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INTERNATIONAL  
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# INTERNATIONAL A-LEVEL FURTHER MATHEMATICS FM04

(9665/FM04) Unit FS2 Statistics

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Mark scheme

January 2020

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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**Key to mark scheme abbreviations**

<b>M</b>	Mark is for method
<b>m</b>	Mark is dependent on one or more M marks and is for method
<b>A</b>	Mark is dependent on M or m marks and is for accuracy
<b>B</b>	Mark is independent of M or m marks and is for method and accuracy
<b>E</b>	Mark is for explanation
<b>✓ or ft</b>	Follow through from previous incorrect result
<b>CAO</b>	Correct answer only
<b>CSO</b>	Correct solution only
<b>AWFW</b>	Anything which falls within
<b>AWRT</b>	Anything which rounds to
<b>ACF</b>	Any correct form
<b>AG</b>	Answer given
<b>SC</b>	Special case
<b>oe</b>	Or equivalent
<b>A2, 1</b>	2 or 1 (or 0) accuracy marks
<b>–x EE</b>	Deduct x marks for each error
<b>NMS</b>	No method shown
<b>PI</b>	Possibly implied
<b>SCA</b>	Substantially correct approach
<b>sf</b>	Significant figure(s)
<b>dp</b>	Decimal place(s)

Q	Answer	Marks	Comments								
1	TT median 0 TH or HT median 5 TT median 10  <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><math>m</math></td><td>0</td><td>5</td><td>10</td></tr> <tr> <td><math>P(M = m)</math></td><td><math>\frac{1}{4}</math></td><td><math>\frac{1}{2}</math></td><td><math>\frac{1}{4}</math></td></tr> </table>	$m$	0	5	10	$P(M = m)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	<b>M1</b>  <b>m1</b>  <b>A1</b>	Identify at least one possible combination and median  Identify all possible combinations and medians  Accept in function form
$m$	0	5	10								
$P(M = m)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$								
	<b>Total</b>	<b>3</b>									

Q	Answer	Marks	Comments
2(a)	$\text{Relative Efficiency} = \frac{100}{\frac{\sigma^2}{10}} \div \frac{\sigma^2}{10}$ $= 10$ Linda's estimator is more efficient than Andrew's estimator	<b>M1</b>  <b>A1</b>  <b>E1</b>	Uses relative efficiency formula oe   oe
2(b)(i)	$E(P) = \frac{E(\overline{X_1}) + E(\overline{X_2})}{2} = \frac{\mu + \mu}{2}$ $= \mu \text{ therefore unbiased}$	<b>M1</b>  <b>A1</b>	
2(b)(ii)	$\text{Var}(P) = \frac{\text{Var}(\overline{X_1}) + \text{Var}(\overline{X_2})}{2^2} = \frac{\frac{\sigma^2}{10} + \frac{\sigma^2}{100}}{2^2}$ $= \frac{11\sigma^2}{400}$	<b>M1</b>  <b>A1</b>	oe
	<b>Total</b>	<b>7</b>	

Q	Answer	Marks	Comments
3(a)	$\bar{x} = 66.404$ $s^2 = \frac{1}{500-1} \left( 2210000 - \frac{33202^2}{500} \right)$ $= 10.5(2984369)$ $z = 2.0537$ $66.404 \pm 2.0537 \sqrt{\frac{10.52...}{500}}$ $(66.106, 66.702)$	<b>B1</b> <b>M1</b> <b>A1</b> <b>B1</b> <b>M1</b> <b>A1</b>	 Attempt at variance formula Allow one slip Implied by correct answer AWRT 10.5 AWRT 2.054 Applies confidence interval formula with their values AWRT
3(b)	Rejects as 67 is outside the confidence interval	<b>E1ft</b>	Follow through their confidence interval
3(c)	Still valid as sample size is large enough for central limit theorem to apply	<b>E1</b> <b>E1</b>	oe oe
	<b>Total</b>	<b>9</b>	

Q	Answer	Marks	Comments
4(a)(i)	$P(X \leq 25) = 0.9427$ $P(X \leq 26) = 0.9686$ Accept if $X \leq 26$ $B(50, 0.5)$ Type II probability = 0.664	<b>M1</b> <b>M1</b> <b>M1</b> <b>A1</b>	Both seen oe Use of PI AWRT
4(a)(ii)	Power = $1 - 0.664$ $= 0.336$	<b>M1</b> <b>A1F</b>	Applies power formula with their Type II probability AWRT
4(b)	The power of the test would increase	<b>E1</b>	
	<b>Total</b>	<b>7</b>	

Q	Answer	Marks	Comments
5	$H_0: \sigma_A = \sigma_B$ $H_1: \sigma_A \neq \sigma_B$  dof 20, 25  $F = \frac{s_A^2}{s_B^2} = \frac{0.3^2}{0.2^2}$  $= 2.25$  $F_{20,25} = 2.70$  Accept $H_0$  Evidence to suggest/support that the population variances of machines <i>A</i> and <i>B</i> are equal	<b>B1</b>   <b>B1</b> <b>M1</b>  <b>A1</b>  <b>B1</b> <b>A1ft</b>  <b>E1</b>	Both hypotheses      AWRT Follow through their $F$ and $F_{20,25}$ Implied by correct conclusion in context  Must be consistent with their conclusion on whether to accept $H_0$ or not or their $F$ and $F_{20,25}$ if not explicitly stated
	<b>Total</b>	<b>7</b>	

[illegible]

	Test statistic = 9.2	<b>B1</b>	AWRT
	Reject $H_0$	<b>A1ft</b>	Follow through their $\sum \frac{(O-E)^2}{E}$ and test statistic Implied by correct conclusion in context
	Evidence to suggest/support that there is an association between the office an employee works in and the transport they use to get to work	<b>E1</b>	Must be consistent with their conclusion on whether to accept $H_0$ or not or their $\sum \frac{(O-E)^2}{E}$ and test statistic if not explicitly stated
	<b>Total</b>	<b>10</b>	



Q	Answer	Marks	Comments
<b>7(a)</b>	$H_0: \mu_A = \mu_B$ $H_1: \mu_A \neq \mu_B$  $z = \frac{ 504 - 502 }{\sqrt{\frac{2^2}{10} + \frac{3^2}{12}}}$  $= 1.87$  $z_{\text{crit}} = 1.96 \text{ or } -1.96$  Accept $H_0$  Evidence to suggest/support that the mean mass of jars of honey is the same for both suppliers	<b>B1</b>   <b>M1</b> <b>M1</b>  <b>A1</b> <b>B1</b> <b>A1ft</b>  <b>E1</b>	Both hypotheses   Numerator Denominator  AWRT Seen anywhere Follow through their $z$ and $z_{\text{crit}}$ Implied by correct conclusion in context  Must be consistent with their conclusion on whether to accept $H_0$ or not or their $z$ and $z_{\text{crit}}$ if not explicitly stated
<b>7(b)</b>	$t$ test/values instead of $z$ test/values with dof $\nu = 20$  Calculate pooled estimate of variance $S_p^2$  Replace $\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}$ with $S_p^2 \left( \frac{1}{n_A} + \frac{1}{n_B} \right)$	<b>E1</b>  <b>E1</b>  <b>E1</b>  <b>E1</b>	oe oe oe oe
	<b>Total</b>	<b>11</b>	

Q	Answer	Marks	Comments														
8(a)	$H_0: \mu_B = \mu_A$ $H_1: \mu_B < \mu_A$	B1	Both hypotheses														
	<table><tr><th>Student</th><th>Difference</th></tr><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>1</td></tr><tr><td>3</td><td>0</td></tr><tr><td>4</td><td>-1</td></tr><tr><td>5</td><td>2</td></tr><tr><td>6</td><td>3</td></tr></table>	Student	Difference	1	4	2	1	3	0	4	-1	5	2	6	3	B1	All differences found
	Student	Difference															
	1	4															
	2	1															
	3	0															
	4	-1															
	5	2															
	6	3															
	$\bar{x} = 1.5$	B1															
$s^2 = \frac{1}{6-1} \left( 31 - \frac{9^2}{6} \right)$	M1	Attempt at variance formula Allow one slip Implied by correct answer															
$= 3.5$	A1	Accept AWRT $s = 1.87$															
dof $\nu = 5$	B1																
$t = \frac{1.5}{\sqrt{\frac{3.5}{6}}}$	M1	Using their $s^2$															
$= 1.96$	A1	AWRT															
$t_5 \text{ cv} = 2.02$	B1	AWRT															
Accept $H_0$	A1ft	Follow through their $t$ and $t_5$ Implied by correct conclusion in context															
	No evidence to suggest/support that the tutorial company's claim is true	E1	Must be consistent with their conclusion on whether to accept $H_0$ or not or their $t$ and $t_5$ if not explicitly stated														
8(b)	The difference between test scores before and after the revision course	E1	oe														
	are normally distributed	E1	oe														
	Total	13															

Q	Answer	Marks	Comments
9(a)	$M'_X(t) = \dots e^t(1 - 0.8e^t)^{-3}$  $M'_X(t) = 0.064e^t(1 - 0.8e^t)^{-3}$  $\text{Mean} = M'_X(0) = 0.064e^0(1 - 0.8e^0)^{-3}$ $= 8$	<b>M1</b>  <b>A1</b>  <b>M1</b> <b>A1</b>	$M'_X(t) = Ae^t(1 - 0.8e^t)^{-3}$ where A is a constant  oe May be unsimplified  Attempts $M'_X(0)$
9(b)	$M''_X(t) = \dots e^t(1 - 0.8e^t)^{-3} + \dots e^{2t}(1 - 0.8e^t)^{-4}$  $M''_X(t) = 0.064e^t(1 - 0.8e^t)^{-3} + 0.1536e^{2t}(1 - 0.8e^t)^{-4}$  $\text{Variance} = M''_X(0) - (M'_X(0))^2$ $= 0.064e^0(1 - 0.8e^0)^{-3} + 0.1536e^{2 \times 0}(1 - 0.8e^0)^{-4} - 8^2$ $= 40$	<b>M1</b>  <b>A1</b>  <b>M1</b>  <b>A1</b>	$M''_X(t) = Ae^t(1 - 0.8e^t)^{-3} + Be^{2t}(1 - 0.8e^t)^{-4}$ where A and B are constants oe May be unsimplified  Applies variance formula with their $M''_X(t)$ and mean
9(c)	$M_{2+3X}(t) = e^{2t} M_X(3t)$ $= e^{2t} \times 0.04(1 - 0.8e^{3t})^{-2}$  $= \left( \frac{0.2e^t}{1 - 0.8e^{3t}} \right)^2$	<b>M1</b>  <b>M1</b>  <b>A1</b>	
9(d)	$M_{X+Y}(t) = M_X(t) M_Y(t)$ $= 0.04(1 - 0.8e^t)^{-2} (0.8 + 0.2e^t)^2$  $= \left( \frac{0.16 + 0.04e^t}{1 - 0.8e^t} \right)^2$	<b>M1</b>  <b>A1</b>	
	<b>Total</b>	<b>13</b>	