INTERNATIONAL A-LEVEL

MATHEMATICS

MA04

(9660/MA04) Unit S2 Statistics

Mark scheme

June 2022

Version 1.0 Final

226XMA04/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from oxfordaqaexams.org.uk

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	A random variable	E1	
	that is a function of known observations from a population	E1	oe
		2	
1(b)	Range of values [of the test statistic]	E1	oe
	that leads us to determine whether or not the null hypothesis is to be rejected or not	E1	oe
		2	
	Total	4	

Q	Answer	Marks	Comments
2(a)	$\lambda = 0.1$	B1	
	$\mathrm{P}(T < 5) = 1 - \mathrm{e}^{-0.1\times 5}$	M1	**ft** their value for λ
	$= 0.3935$	A1	**AWRT** 0.3935
		3	
2(b)	$\mathrm{P}(8 < T < 14)$ $= \left(1 - \mathrm{e}^{-0.1\times 14}\right) - \left(1 - \mathrm{e}^{-0.1\times 8}\right)$ $[= 0.75340... - 0.55067...]$	M1	Difference between two probabilities with at least one correct probability
	$= 0.2027$	A1	**AWRT** 0.2027
		2	
	Total	5	

Q	Answer	Marks	Comments
3(a)	$\int_0^a \frac{4}{(2x+1)^2}\ \mathrm{d}x = 1$	M1	Correct definite integral set equal to 1 Condone missing $\mathrm{d}x$
	$\left[\frac{4}{2\times -1}(2x+1)^{-1}\right]_0^a = 1$	A1	Correct integration **oe**
	$\frac{-2}{2a+1} - (-2) = 1$	M1	**ft** from their integration with correct limits substituted
	$a = 0.5$	A1	**AG** Be convinced
		4	
3(b)	$\mathrm{P}(0.2 < X < 0.4) = \int_{0.2}^{0.4} \frac{4}{(2x+1)^2}\ \mathrm{d}x$	M1	Correct definite integral Condone missing $\mathrm{d}x$ **PI**
	$= \left[\frac{4}{2\times -1}(2x+1)^{-1}\right]_{0.2}^{0.4}$		
	$= (-2)\times(2\times 0.4+1)^{-1} - (-2)\times(2\times 0.2+1)^{-1}$	M1	Applies limits to their integration from **part (a)** **PI**
	$= \frac{20}{63}$	A1	**oe, AWRT** 0.317
		3	
	Total	**7**	

Q	Answer	Marks	Comments
4	H_0: $\mu = 16$ H_1: $\mu < 16$	B1	Both hypotheses
	$\bar{x} = 15.5$	B1	**PI** by correct calculation for z
	$s^2 = \dfrac{1}{200-1}\left(50300 - \dfrac{3100^2}{200}\right)$	M1	Attempt at variance formula Allow one slip **PI** by correct answer
	$= 11.3[0653266]$	A1	**AWRT** $\left(\dfrac{2250}{199}\right)$ Accept $s = 3.36[2518297]$
	$\bar{X} \sim \mathrm{N}\left(16,\ \dfrac{11.3}{200}\right)$	M1	$\bar{X} \sim \mathrm{N}\left(16,\ \dfrac{s^2}{200}\right)$ **PI**
	$z = \dfrac{15.5-16}{\sqrt{\dfrac{11.3}{200}}}$	M1	Calculates z with their s^2
	$= -2.1[0351581]$	A1	**AWRT** -2.1
	$z_{\text{critical}} = -2.3263$	B1	**AWRT** -2.3 or $\mathrm{P}(z < -2.1) = 0.0177$ Accept 2.3
	As $z > z_{\text{critical}}$ we fail to reject H_0	A1ft	Follow through their z and z_{critical} provided signs are consistent or comparing 0.0177 to 1%
	Evidence to suggest that the laptop battery time between charges has not decreased	E1	Must be consistent with their conclusion on whether to fail to reject H_0 or not or their z and z_{critical} if not explicitly stated
	Total	**10**	

Q	Answer	Marks	Comments
5(a)	Symmetrical	**B1**	**oe** such as 'bell-shaped curve' or 'no skew'
	Mode = Mean = Median	**B1**	Accept » instead of =
	95% of the data lies within 2 sd of the mean	**B1**	Accept similar accurate comment eg 99% P 3 sd 68% P 1 sd
		3	
5(b)(i)	$\text{P}(X > 17) = \text{P}\left(Z > \frac{17-14}{1.2}\right)$	**M1**	**PI** Standardises
	$= \text{P}(Z < -2.5)$	**M1**	**PI** by sight or use of 0.99379…
	$= 1 - 0.99379[03346]$		
	$= 0.0062$	**A1**	**AWRT** 0.0062
		3	
5(b)(ii)	$\text{Var}(\bar{X}) = \frac{1.2^2}{50}$ or $\text{sd} = \frac{1.2}{\sqrt{50}}$	**B1**	**PI** Accept $\text{Var}(\bar{X}) = 0.0288$ or **AWRT** $\text{sd} = 0.17$
	$\text{P}(\bar{X} < 13.8) = \text{P}\left(Z < \frac{13.8-14}{\sqrt{0.0288}}\right)$	**M1**	**PI** Standardises using their $\text{Var}(\bar{X})$ but not 1.2^2
	$= \text{P}(Z < -1.1785)$		
	$= 1 - 0.8807[035853]$	**m1**	**PI** by correct answer
	$= 0.119$	**A1**	**AWRT** 0.119
		4	
5(b)(iii)	$X + Y \sim \text{N}(44,\ 1.2^2 + 4^2)$	**B1**	**PI**
	$\text{P}(X + Y < 35) = \text{P}\left(Z < \frac{35-44}{\sqrt{17.44}}\right)$	**M1**	**PI** Standardises with their sum of means and their sum of variances
	$= \text{P}(Z < -2.1551)$		
	$= 1 - 0.9844[233527]$	**m1**	**PI** by correct answer
	$= 0.0156$	**A1**	**AWFW** 0.01539 to 0.01578
		4	
	Total	**14**	

Q	Answer	Marks	Comments
6(a)	$\text{P}(X=3)=\dfrac{e^{-4}\times 4^3}{3!}$	M1	**PI** May use tables: $0.4335-0.2381$
	$=0.195$	A1	**AWRT** 0.195
		2	
6(b)	$\lambda = 2\times(4+2.5)$	M1	**PI**
	$\lambda = 13$	A1	
	H_0: $\lambda = 13$ H_1: $\lambda < 13$	B1	Both hypotheses Allow H_0: $\lambda = 6.5$ H_1: $\lambda < 6.5$
	$\text{P}(X \le 7)$	M1	Attempts $\text{P}(X \le 7)$ or $\text{P}(X < 7)$
	$\text{P}(X \le 7)=0.054$	A1	**AWRT** 0.054
	$0.054 > 0.05$	M1	Compares their probability with 0.05
	Do not reject H_0	A1ft	**ft** their probability compared with 0.05 Implied by correct conclusion in context
	Evidence to suggest that there has not been a reduction in the total number of breakdowns of boats and buses	E1	Must be consistent with their conclusion on whether or not to reject H_0 or on their probability if not explicitly stated
		8	
	Total	10	

Q	Answer	Marks	Comments
7(a)	$X \sim \text{B}(100,\ 0.03)$		
	$100 \times 0.03 = 3$	B1	**PI** by use of $\lambda = 3$ with Poisson distribution
	$Y \sim \text{Po}(3)$	B1	Identifies correct approximate distribution
	$\text{P}(Y \le 3)$	M1	Identifies correct probability
	$= 0.647$	A1	**AWRT** 0.647
		4	
7(b)(i)	H_0: $p = 0.03$ H_1: $p \ne 0.03$	B1	Both hypotheses
	$X \sim \text{B}(20,\ 0.03)$	M1	**PI** by a binomial probability calculation
	$\text{P}(X \ge 3) = 0.021$	A1	**AWRT** 0.021
	$0.021 < 0.025$	M1	Compares their probability with 0.025
	Reject H_0	A1ft	Follow through their probability Implied by correct conclusion in context
	Evidence to suggest that the proportion of viewers watching the local news programme has changed	E1	Must be in context, must not be definite and all the previous 5 marks must have been awarded.
		6	
7(b)(ii)	A Type I error means to reject that the proportion of viewers watching the local news programme is 3% when it is 3%	E2	**oe** **E1** for describing Type I error without context
		2	
7(b)(iii)	$\text{P}(X \ge 3) = 0.021 \quad [< 0.025]$ $\text{P}(X \ge 2) = 0.1198 \quad [> 0.025]$	M1	Considers both probabilities
	$\text{P}(X \le x) < 0.025$	M1	**PI** by calculation $\text{P}(X = 0)$ or $\text{P}(X \le 0)$
	$\text{P}(X = 0) = 0.5438 \quad [> 0.025]$		
	$\text{P}(\text{Type I error}) = 0.021$	A1	**oe**
		3	
	Total	**15**	

Q	Answer	Marks	Comments
8(a)	$\frac{0.4}{2}x$ or $0.2x$	B1	Seen anywhere
	$y-0.4=\frac{0.6}{4}(x-2)$ or $y-1=\frac{0.6}{4}(x-6)$	M1	Use of straight line methods to find second line Allow $y = mx + c$ methods only if a value is found for c
	$0.15x+0.1$	A1	Seen anywhere
	$\mathrm{F}(x)=\begin{cases} 0 & x<0 \\ 0.2x & 0\le x<2 \\ 0.15x+0.1 & 2\le x<6 \\ 1 & x\ge 6 \end{cases}$	B1	0 when $x < 0$ and 1 when $x \geq 6$ Allow either strict or non-strict inequalities
		A1	Completely defined function in **ACF** Allow different but consistent placement of strict inequalities
		5	
8(b)	$\mathrm{f}(x)=\begin{cases} 0.2 & 0\le x<2 \\ 0.15 & 2\le x<6 \\ 0 & \text{otherwise} \end{cases}$	B1	0 and otherwise **oe** seen
		M1	0.2 **oe** or 0.15 **oe** seen anywhere **ft** their equations of lines from **part (a)**
		A1ft	Completely defined function in **ACF** **ft** their equations of lines from **part (a)**
		3	

Q	Answer	Marks	Comments
8(c)	$\mathrm{E}\left(X^3\right)=\int_0^2 0.2x^3\,\mathrm{d}x + \int_2^6 0.15x^3\,\mathrm{d}x$	M1	Identifies correct integral for their $\mathrm{f}(x)$
	$=\left[\dfrac{0.2x^4}{4}\right]_0^2 + \left[\dfrac{0.15x^4}{4}\right]_2^6$	A1	Correct integration
	$\mathrm{E}\left(X^6\right)=\int_0^2 0.2x^6\,\mathrm{d}x + \int_2^6 0.15x^6\,\mathrm{d}x$	M1	Identifies correct integral for their $\mathrm{f}(x)$
	$=\left[\dfrac{0.2x^7}{7}\right]_0^2 + \left[\dfrac{0.15x^7}{7}\right]_2^6$	A1	Correct integration
	$\mathrm{E}\left(X^3\right)=48.8$ or $\mathrm{E}\left(X^6\right)=\dfrac{209984}{35}$	A1	**PI, AWRT** 6000 for $\mathrm{E}\left(X^6\right)$
	$\mathrm{Var}\left(X^3\right)=\mathrm{E}\left(X^6\right)-\mathrm{E}\left(X^3\right)^2$		
	$\mathrm{Var}\left(X^3\right)=\dfrac{209984}{35}-48.8^2$	m1	Uses variance formula with their expectation values for $\mathrm{E}\left(X^6\right)$ and $\mathrm{E}\left(X^3\right)$
	$\mathrm{Var}\left(X^3\right)=3620$	A1	**AWRT** 3620
		7	
	Total	**15**	