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(9665/FM04) Unit FS2 Statistics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	<p>H_0: There is not an association between use of app and passing maths examination</p> <p>H_1: There is an association between use of app and passing maths examination</p>	B1	<p>Must have both H_0 and H_1</p> <p>Condone 'change the chance' in place of 'association'</p>
		1	

Q	Answer	Marks	Comments
1(b)	$\sum \frac{(O - E - 0.5)^2}{E}$ $= \frac{(72 - 66 - 0.5)^2}{66} + \frac{(28 - 34 - 0.5)^2}{66}$ $+ \frac{(60 - 66 - 0.5)^2}{34} + \frac{(40 - 34 - 0.5)^2}{34}$ $\left[= \frac{5.5^2}{66} + \frac{5.5^2}{66} + \frac{5.5^2}{34} + \frac{5.5^2}{34} \right]$ $= 2.6960 [78 \dots]$	<p>M1</p> <p>A1</p>	<p>Allow SC1 for using $\sum \frac{(O - E)^2}{E}$ (i.e. not using Yates correction) which is possibly implied by 3.2085...</p> <p>AG oe $\frac{275}{102}$ each term shown</p>
		2	

Q	Answer	Marks	Comments
1(c)	<p>Degrees of freedom [dof], $\nu = 1$</p> <p>$cv = \chi_1^2(0.9) = 2.706$</p> <p>$2.696 < \chi_1^2(0.9) = 2.706$, do not reject H_0</p> <p>Evidence to suggest there is not an association between use of app and passing maths examination</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>PI</p> <p>Critical value for $\nu = 1$, AWRT 2.71</p> <p>PI, allow 'accept H_0' ft their critical value. If their $cv < 2.696$, must see reject H_0</p> <p>Must be correct contextual statement and consistent with their cv Allow 'support company's belief'</p>
		4	
	Question 1 Total	7	

Q	Answer	Marks	Comments
2	$P(\bar{X} < 0.16 \mid \mu = 0.1468)$ $= P\left(z < \frac{0.16 - 0.1468}{0.04}\right)$ $P(z < 0.33) = 0.6293$ $\text{Power} = 1 - 0.6293 = 0.371 \quad [\text{to 3 sf}]$	<p>M1</p> <p>A1</p> <p>A1</p>	Correct probability statement or standardises correct probability (PI by 0.33 seen)
	Question 2 Total	3	

Q	Answer	Marks	Comments
3(a)	<p>H_0: Distribution is uniform</p> <p>H_1: Distribution is not uniform</p> <p>dof $\nu = 6 - 1 = 5$</p> <p>expected values = 50</p> $\sum \frac{(O-E)^2}{E} = \frac{(50-50)^2}{50} + \frac{(43-50)^2}{50} + \frac{(38-50)^2}{50} + \frac{(63-50)^2}{50} + \frac{(61-50)^2}{50} + \frac{(45-50)^2}{50}$ <p>= 10.16</p> <p>$\chi^2_5(0.99) = 15.086$</p> <p>$10.16 < 15.086$, do not reject H_0</p> <p>Evidence to suggest that the die is fair</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>A1ft</p> <p>E1</p>	<p>PI by correct critical value</p> <p>Seen or used</p> <p>PI</p> <p>oe $\frac{254}{25}$</p> <p>Finds critical value</p> <p>Allow 'accept H_0'</p> <p>ft their test statistic and critical value</p> <p>Implied by correct conclusion in context</p> <p>Must be consistent with their conclusion on whether to accept H_0 or not or their test statistic and critical value if not explicitly stated</p> <p>Must not be definite</p>
		8	

Q	Answer	Marks	Comments
3(b)	<p>$\chi^2_5(0.90) = 9.236$ and rejection of H_0 with their $10.16 > 9.236$</p> <p>For the higher significance level, there is a lower χ^2 value rejection of H_0 or the critical region (tail) is increased</p>	<p>B1ft</p> <p>E1ft</p>	<p>ft their degrees of freedom from (a)</p> <p>[Note $\chi^2_4(0.90) = 7.779$]</p> <p>oe</p>
		2	
	Question 3 Total	10	

Q	Answer	Marks	Comments
4(a)	Both T and V are: Functions of the random variables of a sample and not dependent on population parameters	E2	Must contain emboldened key words E1 for one of the three statements: <ul style="list-style-type: none"> • Uses Random Variables • Calculated from a sample (Allow observations) • Not dependent on any population parameters (Allow unknown parameters)
		2	

Q	Answer	Marks	Comments
4(b)	$E(T) = \sum_{k=1}^n E(X_k) = \sum_{k=1}^n \mu$ $= n\mu \neq \mu$ [therefore not unbiased]	M1 A1	Allow X for X_k Must see $n\mu \neq \mu$
		2	

Q	Answer	Marks	Comments
4(c)(i)	$\text{Var}(X_k) = E(X_k^2) - E(X_k)^2$ $E(X_k^2) = \text{Var}(X_k) + E(X_k)^2$ $[\text{Var}(X_k) = \sigma^2, E(X_k) = \mu]$ $E(X_k^2) = \sigma^2 + \mu^2$	 B1	Allow X for X_k AG Be convinced
		1	

Q	Answer	Marks	Comments
4(c)(ii)	$\text{Var}(T) = E(T^2) - E(T)^2$ $\text{Var}(T) = n\text{Var}(X) = n\sigma^2$ $E(T)^2 = (n\mu)^2 = n^2\mu^2$ $E(T^2) = n\sigma^2 + n^2\mu^2$	<p>M1</p> <p>A1</p>	<p>Either for rearranging or $\text{Var}(T) = n\sigma^2$</p> <p>AG, must see evidence of rearranging and $\text{Var}(T) = n\sigma^2$</p>
		2	

Q	Answer	Marks	Comments
4(d)	$\sum_{k=1}^n E(X_k^2) = n(\sigma^2 + \mu^2)$ $E\left(\frac{nV}{n-1}\right) = \frac{n}{n-1} \left(\frac{1}{n} n(\sigma^2 + \mu^2) - \frac{(n\sigma^2 + n^2\mu^2)}{n^2} \right)$ $= \sigma^2, \text{ therefore unbiased}$	<p>B1</p> <p>M1</p> <p>A1</p>	<p>Seen or used</p> <p>Requires substitution of their $\sum_{k=1}^n E(X_k^2)$</p> <p>Must see σ^2 and conclusion</p>
		3	
	Question 4 Total	10	

Q	Answer	Marks	Comments
5(a)	$\bar{x} = 32.82$ $s^2 = \frac{1}{10-1} \left(10843.9 - \frac{328.2^2}{10} \right)$ $s^2 = 8.0417$ or $s = 2.8358(02845)$ $t_9(0.99) = 2.821$ $32.82 \pm 2.821 \sqrt{\frac{8.0417..}{10}}$ $(30.29, 35.35)$	B1 M1 A1 B1 M1 A1	AWRT 8.04 (s ²) or 2.84 (s) oe $s^2 = \frac{9047}{1125}$ Calculates confidence interval limits with their mean, their sample variance And their t -value. PI by correct answer CAO
		6	

Q	Answer	Marks	Comments
5(b)	$z = [+]$ 2.3263 $\sigma = 3$ $0.5 > 2.3263 \sqrt{\frac{3^2}{n}}$ $n = 195$ [from 194.828....]	B1 B1 M1 A1	Seen or used, AWRT 2.326 PI ft their z Allow = sign Must be 195 and not 194 if z used If t used accept $n > 198$ from $t = 2.345$ to 2.351
		4	
	Question 5 Total	10	

Q	Answer	Marks	Comments
6(a)(i)	$E(\bar{X}) = \mu$	B1	
	$\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$	B1	
		2	

Q	Answer	Marks	Comments
6(a)(ii)	$E(\bar{X}) = \mu$, so estimator is unbiased	B1	Conclusion required
	$\text{Var}(\bar{X}) \rightarrow 0$ as $n \rightarrow \infty$, so estimator is consistent	B1	Conclusion required
		2	

Q	Answer	Marks	Comments
6(b)	Efficiency $\frac{1}{\text{Var}(\bar{X}_A)} = \frac{40}{\sigma^2}$ or $\frac{1}{\text{Var}(\bar{X}_B)} = \frac{60}{\sigma^2}$	M1	Either expression or maybe seen in Relative Efficiency
	Relative Efficiency = $\frac{\left(\frac{1}{\text{Var}(\bar{X}_B)}\right)}{\left(\frac{1}{\text{Var}(\bar{X}_A)}\right)} = \frac{\left(\frac{60}{\sigma^2}\right)}{\left(\frac{40}{\sigma^2}\right)}$ Relative Efficiency = 1.5	A1	
		2	

Q	Answer	Marks	Comments
6(c)	$\text{Var}(T) = p^2 \text{Var}(\bar{X}_A) + (1-p)^2 \text{Var}(\bar{X}_B)$ $\frac{d(\text{Var}(T))}{dp} = 2p \frac{\sigma^2}{40} + 2(p-1) \frac{\sigma^2}{60}$ <p>and $\frac{d(\text{Var}(T))}{dp} = 0$</p> $2p \frac{\sigma^2}{40} + 2(p-1) \frac{\sigma^2}{60} = 0, \text{ leading to } p = 0.4$ $\frac{d^2(\text{Var}(T))}{dp^2} = \frac{\sigma^2}{12} > 0, \text{ so minimum}$ <p>variance or maximum efficiency</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p>	<p>PI by correct derivative</p> <p>May be seen as derivative of Efficiency</p> <p>oe by completing the square</p> <p>Conclusion required with check</p>
		4	
	Question 6 Total	10	

[illegible]

Q	Answer	Marks	Comments
7(b)	$H_0: \mu_m = \mu_f$ $H_1: \mu_m > \mu_f$ $\bar{m} - \bar{f} = 274 - 232.89 = 41.11$ $t_{\text{calc}} = \frac{41.11 - 0}{\sqrt{2120 \left(\frac{1}{11} + \frac{1}{9} \right)}}$ $= 1.986[47]$ $\text{dof } \nu = [11 + 9 - 1 - 1 =] 18$ $t_{\text{crit}} = t_{18}(0.95) = 1.734$ $1.986 > 1.734, \text{ reject } H_0$ Evidence to support that the mean number of platelets is greater for males than females	B1 M1 M1 A1 B1 B1 E1ft	Both hypotheses $\text{oe } 274 - \frac{2096}{9} = \frac{370}{9} \left[= 41\frac{1}{9} \right]$ ft their $\bar{m} - \bar{f}$ and their answer to (a) AWRT 1.986 or 1.987 $p = 0.0312(12)$ PI by correct critical value Must be in context Statement should not be definitive and not contradict any ft accepting H_0
		7	

Q	Answer	Marks	Comments
7(c)	$H_0: \sigma_m^2 = \sigma_f^2$ $H_1: \sigma_m^2 \neq \sigma_f^2$ $F_{\text{calc}} = \frac{S_m^2}{S_f^2} = \frac{2140.8}{2094.36...}$ $= 1.0223...$ $\nu_1 = 10, \nu_2 = 8$ $F_{10,8}(0.95) = 4.295$ 1.0223 < 4.295, insufficient evidence for the variance in number of platelets between males and females to be different from zero. The assumption is supported.	B1 M1 A1 B1 B1 E1	Both hypotheses ft on variances AWRT 1.022 Both dof correct, PI from correct F_{crit} PI , $p = 0.9947$ No need to compare with other lower critical value [0.2594] as $F_{\text{calc}} > 1$ Condone omission of statement context. Statement should not be definite.
		6	
	Question 7 Total	16	

Q	Answer	Marks	Comments
8(a)	$M_{X_k}(t) = E(e^{tX_k}) = \sum_{x=1}^{\infty} e^{tx} p(1-p)^{x-1}$ $= pe^t \sum_{x=1}^{\infty} ((1-p)e^t)^{x-1}$ $[S_{\infty} =] \quad \frac{pe^t}{1-(1-p)e^t} = \frac{p}{e^{-t} - (1-p)}$	<p>M1</p> <p>A1</p> <p>M1 A1</p>	<p>Applies mgf formula</p> <p>Correct formula Must start from $x = 1$ and sum to infinity or imply infinite series</p> <p>M1: Identify as geometric progression, $a = pe^t$, $r = (1-p)e^t$ oe [This may be seen in S_{∞}] A1: AG Be convinced</p>
		4	

Q	Answer	Marks	Comments
8(b)	$M'_{X_k}(t) = \frac{pe^{-t}}{(e^{-t} - (1-p))^2}$ $\mu = M'_{X_k}(0) = \frac{p}{p^2} = \frac{1}{p}$	<p>M1</p> <p>A1</p>	<p>Attempt to differentiate</p> <p>Substitutes $t = 0$ into correct expression and gives correct answer</p>
		2	

Q	Answer	Marks	Comments
8(c)(i)	$\left[\left(M_{X_k}(t) \right)^2 = \right] \frac{\left(\frac{1}{6} \right)^2}{\left(e^{-t} - \left(1 - \frac{1}{6} \right) \right)^2}$ $= \frac{1}{\left(6e^{-t} - 5 \right)^2}$	<p>M1</p> <p>A1</p>	$\frac{p^2}{\left(e^{-t} - (1-p) \right)^2}$ <p>AG</p>
		2	

Q	Answer	Marks	Comments
8(c)(ii)	$\left[\frac{d}{dt} \left(\frac{p^2}{\left(e^{-t} - (1-p) \right)^2} \right) = \right] \frac{12e^{-t}}{\left(6e^{-t} - 5 \right)^3}$ <p>When $t = 0$, $\mu = \frac{12e^0}{\left(6e^0 - 5 \right)^3}$</p> <p>$\mu = 12$</p>	<p>M1</p> <p>A1</p>	<p>Attempt at differentiation</p> <p>AG Be convinced Must see clear evidence of use of $t = 0$</p>
		2	

Q	Answer	Marks	Comments
8(d)	$\left[M_Y(t) = \right] \frac{p^n}{\left(e^{-t} - (1-p) \right)^n} = \frac{1}{\left(6e^{-t} - 5 \right)^n}$ $\left[M'_Y(t) = \right] \frac{np^n e^{-t}}{\left(e^{-t} - (1-p) \right)^{n+1}} = \frac{6ne^{-t}}{\left(6e^{-t} - 5 \right)^{n+1}}$ $\left[M''_Y(t) = \right] \frac{6ne^{-t} (6ne^{-t} + 5)}{\left(6e^{-t} - 5 \right)^{n+2}}$ <p>Use of $\sigma^2 = M''_Y(0) - M'_Y(0)^2$</p> $M'_Y(0) = \frac{6n}{(6-5)^{n+1}} = 6n$ $M''_Y(0) = \frac{6n(6n+5)}{(6-5)^{n+2}} = 36n^2 + 30n$ $\sigma^2 = 36n^2 + 30n - (6n)^2$ $\sigma^2 = 30n$	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>Identifies correct moment generating function</p> <p>Attempt to find first derivative and second derivative</p> <p>Must substitute $t = 0$ correctly</p>
		4	
	Question 8 Total	14	