OXFORD

INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL A-LEVEL
MATHEMATICS
MA05

(9660/MA05) Unit M2 Mechanics

Mark scheme

January 2022

Version: 1.0 Final

221XMA05/MS

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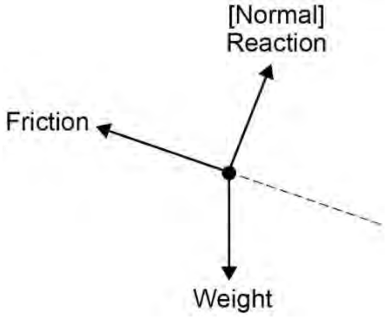
Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$\lvert\mathbf{r}\rvert^2 = \left(\sin(2t)\cos(3t)\right)^2 + \left(\sin(2t)\sin(3t)\right)^2 + \left(\cos(2t)\right)^2$	M1	Squaring each component in terms of t and adding
	$\lvert\mathbf{r}\rvert^2 = \sin^2(2t)\left(\cos^2(3t)+\sin^2(3t)\right) + \cos^2(2t)$	M1	Use of $\cos^2(3t)+\sin^2(3t)=1$
	$\lvert\mathbf{r}\rvert^2 = \sin^2(2t)+\cos^2(2t)$	A1	
	$\lvert\mathbf{r}\rvert = 1$	A1	Shows that the distance or square of the distance is a constant, independent of t
		4	

Q	Answer	Marks	Comments
1(b)	$\mathbf{v} = \begin{bmatrix} 2\cos(2t)\cos(3t)-3\sin(2t)\sin(3t) \\ 2\cos(2t)\sin(3t)+3\sin(2t)\cos(3t) \\ -2\sin(2t) \end{bmatrix}$	B1 M1 A1	Correct **k** component Use of product rule for **i** or **j** component Both **i** and **j** components correct
	$\mathbf{v} = \begin{bmatrix} 2\cos(0)\cos(0)-3\sin(0)\sin(0) \\ 2\cos(0)\sin(0)+3\sin(0)\cos(0) \\ -2\sin(0) \end{bmatrix}$	M1	**PI** ft their velocity evaluated at $t=0$ provided at least one component is correct
	$\mathbf{v} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$	A1	**ft** their velocity evaluated correctly at $t=0$ Do not **ISW**
		5	

	Question 1 Total	**9**	

Q	Answer	Marks	Comments
2(a)		**B1**	Must have names on the three arrows, not symbols unless the symbols are defined Allow gravitational force instead of weight but do not accept 'gravity' instead of weight These three forces and no others
		1	

Q	Answer	Marks	Comments
2(b)	Component of weight down the slope $\left[= mg\sin\alpha = 5\times 9.8\times\sin\left(10^\circ\right)\right]$ $= 8.51$ [N]	**B1**	
	Friction force acting on the particle $\left[\le \mu mg\cos\alpha = 0.25\times 5\times 9.8\times\cos\left(10^\circ\right)\right]$ $[\le]12.1$ [N]	**B1**	If **B0 B0** then allow **SC1** for $[F=]mg\sin\alpha - \mu mg\cos\alpha$ or better
	$8.51 \le 12.1$ [Resultant force = 0] therefore remains at rest	**E1**	Inequality and a correct conclusion that component of weight down the slope is less than the maximum friction force
		3	

Q	Answer	Marks	Comments
2(c)	Resultant force down the slope $F = mg\sin\alpha - \mu mg\cos\alpha$	**M1**	Correct component of weight down the slope or correct friction force **PI**
	$\left[F = 5\times 9.8\sin\left(20^\circ\right) - 0.25\times 5\times 9.8\cos\left(20^\circ\right)\right]$ $= 5.24775...$ [N] or $\left[a = g\sin\alpha - \mu g\cos\alpha\right]$ $a = 9.8\times\sin\left(20^\circ\right) - 0.25\times 9.8\times\cos\left(20^\circ\right)$	**A1**	Correct value for resultant force or correct calculation for acceleration **PI** by correct answer
	$a = 1.05$ $\left[\text{m s}^{-2}\right]$	**A1**	**CAO** to 3 sf
		3	

	Question 2 Total	**7**	

Q	Answer	Marks	Comments
3(a)	(4, 2.5)	**B1**	
	The centre of mass of the uniform rectangular lamina is at the geometric centre	**E1**	Condone 'middle' for geometric centre
		2	

Q	Answer	Marks	Comments
3(b)	$(2+3+4+5)\bar{X}$ $= 2\times4+3\times2+4\times4+5\times7$	**M1**	**M1**: Forming equation for x-coordinate of the centre of mass. Condone one slip
	$\bar{X} = \dfrac{65}{14}$	**A1**	**A1**: Correct and in an exact form, **oe**
	$(2+3+4+5)\bar{Y}$ $= 2\times2.5+3\times2.5+4\times4+5\times1$	**M1**	**M1**: Forming equation for y-coordinate of the centre of mass. Condone one slip
	$\bar{Y} = \dfrac{67}{28}$	**A1**	**A1**: Correct and in an exact form, **oe**
		4	

Q	Answer	Marks	Comments
3(c)	Angle *AC* makes with edge of length 5: $\tan^{-1}\left(\dfrac{8-\frac{65}{14}}{5-\frac{67}{28}}\right) = \tan^{-1}\left(\dfrac{47/14}{73/28}\right) = 52.167...^\circ$	**B1ft**	[Note that *C* represents the centre of mass of the system] or Angle *AC* makes with edge of length 8: $\tan^{-1}\left(\dfrac{73/28}{47/14}\right) = 37.832...^\circ$
	Angle *AR* makes with edge of length 5: $\tan^{-1}\left(\dfrac{1}{4}\right) = 14.036...^\circ$	**B1**	or Angle *AR* makes with edge of length 8: $\tan^{-1}(4) = 75.963...^\circ$
	$52.167...^\circ - 14.036...^\circ$	**M1**	Subtracting their angles – must have scored at least one of the **B1** marks
	38°	**A1**	**CAO** given to the nearest degree
		4	

	Question 3 Total	**10**	

Q	Answer	Marks	Comments
4(a)	No resultant force in vertical direction gives: $T\cos\theta = mg$ $\cos\theta = \dfrac{3\times 9.8}{60}$	**M1**	Forming a correct trigonometric equation
	$\theta = 60.659...^\circ$	**A1**	Finding the correct angle from their trigonometric equation, **PI oe**
	$r = l\sin\theta$ $l = \dfrac{0.6}{\sin\left(60.659...^\circ\right)}$	**M1**	Forming a correct equation for l
	$l = 0.688$	**A1**	**CAO** Ignore inclusion of units on final answer
		4	

Q	Answer	Marks	Comments
4(b)	Magnitude of resultant force $F = 60\sin\left(60.659...^\circ\right)$ $F = 52.303...$ [N]	**M1**	**PI** Calculating resultant force
	Magnitude of acceleration $a = \dfrac{F}{m} = \dfrac{52.303...}{3}$ $a = 17\ \left[\text{m s}^{-2}\right]$	**A1**	Answer to 3 sf is $17.4\ \left[\text{m s}^{-2}\right]$
	Direction: Towards *C*	**B1**	Condone ‘to centre’
		3	

Q	Answer	Marks	Comments
4(c)	$a = r\omega^2 \quad \Rightarrow \quad \omega = \sqrt{\dfrac{a}{r}} = \sqrt{\dfrac{17.434...}{0.6}}$	**M1**	**ft** their acceleration from **part (b)** or for linear speed = 3.23… [m s^{-1}]
	$\omega = 5.4\ \left[\text{rad s}^{-1}\right]$	**A1**	**CAO** Answer to 3 sf is $5.39\left[\text{rad s}^{-1}\right]$
		2	

	Question 4 Total	**9**	

Q	Answer	Marks	Comments
5(a)	Resultant force: $T - \mu mg - 31\sqrt{v} = ma$ $T = ma + \mu mg + 31\sqrt{v}$ $T = 20 \times 2 + 0.5 \times 20 \times 9.8 + 31\sqrt{4}$ $T = 200$	**M1** **m1** **A1**	Forming equation of motion Allow one slip/omission **PI** Substitution into their equation for T Correct value for T Ignore inclusion of units
		3	

Q	Answer	Marks	Comments
5(b)	$P = Fv$ $= 200 \times 8$ $= 1600$ W	**M1** **A1**	Use of $P = Fv$ with their T from **part (a)** **CAO** and must include units Allow J s^{-1}
		2	

Q	Answer	Marks	Comments
5(c)	When in equilibrium: $T - \mu mg - 31\sqrt{v} = 0$ $\Rightarrow \quad v = \left(\frac{T - \mu mg}{31}\right)^2$ $v = \left(\frac{200 - 0.5 \times 20 \times 9.8}{31}\right)^2$ $v = 10.8$	 **M1** **m1** **A1**	Forming a three-term equation for v using equilibrium and making v the subject. Allow one slip **PI** **ft** substituting their T into their formula for v **CAO** Ignore inclusion of units
		3	

	Question 5 Total	**8**	

Q	Answer	Marks	Comments
6(a)	Initial GPE (relative to horizontal ground): $mgh = 12 \times 9.8 \times 8$ $= 940.8$ J	**B1**	May be seen in the calculation for the change in GPE between *A* and *B*
	Work done against the resistive force: $Fd = 16 \times 20$ $= 320$ J	**B1**	
	Conservation of Energy: $940.8 - 12 \times 9.8 \times 3 = 320 + \frac{1}{2}mv^2$	**M1**	
	$v = \sqrt{\frac{2 \times 268}{12}}$ or $v = 6.683...$	**A1**	v or v^2 seen in an exact form or to at least 3 sf. Be convinced
	Therefore $v = 6.7$ $[\text{to } 2 \text{ sf}]$		Accept $\frac{\sqrt{402}}{3}$
		4	

Q	Answer	Marks	Comments
6(b)	At maximum height, vertical component of velocity is zero: $v^2 = u^2 + 2as \quad \Rightarrow \quad s = \frac{v^2 - u^2}{2a}$ $s = \frac{0 - (6.7 \sin 40^\circ)^2}{2 \times -9.8}$	**M1**	Use of SUVAT equation with $v = 0$ or use of conservation of energy, such as $mg\Delta h = \frac{1}{2}m(6.7 \sin 40^\circ)^2$
	$s = 0.946...$ $[\text{m}]$	**A1**	Correct value for s , **PI** (the height above B) Value is 0.941… m if 6.683… is used
	Maximum height above the horizontal ground is: $0.946... + 3 = 3.9$ $[\text{m}]$	**A1ft**	their $s + 3$ **AWFW** [3.9, 4.0] from correct working
		3	

Q	Answer	Marks	Comments
6(c)	Time at which the particle reaches *C*:		
	$s = ut + \frac{1}{2}at^2$	**M1**	Use of SUVAT equation with $s = -3$
	$-3 = 6.7\sin(40^\circ) \times t + \frac{1}{2} \times -9.8 \times t^2$	**m1**	Forming correct quadratic equation in t
	$t = 1.336...$ $[, \; -0.457...]$	**A1**	Correct value for t $t = 1.335\ldots$ if $v = 6.683\ldots$ is used
	$x = 6.7\cos(40^\circ) \times 1.336...$	**M1**	Must use cosine and their $t > 0$
	$x = 6.9$	**A1**	**CAO** Answer is: 6.9 or 6.86 if $v = 6.7$ is used throughout 6.8 or 6.84 if $v = 6.683\ldots$ is used throughout
		5	

Q	Answer	Marks	Comments
6(d)	Vertical component of velocity at *C* $v = u + at$ $v = 6.7\sin(40^\circ) + (-9.8) \times 1.336...$ $= -8.794...$ $[\text{m s}^{-1}]$	**M1**	May use energy considerations Value is –8.789… if $v = 6.683\ldots$ is used **PI** by correct final answer
	Using Pythagoras' theorem speed $= \sqrt{(6.7\cos(40^\circ))^2 + (-8.794...)^2}$ $= 10$ $[\text{m s}^{-1}]$	**A1**	**AWRT** 10 from correct working Answer to 3 sf is 10.2 [m s^{-1}]
		2	

	Question 6 Total	**14**	

Q	Answer	Marks	Comments
7(a)	Frictionless	**B1**	**oe**
		1	

Q	Answer	Marks	Comments
7(b)(i)	Taking moments about the base of the ladder	**M1**	Forming equation using moments
	$4 \times 25g\cos(65^\circ) + 6 \times 75g\cos(65^\circ)$ $= 8 \times R_W \sin(65^\circ)$	**m1 A1**	**m1**: At least one side of equation correct **A1**: Both sides of equation correct R_W = normal reaction on ladder from wall
	$R_W = \dfrac{(4 \times 25 + 6 \times 75) \times 9.8 \times \cos(65^\circ)}{8 \times \sin(65^\circ)}$		
	$R_W = 310\ [\text{N}]$	**A1**	**CAO** Answer is 314 [N] to 3 sf
		4	

Q	Answer	Marks	Comments
7(b)(ii)	310 [N]	**B1ft**	
	Newton's 3rd Law	**E1**	Any reference to Newton's 3rd Law
		2	

Q	Answer	Marks	Comments
7(b)(iii)	310 [N]	**B1ft**	
	Newton's 1st Law	**E1**	Allow any reference to the ladder being in equilibrium in the horizontal direction, eg forces on the ladder to the left have the same magnitude as the forces on the ladder to the right
		2	

Q	Answer	Marks	Comments
7(c)	Equilibrium of forces in the vertical direction: $R_G = 100 \times 9.8 = 980$ N	M1	R_G = normal reaction on ladder from ground
	Equilibrium of forces in the horizontal direction: $f_G = 314.174...$ N		f_G = friction on ladder from ground
	$f_G \leq 0.8\mu R_G$	m1	Forms inequality for the coefficient of friction, including 0.8
	$\mu \geq \dfrac{314.174...}{0.8 \times 980} = 0.40$	A1	**CAO, AWRT** 0.40 Condone 0.4
		3	

Q	Answer	Marks	Comments
7(d)	[As] $0.35 < 0.40$ [the coefficient of friction is now less than the minimum coefficient of friction allowed by the safety reasons]	B1F	Comparison of 0.35 with their minimum value for μ from **part (c)** using the $0.8F$ condition
	It is not safe for the person to use the ladder	E1F	Statement must be consistent with their comparison
		2	

	Question 7 Total	14	

Q	Answer	Marks	Comments
8(a)	$v = \dfrac{2\pi r}{T}$ or $v = \omega r$	M1	**PI** by a calculation
	$v = \dfrac{2\pi \times 6000 \times 10^3}{10 \times 60 \times 60}$	m1	
	$v = 1047.197...\ \left[\mathrm{m\,s^{-1}}\right]$ Therefore $v = 1050\ \left[\mathrm{m\,s^{-1}}\right]\ [\text{to 3 sf}]$	A1	Value seen to at least 4 sf or better. Be convinced
		3	

Q	Answer	Marks	Comments
8(b)	As spacecraft *A* is at the North pole, it is not travelling around a circle and so its speed is zero	E1	Any correct explanation based on $r = 0$
		1	

Q	Answer	Marks	Comments
8(c)	$a = \dfrac{v^2}{r} = \dfrac{1050^2}{6000 \times 10^3}$	M1	**PI**
	$a = 0.18\ \left[\mathrm{m\,s^{-2}}\right]$	A1	**AWRT** $0.18\ \left[\mathrm{m\,s^{-2}}\right]$, accept $\dfrac{147}{800}$
		2	

Q	Answer	Marks	Comments
8(d)	Radius of circle traversed by Spacecraft *C* $r = 6000 \times 10^3 \times \cos(45^\circ)$ $r = 4.242[640687] \times 10^6\ [\mathrm{m}]$	B1	**PI** by correct answer
	$F = m\omega^2 r$ $= 185 \times \left(\dfrac{2\pi}{10 \times 60 \times 60}\right)^2 \times 4.242[...] \times 10^6$	M1	Use of $r = 6000$ km is **M0**
	$F = 24\ [\mathrm{N}]$	A1	**AWRT** 24 [N]
		3	

	Question 8 Total	**9**	