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MA05

(9660/MA05) Unit M2 Mechanics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	$\mathbf{v}=\begin{bmatrix}\frac{5\pi}{6}\cos\left(\frac{\pi t}{6}\right)\\ -\frac{5\pi}{6}\sin\left(\frac{\pi t}{6}\right)\end{bmatrix}$	**M1 A1**	**M1**: at least one component correct **A1**: Both components correct **ACF**
		2	
1(a)(ii)	$\mathbf{a}=\begin{bmatrix}-\frac{5\pi^2}{36}\sin\left(\frac{\pi t}{6}\right)\\ -\frac{5\pi^2}{36}\cos\left(\frac{\pi t}{6}\right)\end{bmatrix}$	**B1**	**ft** their **(a)(i)** **oe**
		1	
1(b)	$\mathbf{v}\cdot\mathbf{a}$	**M1**	Takes scalar product between their velocity and acceleration **PI** by correct products
	$=\begin{bmatrix}\frac{5\pi}{6}\cos\left(\frac{\pi t}{6}\right)\\ -\frac{5\pi}{6}\sin\left(\frac{\pi t}{6}\right)\end{bmatrix}\cdot\begin{bmatrix}-\frac{5\pi^2}{36}\sin\left(\frac{\pi t}{6}\right)\\ -\frac{5\pi^2}{36}\cos\left(\frac{\pi t}{6}\right)\end{bmatrix}$ $=\frac{5\pi}{6}\cos\left(\frac{\pi t}{6}\right)\times\left(-\frac{5\pi^2}{36}\sin\left(\frac{\pi t}{6}\right)\right)$ $+\left(-\frac{5\pi}{6}\sin\left(\frac{\pi t}{6}\right)\right)\times\left(-\frac{5\pi^2}{36}\cos\left(\frac{\pi t}{6}\right)\right)$ $\left[=-\frac{25\pi^3}{216}\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi t}{6}\right)\right.$ $\left.+\frac{25\pi^3}{216}\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi t}{6}\right)\right]$ $=0$	**A1**	Shows the correct scalar product is equal to zero with at least one line of working Be convinced
	[As the] scalar product between the velocity and acceleration is zero [for all values of t, the] velocity [of the particle] is perpendicular to its acceleration.	**E1**	**oe** Must show their scalar product is zero to allow this mark to be awarded Allow 'dot' product for scalar product Do not allow vector product for scalar product
		3	
1(c)	[The particle follows a] circular [path.]	**E1**	**oe**
		1	
	Total	**7**	

Q	Answer	Marks	Comments
2	$\bar{X} = \dfrac{1.25\times3+2.5\times4+3.75\times5+2.5\times1+1.25\times2}{1.25+2.5+3.75+2.5+1.25}$ $\bar{Y} = \dfrac{1.25\times1+2.5\times2+3.75\times3+2.5\times2+1.25\times1}{1.25+2.5+3.75+2.5+1.25}$	M1	Forming an equation for the x-coordinate or y-coordinate of the centre of mass. Condone one slip or omission
	$\bar{X} = \dfrac{37.5}{11.25},\quad \bar{Y} = \dfrac{23.75}{11.25}$		
	$\left(\dfrac{10}{3}, \dfrac{19}{9}\right)$	A1 A1	**oe** for x-coordinate, **AWRT** 3.3 **oe** for y-coordinate, **AWRT** 2.1
	Total	**3**	

Q	Answer	Marks	Comments
3(a)	[Resultant force =] $\begin{bmatrix}2\\-1\\5\end{bmatrix}+\begin{bmatrix}3\\0\\2\end{bmatrix}+\begin{bmatrix}7\\3\\-3\end{bmatrix}$	M1	Adds together the three forces **PI** by resultant force vector with at least two correct components
	$=\begin{bmatrix}12\\2\\4\end{bmatrix}$	A1	Finds the correct resultant force vector
	$\mathbf{a}=\frac{1}{2}\begin{bmatrix}12\\2\\4\end{bmatrix}=\begin{bmatrix}6\\1\\2\end{bmatrix}$ [m s^{-2}]	A1ft	**oe**, finds the correct acceleration from their resultant force vector Do not **ISW** If **M0** awarded**, SC1** for **AWRT** 6.4, such as $\sqrt{41}$ [the magnitude of the acceleration]
		3	
3(b)	$\begin{bmatrix}12\\2\\4\end{bmatrix}+\mathbf{F}_4=\begin{bmatrix}0\\0\\0\end{bmatrix}$		
	$\mathbf{F}_4=\begin{bmatrix}-12\\-2\\-4\end{bmatrix}$	B1ft	**ft** their resultant force vector from **(a)** Do not **ISW**
		1	
	Total	4	

Q	Answer	Marks	Comments
4(a)	Forces up the slope: $F+60\cos(15°)$	M1	At least one of the expressions for the 'forces up the slope' or 'forces down the slope' correct **PI** by correct equation for F
	Forces down the slope: $500\sin(15°)$		$125\left(\sqrt{6}-\sqrt{2}\right)$
	Equilibrium requires: $F=500\sin(15°)-60\cos(15°)$	A1	Correct equation for F
	$F=500\times\dfrac{\sqrt{6}-\sqrt{2}}{4}-60\times\dfrac{\sqrt{6}+\sqrt{2}}{4}$	M1	Uses both of the given sine and cosine relationships with their equation for F **PI**
	$F=110\sqrt{6}-140\sqrt{2}$	A1	**CAO**
		4	
4(b)	Forces into the slope $500\cos(15°)+60\sin(15°)$ $\left[=125\left(\sqrt{6}+\sqrt{2}\right)+15\left(\sqrt{6}-\sqrt{2}\right)\right]$	M1	At least one contribution correct **PI** by correct expression for normal reaction R
	Equilibrium requires: $R=500\cos(15°)+60\sin(15°)$	A1	Correct expression for normal reaction R Note $R=498.49...$ [N]
	$\left\lfloor R=140\sqrt{6}+110\sqrt{2}\right\rfloor$		
	$110\sqrt{6}-140\sqrt{2}$ $\leq\mu\left(500\cos(15°)+60\sin(15°)\right)$	m1	Use of $F\leq\mu R$ with their R and with their F of the form $a\sqrt{6}+b\sqrt{2}$ Note: $F=71.45...$ [N] Condone equality or strict inequality
	$\dfrac{110\sqrt{6}-140\sqrt{2}}{140\sqrt{6}+110\sqrt{2}}\leq\mu$	A1	**CAO** such as $\dfrac{616-317\sqrt{3}}{467}\leq\mu$, $0.14[334...]\leq\mu$ Must be weak inequality for μ and there must be no upper limit
		4	
	Total	**8**	

Q	Answer	Marks	Comments
5(a)(i)	Initial vertical velocity $=15\sin(39°)$ $\lfloor =9.4398...\text{ m s}^{-1}\rfloor$ Time taken to reach maximum height $=\dfrac{0-15\sin(39°)}{-9.8}$ $[=0.9632...\text{ s}]$ Time of flight $=2\times\dfrac{15\sin(39°)}{9.8}$ $=1.93$ $[\text{s}]$	**M1** **m1** **A1**	**PI** by correct time to maximum height **PI** by correct time of flight **oe** **CAO** to 3 sf
		3	
5(a)(ii)	Horizontal velocity $=15\cos(39°)$ $\lfloor =11.657...\text{ m s}^{-1}\rfloor$ Range $=15\cos(39°)\times\left(2\times\dfrac{15\sin(39°)}{9.8}\right)$ $=22$ [metres]	**M1** **A1ft**	Finds the horizontal component of velocity **ft** their **(a)(i)** to correctly find their range to 2 or more sf Note: correct unrounded answer is 22.457... metres
		2	
5(a)(iii)	Air resistance on the pebble is negligible	**E1**	Any plausible assumption
		1	
5(b)(i)	Maximum height $=15\sin(39°)\times\left(\dfrac{15\sin(39°)}{9.8}\right)+\dfrac{1}{2}\times(-9.8)\times\left(\dfrac{15\sin(39°)}{9.8}\right)^2$ $=4.5$ [metres]	**M1** **A1**	Use of $s=ut+\dfrac{1}{2}at^2$ or $v^2=u^2+2as$ with $v=0$, $u=15\sin(39°)$ and $a=-9.8$ Condone one slip **AWRT** 4.5 or 4.55 metres Note: correct unrounded answer is 4.546… metres
		2	
5(b)(ii)	11 [metres]	**B1ft**	**ft** half their answer to **(a)(ii)** to 2 sf
		1	
	Total	**9**	

Q	Answer	Marks	Comments
6(a)	Kinetic energy of cyclist and bicycle $=\frac{1}{2}\times 70\times 6.0^2$ $=1260$ [J]	**M1** **A1**	Use of $E=\frac{1}{2}mv^2$ Accept 1300 [J]
		2	
6(b)(i)	Driving force provided by cyclist $=\frac{150}{6}=25$ [N] As the cyclist and bicycle are not accelerating, the total resistive force must be equal to the driving force.	**B1** **E1**	Must show calculation leading to 25 Explanation based upon acceleration or resultant force being zero
		2	
6(b)(ii)	150 J As the cyclist is not gaining/losing kinetic energy or gravitational potential energy or All the work she does each second is against the resistive forces.	**B1** **E1**	Must include units Explanation based on energy argument
		2	

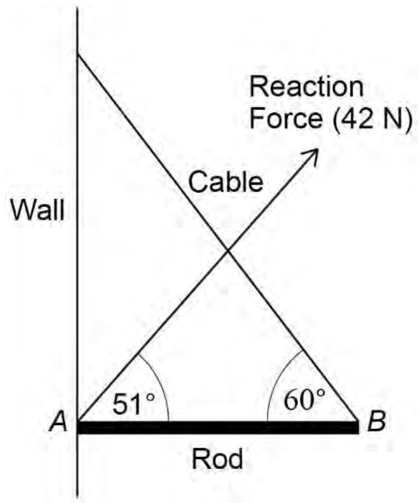
Q	Answer	Marks	Comments
6(c)(i)	Force down the road $= 70 \times 9.8 \times \sin(1.5°) + 25$ $= 42.957...$ [N] Resultant force on cyclist and bicycle $70a = 40 - 42.957...$ $70a = -2.957...$ Constant acceleration of cyclist and bicycle $a = \frac{-2.957...}{70}$ $a = -0.042\lfloor 2...\text{ m s}^{-2}\rfloor$	**M1 A1** **m1** **A1**	**M1**: Sum of 25 and component of weight, condone trigonometric error **A1**: All correct Forms an equation for the resultant force **ft** their forces down the road **AWFW** [–0.043, –0.042] Must be negative
		4	
6(c)(ii)	$v^2 = u^2 + 2as$ $= 6^2 + 2 \times (-0.0422...) \times 100$ $[\Rightarrow v =]\ 5.2\lfloor 488...\text{ m s}^{-1}\rfloor$	 **M1** **A1**	 **ft** their **(c)(i)** even if positive **CAO**
		2	
6(c)(iii)	$E = mg\Delta h$ $= 70 \times 9.8 \times 100 \sin(1.5°)$ $= 1800$ [J]	 **M1** **A1**	 Unrounded answer is 1795.73… [J]
		2	
	Total	**14**	

Q	Answer	Marks	Comments
7(a)(i)	$[x =]\ (u\cos\alpha)t$	B1	
		1	
7(a)(ii)	$[y =]\ (u\sin\alpha)t - \frac{1}{2}gt^2$	B1	
		1	
7(b)	$t = \frac{x}{u\cos\alpha}$ $y = (u\sin\alpha)\times\frac{x}{u\cos\alpha} - \frac{1}{2}g\times\left(\frac{x}{u\cos\alpha}\right)^2$ $(u\sin\alpha)\times\frac{x}{u\cos\alpha} = x\tan\alpha$	B1 M1 A1	Seen or used **M1**: Eliminates t in both terms with at least one term correct **A1**: All correct
	$-\frac{1}{2}g\times\left(\frac{x}{u\cos\alpha}\right)^2 = -\frac{gx^2}{2u^2}\times\frac{1}{\cos^2\alpha}$		
	$-\frac{gx^2}{2u^2}\times\frac{1}{\cos^2\alpha} = -\frac{gx^2}{2u^2}\sec^2\alpha$		
	$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha$	A1	**AG** Be convinced
		4	

Q	Answer	Marks	Comments
7(c)(i)	At maximum height, the vertical component of velocity is zero. $0 = u\sin\alpha - gt$ $t = \dfrac{u\sin\alpha}{g}$	M1	or $u\sin\alpha$ and $0 = u^2 + 2as$
	$y = (u\sin\alpha)t - \dfrac{1}{2}gt^2$ $y = (u\sin\alpha)\times\dfrac{u\sin\alpha}{g} - \dfrac{1}{2}g\left(\dfrac{u\sin\alpha}{g}\right)^2$	m1	Substitutes in correct expression for time to reach maximum height or $s = \dfrac{0-(u\sin\alpha)^2}{2\times -g}$ or better
	$y = \dfrac{u^2\sin^2\alpha}{2g}$	A1	**CAO**
		3	
7(c)(ii)	$16 = \dfrac{20^2\sin^2\alpha}{2\times 9.8}$ $\sin\alpha = [\pm]\dfrac{7\sqrt{10}}{25}$	M1	**PI** by correct answer **ft** their **(c)(i)**
	$\alpha = 62$	A1	**AWRT** 62
		2	
	Total	**11**	

Q	Answer	Marks	Comments
8(a)	The string has zero mass.	**E1**	**oe**
		1	
8(b)	Forces in the vertical plane $T\cos\theta = mg$	**M1 A1**	**M1** for $T\cos\theta$
	Forces in the horizontal plane $T\sin\theta = \dfrac{mv^2}{r}$	**B1**	
	Radius of circle particle is moving on $r = 0.5\sin\theta$	**B1**	Seen or used
	$\tan\theta = \dfrac{v^2}{gr}$	**m1**	Eliminates T or r by combining two of the equations
	$\tan\theta = \dfrac{v^2}{g \times 0.5\sin\theta}$		
	$g\sin\theta\tan\theta = 2v^2$		
	$g\sin^2\theta = 2v^2\cos\theta$	**A1**	**AG** Be convinced
		6	
8(c)(i)	$9.8\sin^2\theta = 2\times 4.0^2 \times \cos\theta$		
	$9.8\left(1-\cos^2\theta\right) = 2\times 4.0^2 \times \cos\theta$	**M1**	Uses $\sin^2\theta + \cos^2\theta = 1$ **PI** by correct quadratic equation in $\cos\theta$
	$9.8\cos^2\theta + 32\cos\theta - 9.8 = 0$	**A1**	Forms a correct quadratic equation in $\cos\theta$
	$\cos\theta = 0.2819...$	**A1**	Correct value for $\cos\theta$ **PI** by correct answer
	$\theta = 74$	**A1**	**CAO, AWRT** 74
		4	
8(c)(ii)	$\omega = \dfrac{v}{r} = \dfrac{4.0}{0.5\sin\left(73.6...^\circ\right)}$	**M1**	**ft** their **(c)(i)** Must use $r = 0.5\sin\theta$
	$\omega = 8.3$ rad s^{-1}	**A1**	**CAO**
		2	
	Total	**13**	

Q	Answer	Marks	Comments
9(a)	No resultant force No resultant moment	**E1** **E1**	**oe** **oe**
		2	
9(b)	Anticlockwise moments about *A* $1.2 \times T \sin(60°)$ Clockwise moments about *A* $0.7 \times 8.0 \times 9.8$ Equilibrium $1.2 \times T \sin(60°) = 0.7 \times 8.0 \times 9.8$ $T = \dfrac{0.7 \times 8.0 \times 9.8}{1.2 \sin(60°)}$ $T = 52.8$ [N]	**M1** **A1** **A1**	**M1** for one moment calculation. 54.88 [N m] Both moments correct and used with the equilibrium condition **CAO** to 3 sf
		3	

Q	Answer	Marks	Comments
9(c)	Vertical component of reaction force (via equilibrium of forces)		
	$=8.0\times9.8-52.8...\times\sin(60°)$	**M1**	**ft** their **(b)**
	$=32.6[66...\text{ N}]$	**A1**	Correct vertical component of reaction force
	Horizontal component of reaction force (via equilibrium of forces)		
	$[52.8...\times\cos(60°)=]\ 26.4[04...\text{ N}]$	**B1**	**ft** their **(b)**
	Magnitude of reaction force		
	$=\sqrt{(26.404...)^2+(32.666...)^2}$		
	$=42\ [\text{N}]$	**B1**	**AWRT** 42 [N]
	Direction of reaction force relative to horizontal		
	$\tan\alpha=\dfrac{32.666...}{26.404...}$	**M1**	**PI**
	$\alpha=51°$ above the horizontal	**A1**	Angle consistent with diagram **AWRT** 51° Allow **AWRT** 39° to the vertical
	 		
		6	
	Total	**11**	