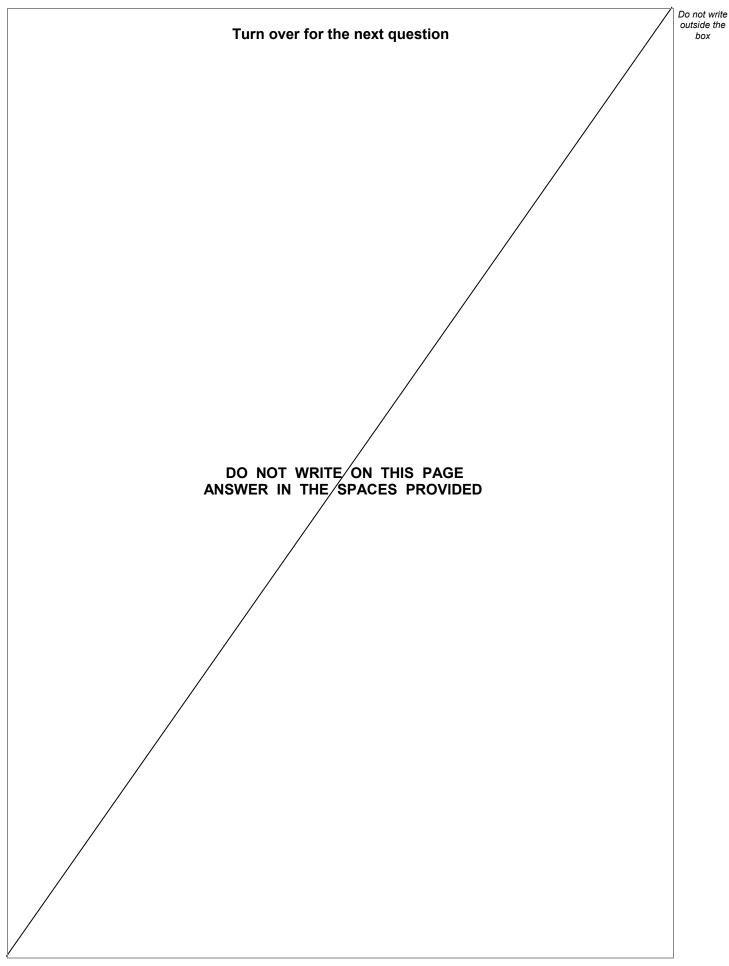
1 A curve passes through the point (5, 7.2) and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{x + 2y}$$

Use Euler's step-by-step method with a step length of 0.05 to estimate the value of y when x = 5.1

Give your answer to four decimal places.	[5 marks]

Answer





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2	The variables x and y are related by an equation of the form		
	$y = ab^x$		
	where a and b are constants.		
	Let $Y = \log_{10} y$		
2 (a)			
2 (a)	Show that there is a linear relationship between x and Y . [3 marks]		



2 (b)	It is given that $y = 27$ when $x = 3$ and that $y = 12$ when $x = 8$			
	Use the linear relationship from part (a) to find the value of a and the value of b .			
	Give your answers to 3 significant figures. [5 marks]			

a = _____

b = _____

Turn over for the next question



3	The matrices A, B and C are defined by
-	The mathematical fit, B and C are defined by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Describe fully the geometrical transformation represented by each of the following matrices:

3	(a) (i)	A		
				[2 marks]
			Answer	
2	(a) (ii)	n		
3	(a) (ii)	В		[2 marks]
			Answer	
			Answer	
3	(a) (iii)	C		[2 marks]
				[2]
			Answer	



3 (b)) The matrix ${\bf M}$ represents a stretch followed by an enlargement, and one of the followir statements is true.					the following
	M = AB	M = BA	M = BC	$\mathbf{M} = \mathbf{C}\mathbf{B}$	M = CA	M = AC
	Calculate the matr	ix M .				[2 marks]
			Answer_			
3 (c)	Triangle $T_{\rm 1}$ has are	ea 1.5 square	units.			
	The transformation	n represented	by M maps tr	iangle $T_{\rm 1}$ to a	triangle $T_{\rm 2}$	
	Find the area of T_2	2				[3 marks]
			Answer			



4	The function f is defined by $f(x) = x^4 - 17x^2 - 5x - 12$
	The equation $f(x) = 0$ has one positive root, α .
4 (a)	Show that α lies in the interval 4.3 < x < 4.4 [2 marks]
4 (b)	Starting from the interval 4.3 < α < 4.4, use interval bisection twice to find an interval of
	width 0.025 within which α must lie. [3 marks]
	[c
	Answer



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Answer Turn over for the next question	Give your answ	er to 3 decimal places.	
			[5 marks]
Turn over for the next question		Answer	
Turn over for the next question			
Turn over for the next question			-
		Turn over for the next quest	ion



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5 The matrices C and D are defined by

$$\mathbf{C} = \begin{bmatrix} 5 & -1 & 0 \\ 0 & 1 & 3 \\ 1 & 2 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{D} = \begin{bmatrix} -4 & 3 & -1 \\ 2 & 10 & -11 \\ -3 & -15 & 5 \end{bmatrix}$$

5 (a) Show that

$$CD^{T} = kI$$

where $\mathbf{D}^{\mathbf{T}}$ is the transpose of \mathbf{D} , \mathbf{I} is the 3×3 identity matrix and k is an integer.	[3 marks]

5 (b) Hence, without doing any further numerical calculations, prove that

$$CD^{T} = DC^{T}$$

[3 marks]

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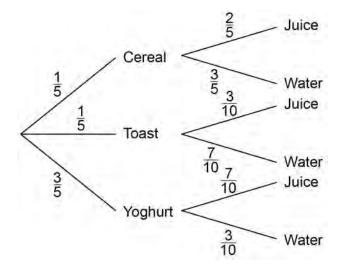
Section B

Statistics

Answer all questions in the spaces provided.

6 For breakfast, Barbara chooses one food item and one drink.

The probability of her making each choice is shown on the tree diagram.



Find the exact probability that Barbara chooses cereal given that she chooses juice.

[3 marks]

Answer



	_
Let X and Y be random variables such that $Var(X) = 2$, $Var(Y) = 8$ and $\rho = 0.78$, where ρ is the product moment correlation coefficient of X and Y .	Do i out:
Find $Var(X + Y)$. [4 marks]	
Answer	_

Turn over for the next question



			Do not write outside the
8	The random variable X has a geometric distribution with parameter p .		box
	The variance of X is 6		
8 (a)	Find p .	[3 marks]	
		[3 marks]	
	Answer		
8 (b)	Find $E(X)$.	F4 17	
		[1 mark]	
	Answer		



8 (c)	Find $P(X \le 2)$	[2 marks]	Do not write outside the box
	Answer		
			6
	Turn over for the next question		



9	The random variable Y has generating function
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$$G_Y(t) = \frac{1}{8} + \frac{1}{16}t + \frac{5}{8}t^2 + \frac{3}{16}t^3$$

9	(a)	Find $P(Y \ge 2)$
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[2 marks]

Answer

- **9 (b)** Using differentiation
- **9 (b) (i)** find the exact value of the mean of Y

[2 marks]

Answer

Question 9 continues on the next page



9 (b) (ii) find the exact value of the variance of Y.	Do not write outside the box
[3 n	narks]
Answer	
	7
Turn over for the next question	



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Section C

Mechanics

Wechanics
Answer all questions in the spaces provided.
Two cyclists, A and B , are travelling on horizontal ground.
Initially A is z metres due south of B .
The velocity of A is $(2\mathbf{i} + 3\mathbf{j})$ m s ⁻¹ and the velocity of B is $(3\mathbf{i} - 4\mathbf{j})$ m s ⁻¹ , where the unit vectors \mathbf{i} and \mathbf{j} are directed east and north respectively.
The distance between the two cyclists is a minimum after 28 seconds.
Find z .
[5 marks]
Answer



11	The escape speed, $v \text{ m s}^{-1}$, for an object is the minimum speed needed for the object to
	escape the gravitational field of a planet.

A model for the escape speed is

$$v = kM^a r^b G^c$$

where: k is a dimensionless constant,

M is the mass of the planet in kilograms, r is the radius of the planet in metres,

and G is the universal gravitational constant, which has units m³ kg⁻¹ s⁻²

Find the value of a , the value of b and the value of c .	[5 marks
a =	

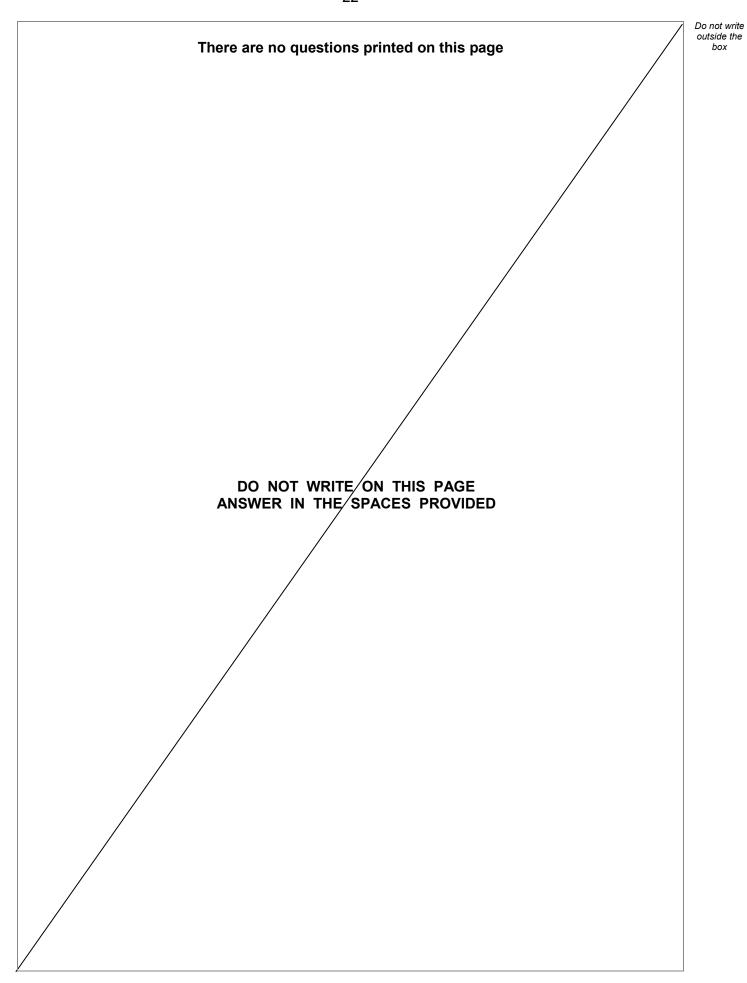
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12	A disc, of mass 0.25 kg, is sliding on a smooth horizontal surface at a speed of 4.8 m s ⁻¹		
	The disc hits a vertical wall that is perpendicular to its path.		
	The disc rebounds and travels in the opposite direction along its original path.		
	At time t seconds after the disc first makes contact with the wall, the magnitude of the force, $F(t)$ newtons, acting on the disc is modelled as		
	$F(t) = kt^2 (t - 0.1)^2$		
	where k is a constant and $0 \le t \le 0.1$		
	The model assumes that the disc is in contact with the wall for 0.1 seconds.		
12 (a)	Calculate, in terms of k , the magnitude of the maximum force on the disc. [2 mar	ks]	
	Answer		
12 (b)	Find, in terms of k , the magnitude of the impulse on the wall. [4 mark	ks]	
	Answer		



12 (c)	The coefficient of restitution between the disc and the wall is 0.6		Do not wr outside th box	
	Find the value of k .	[4 marks]		
	Anguar			
	Answer END OF QUESTIONS		10	







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