OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM05) Unit FM2 Mechanics

Friday 24 January 2020 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables (enclosed).
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to two significant figures, unless stated otherwise.
- Unless stated otherwise, the acceleration due to gravity, g, should be taken as 9.8 m s^{-2}

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Answer **all** questions in the spaces provided.

Do not write outside the box

1 A spring has stiffness 14.7 N m^{-1} and natural length 20 cm

One end of the spring is attached to a fixed point *O*.

A particle of mass 0.6 kg is attached to the other end of the spring.

Find the length of the spring when the particle is in equilibrium directly below *O*.

[3 marks]

Answer _____

3

Turn over for the next question

Do not write outside the box

2 A particle moves with simple harmonic motion between two end points, *A* and *B*, that are 3 metres apart.

The particle takes 2 seconds to move directly from *A* to *B*.

2 (a) Find the maximum speed of the particle.

[3 marks]

Answer _____

2 (b) Find the speed of the particle when it is at the point *C,* which is 1 metre from *A*.

[3 marks]

Answer _____

Do not write outside the box

2 (c) Find the time taken for the particle to move directly from A to C.

[3 marks]

Answer _____

9

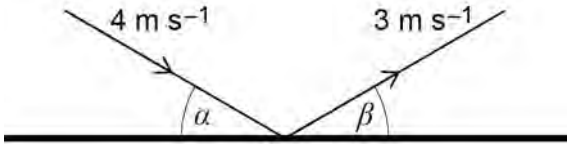
Turn over for the next question

Do not write outside the box

3 A disc, of mass 0.2 kg, moving on a smooth horizontal surface hits a smooth vertical wall.

When it hits the wall, the disc is moving at 4 m s^{-1} at an angle α to the wall.

The disc rebounds with a speed of 3 m s^{-1} at an angle β to the wall.



The coefficient of restitution between the disc and the wall is e.

3 (a) Show that

$$\tan\alpha = \frac{\tan\beta}{e}$$

[4 marks]

Do not write outside the box

3 (b) It is given that $\alpha = 2\beta$, where $0° < \beta < 45°$

3 (b) (i) Find $\tan \beta$ in terms of e.

[3 marks]

Answer _____

3 (b) (ii) Find the set of possible values of e.

[1 mark]

Answer _____

8

Do not write outside the box

4 A particle, of mass 2 kg, slides in a straight line on a horizontal surface.

Initially, at point *A*, the particle has speed 12 m s^{-1}

At time t seconds after leaving *A*, the speed of the particle is v m s^{-1}

The coefficient of friction between the particle and the surface is 0.2

Air resistance also acts on the particle with a magnitude of $4v$ newtons.

The particle comes to rest at point *B*.

4 (a) Show that, as the particle moves between *A* and *B*, the speed of the particle is given by

$$v = 12.98e^{-2t} - 0.98$$

[6 marks]

Do not write outside the box

4 (b) Show that the distance *AB* is 4.73 metres, correct to three significant figures.

[6 marks]

12

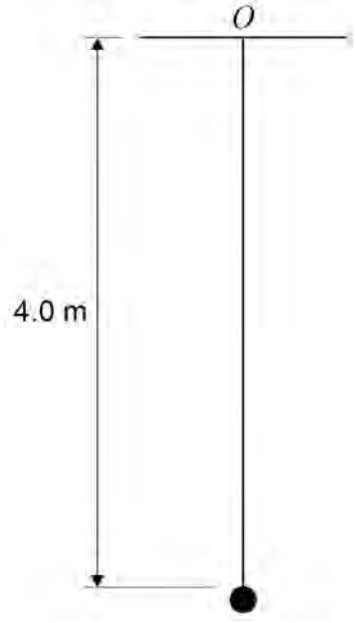
Do not write outside the box

5 A light elastic string has natural length 2.5 metres and modulus of elasticity 10 newtons.

One end of the string is attached to a fixed point, *O*. The other end of the string is attached to a small sphere of mass 0.4 kg

The sphere is held at a point 4 metres below *O* and then released from rest.

The string remains taut in the subsequent motion.



5 (a) Find the minimum distance between *O* and the sphere, giving your answer to three significant figures.

[5 marks]

Do not write outside the box

Answer _____

5 (b) Find the extension of the string when the speed of the sphere is a maximum.

[3 marks]

Answer _____

Question 5 continues on the next page

Do not write outside the box

5 (c) Show that the motion of the sphere is simple harmonic motion.

[5 marks]

5 (d) Find the period of the motion.

[2 marks]

Answer _____

15

Do not write outside the box

Turn over for the next question

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

6 A light inextensible string has length 80 cm

One end of the string is attached to a fixed point, *O*.

A particle, of mass 0.2 kg, is attached to the other end of the string.

Initially the particle is at rest directly below *O*.

The particle is then subject to a horizontal impulse so that it starts to move with speed u m s^{-1}

6 (a) In one case the particle completes vertical circles with centre *O*.

Show that the minimum value of u is 6.3 , correct to two significant figures.

[4 marks]

Do not write outside the box

6 (b) In a **different** case the string becomes slack when the string makes an angle of 30° with the upward vertical.

Find u.

[6 marks]

10

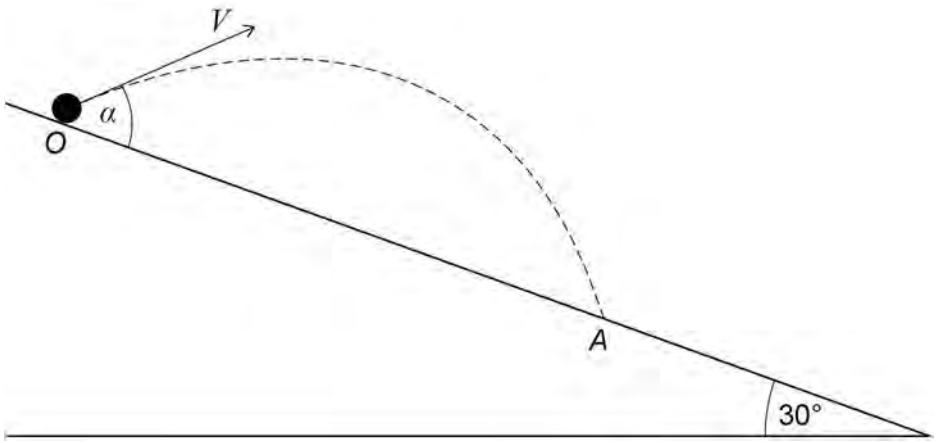
Turn over for the next question

Do not write outside the box

7 A plane is inclined at an angle of 30° to the horizontal.

A ball is projected from the point *O* on the plane and hits the plane again at the point *A*, which is further down the plane than *O*. *OA* is a line of greatest slope of the plane.

The initial velocity of the ball is V m s^{-1} at an angle α above the plane.



7 (a) Show that the distance *OA* is given by

$$\frac{2V^2}{g\cos^2 30°}\sin\alpha\cos(\alpha - 30°)$$

[7 marks]

Do not write outside the box

7 (b) Find the value of α for which the distance OA is a maximum.

[3 marks]

Answer _____

10

Do not write outside the box

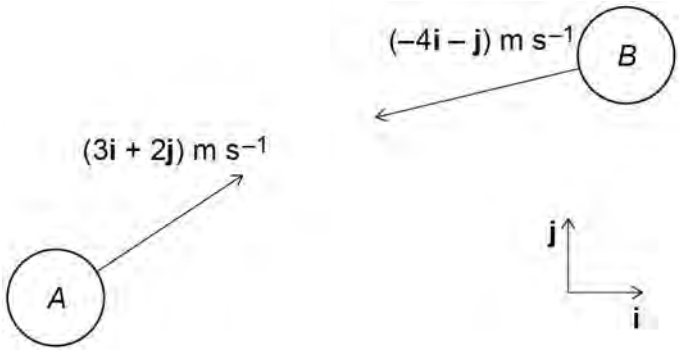
8 Two smooth spheres, *A* and *B*, are moving on a smooth horizontal surface when they collide.

The two spheres have the same radius.

The mass of *A* is 2 kg and the mass of *B* is 4 kg

Before the collision the velocity of *A* is $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1}

Before the collision the velocity of *B* is $(-4\mathbf{i} - \mathbf{j})$ m s^{-1}



After the collision the velocity of *A* is $(-1.5\mathbf{i} - \mathbf{j})$ m s^{-1}

8 (a) Find the velocity of *B* after the collision.

[3 marks]

Answer _____

Do not write outside the box

8 (b) Find the magnitude of the impulse on A during the collision.

[3 marks]

Answer _____

8 (c) Find the coefficient of restitution between the spheres, giving your answer as a fraction.

[7 marks]

Do not write outside the box

Answer _____

13

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**