OXFORD

INTERNATIONAL AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

INTERNATIONAL AS
FURTHER MATHEMATICS

Further Pure Mathematics Unit 1

Thursday 31 May 2018 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the booklet of formulae and statistical tables, which is included as an insert.
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the box at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box on each page or on blank pages. If you require extra space, use a supplementary answer book.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

Answer **all** questions in the spaces provided.

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1 A curve has the equation $y = 2x^2 - 4x$

1 (a) A line passes through two points on the curve, one where $x = 3$ and the other where $x = 3 + h$

Find the gradient of this line in the form $p + qh$, where p and q are integers.

[3 marks]

Answer _____

1 (b) Show how the answer to part **(a)** can be used to find the gradient of the curve at the point where $x = 3$

State the value of this gradient.

[2 marks]

Answer _____

5

Do not write outside the box

2 It is given that $z = x + \mathrm{i}y$, where x and y are real.

Also, $\dfrac{3 - 2\mathrm{i}}{z} = a + \mathrm{i}b$, where a and b are real.

Find a and b in terms of x and y.

[3 marks]

Answer _____

3

Turn over for the next question

Do not write outside the box

3 (a) Given that $f(r) = \dfrac{1}{r+2}$, show that

$$f(r) - f(r+1) = \frac{1}{(r+2)(r+3)}$$

[1 mark]

3 (b) Use the method of differences to:

3 (b) (i) find the exact value of

$$\sum_{r=11}^{30} \frac{1}{(r+2)(r+3)}$$

[4 marks]

Answer _____

Do not write outside the box

3 (b) (ii) show that

$$\sum_{r=18}^{\infty} \frac{1}{(r+2)(r+3)} = \frac{1}{m}$$

where m is an integer.

[4 marks]

9

Turn over for the next question

Do not write outside the box

4 Find the general solution of the equation

$$\cos\left(3x + \frac{\pi}{6}\right) = \frac{1}{\sqrt{2}}$$

giving your answer in terms of π.

[5 marks]

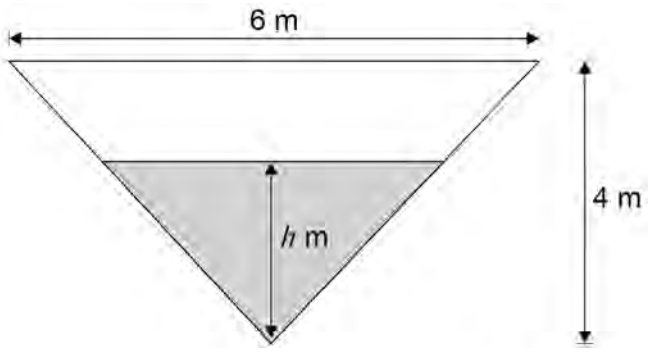
Answer _____

5

Do not write outside the box

5 [The volume V of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the circular base of the cone and h is the height of the cone.]

A water tank in the shape of an inverted cone has height 4 metres and maximum diameter 6 metres. The cross-section of the tank is shown in the diagram.



The tank fills with water at a rate of 0.06 m^3 per minute.

At time t minutes after the tank starts to fill, the depth of water in the tank is h metres.

Find the rate at which h is increasing when $h = 2.5$

Give your answer in terms of π.

[8 marks]

8

Do not write outside the box

6 (a) Find the value of

$$\sum_{r=1}^{45} (2r)^2$$

[2 marks]

Answer _____

6 (b) **Hence** find the sum of the squares of all the odd numbers from 1 to 89

[3 marks]

Answer _____

5

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Turn over for the next question

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ANSWER IN THE SPACES PROVIDED

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7 The quadratic equation

$$x^2 - 4x + 7 = 0$$

has roots α and β.

7 (a) Write down the values of $\alpha + \beta$ and $\alpha\beta$.

[2 marks]

$\alpha + \beta =$ _____

$\alpha\beta =$ _____

7 (b) Find the value of $\alpha^2 + \beta^2$

[2 marks]

Answer _____

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7 (c) Show that

$$\alpha^4 + \beta^4 = -94$$

[2 marks]

7 (d) Find a quadratic equation, with integer coefficients, which has roots

$$\alpha^2 + \frac{\beta}{\alpha} \quad \text{and} \quad \beta^2 + \frac{\alpha}{\beta}$$

[6 marks]

Answer

12

Do not write outside the box

8 A hyperbola H_1 has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

where a and b are positive constants.

H_1 intersects the x–axis at the points (6, 0) and (–6, 0)

The asymptotes of H_1 have equations

$$y = \frac{2}{3}x \quad \text{and} \quad y = -\frac{2}{3}x$$

8 (a) Find the values of a and b.

[2 marks]

$a =$ _____

$b =$ _____

8 (b) The hyperbola H_1 is translated by the vector $\begin{bmatrix} 4 \\ 0 \end{bmatrix}$ to give the hyperbola H_2

8 (b) (i) Write down the equation of H_2

[1 mark]

Answer _____

8 (b) (ii) Show that, if the line $y = mx$ intersects H_2, then the x–coordinates of the points of intersection must satisfy the equation

$$(4 - 9m^2)\,x^2 - 32x - 80 = 0$$

[3 marks]

Question 8 continues on the next page

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8 (b) (iii) Find the equations of the tangents to H_2 which pass through the origin.

[5 marks]

Answer _____

11

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Turn over for the next question

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ANSWER IN THE SPACES PROVIDED

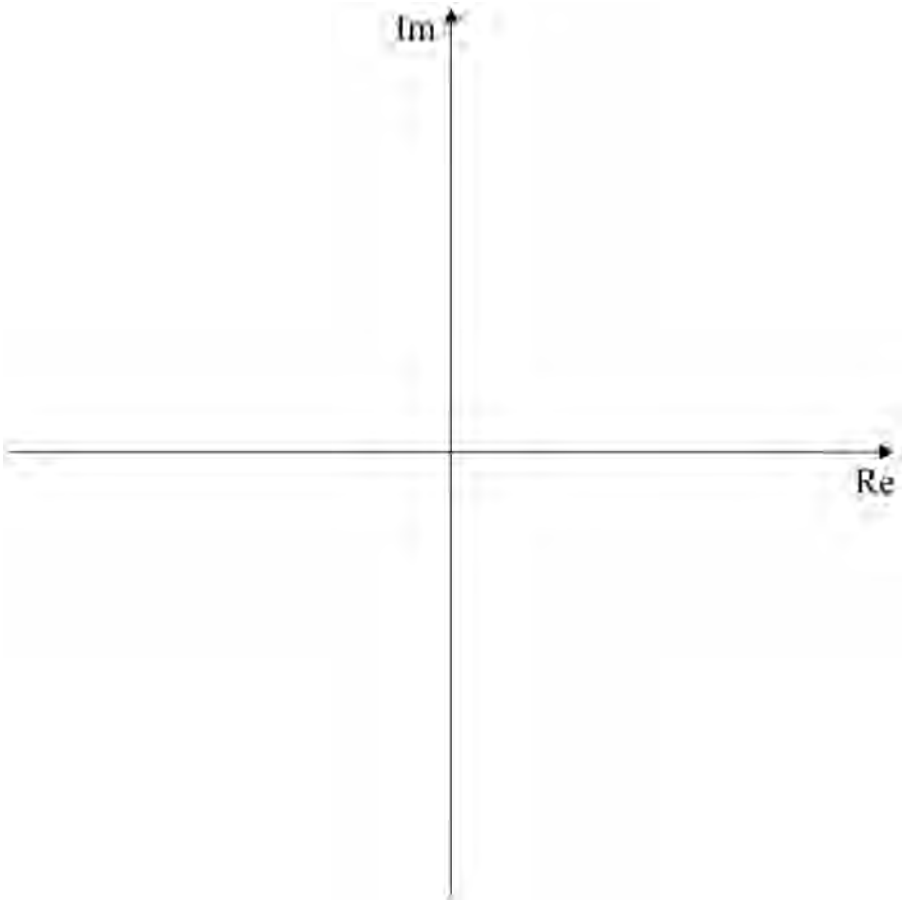
9 The locus L of points satisfies the equation $|z| = |z - 4 - 4\mathrm{i}|$

The point P represents the complex number $4 + 4\mathrm{i}$

The circle C has centre P and touches L.

9 (a) Sketch L and C on the same Argand diagram in the space below.

[4 marks]



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9 (b) Given that z_1 lies on C, find the maximum possible value of $|z_1|$

[3 marks]

Answer _____

9 (c) Given that z_2 lies on C, find the minimum possible value of $\arg(z_2)$

[3 marks]

Answer _____

10

Do not write outside the box

10 A curve C has the equation

$$y = \frac{(x+5)\,(x+1)}{x\,(x-4)}$$

10 (a) State the equations of the asymptotes of C.

[3 marks]

Answer _____

Do not write outside the box

10 (b) The line $y = k$ intersects the curve C.

10 (b) (i) Show that

$$4k^2 + 17k + 4 \geq 0$$

[4 marks]

Question 10 continues on the next page

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10 (b) (ii) **Hence** find the coordinates of the stationary points of the curve C.

No credit will be given for solutions using differentiation.

[5 marks]

Answer _____

12

END OF QUESTIONS