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(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

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2 2 6 X F M 0 1 / M S

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$(6+h)^3 - 4(6+h)^2$ $= 6^3 + 3(36)h + 3(6)h^2 + h^3$ $- 4(36 + 12h + h^2)$ $= 72 + 60h + 14h^2 + h^3$ <p>Gradient</p> $= \frac{72 + 60h + 14h^2 + h^3 - 72}{h}$ $= 60 + 14h + h^2$	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>PI</p> <p>PI</p>
		4	
1(b)	<p>Gradient of curve</p> $= \lim_{h \rightarrow 0} [60 + 14h + h^2]$ $= 60$	<p>M1</p> <p>A1F</p>	<p>SC1 for using $h = 0$ leading to gradient = their 60</p>
		2	
	Total	6	

Q	Answer	Marks	Comments
2	$z = \frac{a+4i}{7+bi} \times \frac{7-bi}{7-bi}$ $= \frac{7a + 4b + 28i - abi}{49 + b^2}$ <p>Re(z) =</p> $\frac{7a + 4b}{49 + b^2}$ <p>Im(z) =</p> $\frac{28 - ab}{49 + b^2}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Alternative method, if using $z = x + iy$: M1 for forming simultaneous equations M1 for solving their simultaneous equations</p> <p>Condone one error</p> <p>Must not include i</p>
	Total	4	

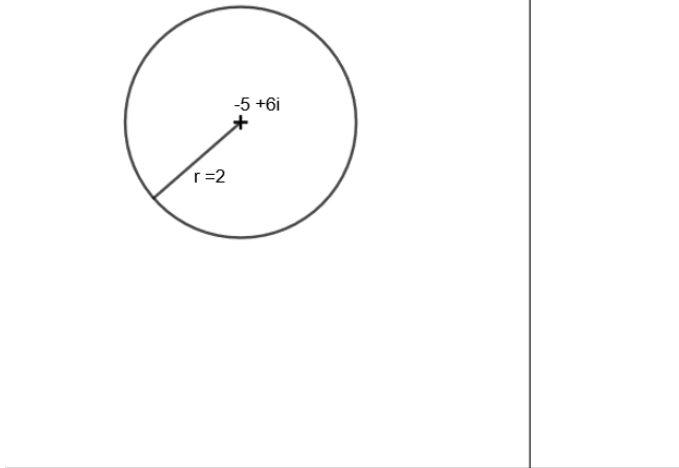
Q	Answer	Marks	Comments
3(a)	$3x - \frac{\pi}{6} = \frac{\pi}{4} + n\pi$ Going from $\left(3x - \frac{\pi}{6}\right)$ to x $x = \frac{5\pi}{36} + \frac{n\pi}{3}$	B1 M1 A1	oe including division of all terms by 3 oe eg $\frac{\pi}{36}(12n + 5)$
		3	
3(b)	Also include solutions to $\tan\left(3x - \frac{\pi}{6}\right) = -1$ $3x - \frac{\pi}{6} = -\frac{\pi}{4} + n\pi$ $x = \frac{5\pi}{36} + \frac{n\pi}{3}, -\frac{\pi}{36} + \frac{n\pi}{3}$	M1 M1 A1	PI oe eg $x = \frac{5\pi}{36} + \frac{n\pi}{3}, \frac{11\pi}{36} + \frac{n\pi}{3}$ or $x = \frac{\pi}{36}(6n + 5)$ (complete solution)
		3	
	Total	6	

Q	Answer	Marks	Comments
4(a)	At $x = 0$, which is a limit of the integral, the integrand $x^{-\frac{1}{4}}$ is not defined	E1	oe
		1	
4(b)	$\int_0^{16} x^{-\frac{1}{4}} dx = \lim_{h \rightarrow 0} \int_h^{16} x^{-\frac{1}{4}} dx$ $= \lim_{h \rightarrow 0} \left[\frac{4x^{\frac{3}{4}}}{\frac{3}{4}} \right]_h^{16}$ $= \lim_{h \rightarrow 0} \left(\frac{4 \left(16^{\frac{3}{4}} \right)}{3} - \frac{4h^{\frac{3}{4}}}{3} \right)$ $= \frac{32}{3}$	M1 M1 A1	For integrating For correct use of limit
		3	
	Total	4	

Q	Answer	Marks	Comments
5(a)	$\sum_{r=2}^n \left(\frac{1}{(r-1)^2} - \frac{1}{(r+1)^2} \right)$ $= \frac{1}{1^2} - \frac{1}{3^2}$ $+ \frac{1}{2^2} - \frac{1}{4^2}$ $+ \frac{1}{3^2} - \frac{1}{5^2}$ $+ \dots$ $+ \frac{1}{(n-3)^2} - \frac{1}{(n-1)^2}$ $+ \frac{1}{(n-2)^2} - \frac{1}{n^2}$ $+ \frac{1}{(n-1)^2} - \frac{1}{(n+1)^2}$ $= \frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{n^2} - \frac{1}{(n+1)^2}$ $= \frac{5}{4} - \frac{(n+1)^2 + n^2}{n^2(n+1)^2}$ <p>or</p> $\frac{5}{4} - \frac{2n^2 + 2n + 1}{n^2(n+1)^2}$ <p>as required</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>Must use method of differences to gain any marks</p> <p>Or equivalent for f(n)</p>
		4	
5(b)	<p>Let $S_{\infty} = \lim_{n \rightarrow \infty} (S_n)$</p> <p>Then $S_{\infty} = \frac{5}{4}$</p> $\sum_{r=11}^{\infty} \left(\frac{1}{(r-1)^2} - \frac{1}{(r+1)^2} \right) = S_{\infty} - S_{10}$ $= \frac{11^2 + 10^2}{11^2 \times 10^2} = \frac{221}{12100}$	<p>M1</p> <p>A1ft</p> <p>M1</p> <p>A1ft</p>	<p>For taking a limit PI</p> <p>oe</p>
		4	
	Total	8	

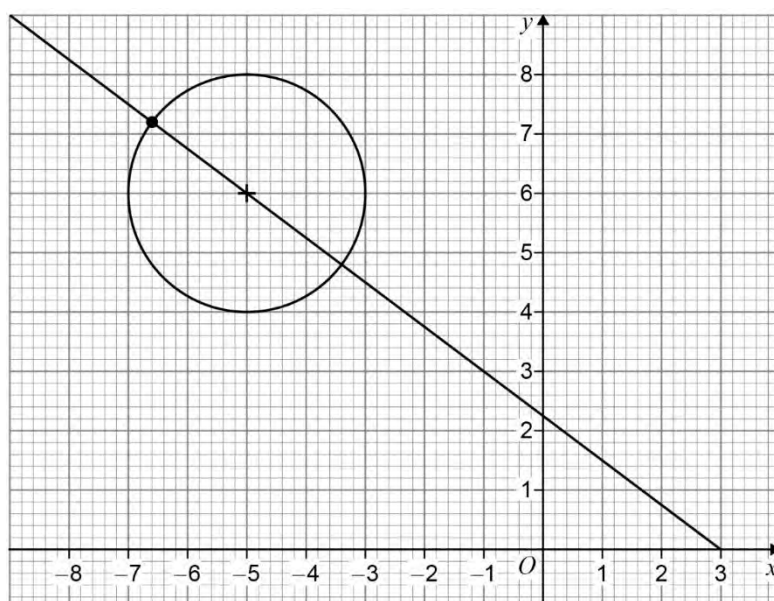
Q	Answer	Marks	Comments
6(a)	$(x-2)(x-5) - 2x = 0$ $x^2 - 9x + 10 = 0$ $\alpha + \beta = 9$ $\alpha\beta = 10$	M1 A1 A1	
		3	
6(b)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= 9^2 - 2 \times 10 = 61$	M1 A1	
		2	
6(c)	Sum of roots $= \alpha + \beta - \frac{1}{\alpha^2} - \frac{1}{\beta^2}$ $= \alpha + \beta - \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$ $= 9 - \frac{61}{100} = \frac{839}{100}$ Product of roots $= \left(\alpha - \frac{1}{\beta^2}\right)\left(\beta - \frac{1}{\alpha^2}\right)$ $= \alpha\beta - \frac{1}{\alpha} - \frac{1}{\beta} + \frac{1}{\alpha^2\beta^2}$ $= \alpha\beta - \frac{\alpha + \beta}{\alpha\beta} + \frac{1}{\alpha^2\beta^2}$ $= 10 - \frac{9}{10} + \frac{1}{100} = \frac{911}{100}$ $100x^2 - 839x + 911 = 0$	M1 A1 M1 M1 A1 A1	PI oe (integer coefficients) cao
		6	
	Total	11	

Q	Answer	Marks	Comments
7(a)	$V = 2h^2$	B1	
		1	
7(b)	$\frac{dV}{dh} = 4h$ $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $-0.006 = 4h \times \frac{dh}{dt}$ $h = 0.75 \text{ so}$ $\frac{dh}{dt} = \frac{-0.006}{4 \times 0.75}$ <p>Rate of decrease = 0.002 (metres per minute)</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1ft</p>	<p>PI</p> <p>Condone omission of minus sign</p> <p>For substituting $h = 0.75$ in their expression for $\frac{dV}{dh}$ or $\frac{dh}{dt}$</p> <p>Condone inclusion of minus sign</p> <p>Follow through from their $V = kh^2$ in part (a)</p>
		5	
	Total	6	

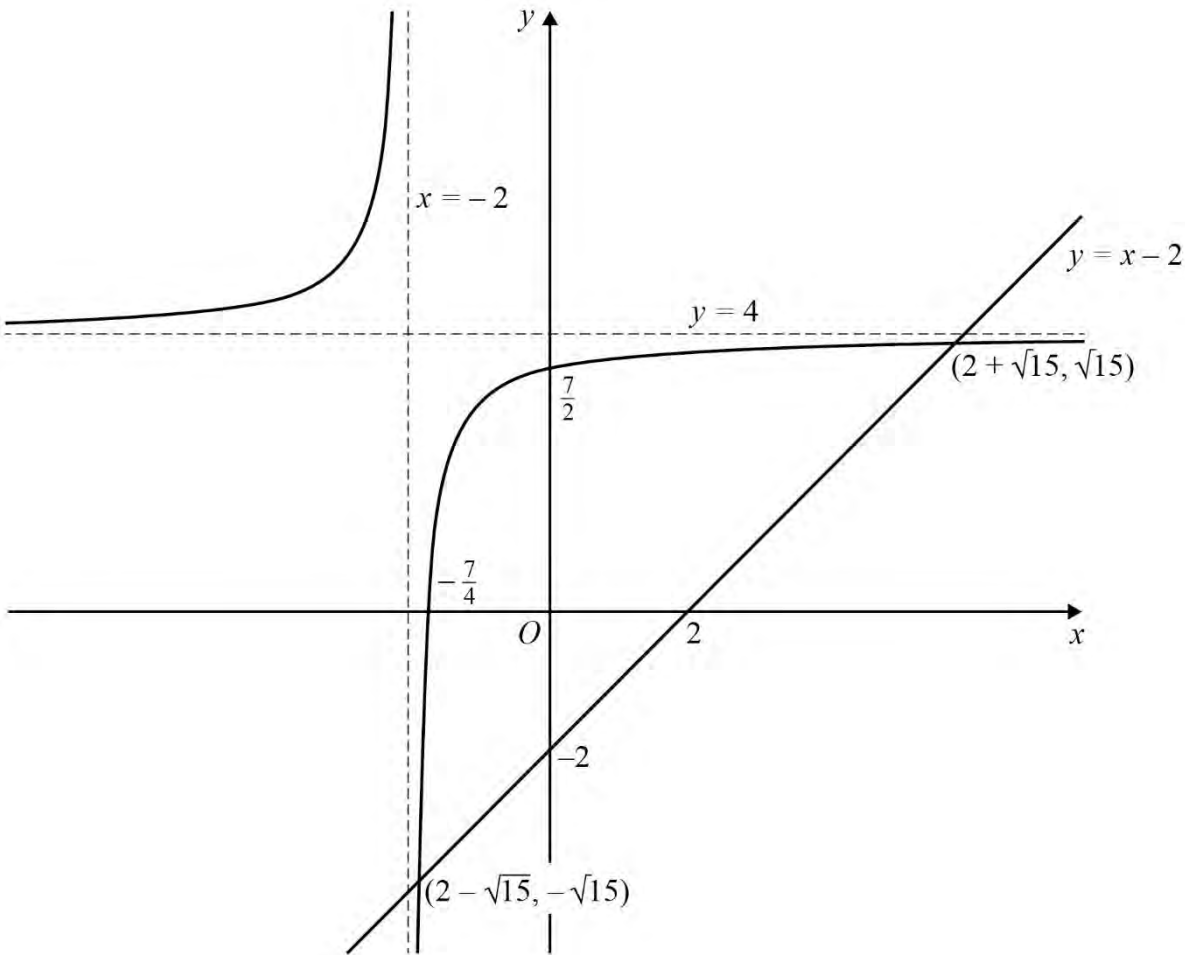
Q	Answer	Marks	Comments
8(a)	Circle with centre $-5 + 6i$, radius = 2	B1 B1	Allow Cartesian coordinates or values shown on axes
			
		2	
8(b)	$-5 + 6i - 3 = -8 + 6i$ $ -8 + 6i = 10$ $\frac{2}{10} \times (-8 + 6i) = -1.6 + 1.2i$ $z_1 = -5 + 6i + -1.6 + 1.2i$ $z_1 = -6.6 + 7.2i$	M1 M1 M1 M1 A1	
		5	
	Total	7	

		2	
8(b) Alt 1	$ z_1 - 3 = 12$ $-5 + 6i - 3 = -8 + 6i$ $[z_1 - 3 =] \quad 12(-0.8 + 0.6i)$ $z_1 - 3 = -9.6 + 7.2i$ $z_1 = -6.6 + 7.2i$	M1 M1 M1 M1 A1	oe
		5	

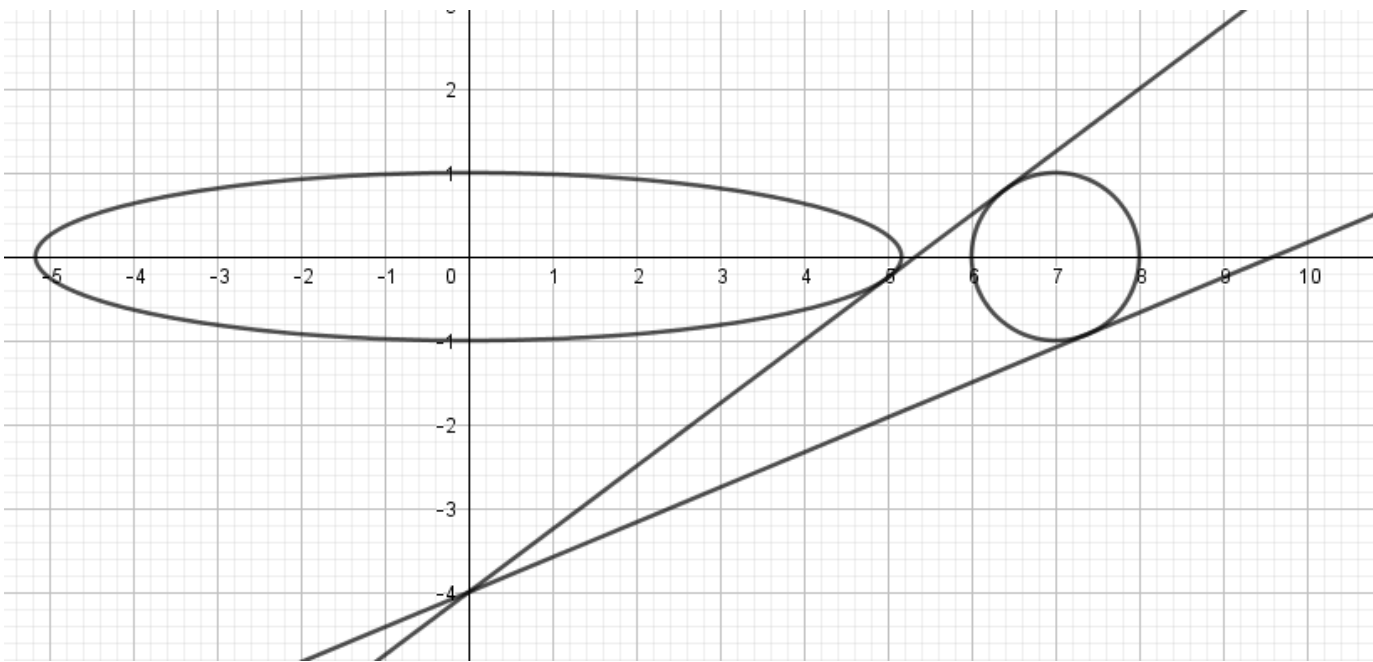
		2	
8(b) Alt 2	Equation of circle: $(x+5)^2 + (y-6)^2 = 4$ Equation of straight line: $y = -\frac{3}{4}x + \frac{9}{4}$ $(x+5)^2 + \left(-\frac{3}{4}x + \frac{9}{4} - 6\right)^2 = 4$ $25x^2 + 250x + 561 = 0$ $x = -6.6$ $z_1 = -6.6 + 7.2i$	B1 B1 M1 M1 A1	For substituting straight line equation into circle equation and forming a quadratic equation in x or y For solving their quadratic equation
		5	



Q	Answer	Marks	Comments
9(a)	$x = -2$	B1	
	$y = 4$	B1	
		2	
9(b)	$x - 2 = \frac{4x + 7}{x + 2}$	M1	
	$(x - 2)(x + 2) = 4x + 7$		
	$x^2 - 4 - 4x - 7 = 0$		
	$x^2 - 4x - 11 = 0$	M1	
	$x = 2 \pm \sqrt{15}$	A1	
	$(2 + \sqrt{15}, \sqrt{15})$		
	and $(2 - \sqrt{15}, -\sqrt{15})$	A1	
		4	

9(c)	Graph of $y = f(x)$, correct shape Asymptotes shown and graph approaches asymptotes Values at axes intercepts shown Line drawn correctly	B1 B1 B1 B1	
			
		4	
9(d)	$-2 < x \leq 2 - \sqrt{15}, x \geq 2 + \sqrt{15}$	B1B1	B1 for either subset (condone use of strict inequality) B2 for exactly both and no more
		2	
	Total	12	

Q	Answer	Marks	Comments
10(a)	Translation parallel to x -axis By $\begin{bmatrix} 7 \\ 0 \end{bmatrix}$	B1 B1	“parallel to x -axis” can be implied by form of vector
		2	
10(b)	$(x - 7)^2 + (mx - 4)^2 = 1$ $x^2 - 14x + 49 + m^2x^2 - 8mx + 16 - 1 = 0$ $(m^2 + 1)x^2 - (8m + 14)x + 64 = 0$ $\Delta \geq 0$ $(8m + 14)^2 - 4(m^2 + 1)(64) \geq 0$ $64m^2 + 224m + 196 - 256m^2 - 256 \geq 0$ $-192m^2 + 224m - 60 \geq 0$ or $48m^2 - 56m + 15 \leq 0$ $48m^2 - 56m + 15 = 0 \text{ has solutions } 5/12 \text{ and } 3/4$ $\frac{5}{12} \leq m \leq \frac{3}{4}$	M1 A1 M1 M1 A1 A1	
		6	

10(c)(i)	An ellipse	E1	
		1	
10(c)(ii)	$\frac{x^2}{a^2} + y^2 = 1$	B1	
		1	
10(d)	$\frac{x^2}{a^2} + (mx - 4)^2 = 1$ $\frac{x^2}{a^2} + m^2x^2 - 8mx + 16 - 1 = 0$ $\left(\frac{1}{a^2} + m^2\right)x^2 - 8mx + 15 = 0$ $\Delta \geq 0$ $64m^2 - 4\left(\frac{1}{a^2} + m^2\right)(15) \geq 0$ $a^2 \geq \frac{15}{m^2}$ <p>The limiting case is $m = \frac{3}{4}$</p> $a > 1 \text{ so } a \geq \frac{4\sqrt{15}}{3}$	M1 A1ft M1 A1 M1 A1	<p>Also possible to use $m = \frac{3}{4}$ from the start</p> <p>Inequality must include both a and m</p> <p>eg see diagram</p> <p>oe eg $a \geq \sqrt{\frac{80}{3}}$</p>
			
		6	
	Total	16	