

Please write clearly in block capitals.

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# INTERNATIONAL AS FURTHER MATHEMATICS

Further Pure, Statistics and Mechanics Unit 1

Monday 4 June 2018

07:00 GMT

Time allowed: 1 hour 30 minutes

## Materials

- For this paper you must have the booklet of formulae and statistical tables, which is included as an insert.
- You may use a graphics calculator.

## Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the box at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box on each page or on blank pages. If you require extra space, use a supplementary answer book.
- Do all rough work in this book. Cross through any work you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for Section A, 20 marks for Section B and 20 marks for Section C.

## Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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6	
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8	
9	
10	
11	
12	
<b>TOTAL</b>	



### Section A

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Answer **all** questions in the spaces provided.

- 1** The variables  $X$  and  $Y$ , where  $X = \frac{1}{x}$  for  $x > 0$  and  $Y = y^2$  for  $y \geq 0$ , are related by the equation

$$Y = aX + b$$

where  $a$  and  $b$  are constants.

- 1 (a)** The table shows some values of  $x$  and  $y$ .

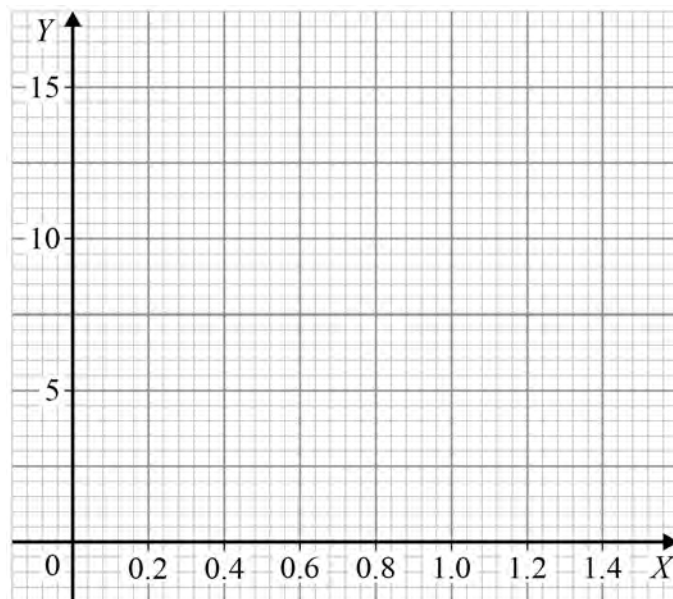
Complete the table to show the values of  $X$  and  $Y$ .

**[2 marks]**

$x$	1	2	5
$y$	3.7	3.1	2.6
$X$			
$Y$			

- 1 (b)** On the grid below, draw a linear graph relating  $X$  and  $Y$ .

**[3 marks]**



- 1 (c)** Use your graph to estimate the values of  $a$  and  $b$ .

**[3 marks]**

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$a =$  \_\_\_\_\_

$b =$  \_\_\_\_\_

- 1 (d)** Hence write down an equation relating  $x$  and  $y$ .

**[1 mark]**

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Answer \_\_\_\_\_

- 1 (e)** Use your equation to estimate the value of  $y$  when  $x = 0.5$

**[1 mark]**

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Answer \_\_\_\_\_

10

Turn over ►



2

$$\frac{dy}{dx} = \sqrt{x^3 - 3x}$$

Use a step-by-step method with a step length of 0.2 to estimate the value of  $y$  at

 $x = 2.4$ 

Give your answer to four decimal places.

**[5 marks]**

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Answer \_\_\_\_\_

5



**3** The matrix **A** is defined by  $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

**3 (a)** Name the type of transformation represented by **A**.

[1 mark]

\_\_\_\_\_

**3 (b)** The matrix **B** represents a clockwise rotation of  $30^\circ$  about the origin.

Write down the matrix **B**.

[1 mark]

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Answer \_\_\_\_\_

**3 (c)** Find the matrix  $\mathbf{B}^{-1}\mathbf{A}^{-1}$ , giving each element as an exact value.

[4 marks]

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Answer \_\_\_\_\_



**4** The function  $f(x)$  is defined as

$$f(x) = -0.1x^4 + x^2 - 2$$

**4 (a)** Show that the equation  $f(x) = 0$  has a real root,  $\alpha$ , in the interval  $2 < x < 3$

**[2 marks]**

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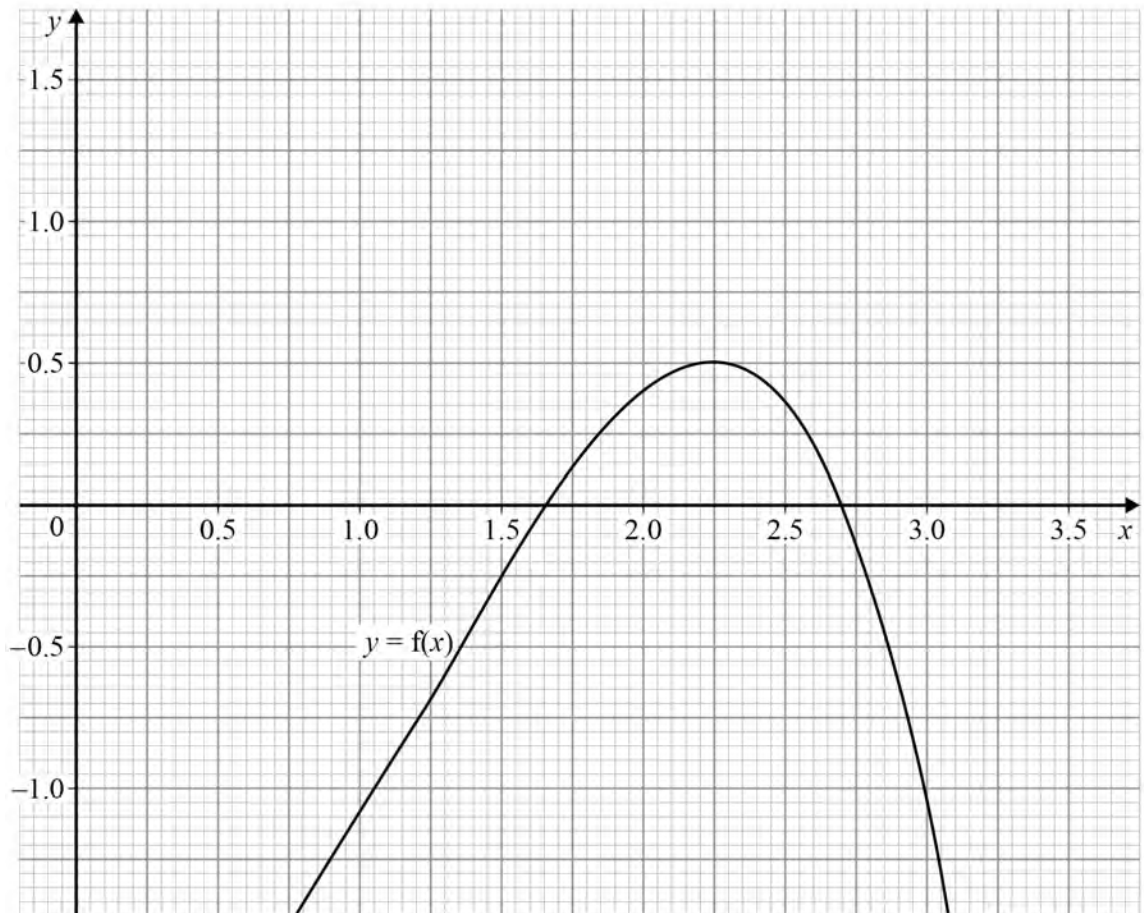
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- 4 (b) The Newton-Raphson method can be used to estimate  $\alpha$ , but it will only work for one of the two initial values  $x_1 = 2$  and  $x_1 = 3$

Explain, with the help of lines which you draw on the diagram below, why this is true.

[4 marks]



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Turn over ►



- 4 (c)** Using  $x_1 = 2.5$  as a first approximation to  $\alpha$ , use the Newton-Raphson method to find a second approximation,  $x_2$ , to  $\alpha$ .

**[4 marks]**

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Answer \_\_\_\_\_

10





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ANSWER IN THE SPACES PROVIDED**

**Turn over ►**



**5** The matrix  $\mathbf{A}$  is defined by  $\mathbf{A} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

**5 (a)** Describe the transformation represented by  $\mathbf{A}$ .

**[1 mark]**

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**5 (b)** It is given that  $y = mx + c$  is an invariant line of the transformation represented by  $\mathbf{A}$ .

Show that  $m$  satisfies the equation

$$m^2 \sin 2\theta + 2m \cos 2\theta - \sin 2\theta = 0$$

**[4 marks]**

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**5 (c)** In the case where  $\theta = \frac{\pi}{6}$

**5 (c) (i)** show that  $m = -\sqrt{3}$  is one solution of the equation in part **(b)** and find the other solution.  
**[3 marks]**

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Answer \_\_\_\_\_

**5 (c) (ii)** find the determinant of **A**.

**[1 mark]**

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Answer \_\_\_\_\_

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**Turn over for the next section**

**Turn over ►**



**Section B**Do not write  
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boxAnswer **all** questions in the spaces provided.

- 6** Jagdeep often catches the bus to school. Sometimes he is late for school.
- The probability that Jagdeep catches the bus is 0.875
- The probability that Jagdeep is late given that he catches the bus is 0.12
- The probability that Jagdeep is late given that he does not catch the bus is 0.34

- 6 (a)** Find the probability that Jagdeep catches the bus and is not late.

**[2 marks]**


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Answer \_\_\_\_\_

- 6 (b)** Find the probability that Jagdeep catches the bus given that he is late.

**[4 marks]**


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Answer \_\_\_\_\_

6



$$P(T=t) = \begin{cases} \frac{1}{4} & t = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

**[2 marks]**

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$E(T) =$  \_\_\_\_\_

$$\text{Var}(T) = \underline{\hspace{2cm}}$$

Find  $E(T + S)$ .

**[4 marks]**

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Answer

- 8** On a production line, components are checked for damage.

The random variable  $X$  represents the number of components that are checked before the first damaged component is found and can be modelled by a geometric distribution with  $p = 0.045$

- 8 (a) (i)** Find the probability that exactly eight products are checked before the first damaged product is found.

**[2 marks]**

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Answer \_\_\_\_\_

- 8 (a) (ii)** Find the probability that more than five products are checked before the first damaged product is found.

**[2 marks]**

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Answer \_\_\_\_\_



**[4 marks]**

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**Section C***Do not write  
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box*Answer **all** questions in the spaces provided.**9**An aeroplane is flying due north at a constant speed of  $150 \text{ m s}^{-1}$  relative to the air.The air is moving south-east at  $40 \text{ m s}^{-1}$ 

Find the resultant speed of the aeroplane.

**[2 marks]**

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Answer

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**2**



- 10** The magnitude of the gravitational attraction,  $F$  newtons, between two planets is given by

$$F = \frac{GMm}{d^2}$$

$G$  is a constant.

$M$  is the mass of the first planet in kilograms.

$m$  is the mass of the second planet in kilograms.

$d$  is the distance between the centres of the planets in metres.

Find the units of  $G$  in the form  $\text{m}^a \text{s}^b \text{kg}^c$

**[3 marks]**

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Answer \_\_\_\_\_

3

**Turn over for the next question**

**Turn over ►**



Boat  $A$  moves with constant velocity of  $(\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$  and is initially at the point with position vector  $(30\mathbf{i} + 90\mathbf{j}) \text{ m}$ .

Boat  $B$  moves with constant velocity of  $(3\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$  and is initially at the point with position vector  $(5\mathbf{i} + 70\mathbf{j}) \text{ m}$ .

Find the shortest distance between the two boats as they move on the lake.

**[6 marks]**

[illegible]

Answer \_\_\_\_\_

**E**



The coefficient of restitution between  $P$  and  $Q$  is 0.8

**[6 marks]**

[illegible]

Answer



**12 (b)** The sphere  $Q$  collides with a vertical wall that is perpendicular to its path.

The coefficient of restitution between  $Q$  and this wall is  $e$ .

After colliding with the wall,  $Q$  collides with  $P$  for a second time.

Deduce an inequality for  $e$ .

**[3 marks]**

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Answer \_\_\_\_\_

9

**END OF QUESTIONS**

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