

INTERNATIONAL A-LEVEL MATHEMATICS

MA03

(9660/MA03) Unit P2 Pure Mathematics

Mark scheme

January 2024

Version: 1.0 Final



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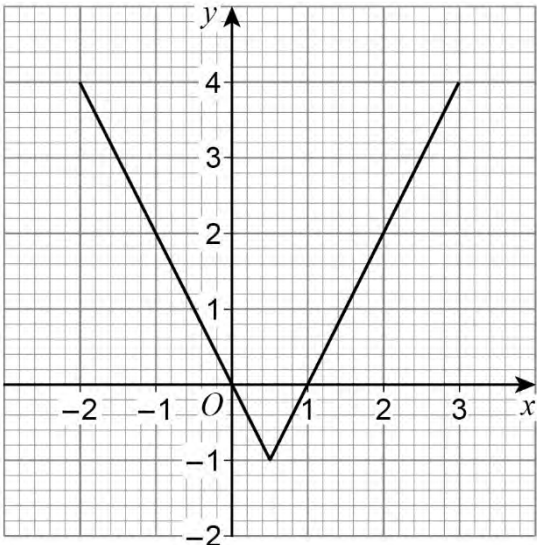
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Key to mark scheme abbreviations

| | |
|----------------|--|
| M | Mark is for method |
| m | Mark is dependent on one or more M marks and is for method |
| A | Mark is dependent on M or m marks and is for accuracy |
| B | Mark is independent of M or m marks and is for method and accuracy |
| E | Mark is for explanation |
| ✓ or ft | Follow through from previous incorrect result |
| CAO | Correct answer only |
| CSO | Correct solution only |
| AWFW | Anything which falls within |
| AWRT | Anything which rounds to |
| ACF | Any correct form |
| AG | Answer given |
| SC | Special case |
| oe | Or equivalent |
| A2, 1 | 2 or 1 (or 0) accuracy marks |
| –x EE | Deduct x marks for each error |
| NMS | No method shown |
| PI | Possibly implied |
| SCA | Substantially correct approach |
| sf | Significant figure(s) |
| dp | Decimal place(s) |
| ISW | Ignore subsequent working |

| Q | Answer | Marks | Comments |
|---|--|--|---|
| 1 | $16(-0.25)^3 + b(-0.25)^2 + c(-0.25) + 12 = 11.5$ $16(0.5)^3 + b(0.5)^2 + c(0.5) + 12 = 17.5$ $b - 4c = -4$ $b + 2c = 14$ $b = 8$ $c = 3$ | M1 A1 m1 A1 | M1: At least one equation correct or use of long division A1: Both equations correct m1: Attempt to solve PI A1: Both values correct ACF |
| | Question 1 Total | 4 | |

| Q | Answer | Marks | Comments |
|------|---|------------------------|---|
| 2(a) |  | B1 B1 | Correct for $-2 \leq x \leq 0.5$ Correct for $0.5 \leq x \leq 3$ |
| | | 2 | |

| Q | Answer | Marks | Comments |
|------|---------------------------|------------------------|---|
| 2(b) | $x \leq 0 \quad x \geq 2$ | M1 A1 | Either correct, condone strict inequalities Both correct and no extras |
| | | 2 | |

| | | | |
|--|-------------------------|----------|--|
| | Question 2 Total | 4 | |
|--|-------------------------|----------|--|

| Q | Answer | Marks | Comments |
|---------|---|-----------|---------------------|
| 3(a)(i) | $\left[\frac{dy}{dx} = \right] ae^{-0.5x} \sin 3x + be^{-0.5x} \cos 3x$ | M1 | Use of product rule |
| | $\left[\frac{dy}{dx} = \right] -0.5e^{-0.5x} \sin 3x + 3e^{-0.5x} \cos 3x$ | A1 | ISW |
| | | 2 | |

| Q | Answer | Marks | Comments |
|----------|---|--------------|--|
| 3(a)(ii) | $\left[\frac{dy}{dx} = \right] \frac{a(3 + \tan 5x) \times (1 - 2x)^2 - (1 - 2x)^3 b \sec^2 5x}{(3 + \tan 5x)^2}$ | M1 A1 | M1: Correct use of quotient/product rule A1: one numerator term and denom correct |
| | $\left[\frac{dy}{dx} = \right] \frac{-6(3 + \tan 5x) \times (1 - 2x)^2 - (1 - 2x)^3 5 \sec^2 5x}{(3 + \tan 5x)^2}$ | A1 | ACF All correct (unsimplified) ISW |
| | | 3 | |

| Q | Answer | Marks | Comments |
|-----------|--|--------------|--|
| 3(a)(iii) | $1 + \frac{1}{xy} \left(x \frac{dy}{dx} + y \right) = 3x^2 + 2y \frac{dy}{dx}$ | M1 A1 | M1: One correct use of implicit differentiation A1: All correct |
| | $\frac{dy}{dx} \left(\frac{1}{y} - 2y \right) = 3x^2 - 1 - \frac{1}{x}$ | m1 | Attempt to isolate $\frac{dy}{dx}$ |
| | $\frac{dy}{dx} = \frac{3x^2 - 1 - \frac{1}{x}}{\frac{1}{y} - 2y}$ or $\frac{dy}{dx} = \frac{y(3x^3 - x - 1)}{x(1 - 2y^2)}$ | A1 | ACF, ISW |
| | | 4 | |

| Q | Answer | Marks | Comments |
|---------|--|-------|----------|
| 3(b)(i) | $\left[\int \frac{x}{4x^2+5} dx = \right] a \ln(4x^2+5)$ | M1 | |
| | $\left[\int \frac{x}{4x^2+5} dx = \right] \frac{1}{8} \ln(4x^2+5) \quad [+c]$ | A1 | ISW |
| | | 2 | |

| Q | Answer | Marks | Comments |
|----------|--|----------|--|
| 3(b)(ii) | $u = x \quad dv = \cos x$ $du = 1 \quad v = \sin x$ | M1 | PI |
| | $\left[\int x \cos x dx = \right] x \sin x - \int \sin x dx$ $= x \sin x + \cos x$ | m1 A1 | Correct use of integration by parts formula All correct |
| | $\left[x \sin x + \cos x \right]_0^{\frac{\pi}{2}}$ | | |
| | $\left[\int_0^{\frac{\pi}{2}} x \cos x dx \right] = \frac{\pi}{2} - 1$ | A1 | ACF, ISW |
| | | 4 | |

| | | | |
|--|------------------|----|--|
| | Question 3 Total | 15 | |
|--|------------------|----|--|

| Q | Answer | Marks | Comments | | | | | | | | | | | | |
|------|---|-------------------------------|---|------|-------------------------------|------|-------------------------------|------|-------------------------------|------|-------------------------------|------|-------------------------------|-----------|---|
| 4(a) | <table><tr><td>x</td><td>y</td></tr><tr><td>0.06</td><td>$4^{-0.06} - 0.25 = 0.670187$</td></tr><tr><td>0.18</td><td>$4^{-0.18} - 0.25 = 0.529165$</td></tr><tr><td>0.30</td><td>$4^{-0.30} - 0.25 = 0.409754$</td></tr><tr><td>0.42</td><td>$4^{-0.42} - 0.25 = 0.308644$</td></tr><tr><td>0.54</td><td>$4^{-0.54} - 0.25 = 0.223029$</td></tr></table> | x | y | 0.06 | $4^{-0.06} - 0.25 = 0.670187$ | 0.18 | $4^{-0.18} - 0.25 = 0.529165$ | 0.30 | $4^{-0.30} - 0.25 = 0.409754$ | 0.42 | $4^{-0.42} - 0.25 = 0.308644$ | 0.54 | $4^{-0.54} - 0.25 = 0.223029$ | B1 | All 5 correct x values (and no extra used) PI by 5 correct y values |
| | x | y | | | | | | | | | | | | | |
| | 0.06 | $4^{-0.06} - 0.25 = 0.670187$ | | | | | | | | | | | | | |
| | 0.18 | $4^{-0.18} - 0.25 = 0.529165$ | | | | | | | | | | | | | |
| 0.30 | $4^{-0.30} - 0.25 = 0.409754$ | | | | | | | | | | | | | | |
| 0.42 | $4^{-0.42} - 0.25 = 0.308644$ | | | | | | | | | | | | | | |
| 0.54 | $4^{-0.54} - 0.25 = 0.223029$ | | | | | | | | | | | | | | |
| | | M1 | At least 3 correct y values in exact form or decimals, rounded or truncated to 3 dp or better (in table or formula) PI by AWRT correct answer | | | | | | | | | | | | |
| | $0.12(0.67... + 0.53... + 0.41... + 0.31... + 0.22...)$ | m1 | Correct sub into formula with $h = 0.12$ oe and at least 3 correct y values either listed, with + signs, or totalled PI by AWRT correct answer | | | | | | | | | | | | |
| | $= 0.2569$ | A1 | CAO Must see this value exactly and no error seen | | | | | | | | | | | | |
| | | 4 | | | | | | | | | | | | | |

| Q | Answer | Marks | Comments |
|---------|--|-------|--------------------------|
| 4(b)(i) | $x = 4^{-y} - 0.25$ | M1 | Interchange x and y |
| | $-y \ln 4 = \ln(x + 0.25)$ | M1 | Attempt to isolate PI |
| | $f^{-1}(x) = \frac{\ln(x+0.25)}{-\ln 4}$ or $\frac{\ln(x+0.25)}{\ln 0.25}$ | A1 | ACF eg $-\log_4(x+0.25)$ |
| | | 3 | |

| Q | Answer | Marks | Comments |
|----------|--|-------|---|
| 4(b)(ii) | $-0.25 < x \leq 0.75$ or $x \in (-0.25, 0.75]$ | B2 | If B2 not awarded, award B1 for at least one of the two limits correct or $-0.25 \leq x < 0.75$ |
| | | 2 | |

| Q | Answer | Marks | Comments |
|------|----------------------------------|-------|----------|
| 4(c) | Reflection [in the line] $y = x$ | B1 | |
| | | 1 | |

| | | | |
|--|------------------|----|--|
| | Question 4 Total | 10 | |
|--|------------------|----|--|

| Q | Answer | Marks | Comments |
|---------|--|---|----------|
| 5(a)(i) | $[10\sin\theta - 24\cos\theta =]$ $R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$ $R = 26$ $\alpha = 1.18$ $26\sin(\theta - 1.18)$ | M1 B1 A1 | PI |
| | | 3 | |

| Q | Answer | Marks | Comments |
|----------|---------------------|-------------|----------|
| 5(a)(ii) | [Min value =] -26 | B1ft | |
| | | 1 | |

| Q | Answer | Marks | Comments |
|-----------|--------|-----------|---------------|
| 5(a)(iii) | 12.17 | B1 | Condone 12.18 |
| | | 1 | |

| Q | Answer | Marks | Comments |
|----------|--|--------------------------------|--|
| 5(a)(iv) | $[26\sin((x - 1.18) - 0.6) = 6.5]$ $\sin(x - 1.78) = 0.25$ $[x - 1.78 =] \quad 0.253$ $x = -1.62, \quad 2.03$ | M1 A1, A1 | Attempt to solve $\sin((x - \text{their } 1.18) - 0.6) = \frac{6.5}{\text{their } 26}$ Condone -1.61 |
| | | 3 | |

| Q | Answer | Marks | Comments |
|------|---|---|--|
| 5(b) | <p>Let $Y = 2y - 10^\circ$</p> <p>$16\tan^2 Y - 14 = 4\sec Y$</p> <p>$16(\sec^2 Y - 1) = 4\sec Y + 14$</p> <p>$8\sec^2 Y - 2\sec Y - 15 = 0$</p> <p>$\sec Y = -1.25, 1.5$</p> <p>$Y = 143.1^\circ \quad \text{and} \quad 48.2^\circ$</p> <p>$y = -67^\circ, -19^\circ, 29^\circ, 77^\circ$</p> | <p>M1</p> <p>m1</p> <p>A1</p> <p>B2,1</p> | <p>oe</p> <p>Correct use of trig identity</p> <p>Attempt to solve <i>their</i> quadratic</p> <p>PI</p> <p>AWRT the correct values Award B1 for 3 correct answers</p> |
| | | 5 | |
| | Question 5 Total | 13 | |

| Q | Answer | Marks | Comments |
|------|--|-----------|--------------------------------------|
| 6(a) | $f(x) = -x^2 + \ln(12 + 24x) \quad \frac{dy}{dx} = -2x + \frac{24}{12 + 24x}$ | M1 | |
| | $\left[\frac{dy}{dx} = 0 \right] \quad 2x = \frac{24}{12 + 24x} \quad 48x^2 + 24x - 24 = 0$ | m1 | Attempt to solve $\frac{dy}{dx} = 0$ |
| | $x = 0.5 \quad [, -1 \therefore \text{reject}]$ | A1 | |
| | $\left[x = 0.5, \quad y = -\frac{1}{4} + \ln 24 \right]$ | | |
| | $x = 1.5, \quad y = -\frac{9}{4} + \ln 48$ | B1 | PI by 1.62 AWRT |
| | $-\frac{9}{4} + \ln 48 \leq f(x) \leq -\frac{1}{4} + \ln 24$ | A1 | Must be in an exact form |
| | | 5 | |

| Q | Answer | Marks | Comments |
|---------|--|-----------|---|
| 6(b)(i) | $g(x) = -x^2 + \ln(12 + 24x) - 2x$ $g(1.1) = 0.24 \quad g(1.2) = -0.13$ | M1 | or reverse Both values rounded or truncated to at least 1sf |
| | Change of sign, $1.1 < \alpha < 1.2$ | A1 | Must have both statement and interval in words or symbols or comparing 2 sides: $f(1.1) = 2.44 > 2.2$ $f(1.2) = 2.27 < 2.4$ (M1) Conclusion as before (A1) |
| | | 2 | |

| Q | Answer | Marks | Comments |
|----------|---|-------|---|
| 6(b)(ii) | $\ln(12+24x) = x^2 + 2x$ $1 + \ln(12+24x) = x^2 + 2x + 1$ $(x+1)^2 = 1 + \ln(12+24x)$ $x = -1 + \sqrt{1 + \ln(12+24x)}$ | B1 | AG No errors seen including correct use of brackets Must be convincingly shown |
| | | 1 | |

| Q | Answer | Marks | Comments |
|-----------|-----------------------------|--------|--|
| 6(b)(iii) | $x_2 = 1.156$ $x_3 = 1.164$ | B1, B1 | If 0 scored, SC1 for AWRT 1.156 and 1,164 |
| | | 2 | |

| Q | Answer | Marks | Comments |
|---------|--|--------------|---|
| 6(c)(i) | Translation $\begin{bmatrix} 0 \\ -\ln 12 \end{bmatrix}$ or $\begin{bmatrix} 0 \\ \ln \frac{1}{12} \end{bmatrix}$ | B1 B1 | Allow $\begin{bmatrix} 0 \\ -2.48 \end{bmatrix}$ AWRT -2.48 |
| | | 2 | |

| Q | Answer | Marks | Comments |
|----------|------------------|-------|----------|
| 6(c)(ii) | $A - 1.5 \ln 12$ | B1ft | oe |
| | | 1 | |

| | | | |
|--|------------------|----|--|
| | Question 6 Total | 13 | |
|--|------------------|----|--|

| Q | Answer | Marks | Comments |
|------|---|--|--|
| 7(a) | $2x - 2y \frac{dy}{dx} = 6 \frac{dy}{dx} - 2$ $\frac{dy}{dx} = \frac{2x+2}{2y+6}$ OR $y = -3 \pm ((x+1)^2 - 12)^{0.5}$ $\frac{dy}{dx} = k((x+1)^2 - 12)^{-0.5} (x+1)$ $\frac{dy}{dx} = \frac{x+1}{((x+1)^2 - 12)^{0.5}}$ At (3, -1) $\frac{dy}{dx} = 2$ $y+1 = 2(x-3)$ or $y = 2x-7$ | M1 A1 (M1) (A1) m1 A1 | LHS or RHS correct Attempt to find gradient at (3, -1) PI by further work ACF |
| | | 4 | |

| Q | Answer | Marks | Comments |
|---------|--|--|---|
| 7(b)(i) | $x = \frac{1+\sqrt{17}\cos\theta}{2}, \quad \cos\theta = \frac{2x-1}{\sqrt{17}}$ $y = -1+\sqrt{17}\sin\theta, \quad \sin\theta = \frac{y+1}{\sqrt{17}}$ $\cos^2\theta + \sin^2\theta = 1$ $(2x-1)^2 + (y+1)^2 = 17$ | M1 A1 | Rearranges for $k\cos\theta$ and $k\sin\theta$ ACF , eg $\left(\frac{2x-1}{\sqrt{17}}\right)^2 + \left(\frac{y+1}{\sqrt{17}}\right)^2 = 1$ $4x^2 - 4x + y^2 + 2y = 15$ |
| | | 2 | |

| Q | Answer | Marks | Comments |
|----------|---|-------------|---------------------------------------|
| 7(b)(ii) | $4(2x-1) + 2(y+1) \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{4-8x}{2y+2}$ | M1 | Attempt at implicit differentiation |
| | | A1 | All correct |
| | OR | | |
| | $\frac{dy}{d\theta} = \sqrt{17} \cos \theta, \quad \frac{dx}{d\theta} = \frac{-\sqrt{17} \sin \theta}{2}$ $\frac{dy}{dx} = -2 \cot \theta$ | (M1) | Attempt at parametric differentiation |
| | | (A1) | All correct |
| | $\theta = \cos^{-1}\left(\frac{1}{\sqrt{17}}\right), \quad x=1, \quad y=3 \quad \frac{dy}{dx} = -0.5$ | m1 | |
| | $y-3 = 2(x-1) \quad \text{or} \quad y = 2x+1$ | A1 | ACF |
| | | 4 | |

| Q | Answer | Marks | Comments |
|------|--------|-------------|--|
| 7(c) | 49:1 | B1ft | ft their answers for part (a) and part (b)(ii) only if both equations have a gradient of 2 |
| | | 1 | |

| | | | |
|--|-------------------------|-----------|--|
| | Question 7 Total | 11 | |
|--|-------------------------|-----------|--|

| Q | Answer | Marks | Comments |
|---|---|--|--|
| 8 | $y = \frac{1}{10-2x} \quad 10-2x = \frac{1}{y}$ $x = 5 - \frac{1}{2y}$ $\left[x^2 = 25 - \frac{5}{y} + \frac{1}{4y^2} \right]$ $[\text{Volume} =] \pi \int_{0.1}^1 \left(25 - \frac{5}{y} + \frac{1}{4y^2} \right) dy$ $\int_{0.1}^1 \left(25 - \frac{5}{y} + \frac{1}{4y^2} \right) dy = 25y - 5\ln y - \frac{1}{4y}$ $\left[25y - 5\ln y - \frac{1}{4y} \right]_{0.1}^1 = (25 - 0.25)$ $- (2.5 - 5\ln 0.1 - 2.5)$ $= 24.75 + 5\ln 0.1$ $[\text{Volume} =] (24.75 - 5\ln 10)\pi$ | <p>M1 A1</p> <p>B1ft</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> | <p>Isolating 'x' Correct</p> <p>PI by later work</p> <p>At least 2 terms integrated correctly All terms integrated correctly</p> <p>Correctly substituting limits into their integration must be in form $py - q\ln y - \frac{r}{y}$</p> <p>ACF</p> |

| | | | |
|--|-------------------------|----------|--|
| | Question 8 Total | 7 | |
|--|-------------------------|----------|--|

| Q | Answer | Marks | Comments |
|------|------------------------------|------------------------|---|
| 9(a) | $\frac{dy}{dx} = k(3x + 4y)$ | M1 A1 | $\frac{dy}{dx} = (3x + 4y)$ or $\frac{dy}{dx} \propto (3x + 4y)$ All correct |
| | | 2 | |

| Q | Answer | Marks | Comments |
|---------|---|--|---|
| 9(b)(i) | $\frac{dy}{dx} = 4xe^{2y} - e^{2y} = e^{2y}(4x - 1)$ $\int e^{-2y} dy = \int 4x - 1 dx$ $-\frac{1}{2}e^{-2y} = 2x^2 - x + c$ $(1, 0) \quad -\frac{1}{2} = 1 + c$ $c = -1.5$ $\left[-\frac{1}{2}e^{-2y} = 2x^2 - x - 1.5 \right]$ $\left[e^{-2y} = -4x^2 + 2x + 3 \right]$ $y = -\frac{1}{2}\ln(3 + 2x - 4x^2)$ | M1 m1 m1 A1 M1 A1 | Separate variables Attempt to integrate eg $ae^{\pm 2y} = 2x^2 + kx$ oe Attempt to find c from equation as above Attempt to isolate y ie $y = a\ln(f(x))$ Must have scored first M1 ACF |
| | | 6 | |

| Q | Answer | Marks | Comments |
|----------|---|--------------------------------|-----------------------------|
| 9(b)(ii) | $y = 0, \quad 1 = 3 + 2x - 4x^2$ $[2x^2 - x - 1 = 0]$ $[(2x + 1)(x - 1) = 0]$ $x = -0.5$ | M1 A1 | $their (3 + 2x - 4x^2) = 1$ |
| | | 2 | |

| | | | |
|--|-------------------------|-----------|--|
| | Question 9 Total | 10 | |
|--|-------------------------|-----------|--|

| Q | Answer | Marks | Comments |
|-------|--|---|--|
| 10(a) | $\frac{x^2}{(3-x)(3+2x)(3-2x)} = \frac{A}{3-x} + \frac{B}{3+2x} + \frac{C}{3-2x}$ $x^2 = A(3+2x)(3-2x) + B(3-x)(3-2x) + C(3-x)(3+2x)$ $x = 3: 9 = A(9)(-3) \quad A = -\frac{1}{3}$ 1 $x = 1.5: 2.25 = C(1.5)(6) \quad C = \frac{1}{4}$ $A = -\frac{1}{3} \quad B = \frac{1}{12} \quad C = \frac{1}{4}$ | <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> | <p>Correctly combining RHS PI</p> <p>Attempt at finding one constant, could equate coefficients</p> <p>Two constants correct</p> <p>All correct</p> |
| | | 4 | |

| Q | Answer | Marks | Comments |
|----------|---|-----------------------------------|---|
| 10(b)(i) | $(3-x)^{-1} = 3^{-1} \left(1 - \frac{x}{3}\right)^{-1}$ $\frac{1}{3} \left(1 + (-1) \times \left(-\frac{x}{3}\right) + \frac{(-1) \times (-2) \times \left(-\frac{x}{3}\right)^2}{2} \right)$ $= \frac{1}{3} + \frac{1}{9}x + \frac{1}{27}x^2$ | <p>M1</p> <p>A1</p> | <p>At least 2 terms correct (unsimplified)</p> <p>All correct, simplified 1</p> |
| | | 2 | |

| Q | Answer | Marks | Comments |
|-----------|---------------------------|-------|----------|
| 10(b)(ii) | $ x < 3$ or $-3 < x < 3$ | B1 | |
| | | 1 | |

| Q | Answer | Marks | Comments |
|-------|--|------------|--|
| 10(c) | $(3-2x)^{-1} = \frac{1}{3} \left(1 + \frac{2}{3}x + \frac{4}{9}x^2 \right)$ | M1 | At least one expansion correct (unsimplified) |
| | $(3+2x)^{-1} = \frac{1}{3} \left(1 - \frac{2}{3}x + \frac{4}{9}x^2 \right)$ | A1 | Both expansions correct (unsimplified) |
| | $[f(x) =]$ $-\frac{1}{3} \left(\frac{1}{3} + \frac{1}{9}x + \frac{1}{27}x^2 \right) + \frac{1}{12} \times \frac{1}{3} \left(1 - \frac{2}{3}x + \frac{4}{9}x^2 \right)$ $+ \frac{1}{4} \times \frac{1}{3} \left(1 + \frac{2}{3}x + \frac{4}{9}x^2 \right)$ | M1 A1ft | Attempt at finding $f(x)$ All correct, ft their A , B and C and their binomial expansions |
| | $= \frac{1}{27}x^2$ | A1 | Must have scored first 4 marks |
| | | 5 | |

| | | | |
|--|-------------------|----|--|
| | Question 10 Total | 12 | |
|--|-------------------|----|--|

| Q | Answer | Marks | Comments |
|-------|--|---|---|
| 12(a) | $\overrightarrow{AB} = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} -6 \\ 4 \\ 12 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} -2 \\ 6 \\ 15 \end{bmatrix} + \lambda \begin{bmatrix} -6 \\ 4 \\ 12 \end{bmatrix}$ $4 - 6\lambda = -3p - 2$ $2 + 4\lambda = 2p + 6$ $3 + 12\lambda = 6p + 15$ $3p - 6\lambda = -6, \quad p - 2\lambda = -2$ $2p - 4\lambda = -4, \quad p - 2\lambda = -2$ $6p - 12\lambda = -12, \quad p - 2\lambda = -2$ <p>Equations are consistent, hence P lies on AB</p> | <p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p> | <p>Correct</p> <p>Equate 3 pairs of equations or solve a pair of equations oe</p> <p>CSO Should be convincingly shown</p> |
| | | 4 | |

| Q | Answer | Marks | Comments |
|-----------|---|---|--|
| 12(b)(i)2 | $\overrightarrow{CP} = \begin{bmatrix} -3p-2 \\ 2p+6 \\ 6p+15 \end{bmatrix} - \begin{bmatrix} -1 \\ 10 \\ 6 \end{bmatrix}$ $\overrightarrow{CP} = \begin{bmatrix} -3p-1 \\ 2p-4 \\ 6p+9 \end{bmatrix}$ $\begin{bmatrix} -3p-1 \\ 2p-4 \\ 6p+9 \end{bmatrix} \cdot \begin{bmatrix} -6 \\ 4 \\ 12 \end{bmatrix} = 0$ $18p + 6 + 8p - 16 + 72p + 108 = 0$ $98p = -98$ $p = -1$ | <p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p> | <p>A1: Correct direction vector oe</p> <p>Correct scalar product</p> |
| | | 4 | |

| Q | Answer | Marks | Comments |
|-----------|---|--|-------------------|
| 12(b)(ii) | $AB = \sqrt{(4 - -2)^2 + (2 - 6)^2 + (3 - 15)^2}$ | B1 M1 A1 A1 | Ft their p |
| | $AB = 14$ | | |
| | $CP = \sqrt{(-1 - 1)^2 + (10 - 4)^2 + (6 - 9)^2}$ | | |
| | $CP = 7$ | | |
| | Area $ABC = 49$ | | |
| | | 4 | |

| Q | Answer | Marks | Comments |
|-------|---|----------------------------|---|
| 12(c) | $AC^2 = (4 - -1)^2 + (2 - 10)^2 + (3 - 6)^2 = 98$ | M1 A1 | $\text{or } \cos BAC = \frac{\begin{bmatrix} -6 \\ 4 \\ 12 \end{bmatrix} \cdot \begin{bmatrix} -5 \\ 8 \\ 3 \end{bmatrix}}{14 \times 7\sqrt{2}} = \frac{1}{\sqrt{2}}$ |
| | $\sin BAC = \frac{7}{\sqrt{98}}$ | | |
| | Angle $BAC = 45^\circ$ | | oe |
| | | 2 | |

| | | | |
|--|--------------------------|-----------|--|
| | Question 12 Total | 14 | |
|--|--------------------------|-----------|--|