OXFORD

INTERNATIONAL AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM01) Pure Maths Unit FP1

Tuesday 22 January 2019 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box on each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Answer **all** questions in the spaces provided.

Do not write outside the box

1 A curve has equation $y = x^3 - 12x$

1 (a) A line passes through two points on the curve.
At one point $x = -2$ and at the other point $x = -2 + h$

Find the gradient of the line in the form $ph + qh^2$, where p and q are integers.

[4 marks]

Answer _____

1 (b) Use your answer to part **(a)** to explain why the point on the curve where $x = -2$ is a stationary point.

[2 marks]

6

Do not write outside the box

2 The series S_n is defined for $n \geq 2$ by

$$S_n = \sum_{r=1}^{n} \left(2r^3 + 3r^2 - 5r\right)$$

2 (a) Show that

$$S_n = kn(n + a)(n + b)(n + c)$$

where k is a fraction and a, b and c are integers.

[5 marks]

2 (b) Explain why S_n is always a multiple of 3

[2 marks]

7

Do not write outside the box

3 One of the roots of the quadratic equation

$$x^2 + 3x + c = 0$$

is $p + 2i$, where p and c are real numbers.

Find the value of p and the value of c.

[5 marks]

p = _____

c = _____

5

Do not write outside the box

4 For each of the improper integrals below, either find its exact value or explain why it has no finite value.

Show all necessary working.

4 (a) $\int_0^{12} \frac{1}{\sqrt{x}}\,\mathrm{d}x$

[3 marks]

Answer _____

4 (b) $\int_0^{12} \frac{1}{x^4}\,\mathrm{d}x$

[3 marks]

Answer _____

6

Do not write outside the box

5 (a) Find the general solution of the equation

$$\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$$

[4 marks]

Answer _____

5 (b) Find the **sum** of all the solutions of the equation

$$\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$$

between -18π and 18π.

Give your answer in terms of π.

[5 marks]

Answer _____

9

Do not write outside the box

6 By considering the derivative of $y = x^{\frac{1}{3}}$ when $x = 8$, find an estimate for $\sqrt[3]{8.06}$

[6 marks]

Answer _____

6

Turn over for the next question

Do not write outside the box

7 For the complex number $z_1 = x + \mathrm{i}y$, where x and y are real numbers,

$$2z_1^* + 3\mathrm{i} = \mathrm{i}z_1$$

7 (a) Find the value of x and the value of y.

[6 marks]

$x =$ _____

$y =$ _____

Do not write outside the box

7 (b) Find the modulus and argument of z_1

[3 marks]

$|z_1| =$ _____

$\arg(z_1) =$ _____

7 (c) Find the shortest distance on an Argand diagram from the point representing z_1 to the half-line $\arg z = \frac{\pi}{4}$

[4 marks]

Answer _____

13

Do not write outside the box

8 The curve C has equation

$$y = \frac{x-2}{x-3}$$

8 (a) State the equations of the asymptotes of C.

[2 marks]

Answer _____

8 (b) The line L has equation

$$y = \frac{1}{2}x$$

Find the coordinates of the points of intersection of L and C.

[4 marks]

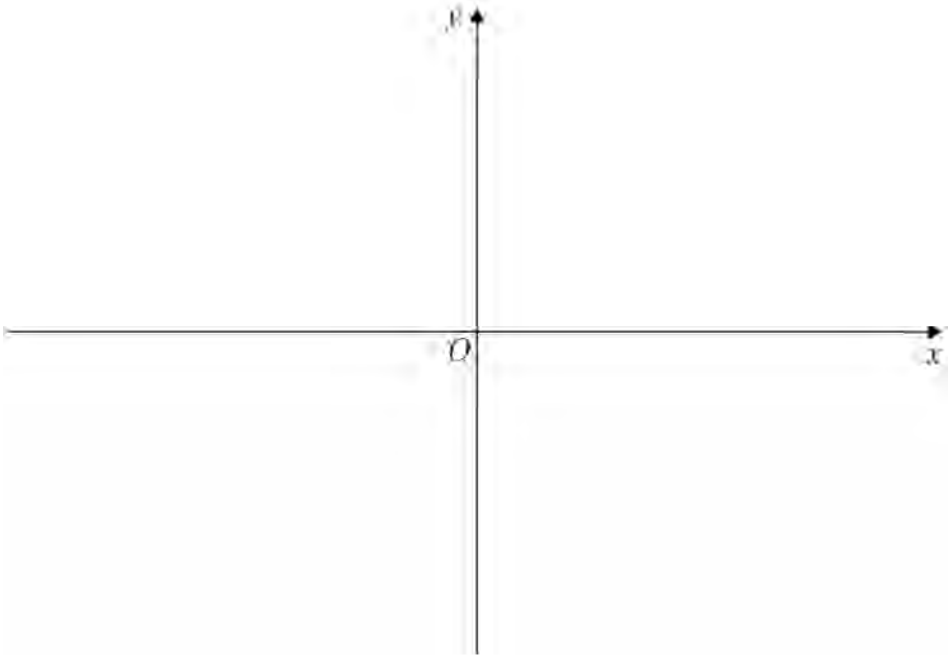
Answer _____

Do not write outside the box

8 (c) Sketch C and L on the same axes.

You are given that C has no stationary points.

[4 marks]



8 (d) Solve the inequality

$$\frac{x-2}{x-3} \leq \frac{1}{2}x$$

[2 marks]

Answer _____

12

Do not write outside the box

9 The ellipse E_1 has equation

$$\frac{x^2}{16}+\frac{y^2}{12}=1$$

The locus of a point P is such that the distance from P to the point (6, 0) is half the distance from P to the y-axis.

9 (a) The locus of P is the curve E_2

Show that E_2 is an ellipse which is a translation of E_1

Write down the vector for this translation.

[7 marks]

Answer _____

Do not write outside the box

9 (b) Find the coordinates of the points where E_2 meets the x-axis.

[1 mark]

Answer _____

9 (c) Show that, if the line $y = mx + c$ meets the ellipse E_2, then

$$(3 + 4m^2)x^2 + (8mc - 48)x + (4c^2 + 144) = 0$$

[3 marks]

Question 9 continues on the next page

Do not write outside the box

9 (d) Hence find the equations of the tangents to the ellipse E_2 which pass through the point (0, 8).

No credit will be given for solutions based on differentiation.

[5 marks]

Answer _____

16

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**