OXFORD AQA
INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

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Candidate signature _____

I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM01) Unit FP1 Pure Mathematics

Wednesday 3 January 2024 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 (a) (i) It is given that

$$w = (a + 3\mathrm{i})(2 - \mathrm{i})$$

where a is a real constant.

Express w in the form $u + \mathrm{i}v$ where u and v are real.

Give your answer in terms of a

[2 marks]

Answer _____

1 (a) (ii) Hence, or otherwise, express the complex number

$$\frac{a + 3\mathrm{i}}{2 + \mathrm{i}}$$

in the form $x + \mathrm{i}y$ where x and y are real.

Give your answer in its simplest form in terms of a

[2 marks]

Answer _____

Do not write outside the box

1 (b) The complex number z satisfies the equation

$$3z^* + \mathrm{i}z = 23 + 13\mathrm{i}$$

Find z

[5 marks]

$z =$ _____

9

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2 (a) Expand $(1+h)^5$

[1 mark]

Answer _____

2 (b) A curve has equation $y = x^5$

2 (b) (i) A line passes through two points on the curve, one where $x = 1$ and the other where $x = 1+h$ with $h > 0$

Find the gradient of this line in the form $a + bh + ch^2 + dh^3 + h^4$ where a, b, c and d are constants.

[3 marks]

Answer _____

Do not write outside the box

2 (b) (ii) Use your answer to **part (b)(i)** to find the gradient of the curve at the point where $x = 1$, showing the limiting process used.

[2 marks]

Answer _____

6

Turn over for the next question

Do not write outside the box

3 (a) Find the general solution of the equation

$$\cos\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2}$$

Give your answer in terms of π

[3 marks]

Answer _____

3 (b) Find the number of solutions of the equation

$$\cos\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2}$$

in the range $-m\pi < x \leq m\pi$ where m is a positive integer.

Give your answer in terms of m

[2 marks]

Answer _____

5

Do not write outside the box

4 By considering the derivative of $y = x^{\frac{1}{4}}$ when $x = 81$, find an estimate for $\sqrt[4]{75}$

Give your answer to three decimal places.

[6 marks]

Answer _____

6

Turn over ►

Do not write outside the box

5 (a) Show that

$$\sum_{r=1}^{n}\left(6r^2-4r+1\right)=n^2\left(an+b\right)$$

where a and b are integers.

[4 marks]

Do not write outside the box

5 (b) Hence show that

$$\sum_{r=p+1}^{2p}\left(6r^2-4r+1\right)=p^2\left(cp+d\right)$$

where c and d are integers.

[3 marks]

7

Turn over for the next question

Do not write outside the box

6 The complex numbers α and β are the roots of the quadratic equation

$$z^2 + bz + c = 0$$

where b and c are real constants.

6 (a) (i) Write down b and c in terms of α and β

[2 marks]

$b =$ _____ $c =$ _____

6 (a) (ii) It is given that $\alpha = x + \mathrm{i}y$ where x and y are real and non-zero.

Write down β in terms of x and y

[1 mark]

Answer _____

6 (b) In the case when $b = 6$, the roots α and β are represented by the points P and Q on an Argand diagram.

The number 8 is represented by the point R on the same Argand diagram.

The area of triangle PQR is $11\sqrt{3}$

6 (b) (i) Find α and β

[4 marks]

Do not write outside the box

Answer _____ and _____

6 (b) (ii) Hence find the value of c

[2 marks]

Answer _____

6 (b) (iii) Express α and β in the form $r(\cos\theta + i\sin\theta)$ where $r \geq 0$ and $-\pi < \theta \leq \pi$

[4 marks]

Answer _____ and _____

13

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7 A curve C has equation

$$y = \frac{2}{x^2 - 3}$$

7 (a) Find the equations of the three asymptotes of C

[3 marks]

Asymptote 1 _____

Asymptote 2 _____

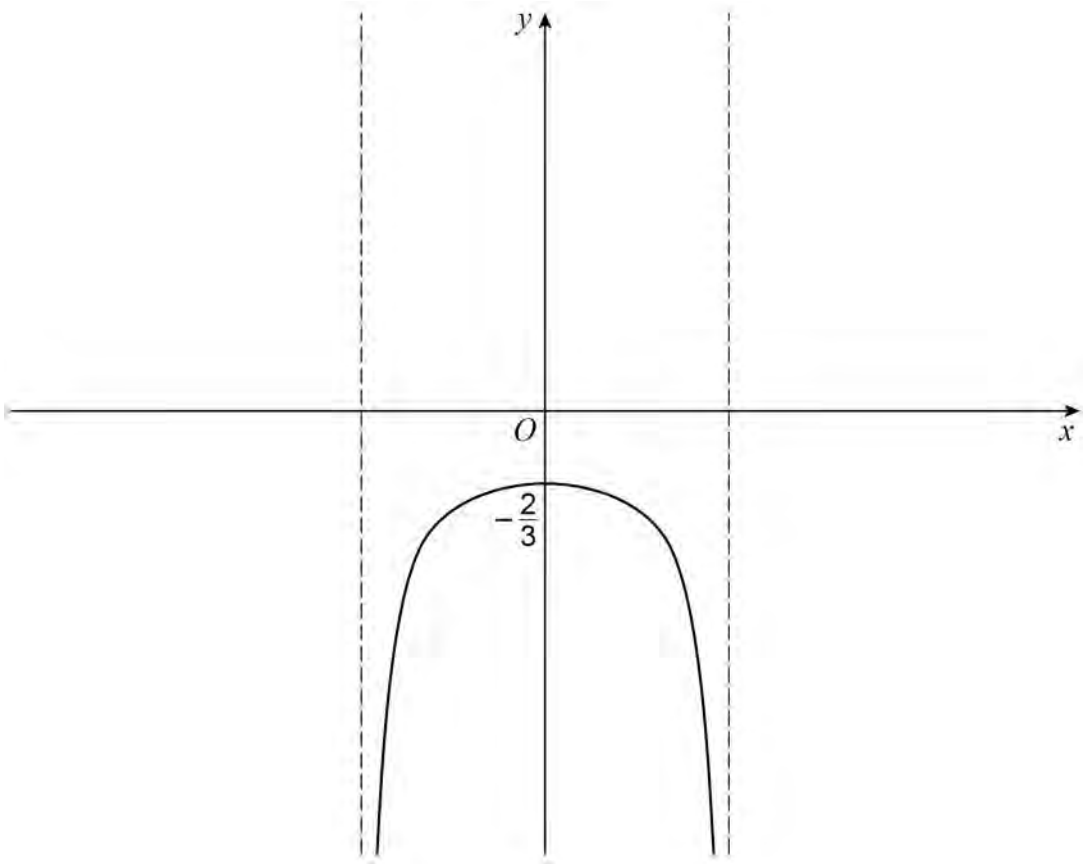
Asymptote 3 _____

7 (b) One section of the graph of C and its vertical asymptotes are shown below.

The y-intercept shown below is the only stationary point of C

On the axes below, complete the sketch of the graph of C

[2 marks]



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7 (c) The point $(2, 2)$ is a point of intersection of C with the line $y = x$

Find the coordinates of the other point where the line $y = x$ meets C

[3 marks]

Answer _____

7 (d) Hence sketch the line $y = x$ on the axes in **part (b)**.

[1 mark]

7 (e) Hence, or otherwise, solve the inequality $\frac{2}{x^2 - 3} \geq x$

[3 marks]

Answer _____

12

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8 The integral I_n is defined by

$$I_n = \int_0^4 x^n \, \mathrm{d}x$$

where n is a constant.

8 (a) Explain why I_n is an improper integral when $n < 0$

[1 mark]

8 (b) Find the exact value of I_n when $n = -\frac{3}{4}$, showing the limiting process.

[3 marks]

Answer _____

Do not write outside the box

8 (c) Write down a value of n for which I_n does **not** have a finite value.

[1 mark]

$n =$ _____

5

Turn over for the next question

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9 The locus of a point P is such that the distance from P to the point $(4, 0)$ is twice the distance from P to the line with equation $x = 1$

The locus of P is the curve C_1

9 (a) Show that the equation of C_1 is

$$\frac{x^2}{m} - \frac{y^2}{n} = 1$$

where m and n are constants.

[4 marks]

9 (b) Write down the equations of the asymptotes of C_1

Give your answers in the form $y = \mathrm{f}(x)$

[2 marks]

Answer _____

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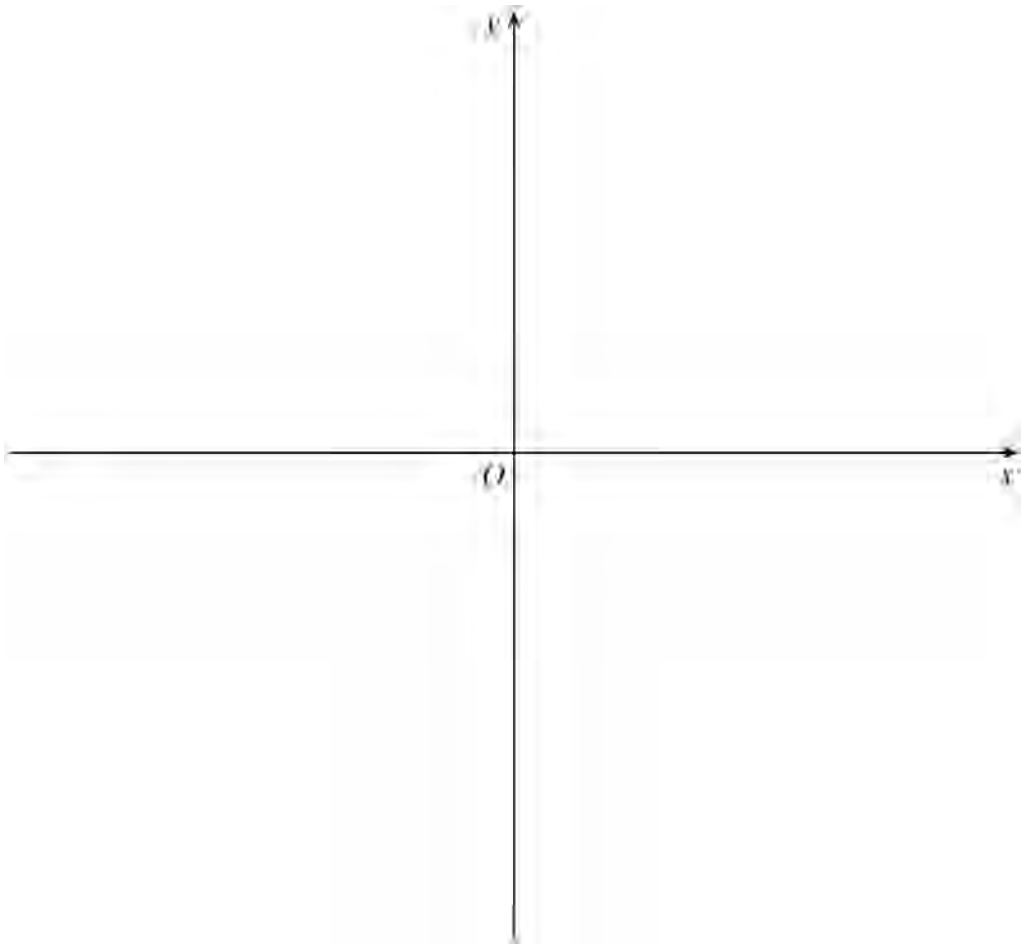
9 (c) Sketch C_1 on the axes below.

Include the asymptotes and label the axis intercepts.

[2 marks]



9 (d) The locus of a point Q is such that the distance from Q to the point $(4, c)$ is twice the distance from Q to the line with equation $x = 1$

The locus of Q is the curve C_2

Write down the equations of the asymptotes of C_2

Give your answer in terms of c

[2 marks]

Answer _____

10

Turn over ►

Do not write outside the box

10 The quadratic equation $2x^2 + x + m = 0$ has roots α and β

The quadratic equation $3x^2 + nx + m = 0$ has roots $\alpha^2\beta$ and $\beta^2\alpha$

The constants m and n are both positive.

10 (a) Find the exact value of m

[3 marks]

Answer _____

Do not write outside the box

10 (b) Find the exact value of n

[4 marks]

Answer _____

7

END OF QUESTIONS

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