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(9660/MA04) Unit S2 Statistics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	standard deviation $= \sqrt{15}$	B1	Allow **AWRT** 3.9
		1	

Q	Answer	Marks	Comments
1(a)(ii)	$\mathrm{P}(W \leq 20) = 0.9170 \quad [> 0.9]$ $\mathrm{P}(W \leq 19) = 0.8752 \quad [< 0.9]$ $a = 20$	M1 A1	For one correct probability **PI** by correct value of a
		2	

Q	Answer	Marks	Comments
1(a)(iii)	$\mathrm{P}(W > 24) = 1 - 0.9888 \quad [= 0.0112 > 0.01]$ $\mathrm{P}(W > 25) = 1 - 0.9938 \quad [= 0.0062 < 0.01]$ $b = 25$	M1 A1	For one correct probability **PI** by correct value of b Allow 0.0111 and 0.0061 from calculator
		2	

Q	Answer	Marks	Comments
1(b)	$\mathrm{P}(X \leq 4) - \mathrm{P}(X \leq 2)$ $[= 0.8153 - 0.4232]$ $= 0.392$	M1 A1	$\mathrm{P}(X \leq m) - \mathrm{P}(X \leq n)$ with at least one of $m = 4$, $n = 2$ **PI** by correct answer **AWRT** 0.392
		2	

Q	Answer	Marks	Comments
1(c)(i)	As n is large **and** p is small	**B1**	**oe eg** $n > 20$ and $p < 0.25$
		1	

Q	Answer	Marks	Comments
1(c)(ii)	$\lambda = 10$	**B1**	**oe**
		1	

Q	Answer	Marks	Comments
1(c)(iii)	$[\lambda = 15 + 3 + 10]$		
	$\lambda = 28$	**B1ft**	**ft** their λ from **(c)(ii)**
	$[\mathrm{P}(C < 3) =]$		
	$\frac{\mathrm{e}^{-28} \times 28^0}{0!} + \frac{\mathrm{e}^{-28} \times 28^1}{1!} + \frac{\mathrm{e}^{-28} \times 28^2}{2!}$	**M1**	**M1** for at least one correct probability in the sum in the form $ne^{-\lambda}$ **ft** their 28
	$= 421\mathrm{e}^{-28}$	**A1**	**CAO NMS** 3/3
		3	

	Question 1 Total	**12**	

Q	Answer	Marks	Comments
2(a)	$\mathrm{P}(X > 50) = \mathrm{P}\left(Z > \frac{50-38}{11}\right)$	**M1**	For standardising $\frac{50-38}{11}$ **PI**
	$\left[\mathrm{P}(Z < 1.09) =\right]$ 0.86214 [from tables]	**M1**	0.862[14] seen or used **PI**
	$\left[\mathrm{P}(Z > 1.09) =\right]$ $1 - 0.86214$		
	$= 0.138$	**A1**	Allow 0.13786 [from use of tables] **NMS** 3/3
		3	

Q	Answer	Marks	Comments
2(b)	$H_0 : \mu = 38$ $H_1 : \mu < 38$	**B1**	Condone 'mu' or other letters, but not $\bar{x}$
	$\bar{X} \sim \mathrm{N}\left(38, \frac{11^2}{30}\right)$	**B1**	**PI** by correct standardisation formulae
	$z = \frac{34-38}{11/\sqrt{30}}$	**M1**	$z = \frac{34-38}{\sigma/\sqrt{30}}$ with their σ **PI** by correct z
	$z = -1.9917...$	**A1**	**AWRT** -1.99 or exact value $-\frac{4\sqrt{30}}{11}$
	$z_{\text{critical}} = \pm 2.0537$	**B1**	or $\mathrm{P}\left(Z < -\frac{4\sqrt{30}}{11}\right) = 0.0232$ to 0.0233 or comparison of $\mathrm{P}(\bar{X} < 34) = 0.0232$ to 0.0233 with 2% Allow 'accept H_0'
	Do not reject H_0 as $z_{\text{critical}} < z$ or $-2.0537 < -1.99\ldots$ or $\lvert z \rvert < 2.0537$	**A1ft**	Comment about H_0 and 0.0232 to $0.0233 > 0.02$ Correct conclusion based upon **ft** their z (signs need to be compatible)
	Insufficient evidence to support the claim that the journey time has reduced from 38 minutes on average [at the 2% level of significance]	**E1**	Correct statement must be in context and must follow from fully correct solution. Condone definite statement.
		7	

	Question 2 Total	**10**	

Q	Answer	Marks	Comments
3(a)	Exponential **and** $\lambda = \frac{1}{8}$	B1	
		1	

Q	Answer	Marks	Comments
3(b)	$\mathrm{P}(T > 7) = 1 - \mathrm{P}(T \leq 7)$	M1	**PI**
	$\left[= 1 - \left(1 - \mathrm{e}^{-\frac{7}{8}} \right) \right] = 0.4169$	A1	Attempts to find correct probability using cdf of exponential or integration of pdf **CAO** to 4 sf
		2	

Q	Answer	Marks	Comments
3(c)	$\left[1 - \mathrm{e}^{-\frac{x}{8}} = 0.95 \Rightarrow \right] \quad x = -8\ln(0.05)$	M1	Attempt to find x by simplifying an equation using a cdf of exponential or integration of pdf **oe**
	$x = 23.9658...$		
	$L = 23970$ [hours]	A1	**oe**
		2	

Q	Answer	Marks	Comments
3(d)	$\mathrm{P}(T<10 \mid T>7)=\mathrm{P}(T<3)$ $\left[=1-\mathrm{e}^{-\frac{3}{8}}\right]$ $=0.3127$	**M1** **A1**	Attempt to find t by simplifying an equation using a cdf of exponential or integration of pdf **AWRT** 0.3127
		2	

Q	Answer	Marks	Comments
3(e)	The no memory property of the exponential distribution suggests the component is memoryless [however given the component is already 7 thousand hours old it is 'more likely' to break]	**E1**	Reference to 'memory' or comment about increasing chance of breakdown
		1	

	Question 3 Total	**8**	

Q	Answer	Marks	Comments
4(a)	$\frac{k-0.4}{5-4}=\frac{0.4-0}{4-2}$ $k=0.6$	**M1** **A1**	Correct equation to find k **M1** for $m=0.2$ **oe**
		2	

Q	Answer	Marks	Comments
4(b)	$\mathrm{F}(x)=\begin{cases}0 & x<2\\ 0.2x-0.4 & 2\le x<5\\ 0.1x+0.1 & 5\le x<9\\ 1 & x\ge 9\end{cases}$	**B1** **M1** **A1** **A1** **B1**	**B1** for correct values of $\mathrm{F}(x)$ for $x<2$ and $x\ge 9$ **M1** for use of a straight line method to find an equation **A1** for one correct equation **A1** for both correct equations **B1** all correct domains
		5	

Q	Answer	Marks	Comments
4(c)	$[\mathrm{F}(6)-\mathrm{F}(4)=]$ $(0.1\times 6+0.1)-(0.2\times 4-0.4)$ $=0.3$	**M1** **A1**	Attempt using their $\mathrm{F}(6)$ and $\mathrm{F}(4)$ **oe**
		2	

	Question 4 Total	**9**	

Q	Answer	Marks	Comments
5(a)	$X \sim \text{B}(50,\ 0.9)$	**M1**	or $Y \sim \text{B}(50,\ 0.1)$
	[p is the probability of a successful repair]		
	$\text{H}_0 : p = 0.9$ $\text{H}_1 : p > 0.9$	**B1**	or $\text{H}_0 : p = 0.1$ $\text{H}_1 : p < 0.1$
	$\left[1 - \text{P}(X \leq 48) = \right]\ 1 - 0.9662$ or 0.0338	**M1**	Sight of **AWRT** 0.0338
	$\text{P}(X \geq 49) = 0.0338$	**A1**	$\text{P}(Y \leq 1) = 0.0338$ Award **M1 A1** for: critical region for Y is $\{0\}$ **oe**, or critical region for X is $\{50\}$ **oe**
	$0.0338 > 0.01$	**M1**	**oe** Comparison of their probability to 0.01
	Do not reject H_0	**A1ft**	Correct conclusion for their value
	Insufficient evidence to suggest that Sam's success rate has improved.	**E1**	Cannot be a definite statement. Must follow a fully correct test
		7	

Q	Answer	Marks	Comments
5(b)	Accepting that the success rate [in repairing water damaged phones] for Sam has increased from 90% when it has not	**B1**	Must be in context
		1	

Q	Answer	Marks	Comments
5(c)	The successful repair of each phone is **not** likely to be **independent** as they were damaged in the same flood and so shouldn't be modelled as a Binomial	**B1**	
		1	

	Question 5 Total	**9**	

Q	Answer	Marks	Comments
6	H_0: $\mu = 6.03$ H_1: $\mu > 6.03$	**B1**	Both hypotheses
	$\bar{x} = \frac{73.08}{12} = 6.09$	**B1**	**PI**
	$s^2 = \frac{1}{12-1}\left(445.2502 - \frac{73.08^2}{12}\right)$	**M1**	Attempt at calculating variance, allow one error **PI** by correct answer
	$\left[= \frac{193}{11000}\right] = 0.017\dot{5}\dot{4}$	**A1**	**AWRT** 0.0175 Accept $s = 0.132$[4592562]
	$\bar{X} \sim \text{N}\left(6.03,\ \frac{0.017\dot{5}\dot{4}}{12}\right)$	**M1**	$\bar{X} \sim \text{N}\left(6.03,\ \frac{s^2}{12}\right)$ with their s^2 **PI**
	$t = \frac{6.09 - 6.03}{\sqrt{\frac{0.017\dot{5}\dot{4}}{12}}}$	**M1**	Calculates t with their s^2
	$t = 1.56$[9132297]	**A1**	**AWRT** 1.57
	$t_{\text{critical}} = 1.363$ [using $\nu = 11$]	**B1**	$p = 0.0725$
	As $1.36 < 1.57$ or $t_{\text{critical}} < t$, reject H_0	**A1ft**	or $0.0725 < 0.1$, reject H_0 **ft** their t and t_{crit} provided signs are consistent
	Evidence to suggest that that Hannah's coach has improved her distance jumped [at the 10% level of significance]	**E1**	Must not be definitive Must be in context
		10	

	Question 6 Total	**10**	

Q	Answer	Marks	Comments
7(a)	$\int_0^1 \frac{3x^{\frac{1}{2}}+k}{6}\,\mathrm{d}x = 1$ $\left[\frac{1}{3}x^{\frac{3}{2}}+\frac{1}{6}kx\right]_0^1 = 1$ $\left[\left(\frac{1}{3}+\frac{1}{6}k\right)-(0+0)\right] = 1$ $k = 4$	**M1** **A1**	Correct integral set equal to 1 with at least 1 correct integration **oe** **AG** Requires at least one intermediate line after integration
		2	

Q	Answer	Marks	Comments
7(b)	$\left[\int x\,\mathrm{f}(x)\,\mathrm{d}x\right] = \int_0^1 \frac{3x^{\frac{3}{2}}+4x}{6}\,\mathrm{d}x$ $= \left[\frac{1}{5}x^{\frac{5}{2}}+\frac{1}{3}x^2\right]_0^1$ $\left[\frac{1}{5}+\frac{1}{3}\right] = \frac{8}{15}$	**M1** **M1** **A1**	Identifies correct integral Correct integration for their $x\,\mathrm{f}(x)\,\mathrm{d}x$ **CAO**
		3	

Q	Answer	Marks	Comments
7(c)	$\left[\mathrm{E}\left(X^2\right)=\int x^2\mathrm{f}\left(x\right)\mathrm{d}x=\right]\ \int_0^1 \frac{3x^{\frac{5}{2}}+4x^2}{6}\,\mathrm{d}x$	M1	Identifies correct integral
	$=\left[\frac{1}{7}x^{\frac{7}{2}}+\frac{2}{9}x^3\right]_0^1$	M1	Correct integration
	$\mathrm{E}\left(X^2\right)=\frac{23}{63}$	A1	**PI**
	$\mathrm{Var}\left(X\right)=\mathrm{E}\left(X^2\right)-\mathrm{E}\left(X\right)^2$		
	$=\frac{23}{63}-\left(\frac{8}{15}\right)^2$	M1	**ft** their $\mathrm{E}\left(X^2\right)$
	$=\frac{127}{1575}$	A1	**AG**
		5	

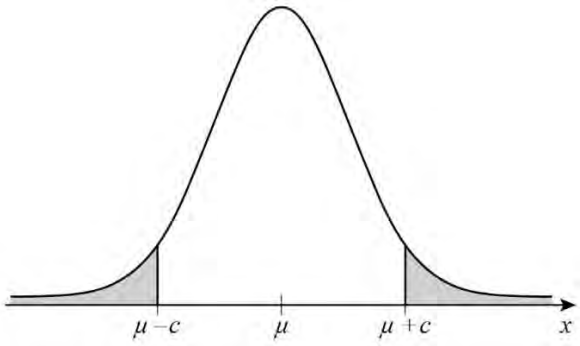
Q	Answer	Marks	Comments
7(d)(i)	$15\mathrm{E}\left(X\right)-9\mathrm{E}\left(Y\right)$	M1	
	$=15\times\frac{8}{15}-9\times2$		
	$=-10$	A1	
		2	

Q	Answer	Marks	Comments
7(d)(ii)	$225\mathrm{Var}\left(X\right)+81\mathrm{Var}\left(Y\right)$	M1	
	$=225\times\frac{127}{1575}+81\times\frac{5}{7}$		
	$=76$	A1	
		2	

	Question 7 Total	**14**	

Q	Answer	Marks	Comments
8(a)(i)	$a = 1.9600\sigma$	B1	Condone **AWRT** 1.96
		1	

Q	Answer	Marks	Comments
8(a)(ii)	$b = \mu + 0.9945\sigma$	B1	Condone $b = \mu + 0.9944\sigma$
		1	

Q	Answer	Marks	Comments
8(a)(iii)		B1	Must be approximately symmetrical and have labels $\mu - c$ and $\mu + c$
		1	

Q	Answer	Marks	Comments
8(b)	$\left[\mathrm{P}(Z < z) = 0.975 \ \Rightarrow\ \right] \ z = 1.96$	B1	
	$1.96 = \dfrac{205 - m}{\sqrt{0.8m}}$	M1	
	$\left(\sqrt{m}\right)^2 + 1.7530\left[77294\right]\sqrt{m} - 205 = 0$	m1	A quadratic equation in m or $\sqrt{m}$ set equal to zero **PI** by 181.38… or 231.68…, or 13.46... or −15.22...
	$\sqrt{m} = 13.4\left[6808822...\right]$	A1	**AWFW** [13.4, 13.5] **PI** by $m = 181.39$
	$m = 181.39$	A1	This value and no others
		5	

	Question 8 Total	8	