OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL A-LEVEL

MATHEMATICS

MA04

(9660/MA04) Unit S2 Statistics

Mark scheme

January 2020

Version: V1 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$E\left(\sum_{i=1}^{3} X_i\right) = 8$	**B1**	
1(b)	$Var\left(\sum_{i=1}^{3} X_i\right) = 4.5$	**B1**	
1(c)	$E(3X_1 - 4X_3) = 3 \times 2 - 4 \times 1$ $= 2$	**M1** **A1**	$E(3X_1 - 4X_3) = 3a - 4b$ where a, b = 2, 5 or 1
1(d)	$Var(5X_1 - 2X_2) = 5^2 \times 3 + 2^2 \times 1$ $= 79$	**M1** **A1**	$Var(5X_1 - 2X_2) = 5^2c + 2^2d$ c, d = 3, 1 or 0.5
	Total	**6**	

Q	Answer	Marks	Comments
2(a)	$\frac{1}{\lambda^2} = 400$ $\lambda = 0.05$	**M1** **A1**	 Do not award if –0.05 seen but not rejected
2(b)	Mean = $\frac{1}{\lambda}$ = 20	**B1ft**	Follow through their λ
2(c)	$P(X > 25) = 1 - F(25)$ $= 1 - (1 - e^{-0.05\times25})$ $= 0.287$	**M1** **A1** **A1**	oe Attempts to find correct probability using cdf of exponential or integration of pdf AWRT, NMS 3/3
2(d)	$1 - e^{-0.05x} = 0.6$ $x = 18.3$	**M1** **A1**	Forms equation using cdf of exponential or integration of pdf AWRT
	Total	**8**	

Q	Answer	Marks	Comments
3(a)	$Y \sim$ Po(1)	**B1**	PI
	P($X > 3$) ≈ P($Y > 3$) = 1 – P($Y \leq 3$)	**M1**	Attempt at correct probability
	= 0.019	**A1**	AWRT
3(b)	n is large	**B1**	oe
	p is small	**B1**	oe
	Total	**5**	

Q	Answer	Marks	Comments
4(a)	$\overline{x} = 2.5$	**B1**	
	$s^2 = \frac{1}{20-1}\left(173 - \frac{50^2}{20}\right)$	**M1**	Attempt at variance formula Allow one slip Implied by correct answer
	= 2.53	**A1**	AWRT
	Because Mean ≈ Variance	**E1**	Values and conclusion must be seen Allow "values of sample mean and variance are close"
4(b)	P($X = 4$) = $\frac{e^{-2.5} \times 2.5^4}{4!}$	**M1**	Applies Poisson formula Implied by correct answer Condone using different value of λ
	= 0.134	**A1**	AWRT
	Total	**6**	

Q	Answer	Marks	Comments
5(a)	H_0: $p = 0.2$ or H_0: $p = 0.8$ H_1: $p < 0.2$ H_1: $p > 0.8$	**B1**	Both hypotheses
	$X \sim B(15, 0.2)$ or $X \sim B(15, 0.8)$	**M1**	PI
	$P(X \leq 1) = 0.1671$ or $P(X \geq 14) = 0.1671$	**A1**	AWRT 0.17
	$0.1671 > 0.1$	**M1**	Compares their probability with 0.1
	Accept H_0	**A1ft**	Follow through their probability provided both M marks awarded Implied by correct conclusion in context
	Evidence to suggest/support that the proportion of customers giving an "Excellent" rating has not increased	**E1**	Must be consistent with their conclusion on whether to accept H_0 or not or their probability if not explicitly stated
5(b)	Concluding that the proportion of customers giving an "Excellent" rating has not increased when it has	**E1**	oe
	Total	**7**	

Q	Answer	Marks	Comments
6(a)	$\frac{83-\mu}{\sigma}=1.6449$ $83-\mu = 1.6449\sigma$ (AG)	**M1** **A1**	Setting up formula with a z-value or $\Phi^{-1}(0.95)$ No errors seen 1.6449 must be seen before the final line
6(b)	1.4051 $\frac{10-\mu}{\sigma}=\pm 1.4051$ $10-\mu = -1.4051\sigma$ $\mu = 43.6$ or $\sigma = 23.9$ $\mu = 43.6$ and $\sigma = 23.9$	**B1** **M1** **A1** **M1** **A1**	Sight of anywhere in part (b) Oe, condone sign error for -1.4051 oe AWRT
6(c)	85 is within two standard deviations of the mean or Negative scores are within two standard deviations of the mean	**E1**	oe Allow non-zero probability (0.042) using normal distribution model of gaining a score higher than 85
	Total	**8**	

Q	Answer	Marks	Comments
7(a)	H_0: $\mu = 14$ H_1: $\mu \neq 14$	**B1**	Both hypotheses
	$\bar{X} \sim N\left(14, \frac{0.25^2}{25}\right)$	**M1**	Use of PI
	$z = \frac{13.892 - 14}{\frac{0.25}{\sqrt{25}}}$	**M1**	Attempts to find z Allow one slip
	z = -2.16	**A1**	AWRT Allow 2.16
	z_{crit} = 2.3263 or -2.3263	**B1**	Seen anywhere AWRT 2.33 Accept P(Z < -2.16) = 0.015
	Accept H_0	**A1ft**	Follow through their z and z_{crit} or their probability and 0.01 Implied by correct conclusion in context
	Evidence to suggest/support that the mean diameter of the pipes produced by machine A has not changed following the power cut	**E1**	Must be consistent with their conclusion on whether to accept H_0 or not or their z and z_{crit} (or their probability and 0.01) if not explicitly stated

Q	Answer	Marks	Comments
7(b)	H_0: $\mu = 14$ H_1: $\mu > 14$	**B1**	Both hypotheses
	dof $\nu = 8$	**M1**	PI
	$t = \dfrac{14.02 - 14}{\frac{0.04}{\sqrt{9}}}$	**M1**	Attempts to find t Allow one slip Condone $z =$
	$t = 1.5$	**A1**	Condone $z =$
	$t_{crit} = 1.397$	**B1**	Seen anywhere
	Reject H_0	**A1ft**	Follow through their t and t_{crit} (but not z_{crit}) Implied by correct conclusion in context
	Evidence to suggest/support that the mean diameter of the pipes produced by machine B is greater than 14 millimetres	**E1**	Must be consistent with their conclusion on whether to accept H_0 or not or their t and t_{crit} if not explicitly stated
	Total	**14**	

Q	Answer	Marks	Comments
8(a)	$\int_1^6 k(x^2-7x+6)\,\mathrm{d}x = 1$	**M1**	Sets integral over whole range equal to 1
	$k\left[\frac{x^3}{3}-\frac{7x^2}{2}+6x\right]_1^6 = 1$	**M1**	Correct integration
	$k\left[\left(\frac{6^3}{3}-\frac{7\times6^2}{2}+6\times6\right)-\left(\frac{1^3}{3}-\frac{7\times1^2}{2}+6\times1\right)\right]=1$	**A1**	Requires intermediate line after integration
	$\Rightarrow k = -\frac{6}{125}$ (AG)		
8(b)	$\mathrm{E}(X) = \int_1^6 kx(x^2-7x+6)\,\mathrm{d}x$	**M1**	Identifies correct integral
	$= k\left[\frac{x^4}{4}-\frac{7x^3}{3}+\frac{6x^2}{2}\right]_1^6$	**M1**	Correct integration
	$= k\left[\left(\frac{6^4}{4}-\frac{7\times6^3}{3}+\frac{6\times6^2}{2}\right)-\left(\frac{1^4}{4}-\frac{7\times1^3}{3}+\frac{1\times1^2}{2}\right)\right]$	**A1**	Requires intermediate line after integration
	$\Rightarrow \mathrm{E}(X) = 3.5$ (AG)		
8(c)	$\mathrm{E}(X^2) = \int_1^6 kx^2(x^2-7x+6)\,\mathrm{d}x$	**M1**	Identifies correct integral
	$= k\left[\frac{x^5}{5}-\frac{7x^4}{4}+\frac{6x^3}{3}\right]_1^6$	**M1**	Correct integration
	$\mathrm{E}(X^2) = 13.5$	**A1**	PI
	$\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$ $= 13.5 - 3.5^2$	**M1**	Applies formula
	$= 1.25$	**A1**	
	Total	**11**	

Q	Answer	Marks	Comments
9(a)(i)	$\bar{X} \sim N\left(20,\ \frac{6.5536}{25}\right)$	**B1**	oe
9(a)(ii)	$\mathrm{P}(\bar{X} > 21) = \mathrm{P}\left(Z > \frac{21-20}{\sqrt{\frac{6.5536}{25}}}\right)$	**M1**	Standardises
	= 1 – Φ(1.95) = 1 – 0.97441	**M1**	PI
	= 0.0254 to 0.0256	**A1**	AWFW
9(b)(i)	$\sum_{i=1}^{25} X_i \sim N(500, 163.84)$	**B1**	oe
9(b)(ii)	$\mathrm{P}\left(\sum_{i=1}^{25} X_i > 525\right) = \mathrm{P}\left(Z > \frac{525-500}{\sqrt{163.84}}\right)$	**M1**	Standardises
	= 1 – Φ(1.95) = 1 – 0.97441	**M1**	PI
	= 0.0254 to 0.0256	**A1**	AWFW
9(c)	The probabilities are the same	**E1**	oe
	This is because a total height of 525 centimetres is the same as a mean height of 21 cm as $\frac{525}{25} = 21$	**E1**	oe
	Total	**10**	

Q	Answer	Marks	Comments
10(a)(i)	All the car owners in the town	**E1**	Must include 'all' or 'every'
10(a)(ii)	A sample where each member of the population has an equal probability of being chosen or Each sample of the required size from the population has an equal probability of being chosen	**E1**	oe
10(b)	average number of events in an interval	**B1**	oe
10(c)	mean standard deviation	**B1** **B1**	 Accept variance SC1 for μ and σ or μ and σ^2
	Total	**5**	