OXFORD AQA
INTERNATIONAL QUALIFICATIONS

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I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM03) Unit FP2 Pure Mathematics

Thursday 11 January 2024 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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10	
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12	
13	
14	
TOTAL	

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Answer **all** questions in the spaces provided.

1 The line L has Cartesian equations $\dfrac{x-1}{2} = \dfrac{y-4}{5} = \dfrac{2z-3}{4}$

The plane Π has Cartesian equation $x + 2y + 3z = 18$

The line L intersects the plane Π at the point A

Find the Cartesian coordinates of A

[3 marks]

Answer ______________________________

3

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2 (a) Express the complex number $-2\sqrt{3}+2\mathrm{i}$ in the form $r\,\mathrm{e}^{\mathrm{i}\theta}$
where $r>0$ and $0\leq\theta<2\pi$

[2 marks]

Answer ______________________

2 (b) Solve the equation

$$z^4=-2\sqrt{3}+2\mathrm{i}$$

giving your solutions in the form $r\,\mathrm{e}^{\mathrm{i}\theta}$ where $r>0$ and $0\leq\theta<2\pi$

[4 marks]

Answer ______________________

6

Turn over ►

Do not write outside the box

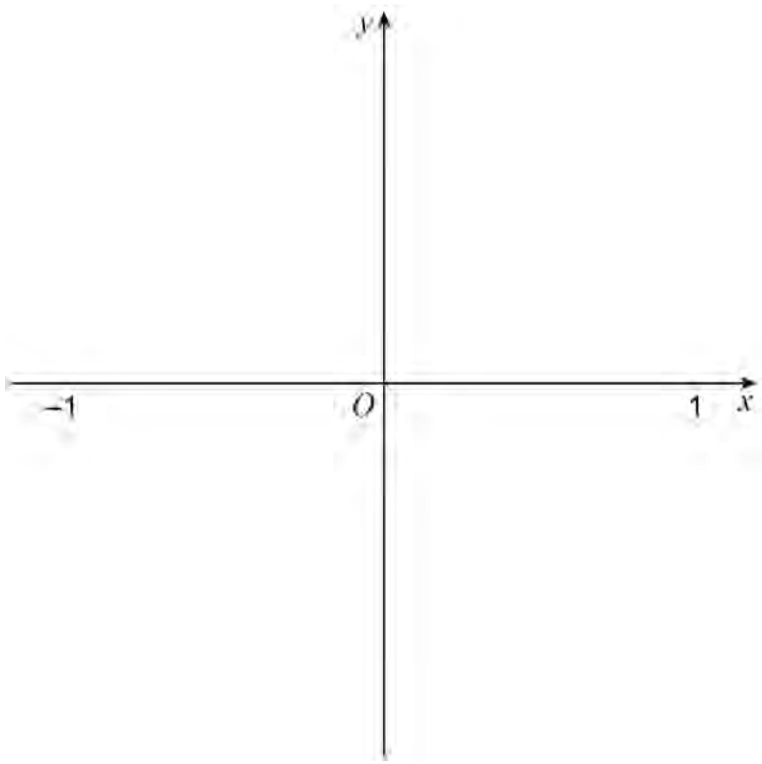
3 (a) (i) Find the gradient of the curve $y = \tanh^{-1} x$ when $x = 0$

[1 mark]

Answer ______________________

3 (a) (ii) Sketch the curve $y = \tanh^{-1} x$ for $|x| < 1$ on the axes below.

[1 mark]



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3 (b) By forming and solving a quadratic equation, find the values of x that satisfy

$$\tanh^{-1}\left(\frac{1+x}{2}\right) + \tanh^{-1}\left(\frac{1-x}{2}\right) = \frac{3}{2}\ln 3 - \frac{1}{2}\ln 2$$

[5 marks]

Answer

7

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4 By expressing $\dfrac{r^2+r+1}{r(r+1)}$ in the form $A+\dfrac{B}{r}+\dfrac{C}{r+1}$ where A, B and C are integers, use the method of differences to show that

$$\sum_{r=1}^{n} \frac{r^2+r+1}{r(r+1)} = 1+n-\frac{1}{n+1}$$

[6 marks]

6

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5 Find the solution of the differential equation

$$\cos x \frac{dy}{dx} + y = \cos^2 x + \sin x \qquad \text{where} \quad 0 \leq x < \frac{\pi}{2}$$

given that $y = 1$ when $x = \frac{\pi}{3}$

Give your answer in the form $y = f(x)$

[6 marks]

$y =$ ______

6

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6 (a) Find the first three non-zero terms in the Maclaurin series expansion in ascending powers of x of $\cos(2x)$

[1 mark]

Answer

6 (b) Hence find the Maclaurin series expansion of $e^{\cos(2x)-1}$ in ascending powers of x up to and including x^4

[2 marks]

Answer

Do not write outside the box

6 (c) Hence show that $\lim_{x \to 0}\left(\frac{e - e^{\cos(2x)}}{x^2}\right) = k\,e$ where k is a constant.

[3 marks]

Answer

6

Turn over for the next question

Do not write outside the box

7 (a) Explain why $\int_3^{\infty} \frac{x-3}{e^x}\, dx$ is an improper integral.

[1 mark]

7 (b) Find the exact value of the improper integral

$$\int_3^{\infty} \frac{x-3}{e^x}\, dx$$

showing the limiting process used.

[5 marks]

Do not write outside the box

Answer

6

Turn over for the next question

Do not write outside the box

8 (a) Prove by induction that, for all integers $n \geq 1$

$$\sum_{r=1}^{n} \left(r^3 + 3r^5\right) = \frac{1}{2}n^3(n+1)^3$$

[5 marks]

Do not write outside the box

8 (b) Hence show that

$$\sum_{r=1}^{3N} r^5 = \frac{3}{4}N^2\left(3N+1\right)^2\left(18N^2+6N-1\right)$$

where N is a positive integer.

[3 marks]

8

Turn over ►

Do not write outside the box

9 The transformation T is represented by the non-singular matrix

$$\mathbf{M} = \begin{bmatrix} -1 & 4 & k \\ 2 & 3 & 6 \\ 1 & 3 & -2 \end{bmatrix}$$

where k is an integer.

9 (a) In the case when T has a line of invariant points:

9 (a) (i) find the value of k

[3 marks]

Answer ____________________

9 (a) (ii) find the Cartesian equations of the line of invariant points.

[2 marks]

Answer ____________________

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9 (b) Find $\mathbf{M}^{-1}$ in terms of k

[6 marks]

Answer

11

Turn over ►

Do not write outside the box

10 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \sin 4x + 38\cos 4x$$

[6 marks]

Answer

Do not write outside the box

10 (b) It is given that $y = \mathrm{f}(x)$ is the solution of the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + y = \sin 4x + 38\cos 4x$$

It is also given that the first two non-zero terms in the Maclaurin series expansion in ascending powers of x of $\mathrm{f}(x)$ are $4 + 17x^2$

Find the value of $\mathrm{f}\left(\frac{\pi}{16}\right)$ giving your answer in a simplified exact form.

[4 marks]

Answer ______________________

10

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11 The plane Π_1 has equation $\mathbf{r} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$

11 (a) Find the sum of the direction cosines of a line perpendicular to the plane Π_1

[5 marks]

Answer ____________________

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11 (b) The plane Π_2 has equation $\mathbf{r} \bullet \begin{bmatrix} 6 \\ c \\ 2 \end{bmatrix} = 14$ where c is a constant.

The line L is the line of intersection of the planes Π_1 and Π_2

The equation of the line L is $\left(\mathbf{r} - \begin{bmatrix} p \\ q \\ 7 \end{bmatrix}\right) \times \begin{bmatrix} 2 \\ -5 \\ 4 \end{bmatrix} = \mathbf{0}$ where p and q are constants.

Find the value of c, the value of p and the value of q

[6 marks]

$c =$ ______________ $p =$ ______________ $q =$ ______________

11

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12 (a) Use de Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta + a\cos^3\theta + b\cos\theta$$

where a and b are integers.

[5 marks]

12 (b) **Hence** prove that the quartic equation whose roots are

$$\cos\frac{2\pi}{5}\ ,\ \cos\frac{4\pi}{5}\ ,\ \cos\frac{6\pi}{5}\ \text{ and }\ \cos\frac{8\pi}{5}$$

is

$$16x^4 + 16x^3 + kx^2 + kx + 1 = 0$$

where k is an integer.

[4 marks]

Do not write outside the box

12 (c) **Hence** use the equation in **part (b)** to find the quadratic equation with integer coefficients whose roots are $\cos\frac{2\pi}{5}$ and $\cos\frac{6\pi}{5}$

[4 marks]

Answer

13

Turn over ►

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13 A curve C is given parametrically by the equations

$$x = \tanh t \quad \text{and} \quad y = \frac{1}{\cosh t} \quad \text{for all real values of } t$$

The length of the arc of C between the points on the curve where $t = -1$ and $t = 1$ is equal to s

13 (a) (i) Prove that $s = \int_{-1}^{1} \operatorname{sech} t \, \mathrm{d}t$

[5 marks]

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13 (a) (ii) **Hence** find the exact value of s giving your answer in terms of e

[4 marks]

Answer ________________________

13 (b) Find the Cartesian equation of the curve C and state as inequalities the possible values of x and the possible values of y

[3 marks]

Cartesian equation ________________________

Values of x ________________________

Values of y ________________________

12

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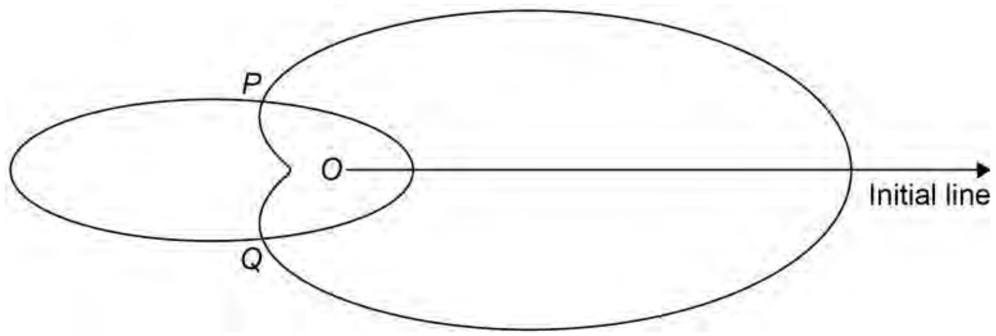
14 **Figure 1** shows an ellipse E and a curve C which intersect at the points P and Q

The pole O and the initial line are also shown.

Figure 1



The Cartesian equation of the ellipse E is

$$5x^2 + 9y^2 = 36 - 24x$$

The polar equation of the curve C is

$$r = 5 + 4\cos\theta \quad \text{where} \quad -\pi \le \theta \le \pi$$

14 (a) Show that the polar equation of the ellipse E is $r = \dfrac{6}{3 + 2\cos\theta}$

[4 marks]

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14 (b) Show that the area of triangle OPQ is $\dfrac{9\sqrt{3}}{4}$

[4 marks]

Question 14 continues on the next page

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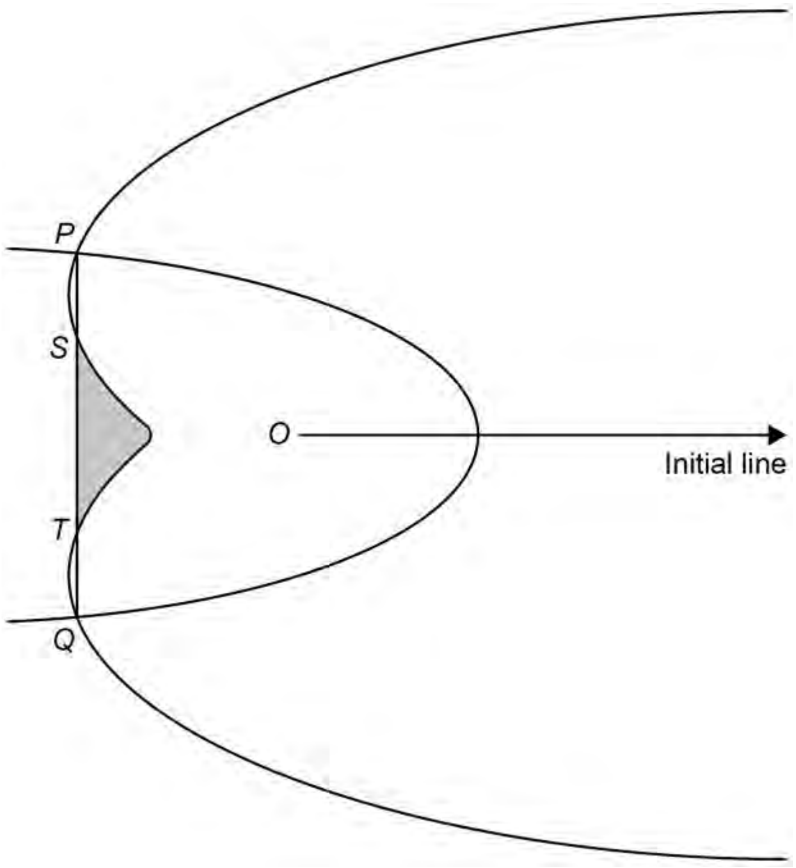
14 (c) **Figure 2** shows an enhanced version of part of **Figure 1**.

The line segment PQ intersects the curve C at the points S and T

The finite region bounded by the line segment ST and the curve C is shaded.

Figure 2



Find the area of the shaded region.

Give your answer in the form $a\sqrt{n} + m\cos^{-1}(b)$ where n and m are integers and a and b are rational.

[7 marks]

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Answer ______________________

15

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