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MATHEMATICS

MA02

(9660/MA02) Unit PSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

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226XMA02/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	$A = 4$	**B1**	
		1	
1(a)(ii)	$20 = 4 \times 3^{8k}$ $k = \frac{1}{8}\log_3(5)$	**M1** **A1**	**oe** Correct substitution of $(8,\ 20)$ and their A from **part 1(a)(i)**. **CSO** Accept $k = 0.125\log_3(5)$
		2	
1(b)	$2x = \log_4(11)$ or $2x = 1.72[971...]$ or $2x\log_{[n]}(4) = \log_{[n]}(11)$ $x = 0.865$	**M1** **A1**	**oe PI** by $x = 0.86[4857...]$ **CAO** to 3 sf
		2	
	Total	**5**	

Q	Answer	Marks	Comments
2	$\frac{1}{2}r^2\theta = 3$ $r\theta + 2r = 8$ $r^2 - 4r + 3 = 0$ or $3\theta^2 - 20\theta + 12 = 0$ $r = 3$ or $\theta = \frac{2}{3}[= 0.66...]$ $r = 3$ and $\theta = \frac{2}{3}[= 0.66...]$	**B1** **B1** **M1** **A1** **A1**	**oe PI** by later working **oe PI** by later working **oe** Eliminates a variable to find a correct equation in either r or θ **PI** by $r = 1$ and $r = 3$, or $\theta = \frac{2}{3}[= 0.66...]$ and $\theta = 6$ For $\theta = \frac{2}{3}$ allow $\theta = 0.67$ In both **A1** marks, allow $\theta = 0.67$ Condone sight of $r = 1$ and/or $\theta = 6$ in addition to correct value(s) This pair of values and no others
	Total	**5**	

Q	Answer	Marks	Comments
3(a)	$[\log_a 2x =] \log_a 4^3 + \log_a 5$ or $[\log_a 2x =] \log_a 64 + \log_a 5$	**M1**	One log rule used
	$[\log_a 2x =] \log_a (4^3 \times 5)$ or $[\log_a 2x =] \log_a (64 \times 5)$ or $[\log_a 2x =] \log_a (320)$	**M1**	Second log rule used
	$[\Rightarrow 2x = 320]$ $x = 160$	**A1**	**CSO** **AG** Be convinced
		3	
3(b)	$\left[\begin{array}{l}\log_a y = 9 + \log_a 10 \\ \Rightarrow \log_a y - \log_a 10 = 9\end{array}\right]$		
	$\log_a \left(\frac{y}{10}\right) = 9$	**M1**	Accept $\log_a a^9$ for 9
	$\frac{y}{10} = a^9$	**M1**	Eliminates log provided an equation involving a single log seen
	$[y =] 10a^9$	**A1**	**CSO**
		3	
3(b) ALT	$\log_a y = \log_a a^9 + \log_a 10$	**M1**	Substitutes $\log_a a^9$ for 9
	$[\log_a y =] \log_a (10a^9)$	**M1**	Log rule used
	$[y =] 10a^9$	**A1**	**CSO**
		3	
	Total	**6**	

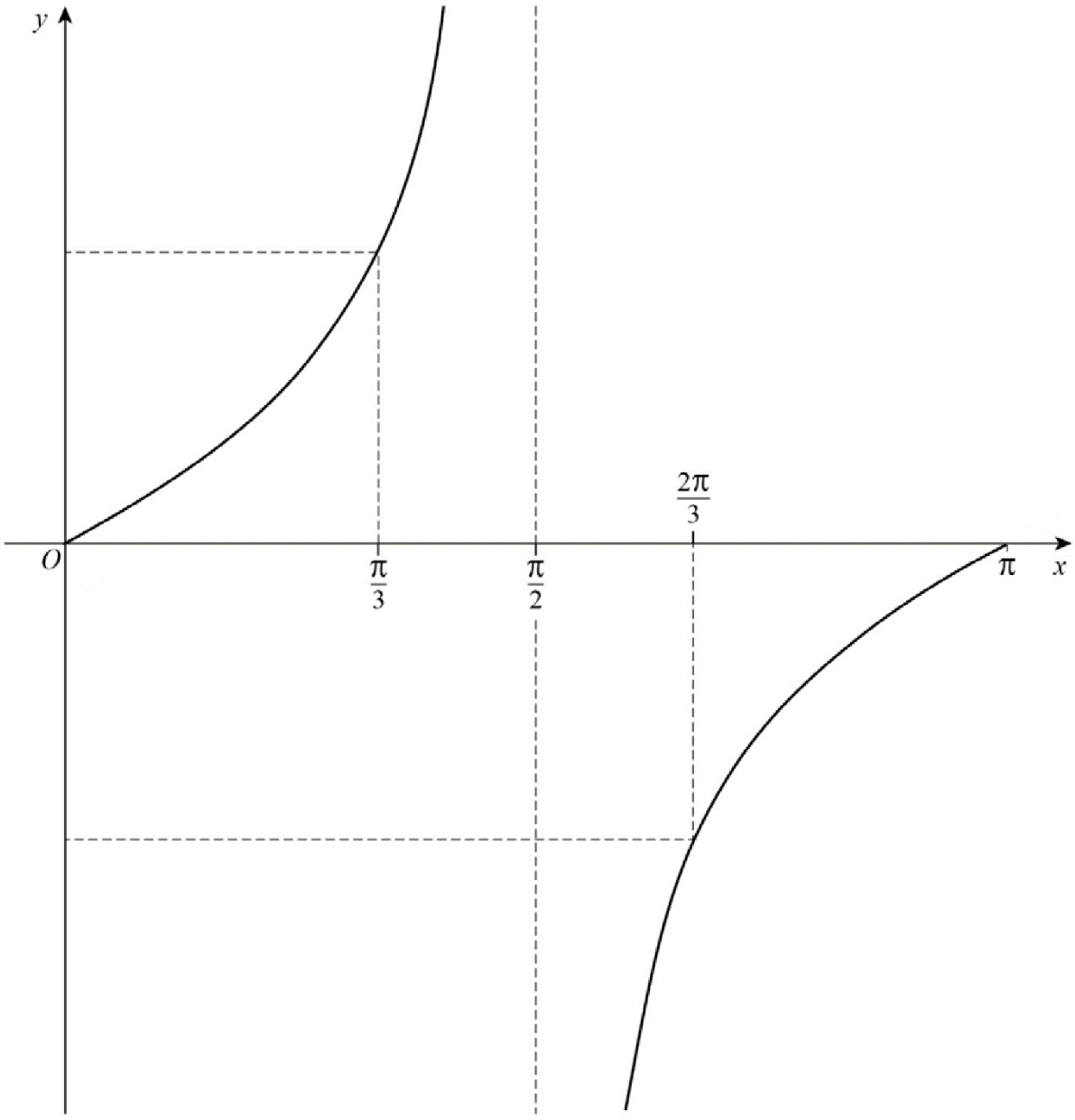
Q	Answer	Marks	Comments
4(a)	$10^2 = 9^2 + 13^2 - 2\times 9\times 13\cos\theta$	**M1**	**oe** cosine rule used with values correctly substituted **PI** by next line
	$\cos\theta = \dfrac{9^2+13^2-10^2}{2\times 9\times 13} \quad \left[= \dfrac{25}{39} = 0.641...\right]$	**M1**	**oe** Correct rearrangement
	$\theta = \left[50.13165...^\circ =\right] \ 50.1^\circ$	**A1**	**CSO** **AWRT** 50.1°
		3	

4(b)	$\frac{\sin PRQ}{20}=\frac{\sin 35^\circ}{15}$	**M1**	Correct equation with all known values substituted
	$\sin PRQ=\frac{20\sin 35^\circ}{15}\ [=0.76476...]$	**M1**	Correct rearrangement **PI** by correct value for $\sin PRQ$
	$49.8[8640...]^\circ$ or [Angle PRQ =] $130.1[1359...]^\circ$	**A1**	**PI** in later working. **AWRT** 49.9°, Allow 49.8° **PI** in later working. **ft** 180 minus their acute angle provided **M1** scored. **AWRT** 130.1°
	$\frac{1}{2}\times 20\times 15\times\sin\left(14.8[8640...]^\circ\right)$	**m1**	**ft** $180^\circ-35^\circ-$ their obtuse angle or **ft** their acute angle -35° Dependent on at least one of the previous **M1** marks
	$38.5\ \left[\text{cm}^2\right]$	**A1**	**CAO** **AWRT** 38.5 or 38.6
		5	

4(b) ALT	$\left[\lvert PR\rvert=x\right]$ $15^2=x^2+20^2-2\times 20\times x\cos 35^\circ$ or $225=x^2+400-\left(40\cos 35^\circ\right)x$	**M1**	**oe** Cosine rule used with values correctly substituted. **PI** by next line.
	$x^2-\left(40\cos 35^\circ\right)x+175=0$ or $x^2-\left(32.7[6608...]\right)x+175=0$	**M1**	**oe** Forms quadratic equation set equal to zero. **PI** by correct value of x
	$[x=]\ 6.7[1846...]$	**A1**	**AWRT** 6.7 **PI** in later working. May also see $[x=]\ 26.0[4761...]$
	$[\text{Area}=]\ \frac{1}{2}\times 20\times 6.7[1846...]\times\sin 35^\circ$	**m1**	**ft** their x Dependent on at least one of the previous **M1** marks
	$38.5\ \left[\text{cm}^2\right]$	**A1**	**CAO** **AWRT** 38.4 or 38.5
		5	
	Total	**8**	

Q	Answer	Marks	Comments
5(a)	$\cos^2 x - 2\cos x \tan x + \tan^2 x$ $+1 + 2\sin x + \sin^2 x [= 5]$	**M1**	Correct expansion of at least one bracket
	$\cos^2 x - 2\cos x \dfrac{\sin x}{\cos x} + \tan^2 x$ $+1 + 2\sin x + \sin^2 x = 5$ or	**M1**	Both brackets correctly expanded and then use of: $\tan x = \dfrac{\sin x}{\cos x}$ or $\cos^2 x + \sin^2 x = 1$ **oe** Must be seen as a correct equation
	$\left[\left(\cos^2 x + \sin^2 x\right) - 2\cos x \tan x + \tan^2 x\right.$ $\left.+1 + 2\sin x = 5\right]$ $1 - 2\cos x \tan x + \tan^2 x$ $+1 + 2\sin x = 5$		
	$1 - 2\sin x + \tan^2 x + 1 + 2\sin x = 5$ $\Rightarrow \tan^2 x = 3$	**A1**	**CSO** **AG** Must see use of both identities Be convinced
		3	
5(b)(i)	$x = \dfrac{\pi}{3}$	**B1**	One correct value **oe** such as 2 sf decimals Condone given in degrees
	$x = \dfrac{2\pi}{3}$	**B1**	A second correct value and no others **oe** such as 2 sf decimals Condone given in degrees
		2	

5(b)(ii)	Correct sketch of $y = \tan x$ for $0 \leq x \leq \pi$ $x = \frac{\pi}{3}$ and $x = \frac{2\pi}{3}$ clearly indicated on horizontal axis or curve in approximately correct positions.	**B1** **B1ft**	Must be of correct form Condone asymptotes not drawn **ft** their solution(s) Condone given in degrees



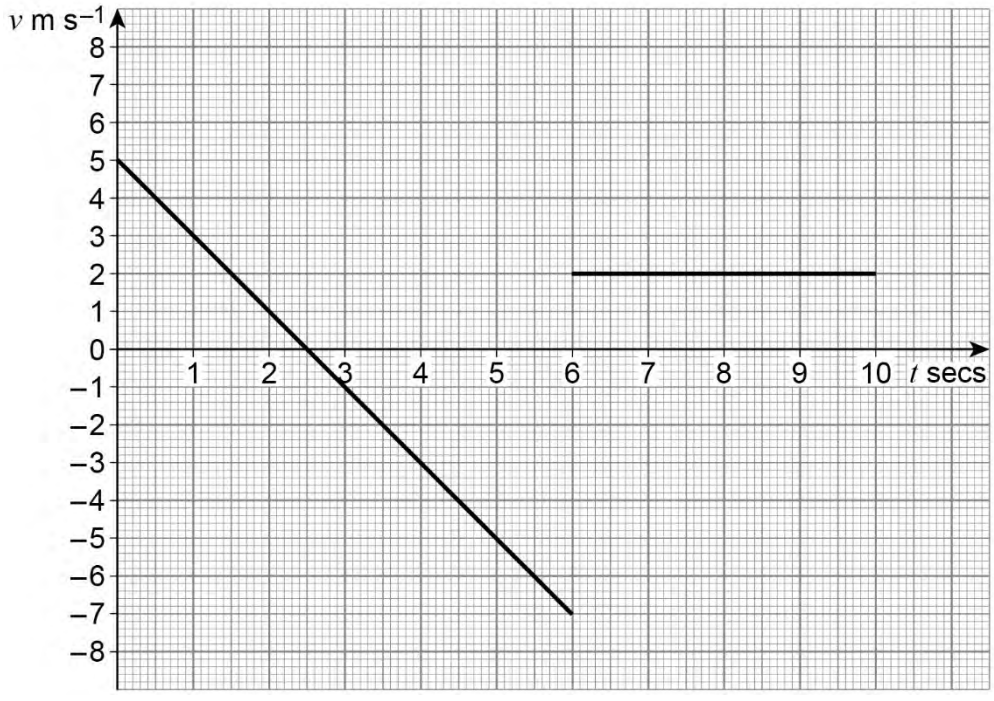
		2	
	Total	**7**	

Q	Answer	Marks	Comments
6(a)	$\left[\text{Radius} =\right]\ 2\sqrt{13}$	**B1**	Accept $\sqrt{52}$ Mark at most accurate
	$\left[\text{Centre} =\right]\ (3, 1)$	**B1**	
		2	
6(b)	$m = \dfrac{7-1}{7-3}\ \left[= \dfrac{3}{2}\right]$	**M1**	
	$(y-1) = \dfrac{3}{2}(x-3)$	**A1**	**ACF** eg $2y-3x=-7$, $y=\dfrac{3}{2}x-\dfrac{7}{2}$, $(y-7)=\dfrac{3}{2}(x-7)$
		2	
6(c)	$\left[2\sqrt{13}+8\sqrt{13} =\right]\ 10\sqrt{13}$	**B1**	**oe** Must be in surd form. Distance between centres of circles. **PI**
	$\sqrt{\left(10\sqrt{13}\right)^2-\left(8\sqrt{13}\right)^2}$	**M1**	Method for finding distance from centre of C_1 to Q
	$\left[\begin{matrix}=\sqrt{1300-832}\\ =\sqrt{468}\end{matrix}\right]$		
	$6\sqrt{13}$	**A1**	**oe** Must be in surd form. Distance from centre of C_1 to Q
	$\left[PQ = 6\sqrt{13}-2\sqrt{13} =\right]\ 4\sqrt{13}$	**B1**	At least one correct distance **oe** Must be single term in surd form
	$\left[PQ = 6\sqrt{13}+2\sqrt{13} =\right]\ 8\sqrt{13}$	**B1**	Both correct distances and no others **oe** Must be single term in surd form
		5	
	Total	**9**	

Q	Answer	Marks	Comments
7(a)	$P(A \cap B) = P(A) \times P(B) = 0.22 \times 0.13$ $= 0.0286$	**M1** **A1**	Uses independence of A and B **oe** such as $\frac{143}{5000}$
		2	
7(b)	$P(A \cap C) = 0$	**B1**	Uses fact A and C mutually exclusive
		1	
7(c)	$P(B\|A) = P(B)$ or $P(B\|A) = \left[\frac{P(A \cap B)}{P(A)} =\right] \frac{0.0286}{0.22}$ $= 0.13$	**M1** **A1**	Uses independence of A and B to find conditional probability **ft** their answer to **(a)** **PI** by correct final answer **CAO**
		2	
	Total	**5**	

Q	Answer	Marks	Comments
8(a)	Bernoulli $p = 0.5$ or Binomial $n = 1, p = 0.5$	**B1** **B1**	Accept $\mathrm{B}(n, p)$ for Binomial Must state both parameter values for Binomial
		2	
8(b)(i)	$\mathrm{E}(H) = 0.5$	**B1**	**oe**
		1	
8(b)(ii)	$\mathrm{Var}(H) = 0.25$	**B1**	**oe**
		1	
8(c)(i)	$\mathrm{Var}(K) = 2$	**B1ft**	**ft** $8 \times$ their $\mathrm{Var}(H)$
		1	
8(c)(ii)	$K \sim \mathrm{B}(8,\ 0.5)$ $[\mathrm{P}(K \geq 7) = 1 - \mathrm{P}(K \leq 6)]$ $= 1 - 0.9648$ or $[\mathrm{P}(K = 7) + \mathrm{P}(K = 8)]$ $= 0.0313 + 0.0039$ or $= \frac{1}{32} + \frac{1}{256}$ $= 0.035$	**B1** **M1** **A1**	May be seen in **(c)(i)** **PI** in later working for example sight of 0.9648 or 0.0313 or 0.9961 or 0.0039 Correct method to calculate probability **AWRT**
		3	
	Total	**8**	

Q	Answer	Marks	Comments
9(a)	$a + 0.2 + 0.34 + b = 1$	**M1**	Forms equation using sum of probabilities $= 1$ Implied by $a + b = 0.46$
	$2a + 3 \times 0.2 + 7 \times 0.34 + 9b = 5.16$	**M1**	Forms equation using $\mathrm{E}(X) = 5.16$ Implied by $2a + 9b = 2.18$
	$a = 0.28$ or $b = 0.18$	**A1**	At least one of a or b correct
	$a = 0.28$ and $b = 0.18$	**A1**	Both a and b correct
		4	
9(b)	$\mathrm{E}(X^2) = 2^2 \times a + 3^2 \times 0.2 + 7^2 \times 0.34 + 9^2 \times b$ $\left[= 34.16 \text{ or } \frac{854}{25}\right]$	**M1**	Applies $\mathrm{E}(X^2)$ formula with their a and b Implied by sight of 34.16
	$\mathrm{Var}(X) = \mathrm{E}(X^2) - \mathrm{E}(X)^2$ $= 34.16 - 5.16^2 \left[= 7.5344 \text{ or } \frac{4709}{625}\right]$	**M1**	Correctly uses $\mathrm{Var}(X)$ formula with values substituted
	$= 2.745$	**A1**	**AWRT**
		3	
	Total	**7**	

Q	Answer	Marks	Comments
10(a)	$\left[s = 2 + 5t - t^2 \Rightarrow\right]$ $\left[v = \frac{\mathrm{d}s}{\mathrm{d}t} =\right] 5 - 2t$	**M1**	Correct expression in terms of t for velocity in the first 6 seconds. **PI** by (part of the) correct line on diagram for first phase of motion.
	$t = 0 \Rightarrow v = 5$ and $t = 6 \Rightarrow v = -7$	**A1**	**PI** by fully correct line on diagram for first phase of motion.
	Fully correct line segment from $(0, 5)$ to $(6, -7)$	**B1**	Ignore velocities not given on diagram if straight-line segment correct.
	$\left[\frac{4-(-4)}{10-6} =\right] 2$	**B1**	Correct gradient for line segment between 6 and 10 seconds on displacement-time graph. **PI** by correct line segment on velocity-time graph.
	Correct line segment from $(6, 2)$ to $(10, 2)$	**B1ft**	Ignore velocity not given on diagram if straight-line segment correct. **ft** their gradient for line segment between 6 and 10 seconds on displacement-time graph provided a correct method for the calculation of the gradient is seen.
			
		5	

10(b)	$\frac{1}{2}\times 2.5\times 5 = 6.25$ [metres] $\frac{1}{2}\times 3.5\times 7 = 12.25$ [metres] $4\times 2 = 8$ [metres] [Total distance =] 26.5 [metres]	**M1** **A1**	Correct values or expressions for at least two relevant areas. **ft** their graph. **CAO**
		2	
	Total	**7**	

Q	Answer	Marks	Comments
11(a)	$19 = u \times 2 + \frac{1}{2} \times a \times 2^2$	**M1**	**oe** Use of $s = ut + \frac{1}{2}at^2$ with correct values substituted. Simplified or unsimplified
	$19 = 2u + 2a$	**A1**	Correct equation in u and a simplified. **PI** in later working.
	$112 = u \times 8 + \frac{1}{2} \times a \times 8^2$	**M1**	**oe** Use of $s = ut + \frac{1}{2}at^2$ with correct values substituted. Simplified or unsimplified
	$14 = u + 4a$	**A1**	**oe** Correct equation in u and a simplified. **PI** in later working.
	$a = 1.5$ and $u = 8$	**B1**	**CAO**
		5	
11(a) ALT	$\left[v_{BC} = u + 2a \Rightarrow\right]$ $19 = \frac{1}{2}\left(u + \left(u + 2a\right)\right) \times 2$ or $19 = u \times 2 + \frac{1}{2} \times a \times 2^2$	**M1**	**oe** Correct use of $v = u + at$ and $s = \frac{1}{2}(u+v)t$ with correct values and their v_{BC} substituted. Or use of $s = ut + \frac{1}{2}at^2$ with correct values substituted. Simplified or unsimplified
	$19 = 2u + 2a$	**A1**	**oe** Correct equation in u and a simplified. **PI** in later working.
	$\left[v_{CD} = \left(u + 2a\right) + a \times 6 = u + 8a \Rightarrow\right]$ $93 = \frac{1}{2}\left(\left(u + 2a\right) + \left(u + 8a\right)\right) \times 6$ or $93 = \left(u + 2a\right) \times 6 + \frac{1}{2} \times a \times 6^2$	**M1**	**oe** Correct use of $v = u + at$ and $s = \frac{1}{2}(u+v)t$ with correct values and their v_{CD} and v_{BC} substituted. Or use of $s = ut + \frac{1}{2}at^2$ with correct values and v_{BC} substituted. Simplified or unsimplified
	$31 = 2u + 10a$	**A1**	**oe** Correct equation in u and a simplified. **PI** in later working.
	$a = 1.5$ and $u = 8$	**B1**	**CAO**
		5	

11(b)	$\left[v = u + at \Rightarrow v = 8 + 1.5 \times 8 \Rightarrow\right]\ \ v = 20$ $\left[mv = 1400 \times 20 \Rightarrow\right]$ Momentum $= 28000$	**B1ft** **B1ft**	**ft** their u and a from **(a)** **ft** their velocity at *D*
		2	
11(b) ALT	$\left[v = u + at \Rightarrow v = 8 + 1.5 \times 8 \Rightarrow\right] v = 20$ or $\left[v = u + at \Rightarrow v = 11 + 1.5 \times 6 \Rightarrow\right] v = 20$ $\left[mv = 1400 \times 20 \Rightarrow\right]$ Momentum $= 28000\ \left[\text{kg m s}^{-1}\right]$	**B1ft** **B1ft**	**ft** their v_{CD}, u and a from **(a)** or **ft** their v_{BC} for 11 and their u and a from **(a)** **ft** their velocity at *D*
		2	
	Total	**7**	

Q	Answer	Marks	Comments
12(a)	$\left[R_{AC} =\right]\ 8.5g\ \ [= 83.3\ \text{N}]$	**B1**	Correct normal reaction between *A* and *C*. **PI** by correct $F_{max(AC)}$
	$\left[F_{max(AC)} =\right]\ 1.7g\ \ [= 16.66\ \text{N}]$	**B1**	Correct maximum possible frictional force between *A* and *C*
	$\left[F_{max(AB)} =\right]\ 2.1g\ \ [= 20.58\ \text{N}]$	**B1**	Correct maximum possible frictional force between *A* and *B*
	$1.7g < 2.1g$ or $16.66 < 20.58$ so box *A* will slide.	**E1**	Comparison of their maximum frictional forces and correct concluding statement.
		4	
12(b)	$19 - 1.7g = 8.5a$ $a = 0.275$	**M1** **A1ft**	**oe** Correct equation of motion, (using their maximum frictional force between *A* and *C*). **AWRT** 0.275, allow $\frac{117}{425}$ **ft** their maximum frictional force between *A* and *C* provided it is less than 19 newtons.
		2	
	Total	**6**	