OXFORD

INTERNATIONAL AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | |

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I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM03) Unit FP2 Pure Mathematics

Monday 20 January 2020 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables (enclosed).
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

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Answer **all** questions in the spaces provided.

1 The matrix $\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

1 (a) Describe fully the single transformation represented by the matrix $\mathbf{A}$

[2 marks]

1 (b) The matrix $\mathbf{B}$ represents a reflection in the plane $y = z$

Find the matrix $\mathbf{A} + \mathbf{B} + \mathbf{B}^{-1}$

[2 marks]

Answer ______________________

4

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2 Evaluate the improper integral

$$\int_0^\infty \left(\frac{2x}{x^2 + 9} - \frac{6}{3x + 2} \right) \mathrm{d}x$$

showing the limiting process used.

Give your answer in the form $\ln p$, where p is a rational number.

[6 marks]

Answer

6

Turn over ►

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3 The points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively relative to an origin O, where

$$\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k} \quad \text{and} \quad \mathbf{b} = -3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$$

3 (a) Use a vector product to show that the area of triangle OAB is $\dfrac{3}{2}\sqrt{10}$

[3 marks]

3 (b) The vector $\mathbf{c}$ is given by $\mathbf{c} = 3\mathbf{i} - \mathbf{j} + 7\mathbf{k}$

Use a scalar triple product to determine whether or not $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are coplanar vectors.

[2 marks]

Answer ______________________

5

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4 Prove by induction that for all integers $n \geq 1$

$$\sum_{r=1}^{n} r \times 4^{r-1} = \frac{1}{9} + \frac{4^n}{9}(3n-1)$$

[6 marks]

6

5 The line L has equation

$$\left(\mathbf{r} - \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}\right) \times \begin{bmatrix} 4 \\ -8 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

5 (a) (i) Find the direction cosines of L

[3 marks]

Answer ______________________________

5 (a) (ii) Find the acute angle between L and the x-axis, giving your answer to the nearest $0.1°$

[1 mark]

Answer ______________________________

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5 (b) The plane Π has equation $\mathbf{r}.\begin{bmatrix}1\\1\\1\end{bmatrix}=12$

Find the position vector of the point of intersection of L and Π

[4 marks]

Answer ________________________

8

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6 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 9y = 9x^2 + 6x + 2\cos 3x$$

[9 marks]

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Answer

9

Turn over ►

7 (a) Using the definition

$$\tanh y = \frac{e^y - e^{-y}}{e^y + e^{-y}}$$

prove that, for $-1 < x < 1$

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

[3 marks]

7 (b) (i) **Hence** find, in terms of r, the coefficient of x^r in the Maclaurin series expansion of $\tanh^{-1} x$

[2 marks]

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Answer

7 (b) (ii) Hence, or otherwise, given that $y = \tanh^{-1} x$, deduce the value of

$$\left(\frac{dy}{dx} + \frac{d^3 y}{dx^3} + \frac{d^5 y}{dx^5} + \frac{d^7 y}{dx^7} \right) \text{ when } x = 0$$

[2 marks]

Answer

7

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8 The matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 \\ 1 & k & 4 \\ 2 & 3 & k \end{bmatrix}$, where k is a real constant.

8 (a) Show that $\mathbf{A}$ is a non-singular matrix.

[3 marks]

8 (b) Find $\mathbf{A}^{-1}$ in terms of k

[5 marks]

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Answer ______

8 (c) Use $\mathbf{A}^{-1}$ to solve the equations

$$x + 2y - z = 1$$

$$x + ky + 4z = 3$$

$$2x + 3y + kz = 6$$

Give your solution in terms of k

[3 marks]

$x =$ ______ $y =$ ______ $z =$ ______

11

Turn over ►

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9 The equation

$$mx^4 + x^3 + (m+n)\,x^2 - x + n = 0\,, \qquad \text{where} \quad m \neq 0 \quad \text{and} \quad n \neq 0$$

has roots α , β , γ and δ

It is given that $\alpha + \beta = 0$

9 (a) (i) Explain why $\gamma + \delta = -\dfrac{1}{m}$

[1 mark]

9 (a) (ii) Show that $n = -m$

[6 marks]

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9 (b) **Hence** find all possible values of m for which the roots α , β , γ and δ are real and distinct.

[4 marks]

Answer

11

Turn over ►

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10 A curve C is defined for $x > 0$

At each point $(x\,,y)$ on the curve C

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2}{x}y = \frac{\cos x}{x}$$

10 (a) By using an integrating factor, find the general solution of this differential equation.

[5 marks]

Answer ______________________________

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10 (b) It is given that, as $x \to 0$, $y \to k$, where k is a constant.

10 (b) (i) Find the value of k

Fully justify your answer.

[4 marks]

$k =$ ____________

10 (b) (ii) A student states that the curve $y = k\cos x$ passes through all the stationary points of C

Determine whether or not the student is correct.

Fully justify your answer.

[2 marks]

11

Turn over ►

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11 (a) Express $-128\,\mathrm{i}$ in the form $r\,\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$

[2 marks]

$-128\,\mathrm{i} =$ ____________________

11 (b) Solve the equation

$$z^7 + 128\,\mathrm{i} = 0$$

giving your solutions in the form $r\,\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$

[4 marks]

Answer ____________________

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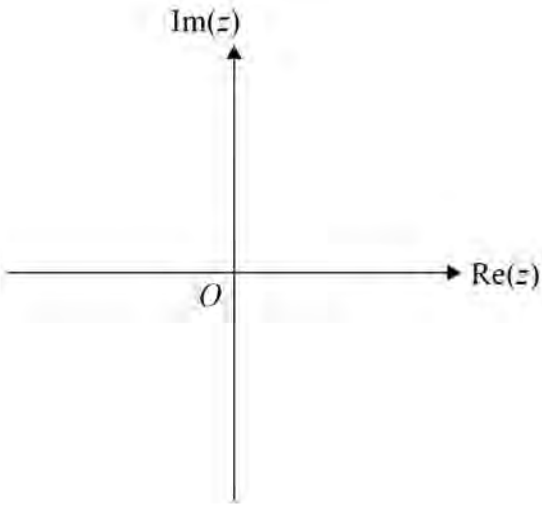
11 (c) It is given that $z^7 + 128\,\mathrm{i} = (z + k\,\mathrm{i})\,Q(z)$, where k is an integer.

11 (c) (i) On the Argand diagram below, show the six roots of the equation $Q(z) = 0$

[3 marks]



11 (c) (ii) Express $Q(z)$ as a product of three quadratic factors, each in the form

$$z^2 + \mathrm{i}(p\sin(q\pi))z + t$$

where p and t are integers and $0 < q < \frac{1}{2}$

[4 marks]

$Q(z) =$ ______________________

13

Turn over ►

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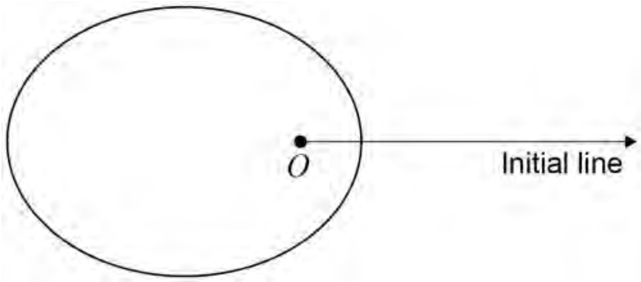
12 The diagram shows a sketch of the curve C_1, the pole O and the initial line.



The curve C_1 has polar equation $r = \dfrac{2}{3+2\cos\theta}$, $0 \le \theta \le 2\pi$

The circle C_2 has polar equation $r = \sin\left(\theta - \dfrac{\pi}{6}\right)$, $\dfrac{\pi}{6} \le \theta \le \dfrac{7\pi}{6}$

12 (a) (i) Verify that the pole O lies on the circle C_2

[1 mark]

12 (a) (ii) Use integration to show that the area of the circle C_2 is $\dfrac{1}{4}\pi$

[3 marks]

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12 (b) The line L has Cartesian equation $\sqrt{3}\,y = 1 - x$

The line L intersects the curve C_1 at the points P and Q, where $OP > OQ$

12 (b) (i) By finding the polar coordinates of the points P and Q, verify that the circle C_2 also passes through the points P and Q

[5 marks]

12 (b) (ii) Explain why OP is a diameter of the circle C_2

[2 marks]

12 (c) Hence find the Cartesian equation of the tangent to the circle C_2 at the point P

[4 marks]

Answer ______________________

15

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13 A curve C has equation $y = a\cosh\left(\frac{x}{a}\right)$, where a is a positive constant.

13 (a) Show that the length of the curve from $x = -d$ to $x = d$ is $2a\sinh\left(\frac{d}{a}\right)$

[4 marks]

13 (b) The ends of a chain are attached to points P and Q such that PQ is horizontal and of length $2d$

The chain hangs below PQ. Its shape is modelled by the curve C

The length of the chain is s

The lowest point of the chain is at a distance $\frac{s}{2n}$ below PQ, where $n > 1$

13 (b) (i) Use a suitable sketch to show that $a + \frac{s}{2n} = a\cosh\left(\frac{d}{a}\right)$

[1 mark]

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13 (b) (ii) Hence show that

$$a + \frac{s}{2n} = \sqrt{a^2 + \frac{s^2}{4}}$$

[2 marks]

13 (b) (iii) Show that $PQ = \frac{s}{2n}\left(n^2 - 1\right)\ln\left(\frac{n+1}{n-1}\right)$

[7 marks]

Turn over ►

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14

END OF QUESTIONS

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