

OXFORD

INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL AS MATHEMATICS MA01

(9660/MA01) Unit P1 Pure Mathematics

Mark scheme

June 2022

Version 1.0 Final



2 2 6 X M A 0 1 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from [oxfordaqaexams.org.uk](https://www.oxfordaqaexams.org.uk)

Copyright information

OxfordAQA retains the copyright on all its publications. However, registered schools/colleges for OxfordAQA are permitted to copy material from this booklet for their own internal use, with the following important exception: OxfordAQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Copyright © 2022 Oxford International AQA Examinations and its licensors. All rights reserved.

Key to mark scheme abbreviations

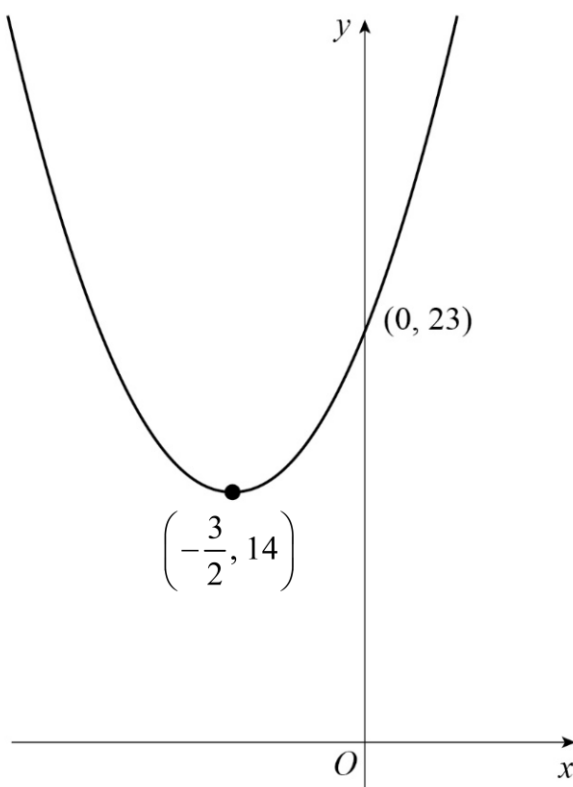
M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	$a = 9$	B1	
		1	
1(a)(ii)	$a = 15$	B1	
		1	
1(b)	$5(x-7)+2(y+2) [=6]$ $5x+2y=37$	M1 A1	oe Clear correct use of $(x-7)$ and $(y+2)$, condone slip with constant. oe but must be in this form.
		2	
	Total	4	

Q	Answer	Marks	Comments
2(a)	$\frac{13-1}{4-(-2)} [= 2]$ $\frac{y-13}{x-4} = 2$ or $\frac{y-1}{x-(-2)} = 2$ $y = 2x + 5$	M1 M1 A1	oe Use of coordinates with method to find gradient of J or a correct pair of simultaneous equations. oe. ft their gradient. May see $y = 2x + c$ and substitution of coordinates to find c Must be in the form of an equation. CAO must come from correct working.
		3	
2(b)(i)	$3x^2 - 6x + 3 [= 0]$ $[b^2 - 4ac =]$ $(-6)^2 - 4 \times 3 \times 3$ or $6^2 - 4 \times 3 \times 3$ $(-6)^2 - 4 \times 3 \times 3 = 0$ One distinct real solution.	B1 M1 A1ft	Correct rearrangement. Correct substitution into $b^2 - 4ac$ Could see $a = 1$, $b = -2$ and $c = 1$ used. ft their a , b and c provided rearrangement attempted isolating terms on LHS. Correct evaluation of their discriminant & consistent conclusion for number of distinct real solutions.
		3	
2(b)(i) ALT	$3x^2 - 6x + 3 [= 0]$ $3(x-1)^2 [= 0]$ or $(x-1)^2 [= 0]$ $[x = 1 \text{ is a repeated root}]$ One distinct real solution.	B1 M1 A1ft	Correct rearrangement. Correct factorisation or substitution into the quadratic formula. ft their quadratic expression. PI by $x = 1$ Correct evaluation of their correct factorisation or use of quadratic formula & consistent conclusion for number of distinct real solutions.
		3	
2(b)(ii)	J is a tangent to the curve or J touches the curve [at one point]	E1ft	Correct geometrical description. ft their final answer to part (b)(i) Allow 'intersect at one point'
		1	
	Total	7	

Q	Answer	Marks	Comments
3	$9 + 5\sqrt{7} = a(3 - \sqrt{7})$ $[a =] \frac{9 + 5\sqrt{7}}{3 - \sqrt{7}}$ $\left[a = \frac{9 + 5\sqrt{7}}{3 - \sqrt{7}} \times \frac{3 + \sqrt{7}}{3 + \sqrt{7}} \right]$ $27 + 9\sqrt{7} + 15\sqrt{7} + 35$ or $27 + 9\sqrt{7} + 15\sqrt{7} + 5\sqrt{7} \cdot \sqrt{7}$ or $27 + 24\sqrt{7} + 35$ or $62 + 24\sqrt{7}$ $3^2 - (\sqrt{7})^2$ or $9 - 7$ or 2 $31 + 12\sqrt{7}$	M1 A1 B1ft B1ft B1	Appreciates that y-coordinate of the object point needs to be multiplied by the scale factor. Correct scale factor. Multiplies their numerator by the conjugate of their denominator and expands. Condone one slip. Multiplies their denominator by its conjugate and expands.
3 ALT	$[(3 - \sqrt{7})(b + c\sqrt{7}) =] 3b + 3c\sqrt{7} - b\sqrt{7} - 7c$ $[(3 - \sqrt{7})(b + c\sqrt{7}) =] (3b - 7c) + (3c - b)\sqrt{7}$ $[(3b - 7c) + (3c - b)\sqrt{7} = 9 + 5\sqrt{7} \Rightarrow]$ $3b - 7c = 9$ and $3c - b = 5$ $b = 31$ or $c = 12$ $31 + 12\sqrt{7}$	M1 A1 B1ft B1 B1	oe Correct expansion. Condone one error. Correct simplification. PI in later working. Forms a correct pair of simultaneous equations. ft their $(3b - 7c) + (3c - b)\sqrt{7}$ At least one value correct.
	Total	5	

Q	Answer	Marks	Comments
4(a)	$\left(x + \frac{3}{2}\right)^2 \dots$ $4\left(x + \frac{3}{2}\right)^2 - 9 + 23$ $4\left(x + \frac{3}{2}\right)^2 + 14$	<p>M1</p> <p>Condone exact decimal equivalents to fractions throughout.</p> <p>Allow</p> $4\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 23$ <p>A1</p> $4\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{23}{4}\right]$ $4\left[\left(x + \frac{3}{2}\right)^2 + \frac{14}{4}\right]$ <p>A1</p> <p>CAO</p>	
		3	

Q	Answer	Marks	Comments
4(b)	<p>Correctly orientated symmetrical quadratic parabola.</p> <p>$(0, 23)$ labelled on y-axis</p> <p>Vertex labelled as $\left(-\frac{3}{2}, 14\right)$</p>	<p>B1</p> <p>B1</p> <p>B1ft</p>	<p>Condone label given as y-value only.</p> <p>ft their $(-b, c)$ from part (a).</p> <p>Accept correctly positioned vertex with $x = -\frac{3}{2}$ and $y = 14$ indicated on axes.</p>
			
		3	
4(c)	$k < 14$	B1ft	Correct inequality stated for their curve and their 14
		1	
4(d)	28	B1ft	ft $2 \times$ their 14 provided the curve does not intersect the x -axis. Final answer must be positive.
		1	
	Total	8	

Q	Answer	Marks	Comments
5(a)	$u_2 = 25k + 15$ $u_3 = k(25k + 15) + 15$ $u_3 = 25k^2 + 15k + 15$	B1 M1 A1	Correct expression for u_2 May be seen embedded. Multiplying their u_2 by k and adding 15, e.g. expression for u_3 unsimplified with expression for u_2 substituted in. AG Be convinced NMS scores 0
		3	
5(b)(i)	$25k^2 + 15k - 18 = 0$ $(5k - 3)(5k + 6) [= 0]$ $k = \frac{3}{5}$ and $k = -\frac{6}{5}$	M1 A1	Value for u_3 substituted, rearranged with zero on RHS and correct factorisation of LHS. PI by correct factorisation only or correct substitution into the quadratic formula unsimplified. oe CAO
		2	
5(b)(ii)	$T = \frac{3}{5}T + 15$ $T = 37.5$	M1 A1	Identifies suitable positive value of k from their part (b)(i) for $0 < k < 1$ oe oe CAO
		2	
	Total	7	

Q	Answer	Marks	Comments
6(a)	$p(m) = m^3 + 8m^2 + 14m - 8$ or $p(2m) = (2m)^3 + 8(2m)^2 + 14(2m) - 8$ $p(2m) = 8m^3 + 32m^2 + 28m - 8$ $(2m)^3 + 8(2m)^2 + 14(2m) - 8 = 4(m^3 + 8m^2 + 14m - 8)$ or $8m^3 + 32m^2 + 28m - 8 = 4(m^3 + 8m^2 + 14m - 8)$ or $8m^3 + 32m^2 + 28m - 8 = 4m^3 + 32m^2 + 56m - 32$ $2m^3 + 8m^2 + 7m - 2 = m^3 + 8m^2 + 14m - 8$ $\Rightarrow m^3 - 7m + 6 = 0$ or $4m^3 - 28m + 24 = 0$ $\Rightarrow m^3 - 7m + 6 = 0$	M1 A1 M1 A1	$p(m)$ or $p(2m)$ attempted correctly. Coefficients correctly evaluated. Forms correct equality relating their $p(m)$ and $p(2m)$ Condone one error in coefficients. AG NMS scores 0
		4	
6(b)(i)	$2^3 - 7(2) + 6 \quad [= 8 - 14 + 6]$ $8 - 14 + 6 = 0$	M1 A1	Substitution of $m = 2$ attempted. Must use Factor Theorem. CSO Power and product evaluated and shown that it equals zero.
		2	
6(b)(ii)	$m^2 + 2m - 3$ $[m^3 - 7m + 6 =] \quad (m - 2)(m^2 + 2m - 3)$ $\quad \quad \quad = (m - 2)(m - 1)(m + 3)$ $[m =] \quad -3, 1, 2$	M1 A1 A1	Quadratic factor by inspection, long division or comparing coefficients. Condone one sign error. Product of correct linear factor and their quadratic factor or Product of three correct linear factors provided the quadratic factor seen in earlier working CSO NMS scores 0
		3	
	Total	9	

Q	Answer	Marks	Comments
7(a)(i)	$2x + \frac{7}{2}x^2 - \frac{1}{3}x^3$ [+c]	M1 A1	M1: two terms correct. A1: all terms correct.
		2	
7(a)(ii)	$\left(2(3) + \frac{7}{2}(3^2) - \frac{1}{3}(3^3)\right) - \left(2(1) + \frac{7}{2}(1^2) - \frac{1}{3}(1^3)\right)$ or $\left(6 + \frac{63}{2} - 9\right) - \left(2 + \frac{7}{2} - \frac{1}{3}\right)$ $\left(6 + \frac{63}{2} - 9\right) - \left(2 + \frac{7}{2} - \frac{1}{3}\right) = 23\frac{1}{3}$	M1 A1	oe Correct substitution into their F(3) – F(1) Terms may be fully or partially evaluated. CSO, be convinced Terms fully evaluated and AG or working shown to $\frac{70}{3} = 23\frac{1}{3}$
		2	
7(b)	$h = 0.5$ [With $f(x) = 6 + 2^x$] $\left[I \approx \frac{h}{2}\{\dots\}\right]$ $\{\dots\} = f(1) + f(3) + 2(f(1.5) + f(2) + f(2.5))$ $\{\dots\} = 8 + 14 + 2(8.8284\dots + 10 + 11.6568\dots)$ $[I \approx] 0.25 \times 82.9705\dots = 20.743$ [to 3dp]	B1 M1 A1 A1	oe Summing the areas of trapezia using five terms. oe Accept 3 dp rounded or truncated. PI by correct final answer. CAO Must be 20.743
		4	
7(c)(i)	$23\frac{1}{3} - 20.743$ [Area \approx] 2.59	M1 A1	ft their $20.743 < 23\frac{1}{3}$ Condone 23.333... CAO
		2	
7(c)(ii)	The area found using the trapezium rule is an over-estimate. Under-estimate.	E1 E1	Explanation implying the value for the area found in part (b) is an over-estimate. Must have been awarded previous E1 to be able to be awarded this mark.
		2	
	Total	12	

Q	Answer	Marks	Comments
8(a)	$y = 3x^{\frac{8}{3}} - 3bx^{\frac{4}{3}} + 12$ $\left[\frac{dy}{dx} = \right] \frac{8}{3} \times 3x^{\frac{5}{3}} - \frac{4}{3} \times 3bx^{\frac{1}{3}} = 8x^{\frac{5}{3}} - 4bx^{\frac{1}{3}}$ $8 \times 8^{\frac{5}{3}} - 4b \times 8^{\frac{1}{3}} = 0$ $[b =] 32$	B1 M1 A1 M1 A1	Condone $12x^0$ M1 : At least one term correct, simplified or unsimplified. ft their expansion provided there are three terms, at least two having fractional powers. A1 : Fully correct derivative, simplified or unsimplified. oe Substitutes $x = 8$ into their derivative and sets it equal to zero. May have powers evaluated. PI CAO
		5	
8(b)	$[c =] 3 \times 8^{\frac{8}{3}} - 3 \times 32 \times 8^{\frac{4}{3}} + 12$ or $[c =] 3 \times 8^{\frac{2}{3}} \left(8^2 - 32 \times 8^{\frac{2}{3}} + 4 \times 8^{-\frac{2}{3}} \right)$ $[c =] -756$	M1 A1	Substitutes $x = 8$ into the equation of the curve with their b substituted. ft their expansion from part (a) . CAO
		2	
	Total	7	

Q	Answer	Marks	Comments
9(a)(i)	$y = 2(5 - 2y)^2 - 10(5 - 2y) + 13$ $y = 50 - 40y + 8y^2 - 50 + 20y + 13$ $8y^2 - 21y + 13 = 0$	M1 M1 A1	Substitutes for x oe Unsimplified. Brackets on RHS expanded. AG NMS scores 0 Must see evidence of both M marks.
		3	
9(a)(ii)	$(8y - 13)(y - 1) [= 0]$ or $(x - 3)(4x - 7) [= 0]$ $y = 1, y = \frac{13}{8}$ or $x = 3, x = \frac{7}{4}$ $(3, 1)$ and $\left(\frac{7}{4}, \frac{13}{8}\right)$	M1 A1 A1	Correct factorisation of x or y PI by one correct x -value or y -value, or correct substitution into quadratic formula unsimplified. oe Both x -values or y -values correct oe Condone not given as coordinates only if correct x and y values correctly linked.
		3	
9(b)	$-\frac{1}{2}$ $\left[\frac{dy}{dx} = \right] 4x - 10$ $x = 3 \Rightarrow \frac{dy}{dx} = 2$ or $x = \frac{7}{4} \Rightarrow \frac{dy}{dx} = -3$ $2 \times -\frac{1}{2} = -1 \Rightarrow L$ is a normal [at (3, 1)]	B1 M1 A1ft E1	oe Correct gradient of L identified. May be seen embedded in rearranged equation of L . Finds value for the gradient at either of the points of intersection. ft x values from part (a) . Correct concluding statement with 2 and $-\frac{1}{2}$ clearly seen and no others. CSO
		4	
9(c)	$4x - 10 = -\frac{1}{2}$ $\frac{19}{8}$ or 2.375	M1 A1	ft their $\frac{dy}{dx}$ and gradient of L CAO
		2	
	Total	12	

Q	Answer	Marks	Comments
10(a)	$\frac{a}{1-\frac{3}{5}} = 20$	M1	oe Use of $S_{\infty} = \frac{a}{1-r}$ with values correctly substituted.
	[Diameter = a] 8	A1	CAO
		2	
10(b)(i)	$\frac{7x^2 + 8x + 3}{x^2 + 1} = \frac{3}{5}$	M1	oe Forms correct equation.
	$8x^2 + 10x + 3 = 0$	M1	oe Forms correct quadratic equation set equal to zero.
	$x = -\frac{1}{2}$ and $x = -\frac{3}{4}$	A1	CAO
		3	
10(b)(ii)	$\frac{7x^2 + 8x + 3}{x^2 + 1} < 1$	B1	oe, PI by correct final answer Correct strict inequality using condition for convergence for positive common ratio. Simplified or unsimplified.
	$6x^2 + 8x + 2 < 0$	M1	oe, correct quadratic expression. PI by correct critical values or correct final answer
	-1 and $-\frac{1}{3}$	A1	Both critical values correct. Condone -0.33 or better for $-\frac{1}{3}$
	$-1 < x < -\frac{1}{3}$	A1	throughout. CAO, ACF
		4	
	Total	9	