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MA03

(9660/MA03) Unit P2 Pure Mathematics

Mark scheme

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211XMA03/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$\mathrm{h}(x) = \dfrac{25}{x+4} - 5$ $\mathrm{h}(x) = \dfrac{5-5x}{x+4}$	M1 A1	
		2	

Q	Answer	Marks	Comments
1(b)(i)	$x = \dfrac{5-5y}{y+4}$ $xy + 4x = 5 - 5y$ $[y = \mathrm{h}^{-1}(x) =]\ \dfrac{5-4x}{x+5}$	M1 M1 A1	‘Swap’ x and y Attempt to rearrange *their* (a) **oe**
		3	

Q	Answer	Marks	Comments
1(b)(ii)	[All values of $\mathrm{h}^{-1}(x)$], $[\mathrm{h}^{-1}(x)] \neq -4$	B1	
		1	

	Question 1 Total	6	

Q	Answer	Marks	Comments
2(a)(i)	$4-\lambda=-1-\mu$ $-2+5\lambda=5-4\mu$	M1	
	$\lambda=3,$ $\mu=-2$	A1	
	$-3+2\lambda=11+\mu c$ $c=4$	A1	
		3	

Q	Answer	Marks	Comments
2(a)(ii)	$(1, 13, 3)$	B1F	Must be coordinates – not a column vector
		1	

Q	Answer	Marks	Comments
2(b)	$\begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} . \begin{pmatrix} -1 \\ -4 \\ c \end{pmatrix} = 0$	M1	
	$1-20+2c=0$	A1	
	$c=9.5$	A1	oe
		3	

	Question 2 Total	7	

Q	Answer	Marks	Comments
3(a)	$[3\sin\theta - 3\cos\theta =]$ $R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$ $R = \sqrt{18}$ $\alpha = 45^\circ$	M1 A1	Seen or used Both correct
		2	

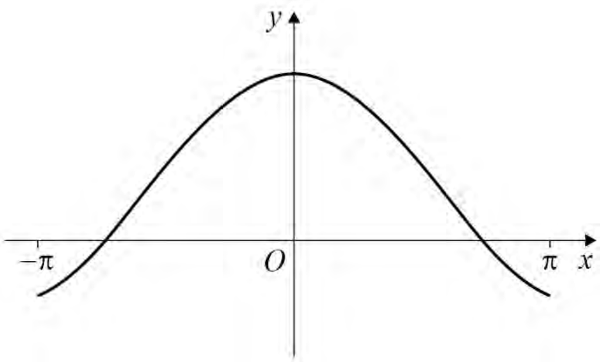
Q	Answer	Marks	Comments
3(b)(i)	$\left[y_{\max}^2 =\right]18$	B1	
		1	

Q	Answer	Marks	Comments
3(b)(ii)	$\left[y_{\min}^2 =\right]0$	B1	
		1	

Q	Answer	Marks	Comments
3(b)(iii)	$\sqrt{18}\sin(\theta - 45) = -\frac{3\sqrt{6}}{2}$ $\sin(\theta - 45) = -\frac{\sqrt{3}}{2}$ $\theta - 45 = -60,\ -120$ $\theta = -15,\ -75$	M1 A1+A1	PI
		3	

	Question 3 Total	7	

Q	Answer	Marks	Comments
4(a)	Stretch $+$ either I **or** II Parallel to y-axis I SF 2 II **Followed by** Translation $\begin{bmatrix}0\\k\end{bmatrix}$ $k=1$	M1 A1 M1 A1	 Including correct terminology Including correct terminology
4(a) ALT	Translation $\begin{bmatrix}0\\k\end{bmatrix}$ $k=0.5$ **Followed by** Stretch in y-direction SF 2	(M1) (A1) (M1) (A1)	Including correct terminology Including correct terminology
		4	

Q	Answer	Marks	Comments
4(b)		B1 B1	Correct shape, symmetric about y-axis (ignore graph outside the given range) $y = 3$ indicated or stated
		2	

Q	Answer	Marks	Comments
4(c)	$[\text{Vol} =]\ \pi \int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} (1+2\cos x)^2\ dx$	B1	Including π, correct limits and dx
	$(1+2\cos x)^2 = 1+4\cos x+4\cos^2 x$	B1	PI
	$[V = \pi\int\ 1+4\cos x+2\cos 2x+2\,dx\,]$	M1	Attempt at integration, must be in form $ax+b\sin x+c\sin 2x$
	$=[\pi]\ (3x+4\sin x+\sin 2x)$	A1	
	$=[\pi]\ \{(2\pi+2\sqrt{3}-\frac{\sqrt{3}}{2})-(-2\pi-2\sqrt{3}+\frac{\sqrt{3}}{2})\}$	m1	Attempt at subst correct limits
	$[V =]\ \pi(4\pi+3\sqrt{3})$	A1	
		6	

	Question 4 Total	**12**	

Q	Answer	Marks	Comments
5(a)	$(1+x^2)^{0.5}=1+0.5(x^2)+\frac{0.5\times-0.5}{2}(x^2)^2$	**M1**	$1+ax^2+bx^4$
	$=1+0.5x^2-0.125x^4$	**A1**	**oe**
		2	

Q	Answer	Marks	Comments
5(b)	$\int_0^{0.5}\sqrt{1+x^2}\ \mathrm{d}x=\int 1+0.5x^2-0.125x^4\ \mathrm{d}x$		
	$=x+\frac{1}{6}x^3-\frac{1}{40}x^5$	**M1** **A1**	$x+cx^3+dx^5$
	$\int_0^{0.5}\ =0.5+\frac{1}{6}\times0.5^3-\frac{1}{40}\times0.5^5$	**m1**	Attempt at subst correct limits
	$=0.52005$	**A1**	
		4	

<table>
<tr><th>Q</th><th>Answer</th><th>Marks</th><th>Comments</th></tr>
<tr><td rowspan="4">5(c)</td><td rowspan="2">

x	y
0	$\sqrt{1+0^2} = 1$
0.125	$\sqrt{1+0.125^2} = 1.007782$
0.25	$\sqrt{1+0.25^2} = 1.0307764$
0.375	$\sqrt{1+0.375^2} = 1.068000$
0.5	$\sqrt{1+0.5^2} = 1.118034$

</td><td>B1</td><td>All five correct x values (and no extra used)
PI by five correct y values</td></tr>
<tr><td>M1</td><td>At least four correct y values in exact form or decimals, rounded or truncated to 3 dp or better (in table or formula)</td></tr>
<tr><td>$\frac{1}{3}\times 0.125\times$
$[1+1.118034+2(1.0307764)+4(1.007782+1.068000)]$</td><td>m1</td><td>Correct sub into formula with $h =$ 0.125 OE and at least four correct y values either listed, with $+$ signs, or totalled</td></tr>
<tr><td>$= 0.52011$</td><td>A1</td><td>CAO, must see this value exactly and no error seen</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
</table>

	Question 5 Total	**10**	

Q	Answer	Marks	Comments
6(a)	$\tan\beta = \tan(45-\alpha)$ $= \dfrac{\tan 45 - \tan\alpha}{1+\tan 45\tan\alpha}$ $= \dfrac{1-\tan\alpha}{1+\tan\alpha}$	M1 A1	PI
		2	

Q	Answer	Marks	Comments
6(b)	$(1+\tan\alpha)(1+\tan\beta)$ $= (1+\tan\alpha)(1+\dfrac{1-\tan\alpha}{1+\tan\alpha})$ $= (1+\tan\alpha)(\dfrac{1+\tan\alpha+1-\tan\alpha}{1+\tan\alpha})$ $= (1+\tan\alpha)(\dfrac{2}{1+\tan\alpha})$ $= 2$	M1 A1	
		2	

Q	Answer	Marks	Comments
6(c)	$\tan\beta = \tan(45-\alpha), \qquad \alpha=\beta$ $x = \tan\beta$ $x = \dfrac{1-x}{1+x}$ $x^2+2x-1=0$ $x = \dfrac{-2\pm\sqrt{4+4}}{2}$ $x = -1+\sqrt{2}$ only	M1 m1 A1	PI No errors seen
		3	

	Question 6 Total	**7**	

Q	Answer	Marks	Comments
7(a)(i)	$f(x)=\sin(\ln(2x))+4x-3$ $f(0.6)=-0.419...$ $f(0.7)=0.130...$	**M1**	**or** reverse
	Change of sign, $0.6 < x < 0.7$	**A1**	Must have both statement and interval in words or symbols **or** comparing 2 sides:
		(M1)	at 0.6, $\sin(\ln(1.2)) = 0.18 < 3 - 2.4 = 0.6$; at 0.7, $\sin(\ln(1.4)) = 0.33 > 3 - 2.8 = 0.2$
		(A1)	Conclusion as before
		2	

Q	Answer	Marks	Comments
7(a)(ii)	$x_2 = 0.705$	**B1**	
	$x_3 = 0.666$	**B1**	
		2	

Q	Answer	Marks	Comments
7(b)	$\frac{dy}{dx} = -\frac{1}{x}\sin(\ln(3x))$	**M1**	$\frac{k}{x}\sin(\ln(3x))$
	$\frac{dy}{dx} = 0, \quad \ln(3x) = 0$	**M1**	
	$3x = 1, \quad x = \frac{1}{3}, \quad y = 1$ $\left(\frac{1}{3}, 1\right)$	**A1**	
		3	

Q	Answer	Marks	Comments
7(c)	$\frac{dy}{dx} = \frac{1}{x} A\cos(\ln(2x)) - \frac{1}{x} B\sin(\ln(3x))$	M1 A1	$\frac{dy}{dx} = \frac{p}{x}\cos(\ln(2x)) + \frac{q}{x}\sin(\ln(3x))$
	$\frac{d^2y}{dx^2} = -\frac{1}{x^2} A\cos(\ln(2x)) - \frac{1}{x^2} A\sin(\ln(2x))$ $+\frac{1}{x^2} B\sin(\ln(3x)) - \frac{1}{x^2} B\cos(\ln(3x))$	M1	$\frac{d^2y}{dx^2} = \frac{r}{x^2}\cos(\ln(2x)) + \frac{s}{x^2}\sin(\ln(2x))$ $+\frac{t}{x^2}\cos(\ln(3x)) + \frac{u}{x^2}\sin(\ln(3x))$
	$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y =$ $-A\cos(\ln(2x)) - A\sin(\ln(2x)) + B\sin(\ln(3x)) - B\cos(\ln(3x))$ $+A\cos(\ln(2x)) - B\sin(\ln(3x)) + A\sin(\ln(2x)) + B\cos(\ln(3x))$ $= 0$	A1	Be convinced
		4	

	Question 7 Total	11	

Q	Answer	Marks	Comments
8(a)	$\frac{d}{dx}(\cot x)=\frac{\sin x\times -\sin x-\cos x\times\cos x}{\sin^2 x}$	M1	$\frac{\mathrm{d}}{\mathrm{d}x}(\cot x)=\frac{A\sin x\times\sin x+B\cos x\times\cos x}{\sin^2 x}$
	$=\frac{-1}{\sin^2 x}$		
	$=-\mathrm{cosec}^2 x$	A1	Must see a middle line
		2	

Q	Answer	Marks	Comments
8(b)(i)	$[\frac{\mathrm{d}x}{\mathrm{d}y}=]-\frac{3}{4}\mathrm{cosec}^2\left(2y-\frac{\pi}{2}\right)\times 2$	M1	$[\frac{\mathrm{d}x}{\mathrm{d}y}=]\ k\,\mathrm{cosec}^2\left(2y-\frac{\pi}{2}\right)$
	$=-\frac{3}{2}\mathrm{cosec}^2\left(2y-\frac{\pi}{2}\right)$	A1	All correct
		2	

Q	Answer	Marks	Comments
8(b)(ii)	At $\left(\frac{3}{4},\frac{3\pi}{8}\right)$		
	$\frac{\mathrm{d}x}{\mathrm{d}y}=-\frac{3}{2}\mathrm{cosec}^2\left(2\times\frac{3\pi}{8}-\frac{\pi}{2}\right)=-3$	M1	**PI** (must have scored M1 in (i))
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{3}$	M1	
	Gradient of normal$=3$	A1	
		3	

	Question 8 Total	**7**	

Q	Answer	Marks	Comments
9(a)	$3x^2+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}=3x\frac{\mathrm{d}y}{\mathrm{d}x}+3y$	**M1**	Either LHS or RHS correct
	$\frac{\mathrm{d}y}{\mathrm{d}x}(y^2-x)=y-x^2$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y-x^2}{y^2-x}$	**A1**	**AG** must see a middle line
		2	

Q	Answer	Marks	Comments
9(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=0,\quad y=x^2$	**M1**	
	$x^3+x^6=3x^3$		
	$x^3(x^3-2)=0$	**m1**	Attempt to solve
	$x=2^{\frac{1}{3}},\quad y=2^{\frac{2}{3}}\quad$ or $\quad(2^{\frac{1}{3}},\ 2^{\frac{2}{3}})$	**A1**	
		3	

Q	Answer	Marks	Comments
9(b)(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y-x^2}{y^2-x}$		
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=$ $\frac{(y^2-x)\left(\frac{\mathrm{d}y}{\mathrm{d}x}-2x\right)-(y-x^2)\left(2y\frac{\mathrm{d}y}{\mathrm{d}x}-1\right)}{(y^2-x)^2}$	**M1** **A1**	
	$\left[\text{As } \frac{\mathrm{d}y}{\mathrm{d}x}=0 \text{ at stationary points, } \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{-2xy^2+x^2+y}{(y^2-x)^2}\right]$		
	Num $=(2^{\frac{4}{3}}-2^{\frac{1}{3}})(0-2^{\frac{4}{3}})-(2^{\frac{2}{3}}-2^{\frac{2}{3}})(0-1)$	**m1**	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\ [=-2]<0$, MAX	**A1**	
		4	

	Question 9 Total	**9**	

Q	Answer	Marks	Comments
10(a)	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)$ rate of change of x is proportional to x, difference in temperature	**E1**	Complete explanation
		1	

Q	Answer	Marks	Comments
10(b)	$\int \frac{\mathrm{d}x}{x} = \int -k\mathrm{d}t$	**M1**	Separate variables
	$\ln x = -kt + c$ $t = 0,\ x = 70$ $c = \ln 70$ [= 4.248...]	**A1**	
	$t = 5,\ x = 50$ $\ln 50 = -5k + \ln 70$	**m1**	Attempt to find k
	$k = \frac{1}{5}\ln\frac{7}{5}$ $[= 0.06729...]$	**A1**	
	$\ln x = -\frac{1}{5}\ln\frac{7}{5} \times 15 + \ln 70$	**M1**	$\ln x = -their(k) \times 15 + their(c)$
	$x = 25.5$ Temp $= 45.5$	**A1**	
		6	

Q	Answer	Marks	Comments
10(c)	$\ln 20 = -\frac{1}{5}\left(\ln\frac{7}{5}\right)t + \ln 70$	**M1**	$\ln 20 = -their(k)t + their(c)$
	$t = 18.6$	**A1**	
		2	

	Question 10 Total	**9**	

Q	Answer	Marks	Comments
11(a)	$u = x \quad dv = e^{-0.5x}$ $du = 1 \quad v = -2e^{-0.5x}$	M1	Correct form **PI**
	$\int \; =] -2xe^{-0.5x} + 2\int e^{-0.5x}dx$	m1	Correct subst into parts formula
	$= -2xe^{-0.5x} - 4e^{-0.5x}$	A1	
	$\int_0^6 \; = (-12e^{-3} - 4e^{-3}) - (-4)$	m1	Subst limits into $axe^{-0.5x} + be^{-0.5x}$
	$= 4 - 16e^{-3}$	A1	**ISW**
		5	

Q	Answer	Marks	Comments
11(b)	$2u\,du = dx$ oe	B1	
	$[\int \frac{\sqrt{x+1}}{x-3}\,dx =] \int \frac{u}{u^2-4} \times 2u\,du$	M1	All in terms of *u*
	$= 2\int 1 + \frac{4}{u^2-4}\,du$	A1	
	$\frac{4}{u^2-4} = \frac{A}{u-2} + \frac{B}{u+2}$ $A = 1, \; B = -1$	m1	Use of partial fractions
	$\int \; = 2\int 1 + \frac{1}{u-2} - \frac{1}{u+2}$		
	$= 2(u + \ln\left(\frac{u-2}{u+2}\right))$	A1	
	$[x]_8^{15} = [u]_3^4$	B1	Or changing back into *x*
	$\int = 2[(4 + \ln\frac{1}{3}) - (3 + \ln\frac{1}{5})]$	M1	Correct subst into $au + b\ln(u-2) - b\ln(u+2)$ oe
	$= 2(1 + \ln\frac{5}{3})$	A1	
	$= 2\ln(\frac{5e}{3})$	A1	
		9	

	Question 11 Total	**14**	

Q	Answer	Marks	Comments
12(a)	$\text{f}(x)=\dfrac{A}{1-x}+\dfrac{B}{2-x}+\dfrac{C}{5-2x}$ $4x^2+5=$ $A(2-x)(5-2x)+B(1-x)(5-2x)+C(1-x)(2-x)$	B1	Correctly eliminating fractions
	$x=1$: $A=3$ $x=2$: $B=-21$	M1 A1	Attempt at finding one constant At least one constant correct
	$x=2.5$: $C=40$	A1	All three constants correct
		4	

Q	Answer	Marks	Comments
12(b)	$(2-x)^{-1}=2^{-1}\left(1-\dfrac{x}{2}\right)^{-1}$	M1	
	$(2-x)^{-1}=\dfrac{1}{2}+\dfrac{1}{4}x+\dfrac{1}{8}x^2$	A1	
		2	

Q	Answer	Marks	Comments
12(c)	$\mathrm{f}(x)$: $(1-x)^{-1} = 1 + x + x^2$ [$(2-x)^{-1} = 2^{-1}\left(1+\frac{1}{2}x+\frac{1}{4}x^2\right)$] $(5-2x)^{-1} = 5^{-1}\left(1+\frac{2}{5}x+\frac{4}{25}x^2\right)$ oe $\mathrm{f}(x) =$ $3(1+x+x^2)-\frac{21}{2}\left(1+\frac{1}{2}x+\frac{1}{4}x^2\right)+\frac{40}{5}\left(1+\frac{2}{5}x+\frac{4}{25}x^2\right)$ $\mathrm{f}(x) = \frac{1}{2}+\frac{19}{20}x+\frac{331}{200}x^2$	**B1** **B1** **M1** **A1**	
		4	

	Question 12 Total	**10**	

Q	Answer	Marks	Comments
13(a)	$t = \frac{x}{c}, \quad y = c \div \frac{x}{c}$ $xy = c^2$	**B1**	**oe**
		1	

Q	Answer	Marks	Comments
13(b)	$\frac{\mathrm{d}x}{\mathrm{d}t} = c \quad \frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{c}{t^2}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{t^2}$	**B1**	
	$y - \frac{c}{p} = -\frac{1}{p^2}(x - cp)$	**M1**	Attempt to find equ of tgt
	$[p^2 y - cp = -x + cp]$		
	$At \ \ A, \ x = 2cp \quad At \ \ B, \ y = \frac{2c}{p}$		
	Midpoint AB $\left(cp, \frac{c}{p}\right)$	**A1**	
	Normal		
	$y - \frac{c}{p} = p^2(x - cp)$	**M1**	Attempt to find equ of normal
	At C, $y = x$, $\quad y - \frac{c}{p} = p^2(y - cp)$		
	$y = \frac{c(1-p^4)}{p(1-p^2)} = \frac{c(1-p^2)(1+p^2)}{p(1-p^2)} = \frac{c(1+p^2)}{p}$	**m1**	Attempt to find C or D
	At D, $y = -x$, $\quad y - \frac{c}{p} = p^2(-y - cp)$		
	$y = \frac{c(1-p^4)}{p(1+p^2)} = \frac{c(1-p^2)(1+p^2)}{p(1+p^2)} = \frac{c(1-p^2)}{p}$	**A1**	Both correct
	Midpoint CD		
	$x = 0.5\left(\frac{c(1+p^2)}{p} + \frac{c(-1+p^2)}{p}\right) = cp$		
	$y = 0.5\left(\frac{c(1+p^2)}{p} + -\frac{c(-1+p^2)}{p}\right) = \frac{c}{p}$		
	Both AB and CD have same midpoint	**A1**	
		7	

Q	Answer	Marks	Comments
13(c)	$\lvert AB \rvert = \lvert CD \rvert = \dfrac{2c}{p} \times \sqrt{1+p^4}$	B1	Either *AB* or *CD*
	As *AB* and *CD* are perp, equilateral and *P* is the midpoint then *ABCD* form a square	E2,1	
		3	

	Question 13 Total	**11**	