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FM05

(9665/FM05) Unit FM2 Mechanics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$10 = 0.2\omega$ $\omega = 50$ Period $= \frac{2\pi}{50} = \frac{\pi}{25}$ [seconds]	**M1** **A1**	Equation based on maximum speed. Correct period.
		2	

Q	Answer	Marks	Comments
1(b)	$v^2 = (50)^2\left(0.2^2 - 0.05^2\right)$ $v = 9.7$ [m s^{-1}]	**M1 A1ft** **A1**	**M1**: Uses the SHM speed formula **A1**: Correct substitutions. **FT** their ω Correct speed **AWRT** 9.7
		3	

Q	Answer	Marks	Comments
1(c)	$0.7 \times 0.2 \times 50^2 = 350$ $-350\ [\text{N}] \le F \le 350\ [\text{N}]$	**M1 A1** **A1**	**M1**: Uses their ω to find magnitude of maximum force. **A1**: Correct force. Correct range in any form.
		3	

	Question 1 Total	**8**	

Q	Answer	Marks	Comments
2(a)	$\begin{bmatrix} -3.6 \\ -6 \end{bmatrix} = 2\mathbf{v}_{\text{A}} - 2\begin{bmatrix} 4 \\ 2 \end{bmatrix}$	**M1 A1**	**M1**: Uses impulse to form a vector equation. **A1**: Correct equation.
	$\mathbf{v}_{\text{A}} = \begin{bmatrix} 2.2 \\ -1 \end{bmatrix}$ $\left[\text{m s}^{-1}\right]$	**A1**	Correct velocity.
		3	

Q	Answer	Marks	Comments
2(b)	$\begin{bmatrix} 3.6 \\ 6 \end{bmatrix}$ $[\text{N s}]$	**B1**	Correct impulse
		1	

Q	Answer	Marks	Comments
2(c)	$\begin{bmatrix} 3.6 \\ 6 \end{bmatrix} = 3\mathbf{v}_{\text{B}} - 3\begin{bmatrix} 1 \\ -3 \end{bmatrix}$	**M1 A1**	**M1**: Uses impulse to form a vector equation. **A1**: Correct equation
	$\mathbf{v}_{\text{B}} = \begin{bmatrix} 2.2 \\ -1 \end{bmatrix}$ $\left[\text{m s}^{-1}\right]$	**A1**	Correct velocity.
		3	

	Question 2 Total	**7**	

Q	Answer	Marks	Comments
3(a)	$0.5\times9.8=\frac{14}{2.5}x$ $x=0.875$ Length of String $=2.5+0.875=3.375$ [metres]	**M1** **A1** **A1**	Uses Hooke's Law Correct extension Correct length
		3	

Q	Answer	Marks	Comments
3(b)(i)	[Let $h=$ height above the equilibrium position when the sphere comes to rest] $\frac{14}{2\times2.5}\times0.875^2+\frac{1}{2}\times0.5\times1.2^2$ $=0.5\times9.8h+\frac{14}{2\times2.5}\times(0.875-h)^2$ $2.14375+0.36=4.9h+2.14375-4.9h+2.8h^2$ $h=0.36\ [\text{m}]$	**M1 M1** **A1** **M1** **A1**	**M1**: Four term energy equation **M1**: At least two terms correct. **A1**: Correct energy equation. **M1**: Solves for h **A1**: Correct conclusion from correct working **AWRT** 0.36
		5	

Q	Answer	Marks	Comments
3(b)(ii)	[Let x = displacement above the equilibrium position] $0.5\dfrac{d^2x}{dt^2} = \dfrac{14}{2.5}(0.875 - x) - 0.5 \times 9.8$ $0.5\dfrac{d^2x}{dt^2} = -5.6x$ $\dfrac{d^2x}{dt^2} = -11.2x$ As the acceleration is proportional to the displacement and in the opposite direction so the motion is SHM.	M1 A1 A1 E1	**M1**: forms differential equation using tension with their extension from **part (a)** **A1**: Correct differential equation. **A1**: Differential equation simplified to correct SHM form. **E1**: Concludes that motion is SHM from correct working
		4	

Q	Answer	Marks	Comments
3(b)(iii)	$x = 0.359\sin\left(\sqrt{11.2}t\right)$	M1 A1	**M1**: Trigonometric expression with one correct value. Allow 0.36 **A1**: Correct expression.
		2	

Q	Answer	Marks	Comments
3(b)(iv)	$-0.2 = 0.359\sin\left(\sqrt{11.2}t\right)$ $\sin\left(\sqrt{11.2}t\right) = -0.557$ $t = \dfrac{\pi + 0.591}{\sqrt{11.2}} = 1.1$[seconds]	M1 A1 A1	Forms an equation using their expression and ± 0.2 Correct equation. Correct time.
		3	

	Question 3 Total	**17**	

Q	Answer	Marks	Comments
4(a)	$\text{WD} = \int_0^a F\,\mathrm{d}x = \int_0^a \left(3 - x^2\right)\mathrm{d}x$ $= \left[3x - \dfrac{x^3}{3}\right]_0^a$ $= 3a - \dfrac{a^3}{3}$	M1 A1	Forms an integral to find the work done. Condone missing or incorrect limits. Correct result from correct working
		2	

Q	Answer	Marks	Comments
4(b)	When $x = 0$, the [initial] KE = 0 Work done $= 3 \times 3 - \dfrac{3^3}{3} = 0$ Work done = 0, so final KE = 0, so at rest when $x = 3$	B1 M1 A1	Explains that the initial KE is zero. Finds work done. Uses WD = Change in KE to explain why student is correct.
		3	

Q	Answer	Marks	Comments
4(c)(i)	When $x = -3$ the work done is zero the change in KE will be zero.	E1	Explains that the work done is zero at $x = -3$
		1	

Q	Answer	Marks	Comments
4(c)(ii)	When $x = 0$, the force is positive and the particle is at rest, so the particle cannot have a negative displacement.	E1 E1	**E1**: States that the force is positive at $x = 0$ **E1**: Explains that as the particle is at rest it cannot have a negative displacement.
		2	

Q	Answer	Marks	Comments
4(d)	$0 \le x \le 3$	B1	Correct range.
		1	

	Question 4 Total	9	

Q	Answer	Marks	Comments
5(a)	$v_A = 4\cos 60° = 2$ $4 \times 2 = 5v_B$ $v_B = \frac{8}{5} = 1.6 \ [\text{m s}^{-1}]$	M1 A1	Equation for conservation of momentum. Correct speed.
		2	

Q	Answer	Marks	Comments
5(b)	$-v_B = -2e$ $\frac{8}{5} = 2e$ $e = \frac{4}{5} = 0.8$	M1 A1	Restitution equation. Correct coefficient.
		2	

Q	Answer	Marks	Comments
5(c)	$\text{Speed} = 4\sin 60° = 2\sqrt{3} \ [\text{m s}^{-1}]$	B1	Correct speed.
		1	

	Question 5 Total	**5**	

Q	Answer	Marks	Comments
6	$m\dfrac{\mathrm{d}v}{\mathrm{d}t} = mg - kv$	M1	Correct differential equation.
	$\displaystyle\int \frac{1}{mg-kv}\,\mathrm{d}v = \int \frac{1}{m}\,\mathrm{d}t$	M1	Separates variables and integrates.
	$-\dfrac{1}{k}\ln(mg-kv) = \dfrac{t}{m} + c_1$	A1	Correct integration. Condone missing constant of integration.
	$t=0, v=0 \Rightarrow c_1 = -\dfrac{1}{k}\ln(mg)$ $\dfrac{kt}{m} = \ln(mg) - \ln(mg-kv)$ $mg - kv = mg\mathrm{e}^{-\frac{kt}{m}}$	A1	Correct constant of integration.
	$v = \dfrac{mg}{k}\left(1-\mathrm{e}^{-\frac{kt}{m}}\right)$ $x = \displaystyle\int \frac{mg}{k}\left(1-\mathrm{e}^{-\frac{kt}{m}}\right)\mathrm{d}t$	M1	Makes v the subject of their equation and integrates
	$= \dfrac{mg}{k}\left(t + \dfrac{m}{k}\mathrm{e}^{-\frac{kt}{m}} + c_2\right)$	A1	Correct integration.
	$x=0, t=0 \Rightarrow c_2 = -\dfrac{m}{k}$ $x = \dfrac{mg}{k}\left(t + \dfrac{m}{k}\mathrm{e}^{-\frac{kt}{m}} - \dfrac{m}{k}\right)$	A1	Correct constant of integration leading to required result.
		7	

	Question 6 Total	7	

Q	Answer	Marks	Comments
7(a)	$v\cos\alpha = u\cos 60° = \frac{u}{2}$ $v\sin\alpha = eu\sin 60° = \frac{eu\sqrt{3}}{2}$ $v^2 = \frac{u^2}{4} + \frac{3e^2u^2}{4}$ $v^2 = \frac{u^2}{4}\left(1+3e^2\right)$	**M1** **M1** **m1** **A1**	Correct equation for motion parallel to the wall. Correct equation for motion perpendicular to the wall. Eliminates α Correct expression from correct working.
		4	

Q	Answer	Marks	Comments
7(b)	$w\cos 15° = v\cos\beta$ $w\sin 15° = ev\sin\beta$ $\tan\beta = \frac{1}{e}\tan 15° = \frac{2-\sqrt{3}}{e}$ $\tan\alpha = e\sqrt{3}$ $\tan 75° = \tan(\alpha+\beta)$ $2+\sqrt{3} = \dfrac{e\sqrt{3}+\frac{2-\sqrt{3}}{e}}{1-e\sqrt{3}\times\frac{2-\sqrt{3}}{e}}$ $\left(2+\sqrt{3}\right)\left(4-2\sqrt{3}\right) = e\sqrt{3}+\frac{2-\sqrt{3}}{e}$ $\sqrt{3}e^2 - 2e + 2 - \sqrt{3} = 0$ $e = 1$ or $\frac{2\sqrt{3}}{3} - 1$	**M1** **A1** **A1** **B1** **B1** **M1** **A1** **A1**	**M1**: Equations for motion parallel and perpendicular to the second wall. **A1**: Both correct. **A1**: Correct expression for $\tan\beta$ Correct expression for $\tan\alpha$ Use of $\alpha+\beta = 75$ Uses $\tan(A+B)$ formula Correct quadratic. Correct values for e Accept 0.15
		8	

Q	Answer	Marks	Comments
7(c)	$w = u$	**B1**	Correct value
		1	

	Question 7 Total	**13**	

Q	Answer	Marks	Comments
8(a)	Let θ be the angle between the vertical and the radius when the particle leaves the sphere. $mg\cos\theta = \frac{mv^2}{2}$ $v^2 = 2g\cos\theta$ $\frac{1}{2}mv^2 = mg(2\cos 30° - 2\cos\theta)$ $g\cos\theta = g\sqrt{3} - 2g\cos\theta$ $\cos\theta = \frac{1}{\sqrt{3}}$ $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.735...$ $= 54.7°$ (to 1 dp)	M1 A1 M1 A1 A1 A1	**M1**: Apply Newton's second law radially when the particle leaves the hemisphere. **A1**: Correct equation. **M1**: Energy equation. **A1**: Correct energy equation. Correct value for $\cos\theta$ Correct angle from correct working
		6	

Q	Answer	Marks	Comments
8(b)	$v = \sqrt{2 \times 9.8 \times \frac{1}{\sqrt{3}}}$ $= 3.364$ $= 3.4\left[\text{m s}^{-1}\right]$ to 2 sf	B1	Correct speed from correct working.
		1	

Q	Answer	Marks	Comments
8(c)	Angle between slope and velocity $= \theta - 30 = 24.7°$	**M1**	Finds angle between slope and velocity
	$0 = -3.4\sin\left((54.7-30)°\right)t - \frac{1}{2}\times 9.8\cos 30° t^2 + 2\sin\left((120-54.7)°\right)$	**M1**	Equation for motion perpendicular to the plane.
	$0 = -3.4\sin(24.7)t - 4.9\cos 30° t^2 + 2\sin(65.3°)$	**A1**	Correct equation.
	$t = 0.508$	**A1**	Correct time **PI**
	$x = 3.4\cos\left((54.7-30)°\right)t + \frac{1}{2}\times 9.8\sin 30° t^2 - \left(2 - 2\cos\left((120-54.7)°\right)\right)$	**M1 A1**	**M1**: Equation for motion parallel to the plane. **A1**: Correct equation
	$= 1.04$ $= 1.0$ [metres to 2 sf]	**A1**	Correct distance.
		7	

	Question 8 Total	**14**	