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FM01

(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

January 2022

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221XFM01/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$\left[\sum_{r=n+1}^{2n} r^3 =\right] \sum_{r=1}^{2n} r^3 - \sum_{r=1}^{n} r^3$	M1	If **M0** awarded, allow **SC1** for sight of $\frac{1}{4}(2n)^2(2n+1)^2$ and $\frac{1}{4}n^2(n+1)^2$
	$=\frac{1}{4}(2n)^2(2n+1)^2-\frac{1}{4}n^2(n+1)^2$	A1	
	$=\frac{1}{4}n^2\left\{4(2n+1)^2-(n+1)^2\right\}$	M1	Factorising at least n^2 using consistent working
	$=\frac{1}{4}n^2\left\{16n^2+16n+4-\left(n^2+2n+1\right)\right\}$	M1	Expands the two squared brackets **or** uses difference of two squares Allow one slip
	$=\frac{1}{4}n^2\left\{15n^2+14n+3\right\}$		
	$=\frac{1}{4}n^2(5n+3)(3n+1)$	A1	

	Question 1 Total	5	

Q	Answer	Marks	Comments
2(a)	$\frac{7-3\text{i}}{k-5\text{i}} \times \frac{k+5\text{i}}{k+5\text{i}}$	M1	**or** $z = x + \text{i}y$ $(x+\text{i}y)(k-5\text{i}) = 7-3\text{i}$ Then multiplies out and equates real and imaginary parts
	Real part $= \frac{7k+15}{k^2+25}$	A1	Seen anywhere
	Imaginary part $= \frac{35-3k}{k^2+25}$	A1	Condone $\left(\frac{35-3k}{k^2+25}\right)\text{i}$
		3	

Q	Answer	Marks	Comments
2(b)	substituting $k=2$ or $\frac{35-3k}{7k+15}$ seen	M1	
	$\left[\frac{7-3\text{i}}{2-5\text{i}} = \frac{29}{29} + \text{i}\left(\frac{29}{29}\right) = \right]$ $1+\text{i}$ or $\frac{35-3\times 2}{7\times 2+15}$	A1	
	$\arg\left(\frac{7-3\text{i}}{2-5\text{i}}\right) = \arg(1+\text{i}) = \left[\tan^{-1}\left(\frac{1}{1}\right) = \right]$ $\frac{\pi}{4}$	A1	**AG** Condone $\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$ where $\theta = \arg\left(\frac{7-3\text{i}}{2-5\text{i}}\right)$
		3	

	Question 2 Total	**6**	

Q	Answer	Marks	Comments
3	$A = \pi r^2$ $\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$	B1	
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}A} \times \frac{\mathrm{d}A}{\mathrm{d}t}$	M1	Seen or used
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{2\pi r} \times 3$	A1ft	their value for r may be substituted in **ft** their $\frac{\mathrm{d}A}{\mathrm{d}r}$
	When $A = 36\pi$, $r = 6$	B1	
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{2\pi(6)} \times 3$	M1	**ft** their $\frac{\mathrm{d}r}{\mathrm{d}t}$ and their value of r
	$= \frac{1}{4\pi}$ [metres/day]	A1	**CAO**

	Question 3 Total	**6**	

Q	Answer	Marks	Comments
4(a)	$2x-\frac{\pi}{2}=2n\pi\pm\frac{2\pi}{3}$	B1	**oe**
	$x=\frac{1}{2}\left(2n\pi\pm\frac{2\pi}{3}+\frac{\pi}{2}\right)$	M1	Rearranging to make x the subject going from $\left(2x-\frac{\pi}{2}\right)$ to x Allow one slip
	$x=n\pi+\frac{\pi}{4}\pm\frac{\pi}{3}$	A1 A1	**oe**, eg $x=n\pi+\frac{7\pi}{12}$ or $x=n\pi-\frac{\pi}{12}$
		4	

Q	Answer	Marks	Comments
4(b)	$k=1$: $\frac{7\pi}{12},\frac{11\pi}{12}$ $k=2$: also $\frac{19\pi}{12},\frac{23\pi}{12},\frac{31\pi}{12},\frac{35\pi}{12}$	M1	For investigating at least one positive value of k with their general solution from **part (a)**
	$k=1$: 2 solutions $k=2$: 6 solutions [etc]	M1	For finding the number of solutions for at least two values of k using their general solution from **part (a)**
	$4k-2$	A1	**CAO**
		3	

	Question 4 Total	**7**	

Q	Answer	Marks	Comments
5(a)	$\alpha+\beta=-5$ $\alpha\beta=9$	B1 B1	
		2	

Q	Answer	Marks	Comments
5(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=25-18$ $\alpha^2+\beta^2=7$	M1 A1	or other valid method
		2	

Q	Answer	Marks	Comments
5(c)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ $=-125-3(9)(-5)=10$	M1 A1	or other valid method
		2	

Q	Answer	Marks	Comments
5(d)	Sum of roots $=\alpha+\dfrac{\beta}{\alpha}+\beta+\dfrac{\alpha}{\beta}$ $=\alpha+\beta+\dfrac{\beta^2+\alpha^2}{\alpha\beta}$	M1	
	$=-5+\dfrac{7}{9}=-\dfrac{38}{9}$	A1ft	**ft** their $\alpha^2+\beta^2$ from **part (b)**
	Product of roots $=\left(\alpha+\dfrac{\beta}{\alpha}\right)\left(\beta+\dfrac{\alpha}{\beta}\right)$ $=\alpha\beta+\dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}+1$	M1	
	$=\alpha\beta+\dfrac{\alpha^3+\beta^3}{\alpha\beta}+1$	m1	Converts $\dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}$ into $\dfrac{\alpha^3+\beta^3}{\alpha\beta}$
	$=9+\dfrac{10}{9}+1=\dfrac{100}{9}$	A1	
	$9x^2+38x+100=0$	A1	Correct quadratic equation with integer coefficients
		6	

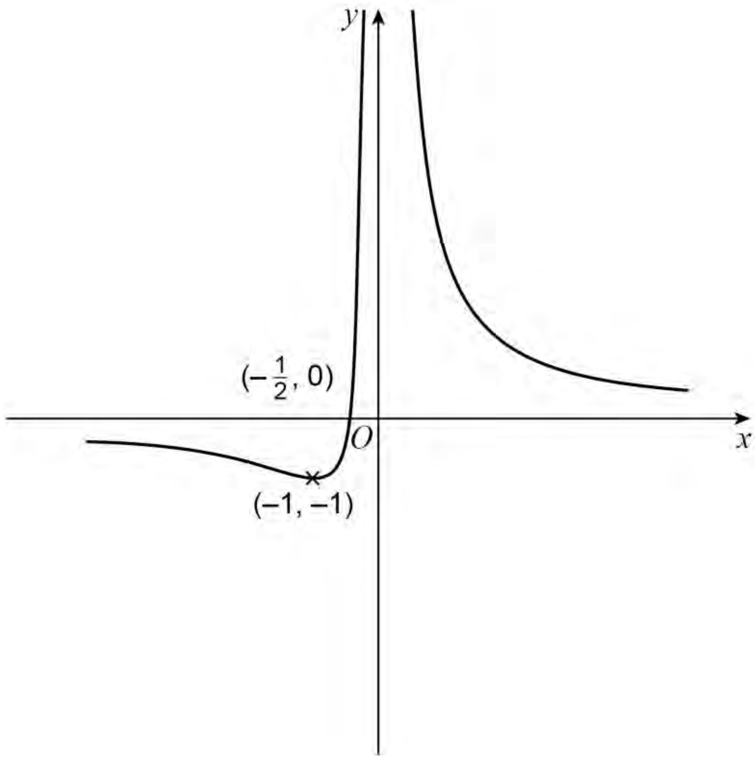
	Question 5 Total	**12**	

Q	Answer	Marks	Comments
6(a)	$x = 0$	B1	
	$y = 0$	B1	
		2	

Q	Answer	Marks	Comments
6(b)	$k = \frac{2x+1}{x^2}$ $kx^2 - 2x - 1 = 0$	M1	
	$\Delta \geq 0$ so $(-2)^2 - 4k(-1) \geq 0$	B1	Explicit use of the discriminant
	$4 + 4k \geq 0$ so $k \geq -1$	A1	**AG**
		3	

Q	Answer	Marks	Comments
6(c)	When $k = -1$, $-x^2 - 2x - 1 = 0$ $(x+1)^2 = 0 \Rightarrow x = -1$	M1	
	Stationary point is $(-1, -1)$	A1	
		2	

Q	Answer	Marks	Comments
6(d)	See below	B1	Correct general shape
	See below	B1	Axis intercept correctly labelled (condone x-coordinate only) and stationary point correctly marked and labelled
	See below	B1	Graph approaches all asymptotes



		3	

Q	Answer	Marks	Comments
6(e)	$\frac{2x+1}{x^2} > 3 \therefore 2x+1 > 3x^2$	M1	Allow equation if followed by attempt to solve inequality
	$3x^2 - 2x - 1 < 0$ $(3x+1)(x-1) < 0$	M1	Or for solving the corresponding equation
	$-\frac{1}{3} < x < 1$	A1	**PI**
	$-\frac{1}{3} < x < 0,\ 0 < x < 1$	A1	**ACF,** e.g. $-\frac{1}{3} < x < 1$ and $x \neq 0$
		4	

	Question 6 Total	**14**	

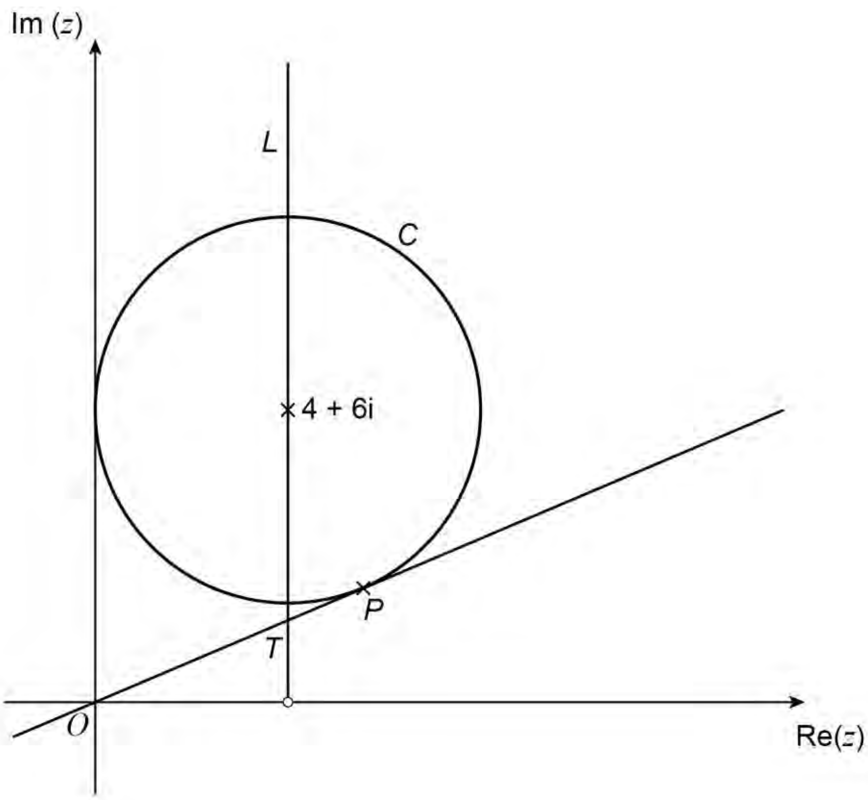
Q	Answer	Marks	Comments
7(a)	Because one of the limits is infinite	**E1**	Or 'the range of integration is infinite'
		1	

Q	Answer	Marks	Comments
7(b)	Because the integrand is not defined at one of the limits of integration **or** Because the integrand is not defined when $x = 0$	**E1**	
		1	

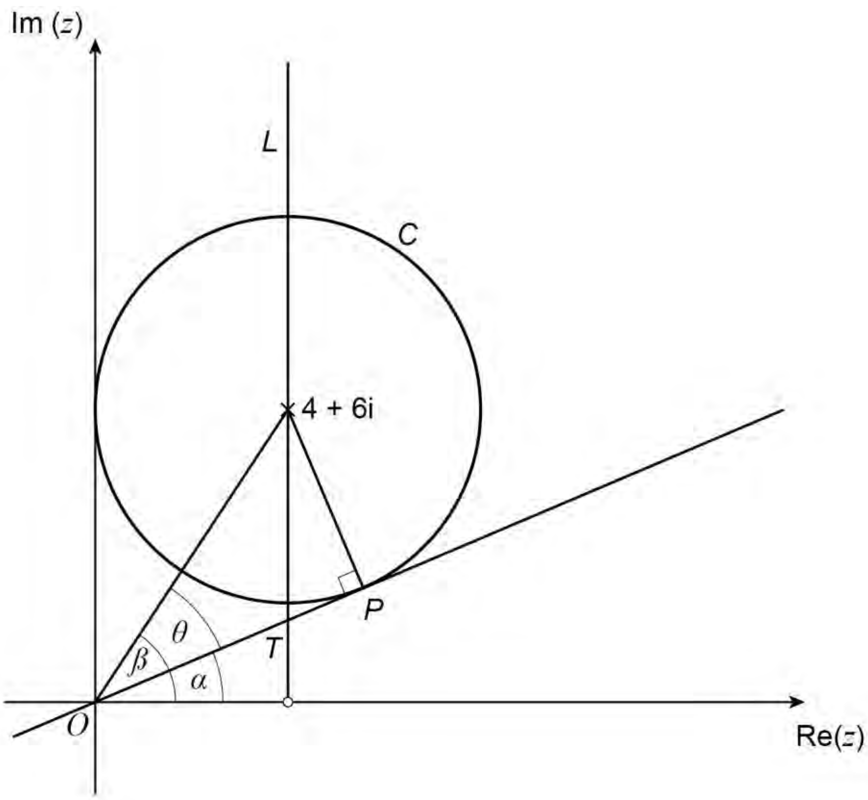
Q	Answer	Marks	Comments
7(c)	$I_2 = \lim_{h\to 0}\int_h^{64} \frac{1}{\left(\sqrt[3]{x}\right)^2}\,dx$	**M1**	Limiting process seen in the solution
	$\left[= \lim_{h\to 0}\int_h^{64} x^{-\frac{2}{3}}\,dx\right]$		
	$= \lim_{h\to 0}\left[3x^{\frac{1}{3}}\right]_h^{64}$	**m1**	Condone 0 as lower limit if 1st **M1** was awarded
	$= 3(4) - 3(0)$ $= 12$	**A1**	Correct answer with no limiting process shown is **SC1**
		3	

	Question 7 Total	**5**	

Q	Answer	Marks	Comments
8(a)	$4+6\text{i}$	B1	
		1	

Q	Answer	Marks	Comments
8(b)	*L* drawn correctly	B1	Condone no indication that end point is not included, but must not be below the real axis
	OP drawn correctly	B1	No need to extend beyond *O* or *P*
	T marked correctly	B1ft	The intersection of their line *OP* with the correct half-line *L* **See artwork below**
			
		3	

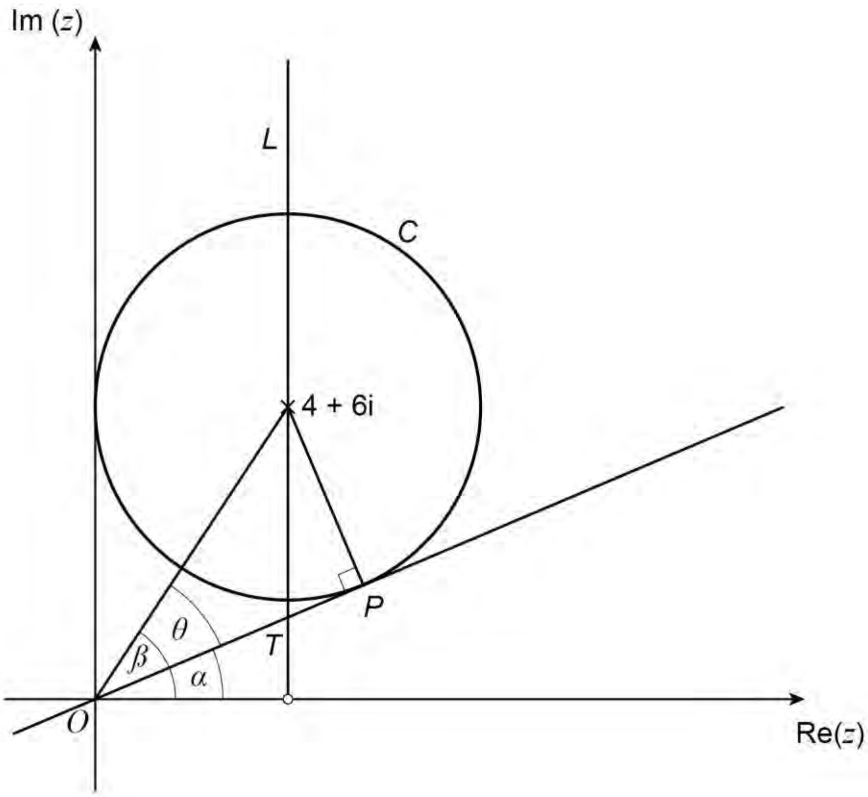
Q	Answer	Marks	Comments
8(c)	$\tan\beta = \dfrac{3}{2}$	B1	See diagram below
	$\left[\sin\theta = \dfrac{2}{\sqrt{13}} \text{ so}\right] \tan\theta = \dfrac{2}{3}$	B1	
	$\arg z = \alpha = \beta - \theta$	M1	
	$\tan\alpha = \dfrac{5}{12}$ [or $\alpha = 0.39479...$]	A1	
	$z = 4 + (4\tan\alpha)\mathrm{i}$	M1	
	$z = 4 + \dfrac{5}{3}\mathrm{i}$	A1	Exact value for real and imaginary parts



		6	

	Question 8 Total	10	

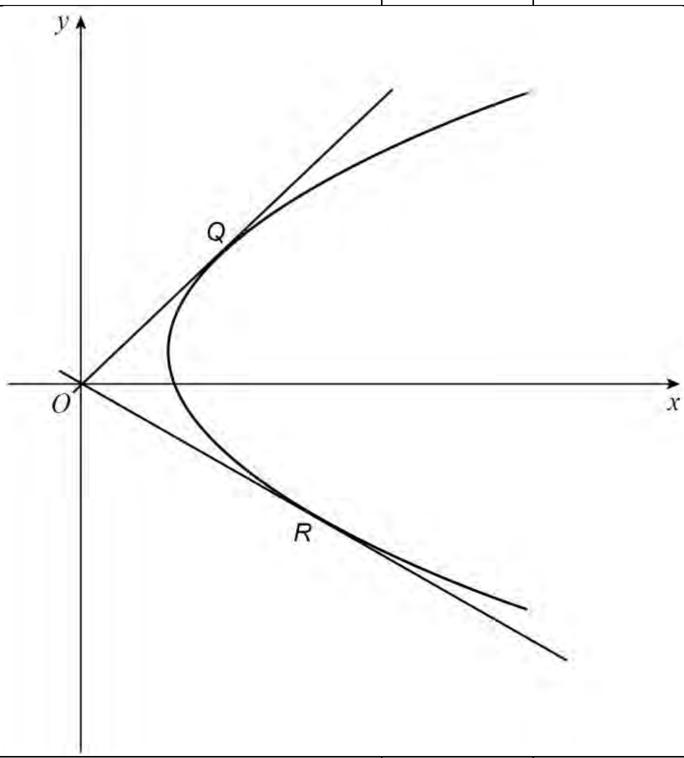
Q	Answer	Marks	Comments
8(c) ALT	Equation of line *OP* is $y = mx$ $(x-4)^2 + (mx-6)^2 = 16$	**M1**	
	$(m^2+1)x^2 - (8+12m)x + 36 = 0$	**A1**	
	$\Delta = 0 \Rightarrow (8+12m)^2 - 4\times(m^2+1)\times 36 = 0$	**M1**	
	$m = \frac{5}{12}$	**A1**	
	y-coordinate of *T* $y = \frac{5}{12}\times 4$	**M1**	**ft** their m
	$z = 4 + \frac{5}{3}\mathrm{i}$	**A1**	Exact value for real and imaginary parts



Q	Answer	Marks	Comments
9(a)	$(x-12)^2+y^2=(x+12)^2$	M1	
	$x^2-24x+144+y^2=x^2+24x+144$ $y^2=48x$	A1	
		2	

Q	Answer	Marks	Comments
9(b)	$(y-4)^2=48(x-5)$	B1 B1	**B1** for LHS, **B1** for RHS
		2	

Q	Answer	Marks	Comments
9(c)(i)	Parabola with vertex facing left	B1	
	Parabola with vertex in 1st quadrant	B1	**ft** their answer to part **(b)** if translation $\begin{bmatrix} 5 \\ -4 \end{bmatrix}$ is used, which leads to a parabola with a vertex in the 4th quadrant
	Lines *OQ* and *OR*, *Q* and *R* marked correctly, *R* to the right of *Q*	B1	**See artwork below**
		3	



Q	Answer	Marks	Comments
9(c)(ii)	$y = mx$ is a tangent so $(mx-4)^2 = 48(x-5)$	**M1**	
	$m^2x^2 - 8mx + 16 - 48x + 240 = 0$ $m^2x^2 - (8m+48)x + 256 = 0$	**A1ft**	**ft** their **part (b)**
	$\Delta = 0 \Rightarrow (8m+48)^2 - 4m^2(256) = 0$	**M1**	
	$960m^2 - 768m - 2304 = 0$ $5m^2 - 4m - 12 = 0$	**A1ft**	for either equation (**oe**) **ft** their **part (b)**
	$m = 2$ or $m = -\dfrac{6}{5}$	**A1ft**	**ft** their **part (b)**
	$m = 2$: $4x^2 - 64x + 256 = 0$ and $x = 8$ or $m = -\dfrac{6}{5}$: $\dfrac{36}{25}x^2 - \dfrac{192}{5}x + 256 = 0$ and $x = \dfrac{40}{3}$	**M1**	
	$Q(8, 16)$	**A1**	
	$R\left(\dfrac{40}{3}, -16\right)$	**A1**	
		8	

	Question 9 Total	**15**	