

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

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Forename(s)

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Candidate signature

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I declare this is my own work.

INTERNATIONAL AS FURTHER MATHEMATICS

(9665/FM01) Unit FP1 Pure Mathematics

Wednesday 4 January 2023 07:00 GMT Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	



J A N 2 3 F M 0 1 0 1

IB/G/Jan23/E6

FM01

- 1 (b)** Show how the answer to **part (a)** can be used to find the gradient of the curve at the point where $x = -\frac{5}{4}$

[2 marks]

6

Turn over for the next question

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4



- 3** For each of the improper integrals below, either find its exact value or explain why it has no finite value.

Show **all** necessary working.

3 (a) $\int_3^{\infty} \frac{1}{\sqrt[5]{x}} dx$

[3 marks]

Answer _____

3 (b) $\int_3^{\infty} \frac{1}{x^2} dx$

[3 marks]

Answer _____



$$\sin\left(5x - \frac{\pi}{12}\right) = 0.5$$

[4 marks]

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Answer



$$\sin\left(5x - \frac{\pi}{12}\right) = 0.5$$

[3 marks]

[illegible]

Answer _____

7

Turn over for the next question

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5 (a) Show that

$$\frac{1}{r!} - \frac{1}{(r+1)!} = \frac{r}{(r+1)!}$$

[1 mark]

5 (b) Use the method of differences to find an expression for the sum of the series

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!}$$

[4 marks]

Answer _____



$$\sum_{r=5}^{\infty} \frac{r}{(r+1)!}$$

[2 marks]

[illegible]

Answer _____

7

Turn over ►



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Find all the solutions of the equation

[7 marks]

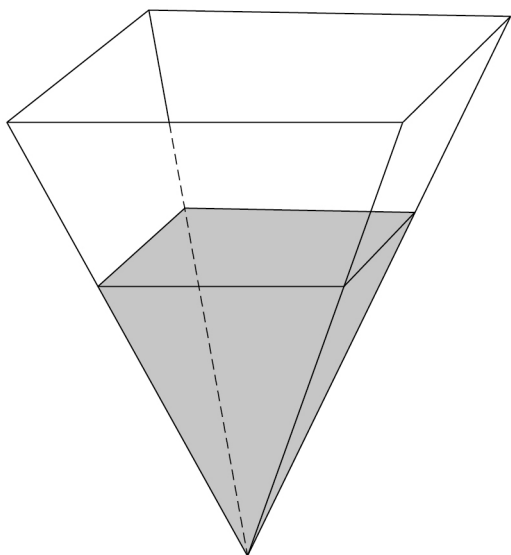
[illegible]

7

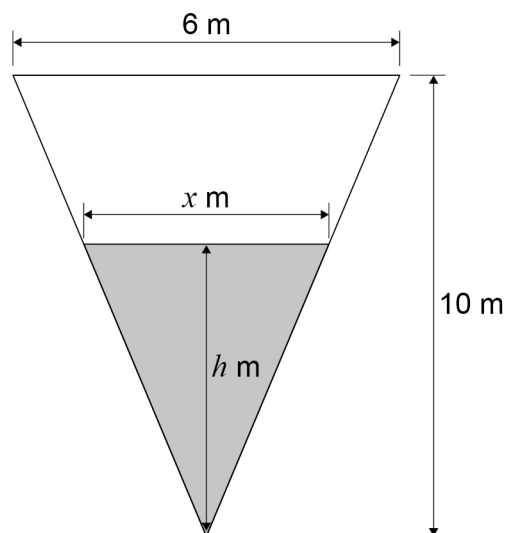
- 7 [The volume of a square-based pyramid is given by the formula $V = \frac{1}{3}a^2h$ where a is the length of the side of the square base, and h is the height.]

A water tank in the shape of an inverted square-based pyramid has height 10 metres, and the length of the side of the square base is 6 metres.

Not drawn to scale



3D image



Cross-section

The tank fills with water at a rate of 0.54 m^3 per minute.

At time t minutes the depth of water in the tank is h metres.

The horizontal surface of the water at time t minutes is in the shape of a square of side x metres.

- 7 (a) Explain why $x = 0.6h$

[1 mark]



[2 marks]

Answer _____

[5 marks]

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Answer _____

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8 The ellipse E_1 has equation

$$\frac{x^2}{4} + y^2 = 1$$

The ellipse E_2 has equation

$$\frac{x^2}{4} + \frac{y^2}{k} = 1$$

where k is a positive constant.

8 (a) Describe fully the transformation that maps E_1 onto E_2

[2 marks]

8 (b) It is given that the line L has equation

$$y = 2x + c$$

where c is a constant.

8 (b) (i) Show that if L intersects E_2 then

$$c^2 - k \leq A$$

where A is a constant to be found.

[5 marks]



[2 marks]

 $k =$

9



9 The function f is defined by

$$f(x) = \frac{4x+5}{x^2+4x+5}$$

9 (a) (i) Explain why the graph of $y = f(x)$ has no vertical asymptotes.

[2 marks]

9 (a) (ii) Write down the equation of the horizontal asymptote of the graph of $y = f(x)$

[1 mark]

Answer _____

9 (b) Prove that if the line $y = k$ where k is a constant, intersects the graph of $y = f(x)$ then

$$k^2 + 3k - 4 \leq 0$$

[4 marks]



[5 marks]

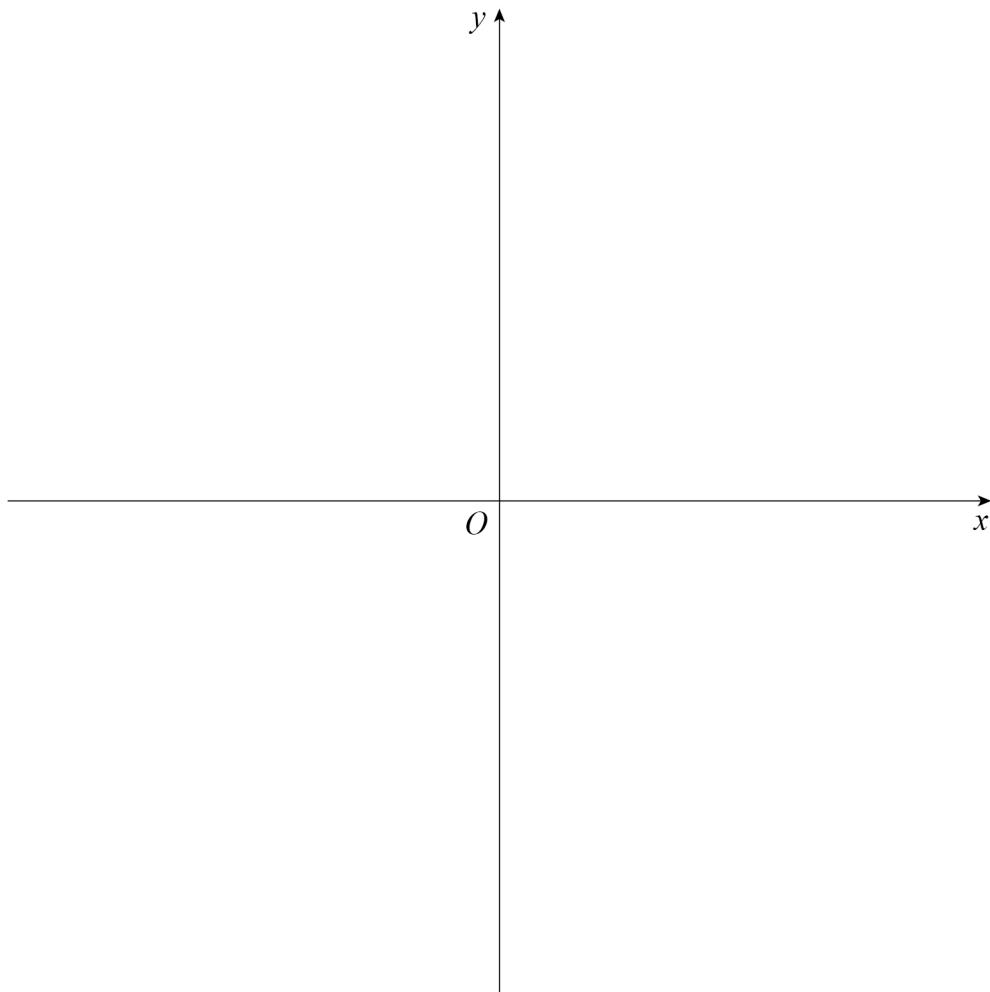
Answer

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- 9 (d) Sketch the graph of $y = f(x)$ on the axes below showing the coordinates of the stationary points.

[3 marks]

15

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- 10** The circle C is the locus of points on an Argand diagram such that

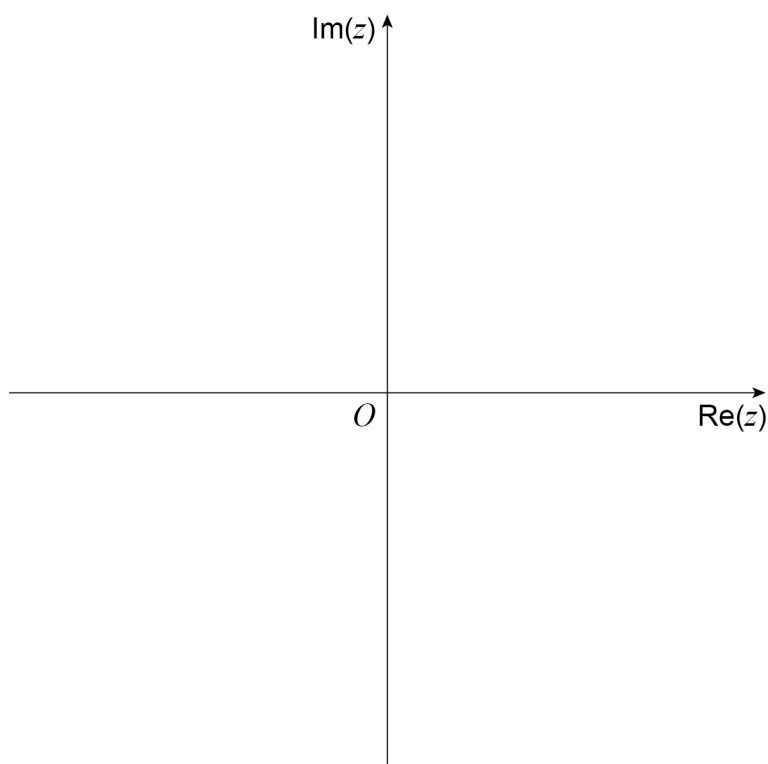
$$|z| = 5$$

The line L is the locus of points on an Argand diagram such that

$$\operatorname{Re}(z) = 3$$

- 10 (a)** Draw C and L on the Argand diagram.

[2 marks]



- 10 (b)** Find all the points which are **both** a distance of exactly one unit from C **and** a distance of exactly one unit from L

[6 marks]



$$\arg(z+5) = \frac{2\pi}{5}$$

[3 marks]

11

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