OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | |    Candidate number | | | | |

Surname ____________________

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Candidate signature ____________________
I declare this is my own work.

# INTERNATIONAL A-LEVEL
# FURTHER MATHEMATICS

(9665/FM03) Unit FP2 Pure Mathematics

Time allowed: 2 hours 30 minutes

**Materials**
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphic calculator.

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

| For Examiner's Use | |
|---|---|
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| **TOTAL** | |

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Answer **all** questions in the spaces provided.

**1** A curve $C$ has equation

$$y = \tan^{-1}(x+1) + \tanh^{-1}\left(\frac{x}{2}\right) \quad \text{where} \quad -2 < x < 2$$

**1 (a)** Find $\dfrac{dy}{dx}$

**[2 marks]**

Answer

**1 (b)** **Hence** find an equation of the normal to $C$ at the point $P$ on the curve given that the $x$-coordinate of $P$ is $0$

**[3 marks]**

Answer

5

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**2** The matrix $\mathbf{A} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

**2 (a)** Describe fully the single transformation represented by the matrix $\mathbf{A}$

**[2 marks]**

______________________________________________

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**2 (b)** For this transformation, state the line of invariant points.

**[1 mark]**

______________________________________________

______________________________________________

______________________________________________

Answer ______________________________

**3**

**Turn over for the next question**

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**3 (a)** Express $\dfrac{6}{(r-1)(r+1)}$ in the form $\dfrac{A}{r-1}+\dfrac{B}{r+1}$, where $A$ and $B$ are integers.

**[2 marks]**

$\dfrac{6}{(r-1)(r+1)} =$ ______________________

**3 (b)** Use the method of differences to show that

$$\sum_{r=2}^{n}\frac{6}{(r-1)(r+1)}=\frac{an^2+bn+c}{2n(n+1)}$$

where $a$, $b$ and $c$ are integers.

**[4 marks]**

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**6**

**Turn over for the next question**

Do not write outside the box

**4** Solve the differential equation

$$4\frac{d^2y}{dx^2}+4\frac{dy}{dx}+y=0$$

given that $y = 4$ and $\frac{dy}{dx}=1$ when $x = 0$

**[6 marks]**

Answer

**6**

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**5 (a)** Explain why $\int_0^{e^2} \ln x \, dx$ is an improper integral.

**[1 mark]**

**5 (b)** Evaluate $\int_0^{e^2} \ln x \, dx$ showing the limiting process used.

**[6 marks]**

Answer

7

**Turn over ►**

Do not write outside the box

**6 (a)** A student states that vectors **r**, **m** and **n** can be found such that

$$\mathbf{r} \times \mathbf{m} = \mathbf{n} \quad \text{and} \quad \mathbf{m} \cdot \mathbf{n} = 12$$

Explain why the student is **not** correct.

**[2 marks]**

**6 (b)** The points $A$, $B$ and $C$ have position vectors **a**, **b** and **c** respectively relative to an origin $O$, where

$$\mathbf{a} = 2\mathbf{i} + p\mathbf{j} - \mathbf{k} \qquad \mathbf{b} = -p\mathbf{i} + 4\mathbf{j} - 7\mathbf{k} \qquad \mathbf{c} = -4\mathbf{i} + 2p\mathbf{j} - 9\mathbf{k}$$

and $p$ is real.

The position vectors **a**, **b** and **c** define the edges of a parallelepiped.

The volume of the parallelepiped is $17$ cubic units.

Use a scalar triple product to find the four possible values of $p$

**[6 marks]**

Answer

8

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**7** The matrix $\mathbf{M} = \begin{bmatrix} 3 & -2 \\ 5 & p \end{bmatrix}$ where $p$ is a constant.

The matrix $\mathbf{M}$ has two distinct eigenvalues.

One of the eigenvalues is $1$

**7 (a)** Find the other eigenvalue.

**[4 marks]**

Answer ____________________

**7 (b)** Find an eigenvector for each eigenvalue.

**[3 marks]**

Eigenvectors ________________ and ________________

7

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**8 (a)** By direct expansion, or otherwise, show that

$$\begin{vmatrix} k & 2 & k-4 \\ 2k-2 & 3k-2 & 4 \\ 2k+3 & 3k & 5 \end{vmatrix} = -8k^2 + pk + q$$

where $p$ and $q$ are positive integers.

**[2 marks]**

**8 (b)** A system of equations is given such that

$$kx + 2y + (k-4)z = a$$

$$(2k-2)x + (3k-2)y + 4z = b$$

$$(2k+3)x + 3ky + 5z = c$$

where $k$ , $a$ , $b$ and $c$ are real constants.

**8 (b) (i)** Find the two values of $k$ for which the system of equations does **not** have a unique solution.

**[2 marks]**

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Answer

**8 (b) (ii)** For the integer value of $k$ found in **part (b)(i)**, find an expression for $b$ in terms of $a$ and $c$ such that the system of equations is consistent.

**[3 marks]**

$b=$

7

Do not write outside the box

**9 (a)** Explain why the cubic equation

$$ax^3 + bx^2 + cx + 8 = 0$$

where $a$ , $b$ and $c$ are real numbers, cannot have exactly one non-real root.

**[1 mark]**

**9 (b)** The equation

$$2z^3 + pz^2 + 4z - 6\mathrm{i} = 0$$

where $p$ is a constant, has roots $\alpha$ , $\beta$ and $\gamma$

**9 (b) (i)** Show that

$$(\alpha\beta + 2)(\alpha\gamma + 2)(\beta\gamma + 2) = k - 3\mathrm{i}p$$

where $k$ is an integer.

**[4 marks]**

Do not write outside the box

**9 (b) (ii)** Find a cubic equation which has roots $\alpha\beta + 2$ , $\alpha\gamma + 2$ and $\beta\gamma + 2$

**[3 marks]**

Answer ______________________

8

**Turn over ►**

Do not write outside the box

**10 (a)** Prove by induction that, for all integers $n \geq 1$

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$$

**[5 marks]**

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**10 (b)** Find, in terms of $\pi$, the two smallest positive values of $\theta$ that satisfy the equation

$$2(\cos\theta + \mathrm{i}\sin\theta)^3 = 1 - \mathrm{i}\sqrt{3}$$

**[4 marks]**

Answer

9

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**11** The plane $\Pi_1$ has equation $\mathbf{r} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + \mu \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

The plane $\Pi_2$ has equation $\mathbf{r} . \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} = 5$

**11 (a)** Find an equation for the plane $\Pi_1$ in the form $\mathbf{r} . \mathbf{n} = d$

**[4 marks]**

Answer ______________________

**11 (b)** Find the acute angle between the planes $\Pi_1$ and $\Pi_2$ giving your answer to the nearest $0.1°$

**[4 marks]**

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Answer

**11 (c)** Write down a Cartesian equation of the plane $\Pi_2$

**[1 mark]**

Answer

**11 (d)** Find a vector equation for the line of intersection of the planes $\Pi_1$ and $\Pi_2$ giving your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$

**[5 marks]**

Answer

14

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**12** It is given that $y = \ln\left[e^{2x}\left(1 + \tan^2 x\right)\right]$

**12 (a) (i)** Show that $\dfrac{dy}{dx} = 2\left(1 + \tan x\right)$

**[2 marks]**

**12 (a) (ii)** Find $\dfrac{d^4 y}{dx^4}$ in terms of $x$

**[3 marks]**

Answer

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**12 (b)** Hence, show that the first three non-zero terms in ascending powers of $x$ in the Maclaurin series of $\ln\left[e^{2x}\left(1+\tan^2 x\right)\right]$ are

$$2x + x^2 + \frac{1}{6}x^4$$

**[3 marks]**

**12 (c)** Show that

$$\lim_{x \to 0}\left[\frac{2\ln\left(\cos x\right) + x\sin x}{2\sqrt{x^8 + x^{10}}}\right]$$

exists and state its value.

**[4 marks]**

Answer ________________________

12

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**13 (a)** Use the definitions of $\cosh\theta$ and $\sinh\theta$ in terms of $e^{\theta}$ and $e^{-\theta}$ to show that

$$1+\sinh^2\theta=\cosh^2\theta$$

**[3 marks]**

**13 (b)** Use an integrating factor to find the general solution of the differential equation

$$\frac{dy}{dx}+\frac{x}{1+x^2}y=2$$

Give your answer in the form $y=f(x)$

**[11 marks]**

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Answer

14

**Turn over ►**

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**14** A curve $C_1$ is given parametrically by the equations

$$x = 2e^{0.5\theta}\cos\theta \quad \text{and} \quad y = 2e^{0.5\theta}\sin\theta$$

The point $P$ on $C_1$ is where $\theta = 0$

The point $Q$ on $C_1$ is where $\theta = \pi$

**14 (a)** Find the length of the arc $PQ$ of the curve $C_1$

Give your answer in an exact form.

**[7 marks]**

Answer ______________________

**14 (b)** A curve $C_2$ has polar equation

$$r = 2e^{0.5\theta} - 1 \quad \text{where} \quad 0 \le \theta \le \pi$$

The point $D$ on $C_2$ is where $\theta = 0$

The point $E$ on $C_2$ is where $\theta = \pi$

**14 (b) (i)** Sketch the curve $C_2$

**[2 marks]**


**14 (b) (ii)** By finding the polar equation of the curve $C_1$, or otherwise, show that the area of the region bounded by $C_1$ and $C_2$ and the line segments $PD$ and $QE$ is

$$\frac{1}{2}\left(a\,\mathrm{e}^{\frac{\pi}{2}} + b + c\,\pi\right)$$

where $a$, $b$ and $c$ are integers.

**[5 marks]**

**14**

**END OF QUESTIONS**

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