OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM03) Unit FP2 Pure Mathematics

Tuesday 12 January 2021 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
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9	
10	
11	
12	
13	
TOTAL	

Answer **all** questions in the spaces provided.

1 A plane transformation is represented by the matrix

$$\mathbf{M} = \begin{bmatrix} 25 & 8 \\ t & 3 \end{bmatrix}$$

where t is a constant.

The eigenvalues of $\mathbf{M}$ are 27 and 1

1 (a) Find the value of t

[2 marks]

$t =$ ________________

1 (b) An eigenvector corresponding to the eigenvalue 27 is $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

An eigenvector corresponding to the eigenvalue 1 is $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

1 (b) (i) State the equations of the invariant lines of the transformation.

[1 mark]

Answer ________________ and ________________

1 (b) (ii) State, with a brief reason, which one of the invariant lines found in **part (b)(i)** is also a line of invariant points.

[2 marks]

5

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2 Evaluate the improper integral

$$\int_{-1}^{\infty} (1+x)\mathrm{e}^{-2x}\,\mathrm{d}x$$

showing the limiting process used.

[6 marks]

Answer ______________________

6

Turn over ►

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3 (a) By direct expansion, or otherwise, show that

$$\begin{vmatrix} 3 & -1 & 1 \\ 5 & k & 3 \\ k+2 & 1 & 2 \end{vmatrix} = k - k^2$$

[2 marks]

3 (b) A set of three planes is given by the system of equations

$$\begin{aligned} 3x - y + z &= 11 \\ 5x + ky + 3z &= k+9 \\ (k+2)x + y + 2z &= -2 \end{aligned}$$

where k is a real constant.

3 (b) (i) Determine the number of solutions of the given system of equations when $k = 1$

[3 marks]

Answer ________________

3 (b) (ii) **Hence** give a geometrical interpretation of the significance of the result in **part (b)(i)** in relation to the three planes when $k = 1$

[1 mark]

6

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4 Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + (\tanh x)\, y = \cosh^2 x + 2\mathrm{e}^x$$

[7 marks]

Answer

7

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5 The cubic equation

$$4z^3 + cz^2 + dz - 12 = 0$$

where c and d are real numbers, has complex roots α and β and a real root γ

It is given that $\alpha = 3 - \sqrt{3}\mathrm{i}$

5 (a) (i) Write down the value of β

[1 mark]

$\beta =$ ____________________

5 (a) (ii) Find the value of γ

[2 marks]

$\gamma =$ ____________________

5 (a) (iii) Find the value of c and the value of d

[3 marks]

$c =$ ________________ $d =$ ________________

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5 (b) (i) Express $3-\sqrt{3}\mathrm{i}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $r>0$ and $-\pi<\theta\leq\pi$

[2 marks]

Answer

5 (b) (ii) Given that n is a positive integer, express $\alpha^n+\beta^n$ as a single trigonometric term.

[4 marks]

Answer

5 (b) (iii) **Hence** find the complete set of positive integer values of n for which

$$\alpha^n+\beta^n=0$$

[2 marks]

Answer

14

Turn over ►

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6 (a) (i) Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{(r+2)(r+3)} = \frac{1}{3} - \frac{1}{n+3}$$

[4 marks]

6 (a) (ii) Prove by induction that, for all integers $n \geq 1$

$$\sum_{r=1}^{n} \frac{2}{(r+1)(r+2)(r+3)} = \frac{1}{6} - \frac{1}{(n+2)(n+3)}$$

[4 marks]

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6 (b) Using the results in **part (a)** show that

$$\sum_{r=1}^{n} \frac{r}{(r+1)(r+2)(r+3)} = \frac{n(n+a)}{b(n+c)(n+d)}$$

where a, b, c and d are integers.

[3 marks]

11

Turn over ►

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7 It is given that y satisfies the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 9e^{-3x} + 18$$

7 (a) Find the values of the constants a and b for which

$$ax^2e^{-3x} + b$$

is a particular integral of this differential equation.

[5 marks]

$a =$ ____________ $b =$ ____________

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7 (b) **Hence** solve the differential equation, expressing y in terms of x

given that $y = 3$ and $\frac{dy}{dx} = 0$ when $x = 0$

[6 marks]

$y =$ ______________________

11

Turn over ►

Do not write outside the box

8 (a) The non-singular matrix

$$\mathbf{M} = \begin{bmatrix} 2 & k+1 & -2 \\ k & 4 & -2 \\ -1 & 3 & 0 \end{bmatrix}$$

where k is an integer.

Find $\mathbf{M}^{-1}$ in terms of k

[6 marks]

Answer

Do not write outside the box

8 (b) The 3×3 matrix $\mathbf{A}$ represents a rotation through an angle of $90°$ about the z-axis.

Write down the matrix $\mathbf{A}^{-1}$

[2 marks]

Answer ______________________________

8

Turn over for the next question

Do not write outside the box

9 (a) Given that

$$\tan y = \frac{1+x}{1-x} \quad \text{and} \quad x \neq 1$$

show that

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

[3 marks]

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9 (b) **Hence**, by expressing $\tan^{-1}\left(\dfrac{1+x}{1-x}\right)$ in terms of $\tan^{-1}x$ describe the single geometrical transformation by which the graph of

$$y = \tan^{-1}x \quad \text{where } x < 1$$

can be transformed onto the graph of

$$y = \tan^{-1}\left(\frac{1+x}{1-x}\right) \quad \text{where } x < 1$$

[4 marks]

7

Turn over for the next question

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10 A curve has Cartesian equation

$$y = 1 + 0.5\sinh^2 2x$$

The arc of the curve from $x = 0$ to $x = 0.5$ is rotated through 2π radians about the x-axis.

10 (a) Show that S, the area of the curved surface generated, is given by

$$S = \frac{\pi}{2}\int_0^{0.5} (3 + \cosh 4x)\cosh 4x \; \mathrm{d}x$$

[6 marks]

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10 (b) **Hence** find the exact value of S leaving your answer in terms of hyperbolic functions.
[3 marks]

Answer

9

Turn over for the next question

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11 The line L has equation

$$\left(\mathbf{r} - \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}\right) \times \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

11 (a) Find the direction cosines of L

[3 marks]

Answer ____________________

11 (b) The plane Π has equation

$$\mathbf{r} \,.\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} = 37$$

The point A has coordinates $(-2, 2, -4)$

11 (b) (i) Verify that A lies on the line L but does not lie on the plane Π

[2 marks]

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11 (b) (ii) The point D is the image of A after reflection in the plane Π

Find the coordinates of D

[5 marks]

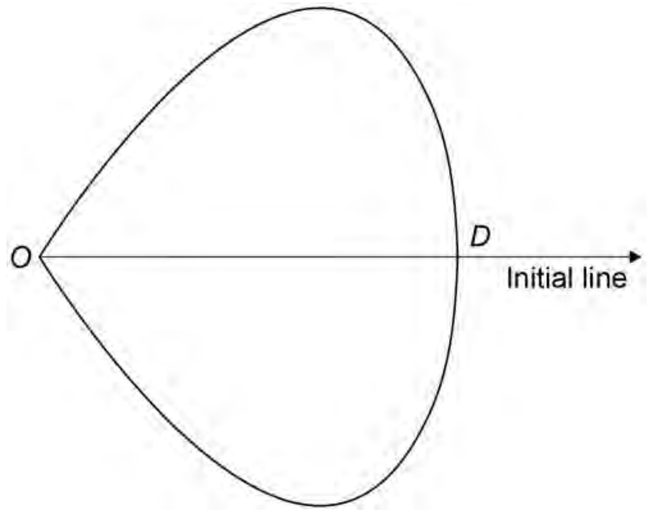
Answer ______________________________

10

Turn over ►

12 The diagram shows a sketch of a curve C_1, the pole O and the initial line.

The curve C_1 intersects the initial line at the point D



The polar equation of C_1 is $r = \left(3 - \tan^2\theta\right)\sec\theta$ where $-\dfrac{\pi}{3} \le \theta \le \dfrac{\pi}{3}$

12 (a) Show that the area of the region bounded by the curve C_1 is $\dfrac{24\sqrt{3}}{5}$

[5 marks]

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12 (b) A circle C_2 has Cartesian equation $x^2 + y^2 = 8$

12 (b) (i) By forming and solving a cubic equation, prove that C_1 and C_2 only intersect at two points, A and B, and find the Cartesian coordinates of A and B

[5 marks]

Answer ______________________

12 (b) (ii) Find the area of the region bounded by the arc ADB of C_1 and the minor arc AB of the circle C_2 giving your answer in an exact form.

[3 marks]

Answer ______________________

13

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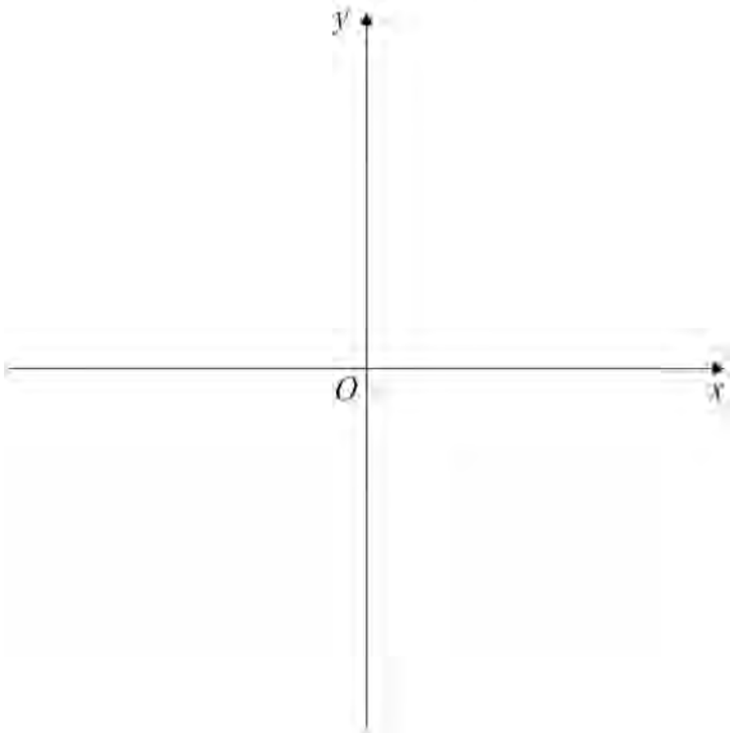
13 A curve C has equation $y = \sinh^{-1} x$

13 (a) Sketch the curve C on the axes below.

[2 marks]



13 (b) Prove that

$$\frac{dy}{dx} = \left(1 + x^2\right)^{-\frac{1}{2}}$$

[3 marks]

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13 (c) It is given that for $|x| < 1$ the first three non-zero terms in the Maclaurin series expansion in ascending powers of x of $\sinh^{-1}x$ are

$$x + ax^3 + bx^5$$

Show that $a = -\frac{1}{6}$ and find the value of b

[4 marks]

$b =$ ______

Question 13 continues on the next page

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13 (d) **Hence** show that

$$\lim_{x \to 0}\left[\frac{x^2 - x\sinh^{-1}x}{(1-\cos 3x)^2}\right]$$

exists and find its value.

[4 marks]

Answer ______________________

13

END OF QUESTIONS

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