OXFORD

INTERNATIONAL AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM04) Unit FS2 – Further Statistics

Monday 24 June 2019 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables (enclosed).
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

Answer **all** questions in the spaces provided.

Do not write outside the box

1 The random variable $X \sim N(\mu, 4^2)$

A random sample of size 50 is taken.

A hypothesis test is conducted at the 1% level of significance with the hypotheses

H_0: $\mu = 75$

H_1: $\mu > 75$

Find the power of the hypothesis test if the true population mean is equal to 77, giving your answer to two significant figures.

[5 marks]

Answer _____

5

Do not write outside the box

2 A business claims that the standard deviation of the mass of their tins of beans is 1.6 grams.

A random sample of 20 tins of beans is taken. The variance of the mass of these tins of beans is 4 grams^2

Assume that the mass of tins of beans is normally distributed.

Investigate the claim at the 5% level of significance.

[6 marks]

6

Do not write outside the box

3 The random variable X_i has a Poisson distribution with mean λ_i

3 (a) Show that the moment generating function of X_i is given by

$$M_{X_i}(t) = e^{\lambda_i(e^t - 1)}$$

[4 marks]

Do not write outside the box

3 (b) The random variables X_1 and X_2 are independent.

Given that $\lambda_1 = 2$ and $\lambda_2 = 3$, find the moment generating function of $X_1 + X_2$

Simplify your answer.

[2 marks]

3 (c) Describe the distribution of $X_1 + X_2$

[1 mark]

7

Turn over for the next question

Do not write outside the box

4 The following table shows the contents of a bag of coins.

Coin (cents)	10	20	50
Frequency	25	55	20

4 (a) Find the mean μ of the value of the coins in the bag.

[1 mark]

4 (b) A random sample of two coins is selected from the bag in the following way.

Step 1: A first coin is taken randomly from the bag, its value is recorded and then the coin is put back into the bag.

Step 2: A second coin is taken randomly from the bag, its value is recorded and then the coin is put back into the bag.

For example, a 10 cent coin followed by a 20 cent coin is recorded as (10, 20).

4 (b) (i) Write down all the distinct possible samples.

[2 marks]

Do not write outside the box

4 (b) (ii) The random variable X is the value of a coin taken randomly from the bag.

Find the sampling distribution of the mean, $\overline{X}$, of the two coins.

[5 marks]

4 (b) (iii) Find $\mathrm{E}(\overline{X})$.

[2 marks]

10

Turn over ►

Do not write outside the box

5 Devon is modelling the number of traffic accidents per day in a town.

Devon records the number of traffic accidents in a town each day for 100 days.

Devon records the results in the following table.

Accidents per day	0	1	2	3	4 or more
Frequency	22	30	33	13	2

There was a day with 4 accidents and a day with 7 accidents.

Devon claims that the number of accidents per day in the town can be modelled by a Poisson distribution.

Investigate Devon's claim using a 5% level of significance.

[10 marks]

Do not write outside the box

10

Do not write outside the box

6 A random variable X has a normal distribution with mean μ and variance σ^2

A sample is taken from X, taking the values $X_1, \ldots, X_n$

6 (a) Determine whether or not

$$\frac{\sum_{i=1}^{n} X_i}{n} + \frac{1}{n}$$

is a consistent estimator of μ.

[4 marks]

6 (b) (i) Show that

$$E(X_i^2) = \sigma^2 + \mu^2$$

[2 marks]

Do not write outside the box

6 (b) (ii) Show that

$$\frac{\sum_{i=1}^{n} X_i^2}{n} - \overline{X}^2$$

is a biased estimator of σ^2

[5 marks]

11

Do not write outside the box

7 Sean models the monthly milk production of a cow on a farm, in kilograms, with a normal distribution with known standard deviation 8 kilograms.

A random sample of the monthly milk production, X, of five cows on the farm was taken.

The results can be summarised as

$$\sum x = 4499 \qquad \text{and} \qquad \sum x^2 = 4048993$$

7 (a) Using Sean's model, construct a 95% confidence interval for the mean monthly milk production of a cow on this farm.

[4 marks]

Do not write outside the box

7 (b) Sean wants the width of the 95% confidence interval to be at most 5 kilograms.

Calculate how many more cows are needed in the sample to achieve this.

[4 marks]

Question 7 continues on the next page

Do not write outside the box

7 (c) Millie assumes the monthly milk production of a cow on the same farm is a normal distribution but with the standard deviation not known.

7 (c) (i) Using Millie's assumption, construct a 95% confidence interval for the mean monthly milk production by a cow on the farm.

[5 marks]

Do not write outside the box

7 (c) (ii) Sean and Millie each carry out a hypothesis test on the mean monthly milk production with the following hypotheses

H_0: $\mu = 890$

H_1: $\mu \neq 890$

Sean and Millie each use their confidence interval to carry out the test.

State whether Sean and Millie reach the same conclusion.

Explain your answer.

[2 marks]

15

Turn over for the next question

Do not write outside the box

8 A chocolate factory has two machines, **A** and **B**.

The chocolate produced in one hour, in tonnes, by the machines can be modelled by normal distributions.

Measurements of the chocolate produced during random 1 hour intervals by Machine **A** and Machine **B** were taken, which are summarised in the table below.

	Number of Measurements	**Mean (tonnes)**	**Standard Deviation (tonnes)**
Machine A	5	5.6	0.20
Machine B	7	6.0	0.24

Beth claims that on average, Machine **B** produces more chocolate in 1 hour than Machine **A**.

8 (a) Using a pooled estimate of variance, investigate at the 1% level of significance whether Beth's claim is valid.

[9 marks]

Do not write outside the box

8 (b) State the assumption required for the test in part **(a)** to be valid.

[1 mark]

Turn over for the next question

Do not write outside the box

8 (c) Investigate at the 10% level of significance whether the assumption stated in part **(b)** is valid.

[6 marks]

16

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED