OXFORD AQA
INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

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I declare this is my own work.

INTERNATIONAL A-LEVEL
MATHEMATICS

(9660/MA03) Unit P2 Pure Mathematics

Friday 12 January 2024 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
TOTAL	

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Answer **all** questions in the spaces provided.

1 The polynomial $f(x)$ is defined by

$$f(x) = 16x^3 + bx^2 + cx + 12$$

where b and c are constants.

When $f(x)$ is divided by $(4x+1)$ the remainder is 11.5

When $f(x)$ is divided by $(2x-1)$ the remainder is 17.5

Find the value of b and the value of c

[4 marks]

$b =$ _____ $c =$ _____

4

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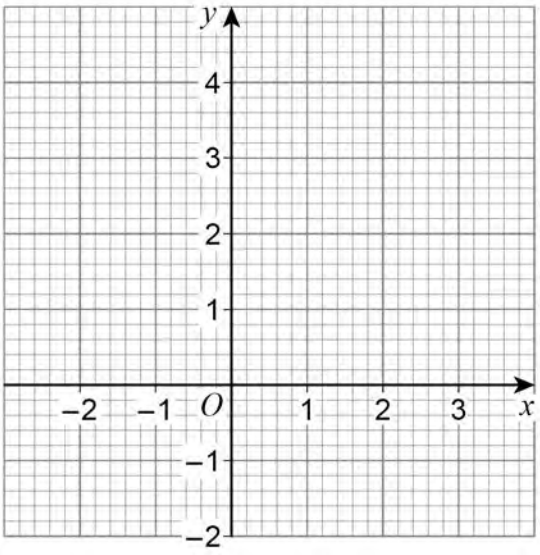
2 (a) On the axes below, draw the graph of

$$y = |2x - 1| - 1 \quad \text{for} \quad -2 \leq x \leq 3$$

[2 marks]



2 (b) Solve the inequality

$$|2x - 1| - 1 \geq |x|$$

[2 marks]

Answer _____

4

Turn over ►

Do not write outside the box

3 (a) For each of the following find $\frac{\mathrm{d}y}{\mathrm{d}x}$

3 (a) (i) $y = \mathrm{e}^{-0.5x} \sin 3x$

[2 marks]

$\frac{\mathrm{d}y}{\mathrm{d}x} =$

3 (a) (ii) $y = \frac{(1-2x)^3}{3+\tan 5x}$

[3 marks]

$\frac{\mathrm{d}y}{\mathrm{d}x} =$

Do not write outside the box

3 (a) (iii) $x + \ln(xy) = x^3 + y^2$

[4 marks]

$$\frac{dy}{dx} =$$

Question 3 continues on the next page

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3 (b) (i) Find $\int \frac{x}{4x^2+5}\,\mathrm{d}x$

[2 marks]

Answer _____

Do not write outside the box

3 (b) (ii) Find $\int_0^{\frac{\pi}{2}} x \cos x \, \mathrm{d}x$

Give your answer in an exact form.

[4 marks]

Answer _____

15

Turn over for the next question

Do not write outside the box

4 (a) Use the mid-ordinate rule with 5 strips, to find an estimate for

$$\int_0^{0.6} \left(4^{-x} - \frac{1}{4}\right) \mathrm{d}x$$

Give your answer to four decimal places.

[4 marks]

Answer _____

Do not write outside the box

4 (b) The function f is defined by

$$\mathrm{f}(x) = 4^{-x} - \frac{1}{4} \qquad \text{for } x \geq 0$$

The inverse of f is f^{-1}

4 (b) (i) Find $\mathrm{f}^{-1}(x)$

[3 marks]

Answer _____

4 (b) (ii) Find the domain of f^{-1}

[2 marks]

Answer _____

4 (c) Describe the **single** geometrical transformation that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}^{-1}(x)$

[1 mark]

10

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5 (a) (i) Find R and α such that $10\sin\theta - 24\cos\theta = R\sin(\theta - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give your value of α in radians to three significant figures.

[3 marks]

Answer _____

5 (a) (ii) Write down the minimum value of $10\sin\theta - 24\cos\theta$

[1 mark]

Answer _____

5 (a) (iii) Find the value of θ in the interval $2\pi < \theta < 4\pi$ at which the minimum value of $10\sin\theta - 24\cos\theta$ occurs.

Give your answer to two decimal places.

[1 mark]

Answer _____

Do not write outside the box

5 (a) (iv) Solve the equation

$$10\sin(x-0.6)-24\cos(x-0.6)=6.5 \quad \text{for} \quad -\pi < x < \pi$$

Give all values of x to two decimal places.

[3 marks]

Answer _____

5 (b) Solve the equation

$$16\tan^2(2y-10°)-14=4\sec(2y-10°) \quad \text{for} \quad -90° < y < 90°$$

Give all values of y to the nearest degree.

[5 marks]

Answer _____

13

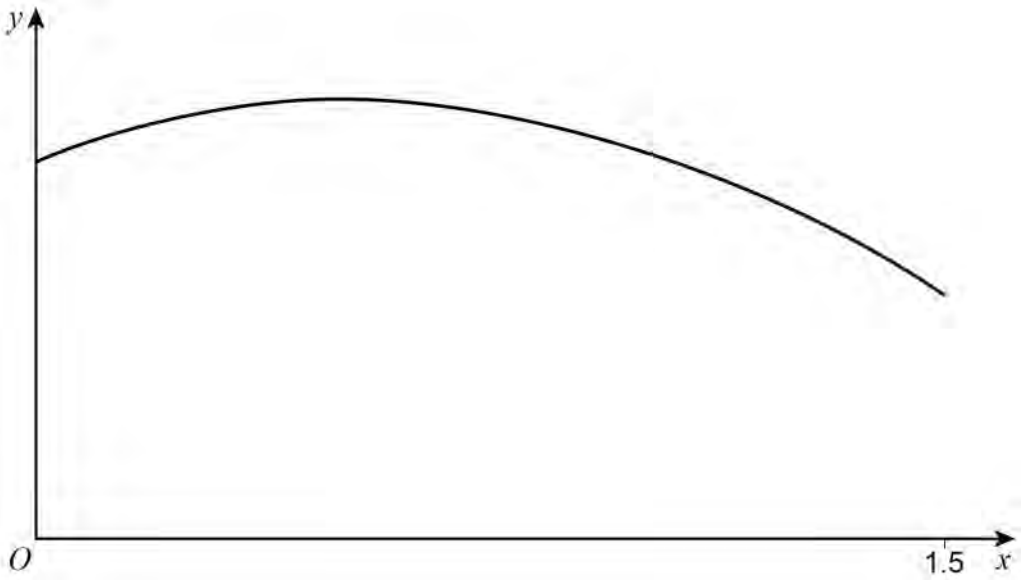
Turn over ►

Do not write outside the box

6 The function f is defined by

$$\mathrm{f}(x) = -x^2 + \ln(12 + 24x) \quad \text{for} \quad 0 \leq x \leq 1.5$$

The graph of $y = \mathrm{f}(x)$ is shown below.



6 (a) Find the range of f

Give your answer in an exact form.

[5 marks]

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Answer _____

Question 6 continues on the next page

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6 (b) The graph of $y = f(x)$ intersects the graph of $y = 2x$ at the point where $x = \alpha$

6 (b) (i) Show that α lies between 1.1 and 1.2

[2 marks]

6 (b) (ii) Show that the equation $-x^2 + \ln(12 + 24x) = 2x$ can be rearranged into the form

$$x = -1 + \sqrt{1 + \ln(12 + 24x)}$$

[1 mark]

6 (b) (iii) Use the iterative formula

$$x_{n+1} = -1 + \sqrt{1 + \ln(12 + 24x_n)}$$

with $x_1 = 1.1$ to find the value of x_2 and the value of x_3

Give your answers to three decimal places.

[2 marks]

$x_2 =$ _____ $x_3 =$ _____

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6 (c) (i) Describe the **single** geometrical transformation that maps the graph of $y = -x^2 + \ln(12 + 24x)$ onto the graph of $y = -x^2 + \ln(1 + 2x)$

[2 marks]

6 (c) (ii) It is given that

$$\int_0^{1.5} \left(-x^2 + \ln(12 + 24x)\right) \mathrm{d}x = A$$

where A is a constant.

Find, in terms of A, the exact value of $\int_0^{1.5} \left(-x^2 + \ln(1 + 2x)\right) \mathrm{d}x$

[1 mark]

Answer _____

13

Turn over ►

Do not write outside the box

7 (a) The curve C_1 has Cartesian equation

$$x^2 - y^2 = 6y - 2x + 20$$

Find an equation of the tangent to C_1 at the point $(3, -1)$

[4 marks]

Answer _____

7 (b) The curve C_2 has parametric equations

$$x = \frac{1 + \sqrt{17}\cos\theta}{2} \quad \text{and} \quad y = -1 + \sqrt{17}\sin\theta \quad \text{for} \quad 0 \leq \theta \leq 2\pi$$

7 (b) (i) Find a Cartesian equation of C_2

[2 marks]

Answer _____

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7 (b) (ii) Find an equation of the normal to C_2 at the point where $\theta = \cos^{-1}\left(\frac{1}{\sqrt{17}}\right)$

[4 marks]

Answer

7 (c) The tangent found in **part (a)** intersects the coordinate axes at the points A and B

The normal found in **part (b)(ii)** intersects the coordinate axes at the points P and Q

Find the ratio

Area of triangle OAB : Area of triangle OPQ

where O is the origin of the coordinate axes.

[1 mark]

Answer

11

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8 The region bounded by the curve $y = \frac{1}{10 - 2x}$, the line $y = 1$ and the y-axis from $y = 0.1$ to $y = 1$ is rotated through 2π radians about the ***y*-axis** to form a solid.

Find the volume of the solid generated.

Give your answer in an exact form.

[7 marks]

Do not write outside the box

Answer _____

7

Turn over for the next question

Do not write outside the box

9 (a) The gradient of a curve at the point (x, y) is directly proportional to the sum of 3 times the x-coordinate and 4 times the y-coordinate.

Construct a differential equation for the curve.

[2 marks]

Answer _____

9 (b) (i) Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{e}^{2y} = 4x\mathrm{e}^{2y} \quad \text{for} \quad -0.6 < x < 1.1$$

such that $y = 0$ when $x = 1$

Give your answer in the form $y = \mathrm{f}(x)$

[6 marks]

Do not write outside the box

Answer _____

9 (b) (ii) Find a second value for x when $y = 0$ for the solution $y = f(x)$ found in **part (b)(i)**.

[2 marks]

Answer _____

10

Turn over ►

Do not write outside the box

10 (a) Find the values of A, B and C such that

$$\frac{x^2}{(3-x)(3+2x)(3-2x)} = \frac{A}{3-x} + \frac{B}{3+2x} + \frac{C}{3-2x}$$

[4 marks]

$A =$ _____ $B =$ _____ $C =$ _____

10 (b) (i) Find the binomial expansion of $(3-x)^{-1}$ up to and including the term in x^2

[2 marks]

Answer _____

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10 (b) (ii) State the values of x for which the full binomial expansion of $(3-x)^{-1}$ is valid.

[1 mark]

Answer _____

10 (c) Use your answers to **parts (a)** and **(b)(i)** to show that for small values of x

$$\frac{x^2}{(3-x)(3+2x)(3-2x)} = Dx^2$$

where D is a rational number.

[5 marks]

12

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11 Use the substitution $u^2 = 9 - x^2$ to find

$$\int_0^1 \frac{x^3}{\sqrt{(9 - x^2)}}\, \mathrm{d}x$$

Give your answer in the form $a + b\sqrt{2}$ where a and b are constants.

[7 marks]

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Answer _____

7

Turn over for the next question

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12 The points A, B and C have coordinates $(4, 2, 3)$, $(-2, 6, 15)$ and $(-1, 10, 6)$ respectively.

12 (a) Show that for all values of p, the point $P(-3p - 2,\ 2p + 6,\ 6p + 15)$ lies on the line passing through A and B

[4 marks]

12 (b) (i) Find the value of p for which CP is perpendicular to AB

[4 marks]

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Answer

12 (b) (ii) Hence find the area of triangle *ABC*

[4 marks]

Answer

12 (c) Find the angle *BAC*

[2 marks]

Answer

14

END OF QUESTIONS

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