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# INTERNATIONAL AS MATHEMATICS MA02

(9660/MA02) Unit PSM1 Pure Mathematics, Statistics and Mechanics

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Mark scheme

January 2021

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Version: 1.0 Final

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### Key to mark scheme abbreviations

<b>M</b>	Mark is for method
<b>m</b>	Mark is dependent on one or more M marks and is for method
<b>A</b>	Mark is dependent on M or m marks and is for accuracy
<b>B</b>	Mark is independent of M or m marks and is for method and accuracy
<b>E</b>	Mark is for explanation
✓ <b>or ft</b>	Follow through from previous incorrect result
<b>CAO</b>	Correct answer only
<b>CSO</b>	Correct solution only
<b>AWFW</b>	Anything which falls within
<b>AWRT</b>	Anything which rounds to
<b>ACF</b>	Any correct form
<b>AG</b>	Answer given
<b>SC</b>	Special case
<b>oe</b>	Or equivalent
<b>A2, 1</b>	2 or 1 (or 0) accuracy marks
<b>–x EE</b>	Deduct x marks for each error
<b>NMS</b>	No method shown
<b>PI</b>	Possibly implied
<b>SCA</b>	Substantially correct approach
<b>sf</b>	Significant figure(s)
<b>dp</b>	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$360^\circ$	B1	Condone lack of units. Condone $2\pi$ for $360^\circ$
		1	

Q	Answer	Marks	Comments
1(b)	$(a + 180^\circ, b - 1)$	B1	Correct $x$ -coordinate. Condone lack of units. Condone $a + \pi$ for $a + 180^\circ$
		B1	Correct $y$ -coordinate. For both marks condone not given as coordinates but must be identified as $x$ -coordinate and $y$ -coordinate.
		2	

Q	Answer	Marks	Comments
1(c)	$a + 90^\circ$	B1	Condone lack of units. Condone $a + \frac{\pi}{2}$ for $a + 90^\circ$ Can be seen as pair of coordinates but ignore the $y$ -coordinate.
		1	

	Question 1 Total	4	
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Q	Answer	Marks	Comments
2(a)	$y = 5 \Rightarrow 5 = 23 - 3x$	<b>M1</b>	<b>PI</b> by correct $x$ -coordinate of centre.
	$(6, 5)$	<b>A1</b>	
		<b>2</b>	

Q	Answer	Marks	Comments
2(b)	$r^2 = (9-6)^2 + (11-5)^2$ or $r^2 = (6-0)^2 + (5-8)^2$ or $r^2 = (6-0)^2 + (5-2)^2$ or $r^2 = 36 + 9$	<b>M1</b>	<b>oe PI</b> Uses coordinates of centre and coordinates of either $P$ , $Q$ or $R$ <b>ft</b> their centre.  <b>PI</b> by correct value of $k$ in the equation of the circle.  <b>ft</b> their centre. <b>ft</b> their $r^2$ provided <b>M1</b> scored.
	$r = \sqrt{45}$ or $r = 3\sqrt{5}$ or $r^2 = 45$	<b>A1</b>	
	$(x-6)^2 + (y-5)^2 = 45$	<b>B1ft</b>	
		<b>3</b>	

Q	Answer	Marks	Comments
2(c)	$\frac{11-5}{9-6} [= 2]$  $[\text{Gradient of Tangent} =] -\frac{1}{2}$  $(y-11) = -\frac{1}{2}(x-9)$  $y = -\frac{1}{2}x + \frac{31}{2}$  $x + 2y = 31$	<b>M1</b>	<b>oe ft</b> their centre.  <b>ft</b> their gradient of tangent provided <b>M1</b> scored and any form.  Correct equation in correct form.
		<b>A1ft</b>	
		<b>A1</b>	
		<b>3</b>	

	<b>Question 2 Total</b>	<b>8</b>	
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Q	Answer	Marks	Comments
3(a)	$13^2 = (x-3)^2 + (2x-5)^2 - 2(x-3)(2x-5)\cos\frac{\pi}{3}$ <p>or</p> $13^2 = (x-2)^2 + (x-3)^2 - 2(x-2)(x-3)\cos\frac{2\pi}{3}$ $169 = (x^2 - 6x + 9) + (4x^2 - 20x + 25) - (2x^2 - 11x + 15)$ <p>or</p> $169 = (x^2 - 4x + 4) + (x^2 - 6x + 9) + (x^2 - 5x + 6)$ $169 = 3x^2 - 15x + 19$ <p>or</p> $3x^2 - 15x - 150 = 0$ <p>and</p> $x^2 - 5x - 50 = 0$	<div style="text-align: center;">M1</div>          <div style="text-align: center;">M1</div>          <div style="text-align: center;">A1</div>	<p>Correct substitution into Cosine Rule. Condone 169 for <math>13^2</math> <b>PI</b></p>          <p><b>oe</b> Expansion of brackets. Condone one error.</p>          <p><b>AG</b> Must be convinced. Must have further simplification before required result stated.</p>
		<b>3</b>	

Q	Answer	Marks	Comments
3(b)	$[x^2 - 5x - 50 = 0 \Rightarrow] \quad x = 10 \text{ or } x = -5$  $[\text{Arc Length} =] \quad (10 - 3) \times \frac{\pi}{3}$  $[\text{Arc Length} =] \quad \frac{7\pi}{3}$ or $[\text{Arc Length} =] \quad 7.3[303\dots]$  $[\text{Perimeter} =] \quad 13 + ((2 \times 10 - 5) - (10 - 3)) + \frac{7\pi}{3}$ or $[\text{Perimeter} =] \quad 13 + 8 + \frac{7\pi}{3}$  $[\text{Perimeter} =] \quad 28.3[\text{cm}]$	<b>B1</b>  <b>M1</b>  <b>A1</b>  <b>m1</b>  <b>A1</b>	Condone omission of $x = -5$ <b>PI</b> by correct further working.  <b>oe ft</b> their 10 provided their $(10 - 3)$ is positive.  <b>CAO</b>  <b>oe ft</b> their 10 provided their length of $CD$ is positive. <b>ft</b> their arc length.  <b>CAO AWRT</b>
		5	
	Question 3 Total	8	

Q	Answer	Marks	Comments
4	$(x+5)^2 + \dots + (y-7)^2 + \dots [= 0]$  $\left[ (x+5)^2 + (y-7)^2 = \right] 64 - k^2$  $64 - k^2 > 0$  $-8 < k < 8$	<b>M1</b>   <b>A1</b>   <b>m1</b>   <b>A1</b>	<b>oe</b> <b>PI</b> by $(x^2 + 10x + 25) + (y^2 - 14y + 49) + \dots [= 0]$  Rearranges to find expression for square of the radius. Simplified or unsimplified. <b>PI</b> by both correct critical values.  Forms inequality. <b>ft</b> their expression for the square of the radius. Condone $64 - k^2 \geq 0$  <b>CAO oe</b> such as $ k  < 8$ Condone $-8 \leq k \leq 8$ <b>oe</b>
		<b>4</b>	
	<b>Question 4 Total</b>	<b>4</b>	



Q	Answer	Marks	Comments
5(a)	$\left[ \log_{16} x^3 = \right] 3y$	B1	
		1	

Q	Answer	Marks	Comments
5(b)	$\left[ y = \log_{16} x \Rightarrow \right] 16^y = x$ $2^{4y} = x$ or $y = \frac{\log_2 x}{\log_2 16} \quad \text{oe}$ or $\log_2 x = \frac{\log_{16} x}{\log_{16} 2} \left[ = 4 \log_{16} x \right] \quad \text{oe}$ $\left[ \log_2 x = \right] 4y$	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>PI</b></p> <p><b>PI</b> by subsequent working.</p>
		<b>3</b>	

Q	Answer	Marks	Comments
5(c)	$\log_3 81 = 4$	M1	PI
	$4 \times 3y + 5 \times 4y - 4 = 60 \Rightarrow y = 2$	M1	PI Condone $\log_{16} x$ used instead of $y$
	$[x = 16^2 = ] \text{ 256}$	A1	NMS scores M0M0A0
		3	

	<b>Question 5 Total</b>	<b>7</b>	
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Q	Answer	Marks	Comments
6(a)	$6 \frac{\sin^2 x}{\cos x} = 5 \left( 1 + \frac{\sin^2 x}{\cos^2 x} \right) \cos^2 x$ $\left[ 5 \left( 1 + \frac{\sin^2 x}{\cos^2 x} \right) \cos^2 x = \right] 5 (\cos^2 x + \sin^2 x) = 5$ $\left[ 6 \frac{\sin^2 x}{\cos x} = 5 \Rightarrow \right] 6 \sin^2 x = 5 \cos x$ $\Rightarrow 6(1 - \cos^2 x) = 5 \cos x$ $6 - 6 \cos^2 x = 5 \cos x$ $\Rightarrow 6 \cos^2 x + 5 \cos x - 6 = 0$	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>PI</b> Uses <math>\tan x = \frac{\sin x}{\cos x}</math> to write equation in terms of <math>\cos x</math> and <math>\sin x</math> only.</p> <p><b>PI</b> Clear use of <math>\cos^2 x + \sin^2 x = 1</math> to show <math>\left( 1 + \frac{\sin^2 x}{\cos^2 x} \right) \cos^2 x = 1</math></p> <p>Clear use of <math>\cos^2 x + \sin^2 x = 1</math> on LHS to form an equation in <math>\cos x</math> only.</p> <p><b>AG</b> Must come from a convincing argument.</p>
		<b>4</b>	

Q	Answer	Marks	Comments
<b>6(b)</b>	$\cos x = \frac{2}{3}$ , $\cos x = -\frac{3}{2}$  Indicates that $\cos x$ cannot take a value less than $-1$ [Hence the only real solutions satisfy $\cos x = \frac{2}{3}$ ]	<b>B1</b>	Considers the two possible values for $\cos x$
		<b>E1</b>	Eliminates $\cos x = -\frac{3}{2}$ by logical argument.
		<b>2</b>	

Q	Answer	Marks	Comments
<b>6(c)</b>	$\cos(x + 35^\circ) = \frac{2}{3}$ or $x + 35^\circ = \cos^{-1}\left(\frac{2}{3}\right)$  $x = 13.2^\circ$  $x = -83.2^\circ$	<b>M1</b>	<b>PI</b>
		<b>A1</b>	<b>CAO</b> Condone more accurate figures given.
		<b>A1</b>	
		<b>3</b>	

	<b>Question 6 Total</b>	<b>9</b>	
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Q	Answer	Marks	Comments
7(a)	$[E(2X + 100) =]$ $2 \times 200 + 100$ $= 500 \text{ [dollars]}$	<b>M1</b>  <b>A1</b>	Applies expectation formula <b>PI</b>
		<b>2</b>	

Q	Answer	Marks	Comments
7(b)(i)	100	<b>B1</b>	
		<b>1</b>	

Q	Answer	Marks	Comments
7(b)(ii)	$[\text{Var}(X - Y) =]$ $25.2 + 4.7^2 \text{ or } 25.2 + 22.09$ $= 47.29$	<b>M1</b>  <b>A1</b>	Applies variance formula <b>PI</b> Condone $25.2 + 4.7$
		<b>2</b>	

	<b>Question 7 Total</b>	<b>5</b>	
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Q	Answer	Marks	Comments
8(a)(i)	$[P(A \cup B) = 0.45 + 0.32 = ] 0.77$	B1	
		1	

Q	Answer	Marks	Comments
8(a)(ii)	$[P(A \cap B) = ]$ $0.45 \times 0.32 [ = 0.144 ]$  $[P(A \cup B) = 0.77 - 0.144 = ]$ $0.626$	M1   A1	PI
		2	

Q	Answer	Marks	Comments
8(b)	$0.74 = 0.65 + 0.18 - P(C \cap D)$  $[P(C \cap D) = ] 0.09$  $[P(D C) = ] \frac{0.09}{0.65}$  $\frac{9}{65}$	M1  A1  M1  A1ft	Uses addition formula to form equation to find $P(C \cap D)$  PI  Applies conditional probability formula with their $P(C \cap D)$  Anything which rounds or truncates to 0.138 ft their 0.09 provided full method seen.
		4	

	Question 8 Total	7	
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Q	Answer	Marks	Comments
9(a)	$np = 22.5, \quad np(1-p) = 12.375$	<b>B1</b>	Forms two equations using mean and variance.
	$22.5(1-p) = 12.375$	<b>M1</b>	Obtains equation in one variable.
	$1-p = 0.55$		
	$p = 0.45$	<b>A1</b>	<b>AG</b> Be convinced.
	$n = 50$	<b>B1</b>	
		<b>4</b>	

Q	Answer	Marks	Comments
9(b)	$\left[ P(X = 24) = \right] \left( \begin{matrix} 50 \\ 24 \end{matrix} \right) 0.45^{24} (1-0.45)^{50-24}$	<b>M1</b>	Applies probability formula or uses cumulative probability tables with their $n$ <b>or</b> 0.7160 – 0.6134
	$= 0.1026$	<b>A1</b>	<b>PI</b> <b>AWRT</b>
		<b>2</b>	

Q	Answer	Marks	Comments
9(c)	$\left[ P(X > 19) = \right] 1 - 0.1974$	<b>M1</b>	<b>PI</b> Follow through their $n$
	$= 0.8026$	<b>A1</b>	<b>AWRT</b>
		<b>2</b>	

	<b>Question 9 Total</b>	<b>8</b>	
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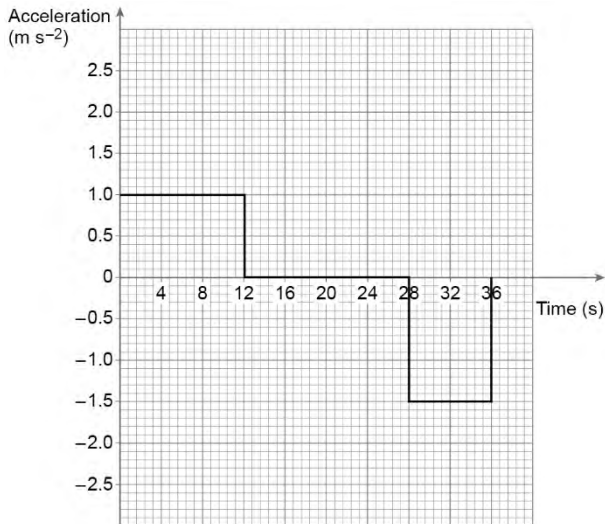
Q	Answer	Marks	Comments
10(a)	$0^2 = 32^2 + 2 \times (-9.8) \times s$	M1	Substitutes correct values into $v^2 = u^2 + 2as$ or a correct pair of suvat equations. Condone sign error with $-9.8$
	$[s = ] 52.2 [\text{metres}]$	A1	Anything that rounds or truncates to 52 (52.24489...)
		2	

Q	Answer	Marks	Comments
10(b)(i)	$[52.2 \times 2 - 2 = ] 102.4 [\text{metres}]$	B1ft	ft their value in <b>Part (a)</b> as long as it is larger than 2 Allow for answers between 102.4 and 102.5 (inclusive).
		1	

Q	Answer	Marks	Comments
10(b)(ii)	$2 = 32 \times t + 0.5 \times (-9.8) \times t^2$ or $0 = 32 - 9.8t_{up}$ and $50.2 = \frac{1}{2} \times 9.8t_{down}^2$  $4.9t^2 - 32t + 2 = 0$ or $0 = 32 - 9.8t_{up}$ and $50.2 = \frac{1}{2} \times 9.8t_{down}^2$ and $t = 3.2[6530...] + 3.2[02196...]$  $t = 6.47 [\text{seconds}]$	M1	PI Substitutes correct values into $s = ut + \frac{1}{2}at^2$ or a correct pair of suvat equations. Condone sign error with $-9.8$
		M1	Forms a completely correct quadratic equation or pair of suvat equations and attempts to find a value of $t$ PI By correct final answer.
		A1	AWRT 6.5 [May see $t = 0.063$ s rejected PI]
		3	

	Question 10 Total	6	
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Q	Answer	Marks	Comments
11(a)	$[\text{Total Distance} =] \frac{12 \times (16 + 36)}{2}$	<b>M1</b>	Attempt to find total distance travelled. Minimum of either one fully correct suvat formula, or intention to add at least two correct, relevant areas if area not treated as a trapezium. <b>PI</b>
	$[\text{Total Distance} =] \frac{312}{36}$	<b>M1</b>	<b>oe</b> Divides their 'total distance' by 36
	$\frac{26}{3} \text{ [m s}^{-1}\text{]}$	<b>A1</b>	<b>CAO</b> Correct answer implies <b>M1M1A1</b> Fraction must be simplified. Could be a mixed number. <b>AWRT</b> 8.7 if given as a decimal.
		<b>3</b>	

Q	Answer	Marks	Comments
11(b)		<b>B1</b>	3 horizontal line segments between correct time intervals (give bod for unclear, or missing, segment in the interval $12 < t < 28$ ). Ignore graph for values of $t$ greater than 36.
		<b>B1</b>	or $a = 1$ <u>and</u> $a = -1.5$ and no other values seen (could include $a = 0$ ).  A fully correct graph (give bod for unclear, or missing, segment in the interval $12 < t < 28$ ) Condone omission of vertical lines.
		<b>2</b>	

	<b>Question 11 Total</b>	<b>5</b>	
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Q	Answer	Marks	Comments
12(a)	<p>Statement that acceleration is zero.</p> <p>or</p> <p>Statement that resultant force is zero.</p> <p><math>5000 - 1500 - 3500 = 0</math></p>	<p><b>B1</b></p> <p><b>B1</b></p>	<p>Condone <math>a = 0</math></p> <p><b>oe</b> Shows that the resultant force is zero by giving numerical values in a suitable equation. May form an equation for <math>R</math> and solve.</p>
		<b>2</b>	

Q	Answer	Marks	Comments
12(b)	Thrust: $-3500 - 500 = 10000a$ $a_1 = -0.4$	M1	Considers correct forces on the trailer with 500 [N] as thrust. [a may be taken in either direction] Allow one error but must lead to a value for a
	Tension: $-3500 + 500 = 10000a$ $a_2 = -0.3$	M1	Considers correct forces on the trailer with 500 [N] acting as tension. [a may be taken in either direction] Allow one error but must lead to a value for a
	Thrust: $D - 1500 + 500 = 2000 \times (-0.4)$ Tension: $D - 1500 - 500 = 2000 \times (-0.3)$	M1	Considers forces on the tractor with their value for a from considering thrust and/or tension. Condone a written instead of its value. Must be a correct equation.
	Thrust: [D =] 200 [N]	A1	CAO
	Tension: [D =] 1400 [N]	A1	CAO
			Candidates do not need to identify the 'type' of force in the rod at any point in this question. Whether they are considering thrust or tension should be ascertained by their shown working.

<p><b>12(b)</b> <b>ALT</b></p>	<p><math>D - 1500 - 3500 = 12000a</math> or <math>D - 5000 = 12000a</math></p> <p>Thrust: <math>D + 500 - 1500 = 2000a</math> or <math>D - 1000 = 2000a</math></p> <p>Thrust:  <math display="block">\left[ \begin{array}{l} D + 500 - 1500 = 2000a \\ \text{and } D - 1000 = 2000a \Rightarrow \end{array} \right]</math> <math>D = 200 \text{ [N]}</math> </p> <p>Tension: <math>D - 500 - 1500 = 2000a</math> or <math>D - 2000 = 2000a</math></p> <p>Tension:  <math display="block">\left[ \begin{array}{l} D - 500 - 1500 = 2000a \\ \text{and } D - 2000 = 2000a \Rightarrow \end{array} \right]</math> <math>D = 1400 \text{ [N]}</math> </p>	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>oe</b> Considers tractor and trailer as a single particle and forms equation of motion simplified or unsimplified.</p> <p><b>oe</b> Forms equation of motion for tractor with 500 [N] as thrust simplified or unsimplified.</p> <p><b>CAO</b></p> <p><b>oe</b> Forms equation of motion for tractor with 500 [N] as tension simplified or unsimplified.</p> <p><b>CAO</b></p> <p>Candidates do not need to identify the 'type' of force in the rod at any point in this question. Whether they are considering thrust or tension should be ascertained by their shown working.</p>
		<p><b>5</b></p>	

Q	Answer	Marks	Comments
12(c)	$-3500 = 10\,000a$ $a = -0.35$ $0 = 2 + (-0.35)t$  $t = 5.7$	<p><b>M1</b></p>          <p><b>A1</b></p>	<p>Considers horizontal forces to find the correct value of acceleration (allow positive or negative) <u>and</u> substitutes their value for the acceleration into a correct suvat formula involving <math>t</math></p> <p><b>PI</b> by <math>\frac{20\,000}{3500}</math></p> <p>or better (eg 5.714) <b>oe</b></p>
		<b>2</b>	
	<b>Question 12 Total</b>	<b>9</b>	