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# INTERNATIONAL AS MATHEMATICS

## 9665

FM02 Further Pure Mathematics Unit 2

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Mark scheme

January 2019

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Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

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Further copies of this mark scheme are available from [oxfordaqaexams.org.uk](http://oxfordaqaexams.org.uk)

| Q | Answer   | Mark         | Comments                              |
|---|--|--------------|---------------------------------------|
| 1 | $hf(x, y) = \frac{0.1}{1.2^2 + 0.8^2}$ $= 0.048077$              | M1<br><br>A1 | PI                                    |
|   | $y_2 = 0.8 + 0.048077 = 0.848077$                                | m1           | 0.8+ their value of $hf(x, y)$        |
|   | $y_3 = 0.848077 + \frac{0.1}{1.3^2 + 0.848077^2}$<br>(= 0.88958) | m1           |                                       |
|   | 0.890  | A1           | CAO – candidate's <b>final</b> answer |
|   | <b>Total</b>   | <b>5</b>     |                                       |

| Q | Answer  | Mark     | Comments |
|---|---|----------|----------|
| 2 | $[f(3) =] -9 \text{ and } [f(4) =] 20$  | B1       | PI       |
|   | $x_1 = 3 + \frac{9}{20 + 9}$  | M1       | PI       |
|   | $x_1 = \frac{96}{29}$   | A1       |          |
|   | $f(x_1)[ = -2.2067] < 0$  | B1       |          |
|   | $f(x_1) < 0 \text{ so } \alpha > \frac{96}{29} \text{ and } \frac{96}{29} < \alpha < 4$ | E1       |          |
|   | <b>Total</b>  | <b>5</b> |          |

|             | Answer   | Mark      | Comments   |
|-------------|--|-----------|--|
| <b>3(a)</b> | $y = ax^n \Rightarrow \log_{10} y = \log_{10} ax^n$<br>$\log_{10} y = \log_{10} a + \log_{10} x^n$ | M1        | Take logs and apply one log law correctly PI   |
|             | $\log_{10} y = \log_{10} a + n\log_{10} x$   | m1        | Apply a further log law correctly.   |
|             | $Y = \log_{10} a + nX$<br>(which is a linear relationship between Y and X.)                        | A1        | Correct eqn. with base 10 (or lg or later evidence of use of base 10 if log without base here) |
| <b>3(b)</b> | 0.602, 0.778, 0.903  | B1        | At least 2 sig fig   |
|             | 1.96, 2.37, 2.66   | B1        | At least 2 sig fig   |
| <b>3(c)</b> | Their four points plotted correctly  | B1ft      |  |
|             | Straight line drawn through their points   | B1ft      |  |
| <b>3(d)</b> | $a = 10^{0.58}$  | M1        | 10 to the power of their intercept   |
|             | $a = 3.8$  | A1        | [3.3, 4.2]   |
|             | Gradient found   | M1        |  |
|             | $n = 2.3$  | A1        | [2.1, 2.5]   |
|             | <b>Total</b>   | <b>11</b> |  |

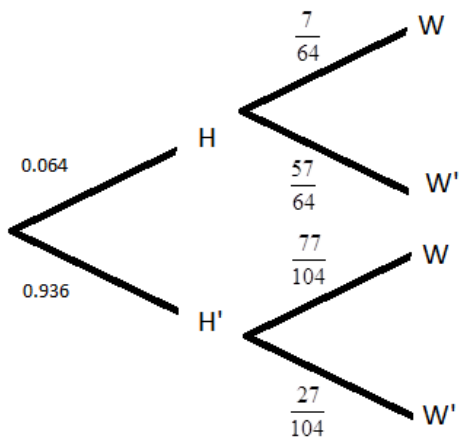
| Q    | Answer   | Mark     | Comments                  |
|------|--|----------|---------------------------|
| 4(a) | $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$                 | B1       |                           |
| 4(b) | W in correct place<br>$(-3, -3)$ , $(-1, -3)$ and $(-3, -6)$   | B1<br>B1 | Two vertices correct B1B0 |
| 4(c) | <b>Method 1</b>  |          |                           |
|      | $\det(\mathbf{P}) = 3$   | B1ft     |                           |
|      | $\det(\mathbf{Q}) = -1$  | B1       | PI                        |
|      | $\det(\mathbf{QP}) = -3$                                       | B1       |                           |
|      | <b>Alternative method</b>                                      |          |                           |
|      | $\mathbf{Q} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$  | B1       |                           |
|      | $\mathbf{QP} = \begin{bmatrix} 0 & -1 \\ -3 & 0 \end{bmatrix}$ | M1       |                           |
|      | $\det(\mathbf{QP}) = -3$                                       | A1       |                           |
|      | <b>Total</b>   | <b>6</b> |                           |

| Q    | Answer  | Mark | Comments                          |
|------|---|------|-----------------------------------|
| 5(a) | <b>Method 1</b>   |      |                                   |
|      | $\det(\mathbf{B}) = 2$  | M1   |                                   |
|      | $\mathbf{B}^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$  | A1   |                                   |
|      | $\mathbf{A} = \mathbf{CB}^{-1}$   | M1   |                                   |
|      | $\mathbf{A} = \begin{bmatrix} 1 & -3 \\ 2 & -4 \end{bmatrix}$   | A1   |                                   |
|      | <b>Alternative method</b>   |      |                                   |
|      | Let $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then<br>$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 2 & 6 \end{bmatrix}$ | M1   |                                   |
|      | $\begin{aligned} -a - b &= 2 \\ a - b &= 4 \\ -c - d &= 2 \\ c - d &= 6 \end{aligned}$  | M1   | For four equations                |
|      | $a = 1$ and $b = -3$<br>or $c = 2$ and $d = -4$   | M1   | For solving one pair of equations |
|      | $\mathbf{A} = \begin{bmatrix} 1 & -3 \\ 2 & -4 \end{bmatrix}$   | A1   |                                   |

|                 |  |           |  |
|-----------------|--|-----------|--|
| <b>5(b)</b>     | $\mathbf{B}^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$                     | M1        |  |
|                 | $\mathbf{B}^4 = (\mathbf{B}^2)^2 = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$ | M1<br>A1  |  |
|                 | $\mathbf{B}^4 = k\mathbf{I}$ where $k = -4$  | E1        |  |
| <b>5(c)(i)</b>  | Scale factor = $\sqrt{2}$  | B1        |  |
| <b>5(c)(ii)</b> | $-135^\circ$   | B1        | Or $135^\circ$ clockwise oe.<br>Do not accept just $135^\circ$ |
| <b>5(d)</b>     | $\mathbf{B}^{21} = (\mathbf{B}^4)^5 \times \mathbf{B}$                             | M1        |  |
|                 | $= (-4)^5 \times \mathbf{B}$ or $-1024\mathbf{B}$                                  | M1        |  |
|                 | $\mathbf{B}^{21} = \begin{bmatrix} 1024 & -1024 \\ 1024 & 1024 \end{bmatrix}$      | A1        |  |
|                 | <b>Total</b>   | <b>13</b> |  |



| Q    | Answer  | Mark     | Comments  |
|------|---|----------|---|
| 6(a) | $\frac{2}{3}$   | B1       |   |
| 6(b) | $G_X(t) = \sum_{n=1}^3 t^n \left( \frac{1}{3} \right)$                              | M1       | Applies formula for $G_X(t)$  |
|      | $= \frac{1}{3} (t + t^2 + t^3)$ oe  | A1       | Ignore subsequent incorrect working   |
| 6(c) | <b>Alternative Method 1</b>   |          |   |
|      | $G_{X+Y}(t) = \frac{1}{3} (t + t^2 + t^3)(0.6 + 0.4t)$                              | M1       | Multiplies their $G_X(t)$ and $G_Y(t)$  |
|      | $\frac{1}{3} t \times 0.6$  | M1       | Multiplies out the required terms to find the coefficient of t<br>Implied by correct answer |
|      | $= 0.2$   | A1       | Accept 1/5 oe   |
|      | <b>Alternative method 2</b>   |          |   |
|      | Y Bernoulli, $p = 0.4$<br>or $Y \sim B(1, 0.4)$                                     | M1       | Identifies distribution of Y  |
|      | $P(X + Y = 1)$ means $X = 1$ and $Y = 0$<br>or<br>$P(X + Y = 1) = P(X = 1)P(Y = 0)$ | M1       | Identifies possible combinations of X and Y   |
|      | $= \frac{1}{3} (1 - 0.4) = 0.2$   | A1       | Accept 1/5 oe   |
|      | <b>Total</b>  | <b>6</b> |   |

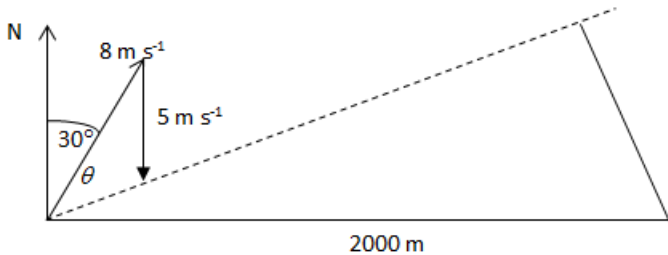
| Q    | Answer  | Mark     | Comments  |
|------|---|----------|---|
| 7(a) | $(1 - 0.6)^3 = 0.064$   | B1       | Accept 8/125 oe   |
| 7(b) | $P(W   H) = \frac{P(H   W)P(W)}{P(H)}$ $= \frac{0.01 \times 0.7}{0.064}$  | M1       | Applies Bayes Theorem to find P(W H)                    |
|      | $= \frac{7}{64}$ or AWRT 0.109  | A1ft     | ft their P(H) from (a) provided $0 < P(H) < 1$          |
|      | $P(H' \cap W) = P(W) - P(H \cap W)$<br>$P(H')P(W H') = P(W) - P(H)P(W H)$<br>$(1 - 0.064)P(W H') = 0.7 - 0.064 \times \frac{7}{64}$ | M1       | Method to find P(W H')                                  |
|      | $P(W H') = \frac{77}{104}$ or 0.740 AWRT  | A1ft     | ft their P(H) from (a) provided $0 < P(H) < 1$          |
|      |    | A1ft     | ft their probabilities<br>Need to score at least one M1 |
|      | <b>Total</b>  | <b>6</b> |   |

| Q    | Answer  | Mark     | Comments   |
|------|---|----------|--|
| 8(a) | $E(2P^2 - 5) = 2E(P^2) - 5$<br>or<br>$E(PQ) = 2E(P^3) - 5E(P)$  | M1       | One correct formula seen   |
|      | $E(2P^2 - 5) = 2 \times 5 - 5 = 5$<br>or<br>$E(PQ) = 2 \times 14.6 - 5 \times 2 = 19.2$                   | A1       | One correct value<br>Accept 96/5 oe for 19.2<br>Can be implied by correct final answer   |
|      | $E(2P^2 - 5) = 2 \times 5 - 5 = 5$<br>and<br>$E(PQ) = 2 \times 14.6 - 5 \times 2 = 19.2$                  | A1       | Both correct values<br>Accept 96/5 oe for 19.2<br>Can be implied by correct final answer |
|      | $\text{Cov}(P, Q) = E(PQ) - E(P)E(Q)$<br>$= 19.2 - 2 \times 5$  | M1       | Applies formula for Cov (P, Q)   |
|      | $= 9.2$   | A1       | Accept 46/5 oe   |
| 8(b) | $\text{Var}(P) = E(P^2) - (E(P))^2$<br>$= 5 - 2^2$<br>$= 1$   | B1       | Finds Var (P)<br>Can be implied by correct final answer                                  |
|      | $\text{Var}(P + Q)$<br>$= \text{Var}(P) + \text{Var}(Q) + 2 \text{Cov}(P, Q)$<br>$= 1 + 8 + 2 \times 9.2$ | M1       | Applies formula for Var (P + Q)  |
|      | $= 27.4$  | A1       | Accept 137/5 oe  |
|      | <b>Total</b>  | <b>8</b> |  |

| Q | Answer  | Mark     | Comments |
|---|---|----------|----------|
| 9 | $MLT^{-2} = [k]L^{\frac{3}{2}}T^{-\frac{3}{2}}$ $[k] = ML^{-\frac{1}{2}}T^{-\frac{1}{2}}$ | M1<br>A1 |          |
|   | Total   | 2        |          |

| Q     | Answer  | Mark           | Comments   |
|-------|---|----------------|--|
| 10(a) | $I = \frac{1}{2} \times 2500 \times 10$ $= 12500 \text{ Ns}$                            | B1             |  |
| 10(b) | $-12500 = 2000U - 2000 \times 20$ $U = \frac{2000 \times 20 - 12500}{2000}$ $U = 13.75$ | M1<br>A1<br>A1 | M1: Equation with correct terms but any signs.<br>A1: Correct equation.<br>A1: Correct $U$ . |
|       | <b>Total</b>  | <b>4</b>       |  |

| Q     | Answer   | Mark  | Comments |
|-------|--|---|----------|
| 11(a) | $5mU + 4mU = mv_A + 4mv_B$ $9U = v_A + 4v_B$ $v_A - v_B = -e(5U - U)$ $v_A - v_B = -4eU$ $9U = v_B - 4eU + 4v_B$ $v_B = \frac{U(9 + 4e)}{5}$ | <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> |          |
| 11(b) | $v_A = \frac{U(9 + 4e)}{5} - 4eU$ $v_A = \frac{U(9 - 16e)}{5}$ $9 - 16e < 0$ $e > \frac{9}{16}$  | <p>B1</p> <p>M1</p> <p>A1</p>                     |          |
|       | <b>Total</b>   | <b>8</b>  |          |

| Q  | Answer   | Mark                                     | Comments   |
|----|--|--|------------|
| 12 | $v_{MY}^2 = 8^2 + 5^2 - 2 \times 8 \times 5 \cos 30^\circ$ $v_{MY}^2 = 89 - 40\sqrt{3}$ $\frac{\sin \theta}{5} = \frac{\sin 30^\circ}{\sqrt{89 - 40\sqrt{3}}}$ $\theta = \sin^{-1} \frac{5}{2\sqrt{89 - 40\sqrt{3}}}$ $\theta = 34.26357 \dots^\circ$ $\text{Min Sep} = 2000 \times \sin(60 - \theta) = 868 \text{ m}$ | M1<br>A1<br>M1<br><br>A1<br><br>M1<br>A1 | Accept 869 |
|    | <b>Additional Guidance</b>   |  |            |
|    |   |  |            |
|    | <b>Total</b>   | <b>6</b>                                 |            |