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Centre number	Candidate number	
Surname		
Forename(s)		
Candidate signature	I declare this is my own work.	/

INTERNATIONAL AS FURTHER MATHEMATICS

(9665/FM01) Unit FP1 Pure Mathematics

Wednesday 4 January 2023 07:00 GMT Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions

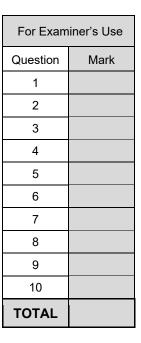
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.





FM01

Answer **all** questions in the spaces provided.

- **1** A curve has equation $y = 2x^2 + 3x$
- 1 (a) A line passes through two points on the curve, one where $x = -\frac{5}{4}$ and the other where $x = -\frac{5}{4} + h$

Find the gradient of this line in the form a+bh where a and b are integers.

[4 marks]

Answer _____

1 (b)	Show how the answer to part (a) can be used to find the gradient of the curve at the point where $x = -\frac{5}{4}$	Do not write outside the box
	[2 marks]	
		6

Turn over for the next question



2	The quadratic	equation
2	The quadratic	equation

$$x^2 + bx + c = 0$$

where b and c are real, has -4+7i as one of its roots.

Find the value of $\,b\,$ and the value of $\,c\,$

[4 marks]

b =

c =

4



3		For each of the improper integrals below, either find its exact value or explain who finite value.	y it has
		Show all necessary working.	
3	(a)	$\int_{3}^{\infty} \frac{1}{\sqrt[5]{x}} \mathrm{d}x$	3 marks]
		Answer	
3	(b)	$\int_3^\infty \frac{1}{x^2} \mathrm{d}x$	3 marks]
		Answer	

6



4	(a)	Find the general solution of the equation
		$\sin\left(5x-\frac{\pi}{12}\right)=0.5$
		giving your answer in terms of π [4 marks]
		Answer



4	(b)	Hence find the solution of the equation
-	(~ <i>)</i>	richee ind the solution of the equation

$$\sin\left(5x - \frac{\pi}{12}\right) = 0.5$$

which is closest to $\frac{3\pi}{2}$ giving your answer in terms of π

[3 marks]

Answer____

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Turn over for the next question



5	(a)	Show that $\frac{1}{r!} - \frac{1}{(r+1)!} = \frac{r}{(r+1)!}$	[1 mark]
5	(b)	Use the method of differences to find an expression for the sum of the series $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + + \frac{n}{(n+1)!}$	[4 marks]
		Answer	



5	(c)	Use you	r answer to	part (b) to find	the value of
	\ · /			P 21. 2 (.2)	,	

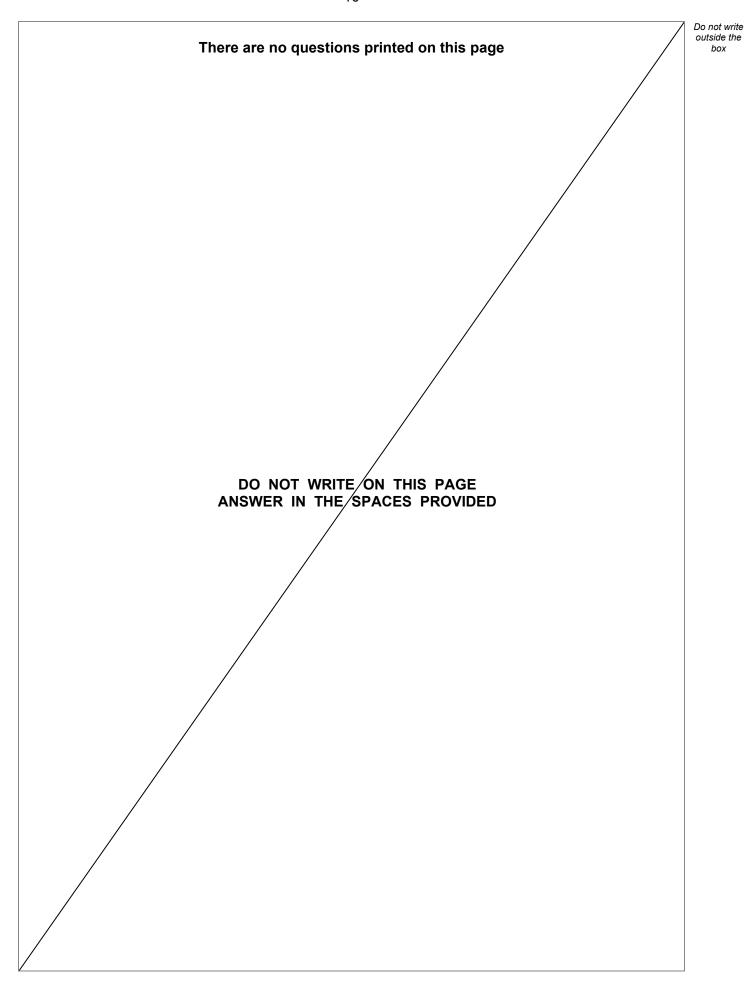
$$\sum_{r=5}^{\infty} \frac{r}{(r+1)!}$$

Give your answer in the form $\frac{1}{k}$ where k is an integer. [2 marks]

Answer

Turn over for the next question







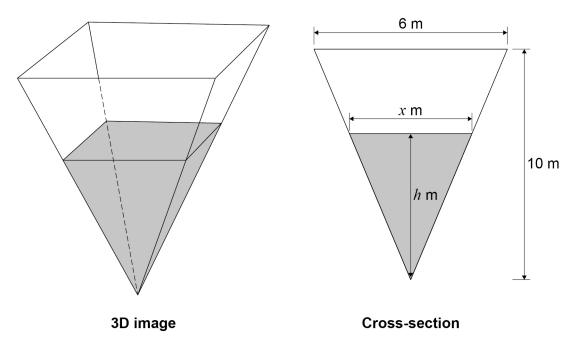
6	Find all the solutions of the equation	Do not write outside the box
	$3z^2=z^*$ [7 marks]	
	Answer	7



7 [The volume of a square-based pyramid is given by the formula $V = \frac{1}{3}a^2h$ where a is the length of the side of the square base, and h is the height.]

A water tank in the shape of an inverted square-based pyramid has height 10 metres, and the length of the side of the square base is 6 metres.

Not drawn to scale



The tank fills with water at a rate of 0.54 m³ per minute.

At time t minutes the depth of water in the tank is h metres.

The horizontal surface of the water at time t minutes is in the shape of a square of side x metres.

7 (a)	Explain why $x = 0.6h$	[1 mark]

7	(b)	Find an expression in terms of h for the volume of water in the tank at time t minutes. [2 marks]	Do not write outside the box
			-
		Answer	-
7	(c)	Find the rate at which the depth of water in the tank is increasing when $h=4$ [5 marks]	
			-
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$$\frac{x^2}{4} + y^2 = 1$$

The ellipse E_2 has equation

$$\frac{x^2}{4} + \frac{y^2}{k} = 1$$

where $\,k\,$ is a positive constant.

8	(a)	Describe fully	the	transformation	that maps	E_1	onto	$E_{\mathbf{z}}$
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[2 marks]

8 (b) It is given that the line L has equation

$$y = 2x + c$$

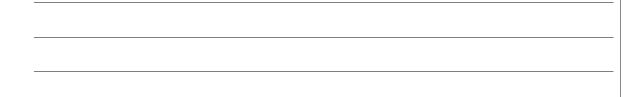
where c is a constant.

8 (b) (i) Show that if L intersects E_2 then

$$c^2 - k \le A$$

where A is a constant to be found.

[5 marks]





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8 (b) (ii) Given that the line $y = 2x + 7$ is a tangent to E_2 find the value of k	
(2) (1)	To order that the line $y = 2x + 1$ is a tangent to E_2 find the value of x	[2 marks]
	k =	9

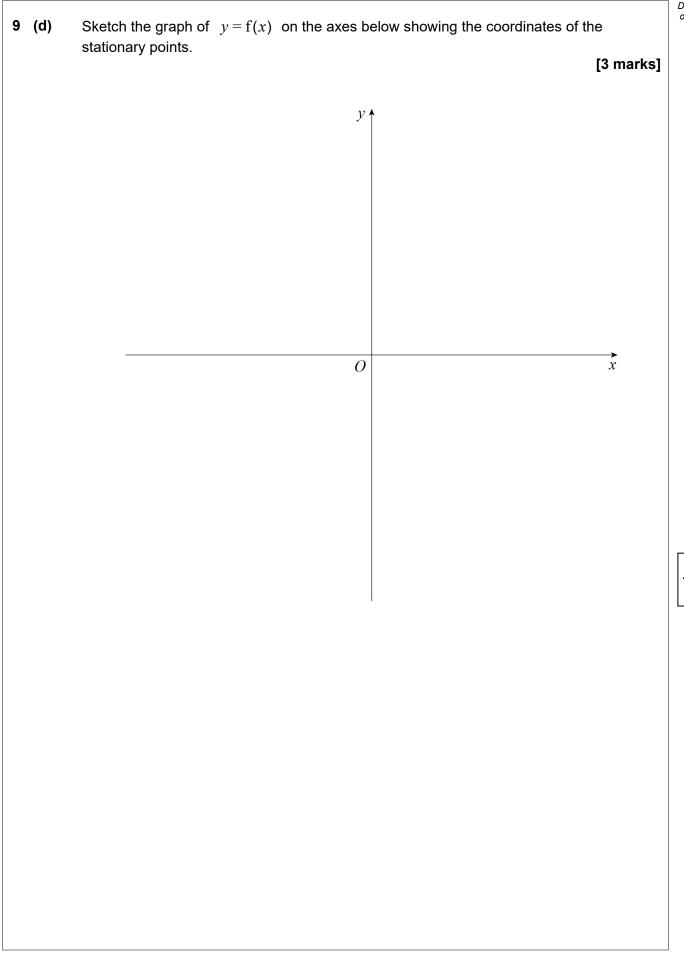


9		The function $\ f$ is defined by
		$f(x) = \frac{4x+5}{x^2+4x+5}$
9	(a) (i)	Explain why the graph of $y = f(x)$ has no vertical asymptotes. [2 marks]
9	(a) (ii)	Write down the equation of the horizontal asymptote of the graph of $y = f(x)$ [1 mark]
		Answer
9	(b)	Prove that if the line $y = k$ where k is a constant, intersects the graph of $y = f(x)$ then
		$k^2 + 3k - 4 \le 0$ [4 marks]



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9	(c)	Use the result in part (b) to find the coordinates of the stationary points of the graph of $y = f(x)$	
		[5 marks]	
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		Answer	-
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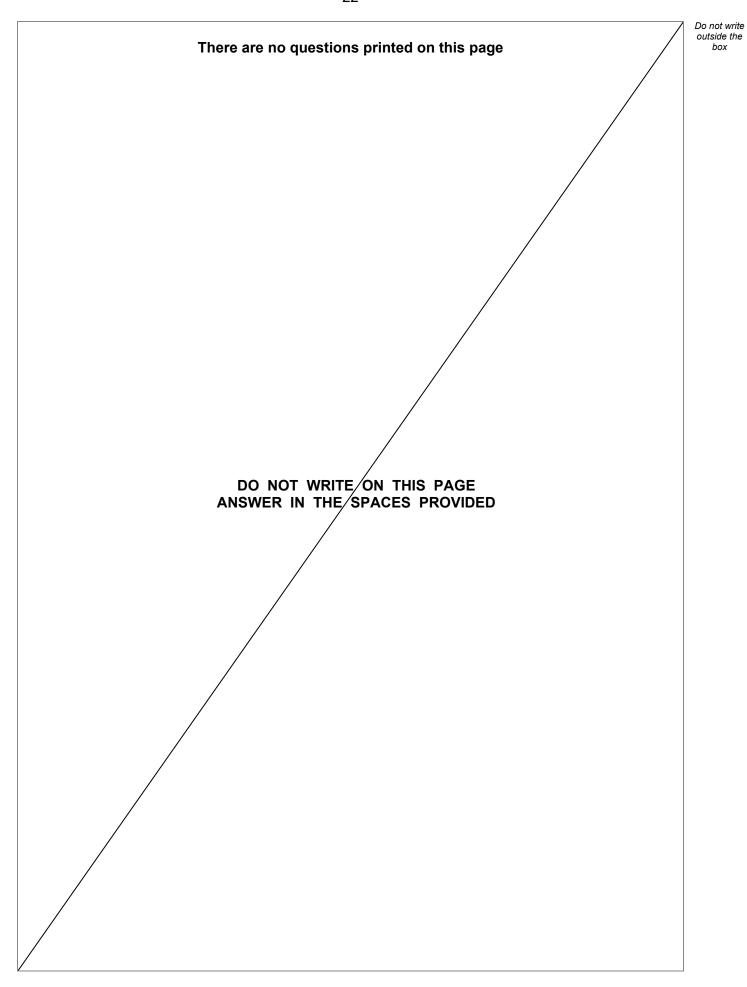
10	The circle C is the locus of points on an Argand diagram such the	ıat
	z = 5	
	The line L is the locus of points on an Argand diagram such that	t
	Re(z) = 3	
10 (a)	Draw C and L on the Argand diagram.	[2 marks]
	$\operatorname{Im}(z)$	
	O Re	$\overrightarrow{e}(z)$
10 (b)	Find all the points which are ${\bf both}$ a distance of exactly one unit f of exactly one unit from L	
		[6 marks]



		Do l out
Answer		
The half-line H is the locus of points on an Argand diagram such that		
$\arg(z+5) = \frac{2\pi}{5}$		
The half-line H meets the circle C at the point z_1		
Find z_1 giving your answer in the form $r(\cos\theta + i\sin\theta)$		
	[3 marks]	
$z_1^{}=$		

END OF QUESTIONS







Question number	Additional page, if required. Write the question numbers in the left-hand margin.



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