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FM04

(9665/FM04) Unit FS2 Statistics

Mark scheme

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2 2 6 X F M 0 4 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	H_0: $\sigma = 1$ H_1: $\sigma > 1$	**B1**	Both hypotheses, **oe**
	$v = 100$	**B1**	**PI** by correct critical value
	$\dfrac{(n-1)s^2}{\sigma^2} = \dfrac{(101-1)\times 1.2^2}{1^2}$	**M1**	**PI** Condone one slip
	$= 144$	**A1**	
	$X^2_{100}(0.99) = 135.807$	**B1**	Finds correct critical value or correct probability, 0.0026
	144 > 135.807	**A1ft**	Correctly compares their X^2 test statistic and their critical value or their probability and 0.01
	Reject H_0. Evidence to suggest that the standard deviation of the diameters of metal discs produced by the machine has increased	**E1ft**	States that null hypothesis is rejected and gives a conclusion in context Conclusion must not be definite ft their comparison using a X^2 model
	Total	**7**	

Q	Answer	Marks	Comments
2(a)	$z = 1.8808$	B1	**AWRT** 1.88
	$1.8808 \times \frac{1.6}{\sqrt{n}} = 0.2$	M1	Set up an equation with their $z \times \frac{1.6}{\sqrt{n}}$ **oe**
	$n = 226$	A1	226 or 227
		3	
2(b)	$\overline{x} = 10.18$	B1	**AWRT**
	10.18 ± 0.2	M1	Follow through their $\overline{x}$
	(9.98, 10.38)	A1	**AWRT**
		3	
	Total	**6**	

Q	Answer	Marks	Comments
3	H_0: There is not an association between gender and survey response H_1: There is an association between gender and survey response	B1	Both hypotheses, variables must be stated in at least the null hypothesis **oe**
	<table><tr><td>Expected</td><td>S</td><td>N</td></tr><tr><td>M</td><td>24</td><td>16</td></tr><tr><td>F</td><td>24</td><td>16</td></tr></table>	M1 A1	At least two expected values correct All correct
	$\sum \frac{(\lvert O-E\rvert-0.5)^2}{E} = \frac{(\lvert 21-24\rvert-0.5)^2}{24} + \frac{(\lvert 27-24\rvert-0.5)^2}{24} + \frac{(\lvert 19-16\rvert-0.5)^2}{16} + \frac{(\lvert 13-16\rvert-0.5)^2}{16}$	M1	Attempts to calculate test statistic Condone use of $\sum \frac{(O-E)^2}{E}$
	= 1.30	A1	**AWRT** 1.3
	$v = 1$	B1	**PI** by correct critical value
	Critical value = 2.71	B1	**AWRT** 2.7
	1.30 < 2.71	A1ft	Correctly compares their X^2 test statistic and their critical value
	Accept H_0. Evidence to suggest that there is not an association between gender and survey response	E1ft	States that null hypothesis is not rejected and gives a conclusion in context Conclusion must not be definite ft their comparison using a X^2 model
	Total	**9**	

Q	Answer	Marks	Comments
4(a)(i)	$P(X \le 1) = 0.0266$ [< 0.03] $P(X \le 2) = 0.0884$ [> 0.03]	**M1**	Both seen
	$P(X \ge 11) = 0.0253$ [< 0.03] $P(X \ge 10) = 0.0538$ [> 0.03]	**M1**	Both seen or $P(X \le 10) = 0.9747$ and $P(X \le 9) = 0.9462$
	Accept if $2 \le X \le 10$	**A1**	**oe, PI**
	$X \sim \text{Po}(5)$	**M1**	Use of
	Type II probability = $P(2 \le X \le 10)$ $= P(X \le 10) - P(X \le 2)$ $= 0.9863 - 0.0404$	**M1**	Follow through their region
	$= 0.946$	**A1**	**AWRT**
		6	
4(a)(ii)	Power = $1 - 0.946$	**M1**	Applies power formula with their Type II probability
	$= 0.054$	**A1ft**	**AWRT** **ft** their probability of Type II error
		2	
4(b)	$X \le 1$ or $X \ge 11$	**B1ft**	**ft** their acceptance region in part **(a)**
		1	
	Total	**9**	

Q	Answer	Marks	Comments
5	H_0: $\mu_{2008} = \mu_{2018}$ H_1: $\mu_{2008} \neq \mu_{2018}$	B1	Both hypotheses
	<table><tr><th>Household</th><th colspan="3">Difference</th></tr><tr><td>A</td><td>10</td><td rowspan="5">or</td><td>–10</td></tr><tr><td>B</td><td>3</td><td>–3</td></tr><tr><td>C</td><td>0</td><td>0</td></tr><tr><td>D</td><td>–2</td><td>2</td></tr><tr><td>E</td><td>0</td><td>0</td></tr></table>	B1	All differences
	$\bar{x} = \frac{11}{5}$ or $-\frac{11}{5}$	B1	Sight of 2.2 or –2.2 Must be consistent with their differences
	$s^2 = \frac{1}{5-1}\left(113 - \frac{11^2}{5}\right)$	M1	Attempt at variance formula Allow one slip **PI**
	$= 22.2$	A1	Accept $s =$ **AWRT** 4.71
	$t = \frac{2.2}{\sqrt{\frac{22.2}{5}}}$ or $\frac{-2.2}{\sqrt{\frac{22.2}{5}}}$	M1	Using their mean and variance Condone use of $\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{S_p^2\left(\frac{1}{n_x}+\frac{1}{n_y}\right)}}$
	$= 1.04$ or -1.04	A1	**AWRT** Must be consistent with their differences
	$v = 4$	B1	**PI** by correct critical value
	t_4 critical value $= 2.13$ or -2.13	B1	**AWRT** Must be consistent with their differences unless changes -1.04 to 1.04 first
	1.04 < 2.13	A1ft	Correctly compares their t test statistic and their critical value
	Accept H_0. Evidence to suggest that average weekly household income has not changed between 2008 and 2018	E1ft	States that null hypothesis is not rejected and gives a conclusion in context Conclusion must not be definite ft their comparison using either a t or z test statistic
	Total	**11**	

Q	Answer	Marks	Comments
6	H$_0$: $\mu_X = \mu_Y$ H$_1$: $\mu_X > \mu_Y$	B1	Both hypotheses
	$\bar{x} = \frac{65760}{400}$	B1	Implied by sight of 164.4
	$\bar{y} = \frac{49197}{300}$	B1	Implied by sight of 163.99
	$s_X^2 = \frac{1}{400-1}\left(10814020 - \frac{65760^2}{400}\right)$ or $s_Y^2 = \frac{1}{300-1}\left(8070022 - \frac{49197^2}{300}\right)$	M1	Attempt at one of the variance formulae Allow one slip Implied by one correct answer
	$s_X^2 = 7.71$	A1	**AWRT**
	$s_Y^2 = 7.38$	A1	**AWRT**
	$z = \frac{164.4 - 163.99}{\sqrt{\frac{7.7...}{400} + \frac{7.3...}{300}}}$	M1	Correct numerator for their means, subtracted either way round
		M1	Correct denominator for their variances
	$= 1.96$	A1	**AWRT**
	z critical value $= 1.64$	B1	Allow 1.645 **AWRT**
	1.96 > 1.64	A1ft	Correctly compares their z or t test statistic and critical value
	Reject H$_0$. Evidence to suggest that the average height of women who play basketball is larger than the average height of women who don't play basketball	E1ft	States that null hypothesis is rejected and gives a conclusion in context Conclusion must not be definite **ft** their comparison using z or t test statistic
	Total	**12**	

Q	Answer	Marks	Comments
7(a)	$\mathrm{E}(R) = \frac{1}{n}\mathrm{E}(B_1) = \frac{np}{n}$	M1	Finds $\mathrm{E}(R)$ in terms of n and p
	$= p$ therefore unbiased	A1	Must see p and conclusion n must not be assigned a value
		2	
7(b)	$\mathrm{Var}(T) = \mathrm{Var}\left(\frac{\sum_{i=1}^{k} B_i}{kn}\right) = \frac{knp(1-p)}{k^2n^2}$	M1	Finds $\mathrm{Var}(T)$ in terms of k,n and p, condoning slips May be seen in part **(c)** if not attempted in this part
	$= \frac{p(1-p)}{kn}$ or $\frac{kp(1-p)}{k^2n}$	A1	Correct simplification May be seen in part **(c)** if not attempted in this part
	As $n \to \infty$, $\mathrm{Var}(T) \to 0$	M1	Applies limiting process to their $\mathrm{Var}(T)$
	Therefore consistent estimator	A1	**CSO**
		4	
7(c)	$\mathrm{Var}(R) = Var\left(\frac{B_1}{n}\right) = \frac{np(1-p)}{n^2}$	M1	Finds $\mathrm{Var}(R)$ in terms of n and p, condoning slips **PI** by correct simplified Var(R) or relative efficiency
	$= \frac{p(1-p)}{n}$	A1	Correct simplification **PI** by correct relative efficiency
	Relative Efficiency $= \frac{\frac{1}{Var(T)}}{\frac{1}{Var(R)}} = \frac{\frac{kn}{p(1-p)}}{\frac{n}{p(1-p)}}$	M1	Applies formula with either the correct Var(T) or their Var(T) from part **(b)** and their Var(R)
	$= k$	A1	Correct simplification
	Estimator T is more efficient than estimator R	E1	**CSO**
		5	
	Total	**11**	

Q	Answer	Marks	Comments
8(a)	$\mathrm{M}_{X_i}(t) = \mathrm{E}\left(e^{tX_i}\right) = \int_0^\infty e^{tx}\lambda e^{-\lambda x}\ dx$	M1	Applies mgf formula with any or missing limits Condone spurious notation
	$= \left[\dfrac{\lambda e^{(t-\lambda)x}}{t-\lambda}\right]_0^\infty$	M1	Integrates to $A\mathrm{e}^{(t-\lambda)x}$
	$= -\dfrac{\lambda}{t-\lambda}$	A1	Applies limits to obtain a correct form **oe**
	$= \left(\dfrac{\lambda - t}{\lambda}\right)^{-1}$ $= \left(1-\dfrac{t}{\lambda}\right)^{-1}$	A1	Reaches correct form with no errors and at least one intermediate line Condone interchangeable x and x_i **AG CSO**
		4	
8(b)	$\mathrm{M}_X(t) = \left(1-\dfrac{t}{\lambda}\right)^{-n}$	B1	
		M1	Differentiates to $A\left(1-\dfrac{t}{\lambda}\right)^{-n-1}$
	$\mathrm{M}_X'(t) = \dfrac{n}{\lambda}\left(1-\dfrac{t}{\lambda}\right)^{-n-1}$	A1	$\dfrac{n}{\lambda}\left(1-\dfrac{t}{\lambda}\right)^{-n-1}$ **oe**
		M1	Differentiates to $A\left(1-\dfrac{t}{\lambda}\right)^{-n-2}$
	$\mathrm{M}_X''(t) = \dfrac{n(n+1)}{\lambda^2}\left(1-\dfrac{t}{\lambda}\right)^{-n-2}$	A1	$\dfrac{n(n+1)}{\lambda^2}\left(1-\dfrac{t}{\lambda}\right)^{-n-2}$ **oe**
	$\mathrm{Var}(X) = \mathrm{M}_X''(0) - \left(\mathrm{M}_X'(0)\right)^2$ $= \dfrac{n(n+1)}{\lambda^2}\left(1-\dfrac{0}{\lambda}\right)^{-n-2} - \left(\dfrac{n}{\lambda}\left(1-\dfrac{0}{\lambda}\right)^{-n-1}\right)^2$	M1	Applies variance formula with their $\mathrm{M}_X''(0)$ and $\mathrm{M}_X'(0)$
	$= \dfrac{n}{\lambda^2}$	A1	**CAO**
		7	

Q	Answer	Marks	Comments
8(b) ALT	$\mathrm{M}'_{X_i}(t) = \frac{1}{\lambda}\left(1-\frac{t}{\lambda}\right)^{-2}$	B1	Condone X for X_i **oe**
		M1	Differentiates to $A\left(1-\frac{t}{\lambda}\right)^{-3}$ Condone X for X_i
	$\mathrm{M}''_{X_i}(t) = \frac{2}{\lambda^2}\left(1-\frac{t}{\lambda}\right)^{-3}$	A1	$\frac{2}{\lambda^2}\left(1-\frac{t}{\lambda}\right)^{-3}$ **oe** Condone X for X_i
	$\mathrm{M}'_{X_i}(0) = \frac{1}{\lambda}$ and $\mathrm{M}''_{X_i}(t) = \frac{2}{\lambda^2}$	M1	X_i must be seen and used
	$\mathrm{Var}(X_i) = \frac{1}{\lambda^2}$	A1	Must be $\mathrm{Var}(X_i) = \frac{1}{\lambda^2}$ not $\mathrm{Var}(X) = \frac{1}{\lambda^2}$
	$\mathrm{Var}(X) = n\mathrm{Var}(X_i) = n \times \frac{1}{\lambda^2}$	M1	Multiplies their $\mathrm{Var}(X_i)$ by n
	$= \frac{n}{\lambda^2}$	A1	**CAO**
		7	
8(c)(i)	$\mathrm{E}(Y) = 2\lambda\mathrm{E}(X) = 2\lambda \times \frac{n}{\lambda}$	M1	2λ × their $\mathrm{E}(X)$
	$= 2n$	A1	**CAO**
		2	
8(c)(ii)	$\mathrm{Var}(Y) = 4\lambda^2\,\mathrm{Var}(X) = 4\lambda^2 \times \frac{n}{\lambda^2}$	M1	$4\lambda^2$ × their $\mathrm{Var}(X)$
	$= 4n$	A1	**CAO**
		2	
	Total	**15**	