OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

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I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM01) Unit FP1 Pure Mathematics

Monday 13 January 2020 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables (enclosed).
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
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6	
7	
8	
9	
TOTAL	

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Answer **all** questions in the spaces provided.

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1 Find the complex number z such that

$$4z - \mathrm{i}z^* - 3\mathrm{i} = 7$$

Give your answer in the form $a + b\mathrm{i}$, where a and b are real.

[6 marks]

Answer _____

6

Turn over ►

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2 It is given that

$$\cos\left(2x - \frac{\pi}{4}\right) = -\frac{\sqrt{3}}{2}$$

2 (a) Find the general solution of the equation, giving your answer in terms of π.

[5 marks]

Answer _____

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2 (b) Show that the mean of the solutions which lie between $2k\pi$ and $2(k+1)\pi$, where k is an integer, is

$$2k\pi + \frac{9\pi}{8}$$

[3 marks]

8

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3 A curve has the equation $y = x + \frac{1}{x}$

3 (a) A line passes through two points on the curve, one where $x = 5$ and the other where $x = 5 + h \quad (h > 0)$

Find the gradient of this line in the form

$$1 - \frac{1}{\mathrm{f}(h)}$$

where f is a function of h.

[4 marks]

Answer _____

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3 (b) Show how the answer to part **(a)** can be used to find the gradient of the curve at the point where $x = 5$

State the value of this gradient.

[2 marks]

Answer _____

6

Turn over for the next question

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4 The quadratic equation

$$2x^2 - 7x + 10 = 0$$

has roots α and β.

4 (a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$.

[2 marks]

$\alpha + \beta =$ _____ $\alpha\beta =$ _____

4 (b) Find a quadratic equation, with integer coefficients, which has roots α^3 and β^3

[6 marks]

Answer _____

8

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5 [The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$ where r is the radius.]

A spherical balloon is increasing in size.

Its volume increases at a rate of 50 cm^3 per second.

Show that, when the volume of the balloon is $\frac{500\pi}{3}$ cm^3, the **radius** of the balloon is increasing at a rate of $\frac{1}{2\pi}$ cm per second.

[6 marks]

6

Turn over ►

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6 The function f is defined by

$$\mathrm{f}(x) = \frac{x-3}{(x-2)(x-1)}$$

6 (a) Write down the equations of the asymptotes of the graph of $y = \mathrm{f}(x)$.

[2 marks]

Answer _____

6 (b) Find the possible values of $\mathrm{f}(x)$.

[6 marks]

Answer _____

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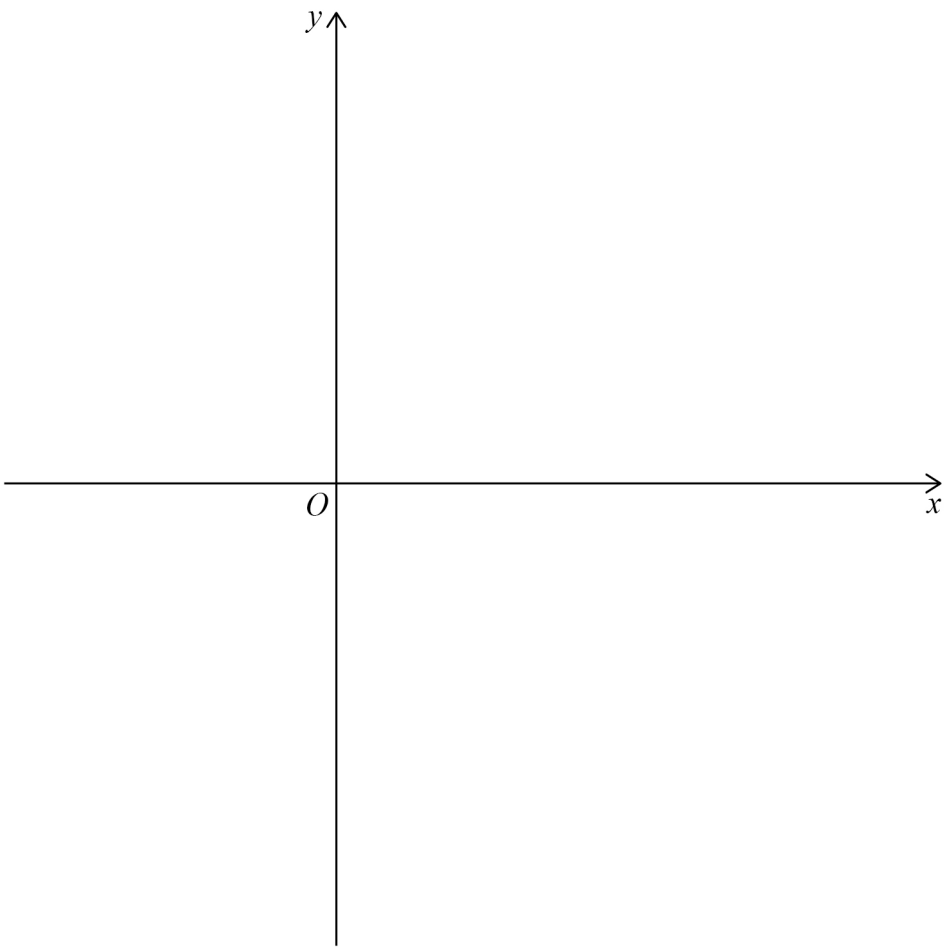
6 (c) Sketch the graph of $y = f(x)$ on the axes below, showing the y-coordinates of any stationary points.

[You do **not** need to find the x-coordinates of any stationary points.]

[4 marks]



12

Turn over for the next question

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7 (a) Show that

$$(x+1)^4-(x-1)^4=k\left(x^3+x\right)$$

where k is an integer.

[2 marks]

7 (b) Use the method of differences to show that

$$\sum_{r=1}^{n}\left(r^3+r\right)=\frac{1}{8}\left(n^4+(n+1)^4-1\right)$$

[5 marks]

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7 (c) Hence prove that if n is a positive integer, $n^4 + (n+1)^4 - 1$ is a multiple of 16

[2 marks]

9

Turn over for the next question

8 The circle C is the locus of points on an Argand diagram such that

$$\left|z+3+4\mathrm{i}\right|=5$$

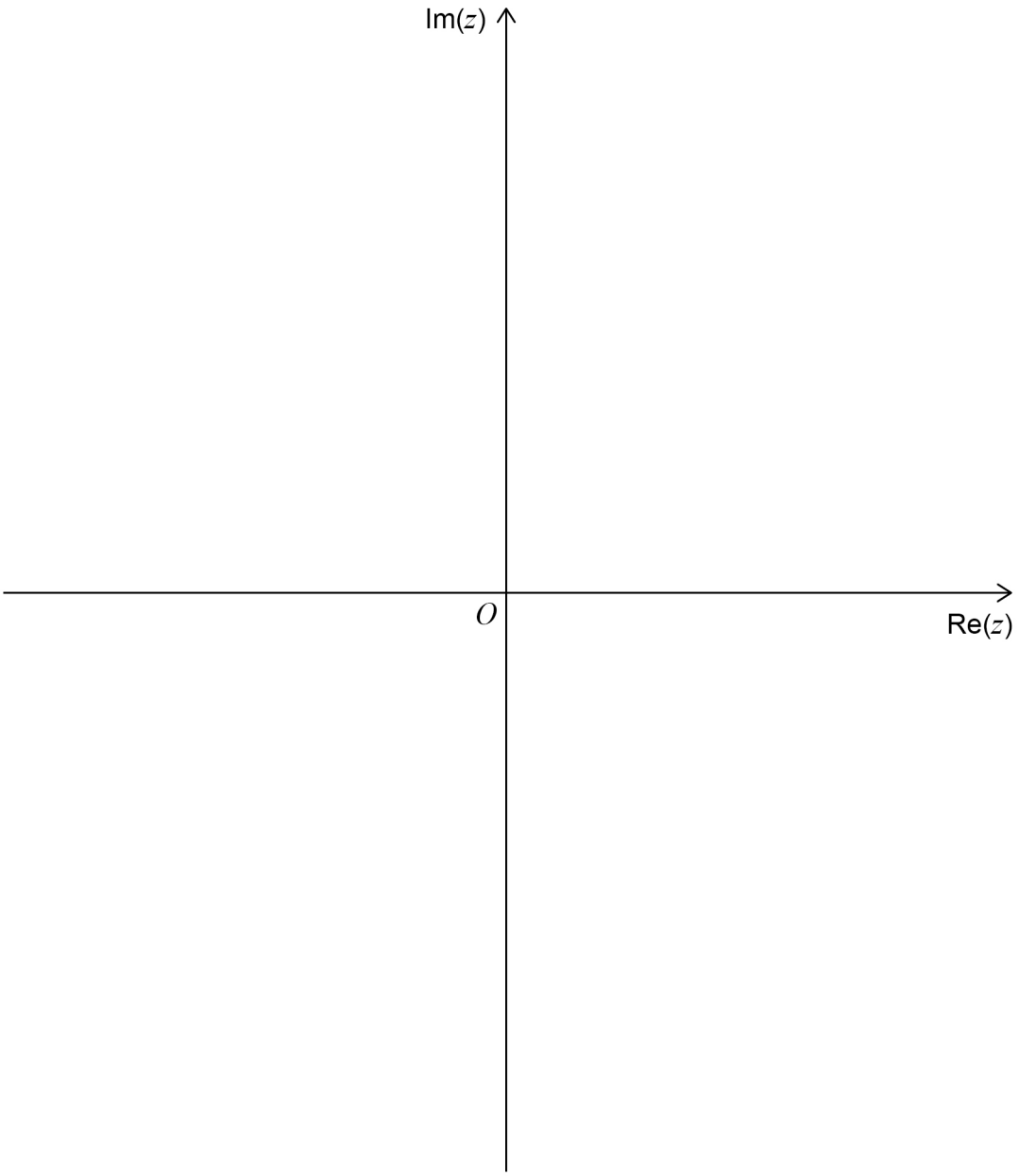
The half-line L is the locus of points on an Argand diagram such that

$$\arg\left(z+10\mathrm{i}\right)=\alpha, \quad \text{where } 0<\alpha<\frac{\pi}{2}$$

L is a tangent to C.

8 (a) Draw L and C on the Argand diagram.

[4 marks]



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8 (b) Calculate the value of α, giving your answer to three significant figures.

[6 marks]

Answer _____

10

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9 The rectangular hyperbola H has equation $xy = 8$

The parabola P has equation $y^2 = 8x$

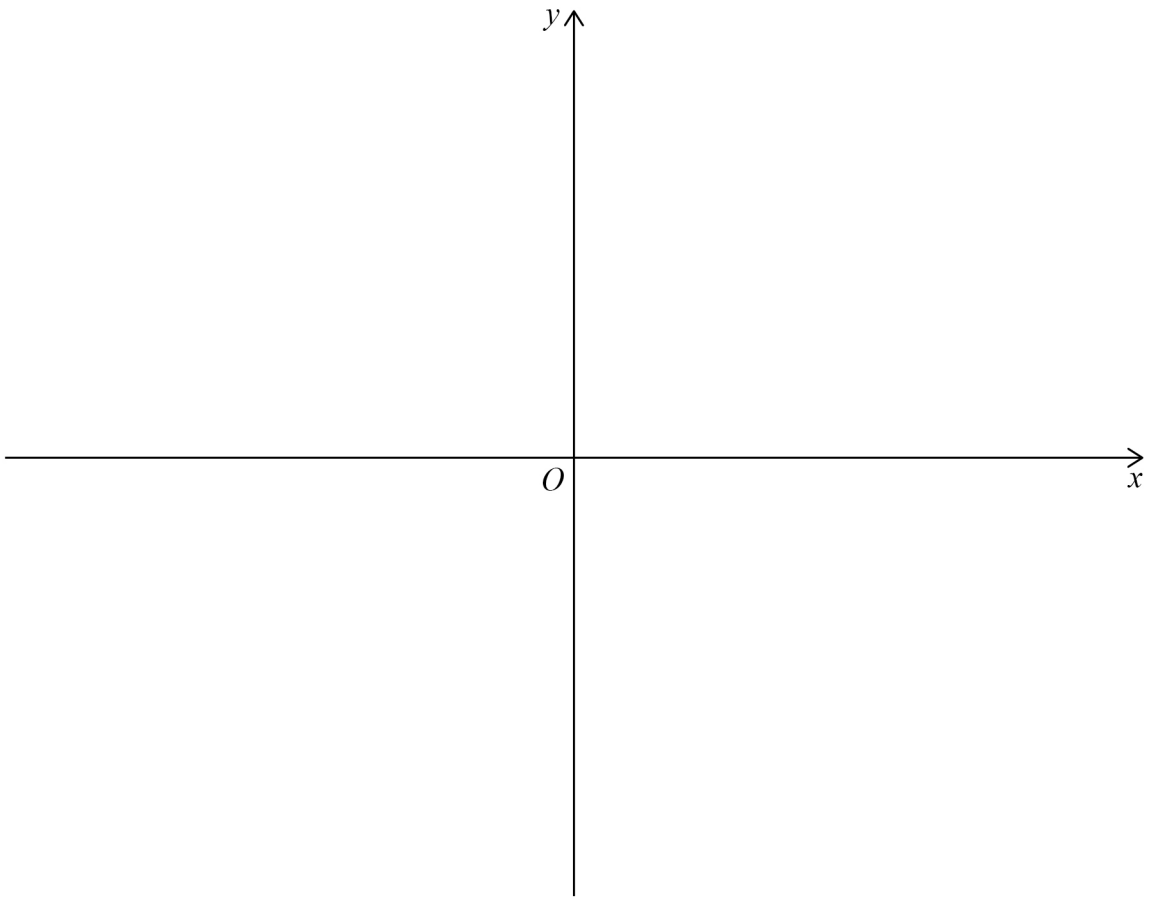
9 (a) Find the coordinates of the point of intersection of H and P.

[2 marks]

Answer _____

9 (b) Sketch the graphs of H and P on the axes below.

[2 marks]



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9 (c) The line $y = mx + c$ is a tangent to H.

Show that $c^2 + 32m = 0$

[4 marks]

9 (d) Find an equation of the line that is a tangent to **both** H **and** P.

[7 marks]

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Answer _____

15

END OF QUESTIONS

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