

Please write clearly in block capitals.

Centre number

Candidate number

Surname

Forename(s)

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I declare this is my own work.

INTERNATIONAL A-LEVEL

FURTHER MATHEMATICS

(9665/FM04) Unit FS2 Statistics

Wednesday 17 January 2024 07:00 GMT Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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6	
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8	
9	
10	
TOTAL	



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6

2

x	-1	2	5	7
$P(X = x)$	0.1	0.2	0.6	0.1

The moment generating function of X is $M_X(t)$

2 (a) Use differentiation to find the value of $M'_X(0)$

[3 marks]

[illegible]

Answer _____



2 (b) Use differentiation to find the value of $M_X''(0)$

[2 marks]

Answer _____

2 (c) Hence find $\text{Var}(X)$

[2 marks]

Answer _____

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4

She takes a random sample of 101 moths. For this sample the standard deviation for the lifetime of the moths is 5.6 days.

[1 mark]

[6 marks]

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[6 marks]

[illegible]

5

The number of errors made each day by the employees of a company can be modelled by a normal distribution.

Each employee participates in a training course designed to reduce the number of errors they make.

The company takes a random sample of seven employees and records the number of errors they each make on a randomly selected day before they have participated in the training course.

The company also records the number of errors the same seven employees each make on a randomly selected day after they have participated in the training course.

The results are given in the following table.

Employee	Number of errors	
	Before training	After training
1	12	4
2	21	5
3	8	8
4	7	6
5	9	12
6	10	9
7	13	10

Using the 5% level of significance, investigate whether the employees make fewer errors in a day after they have participated in the training course.

[9 marks]



[illegible]

$\frac{\quad}{9}$

- 6** Ayegbeni models the mass in kilograms of adult female African bush elephants using a normal distribution with standard deviation 300 kg

He takes a random sample of n African bush elephants and uses it to construct a 95% confidence interval of width 58.8 kg for the population mean mass of adult female African bush elephants.

- 6 (a)** Find the value of n

[3 marks]

Answer _____

- 6 (b)** Rashida constructs a 99% confidence interval for the population mean mass of adult female African bush elephants using Ayegbeni's data.

Find the width of Rashida's confidence interval.

Give your answer to three significant figures.

[3 marks]

Answer _____



6 (c) The sample mean of Ayegbeni's sample is 4450 kg

Using a 1% level of significance, Rashida conducts a hypothesis test with her confidence interval and the hypotheses

$$H_0 : \mu = 4500$$

$$H_1 : \mu \neq 4500$$

State with a reason whether Rashida rejects the null hypothesis.

[2 marks]

8

Turn over for the next question

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- 7** A hypothesis test is carried out to test for association between time of day and number of snacks eaten during the working hours of a large office.

The **expected** frequencies calculated for the test are given in the following table.

		Number of snacks eaten		
		0	1	2
Time of day	Morning	11.44	8.8	1.76
	Afternoon	14.56	11.2	2.24

- 7 (a)** State the hypotheses for the test.

[1 mark]

- 7 (b)** Explain why the number of degrees of freedom for the hypothesis test is equal to 1

[2 marks]

- 7 (c)** Write down the formula used to calculate the test statistic for this test.

[1 mark]



- 7 (d)** The test statistic is calculated correctly to be 5.05 given to three significant figures.
The hypothesis test is carried out using the 5% level of significance.
Explain whether the null hypothesis will be rejected.

[3 marks]

7**Turn over for the next question****Turn over ►**

- 8** The random variables $A, B, X_1, X_2, X_3, \dots, X_n$ are independent, unbiased estimators of the mean μ of a population and each have variance σ^2

- 8 (a)** Show that

$$R = \frac{1}{n+2} \left(A + B + \sum_{i=1}^n X_i \right)$$

is an unbiased estimator of μ

[3 marks]

- 8 (b)** Show that R is a consistent estimator of μ

[3 marks]



8 (c) The random variable

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

is also an unbiased estimator of μ

A student claims that \bar{X} is a more efficient estimator than R

By calculating the relative efficiency, determine whether the student is correct.

[4 marks]

[illegible]

10

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- 9** The random variable X has a normal distribution with population mean μ and population variance 10.24

A random sample of six observations is taken from X

The random sample is used to carry out a hypothesis test at the 2% level of significance with the hypotheses

$$H_0 : \mu = 8$$

$$H_1 : \mu \neq 8$$

- 9 (a)** Find the critical region of the hypothesis test.

Give the values in your answer to three decimal places.

[3 marks]

Answer _____



9 (b) Subsequently μ is found to be 11.4

Find the power of the hypothesis test.

Give your answer to two significant figures.

[2 marks]

Answer _____

5

Turn over for the next question

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The lifetime T billion years of a particular type of star is to be modelled by a normal distribution.

t	$t \leq 9.25$	$9.25 < t \leq 9.5$	$9.5 < t \leq 9.75$	$t > 9.75$
Frequency	6	22	13	9

$$\sum t = 475 \quad \text{and} \quad \sum (t - \bar{t})^2 = 1.96$$

[11 marks]

[illegible]

[illegible]

11



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