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MATHEMATICS
MA05

Mechanics Unit M2

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

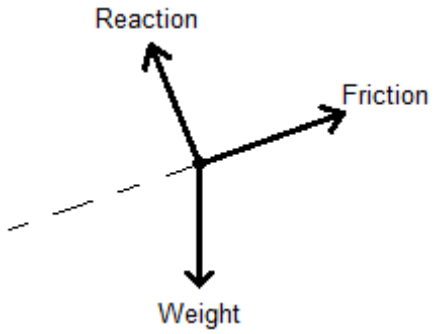
Q	Answer	Mark	Comments
1(a)	$\boldsymbol{v} = \dfrac{\mathrm{d}\boldsymbol{r}}{\mathrm{dt}}$	**M1**	Attempt at differentiating either of the two components
	$\boldsymbol{v} = -2e^{-0.5t}\mathbf{i} + \left(1+\cos t\right)\mathbf{j}$	**A1** **A1**	Correct **i** component Correct **j** component
1(b)	$\boldsymbol{v} = -2e^{-0.5\times 5}\mathbf{i} + \left(1+\cos\left(5\right)\right)\mathbf{j}$	**M1**	oe, e.g. column vector FT their answer to **(a)**
	$\left\|\boldsymbol{v}\right\| = 1.29\ [\mathrm{m\ s^{-1}}]$	**A1**	AWRT 1.3 m s^{-1} from below
1(c)	$\boldsymbol{a} = e^{-0.5t}\ \mathbf{i} - \sin t\ \mathbf{j}$	**M1**	FT their answer to part **(a)** in the form $\boldsymbol{v} = ae^{-0.5t}\mathbf{i} + \left(b \pm \cos t\right)\mathbf{j}$ where $a \neq 0$
	$\left\|\boldsymbol{a}\right\| = 0.962\left[\mathrm{m\ s^{-2}}\right]$	**A1**	CAO Condone 0.96 [m s^{-2}] but not 0.963 [m s^{-2}]
	Total	**7**	

Q	Answer	Mark	Comments
2(a)	$31\cos(15) = 29.9$[437 m s^{-1}]		
	$\dfrac{25}{31\cos(15)} = 0.834$[9 s]	**B1**	Must show working, such as $\dfrac{25}{29.9...}$, $\dfrac{25}{31\left(\dfrac{\sqrt{6}+\sqrt{2}}{4}\right)}$, $\dfrac{25\left(\sqrt{6}-\sqrt{2}\right)}{31}$ or gives the time to at least 3 significant figures.
2(b)	$31\sin(15)$ or 8.0[234 m s^{-1}]	**M1**	PI or seen in any working
	$s = 8.0234 \times 0.8349 - 0.5 \times 9.8 \times 0.8349^2$	**m1**	Use of $s = ut + \frac{1}{2}at^2$ and $a = \pm 9.8$ with $t = 0.83$ or better AWRT 3.3 m
	$s = 3.28$[31 m]	**A1**	The answer must come from considering the height of ball at the goal line and not from a calculation of the maximum height reached by the ball during its flight. Must have scored at least **M1 m0**
	No goal is scored as $3.28 > 2.4$	**E1F**	Must compare their 3.28 (or 3.3) with 2.4 or state '0.88.. m over' and give correct conclusion.
	Total	**5**	

Q	Answer	Mark	Comments
3(a)	[GPE $= mgh$ =] 0.17 × 9.8 × 2.5 $= 4.165$ [J]	**B1**	Condone 4.17 [J] and 4.2 [J], but not 4.16 [J]
3(b)	$\frac{1}{2}mv^2 = 4.165$ $v^2 = 49$ $v = 7.0$ [m s^{-1}]	**M1** **A1**	Use of SUVAT scores 0/2 FT their GPE from part **(a)** CAO, AWRT 7.0 [m s^{-1}] Condone 1 significant figure answer
3(c)	[The speed the apple hits the ground with will be] less [than 7.0 m s^{-1}] [Work is done against] air resistance or [Apple is] not a particle	**E1** **E1**	Not 'different' Allow 'drag' instead of air resistance
	Total	**5**	

Q	Answer	Mark	Comments
4(a)	$\bar{X} = \frac{(2.1\times1)+(1.2\times2)+(2.7\times3)+(1.8\times5)+(2.6\times5)}{10.4}$	**M1**	M1 for at least 3 multiplications and additions
	$\bar{X} = \frac{173}{52}$	**A1**	oe. AWRT 3.3
	$\bar{Y} = \frac{(2.1\times1.5)+(1.2\times5)+(2.7\times3)+(1.8\times1.5)+(2.6\times5)}{10.4}$	**M1**	M1 for at least 3 multiplications and additions
	$\bar{Y} = \frac{659}{208}$	**A1**	oe. AWRT 3.2
4(b)	$\bar{X} = \frac{1.5m\times1.9+m\times5.7}{2.5m}$	**M1**	PI, allow one slip
	$\bar{X} = \frac{171}{50}$	**A1**	oe (may be seen on diagram)
	$\bar{Y} = \frac{13}{4}$	**B1**	oe (may be seen on diagram)
	$\tan^{-1}(3.25/3.8) = 40.5[39\ldots]^\circ$	**B1**	or $\tan^{-1}(3.8/3.25) = 49.4[608\ldots]^\circ$
	$\tan^{-1}(3.25/3.42) = 43.5[40\ldots]^\circ$	**B1**	or $\tan^{-1}(3.42/3.25) = 46.4[599\ldots]^\circ$
	$43.5[40\ldots] - 40.5[39\ldots]$	**m1**	PI by correct final answer oe, such as $49.4[608\ldots] - 46.4[599\ldots]$
	$= 3^\circ$	**A1**	CAO
	Total	**11**	

ALT			
4(b)	$\bar{X} = \frac{1.5m\times1.9+m\times5.7}{2.5m}$	**M1**	PI, allow one slip
	$\bar{X} = \frac{171}{50}$	**A1**	oe (may be seen on diagram)
	$\bar{Y} = \frac{13}{4}$	**B1**	oe (may be seen on diagram)
	[Length BP =] 5.00025 [m]	**B1**	
	[Length B to COM =] 4.71793 [m]	**B1**	
	$\cos\theta = \frac{5.00025^2 + 4.71793^2 - 0.38^2}{2\times5.00025\times4.71783}$	**m1**	PI by correct final answer
	$[\theta] = 3^\circ$	**A1**	CAO

Q	Answer	Mark	Comments
5(a)		**B1**	Must have names on the three arrows, not symbols unless the symbols are defined. Do not accept 'gravity' in place of weight.
5(b)(i)	[Component of weight down the slope $= 35 \times 9.8 \sin(25^o)$] $= 144.9$[58 N]	**B1**	oe, AWRT 145 N
	[Normal reaction force on child $= 35 \times 9.8 \cos(25^o)$] $= 310.8$[64 N]	**M1**	oe, AWRT 311 N PI
	[Friction on child $= 0.2 \times 310.863\ldots$] $= 62.1$[73 N]	**A1**	oe
	Resultant force down the slope $= 144.958 - 62.173$ $= 82.7$[85 N]	**M1**	FT their force up and down the slope
	Resultant force $= ma$ $a = 82.785 / 35$ $= 2.37$ [m s^{-2}]	**A1**	CAO, ISW Allow 2.4 m s^{-2} but not 2.36 m s^{-2}
5(b)(ii)	[Distance covered down the slope $= 2.2 / \sin(25^o)$] $= 5.2$[06 m]	**M1**	AWRT 5.2 metres PI
	[$W = Fd$ $= 62.173 \times 5.206$] $= 324$ [J]	**A1F**	FT their friction force from **(b)(i)** with the correct distance Condone negative answers No ISW
5(c)	[Acceleration would be] less	**E1**	Not 'different' Not just 'air resistance'
	[Greater] air resistance would reduce the resultant force [acting on the child due to the child now having a larger surface area]	**E1**	Allow any plausible explanation related to the resultant force reducing or more work done against resistive forces.
	Total	**10**	

Q	Answer	Mark	Comments
6(a)	$F = P / v$	**M1**	Use of $P = Fv$ and attempt at rearranging or substituting.
	[Driving force =] 160 000 / v	**A1**	
	[Resultant force =] $\frac{160\,000}{v} - 4v^{3/2}$	**A1**	oe, ISW
6(b)	Resultant force = $\frac{160\,000}{20} - 4 \times 20^{3/2}$ = 7642 [N]	**M1**	FT their expression for the resultant force.
	Acceleration = 7642 / 1300 = 5.88 [m s^{-2}]	**A1**	CAO, AWRT 5.9 [m s^{-2}]
6(c)	At max speed, resultant force = 0	**M1**	Sets their expression for the resultant force from **(a)** equal to zero. Not from incorrect working.
	$\frac{160000}{v} - 4v^{3/2} = 0$		
	$v^{5/2} = 40000$	**A1**	
	v = 69.3 [m s^{-1}]	**A1**	
	Total	**8**	

Q	Answer	Mark	Comments
7(a)(i)	Impulse = change in momentum $= 0.35 \times 6.4 [- 0.35 \times 0]$ 2.24 [kg m s^{-1}]	**M1** **A1**	 Condone –2.24 [kg m s^{-1}]
7(a)(ii)	2.24 [kg m s^{-1}] Cart A and B are in contact for the same amount of time <u>and</u> experience the same magnitude of force	**B1F** **E1**	FT their impulse to **(a)(i)** Condone –2.24 [kg m s^{-1}] or ± correct calculation Explanation based on Newton's third law
7(b)	[F = Change in momentum / time taken] = 2.24 / 0.25 8.96 [N]	**M1** **A1**	Use of Newton's second law FT their impulse from **(a)** CAO
7(c)	Momentum of cart A after collision = $0.55 \times 8.2 - 2.24$ = 2.27 [kg m s^{-1}] Speed = 2.27 / 0.55 = 4.1273 [m s^{-1}] Kinetic energy before the collision $= 0.5 \times 0.55 \times 8.2^2$ = 18.491 [J] Kinetic energy after the collision $= 0.5 \times 0.55 \times 4.1273^2$ $+ 0.5 \times 0.35 \times 6.4^2$ = 11.853 [J] 18.491 – 11.853 = 6.64 [J]	**M1** **A1** **B1** **B1F** **A1**	PI oe, AWRT 4.1 m s^{-1} May be seen as part of a calculation AWRT 18.5 May be seen as part of a calculation FT their 4.1273 [m s^{-1}] May be seen in two separate calculations as 4.68[45… J] and 7.168 [J] AWRT 11.9 CAO AWRT 6.6 [J], condone 6.65 [J]
	Total	**11**	

Q	Answer	Mark	Comments
8	[vertical comp. of velocity =] $u\sin\theta$	**B1**	May be seen as part of a correct formula for 'vertical distance' involving t
	[$v = u + at$ At max. height $v = 0$]	**M1**	Allow one slip
	$t = \frac{u\sin\theta}{g}$		
	[Time of flight =] $2 \times \frac{u\sin\theta}{g}$	**A1**	oe, for instance $\frac{u\sin\theta}{4.9}$
	[horizontal comp. of velocity =] $u\cos\theta$	**B1**	May be seen as part of a correct formula for 'horizontal distance' involving t
	[Range $= 2 \times \frac{u\sin\theta}{g} \times u\cos\theta$ =] $\frac{2u^2\sin\theta\cos\theta}{g}$	**A1**	ISW, but must get to a correct formula involving u^2 Allow 9.8 in place of g No errors seen
	Total	**5**	

Q	Answer	Mark	Comments
9(a)	Vertical component of tension, $T\cos\theta = 0.35g$	**M1**	oe, for instance $T\cos\theta = 3.43$
	Horizontal component of tension, $T\sin\theta = F$ and eliminates T		
	$F = 0.35g\tan\theta$	**A1**	oe, for instance $F = 3.43\tan\theta$ Condone answer left in terms of m, i.e. $F = mg\tan\theta$
9(b)	[The centripetal (or resultant)] force [on the particle acts] perpendicular [to the] velocity [of the particle]	**E1**	Condone 'speed'
	[The centripetal (or resultant) force does] no work [on the particle, so does not change the kinetic energy]	**E1**	
9(c)	Speed of particle $[v =]$ 2 × π × 0.62 / 0.48	**M1**	Angular speed of particle $[\omega =]$ 2 × π / 0.48
	$[v =]$ 8.11[578 m s^{-1}] oe	**A1**	$[\omega =]$ 13.0[899… rad s^{-1}] oe
	$0.35g\tan\theta = 0.35v^2/r$		oe
	or		accept 3.43 or mg for $0.35g$
	$0.35g\tan\theta = 0.35\omega^2 r$	**B1**	
	or		
	[Resultant force =] 37.18… [N]		
	$\tan\theta = v^2/gr$ or $\tan\theta = \omega^2 r/g$	**M1**	Sight or attempt of isolating for $\tan\theta$
	$\tan\theta = 8.11578^2/(9.8 \times 0.62)$ oe		
	or	**m1**	Only FT their speed, no other slips PI by correct answer
	$\tan\theta = 13.0899\ldots^2 \times 0.62/9.8$ oe		
	84.7[°]	**A1**	CAO, AWRT 85[°] oe, for instance 1.48 [rad]
	Total	**10**	

Q	Answer	Mark	Comments
10	Taking moments about *A* or *B* [Moments about *B*] $LMg\sin\alpha = 2LR_1\cos\alpha + 2L\,\mu R_1\sin\alpha$ $\tan\alpha = \dfrac{2R_1}{Mg - 2\mu R_1}$ Forces in equilibrium Horizontal: $\mu R_2 = R_1$ Forces in equilibrium Vertical: $Mg = R_2 + \mu R_1$ $\mu Mg = R_1 + \mu^2 R_1$ $R_1 = \dfrac{\mu Mg}{1+\mu^2}$ $\tan\alpha = \dfrac{\frac{2\mu Mg}{1+\mu^2}}{Mg - 2\mu \times \frac{\mu Mg}{1+\mu^2}}$ $\tan\alpha = \dfrac{2\mu}{1-\mu^2}$	**M1** **A1** **A1** **B1** **B1** **m1** **m1** **A1**	Any attempt L = length of rod R_1 = normal reaction on rod from wall Condone cancelled L R_2 = normal reaction on rod from ground Accept 'f_1' and 'f_2' in place of μR_1 and μR_2 respectively. Attempt at eliminating R_2 Substituting into correct relation for tanα CAO
	Total	**8**	