OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL AS
MATHEMATICS
MA01

(9660/MA01) Unit P1 Pure Mathematics

Mark scheme

January 2020

Version: V1 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from oxfordaqaexams.org.uk

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	$\frac{5}{2}$	B1	
1(a)(ii)	3	B1	
1(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5}{2} \times 8x^{\frac{3}{2}} - 3 \times 2x^2$	M1	oe ft p and q from part **(a).** Must use coefficients 8 and 2 at this stage. One correct term, simplified or unsimplified.
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 20x^{\frac{3}{2}} - 6x^2$	A1	CAO
	Total	**4**	

Q	Answer	Marks	Comments
2(a)	$(x-5)^2$ or $-\frac{b}{2a} = 5$ or $-\frac{b}{2} = 5$ or $x^2 - 10x + 25$ or Indicates there is a stationary point at $(5, 0)$	M1	
	$b = -10 \quad c = 25$	A1	CSO. Both values. Correct b and c but NMS scores 2/2.
2(b)	Correctly orientated symmetrical quadratic parabola tangential to x-axis or symmetrical about the line $x = 5$	B1	
	$(0, 25)$ labelled on y-axis.	B1ft	ft their c Condone label given as the y-value only.
	Correctly positioned vertex labelled as $(5, 0)$	B1	Accept correctly positioned vertex with 5 indicated or implied on the x-axis.
	Total	**5**	

Q	Answer	Marks	Comments
3(a)(i)	$y = \frac{3}{2}x + 3$ (Gradient =) $\frac{3}{2}$	**M1** **A1**	Attempt at $y = mx + c$ or Attempt to find gradient using two correct points on the line. oe
3(a)(ii)	(y-coordinate =) 3	**B1**	Condone given as coordinates.
3(b)(i)	$(m =)\ -\frac{2}{3}$	**B1ft**	ft their gradient for L_1.
3(b)(ii)	$0 = -\frac{2}{3}x + \frac{22}{3}$ or $\frac{2}{3}x = \frac{22}{3}$ and $(x =)$ 11	**B1**	AG. Zero substituted for y in the equation of L_2 and correct answer.
3(c)	$(D =)\left(\frac{11}{2}, \frac{3}{2}\right)$ $y - \frac{3}{2} = \text{their gradient} \times \left(x - \frac{11}{2}\right)$ $y - \frac{3}{2} = \frac{3}{2}\left(x - \frac{11}{2}\right)$	**B1ft** **M1** **A1**	ft their y-coordinate of B. PI in later working. ft their gradient of L_1. May see $2y - 3x = k$ and substitution of coordinates of D to find k. But must be full method. CAO. oe
	Total	**8**	

Q	Answer	Marks	Comments
4(a)	$(2-x)^4 =$ $\left[(2)^4\right]+\left[4(2)^3(-x)\right]+6(2)^2(-x)^2+4(2)(-x)^3+\left[(-x)^4\right]$	**M1**	For either (1), [4], 6, 4, (1) oe unsimplified. or $\binom{4}{2}(2)^2(-x)^2$ or $\binom{4}{3}(2)(-x)^3$ oe, x not needed, PI.
	$a=24$	**A1**	If correct working for a seen condone 24 seen in expansion but not explicitly stated.
	$b=8$	**A1**	a not shown but correct b seen but NMS merits M1A0A1. Condone $24x^2$ or $8x^3$.
4(b)	$\left(\left(2-\frac{1}{\sqrt{2}}\right)^4=\right)$ $16-32\left(\frac{1}{\sqrt{2}}\right)+24\left(\frac{1}{\sqrt{2}}\right)^2-8\left(\frac{1}{\sqrt{2}}\right)^3+\left(\frac{1}{\sqrt{2}}\right)^4$	**M1**	Substitution of $x=\frac{1}{\sqrt{2}}$ into their expansion.
	$16-\frac{32}{\sqrt{2}}+12-\frac{4}{\sqrt{2}}+\frac{1}{4}$	**M1**	Powers correctly evaluated in an expression.
	$16-16\sqrt{2}+12-2\sqrt{2}+\frac{1}{4}$	**M1**	Denominators rationalised in an unsimplified expression.
	$\frac{113-72\sqrt{2}}{4}$	**A1**	Accept other correct forms, e.g. $\frac{226-144\sqrt{2}}{8}$ NMS = 0
	Total	**7**	

4(b) ALT	$\left((2-x)^4=\right)$ $16-32\left(\frac{\sqrt{2}}{2}\right)+24\left(\frac{\sqrt{2}}{2}\right)^2-8\left(\frac{\sqrt{2}}{2}\right)^3+\left(\frac{\sqrt{2}}{2}\right)^4$	**M2**	Substitution of $x=\frac{\sqrt{2}}{2}$ into their expansion.
	$16-16\sqrt{2}+12-2\sqrt{2}+\frac{1}{4}$	**M1**	Powers correctly evaluated in an expression.
	$\frac{113-72\sqrt{2}}{4}$	**A1**	Accept other correct forms, e.g. $\frac{226-144\sqrt{2}}{8}$ NMS = 0

Q	Answer	Marks	Comments
5(a)	$x-15+\frac{14}{x}$ or $x-15+14x^{-1}$	**M1**	Expansion of numerator and division by x. Allow error in one term if two other terms correct.
	$(y=)\ x-15+14x^{-1}$	**A1**	oe. Correct simplification
	$\frac{dy}{dx}=1-14x^{-2}$	**A1ft**	oe. ft their rearrangement of y provided their rearrangement contains a term in x^{-1}
	$x=2\Rightarrow\frac{dy}{dx}=1-\frac{14}{4}=-\frac{5}{2}$	**B1ft**	ft their $\frac{dy}{dx}$
	$x=2\Rightarrow y=-6$	**B1**	ft their $-\frac{5}{2}$ provided it came from a correct method. ft their -6
	$(y-(-6))=-\frac{5}{2}(x-2)$	**M1ft**	May see substitution of $x=2$, $y=-6$ and $m=-\frac{5}{2}$ into $y=mx+c$ but must be a complete method to find c.
	$(y-(-6))=-\frac{5}{2}(x-2)$	**A1**	ACF. Correct equation simplified or unsimplified.
5(b)	$\frac{dy}{dx}=1-14\times(-4)^{-2}\left[=0.125>0\right]$	**B1ft**	ft their $\frac{dy}{dx}$ Substituting $x=-4$ into their $\frac{dy}{dx}$ to show it has a positive value.
	Increasing	**E1ft**	ft. Allow 'decreasing' if their $\frac{dy}{dx}<0$ for their $\frac{dy}{dx}$ Dependent on B1 scored.
	Total	**9**	

Q	Answer	Marks	Comments
6(a)(i)	Arithmetic (sequence)	**E1**	With an attempt at an explanation.
	The sequence increases by 22 each time.	**E1**	Explanation implying that the difference between each term is constant.
6(a)(ii)	$36+(n-1)d$ or $a+(n-1)22$	**M1**	$a+(n-1)d$ with a or d correct and the other a letter.
	$36+(n-1)22$	**A1**	CAO oe
6(b)	$(\text{Total boxes sold}=)$ 7500	**B1**	PI in later working.
	$\frac{1}{2}N(2\times 36+(N-1)\times 22)$	**M1**	oe. Expression for total number of boxes sold in N months. Condone other letters used instead of N throughout.
	$11N^2+25N-7500=0$	**A1**	oe. Correct quadratic equation in N. PI.
	$(N=)\ \frac{-25\pm\sqrt{25^2-4\times 11\times(-7500)}}{2\times 11}$ or $(11N+300)(N-25)\ \ (=0)$	**M1**	PI by 25 and $-27.27...$ oe. Must have both values. Correct use of quadratic formula to solve their quadratic equation, condoning one error in signs or Correct factorisation ignoring signs.
	25	**A1**	Both values given as final answer scores A0
	Total	**9**	

Q	Answer	Marks	Comments
7(a)	$\int\left(3x^2+ax-36\right)\mathrm{d}x$ $=x^3+\frac{a}{2}x^2-36x\ \ (+c)$	M1	Attempt to integrate $\frac{\mathrm{d}y}{\mathrm{d}x}$ with at least two terms correct.
	$\left(y=\right)x^3+\frac{a}{2}x^2-36x\left(+c\right)$	A1	oe.
	$1+\frac{a}{2}-36+c=-7$ or $27+\frac{9a}{2}-108+c=-5$	m1	oe. Substituting either $x=1$ and $y=-7$ or $x=3$ and $y=-5$ into their equation for y in an attempt to find an equation in a and c.
	$\frac{a}{2}+c=28$	A1	oe. For each correct equation simplified or unsimplified. PI.
	$\frac{9a}{2}+c=76$	A1	
	$a=12$ and $c=22$	B1	Both correct. PI
	$\left(y=\right)\ x^3+6x^2-36x+22$	A1ft	ft their a and c provided both method marks scored.
7(b)(i)	$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\right)\ 6x+12$	B1ft	ft their $\frac{\mathrm{d}y}{\mathrm{d}x}$ with their a substituted.
7(b)(ii)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\ 3x^2+12x-36=0$ and $(x+6)(x-2)=0$	M1	oe. Sets their $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero with their a and attempts to solve for x. PI by $x=2$
	$x=2\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=6\times2+12\left[=24>0\right]$	M1	Establishes that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ is positive at $x=2$ Must be working with correct $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $x=2$ at this stage.
	$x=2$	A1	CSO. Failure to show that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ is positive at $x=2$ scores A0.
	Total	**11**	

Q	Answer	Marks	Comments
8(a)(i)	$\frac{a}{1-r} = 425$ $a + ar = 408$ $425(1-r) + 425(1-r)r = 408$ or $\frac{408}{1-r^2} = 425$ $r = \frac{1}{5}$	**M1** **A1** **M1** **A1**	oe. One correct equation. oe. Both correct equations. Attempts to solve simultaneously by elimination and forms equation in r. Simplified or unsimplified. AG. From correct working.
8(a)(ii)	$\frac{a}{1-\frac{1}{5}} = 425$ or $a + \frac{1}{5}a = 408$ $(a =)\ 340$	**M1** **A1**	oe. Substitutes $r = \frac{1}{5}$ into either of their equations in part (a).
8(b)	$\left(\sum_{n=k}^{k+1} u_n =\right) \sum_{n=1}^{k+1} u_n - \sum_{n=1}^{k-1} u_n$ or $ar^{k-1} + ar^k$ $\frac{340\left(1-\left(\frac{1}{5}\right)^{k+1}\right)}{1-\frac{1}{5}} - \frac{340\left(1-\left(\frac{1}{5}\right)^{k-1}\right)}{1-\frac{1}{5}}$ or $340\left(\frac{1}{5}\right)^{k-1} + 340\left(\frac{1}{5}\right)^{k}$ $425\left(\frac{1}{5}\right)^{k-1}\left(1-\left(\frac{1}{5}\right)^2\right)$ or $340\left(\frac{1}{5}\right)^{k-1}\left(1+\frac{1}{5}\right)$ $\left(\sum_{n=k}^{k+1} u_n =\right) 408\left(\frac{1}{5}\right)^{k-1}$	**B1** **M1** **M1** **A1**	PI in later working. oe. Correct substitution of r and a. Simplified or unsimplified. ft their a. Simplification and factorisation, unsimplified. CSO
	Total	**10**	

Q	Answer	Marks	Comments
9(a)	$(y=)\ x^{\frac{2}{3}}-8x^{\frac{1}{3}}+5$	**B1**	Correct expansion simplified or unsimplified.
	$\int\left(\left(x^{\frac{1}{3}}-4\right)^2-11\right)\mathrm{d}x=$ $\frac{3}{5}x^{\frac{5}{3}}-6x^{\frac{4}{3}}+5x(+c)$	**B2ft**	Simplified or unsimplified. For B2 condone omission of $+c$ Two correct terms only scores B1B0 ft their expansion.
9(b)	$\left(\frac{3}{5}(8)^{\frac{5}{3}}-6(8)^{\frac{4}{3}}+5(8)\right)-\left(\frac{3}{5}(1)^{\frac{5}{3}}-6(1)^{\frac{4}{3}}+5(1)\right)$	**M1ft**	Identifies correct definite integral and attempts $\mathrm{F}(8)-\mathrm{F}(1)$. ft their indefinite integral from part **(a)**.
	$\frac{-182}{5}$ or -36.4	**A1**	oe. CSO PI by correct final answer.
	$\frac{182}{5}$ or 36.4	**A1ft**	oe. Correct area ft the value of their definite integral provided it was negative.
9(c)(i)	$\left(\frac{182}{5}\right)+14$	**M1**	oe. Recognises that they need to increase the area in part **(b)** by 14.
	$\frac{252}{5}$ or 50.4	**A1ft**	oe. ft their area from part **(b)** provided it was positive. Re-integrating the equation of C_2 scores M0A0.
9(c)(ii)	$(y=)\left(x^{\frac{1}{3}}-4\right)^2-13$	**B1**	ACF
	Total	**9**	

Q	Answer	Marks	Comments
10(a)	$16(p+3)^2 - 4(2)(12p+q+12)\ [>0]$	**M1**	Use of $b^2 - 4ac$ with a, b and c substituted.
	$16p^2 + 96p + 144 - 96p - 8q - 96(>0)$	**A1**	oe. Correct expansion.
	$16p^2 - 8q + 48 > 0$ $2p^2 - q + 6 > 0$	**A1**	AG. From correct working. Brackets must be seen to be expanded before final answer given. >0 must appear before final line.
10(b)	$2(0)^2 + 4(p+3)(0) + 12p + q + 12 = 32$	**M1**	Attempts to use $(0,32)$ to find an equation in p and q.
	$q = 20 - 12p$	**A1**	oe.
	$2p^2 - (20 - 12p) + 6\ [>0]$	**m1**	oe. Uses their equation in p and q to eliminate q in the inequality.
	$p = -7$ and $p = 1$	**A1**	Correct critical values.
	$p < -7$ or $p > 1$	**A1**	Condone 'or' omitted but do not condone inclusion of 'and'.
	Total	**8**	