OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

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I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM04) Unit FS2 Statistics

Wednesday 22 January 2020 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables (enclosed).
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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8	
9	
TOTAL	

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Answer **all** questions in the spaces provided.

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1 Fiona is tossing an unbiased coin with 'heads' on one side and 'tails' on the other.

She is playing a game where she gets 10 points if the coin lands with 'heads' facing up and 0 points if it lands with 'tails' facing up.

She tosses the coin twice.

Find the sampling distribution of Fiona's median score, M.

[3 marks]

Answer _____

3

Turn over ►

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2 The random variable X represents the weights of cats in a town.

Andrew estimates the mean by taking a random sample, X_1, of size 10 from X and calculating the sample mean $\overline{X_1}$

Linda estimates the mean by taking a random sample, X_2, of size 100 from X and calculating the sample mean $\overline{X_2}$

X_1 and X_2 are independent.

X has mean μ kilograms and standard deviation σ kilograms.

Both $\overline{X_1}$ and $\overline{X_2}$ are unbiased estimators of μ.

2 (a) Evaluate the efficiency of Linda's estimator relative to Andrew's estimator.

Interpret your answer.

[3 marks]

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2 (b) (i) Show that $P = \dfrac{\overline{X_1} + \overline{X_2}}{2}$ is an unbiased estimator of μ.

[2 marks]

2 (b) (ii) Find $\mathrm{Var}(P)$.

[2 marks]

Answer _____

7

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3 Abdul is studying the age, X years, of people at retirement.

He takes a random sample of 500 people. The results are

$$\sum x = 33\,202 \qquad \text{and} \qquad \sum x^2 = 2\,210\,000$$

3 (a) Abdul assumes that age at retirement is a normal distribution.

Construct a 96% confidence interval for the mean age at retirement, giving your values to three decimal places.

[6 marks]

Answer _____

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3 (b) Abdul uses his random sample to conduct a hypothesis test at the 4% level of significance with hypotheses

$$H_0: \mu = 67$$

$$H_1: \mu \neq 67$$

Using the confidence interval found in part **(a)**, explain whether Abdul accepts or rejects the null hypothesis.

[1 mark]

3 (c) After further investigation, Abdul concludes that age at retirement is not a normal distribution.

Explain whether or not the confidence interval found in part **(a)** is still valid.

[2 marks]

9

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4 A hypothesis test, using a 5% level of significance, is performed on the population of a binomial distribution using a random sample of size 50. The hypotheses are

$$H_0: p = 0.4$$

$$H_1: p > 0.4$$

The actual value of p is 0.5

4 (a) (i) Find the probability that a Type II error is made, giving your answer to three significant figures.

[4 marks]

Answer _____

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4 (a) (ii) Find the power of the test, giving your answer to three significant figures.

[2 marks]

Answer _____

4 (b) State how increasing the level of significance for the hypothesis test will affect the power of the test.

[1 mark]

7

Turn over for the next question

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5 A fabrics company has two machines, A and B.

The length of fabric produced per minute by machine *A* and the length of fabric produced per minute by machine B both have normal distributions which are independent of each other.

The company claims that the population variances for the two machines are equal.

The company takes a random sample of 21 lengths of fabric, each produced in a minute, from machine A.

The company also takes a random sample of 26 lengths of fabric, each produced in a minute, from machine B.

The sample standard deviation for machine A is 0.3 metres.

The sample standard deviation for machine B is 0.2 metres.

Test the company's claim that the population variances for the two machines are equal, using the 2% level of significance.

[7 marks]

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7

Turn over ►

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6 A company performed a survey of its employees in both its northern offices and its southern offices about the transport that the employees use to get to work.

The results are summarised in the following table.

	Walk	**Car**	**Bus**	**Train**	**Total**
Northern	4	32	30	2	68
Southern	22	15	20	5	62
Total	26	47	50	7	130

Test if there is an association between the office in which an employee works and the transport they use to get to work, using the 1% level of significance.

[10 marks]

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10

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7 A supermarket sells jars of honey from two suppliers.

The masses of jars of honey supplied by the two suppliers are independent.

The mass of the jars of honey from supplier A has a normal distribution with standard deviation 2 grams.

The mass of the jars of honey from supplier B has a normal distribution with standard deviation 3 grams.

A random sample of size 10 is taken from the jars of honey from supplier A and the mean mass is 504 grams.

A random sample of size 12 is taken from the jars of honey from supplier B and the mean mass is 502 grams.

The supermarket claims that the mean mass of jars of honey is different for the two suppliers.

7 (a) Test the supermarket's claim, using a 5% level of significance.

[7 marks]

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7 (b) Explain what changes would need to be made to the test carried out in part **(a)** if only sample standard deviations were available and assuming that the population standard deviations were equal.

[4 marks]

11

Turn over ►

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8 A tutorial company offers a four-week revision course to students taking an examination.

The students attempt a practice paper before the course and after the end of the course.

The results are shown in the table below.

Student	Before	After
1	20	24
2	18	19
3	16	16
4	10	9
5	5	7
6	3	6

The tutorial company claims that students score more on average on the practice paper after the revision course than before.

8 (a) Test the tutorial company's claim, using the 5% level of significance.

[11 marks]

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8 (b) State a necessary assumption for the test in part **(a)** to be valid.

[2 marks]

13

Turn over ►

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9 The random variable X has moment generating function $M_X(t)$ where

$$M_X(t) = 0.04(1 - 0.8e^t)^{-2}$$

9 (a) Find the mean of X.

[4 marks]

Answer _____

9 (b) Find the variance of X.

[4 marks]

Answer _____

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9 (c) Find $M_{2+3X}(t)$, giving your answer in the form $\left(\frac{a\mathrm{e}^{t}}{1-0.8\mathrm{e}^{bt}}\right)^{2}$, where a and b are constants.

[3 marks]

Answer _____

Question 9 continues on the next page

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9 (d) The random variable Y has moment generating function $M_Y(t)$ where

$$M_Y(t) = \left(0.8 + 0.2e^t\right)^2$$

X and Y are independent.

Find $M_{X+Y}(t)$, giving your answer in the form $\left(\frac{a + be^t}{1 - 0.8e^t}\right)^2$, where a and b are constants.

[2 marks]

Answer _____

13

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