OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM05) Unit FM2 Mechanics

Wednesday 19 January 2022 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to two significant figures, unless stated otherwise.
- Unless stated otherwise, the acceleration due to gravity, g, should be taken as 9.8 m s^{-2}

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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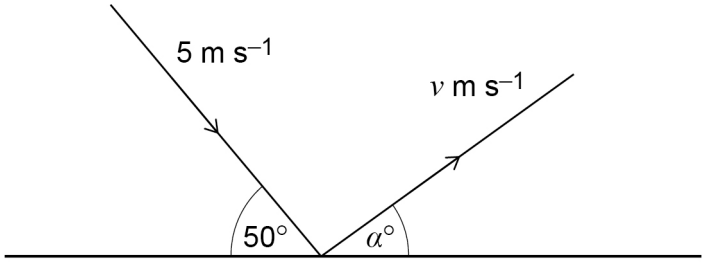
Answer **all** questions in the spaces provided.

1 A disc, of mass 0.3 kg, is sliding on a smooth horizontal surface when it collides with a smooth fixed vertical wall.

When the disc collides with the wall it has velocity $5\ \text{m s}^{-1}$ at an angle of 50° to the wall.

When the disc leaves the wall it has velocity $v\ \text{m s}^{-1}$ at an angle of α degrees to the wall.

The velocities of the disc are shown in the diagram below.



The coefficient of restitution between the disc and the wall is 0.8

1 (a) Find the value of v giving your answer correct to two decimal places.

[5 marks]

Do not write outside the box

Answer _____

1 (b) Find the value of α giving your answer to the nearest integer.

[2 marks]

Answer _____

1 (c) Find the magnitude of the impulse exerted on the disc by the wall.

Give your answer to two significant figures and clearly state the units of your answer.

[3 marks]

Answer _____ Units _____

10

Turn over ►

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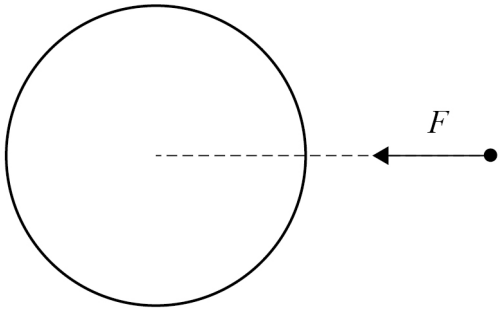
2 A particle of mass 4 kg is released from rest at a distance of 0.02 metres from the centre of a fixed sphere of radius 0.01 metres.

A force, F newtons, acts on the particle and is directed towards the centre of the sphere.

The magnitude of F is $\frac{8}{x^2}$ when the particle is x metres from the centre of the sphere.

Assume that F is the only force that acts on the particle.



2 (a) Show that the work done by the force F newtons as the particle moves from its initial position to the surface of the sphere is 400 joules.

[4 marks]

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2 (b) Find the speed of the particle when it reaches the surface of the sphere, giving your answer to three significant figures.

[2 marks]

Answer _____

6

Turn over for the next question

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3 A particle moves with simple harmonic motion on a straight line between the points P and Q which are 3 metres apart.

The period of the motion is 5 seconds.

3 (a) Find the maximum speed of the particle, giving your answer in terms of π

[2 marks]

Answer _____

3 (b) Find the speed of the particle when it is 1 metre from P giving your answer in exact form in terms of π

[3 marks]

Answer _____

Do not write outside the box

3 (c) The point O is the mid-point of PQ

The particle starts from rest at P

3 (c) (i) Write down an expression for the displacement of the particle from O at time t seconds after leaving P

[1 mark]

Answer _____

3 (c) (ii) By using calculus with your answer to **part (c)(i)**, find the maximum acceleration of the particle giving your answer in terms of π

[3 marks]

Answer _____

9

Turn over for the next question

Do not write outside the box

4 A simple pendulum consists of a string of length 0.7 metres and a small sphere.

4 (a) (i) Show that, for small oscillations, the pendulum moves with approximately simple harmonic motion.

[5 marks]

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4 (a) (ii) State two assumptions that you have made about the properties of the string.

[2 marks]

Assumption 1 _____

Assumption 2 _____

4 (b) Find the period of the motion of the pendulum for small oscillations, giving your answer in terms of π

[1 mark]

Answer _____

8

Turn over ►

Do not write outside the box

5 A particle of mass 2.5 kg moves in a straight line on a smooth horizontal surface.

The speed of the particle is v m s^{-1}

A resistance force of magnitude $10v^2$ newtons acts on the particle.

The speed of the particle is 20 m s^{-1} when it is at the origin O

Find the distance the particle has travelled from O when its speed is 10 m s^{-1}

Give your answer in the form $\frac{1}{b}\ln a$ where a and b are integers.

[5 marks]

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Answer _____

5

Turn over for the next question

Do not write outside the box

6 A particle of mass 2 kg is attached to one end of a spring of stiffness k Nm^{-1}

The other end of the spring is attached to a fixed point *O*

6 (a) Find, in terms of g and k, the extension of the spring when the particle hangs in equilibrium below *O*

[2 marks]

Answer _____

6 (b) The particle is pulled down and released from rest at a point vertically below the equilibrium position.

6 (b) (i) Show that the subsequent motion of the particle is simple harmonic motion.

[5 marks]

Do not write outside the box

6 (b) (ii) The particle is released from rest at a point which is a distance $2a$ metres vertically below the equilibrium position.

The particle then takes $\frac{\pi}{8}$ seconds to move directly to a point $\frac{3a}{2}$ metres vertically below the equilibrium position.

Find the value of k

[4 marks]

Answer _____

11

7 A particle of mass m kg is attached to one end of a light inextensible string of length $2a$ metres.

The other end of the string is attached to a fixed point O

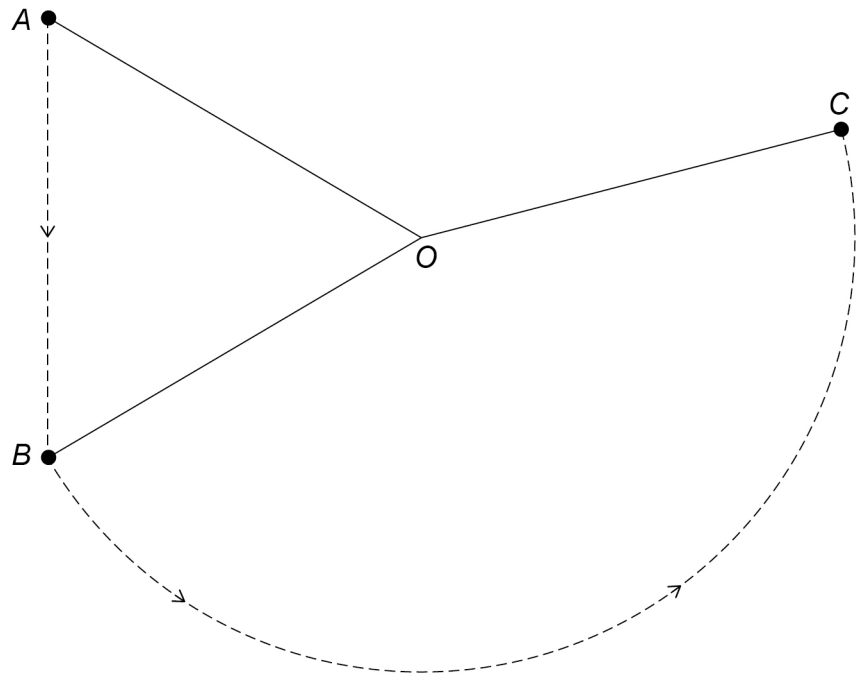
The points A, B and O lie in the same vertical plane and are the vertices of an equilateral triangle.

The particle is released from rest at the point A where the string is taut.

The particle falls vertically until the string becomes taut again at the point B

Assume that the motion of the particle changes instantaneously at B from vertical motion to circular motion.

From B the particle moves on the arc of a circle until the string becomes slack again at the point C as shown in the diagram below.



7 (a) Find, in terms of a and g, the speed of the particle at B

[2 marks]

Answer _____

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7 (b) At B an impulse exerted on the particle by the string changes the motion of the particle, so that its velocity is perpendicular to OB

Show that this impulse changes the speed of the particle to $\sqrt{3ag}$ and find the magnitude of the impulse.

[5 marks]

Answer _____

Question 7 continues on the next page

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7 (c) Find the acute angle between OC and the vertical, giving your answer to the nearest degree.

[6 marks]

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Answer _____

13

Turn over for the next question

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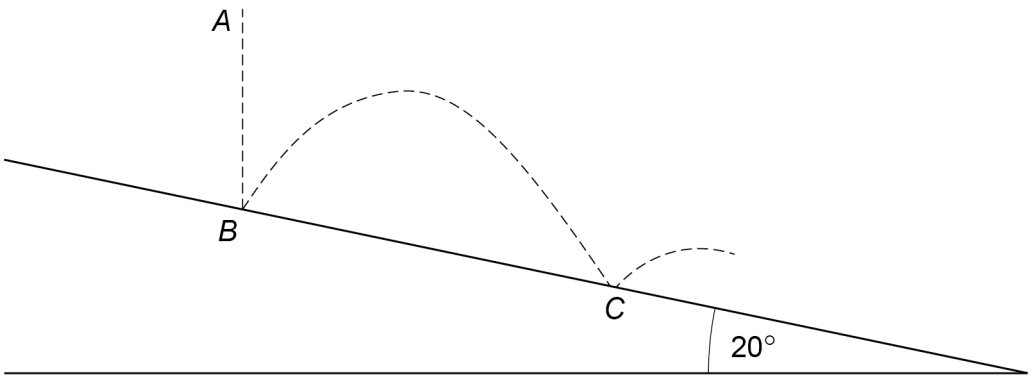
8 A smooth plane is inclined at 20° to the horizontal.

A ball is released from rest from the point A and falls a distance of 3 metres vertically before bouncing on the plane at the point B

The ball next bounces on the plane at the point C

The plane, the points and the motion of the ball are shown in the diagram below.



The coefficient of restitution between the ball and the plane is 0.6

The line BC is a line of greatest slope of the plane.

Find the distance BC

[9 marks]

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Answer _____

9

Turn over ►

Do not write outside the box

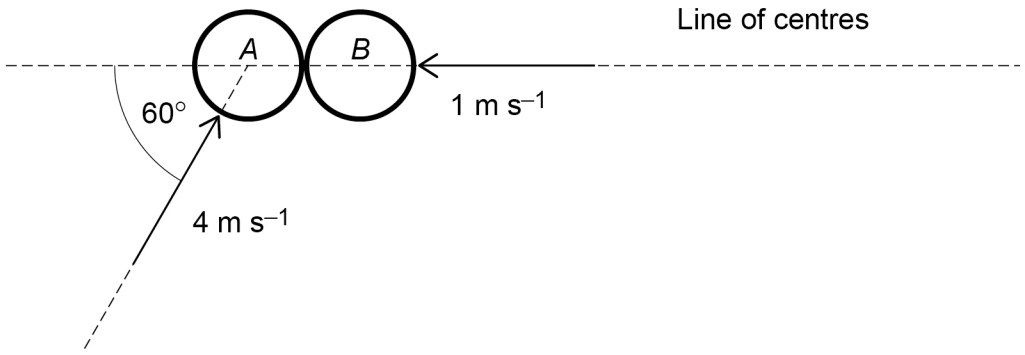
9 Two smooth spheres, A and B, with the same radius are moving on a horizontal surface when they collide.

The mass of A is 2 kg and the mass of B is 3 kg

As the spheres collide:

- sphere A is moving at $4\ \text{m s}^{-1}$ at an angle of 60° to the line of centres.
- sphere B is moving at $1\ \text{m s}^{-1}$ along the line of centres.

The velocities of the spheres as they collide are shown in the diagram below.



The coefficient of restitution between the two spheres is e

The direction of motion of A is changed by an angle α degrees as a result of the collision.

Find the range of possible values of α

[9 marks]

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Answer _____

9

END OF QUESTIONS

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