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MATHEMATICS
MA01

Paper 1 Pure Mathematics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Mark	Comments
1(a)(i)	$-\frac{5}{3}$	B1	
1(a)(ii)	$\frac{3}{2}$	B1	
1(b)	$y = -\frac{2}{3}x + \frac{5}{2}$	B1ft B1	B1 for each of m and c in equation of correct form. ft minus $\frac{1}{their\ gradient\ of\ L_1}$ for gradient of L_2. Equation must be in the correct form for both marks. If correct m and c given but not in an equation, or final answer is a correct equation but in an incorrect form award B1 only. ft their gradient from part (a).
	Total	4	

Q	Answer	Mark	Comments
2(a)	$(p =)\ 2a^5b^2$	B2	B2 for fully correct answer. B1 for any two terms correct expressed as a product.
2(b)	$\left(\sqrt[3]{x} =\right) x^{\frac{1}{3}}$	B1	PI
	$\left(\frac{x}{x^{\frac{1}{3}}}\right)^2$ or $\left(x^{\frac{2}{3}}\right)^2$ or $\frac{x^2}{x^{\frac{2}{3}}}$	M1ft	Substituting for y in z. ft their $x^{\frac{1}{3}}$. Must be correctly simplified if expressed as a single power of x.
	$(z =)\ x^{\frac{4}{3}}$	A1	CAO
	Total	5	

Q	Answer	Mark	Comments
3(a)	$(x-4)^2 \ldots$	M1	
	$2(x-4)^2 - 32 + 38$	A1	Allow $2[(x-4)^2 - 16] + 38$ $2[(x-4)^2 - 16 + 19]$ $2[(x-4)^2 + 3]$
	$2(x-4)^2 + 6$	A1	CAO
3(b)	Correctly orientated symmetrical quadratic parabola.	B1	
	(0, 38) labelled on y-axis.	B1	Condone label given as y-value only.
	Vertex labelled as (4, 6)	B1ft	ft their (b, c) from part (a). Accept correctly positioned vertex with $x = 4$ and $y = 6$ indicated on axes.
3(c)(i)	$2x^2 - 16x + 38 = 4x + 20$ $2x^2 - 20x + 18 = 0$ $x^2 - 10x + 9 = 0$	B1	Must equate $\mathrm{f}(x)$ to $4x + 20$ and show correctly rearranged intermediate step equated to zero prior to quoting required result. If completed square form of $\mathrm{f}(x)$ used then it must be correctly expanded. Be convinced.
3(c)(ii)	$(x-1)(x-9) \quad (= 0)$	M1	Correct factorisation of the quadratic expression or correct unsimplified quadratic equation formula $x = \dfrac{-(-10) \pm \sqrt{(-10)^2 - 4 \times 1 \times 9}}{2 \times 1}$ PI by correct x-coordinates of P and Q.
	(1, 24) and (9, 56)	A1	CAO for coordinates of P and Q. Condone x and y values not given as coordinates. PI in later working.
	$\sqrt{(9-1)^2 + (56-24)^2}$ oe	M1ft	ft their coordinates of P and Q.
	$8\sqrt{17}$	A1	CAO NMS award M1A0 for $\sqrt{1088}$ unsimplified or not completely simplified as final answer.
	Total	11	

Q	Answer	Mark	Comments
4(a)	$(1-3x)^6 =$ $[1] + [6(-3x)] + 15(-3x)^2 + 20(-3x)^3 \ldots$	M1	For either (1), [6], 15, 20 oe unsimplified, or $\binom{6}{2}(-3x)^2$ or $\binom{6}{3}(-3x)^3$ oe, x not needed, PI.
	$p = 135$	A1	If correct working for p seen condone 135 seen in expansion but not explicitly stated. p not shown but correct q seen but NMS merits M1A0A1. Condone $135x^2$ or $-540x^3$.
	$q = -540$	A1	
4(b)	$-540x^3$ or $\frac{1}{5}x(135x^2)$ oe	M1	For either x^3 term from the expansion of $\left(1+\frac{x}{5}\right)(1-3x)^6$
	$-540x^3 + \frac{1}{5}x(135x^2)$ or $-513x^3$	A1ft	oe. ft their q or $q+27$ from part (a).
	(Coefficient of x^3 =) -513	A1ft	ft $q+27$ for their q from part (a). NMS scores 3 marks.
	Total	6	

Q	Answer	Mark	Comments
5(a)	$\frac{dy}{dx} = 3x^2 + 4x - 15$	B2	B2 fully correct. B1 two terms correct.
	$\left(\frac{dy}{dx} =\right) 12 + 8 - 15 = 5$	M1ft	Attempt to find value of $\frac{dy}{dx}$ at *P*. ft their expression for $\frac{dy}{dx}$
	(Gradient of normal =) $-\frac{1}{5}$	M1ft	Stated, or use of $m \times m' = -1$. ft their 5
	$y - 6 = -\frac{1}{5}(x - 2)$	A1ft	oe. ft their $-\frac{1}{5}$
5(b)(i)	$\left(x = -3 \implies \frac{dy}{dx} =\right) \quad 27 - 12 - 15 = 0$	B1	Shows $\frac{dy}{dx} = 0$ when $x = -3$. Must show working. If solving $\frac{dy}{dx} = 0$ then must set $\frac{dy}{dx} = 0$, factorise and then state $x = -3$. If no factorisation seen then must state both $x = -3$ and $x = \frac{5}{3}$
5(b)(ii)	$\frac{d^2y}{dx^2} = 6x + 4$	M1	
	$\frac{d^2y}{dx^2}$ = -14	A1	
5(b)(iii)	Stating $\frac{d^2y}{dx^2} < 0$ so maximum.	E1ft	Valid conclusion based on their answer to (b)(ii)
5(c)	$-3 < x < 1\frac{2}{3}$	B1	Accept $-3 \leq x \leq 1\frac{2}{3}$ Both inequalities must be consistent
	Total	10	

Q	Answer	Mark	Comments
6(a)	$h = 0.5$	B1	PI by correct later working or all four correct ordinates seen (and no others)
	(With $\mathrm{f}(x) = 1 + 0.3^x$) $(\mathrm{I} \approx \frac{h}{2}\{\ldots\})$ $\{\ldots\} = \mathrm{f}(0.5) + \mathrm{f}(2) + 2(\mathrm{f}(1) + \mathrm{f}(1.5))$	M1	oe. Summing the areas of the 'trapezia'
	$\{\ldots\} = 1.5477\ldots + 1.09 + 2(1.3 + 1.1643\ldots)$	A1	oe. Accept 3dp rounded or truncated.
	$(\mathrm{I} \approx)\ 0.25 \times 7.56635\ldots = 1.892$ (to 3dp)	A1	CAO Must be 1.892
6(b)(i)	Over-estimate.	E1	Over-estimate stated.
	The curve is concave (between $x = 0.5$ and $x = 2$). Or The tops of the trapezia/strips are above the curve.	E1	oe. Valid explanation.
6(b)(ii)	Increase the number of strips/ordinates/trapezia.	E1	oe. Valid explanation.
	Total	7	

Q	Answer	Mark	Comments
7(a)	$y-4=2a(x-1)^3-7b(x-1)$	M1	Substituting $y-4$ for *y*, (or for adding 4 to RHS), and $x-1$ for *x* in the given equation.
	$2a(x^3-3x^2+3x-1)-7b(x-1)$ $2ax^3-6ax^2+6ax-2a-7bx+7b$	M1	Expanding brackets in x. Allow one error in signs.
	$y=2ax^3-6ax^2+(6a-7b)x-2a+7b+4$	A1	CSO. Must come from completely correct working. Be convinced.
7(b)	$8=16a-24a+2(6a-7b)-2\mathrm{a}+7\mathrm{b}+4$	M1	oe. Substitutes coordinates of *P* into equation of *C*.
	$2a-7b=4$	A1	oe. Obtains fully simplified equation in *a* and *b*.
	$\left(\frac{dy}{dx}=\right)6ax^2-12ax+6a-7b$	B1	Differentiates equation of *C*, PI.
	$24a-24a+6a-7b=20$	M1ft	Substitutes $x=2$ into their $\frac{dy}{dx}$ and sets it equal to 20. Simplified or unsimplified. ft their $\frac{dy}{dx}$.
	$6a-7b=20$	A1	oe. Correct fully simplified equation in *a* and *b*.
	$a=4$ and $b=\frac{4}{7}$	B2	B1 for each correct.
	Total	10	

Q	Answer	Mark	Comments
8(a)	$u_5 = 3 + (5-1)d$ $p + 4 = 3 + 4d$	B1	oe. Uses 3 and expression for u_5 to gain correct equation in p and d.
	$u_9 = 3 + (9-1)d$ $(2p-1)^2 = 3 + 8d$	B1	oe. Uses value for a and expression for u_9 to gain correct equation in p and d.
	$4p^2 - 6p - 7 = -3$ or $4p^2 - 6p - 4 = 0$	M1	oe. Uses both equations in p and d to eliminate d and obtain an unsimplified equation in p. ft equations in p and d provided two equations derived.
	$2p^2 - 3p - 2 = 0$	A1	Dependent on M1 scored. Must follow from completely correct working.
8(b)(i)	$(2p+1)(p-2) \quad (=0)$	M1	Correct factorisation of the quadratic expression or correct unsimplified quadratic equation formula $p = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 2 \times (-2)}}{2 \times 2}$ PI by correct p values.
	$p = 2$	A1	Ignore $p = -\frac{1}{2}$ if given.
	$d = \dfrac{3}{4}$	B1ft	oe. Uses $p = 2$ and equation for u_5 or u_9 from part (a) to obtain d. B0 scored if two values of d given as final answer. ft their p provided $p > 0$.

Q	Answer	Mark	Comments
8(b)(ii)	$\frac{1}{2}k\left(2\times 3+(k-1)\times\frac{3}{4}\right)$	M1	For expression in k for sum of the first k terms of the series with values substituted. Can be simplified or unsimplified. PI by later working. ft their d from part (b). Condone n used for k throughout.
	$\frac{1}{2}k\left(2\times 3+(k-1)\times\frac{3}{4}\right)=138$ or $k\left(6+\frac{3}{4}(k-1)\right)=276$ or $24k+3k(k-1)=1104$	M1	oe. Forms equation to solve for k. Simplified or unsimplified. ft their d from part (b).
	$k^2+7k-368\ \ (=0)$	M1	Forms quadratic in k to be solved.
	$k=16$	A1	Correct value. Scores A0 if two values of k given. If build-up method used scores M1M1M1A1 for correct value of k obtained. Otherwise scores M0M0M0A0.
	Total	11	

8(a). SC B2, (in place of B1B1). Correct relationship between u_5 and u_9 used to eliminate d

Q	Answer	Mark	Comments
9(a)	$(\mathrm{p}(-2) =)$ $(-2)^3 + a(-2)^2 - (-2) - 21 = -7$ or $-8 + 4a + 2 - 21 = -7$	M1	oe. Uses Remainder Theorem with $x = -2$ to form equation in a, simplified or unsimplified. Allow one error.
	$a = 5$	A1	CSO. Be convinced. Must follow from completely correct working.
9(b)	$(\mathrm{p}(-3) =)\ \ (-3)^3 + 5(-3)^2 - (-3) - 21$	M1	$\mathrm{p}(-3)$ attempted. Must use Factor Theorem.
	$(\mathrm{p}(-3) =)\ \ -27 + 45 + 3 - 21 = 0$	A1	CSO. Correctly shows $\mathrm{p}(-3) = 0$.
9(c)	Quadratic factor $(x^2 + 2x - 7)$	M1	Attempt to express numerator as product of $x + 3$ and a quadratic factor. Allow one error in quadratic factor. Can use inspection, long division or compare coefficients.
	$(x + 3)(x^2 + 2x - 7)$	A1	Numerator correctly expressed as product of $x + 3$ and a quadratic factor. May be implied by further correct working.
	$x^{\frac{3}{2}} + bx^{\frac{1}{2}} + dx^{-\frac{1}{2}}$ or $x\sqrt{x} + b\sqrt{x} + \frac{d}{\sqrt{x}}$	M1	Attempt to write their quotient as the sum of powers of x. $x + 3$ cancelled and division by $\sqrt{x}$ attempted. Allow one error in dividing quadratic term by $\sqrt{x}$. Allow sum of powers of x given in surd form with final term $\frac{d}{\sqrt{x}}$. Sum must be of three powers of x. ft if numerator was expressed in the form $(x + 3)(x^2 + bx + d)$
	$x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 7x^{-\frac{1}{2}}$ or $x\sqrt{x} + 2\sqrt{x} - \frac{7}{\sqrt{x}}$	A1	Correct expression for quotient as the sum of powers of x. Allow answer given in surd form.
	$\frac{2}{5} \times\ x^{\frac{5}{2}} + 2 \times \frac{2}{3} \times\ x^{\frac{3}{2}} - 7 \times\ 2\ \times\ x^{\frac{1}{2}} + c$ or $\frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} - 14x^{\frac{1}{2}} + c$	B2ft	Correct answer simplified or unsimplified. Can be given in index or surd form. ft their integral provided previous M1 scored. B2 for fully correct answer. B1B0 for two terms correct including

	or $\frac{2}{5}x^2\sqrt{x}+\frac{4}{3}x\sqrt{x}-14\sqrt{x}+c$		signs. Condone omission of $+c$.
	Total	10	

10(a)	$k^4-4k^2-11\geq 1$ or $y^2-4y-11\geq 1$ or $(k^2)^2-4k^2-11\geq 1$	M1	States condition for divergence of geometric series for positive r.
	$\Rightarrow\ y^2-4y-12\geq 0$	A1	Required inequality.
10(b)	$(y-6)(y+2)$	B1	Correct factorisation of quadratic expression. Condone k^2 instead of y. PI by correct critical values.
	Rejects $y=-2$ as a critical value.	E1ft	Must indicate that it is being rejected, ft their $(y+2)$ as $(y+e)$ provided their factorisation is of form $(y-d)(y+e)$ where $d, e>0$. Condone k^2 instead of y.
	$k^2-6\geq 0$ (for divergence). or $k^2\geq 6$ (for divergence).	E1ft	Stating inequality. Must be $\geq$. ft their (k^2-6) as (k^2-d) provided their factorisation is of form $(k^2-d)(k^2+e)$ where $d, e>0$. Must be considering one inequality only at this stage.
	$k\leq-\sqrt{6}$ or $k\geq\sqrt{6}$	A1	CSO. ‘and’ instead of ‘or’ scores A0. SC2 for correct final solution if E0 scored.
	Total	6	