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(9660/MA03) Unit P2 Pure Mathematics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$\left[\mathrm{f}(x+1)-\mathrm{f}(x-2)=\right]\ 3^{x+1}-3^{x-2}$	**B1**	
	$=3^{x}(3-3^{-2})$	**M1**	**oe**, e.g. $3^{x-2}(3^{3}-1)$ Factorises
	$=\dfrac{26}{9}\mathrm{f}(x)$	**A1**	Correct simplified value of k
		3	

Q	Answer	Marks	Comments
1(b)(i)	$x=\dfrac{3-y}{5+2y}$	**M1**	Interchanges x and y
	$5x+2xy=3-y$		
	$2xy+y=3-5x$	**M1**	Attempt to rearrange
	$\left[y=\mathrm{g}^{-1}(x)=\right]\ \dfrac{3-5x}{1+2x}$	**A1**	**ACF**, e.g. $3-\dfrac{11x}{1+2x}$
		3	

Q	Answer	Marks	Comments
1(b)(ii)	$\mathrm{g}^{-1}(x)\in\square$, $\mathrm{g}^{-1}(x)\neq-2.5$	**B1**	**oe** Condone omission of $\mathrm{g}^{-1}(x)\in\square$ Allow $y\neq-2.5$ and no other values
		1	

	Question 1 Total	**7**	

Q	Answer	Marks	Comments
2(a)	$[8\cos\theta + 15\sin\theta =]$ $R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$	**M1**	
	$R = 17$	**B1**	
	$\alpha = 62°$	**A1**	**AWRT** 62°
	$[8\cos\theta + 15\sin\theta =]\quad 17\cos(\theta - 62°)$		
		3	

Q	Answer	Marks	Comments
2(b)(i)	0	**B1**	
		1	

Q	Answer	Marks	Comments
2(b)(ii)	152°	**B1**	**AWRT** 152° Any correct value eg 332°, 512°
		1	

Q	Answer	Marks	Comments
2(c)	$\left[\begin{array}{l}\text{Let } X = 2y + 10° \\ 8\operatorname{cosec} X + 15\sec X = 8.5\tan X + 8.5\cot X\end{array}\right]$		
	$\dfrac{8}{\sin X} + \dfrac{15}{\cos X} = 8.5\left(\dfrac{\sin X}{\cos X} + \dfrac{\cos X}{\sin X}\right)$	**B1**	**PI**
	$8\cos X + 15\sin X = 8.5\left(\sin^2 X + \cos^2 X\right)$	**M1**	Eliminate fractions
	$17\cos\left(X - 62\right) = 8.5$	**A1ft**	**ft** their **part (a)**
	$17\cos\left(2y + 10 - 62\right) = 8.5$		
	$\left[\cos\left(X - 62\right) = 0.5\right]$		
	$X - 62 = \pm 60$		
	$2y + 10 = -238°, 2°, 122°, 362°$		
	$y = -124°, -4°, 56°, 176°$	**B1** **B1**	At least one correct answer All four correct and no others
		5	

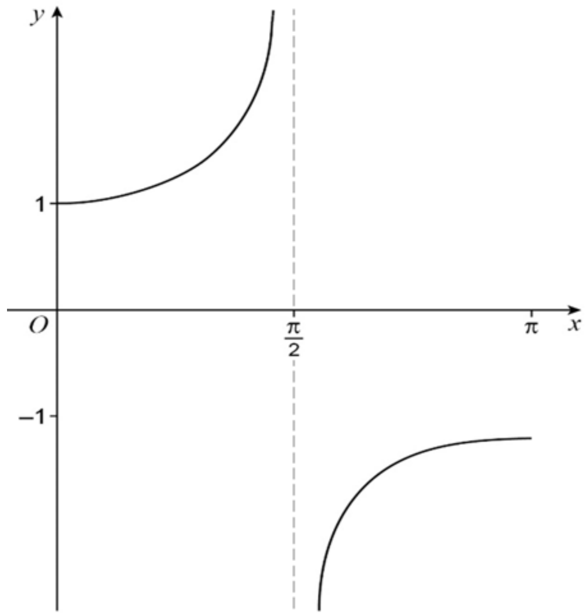
	Question 2 Total	**10**	

Q	Answer	Marks	Comments
3(a)(i)	$16(-1.5)^3+b(-1.5)^2+c(-1.5)=-45$ $16(1.25)^3+b(1.25)^2+c(1.25)=10$	**M1**	One correct substitution or **M1** for clear use of long division
	$\frac{9}{4}b-\frac{3}{2}c=9$ $\frac{25}{16}b+\frac{5}{4}c=-21.25$	**A1**	Correct equations **oe**, e.g. $3b-2c=12$ $5b+4c=-68$
	$11b=-44$	**m1**	Attempt to solve for b or c **PI** by correct final answers
	$b=-4$ $c=-12$	**A1**	Both answers
		4	

Q	Answer	Marks	Comments
3(a)(ii)	$\left[f(x)=\right]\ 4x(4x+3)(x-1)$	**M1 A1**	**M1**: $\left[f(x)=\right]\ kx(px+q)(rx+s)$ **A1**: Any correct form, **ISW**
		2	

Q	Answer	Marks	Comments
3(b)	$\frac{f(x)}{16x^2-9}=\frac{4x(4x+3)(x-1)}{(4x+3)(4x-3)}\ =\frac{4x(x-1)}{4x-3}$	**M1**	or **M1** for correct use of long division
	$\frac{4x^2-4x}{4x-3}=x-\frac{x}{4x-3}$	**M1**	**PI** by correct final answer
	$\left[\frac{x}{4x-3}=\frac{(4x-3)+3}{16x-12}=\frac{1}{4}+\frac{3}{16x-12}\right]$		
	$=x-\frac{1}{4}-\frac{3}{16x-12}$	**A1**	Condone $x-\frac{1}{4}-\frac{3}{4(4x-3)}$
		3	

	Question 3 Total	**9**	

Q	Answer	Marks	Comments
4(a)		M1 A1	Two sections with approx. correct curvature End points correct (approx.) and asymptote correct (approx.)
		2	

Q	Answer	Marks	Comments
4(b)(i)	$f(x) = \sec x - 10x + 5$ $f(0.6) = 0.21...$ $f(0.7) = -0.69...$ Change of sign, $0.6 < \alpha < 0.7$	M1 A1	or reverse Both values rounded or truncated to at least 1sf Must have both statement and interval in words or symbols **or** comparing 2 sides: at 0.6, $\sec 0.6 > 6-5$; at 0.7, $\sec 0.7 < 7-5$ **(M1)** Conclusion as before **(A1)**
		2	

Q	Answer	Marks	Comments
4(b)(ii)	$[x_2 =]$ 0.621 $[x_3 =]$ 0.623	B1 B1	
		2	

Q	Answer	Marks	Comments
4(c)	<table><tr><td>x</td><td>y</td></tr><tr><td>0.61</td><td>1.22003589</td></tr><tr><td>0.63</td><td>1.2375816</td></tr><tr><td>0.65</td><td>1.2561492</td></tr><tr><td>0.67</td><td>1.2758004</td></tr><tr><td>0.69</td><td>1.2966031</td></tr></table>	**B1** **M1**	All five correct x values (and no extras used) **PI** by four correct y values to 3 dp At least four correct y values in exact form or as decimals which are rounded or truncated correct to 2 dp or better May be seen in a table or a formula **PI** by **AWRT** 1.2572
	$0.02\times(1.22003589+1.2375816+1.2561492$ $+1.2758004+1.2966031)$	**m1**	Correct sub into formula with $h = 0.02$ **oe** and at least four correct y values either listed, with + signs, or totalled
	$=0.125723$	**A1**	**CAO** Must see this value exactly and no errors made
		4	

	Question 4 Total	**10**	

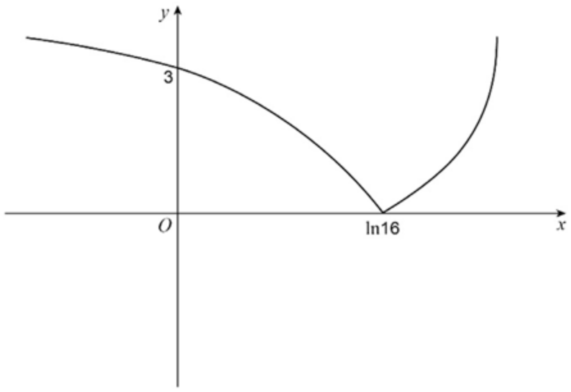
Q	Answer	Marks	Comments
5(a)	$\left[(1-px)^{-\frac{1}{2}}=\right]$		
	$1+\left(-\frac{1}{2}\right)(-px)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(-px)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{6}(-px)^3$	M1	At least 2 terms in x correct
	$=1+\frac{1}{2}px+\frac{3}{8}p^2x^2+\frac{5}{16}p^3x^3$	A1	**AG** Must be convincingly shown
		2	

Q	Answer	Marks	Comments
5(b)	$(4+px)^{\frac{1}{2}}=2\left(1+\frac{px}{4}\right)^{\frac{1}{2}}$	B1	
	$=2\left(1+\left(\frac{1}{2}\right)\left(\frac{px}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}\left(\frac{px}{4}\right)^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6}\left(\frac{px}{4}\right)^3\right)$	M1	At least 2 terms in x correct
	$=2+\frac{1}{4}px-\frac{1}{64}p^2x^2+\frac{1}{512}p^3x^3$	A1	
		3	

Q	Answer	Marks	Comments
5(c)(i)	$[\text{LHS} =]\ \frac{3}{4}px + \left(2 + \frac{1}{4}px - \frac{1}{64}p^2x^2 + \frac{1}{512}p^3x^3\right)$ $-2\left(1 + \frac{1}{2}px + \frac{3}{8}p^2x^2 + \frac{5}{16}p^3x^3\right)$	M1	Use of their **part (b)**
	$= (2-2) + \left(\frac{3}{4} + \frac{1}{4} - 1\right)px - \left(\frac{1}{64} + \frac{3}{4}\right)p^2x^2$ $+\left(\frac{1}{512} - \frac{5}{8}\right)p^3x^3$	A1ft	Correctly collecting their x^2 terms
	$-\frac{49}{64}p^2x^2\ \left[-\frac{319}{512}p^3x^3\right] = -x^2\ \left[+qx^3\right]$	m1	Equating their x^2 terms and attempting to solve
	$\left[-\frac{49}{64}p^2 = -1 \quad \Rightarrow \quad\right]\ p^2 = \frac{64}{49}$		
3	$p = \pm\frac{8}{7}$	A1	**AG** Must be convincingly shown
		4	

Q	Answer	Marks	Comments
5(c)(ii)	$-\frac{319}{512}\left(\pm\frac{8}{7}\right)^3 x^3 = qx^3$	M1	Equating their x^3 terms and attempting to solve **PI** by at least one correct value for q (which may be a truncated decimal)
	$q = \pm\frac{319}{343}$	A1	**CAO**
		2	

	Question 5 Total	11	

Q	Answer	Marks	Comments
6(a)		B1 B1 B1	Graph in first and second quadrant only Correct curvature Correct intercepts (ln16, 0) and (0, 3) shown or stated Allow (2.8, 0) or better
		3	

Q	Answer	Marks	Comments
6(b)	$\frac{dy}{dx} = 0.5e^{0.5x}$ When $x = \ln 25$ $\frac{dy}{dx} = 0.5e^{0.5\times\ln 25}$ $\frac{dy}{dx} = 0.5e^{\ln 5}$ $\frac{dy}{dx} = 2.5$ When $x = \ln 25$ $y = \left\|e^{0.5\ln 25} - 4\right\| = 1$ $\frac{dy}{dx}_{\text{normal}} = -\frac{2}{5}$ $y - 1 = -\frac{2}{5}(x - \ln 25)$ $2x + 5y = 5 + 2\ln 25$	M1 A1 B1 M1 A1	$ke^{0.5x}$ **oe** **M1** for the negative reciprocal of their 2.5 **oe**
		5	

Q	Answer	Marks	Comments
6(c)	$x=0,\ y=\dfrac{(5+\ln 625)}{5}$ $y=0,\ x=\dfrac{(5+\ln 625)}{2}$	**M1**	Either correct
	$A=\dfrac{1}{2}\times\dfrac{(5+\ln 625)}{5}\times\dfrac{(5+\ln 625)}{2}$	**M1**	
	$A=\dfrac{1}{20}(5+\ln 625)^2$	**A1**	**oe**, e.g. $A=\dfrac{1}{5}\left(\dfrac{5}{2}+\ln 25\right)^2$ or $A=\dfrac{4}{5}\left(\dfrac{5}{4}+\ln 5\right)^2$
		3	

	Question 6 Total	**11**	

Q	Answer	Marks	Comments
7(a)(i)	$\overrightarrow{AB} = \begin{bmatrix} -3 \\ -2 \\ 7 \end{bmatrix}$	**B1**	
		1	

Q	Answer	Marks	Comments
7(a)(ii)	$\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{(-3)^2 + (-2)^2 + 7^2}$	**M1**	
	$= \sqrt{62}$	**A1**	
		2	

Q	Answer	Marks	Comments
7(a)(iii)	$\begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ -2 \\ 7 \end{bmatrix} \quad [= -20]$	**M1**	**PI** by –20 seen or used
	$\cos\theta = \dfrac{\begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ -2 \\ 7 \end{bmatrix}}{\sqrt{62} \times \sqrt{1^2 + 2^2 + 3^2}}$	**M1**	
	$\theta = 132.75...^\circ$		
	$[\Rightarrow \text{ acute angle} =] \;\; 47.2^\circ$	**A1**	
		3	

Q	Answer	Marks	Comments
7(a)(iv)	The line *AB* has vector equation $\mathbf{r} = \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix} + \mu \begin{bmatrix} -3 \\ -2 \\ 7 \end{bmatrix}$		
	$4 + \lambda = 1 - 3\mu$ $-1 - 2\lambda = 5 - 2\mu$	**M1**	
	$\mu = 0, \;\; \lambda = -3$	**A1**	
	$-3 = c + (-3)(-3)$ $c = -12$	**A1**	
		3	

Q	Answer	Marks	Comments
7(b)(i)	$\begin{bmatrix} 4+p \\ -1-2p \\ -12-3p \end{bmatrix} . \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} = 0$	**M1**	
	$4+p+2+4p+36+9p=0$		
	$p=-3$	**A1**	
	$\lvert OP \rvert = \sqrt{1^2+5^2+(-3)^2}$	**m1**	
	$=\sqrt{35}$	**A1**	
		4	

Q	Answer	Marks	Comments
7(b)(ii)	$\lvert OB \rvert = \sqrt{29}$	**B1**	**2** **oe** Note, shortest distance from line *AB* to origin is $\frac{13\sqrt{93}}{31}=4.044...$
	The shortest distance from l to the origin is $\sqrt{35}$, so the line *AB* must be nearer	**E1ft**	**ft** their $\sqrt{35}$ and their equivalent of $\sqrt{29}$ with a consistent conclusion
		2	

	Question 7 Total	**15**	

Q	Answer	Marks	Comments
8	$1+\frac{\mathrm{d}y}{\mathrm{d}x}=2(x-2y)\left(1-2\frac{\mathrm{d}y}{\mathrm{d}x}\right)$	M1 A1	**M1**: LHS or RHS correct **A1**: Both correct $\left[\frac{\mathrm{d}y}{\mathrm{d}x}(1+4(x-2y))=2(x-2y)-1\right.$ $\left.\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2(x-2y)-1}{(1+4(x-2y))}\right]$ **oe**
	At $(2, 2)$ $1+\frac{\mathrm{d}y}{\mathrm{d}x}=-4+8\frac{\mathrm{d}y}{\mathrm{d}x}$	M1	Attempt to find $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5}{7}$	A1	
	$y=\frac{5}{7}x+c$		
	$2=\frac{5}{7}\times 2+c$	m1	Attempt to find c
	$c=\frac{4}{7}$		
	$y=\frac{5}{7}x+\frac{4}{7}$	A1	
		6	

	Question 8 Total	**6**	

Q	Answer	Marks	Comments
9(a)	Stretch + either I or II Parallel to x-axis I SF 0.5 II	**M1** **A1**	or Translation $\begin{bmatrix}0\\k\end{bmatrix}$ **M1** $k = \ln 2$ **A1**
		2	

Q	Answer	Marks	Comments
9(b)	$V = \pi\int_{0.5}^{4}\left(\ln\left(2x\right)\right)^2 \mathrm{d}x$	**B1**	Complete correct statement
	$u = \left(\ln\left(2x\right)\right)^2, \quad \dfrac{\mathrm{d}v}{\mathrm{d}x} = 1$	**M1**	Attempt at parts
	$\dfrac{\mathrm{d}u}{\mathrm{d}x} = 2\ln\left(2x\right)\times\dfrac{1}{x}, \quad v = x$	**A1**	All 4 terms correct
	$\int \ln\left(2x\right)^2 \mathrm{d}x = x\left(\ln\left(2x\right)\right)^2 - \int x \times \dfrac{2\ln\left(2x\right)}{x}\mathrm{d}x$	**m1**	Correct substitution into parts formula
	$\int \ln\left(2x\right)\, \mathrm{d}x$		
	$u = \ln\left(2x\right), \quad \dfrac{\mathrm{d}v}{\mathrm{d}x} = 1$	**M1**	Attempt at parts
	$\dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{1}{x}, \quad v = x$		
	$\int \ln\left(2x\right)\, \mathrm{d}x = x\ln\left(2x\right) - \int x \times \dfrac{1}{x}\, \mathrm{d}x$	**m1**	Correct substitution into parts formula
	$= x\ln 2x - x$	**A1**	
	$\left[\int \ln\left(2x\right)^2 \mathrm{d}x = x\left(\ln\left(2x\right)\right)^2 - 2x\ln\left(2x\right) + 2x\right]$		
	$V = \pi\int_{0.5}^{4} \ln\left(2x\right)^2 \mathrm{d}x$		
	$= \pi\left(4\left(\ln 8\right)^2 - 8\ln 8 + 8 - 1\right)$	**M1**	Subst limits into their expression (must be in form $ax\left(\ln(2x)^2\right) + bx\ln(2x) + cx$)
	$= \pi\left(4\left(\ln 8\right)^2 - 8\ln 8 + 7\right)$	**A1**	**ACF** eg $\pi\left(36\left(\ln 2\right)^2 - 24\ln 2 + 7\right)$
		9	

	Question 9 Total	**11**	

Q	Answers	Marks	Comments
10	$\displaystyle\int \frac{\mathrm{d}y}{(3a-2y)(a-y)} = \int b\,\mathrm{d}x$	**M1**	Separate variables
	$\displaystyle\frac{1}{(3a-2y)(a-y)} = \frac{A}{3a-2y} + \frac{B}{a-y}$	**M1**	Use of partial fractions
	$1 = A(a-y) + B(3a-2y)$		
	$A = -\dfrac{2}{a}, \quad B = \dfrac{1}{a}$	**A1**	
	$-\dfrac{2}{a} \times \left(\dfrac{1}{-2}\right) \ln(3a-2y) + \dfrac{1}{a} \times (-1) \ln(a-y) = bx + c$	**M1 A1**	**M1**: Attempt to integrate **A1**: Fully correct integration
	$x = 0, \ \ y = 0$		
	$\dfrac{1}{a}\ln 3a - \dfrac{1}{a}\ln a = c$	**m1**	Attempt to find c
	$c = \dfrac{1}{a}\ln 3$	**A1**	
	$\left[\begin{array}{l} \ln\left(\dfrac{3a-2y}{a-y}\right) = abx + \ln 3 \\ \ln\left(\dfrac{2y-3a}{3(y-a)}\right) = abx \end{array}\right]$		
	$\dfrac{2y-3a}{3(y-a)} = \mathrm{e}^{abx}$	**M1**	Eliminates logarithms
	$2y - 3a = 3y\,\mathrm{e}^{abx} - 3a\,\mathrm{e}^{abx}$		
	$y\left(2 - 3\mathrm{e}^{abx}\right) = 3a\left(1 - \mathrm{e}^{abx}\right)$	**M1**	Attempt to find y
	$y = \dfrac{3a\left(1-\mathrm{e}^{abx}\right)}{2-3\mathrm{e}^{abx}}$	**A1**	**oe**
		10	

	Question 10 Total	**10**	

Q	Answer	Marks	Comments
11(a)	$\left[\cos 2\theta = 2\cos^2\theta - 1\right]$ $\int 4\cos^2\theta \,\mathrm{d}\theta = \int (2\cos 2\theta + 2)\,\mathrm{d}\theta$ $= \sin 2\theta + 2\theta \;[+c]$	**M1A1**	**M1** for $a\sin 2\theta + b\theta$ **A1** correct with no errors seen
		2	

Q	Answer	Marks	Comments
11(b)	$t = \sin x, \quad \mathrm{d}t = \cos x \;\mathrm{d}x$	**B1**	**oe**
	$[t]_0^{\frac{1}{2}} = [\sin x]_0^{\frac{\pi}{6}}$	**B1**	Change of limits
	$\int \frac{\sin 2x}{3 + \cos^2 x}\,\mathrm{d}x = \int \frac{2t}{4 - t^2}\,\mathrm{d}t$	**M1**	
	$\int \frac{2t}{4 - t^2}\,\mathrm{d}t = -\ln\left(4 - t^2\right)$	**m1 A1**	**m1:** $k\ln\left(4 - t^2\right)$ **A1:** Correct, or $-\ln(2-t) - \ln(2+t)$ **oe**
	$= -\ln\left(4 - \frac{1}{4}\right) - \left(-\ln(4)\right)$	**M1**	Substituting into $k\ln\left(4 - t^2\right)$ **oe**
	$= \ln\left(\frac{16}{15}\right)$	**A1**	
		7	

	Question 11 Total	**9**	

Q	Answer	Marks	Comments
12(a)	$\cos\theta = \dfrac{x}{2}, \quad \sin\theta = \dfrac{y}{3}$	**M1**	
	$\left(\dfrac{x}{2}\right)^2 + \left(\dfrac{y}{3}\right)^2 = 1$	**A1**	**oe**
		2	

Q	Answer	Marks	Comments
12(b)	$\theta = \dfrac{\pi}{6}, \quad x = \sqrt{3}, \quad y = \dfrac{3}{2}$	**B1**	$\left[\dfrac{2x}{4} + \dfrac{2y}{9}\dfrac{dy}{dx} = 0\right]$
	$\dfrac{dx}{d\theta} = -2\sin\theta \quad \dfrac{dy}{d\theta} = 3\cos\theta$		
	$\dfrac{dy}{dx} = -\dfrac{3\cos\theta}{2\sin\theta}\ \left[= -1.5\cot\theta\right]$	**M1**	$\dfrac{dy}{dx} = -\dfrac{9x}{4y}$ **PI**
	$\theta = \dfrac{\pi}{6}, \quad \dfrac{dy}{dx} = -\dfrac{3\sqrt{3}}{2}$	**A1**	
	$y - 1.5 = -\dfrac{3\sqrt{3}}{2}\left(x - \sqrt{3}\right)$		
	$y + \dfrac{3\sqrt{3}}{2}x - 6 = 0$	**A1**	
		4	

Q	Answer16	Marks	Comments
12(c)	$xy = k^2 \quad \Rightarrow \quad y = \dfrac{k^2}{x}$	M1	$xy = k^2 \quad \Rightarrow \quad x = \dfrac{k^2}{y}$
	$\dfrac{x^2}{4} + \left(\dfrac{k^2}{3x}\right)^2 = 1$		$\dfrac{k^4}{4y^2} + \dfrac{y^2}{9} = 1$
	$9x^4 + 4k^4 = 36x^2$		$9k^4 + 4y^4 = 36y^2$
	$9x^4 - 36x^2 + 4k^4 = 0$	A1	$4y^4 - 36y^2 + 9k^4 = 0$
	$(-36)^2 - 4 \times 9 \times 4k^4 > 0$	B1	$(-36)^2 - 4 \times 9 \times 4k^4 > 0$
	$x^2 = 2 \pm \dfrac{2}{3}\sqrt{9 - k^4}$ Given that k is positive, for x^2 to have two distinct positive real values then $x^2 = 2 + \dfrac{2}{3}\sqrt{9 - k^4} > 0 \Rightarrow k^2 < 3$ or $x^2 = 2 - \dfrac{2}{3}\sqrt{9 - k^4} > 0 \Rightarrow k^2 < 3$ $\therefore\ k^2 < 3$	M1	$y^2 = \dfrac{9}{2} \pm \dfrac{3}{2}\sqrt{9 - k^4}$ Given that k is positive, for y^2 to have two distinct positive real values then $y^2 = \dfrac{9}{2} + \dfrac{3}{2}\sqrt{9 - k^4} > 0 \Rightarrow k^2 < 3$ or $y^2 = \dfrac{9}{2} - \dfrac{3}{2}\sqrt{9 - k^4} > 0 \Rightarrow k^2 < 3$ $\therefore\ k^2 < 3$
	then there will be 4 distinct points of intersection.	A1	then there will be 4 distinct points of intersection. **oe**
		5	

	Question 12 Total	**11**	