OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM05) Unit FM2 Mechanics

Monday 19 June 2023 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to two significant figures, unless stated otherwise.
- Unless stated otherwise, the acceleration due to gravity, g, should be taken as 9.8 m s^{-2}

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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Answer **all** questions in the spaces provided.

1 A particle of mass 0.7 kg moves with simple harmonic motion on a straight line between the two points A and B

The distance between A and B is 0.4 metres.

The maximum speed of the particle during the motion is 10 m s^{-1}

1 (a) Find the period of the motion, giving your answer in terms of π

[2 marks]

Answer _____

1 (b) Find the speed of the particle when it is 0.15 metres from A

[3 marks]

Answer _____

1 (c) Find the range of values of the resultant force acting on the particle during the motion.

[3 marks]

Answer _____

8

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2 Two particles A and B are moving on a smooth horizontal surface and collide.

The mass of A is 2 kg and before the collision it has velocity $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ m s^{-1}

The mass of B is 3 kg and before the collision it has velocity $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$ m s^{-1}

The impulse on A during the collision is $\begin{bmatrix} -3.6 \\ -6 \end{bmatrix}$ N s

2 (a) Find the velocity of A after the collision.

[3 marks]

Answer _____

2 (b) State the impulse on B during the collision.

[1 mark]

Answer _____

2 (c) Show that A and B have the same velocity after the collision.

[3 marks]

7

Turn over ►

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3 An elastic string has natural length 2.5 metres and modulus of elasticity 14 newtons.

A sphere of mass 0.5 kg is attached to one end of the string.

The other end of the string is attached to a fixed point O

3 (a) Find the length of the string when the sphere hangs in equilibrium below O

[3 marks]

Answer _____

3 (b) The sphere is projected vertically **upwards** from its equilibrium position with a speed of 1.2 m s^{-1}

3 (b) (i) Use an energy method to find the height of the sphere above the equilibrium position when it first comes to rest.

[5 marks]

Answer _____

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3 (b) (ii) At time t seconds after the sphere has been set in motion, its displacement above the equilibrium position is x metres.

Show that the sphere moves with simple harmonic motion.

[4 marks]

3 (b) (iii) Hence find x in terms of t

[2 marks]

Answer _____

3 (b) (iv) Find the time when the sphere is 0.2 metres below its equilibrium position for the first time.

[3 marks]

Answer _____

17

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4 A particle moves in a straight line on a smooth horizontal surface.

The displacement of the particle from its initial position is x metres.

A horizontal force F newtons acts on the particle where

$$F = 3 - x^2$$

and no other horizontal forces act on the particle.

The particle is initially at rest.

4 (a) Show that the work done by the force as the particle moves from its initial position to the point where $x = a$ is given by

$$3a - \frac{a^3}{3}$$

[2 marks]

4 (b) A student claims that the particle will come to rest for an instant at the point where $x = 3$

Explain why the student is correct.

[3 marks]

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4 (c) The student claims that the particle will also come to rest at the point where $x = -3$

4 (c) (i) With reference to the work done by the force, explain why the student thinks that this is the case.

[1 mark]

4 (c) (ii) Explain why the student is incorrect.

[2 marks]

4 (d) State the range of values of x during the motion.

[1 mark]

Answer _____

9

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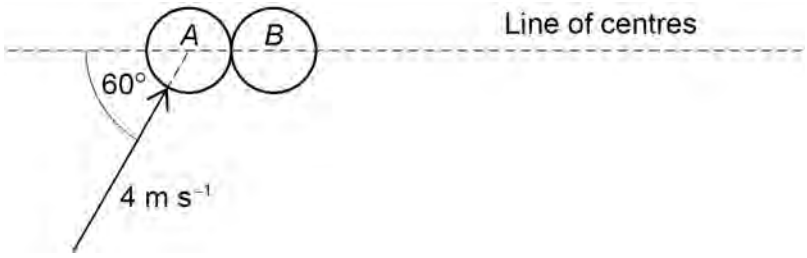
5 A smooth sphere A is moving on a horizontal surface when it collides with a stationary smooth sphere B

The spheres have equal radii.

Sphere A has mass 4 kg and sphere B has mass 5 kg

When the two spheres collide, A is moving at 4 m s^{-1} at an angle of 60° to the line of centres, as shown in the diagram below.



After the collision the velocity of A is perpendicular to the line of centres.

The coefficient of restitution between the two spheres is e

5 (a) Find the speed of B after the collision.

[2 marks]

Answer _____

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5 (b) Find the value of e

[2 marks]

Answer _____

5 (c) Find the speed of A after the collision, giving your answer in exact form.

[1 mark]

Answer _____

5

Turn over for the next question

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6 A sphere of mass m kg is released from rest in a fluid and allowed to fall vertically.

At time t seconds the speed of the sphere is v m s^{-1}

A resistance force of magnitude kv newtons, where k is a constant, acts on the sphere as it falls.

At time t seconds the displacement of the particle from its initial position is x metres.

Show that

$$x = \frac{mg}{k}\left(t + \frac{m}{k}\mathrm{e}^{\frac{-kt}{m}} - \frac{m}{k}\right)$$

[7 marks]

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7

Turn over ►

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7 A disc is sliding on a smooth horizontal surface.

There are two smooth fixed vertical walls A and B on the surface.

The obtuse angle between the walls A and B is 105°

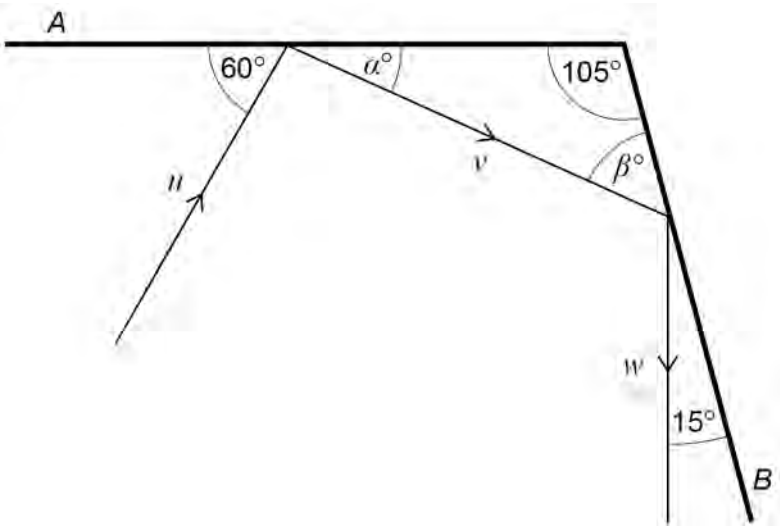
When the disc collides with Wall A it has velocity u m s^{-1} at an angle of 60° to Wall A

When the disc leaves Wall A it has velocity v m s^{-1} at an angle of α degrees to Wall A

When the disc collides with Wall B it has velocity v m s^{-1} at an angle of β degrees to Wall B

When the disc leaves Wall B it has velocity w m s^{-1} at an angle of 15° to Wall B

The walls and velocities are shown in the diagram below.



The coefficient of restitution between the disc and both walls is e

7 (a) Show that

$$v^2 = \frac{u^2}{4}\left(1+3e^2\right)$$

[4 marks]

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7 (b) In this question you may use the following

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan 75°=2+\sqrt{3} \quad \text{and} \quad \tan 15°=2-\sqrt{3}$$

Find the two possible values of e

[8 marks]

Question 7 continues on the next page

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$e =$ _____ or $e =$ _____

7 (c) For the larger value of e, state the value of w in terms of u

[1 mark]

Answer _____

13

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Turn over for next question

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8 A plane is inclined at an angle of $30°$ to the horizontal.

A smooth hemisphere of radius 2 metres and centre O is fixed to the plane.

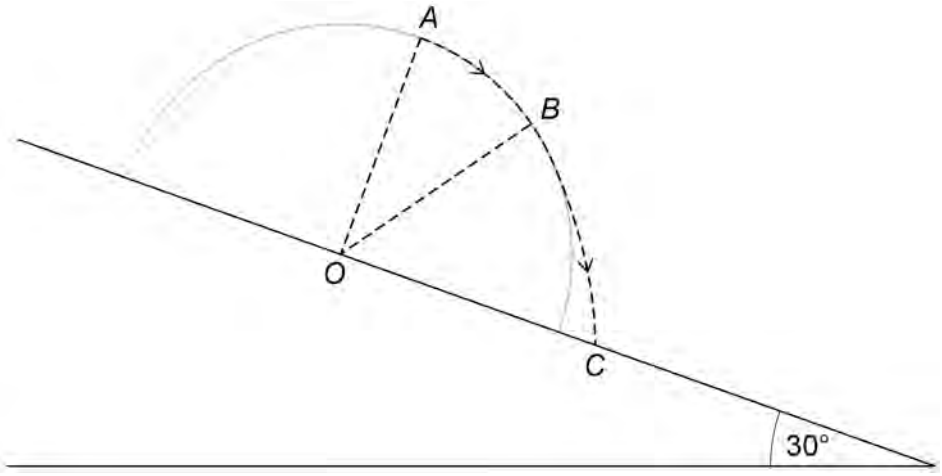
The point A is located on the hemisphere so that it is 2 metres from the plane.

A particle is released from rest at the point A and moves on the hemisphere.

The particle leaves the hemisphere at the point B and hits the plane at the point C

The acute angle between OB and the **vertical** is θ degrees.

The hemisphere and the plane are shown in the diagram below.



8 (a) Show that $\theta = 54.7°$ correct to one decimal place.

[6 marks]

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8 (b) Show that the speed of the particle when it leaves the hemisphere is 3.4 m s^{-1} correct to two significant figures.

[1 mark]

8 (c) Find the shortest distance between the hemisphere and *C*

[7 marks]

Answer _____

14

END OF QUESTIONS

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