OXFORD

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INTERNATIONAL A-LEVEL
MATHEMATICS
MA04

Statistics Unit S2

Mark scheme

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196yMA04/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Mark	Comments
1	W~Exp(60) or W~Exp(1)	B1	Identifies correct distribution May be implied by later correct working
	$P(W \geqslant \frac{2}{60})$ or P(W ≥ 2)	M1	Identifies correct probability May be implied by later correct working
	$= 1 - \left(1 - e^{-60 \times \frac{2}{60}}\right)$ or $1 - (1 - e^{-1 \times 2})$	M1	Uses cdf of exponential to calculate probability
	= 0.135 AWRT	A1	
	Total	**4**	

Q	Answer	Mark	Comments
2 (a)	$P(X < 152) = P\left(Z < \frac{152 - 162}{5}\right)$ or = P(Z < -2)	M1	Standardise Can be implied by correct final answer
	= 1 – P(Z < 2)= 1 – 0.977725 = 0.0228 AWRT	A1	
2(b)(i)	1.645 AWRT	B1	z value to at least 4 s.f. Can be implied by correct equation
	P(Y < 185) = 0.95 $P\left(Z < \frac{185 - 175}{\sigma}\right) = 0.95$	M1	Standardise Can be implied by correct equation
	$\left(\frac{185 - 175}{\sigma}\right) = 1.645$	m1	Forms equation
	σ = 6.08	A1	CSO
2(b)(ii)	X + Y ~ N(337, 62.0)	B2	B1 N(337, …) or 62.0 AWRT 62.0 for variance
	Total	**8**	

Q	Answer	Mark	Comments
3 (a)	$5^4k = 1$	M1	Form correct equation involving k Condone $5^4k - 0^4k = 1$
	$k = \dfrac{1}{625}$ AG	A1	CSO but condone starting with $5^4k - 0^4k = 1$
3(b)	0	B1	
3(c)	$kc^4 = 0.75$	M1	Correct equation in c Accept t used for c
	c = 4.653	A1	CAO Accept t =
	Total	**5**	

4 (a)	Var(sample mean) = $\frac{0.3^2}{20}$ oe or SD(sample mean) = $\frac{0.3}{\sqrt{20}}$	B1	Accept 0.0045 oe or AWRT 0.067
	P($\overline{B}$ > 3.65) = P $\left(Z > \frac{3.65-3.5}{\frac{0.3}{\sqrt{20}}}\right)$ or = P(Z > 2.24)	M1	Standardise Can be implied by correct final answer Accept P(Z > $\sqrt{5}$)
	= 1 – P(Z < 2.24) or = 1 – 0.98745	m1	Rearrange to form that can be found in standard table Can be implied by correct final answer
	= 0.0125 to 0.0127	A1	AWFW
4(b)	2.33 AWRT	B1	z value to at least 3 s.f. Can be implied by correct final answer
	P($\overline{B}$ ≤ 3.6) < 0.99 P $\left(Z \leqslant \frac{3.6-3.5}{\frac{0.3}{\sqrt{n}}}\right)$ > 0.99	M1	Standardise Can be implied by correct final answer Accept use of =
	$\frac{3.6-3.5}{\frac{0.3}{\sqrt{n}}}$ > 2.33 $\Rightarrow$ n > …	m1	Forms equation and attempt to solve to reach n > Accept use of =
	n = 49	A1	CAO
	Total	**8**	

5 (a)	H_0: μ = 500 H_1: μ < 500	B1	Both hypotheses
	$X \sim N\left(500, \frac{38.2^2}{100}\right)$	M1	Use of May be implied
	$z = \frac{492-500}{\frac{38.2}{\sqrt{100}}}$	M1	Method to calculate z value for test
	z = -2.09 AWRT	A1	Accept +2.09 M1M1A1 for p = 0.018
	$z_{crit} = \pm\ 2.0537$	B1	AWRT 2.05 B1 for p = 0.018, comparison made with 0.02
	Reject H_0	A1ft	ft their z Use of signs must be consistent
	Evidence to suggest/support that the mean score on the computer game has decreased since the new version	E1ft	Comment in context ft their decision to accept or reject the null hypothesis if stated or their z value if not Must come from consistent signs Must not be definite
5(b)	Central Limit Theorem states that when the sample size is large enough, the sample mean will be [approximately] normally distributed	E1	General description of CLT
	Sample size of 100 is large enough for Central Limit Theorem to apply	E1	Comment relating sample size to CLT
	Total	**9**	

Q	Answer	Mark	Comments
6(a)(i)	$\bar{x} = 3.89$	B1	Calculates mean
6(a)(ii)	$s^2 = \frac{1}{5}\left(92.4488 = \frac{23.34^2}{6}\right)$	M1	Attempts to calculate s^2 or s
	$s^2 = 0.33124$	A1	Accept 8281/25000 oe
6(b)	H_0: μ = 4.56 H_1: μ ≠ 4.56	B1	Both hypotheses
	d.o.f, v = 5	M1	Use of May be implied
	$t = \dfrac{3.89 - 4.56}{\sqrt{\dfrac{0.33124}{6}}}$	M1	Method to calculate t value for test Condone z =
	t = -2.85 AWRT	A1ft	Accept +2.85 ft their s^2 but mean must be correct p = 0.0358 AWRT Condone z =
	$t_{crit} = \pm 4.032$	B1ft	AWRT 4.03 ft their d.o.f. B1 for p = 0.0358, comparison made with 0.005
	Accept H_0 / Reject H_1	A1ft	ft their t (or even z) Use of signs must be consistent
	Evidence to suggest/support that the manufacturer's claim is true	E1ft	Comment in context ft their decision to accept or reject the null hypothesis if stated or their t value if not Must not be definite
	Total	**10**	

8

Q	Answer	Mark	Comments
7(a)	$\frac{e^{-6.5}6.5^4}{4!}$ or $P(X \le 4) - P(X \le 3)$ $= 0.2237 - 0.1118$	M1	Method to calculate $P(X = 4)$
	= 0.112 AWRT	A1	
7(b)(i)	$X + Y \sim Po(7)$	B1	May be implied
	$P(X + Y < 3) = P(X + Y \le 2)$	M1	Identifies correct probability from table or for use in calculator May be implied by correct answer
	= 0.0296 AWRT	A1	
7(b)(ii)	H_0: $\lambda = 0.5$ H_1: $\lambda > 0.5$	B1	Both hypotheses
	$Y \sim Po(0.5)$	M1	Use of May be implied
	$P(Y \ge 3) = 1 - P(Y \le 2)$ or $1 - 0.9856$	M1	Method to calculate probability
	= 0.0144 AWRT	A1	
	Reject H_0	A1ft	ft their probability, compare with 5%
	Evidence to suggest/support that Emily's claim is true	E1ft	Comment in context ft their decision to accept or reject the null hypothesis if stated or their probability if not Must not be definite
7(b)(iii)	Reject H_0 when it is true	E1	General description of Type I error, specific comment also gains this mark
	Believing that Emily is correct that the mean has increased when it has not	E1	Specific comment
7(b)(iv)	$P(X \ge 2) = 0.0902$ and $P(X \ge 3) = 0.0144$	M1	At least one of the probabilities
	0.0144 or 1.44% AWRT	A1	Both probabilities need to be seen before final answer given
	Total	**15**	

Q	Answer	Mark	Comments
8(a)	E(X) = $\int_0^2 x\left(\frac{x^3}{32}\right)dx + \int_2^4 x\left(\frac{1}{8}x+\frac{1}{16}\right)dx$	M1	Correct integrals but condone missing dx May be done separately and the results added together May be implied
	= $\left[\frac{x^5}{32\times5}\right]_0^2 + \left[\frac{x^3}{8\times3}+\frac{x^2}{16\times2}\right]_2^4$	A1	Correct integration Limits not required
	= $\frac{349}{120}$ AG	A1	CSO but condone missing dx
8(b)	E(X^2) = $\int_0^2 x^2\left(\frac{x^3}{32}\right)dx + \int_2^4 x^2\left(\frac{1}{8}x+\frac{1}{16}\right)dx$	M1	Correct integrals but condone missing dx May be done separately and the results added together
	= $\left[\frac{x^6}{32\times6}\right]_0^2 + \left[\frac{x^4}{8\times4}+\frac{x^3}{16\times3}\right]_2^4$	A1	Correct integration Limits not required
	= $\left(\frac{2^6}{32\times6}\right)+\left(\frac{4^4}{8\times4}+\frac{4^3}{16\times3}\right)$ $-\left(\frac{2^4}{8\times4}+\frac{2^3}{16\times3}\right)$ [E(X^2) = 9]	m1	Applies limits Accept 9 oe NMS for E(X^2) = 9 scores M1A1m1
	Var (X) = E(X^2) – (E(X))^2 = 9 - $\left(\frac{349}{120}\right)^2$	M1	Applies variance formula using their E(X^2)
	= 0.542 AWRT	A1	Accept 7799/14400 oe
8(c)	Var (2X + 3) = 2^2 Var (X)	M1	Applies Var(aX + b) formula
	= 2.17 AWRT	A1ft	Accept 7799/3600 oe ft their answer to (c)
8(d)	0 < x ≤ 2 $\int_0^x \frac{1}{32}t^3dt = \left[\frac{t^4}{32\times4}\right]_0^x$	M1	Integrates pdf for 0 < x ≤ 2

	$= \dfrac{x^4}{128}$	A1	
	$2 < x \leq 4$ $\dfrac{2^4}{128} + \int_2^x \dfrac{1}{8}t + \dfrac{1}{16}\,dt$	M1	Correct expression for $2 < x \leq 4$
	$\int_2^x \dfrac{1}{8}t + \dfrac{1}{16}\,dt \left[\dfrac{t^2}{8\times 2} + \dfrac{t}{16}\right]_2^x$	M1	Integrates pdf for $2 < x \leq 4$ Limits not required
	$= \dfrac{x^2}{16} + \dfrac{x}{16} - \dfrac{1}{4}$	A1	
	$F(x) = 0$ for $x \leq 0$ and $F(x) = 1$ for $x > 4$	B1	
	$f(x) = \begin{cases} 0 & x \leqslant 0 \\ \dfrac{x^4}{128} & 0 < x \leqslant 2 \\ \dfrac{x^2}{16} + \dfrac{x}{16} - \dfrac{1}{4} & 2 < x \leqslant 4 \\ 1 & x > 4 \end{cases}$	B1ft	Fully specified cdf Follow through their functions for $0 < x \leq 2$ and $2 < x \leq 4$
	Total	**17**	

9 (a)	Is a statistic, ….	B1	States is a statistic and **any** attempted reason (even incorrect) Condone yes
	It is a random variable consisting of known observations	E1	Correct reason Condone all values are known/found
9(b)	Isn't a statistic, …	B1	States isn't a statistic and **any** attempted reason (even incorrect) Condone no
	It includes a population parameter μ	E1	Correct reason Condone includes unknown μ
	Total	**4**	