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MA04

(9660/MA04) Unit S2 Statistics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	$\mathrm{E}\left(\sum_{i=1}^{3} X_i\right) = 16$	**B1**	
		1	

Q	Answer	Marks	Comments
1(b)	$4 + 1 + a = 6a$	**M1**	Forms an equation and attempts to solve **PI**
	$5 = 5a$		
	$a = 1$	**A1**	
		2	

Q	Answer	Marks	Comments
1(c)	$2\mathrm{E}(X_1) - 3\mathrm{E}(X_2) = 2 \times 5 - 3 \times 4$	**M1**	**PI**
	$= -2$	**A1**	**CAO**
		2	

Q	Answer	Marks	Comments
1(d)	$4\mathrm{Var}(X_1) + 9\mathrm{Var}(X_2) = 4 \times 4 + 9 \times 1$	**M1**	**PI** Allow $\pm$
	$= 25$	**A1**	**CAO**
		2	

	Question 1 Total	**7**	

Q	Answer	Marks	Comments
2(a)	$\text{P}(X=8)=\dfrac{e^{-15}\times 15^{8}}{8!}$	**M1**	**PI**
	$=0.0194\ \ [3\text{ sf}]$	**A1**	**AWRT** 0.0194
		2	

Q	Answer	Marks	Comments
2(b)(i)	$\mu=15$ $\sigma=\sqrt{15}$	**B1**	Allow **AWRT** 3.87 for σ
		1	

Q	Answer	Marks	Comments
2(b)(ii)	$\text{P}\left(15-\sqrt{15}<X<15+\sqrt{15}\right)$		
	$=\text{P}(11.127<X<18.873)$	**M1**	For substitution of their μ and σ
	$=\text{P}(12\le X\le 18)$	**A1**	For rounding to integer values **oe**
	$=0.81947-0.18475$	**M1**	For sight of 1 correct limit **PI**
	$=0.635\ \ [3\text{ sf}]$	**A1**	**AWRT** 0.635
		4	

Q	Answer	Marks	Comments
2(c)	More than one person could be injured in a single accident The model will only hold for the day as skiing is unlikely to take place at night (unless floodlit!) The model will only work for the ski season (not the summer season)	**E1**	Any plausible explanation in context eg: The number of injured people is unlikely to be **independent** [which is a requirement for data to be modelled as Poisson distrbution]
		1	

	Question 2 Total	**8**	

Q	Answer	Marks	Comments
3	$H_0: \mu = 25.1$ [megajoules per litre] $H_1: \mu > 25.1$	B1	
	$\bar{x} = 26.2$	B1	**PI**
	$s^2 = \frac{1}{59} \times \left(41850 - \frac{1572^2}{60}\right)$	M1	Attempt at variance formula Allow one slip
	$= 11.2[474...]$	A1	**PI** by correct z or probability Allow 3318/295 **AWRT** accept s = 3.35[372295]
	$\bar{X} \sim \mathrm{N}\left(25.1, \frac{11.2[474...]}{60}\right)$	M1	$\bar{X} \sim \mathrm{N}\left(25.1, \frac{s^2}{60}\right)$ **PI**
	$z = \frac{26.2 - 25.1}{\sqrt{\frac{11.2[474...]}{60}}}$	M1	Calculates z with their s^2 **or** for se = 0.433…. **or** $\mathrm{P}(\bar{X} \geq 26.2) = 0.006$
	= 2.5[40628277]	A1	**AWRT** 2.5 **or** $\mathrm{P}\left(\bar{X} \geq 26.2\right) = 0.006$ (1sf)
	$z_{\text{crit}} = 2.3[26347931]$	B1	**AWRT** 2.3 $\mathrm{P}\left(\bar{X} \geq 26.2\right) = 0.00553$ **or** for CR is $\bar{x} > 26.1$
	Reject H_0 as 2.5[…] > 2.3[…] **or** $z > z_{\text{crit}}$	A1ft	Accept H_1 Follow through their z and z_{crit} provided signs are consistent or comparison of 26.2 > 26.1 **or** comparison of their ‘0.00553’ to 0.01
	Evidence to suggest that the mean amount of energy per litre has increased [at the 1% level of significance]	E1ft	Must be consistent with their conclusion on whether to accept H_1 or not based on their z and z_{crit} if not explicitly stated

	Question 3 total	**10**	

Q	Answer	Marks	Comments
4(a)	$\int_1^4 \left(\frac{1}{k}x^2\right)dx = 1$	B1	Setting an integral and fraction summing to 1 **oe PI**
	$\left[\frac{x^3}{3k}\right]_1^4 = 1$	M1	For correct integration with attempt to substitute limits **oe PI** By later working
	$\frac{1}{3k}(64-1) = 1$		**SC** 2/3 for substitution of k into a correct integral which sums to 1
	$\frac{63}{3} = k$		
	$k = 21$	A1	**AG** Must be convincingly shown
		3	

Q	Answer	Marks	Comments
4(b)	$\int_3^4 \frac{x^2}{21}dx$	M1	**PI** by 0.587… Correct integration and limits
	$= \left[\frac{x^3}{63}\right]_3^4$		
	$= \frac{4^3}{63} - \frac{3^3}{63}$		
	$= \frac{37}{63}$	A1	**CAO** in an exact form
		2	

Q	Answer	Marks	Comments
4(c)(i)	$\mathrm{E}(5)+2\mathrm{E}\left(\frac{1}{X}\right)$	B1	**PI**
	$\left[\mathrm{E}\left(\frac{1}{X}\right)\right]=\int_1^4 \frac{1}{x}\times\frac{1}{21}x^2\,\mathrm{d}x$		
	$=\left[\frac{x^2}{42}\right]_1^4$	M1	For correct integration with attempt to substitute limits **oe PI** by later working
	$=\frac{1}{42}(16-1)$		$\frac{1}{21}\int_1^4\left(\frac{5x+2}{x}\right)x^2\,\mathrm{d}x=\frac{1}{21}\int_1^4 5x^2+2x\,\mathrm{d}x$
	$=\frac{5}{14}$		$\left[\frac{1}{21}\left(\frac{5}{3}x^3+x^2\right)\right]_1^4$
	$\left[\mathrm{E}(5)+2\mathrm{E}\left(\frac{1}{X}\right)=\right]\quad 5+2\times\frac{5}{14}$	A1	$\frac{1}{21}\left[\left(\frac{5}{3}\times4^3+4^2\right)-\left(\frac{5}{3}\times1^3+1^2\right)\right]$ **oe**
	$=\frac{40}{7}$	A1	**CAO** in an exact form
		4	

Q	Answer	Marks	Comments
4(c)(ii)	$\text{Var}\left(5+\frac{2}{X}\right)=4\text{Var}\left(\frac{1}{X}\right)$	B1	**PI**
	$\left[\text{E}\left(\frac{1}{X^2}\right)\right]=\int_1^4 \frac{1}{x^2}\times\frac{1}{21}x^2\,\text{d}x$		$\frac{1}{21}\int_1^4\left(\frac{(5x+2)}{x}\right)^2 x^2\,\text{d}x$
	$=\left[\frac{x}{21}\right]_1^4$	M1	$\frac{1}{21}\int_1^4 25x^2+20x+4\,\text{d}x$ **PI** by later working
	$=\frac{1}{21}(4-1)$		
	$\left[\text{E}\left(\frac{1}{X^2}\right)\right]=\frac{1}{7}$	A1	$\left[\frac{25}{3}x^3+10x^2+4x\right]_1^4$
	$\text{Var}\left(\frac{1}{X}\right)=\text{E}\left(\frac{1}{X^2}\right)-\left(\text{E}\left(\frac{1}{X}\right)\right)^2$		$\frac{1}{21}\left(\left(\frac{1600}{3}+160+16\right)-\left(\frac{25}{3}+10+4\right)\right)$ Allow **AWRT** 32.7
	$=\frac{1}{7}-\left(\frac{5}{14}\right)^2$	M1	For their $\text{E}\left(\frac{1}{X^2}\right)-\left[\text{E}\left(\frac{1}{X}\right)\right]^2$
	$\frac{3}{196}$	A1	Or $\frac{229}{7}-\left(\frac{40}{7}\right)^2$
	$\left[\text{Var}\left(5+\frac{2}{X}\right)\right]=4\times\frac{3}{196}=\frac{3}{49}$	A1	**CAO** in an exact form
		6	

	Question 4 total	**15**	

Q	Answer	Marks	Comments
5(a)	Yes, …	**B1**	‘Yes’ with a reason
	It is a **random variable** consisting of known observations	**E1**	Correct reason or X_1 , X_2 and X_3 are random samples
		2	

Q	Answer	Marks	Comments
5(b)	No, …	**B1**	‘No’ with a reason
	It includes a **population parameter**	**E1**	Correct reason Allow it includes μ
		2	

	Question 5 total	**4**	

Q	Answer	Marks	Comments
6(a)	$8r - \frac{3r}{t}x$	**M1**	pdf in terms of x, r and t
	$\int_0^t \left(8r - \frac{3r}{t}x\right) \mathrm{d}x = 1$	**A1**	
	$\Rightarrow \left[8rx - \frac{3r}{2t}x^2\right]_0^t = 1$	**M1**	
	$8rt - \frac{3r}{2t}(t)^2 = 1$		
	$r = \frac{2}{13t}$	**A1**	
	$8 \times \frac{2}{13t} \times x \; - \; \frac{3}{2t} \times \frac{2}{13t} \times x^2$		
	$\mathrm{F}(x) = \frac{16}{13t}x - \frac{3}{13t^2}x^2$	**A1**	**AG** Must be convincingly shown
		5	

Q	Answer	Marks	Comments
6(a) ALT	$\frac{1}{2} \times t \times (8r + 5r) = 1$	**M1**	**oe**
	$r = \frac{2}{13t}$	**A1**	
	$\frac{16}{13t} - \frac{6}{13t^2}x$	**M1**	pdf in terms of x and t only
	$\left[\int \left(\frac{16}{13t} - \frac{6}{13t^2}x\right) \mathrm{d}x = \right]$		
	$\frac{16}{13t}x - \frac{3}{13t^2}x^2 + c$	**A1**	
	$\mathrm{F}(x) = \frac{16}{13t}x - \frac{3}{13t^2}x^2$	**A1**	**AG** Must be convincingly shown by considering the value of the constant of integration, possibly by definite integration
		5	

Q	Answer	Marks	Comments
6(b)	$\frac{16}{13t}\times 4-\frac{3}{13t^2}\times 4^2=\frac{1}{13t}\Rightarrow 64t-48=t$ $t=\frac{16}{21}\ [<4]$ $\mathrm{f}(x)=0$ for $x>\frac{16}{21}\Rightarrow \mathrm{F}(4)=1$ $\Rightarrow t=\frac{1}{13}$	**B1**	If **B0** awarded, allow **SC1** for $t=\frac{16}{21}$ from correct working
		1	

	Question 6 total	**6**	

Q	Answer	Marks	Comments
7(a)	$\left[\text{P}\left(0.805 < B < 0.815\right) = \right]$ $\text{P}\left(\frac{0.805-0.8}{0.006} < Z < \frac{0.815-0.8}{0.006}\right)$ $= \text{P}\left(\frac{5}{6} < Z < \frac{5}{2}\right)$ $\left[= \text{P}\left(Z < \frac{5}{2}\right) - \text{P}\left(Z < \frac{5}{6}\right)\right]$	M1	Standardises both Allow 0.83
	$= 0.99379 - 0.79673$ [from tables]	M1	**PI**
	$= 0.197$ [3 sf]	A1	Allow 0.196 [from calculator] or 0.197 [from tables] for the final answer
		3	

Q	Answer	Marks	Comments
7(b)(i)	$\text{Var(sample mean)} = \frac{0.005^2}{25}$ $= 1\times10^{-6}$ or SD (sample mean) $= \frac{0.005}{\sqrt{25}}$ $= 1\times10^{-3}$	B1	**PI**
	$\left[\text{P}\left(\overline{W} > 1.0015\right) = \right]$ $\text{P}\left(Z > \frac{1.0015-1}{\frac{0.005}{\sqrt{25}}}\right)$ $= \text{P}\left(Z > \frac{3}{2}\right)$	M1	For standardising with their SD
	$\left[= 1 - \text{P}\left(Z < \frac{3}{2}\right)\right]$		**PI**
	$= 1 - 0.93319$ [from tables]	M1	
	$= 0.06681$	A1	**AWRT** 0.0668
		4	

Q	Answer	Marks	Comments
7(b)(ii)	$\left[\mathrm{P}(Z < z) = 0.05\right] \Rightarrow z = -1.6449$	**B1**	Allow ± 1.64 **PI** By answer correct to 3sf before rounding
	$\dfrac{0.9992 - 1}{\frac{0.005}{\sqrt{n}}} < -1.6449$	**M1**	For standardising
	$\sqrt{n} > 10.28...$ $n > 105.69...$	**M1**	**PI** For attempt at an equation to solve for n
	$n = 106$	**A1**	**CAO**
		4	

Q	Answer	Marks	Comments
7(c)(i)	$\left[\mu = 100 \times (1 + 0.8)\right] = 180$ [grams]	**B1**	
	$\left[\sigma^2 = 100 \times \left(0.005^2 + 0.006^2\right)\right]$ $= 0.0061 \left[\text{grams}^2\right]$	**B1**	
		2	

Q	Answer	Marks	Comments
7(c)(ii)	$\mathrm{P}\left(Z < \dfrac{m - 180}{\sqrt{0.0061}}\right) = 0.98$	**M1**	For standardising with their mean and SD
	$\dfrac{m - 180}{\sqrt{0.0061}} = 2.0537$	**B1**	Allow ± 2.05
	$= 180.2$ [4 sf]	**A1**	**AWRT** 180.2
		3	

	Question 7 total	**16**	

Q	Answer	Marks	Comments
8(a)	$Y \sim \text{Po}(4)$	**B1**	**PI**
	$\left[\text{P}(Y \geq 1) =\right]$ $1 - \text{P}(Y = 0)$		
	$= 1 - 0.01832$	**M1**	**PI**
	$= 0.9817$	**A1**	**AWRT** 0.9817
		3	

Q	Answer	Marks	Comments
8(b)(i)	Stage 6	**B1**	Correctly identifying Stage 6
	[critical region is] {0, 1, 2, 3, 14, …}	**B1**	**oe**, such as critical region should not include 13
		2	

Q	Answer	Marks	Comments
8(b)(ii)	As 12 is not in the critical region, we do not reject H_0	**E1ft**	**ft** Their critical region
	Insufficient evidence to support the universities claim that the number of meteors seen per 15 minute period has changed from 2	**E1ft**	**ft** Their comment about H_0
		2	

Q	Answer	Marks	Comments
8(b)(iii)	$[0.0342 + 0.0424 =]$ 0.0766	**B1**	**CAO**
		1	

Q	Answer	Marks	Comments
8(c)(i)	$\mathrm{F}(t)=\begin{cases}0 & t<0\\ 1-\mathrm{e}^{-\frac{2}{15}t} & t\geq 0\end{cases}$	B2	**B1** For use of $\lambda=\dfrac{2}{15}$
		2	

Q	Answer	Marks	Comments
8(c)(ii)	$\left[\mathrm{P}(T>a)=\right]\ \mathrm{e}^{-\frac{2}{15}a}=0.6$	M1	**oe, PI**
	3 mins 50 seconds	A1	**oe** (eg 230 seconds) **CAO**
		2	

Q	Answer	Marks	Comments
8(c)(iii)	$\left[\mathrm{P}(T<b+30 \mid T>30)=\right]$ $\mathrm{P}(T<b)=3\mathrm{e}^{-\frac{2}{15}b}$	M1	For setting up and attempting to solve by obtaining a single unknown **oe**
	$1-\mathrm{e}^{-\frac{2}{15}b}=3\mathrm{e}^{-\frac{2}{15}b}$ $\dfrac{1}{4}=\mathrm{e}^{-\frac{2}{15}b}$		
	$b=15\ln 2$	A1	**oe**
		2	

	Question 8 Total	**14**	