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MATHEMATICS
MA03

(9660/MA03) Unit P2 Pure Mathematics

Mark scheme

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2 3 6 X M A 0 3 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
$\surd$ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]kx^4(3x^5-4)^7$	M1	k is an integer or product
	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]120x^4(3x^5-4)^7$	A1	Accept $5\times3\times8\times x^4(3x^5-4)^7$
		2	

Q	Answer	Marks	Comments
1(a)(ii)	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]\frac{p\times x^3(7x-5)-qx^4}{(7x-5)^2}$	M1	p, q integers
	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]\frac{12x^3(7x-5)-21x^4}{(7x-5)^2}$	A1	oe
	or $\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]px^4(7x-5)^{-2}+qx^3(7x-5)^{-1}$	(M1)	
	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]3x^4(-1)(7)(7x-5)^{-2}+(7x-5)^{-1}12x^3$	(A1)	
	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{63x^4-60x^3}{(7x-5)^2}\right]$		
		2	

Q	Answer	Marks	Comments
1(a)(iii)	$\frac{2x}{x^2}+2y\frac{\mathrm{d}y}{\mathrm{d}x}=y+x\frac{\mathrm{d}y}{\mathrm{d}x}$	M1	Correct differentiation of y^2 or xy
		B1	Correct differentiation of $\ln(x^2)$
	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]\frac{y-\frac{2x}{x^2}}{2y-x}$	A1	oe
	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{xy-2}{x(2y-x)}\right]$		
		3	

Q	Answer	Marks	Comments
1(b)(i)	$\int \frac{x-6}{x^2-12x+5}\,\mathrm{d}x = k\ln\left\|x^2-12x+5\right\|$	**M1**	$k \neq 0$
	$= \frac{1}{2}\ln\left\|x^2-12x+5\right\| \quad [+c]$	**A1**	**oe** Must be in terms of x condone brackets as moduli signs
		2	

Q	Answer	Marks	Comments
1(b)(ii)	$\left[\frac{\mathrm{d}}{\mathrm{d}x}(2x^2+3x-1)=\right] 4x+3$	**B1**	**PI**
	$\left[\int \frac{8x+6}{(2x^2+3x-1)^3}\,\mathrm{d}x=\right] k(2x^2+3x-1)^{-2}$	**M1**	$k \neq 0$
	$= \frac{-1}{\left(2x^2+3x-1\right)^2} \quad [+c]$	**A1**	**oe** Must be in terms of x
		3	

	Question 1 Total	**12**	

Q	Answer	Marks	Comments
2(a)(i)	$f(x) = 2^{-x} - 4 + 2x$ $f(1.8) = 2^{-1.8} - 4 + 3.6 = -0.1[12825]...$ $f(1.9) = 2^{-1.9} - 4 + 3.8 = 0.06[794]...$	**M1**	Or reverse Both values rounded or truncated to at least 1sf
	Change of sign, $1.8 < \alpha < 1.9$	**A1**	Must have both statement and interval in words or symbols **or** comparing 2 sides: at 1.8, $2^{-1.8} = 0.2[871].. < 0.4$; at 1.9, $2^{-1.9} = 0.26[79]... > 0.2$ Accuracy as before **(M1)** Conclusion as before **(A1)**
		2	

Q	Answer	Marks	Comments
2(a)(ii)	$2x = 4 - 2^{-x}$ $x = 2 - \dfrac{2^{-x}}{2}$ $x = 2 - 2^{-x-1}$ $x = 2 - 2^{-(x+1)}$	**B1**	**AG** Must be convincingly shown
		1	

Q	Answer	Marks	Comments
2(a)(iii)	$x_2 = 1.856$	**B1**	**AWRT** 1.856
	$x_3 = 1.862$	**B1**	**CAO**
		2	

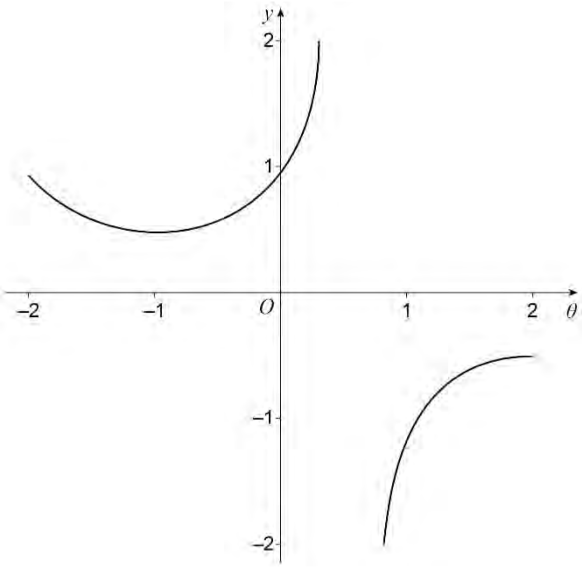
<table>
<tr><th>Q</th><th>Answer</th><th>Marks</th><th>Comments</th></tr>
<tr><td>2(b)</td><td>
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>1</td><td>$2^{-1} = 0.5$</td></tr>
<tr><td>1.75</td><td>$2^{-1.75} = 0.2973018$</td></tr>
<tr><td>2.5</td><td>$2^{-2.5} = 0.1767767$</td></tr>
<tr><td>3.25</td><td>$2^{-3.25} = 0.1051121$</td></tr>
<tr><td>4.0</td><td>$2^{-4} = 0.0625$</td></tr>
</table>
</td><td>B1

M1</td><td>All five correct x values (and no extra used) PI by five correct y values

At least four correct y values in exact form or decimals, rounded or truncated to three dp or better (in table or formula)
(PI by AWRT correct answer)</td></tr>
<tr><td></td><td>$\frac{1}{3}\times 0.75[0.5+0.0625+4(0.2973018$
$+0.1051121)+2\times 0.1767767]$</td><td>m1</td><td>Correct sub into formula with $h = 0.75$ OE and at least four correct y values either listed, with + signs, or totalled.
(PI by AWRT correct answer)</td></tr>
<tr><td></td><td>0.6314</td><td>A1</td><td>CAO, must see this value exactly and no error seen</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
</table>

	Question 2 Total	**9**	

Q	Answer	Marks	Comments
3(a)	Translation $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ Stretch SF 0.5, parallel to y-axis	B1 B1 B1 B1	
		4	

Q	Answer	Marks	Comments
3(b)	 	B1 B1 B1	Correct shape and position Min at (–1, 0.5) may be written as coordinates or indicated on the diagram y-intercept as (0, 0.9) AWRT **oe** may be written as coordinates or indicated on the diagram
		3	

Q	Answer	Marks	Comments
3(c)(i)	$\left[\frac{\mathrm{d}x}{\mathrm{d}y}=\right]\frac{1}{2}\sec y\tan y$	**B1**	**oe**
		1	

Q	Answer	Marks	Comments
3(c)(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}\left[=\frac{2}{\sec y\tan y}\right]=\frac{2\cos^2 y}{\sin y}$	**M1**	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\frac{\mathrm{d}x}{\mathrm{d}y}}$
	$=\frac{2(1-\sin^2 y)}{\sin y}$	**A1**	**oe**
		2	

	Question 3 Total	**10**	

Q	Answer	Marks	Comments
4(a)	$[\mathrm{f}(x)] \geq 0$	B1	Do not allow $x \geq 0$
		1	

Q	Answer	Marks	Comments
4(b)(i)	$\mathrm{fg}(x) = \sqrt{1 - \dfrac{2}{x-1}}$	B1	oe eg $\sqrt{\dfrac{x-3}{x-1}}$
		1	

Q	Answer	Marks	Comments
4(b)(ii)	$\left[1 - \dfrac{2}{x-1} \geq 0\right]$ $x \geq 3$ $x < 1$	 B1 B1	 oe oe
		2	

Q	Answer	Marks	Comments
4(b)(iii)	$1 - \dfrac{2}{x-1} = 9$ $[x - 1 = -0.25]$ $x = 0.75$	M1 A1	Attempt to solve by eliminating square root. only
		2	

Q	Answer	Marks	Comments
4(c)(i)	$x = \sqrt{1 - \dfrac{2}{y-1}}$ $x^2 = 1 - \dfrac{2}{y-1}$ $\mathrm{h}(x) = \dfrac{3 - x^2}{1 - x^2}$	M1 M1 A1	Interchange x and y Attempt to solve Allow p = 3, q = 1
		3	

Q	Answer	Marks	Comments
4(c)(ii)	$\left[9 - 3x^2 = 11 - 11x^2\right]$ $8x^2 = 2$ $x = [\pm]0.5$	 M1 A1	 At least one correct
		2	

	Question 4 Total	**11**	

Q	Answer	Marks	Comments
5(a)	$[7\cos\theta + 24\sin\theta =]$ $R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$	M1	PI
	$R = 25$	B1	
	$\alpha = 1.29$	A1	AWRT
	$[7\cos\theta + 24\sin\theta = 25\cos\left(\theta - 1.29\right)]$		
		3	

Q	Answer	Marks	Comments
5(b)	$2\operatorname{cosec}4x + 2\cot 4x = \dfrac{2}{\sin 4x} + \dfrac{2\cos 4x}{\sin 4x}$		
	$= \dfrac{2 + 2(2\cos^2 2x - 1)}{2\sin 2x\cos 2x}$	M1	Correct use of both double angle formulae
	$= \dfrac{4\cos^2 2x}{2\sin 2x\cos 2x}$ 4	A1	
	$= \dfrac{2\cos 2x}{\sin 2x}$		
	$= \dfrac{\cos^2 x - \sin^2 x}{\sin x\cos x}$	M1	Correct use of both double angle formulae
	$= \dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x}$		
	$= \cot x - \tan x$	A1	**AG** no errors seen
		4	

Q	Answer	Marks	Comments
5(c)	$5\left(\sec^2 Y - 1\right) = 7 - 4\sec Y$	**M1**	Correct use of trig identity **PI**
	$5\sec^2 Y + 4\sec Y - 12 = 0$ $(5\sec Y - 6)(\sec Y + 2)[=0]$		
	$\sec Y = -2, \dfrac{6}{5}$	**A1**	Both values correct
	$\left[\cos Y = \dfrac{5}{6}, -0.5\right]$		
	$Y = 0.585, -0.585, 5.697, 6.868, 2.094,$ $4.188..., 8.377...$	**A1**	For **any** of these values, or rounded or truncated to 2 dp **PI**
	$y = 0.59, 1.35, 1.93, 2.69$	**B1**	Sight of **any** of these values
		B1	All 4 correct and no extras in interval (ignore answers outside interval)
	or $5\dfrac{\sin^2 Y}{\cos^2 Y} = 7 - \dfrac{4}{\cos Y}$ $5(1-\cos^2 Y) = 7\cos^2 Y - 4\cos Y$	**[M1]**	
	$12\cos^2 Y - 4\cos Y - 5 = 0$ $(6\cos Y - 5)(2\cos Y + 1)[=0]$		
	$\cos Y = \dfrac{5}{6}, -0.5$	**[A1]**	
		5	
	Question 5 Total	**12**	

Q	Answer	Marks	Comments
6(a)	$4(0.5)^3 + a(0.5)^2 + b \times 0.5 + c = 0$ $4(1.5)^3 + a(1.5)^2 + b \times 1.5 + c = 15$	**M1**	One correct substitution
	$4\left((1.5)^3 - (0.5)^3\right) + a\left((1.5)^2 - (0.5)^2\right)$ $+ b\left(1.5 - 0.5\right) = 15$ or $a + 2b + 4c = -2$ $9a + 6b + 4c = 6$	**m1**	Attempt to eliminate c
	$13 + 2a + b = 15$ or $8a + 4b = 8$ leading to $2a + b = 2$	**A1**	**AG** no errors seen
		3	

Q	Answer	Marks	Comments
6(b)	$4(-0.5)^3 + a(-0.5)^2 + b \times (-0.5) + c = 9$	**M1**	Correct substitution
	$a - 2b + 4c = 38$ $4b = -40 \quad \Rightarrow \quad b = -10$	**m1**	Attempt to solve simultaneous equations in a, b and c
	$a = 6, \quad c = 3$	**A1**	All 3 values correct
	$\mathrm{f}(x) = 4x^3 + 6x^2 - 10x + 3$ $\left[\mathrm{f}(-1.5)\right] = 18$	**B1**	
		4	

	Question 6 Total	**7**	

Q	Answer	Marks	Comments
7(a)	$\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{2t-1-2t}{(2t-1)^2}$ or $(-t)(2t-1)^{-2}+(2t-1)^{-1}$	M1	**oe** Either derivative correct
	$\dfrac{\mathrm{d}y}{\mathrm{d}t}=2-\dfrac{1}{2\sqrt{t}}$	A1	**oe** Both correct
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{(2t-1)^2(4\sqrt{t}-1)}{2\sqrt{t}}$	m1	
	$t=1,\ \dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{3}{2},\ x=1,\ y=1$	A1	All correct **PI**
	Equation of normal: $y-1=\dfrac{2}{3}(x-1)$	M1	Correct attempt to find *their* normal
	$3y-2x=1$	A1	Allow p = 3, q = –2, r = 1, Allow integer multiples
		6	

Q	Answer	Marks	Comments
7(b)(i)	$x+y=2\mathrm{e}^{2m}$ $x-y=2\mathrm{e}^{-2m}$	M1	Both correct
	$\dfrac{x+y}{2}=\dfrac{2}{x-y}$	m1	Attempt to eliminate m
	$x^2=y^2+4$	A1	
	or		
	$x^2=\left(\mathrm{e}^{2m}\right)^2+\left(\mathrm{e}^{-2m}\right)^2+2$	[M1]	Either equation correct
	$y^2=\left(\mathrm{e}^{2m}\right)^2+\left(\mathrm{e}^{-2m}\right)^2-2$	[m1]	Both correct
	$x^2=y^2+4$	[A1]	
		3	

Q	Answer	Marks	Comments
7(b)(ii)	$\left[\begin{array}{ll} x+y=2\mathrm{e}^{kn} & x-y=2\mathrm{e}^{-kn} \\ \dfrac{x+y}{2}= & \dfrac{2}{x-y} \end{array}\right]$ $x^2 = y^2 + 4$	B1ft	**oe**
		1	

Q	Answer	Marks	Comments
7(c)	$x^2 = a^2 \sin^2\theta + b^2\cos^2\theta + 2ab\cos\theta\sin\theta$ $y^2 = a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta\sin\theta$	M1	Squaring one expression (condone one slip)
	$x^2 + y^2$ $= a^2(\sin^2\theta + \cos^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$	m1	Attempt to eliminate θ
	$x^2 + y^2 = a^2 + b^2$	A1	**oe**
	or		
	$ax - by = \sin\theta(a^2 + b^2)$ $ay + bx = \cos\theta(a^2 + b^2)$	[M1]	One expression in either $\sin\theta$ or $\cos\theta$ (condone one slip)
	$\left(\dfrac{ax-by}{a^2+b^2}\right)^2 + \left(\dfrac{ay+bx}{a^2+b^2}\right)^2 = 1$ $\left[(ax-by)^2 + (ay+bx)^2 = \left(a^2+b^2\right)^2\right]$	[m1] [A1]	Attempt to eliminate θ
		3	

	Question 7 Total	**13**	

Q	Answer	Marks	Comments
8(a)	$\frac{\mathrm{d}P}{\mathrm{d}t} = kP$	M1	
	$[3000 = k \times 1000000]$		
	$\frac{\mathrm{d}P}{\mathrm{d}t} = 0.003P$	A1	**oe**
		2	

Q	Answer	Marks	Comments
8(b)	$2x\frac{\mathrm{d}y}{\mathrm{d}x} = 4 - y^2$		
	$\int \frac{\mathrm{d}y}{4-y^2} = \int \frac{\mathrm{d}x}{2x}$	B1	**PI** Correctly separate variables
	$\frac{1}{4-y^2} = \frac{A}{2-y} + \frac{B}{2+y}$	M1	Using partial fractions
	$1 = A(2+y) + B(2-y)$		
	$A = 0.25, \quad B = 0.25$	A1	Both correct
	$\frac{1}{4}\int \frac{1}{2-y} + \frac{1}{2+y}\,\mathrm{d}y = \frac{1}{2}\int \frac{\mathrm{d}x}{x}$		
	$-\ln(2-y) + \ln(2+y) = 2\ln x + \ln A \text{ (or } +c)$	m1	Correct integration (at least 3 terms correct)
	$\ln\frac{2+y}{2-y} = \ln Ax^2$		$\int \frac{\mathrm{d}y}{4-y^2} = \frac{1}{4}\ln\left\|\frac{2+y}{2-y}\right\|$ scores **M1A1m1**
	$Ax^2 = \frac{2+y}{2-y}$	m1	Eliminating 'ln' from *their* equation – must have scored **M1m1**
	$(1,1) \Rightarrow A = 3$		or $(1,1) \Rightarrow c = \pm\frac{1}{4}\ln 3$
	$3x^2(2-y) = 2+y$		
	$y = \frac{2(3x^2-1)}{(3x^2+1)}$	A1	**ACF**
		6	

	Question 8 Total	**8**	

Q	Answer	Marks	Comments
9(a)	$\frac{dy}{dx} = Ax^{0.5}e^{-0.5x} + Bx^{-0.5}e^{-0.5x}$	M1	
	$\frac{dy}{dx} = x^{0.5}(-0.5)e^{-0.5x} + 0.5x^{-0.5}e^{-0.5x}$	A1	
	$x^{0.5}(-0.5)e^{-0.5x} + 0.5x^{-0.5}e^{-0.5x} = 0$, $0.5x^{0.5} = 0.5x^{-0.5}$ $x = 1$	m1	Equating to 0 and attempt to solve
	$\left(1,\ e^{-0.5}\right)$	A1	Allow $x = 1,\ \ y = e^{-0.5}$
		4	

Q	Answer	Marks	Comments
9(b)	$V = \pi\int_1^2 \left(\sqrt{x}e^{-0.5x}\right)^2 dx$	B1	**PI** by final answer
	$\int xe^{-x}dx = -xe^{-x} - \int -e^{-x}dx$	M1	Use of parts formula
	$= -xe^{-x} - e^{-x}$	A1	
	$\int_1^2 \ = [\pi]\left(-xe^{-x} - e^{-x}\right)_1^2$ $= [\pi]\left(\left(-2e^{-2} - e^{-2}\right) - (-e^{-1} - e^{-1})\right)$	m1	Correct substitution of *their* 1, 2 into *their* expression in the form $axe^{-x} + be^{-x}$
	$= \pi\left(\frac{2}{e} - \frac{3}{e^2}\right)$	A1	Allow a = 2, b = –3
		5	

	Question 9 Total	**9**	

Q	Answer	Marks	Comments
10	$\frac{\mathrm{d}u}{\mathrm{d}\theta} = \cos\theta$	B1	oe PI
	$\left[\int \frac{\cos^3\theta}{(1+\sin\theta)^{1.5}}\mathrm{d}\theta = \right] \int \frac{1-(u-1)^2}{(u)^{1.5}}\mathrm{d}u$	M1	All in terms of u, condone omission of $\mathrm{d}u$
	$\int \frac{2u-u^2}{(u)^{1.5}}\mathrm{d}u$		
	$= \int 2u^{-0.5} - u^{0.5}\mathrm{d}u$	A1	Must see $\mathrm{d}u$ here, or earlier
	$= 4u^{0.5} - \frac{2}{3}u^{1.5} \quad [+c]$	M1	$Au^{0.5} + Bu^{1.5}$
		A1ft	Correct integration
	$[\theta]_0^{\frac{\pi}{2}} = [u]_1^2$ $\left[\int_0^{\frac{\pi}{2}} \frac{\cos^3\theta}{(1+\sin\theta)^{1.5}}\mathrm{d}\theta = \left(4u^{0.5} - \frac{2}{3}u^{1.5}\right)_1^2\right]$	B1	Change of limits, maybe seen earlier (may change back to θ and not change limits)
	$= \left(4\times 2^{0.5} - \frac{2}{3}\times 2^{1.5}\right) - \left(4 - \frac{2}{3}\right)$ $= \frac{8}{3}\times 2^{0.5} - \frac{10}{3}$	M1	Correct substitution of *their* limits into *their* expression of the form $Au^{0.5} + Bu^{1.5}$
	$= \frac{2}{3}(4\sqrt{2} - 5)$	A1	Allow p = 4, q = –5
		8	

	Question 10 Total	**8**	

Q	4Answer	Marks	Comments
11(a)	$\mathrm{f}(x)=\dfrac{A}{(3-x)}+\dfrac{B}{(3-x)^2}+\dfrac{C}{(1-3x)}$		
	$7x^2-17x+12=A(3-x)(1-3x)+B(1-3x)+C(3-x)^2$	**B1**	Correctly eliminating fractions
	$x=3,\ \ 24=-8B,\ \ B=-3$	**M1**	Attempt at finding one constant
	$x=\dfrac{1}{3},\ \ \dfrac{64}{9}=\dfrac{64}{9}C,\ \ C=1$	**A1**	Two constants correct
	$x=0,\ \ 12=3A+B+9C,\ \ A=2$		
	$\mathrm{f}(x)=\dfrac{2}{(3-x)}-\dfrac{3}{(3-x)^2}+\dfrac{1}{(1-3x)}$	**A1**	Allow A = 2, B = –3, C = 1 Allow equivalent methods
		4	

Q	Answer	Marks	Comments
11(b)	$(3-x)^{-1}=\dfrac{1}{3}(1-\dfrac{1}{3}x)^{-1}$	**M1**	
	$=\dfrac{1}{3}+\dfrac{1}{9}x+\dfrac{1}{27}x^2$	**A1**	
		2	

Q	Answer	Marks	Comments
11(c)	$(3-x)^{-2}=\left(\dfrac{1}{3}\right)^2(1+\dfrac{2}{3}x+\dfrac{1}{3}x^2)$	**M1**	**oe** either expansion correct
	$(1-3x)^{-1}=1+3x+9x^2$	**A1**	Both correct
	$\mathrm{f}(x)$: $2(\dfrac{1}{3}+\dfrac{1}{9}x+\dfrac{1}{27}x^2)-3\times\dfrac{1}{9}(1+\dfrac{2}{3}x+\dfrac{1}{3}x^2)+1(1+3x+9x^2)$	**m1**	Correct substitutions of *their* expansions into *their three term part (a)*
	$\mathrm{f}(x)=\dfrac{4}{3}+3x+\dfrac{242}{27}x^2$ $\left[D=\dfrac{4}{3},\ \ E=3,\ \ F=\dfrac{242}{27}\right]$	**A1**	
		4	

	Question 11 Total	**10**	

	Answer	Marks	Comments
12(a)	$\left[l_1: \mathbf{r} =\right] \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -1 \\ -4 \end{bmatrix}$	**B1**	**oe** $\left[l_1: \mathbf{r} =\right] \begin{bmatrix} 5 \\ -2 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -1 \\ -4 \end{bmatrix}$
		1	

Q	Answer	Marks	Comments
12(b)(i)	$2+3\lambda = -2-\mu$ $-1-\lambda = 5-2\mu$	**M1**	Both equations correct, **ft** from (a)
	$[7\lambda = -14 \quad \Rightarrow \quad \lambda = -2,] \quad \mu = 2$	**A1**	
	$[z: 3-4\times -2 = 11 = 7+2k] \quad k = 2$	**A1**	
		3	2

Q	Answer	Marks	Comments
12(b)(ii)	(–4, 1, 11)	**B1**	
		1	

Q	Answer	Marks	Comments
12(c)	Coords of D $(-2-d, 5-2d, 7+2d)$	**B1ft**	Ft *their ‘k’* in (b)(i)
	$\overrightarrow{CD} = \begin{bmatrix} -5-d \\ 1-2d \\ 3+2d \end{bmatrix}$	**M1**	**oe** Seen or used, Ft *their ‘D’*
	$\begin{bmatrix} -5-d \\ 1-2d \\ 3+2d \end{bmatrix} . \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix} [=0]$	**m1**	Correct use of dot product with their CD and their k
	$9d = -9 \quad \Rightarrow \quad d = -1$	**A1**	
	Coords of D $(-1, 7, 5)$		
	Dist $= \sqrt{(-1-3)^2 + (7-4)^2 + (5-4)^2}$	**M1**	Must have scored **m1**
	$= \sqrt{26}$	**A1**	
		6	

	Question 12 Total	**11**	