OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS
FM05

(9665/FM05) Unit FM2 Mechanics

Mark scheme

January 2020

Version: 1.0 Final

20IXfm05/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from oxfordaqaexams.org.uk

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$14.7e = 0.6 \times 9.8$ $e = 0.4$ $\text{Length} = 0.2 + 0.4 = 0.6\ \text{m}$	**M1** **A1** **A1**	M1: Forms equation to find extension. A1: Obtains extension. A1: Correct length.
	Total	**3**	

Q	Answer	Marks	Comments
2(a)	$4 = \frac{2\pi}{\omega}$ $\omega = \frac{\pi}{2}$ $v_{\max} = \frac{3}{2} \times \frac{\pi}{2}$ $v_{\max} = \frac{3\pi}{4}\ \text{m s}^{-1}$	**B1** **M1** **A1**	B1: Obtains correct ω. M1: Use of *a* multiplied by their ω. A1: Accept AWRT 2.4 from correct working.
2(b)	$v^2 = \frac{\pi^2}{4}\left(\frac{9}{4} - \frac{1}{4}\right)$ $v = \frac{\pi}{\sqrt{2}}\ \text{m s}^{-1}$	**M1** **A1** **A1**	M1: Using SHM equation with their ω and correct distances. A1: Correct equation. A1: Accept AWRT 2.2 from correct working.
2(c)	$x = \frac{3}{2}\cos\left(\frac{\pi t}{2}\right)$ $x = \frac{3}{2}$ $\frac{3}{2} = \frac{3}{2}\cos\left(\frac{\pi t}{2}\right)$ $t = 0$ $x = \frac{1}{2}$ $\frac{1}{2} = \frac{3}{2}\cos\left(\frac{\pi t}{2}\right)$ $t = 0.78$ $\text{Time} = 0.78\ \text{s}$	**M1** **M1** **A1**	M1: Forms equation for displacement in terms of time. M1: Substitutes both values for *t*. A1: Accept 0.78
	Total	**9**	

Q	Answer	Marks	Comments
3(a)	$4\cos\alpha = 3\cos\beta$ $3\sin\beta = 4e\sin\alpha$ $\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{3\sin\beta}{4e} \times \frac{4}{3\cos\beta}$ $\tan\alpha = \frac{\tan\beta}{e}$	**M1** **A1** **M1** **A1**	M1: Forms equations for both perpendicular and parallel motion. A1: Both equations correct. M1: Uses $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ A1: Obtains required result from correct working.
3(b)(i)	$\tan 2\beta = \frac{\tan\beta}{e}$ $\frac{2\tan\beta}{1-\tan^2\beta} = \frac{\tan\beta}{e}$ $2e = 1 - \tan^2\beta$ $\tan\beta = \sqrt{1-2e}$	**M1** **M1** **A1**	M1: Substitutes 2β M1: Uses double angle formula. A1: Correct expression.
3(b)(ii)	$0 < e < \frac{1}{2}$	**B1**	B1: Obtains correct inequality or makes correct statement. Condone $e < \frac{1}{2}$
	Total	**8**	

Q	Answer	Marks	Comments
4(a)	$2\frac{dv}{dt} = -0.4 \times 9.8 - 4v$	**M1**	M1: Forms a three term differential equation.
	$\frac{1}{1.96+2v} \times \frac{dv}{dt} = -1$	**M1**	M1: Separates variables.
	$\frac{1}{2}\ln(1.96+2v) = -t + c$		
	$1.96 + 2v = A\mathrm{e}^{-2t}$	**A1**	A1: Integrates and obtains a correct result.
	$v = \frac{A\mathrm{e}^{-2t}}{2} - 0.98$	**M1**	M1: makes *v* the subject.
	$t = 0, v = 12 \Rightarrow A = 25.96$	**M1**	M1: Finds constant of integration.
	$v = 12.98\mathrm{e}^{-2t} - 0.98$	**A1**	A1: Obtains required result from correct working.
4(b)	$12.98\mathrm{e}^{-2t} - 0.98 = 0$	**M1**	M1: Forms equation for zero speed.
	$t = -\frac{1}{2}\ln\left(\frac{0.98}{12.98}\right) = 1.2918$	**A1**	A1: Finds correct time (PI)
	$s = -\frac{12.98}{2}\mathrm{e}^{-2t} - 0.98t + c$	**M1**	M1: Integrates velocity to obtain displacement.
		A1	A1: Correct displacement with coefficients given in unrounded form.
	$t = 0, s = 0 \Longrightarrow c = 6.49$		
	$s = -\frac{12.98}{2}\mathrm{e}^{-2\times1.2918}$	**M1**	M1: Finds constant of integration.
	$-0.98 \times 1.2918 + 6.49$		
	$s = 4.73\ \mathrm{m}$	**A1**	A1: AG, CSO
	Total	**12**	

Q	Answer	Marks	Comments
5(a)	$\frac{10 \times 1.5^2}{2 \times 2.5} = 0.4 \times 9.8 \times (4-d) + \frac{10 \times (d-2.5)^2}{2 \times 2.5}$ $4.5 = 15.68 - 3.92d + 2d^2 - 10d + 12.5$ $2d^2 - 13.92d + 23.68 = 0$ $d = 4 \text{ or } 2.96$ $d = 2.96 \text{ m}$	**M1** **A1** **A1** **M1** **A1**	M1: Four term energy equation. A1: Three terms correct. A1: Correct equation. M1: Simplified quadratic. A1: Correct distance.
5 (b)	$0.4 \times 9.8 = \frac{10e}{2.5}$ $e = 0.98 \text{ m}$	**M1** **A1** **A1**	M1: Equation for extension. A1: Correct equation. A1: Correct extension.
5 (c)	Let x be the displacement of the sphere from the equilibrium position $0.4\frac{d^2x}{dt^2} = 0.4g - T$ $T = \frac{10}{2.5}(0.98 + x)$ $0.4\frac{d^2x}{dt^2} = 0.4g - 3.92 - 4x$ $\frac{d^2x}{dt^2} = -10x$ $\therefore$SHM	**M1** **A1** **M1** **M1** **A1**	M1: Three term differential equation. A1: Correct equation. M1: makes second derivative the subject. M1: Simplified to $\pm kx$ A1: Concludes SHM with correct value of *k*.
5 (d)	$\omega = \sqrt{10}$ $\text{Period} = \frac{2\pi}{\sqrt{10}} = 2.0 \text{ seconds}$	**M1** **A1**	M1: Uses their ω to find period. A1: Correct period.
	Total	**15**	

Q	Answer	Marks	Comments
6(a)	At top of circle $T = 0$ $\frac{0.2v^2}{0.8} = 0.2 \times 9.8$ $v^2 = 7.84$	**M1** **A1**	M1: Newton's second law applied at the top of the circle. A1: Correct speed at top of the circle.
	$\frac{1}{2} \times 0.2u^2 = 0.2 \times 9.8 \times 1.6 + \frac{1}{2} \times 0.2 \times 7.84$ $u = \sqrt{39.2} = 6.3 \text{ m s}^{-1}$	**M1** **A1**	M1: Uses conservation of energy. A1: Obtains correct speed from correct working.
6(b)	$0.2g \cos 30° = \frac{0.2v^2}{0.8}$	**M1**	M1: Newton's second law applied at 30°.
	$v^2 = 7.84 \cos 30°$	**A1**	A1: Correct equation.
	$\frac{1}{2} \times 0.2u^2$ $= 0.2 \times 9.8 \times 0.8(1 + \cos 30°) + \frac{1}{2} \times 0.2 \times 7.84 \cos 30°$ $u = \sqrt{36.04 \ldots} = 6.0 \text{ m s}^{-1}$	**A1** **M1** **A1** **A1**	A1: Correct speed. M1: Uses conservation of energy. A1: Correct equation. A1: Obtains correct speed. AWRT 6.0
	Total	**10**	

Q	Answer	Marks	Comments
7(a)	$0 = V\sin\alpha\, t - \frac{1}{2}g\cos 30°\, t^2$ $t = \frac{2V\sin\alpha}{g\cos 30°}$ $OA = V\cos\alpha \times \frac{2V\sin\alpha}{g\cos 30°} + \frac{1}{2}g\sin 30° \times \left(\frac{2V\sin\alpha}{g\cos 30°}\right)^2$ $= \frac{2V^2\sin\alpha}{g\cos^2 30°}(\cos\alpha\cos 30° + \sin 30°\sin\alpha)$ $= \frac{2V^2}{g\cos^2 30°}\sin\alpha\cos(\alpha - 30°)$	**M1** **A1** **A1** **M1** **A1** **M1** **A1**	M1: Forms equation for motion perpendicular to the plane. A1: Correct equation. A1: Correct time. M1: Forms equation for motion parallel to the plane. A1: Correct equation. M1: Uses trig identity. A1: Obtains correct result from correct working.
7(b)	$\frac{\mathrm{d}(OA)}{\mathrm{d}\alpha} = \frac{2V^2}{g\cos^2 30°}$ $[\sin\alpha\sin(\alpha - 30) - \cos\alpha\cos(\alpha - 30)]$ $\sin\alpha\sin(\alpha - 30) - \cos\alpha\cos(\alpha - 30) = 0$ $\cos(2\alpha - 30) = 0$ $2\alpha - 30° = 90$ $\alpha = 60°$	**M1** **A1** **A1**	M1: Takes derivative of distance OA with respect to α A1: Obtains correct result. A1: Obtains correct angle.
	Total	**10**	

Q	Answer	Marks	Comments
8(a)	$2(3\mathbf{i}+2\mathbf{j})+4(-4\mathbf{i}-\mathbf{j})=2(-1.5\mathbf{i}-\mathbf{j})+4\mathbf{v}_B$ $4\mathbf{v}_B=-7\mathbf{i}+2\mathbf{j}$ $\mathbf{v}_B=-1.75\mathbf{i}+0.5\mathbf{j}$	**M1** **A1** **A1**	M1: Equation for conservation of momentum. A1: Correct equation. A1: Correct velocity.
8(b)	$\mathbf{I}=2(-1.5\mathbf{i}-\mathbf{j})-2(3\mathbf{i}+2\mathbf{j})$ $\mathbf{I}=-9\mathbf{i}-6\mathbf{j}$ $I=\sqrt{9^2+6^2}=\sqrt{117}=10.8\text{ Ns}$	**M1** **A1** **A1**	M1: Finding impulse with a pair of velocities. A1: Correct simplified vector. A1: Correct magnitude.
8(c)	$u_A=\frac{1}{\sqrt{117}}\begin{pmatrix}3\\2\end{pmatrix}\cdot\begin{pmatrix}9\\6\end{pmatrix}=\frac{39}{\sqrt{117}}$ $u_B=\frac{1}{\sqrt{117}}\begin{pmatrix}-4\\-1\end{pmatrix}\cdot\begin{pmatrix}9\\6\end{pmatrix}=\frac{-42}{\sqrt{117}}$ $v_A=\frac{1}{\sqrt{117}}\begin{pmatrix}-1.5\\-1\end{pmatrix}\cdot\begin{pmatrix}9\\6\end{pmatrix}=\frac{-19.5}{\sqrt{117}}$ $v_B=\frac{1}{\sqrt{117}}\begin{pmatrix}-1.75\\0.5\end{pmatrix}\cdot\begin{pmatrix}9\\6\end{pmatrix}=\frac{-12.75}{\sqrt{117}}$ $\frac{-12.75}{\sqrt{117}}-\left(\frac{-19.5}{\sqrt{117}}\right)=-e\left(\frac{-42}{\sqrt{117}}-\frac{39}{\sqrt{117}}\right)$ $6.75=81e$ $e=\frac{6.75}{81}=\frac{1}{12}$	**M1** **A1** **M1** **A1** **M1** **A1** **A1**	M1: Finds both components parallel to the line of centres before the collision. A1: Both correct. M1: Finds both components parallel to the line of centres after the collision. A1: Both correct. M1: Uses coefficient of restitution equation with their speeds. A1: Correct equation. A1: Correct coefficient of restitution.
	Total	**13**	