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MA05

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Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$[\mathbf{v}=]\begin{bmatrix} 6t \\ -4\sin(4t) \\ 2te^{t^2} \end{bmatrix}$	M1 A1	**M1**: At least one component correct **PI** **A1**: All components correct **PI**
	$[\mathbf{a}=]\begin{bmatrix} 6 \\ -16\cos(4t) \\ 2e^{t^2}+4t^2e^{t^2} \end{bmatrix}$	M1 A1	**M1**: At least two components correct **A1**: All components correct
	$[\mathbf{F}=]\begin{bmatrix} 3 \\ -8\cos(4t) \\ e^{t^2}+2t^2e^{t^2} \end{bmatrix}$	A1ft	**oe** **ft** their acceleration
		5	

	Question 1 Total	**5**	

Q	Answer	Marks	Comments
2(a)	Clockwise moments about A $= 60\times 9.8\times 4.00 + 25\times 9.8\times 2.00$ $= 2842\ [\text{N m}]$ Anticlockwise moments about A [due to normal reaction force at B, R_B] $= 1.25R_B$ Equilibrium $2842 = 1.25R_B$ $N_B = 2300\ [\text{N}]$	**M1** **M1** **A1**	At least one term correct **CAO, AWRT** 2300 N Exact answer is 2273.6 N
		3	

Q	Answer	Marks	Comments
2(b)	Equilibrium [where R_A is the normal reaction force at A] $60\times 9.8 + 25\times 9.8 + R_A = 2273.6$ $R_A = 1400\ [\text{N}]$ [Direction = Vertically] Downwards	 **M1** **A1ft** **B1**	 Note $60\times 9.8 + 25\times 9.8 = 833$ **AWRT** 1400 N Exact answer is 1440.6 N Allow 1500 N (1467 N) if 2300 N is used **ft** their **(a)** provided moments taken in **part (a)**
		3	

	Question 2 Total	**6**	

Q	Answer	Marks	Comments
3(a)	Change in GPE of cart $=16\times 9.8\times(4.0-2.5)$ $=235.2\,[\text{J}]$ Work done against resistance force $235.2=F\times 48$ $F=4.9\ [\text{N}]$	 **M1** **A1** **M1** **A1**	 Correct change in GPE, **PI** **ft** their change in GPE **CAO** **SC2** for a correct answer from a method that assumes equivalence with constant acceleration
		4	

Q	Answer	Marks	Comments
3(b)	Work done against resistance force $=4.9\times 30$ $=147\,[\text{J}]$ Kinetic energy of cart at *Y* $=16\times 9.8\times 4.0-147$ $=480.2\,[\text{J}]$ Speed of cart at *Y* $\left[v=\sqrt{\frac{2KE}{m}}=\right]\ \sqrt{\frac{2\times 480.2}{16}}$ $[v=]7.7\,[\text{m s}^{-1}]$	 **B1ft** **M1** **A1**	 **ft** their answer to **part (a)** **ft** their work done against friction **oe**, e.g. 7.74758… [m s^{-1}] Exact answer is $\frac{49\sqrt{10}}{20}$ [m s^{-1}]
		3	

	Question 3 Total	**7**	

Q	Answer	Marks	Comments
4(a)	30 [cm]	**B1**	
	The line joining the midpoint of OC and the midpoint of AB is a line of symmetry	**E1**	Allow any mention of symmetry or that the centres of mass of the square and rectangle lie on the line connecting the midpoints of *OC* and *AB*
		2	

Q	Answer	Marks	Comments
4(b)	**Area / cm²** | **COM from *OC* / cm** $4200\ [=60\times70]$ | 35 $256\ [=16\times16]$ | 56	**B1**	Use or sight of all four values
	$4200\sigma\times35-256\sigma\times56=3944\sigma\overline{Y}$	**M1**	At least two terms correct Condone omission of σ
	$\left[\overline{Y}=\right]\ 33.6\,[\text{cm}]$	**A1**	**CAO** to 3 sf
		3	

Q	Answer	Marks	Comments
4(c)	Angle *OBA* $\tan^{-1}\left(\frac{35}{30}\right)=49.39...^\circ$	**B1**	or Angle *OBC* $\tan^{-1}\left(\frac{30}{35}\right)=40.60...^\circ$
	Angle *MBA* $\tan^{-1}\left(\frac{70-33.636...}{30}\right)=50.47...^\circ$	**B1**	or Angle *MBC* $\tan^{-1}\left(\frac{30}{70-33.636...}\right)=39.52...^\circ$
	$50.47...^\circ-49.39...^\circ$	**M1**	or $40.60...^\circ-39.52...^\circ$
	$=1.1^\circ$	**A1**	**CAO**
		4	

	Question 4 Total	**9**	

Q	Answer	Marks	Comments
5(a)(i)	$\left[ma = mg\sin\left(25^\circ\right) = 24.849...\,\text{N}\right]$ $a = \left[9.8\sin\left(25^\circ\right) =\right]$ 4.1 $\left[\text{m s}^{-2}\right]$	**B1**	Answer to 3 sf is 4.14 [m s^{-2}]
		1	

Q	Answer	Marks	Comments
5(a)(ii)	$v^2 = u^2 + 2as$ $v^2 = 0^2 + 2\times 4.141...\times 10$ Speed of particle at Y $v^2 = 82.833...$ $v = 9.101...\left[\text{m s}^{-1}\right]$	**M1** **A1**	$\frac{1}{2}mv^2 = mg\Delta h \quad \Rightarrow \quad v^2 = 2g\Delta h$ or $v^2 = 2\times 9.8\times\left(10\times\sin\left(25^\circ\right)\right)$ **AG** Must be convincingly shown Value of speed shown to more than 2 sf or correct unsimplified equation for v leading to the given result
		2	

Q	Answer	Marks	Comments
5(a)(iii)	Speed at Y would be greater Component of weight down the slope would increase [therefore acceleration increases]	**B1** **E1**	 Allow explanations based upon conversion of GPE to KE
		2	

Q	Answer	Marks	Comments
5(b)	Friction acting on particle between Y and Z $= \mu mg\cos\left(25^\circ\right)$ $= 0.5\times 6\times 9.8\times\cos\left(25^\circ\right)$ $= 26.645...\,\text{N}$	**M1** **A1**	Attempt to find friction acting on particle. Correct value for friction to at least 2 sf, **PI**
	Resultant force acting on particle between Y and Z $24.849... - 26.645... = -1.795...\,[\text{N}]$	**m1**	**ft** their friction, **PI**
	Acceleration of particle between Y and Z $a = \dfrac{-1.795...}{6} = -0.299...\,\text{m s}^{-2}$	**A1ft**	**ft** their resultant force
	Distance YZ $v^2 = u^2 + 2as \;\Rightarrow\; s = \dfrac{v^2-u^2}{2a}$ $s = \dfrac{0^2 - 9.1^2}{2\times -0.299...} = 138.362...\,[\text{m}]$	**M1**	Uses their acceleration and the initial velocity of 9.1 m s^{-1} to find their distance YZ
	Distance XZ $[138.362... + 10 =]\;\; 150\,[\text{m}]\;\; (2\,\text{sf})$	**A1**	**AWRT** 150 [m] Answer to 3 sf is 148 m
		6	

Q	Answer	Marks	Comments
5(c)	Resultant force on rough part of slope must not be negative $mg\sin\alpha - \mu mg\cos\alpha \geq 0$ $\tan\alpha \geq \mu$ $\alpha \geq \tan^{-1}(0.5)$ $\alpha \geq 26.565...$	**M1**	**PI**
	$\alpha = 26.6\;\; (1\text{dp})$	**A1**	**CAO**
		2	

	Question 5 Total	**13**	

Q	Answer	Marks	Comments
6(a)	$[\mathbf{v} =] \left(-\frac{1}{2}\mathrm{e}^{-2t} + c_1\right)\mathbf{i} + \left(-\frac{1}{1+t} + c_2\right)\mathbf{j}$	M1 A1	**M1**: At least one term correct **A1**: Both components correct Condone omission of constants of integration for **M1 A1**
	$[\mathbf{v} =] \frac{1}{2}\left(1-\mathrm{e}^{-2t}\right)\mathbf{i} + \left(1-\frac{1}{1+t}\right)\mathbf{j}$	m1 A1	**m1**: At least one component correct **A1**: Both components correct
		4	

Q	Answer	Marks	Comments
6(b)	$[\mathbf{r} =] \left(\frac{1}{4}\mathrm{e}^{-2t} + \frac{1}{2}t + c_3\right)\mathbf{i} + \left(t - \ln(1+t) + c_4\right)\mathbf{j}$	M1 A1	**M1**: At least correct exponential term in **i** component or correct logarithm term in **j** component **A1**: Both components fully correct Condone omission of constants of integration for **M1 A1**
	$[\mathbf{r} =] \left(\frac{1}{4}\mathrm{e}^{-2t} + \frac{1}{2}t + \frac{11}{4}\right)\mathbf{i} + \left(t - \ln(1+t) + 2\right)\mathbf{j}$	B1	Correct position vector
	[When $t = 5$] $\left[\lvert\mathbf{r}\rvert =\right] \sqrt{\left(\frac{1}{4}\mathrm{e}^{-10} + \frac{5}{2} + \frac{11}{4}\right)^2 + \left(5 - \ln(6) + 2\right)^2}$	M1	Evaluates their distance at $t = 5$
	$\left[\lvert\mathbf{r}\rvert =\right]$ 7.4 [m]	A1	Answer to 3 sf is 7.40
		5	

	Question 6 Total	**9**	

Q	Answer	Marks	Comments
7	Time to cover the 12 m to P $T = \dfrac{12}{15\cos\alpha}$ or $T = \dfrac{x}{v\cos\alpha}$	B1	
	Vertical displacement $s = 15\sin\alpha \times T - 0.5\times 9.8\times T^2$ or $s = v\sin\alpha \times T - 0.5\times g\times T^2$	M1 A1	**M1**: Use of $s = ut + \frac{1}{2}at^2$ with $u = 15\sin\alpha$ and $a = \pm 9.8$ **PI** **A1**: Correct result for s in terms of T **PI**
	$s = 15\sin\alpha \times \dfrac{12}{15\cos\alpha} - 0.5\times 9.8\times\left(\dfrac{12}{15\cos\alpha}\right)^2$	m1	Eliminates T **PI**
	$5 = 15\sin\alpha \times \dfrac{12}{15\cos\alpha} - 0.5\times 9.8\times\left(\dfrac{12}{15\cos\alpha}\right)^2$	m1	Use of $s = 5$ **PI** Dependent on **M1**
	$3.136\tan^2\alpha - 12\tan\alpha + 8.136 = 0$	m1	Forms quadratic equation in $\tan\alpha$ **PI** Note $3.136 = \dfrac{392}{125}$ and $8.136 = \dfrac{1017}{125}$
	$\alpha = \left[41.370...^\circ,\right]\ 71.249...^\circ$	A1	**PI** Solves equation to find the required value of α
	$v^2 = u^2 + 2as \quad \Rightarrow \quad s = \dfrac{v^2 - u^2}{2a}$ $s = \dfrac{0^2 - \left(15\sin\left(71.249...^\circ\right)\right)^2}{2\times -9.8}$	M1	Uses their (larger) angle [< 90°] with $v^2 = u^2 + 2as$
	$[s =]\ 10.3\,[\text{m}]$	A1	**CAO**
		9	

	Question 7 Total	9	

Q	Answer	Marks	Comments
8(a)	$\omega = \frac{2\pi}{T}$ or $v = \frac{2\pi r}{T}$	B1	May be seen in elimination of ω or v
	$\frac{GMm}{r^2} = m\omega^2 r$ or $\frac{GMm}{r^2} = \frac{mv^2}{r}$	M1	Relates given force to resultant force
	$\frac{GM}{r^3} = \left(\frac{2\pi}{T}\right)^2$ or $\frac{GM}{r} = \left(\frac{2\pi r}{T}\right)^2$	m1	Eliminates ω or v
	$\frac{GM}{r^3} = \frac{4\pi^2}{T^2}$ or $\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$		
	$T^2 = \frac{4\pi^2 r^3}{GM}$	A1	**AG** Must be convincingly shown Must see correct expansion of bracket
		4	

Q	Answer	Marks	Comments
8(b)(i)	$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2 \times \left(4.2\times10^7\right)^3}{6.7\times10^{-11}\times6.0\times10^{24}}}$		
	$T = 85298.3...\,[\text{s}]$	B1	**PI** by correct answer Value is 85000 [s] to 2 sf
	$\omega = \frac{2\pi}{T} = \frac{2\pi}{85298.3...}$	M1	**ft** their T
	$[\omega =]\ 7.4\times10^{-5}\ \left[\text{rad s}^{-1}\right]$	A1ft	**ft** their T
		3	

Q	Answer	Marks	Comments
8(b)(ii)	$v = r\omega = 4.2\times10^7 \times 7.366...\times10^{-5}$ $v = 3093.7...\left[\text{m s}^{-1}\right]$	B1ft	**AWRT** 3100 **ft** their ω from **part (b)(i)**
	$[\mathbf{v} =]\left(-3100\cos\left(7.4\times10^{-5}t\right)\mathbf{i}\right.$ $\left.-3100\sin\left(7.4\times10^{-5}t\right)\mathbf{j}\right)\ \left[\text{m s}^{-1}\right]$	M1 A1	**M1**: **ft** their ω with use of sine and cosine **A1**: Both components fully correct
		3	

	Question 8 Total	**10**	

Q	Answer	Marks	Comments
9(a)	Particle is in equilibrium as the particle is not accelerating [so there is no resultant force acting on the particle] and there is no resultant moment on a particle	**B1** **E1** **E1**	Condone 'Yes' for **B1**
		3	

Q	Answer	Marks	Comments
9(b)	Forces parallel to slope $T\cos\left(5^\circ\right) = mg\sin\left(35^\circ\right) + \mu N$ Forces perpendicular to slope $T\sin\left(5^\circ\right) + N = mg\cos\left(35^\circ\right)$	**M1 A1**	**M1**: At least two terms in an equation correct **A1**: Both equations fully correct
	$T\cos\left(5^\circ\right) = mg\sin\left(35^\circ\right) + \mu\left(mg\cos\left(35^\circ\right) - T\sin\left(5^\circ\right)\right)$	**M1 A1**	**M1**: Eliminating normal reaction using their two equations **A1**: Correct elimination of normal reaction
	$T = \dfrac{mg\left(\sin\left(35^\circ\right) + \mu\cos\left(35^\circ\right)\right)}{\cos\left(5^\circ\right) + \mu\sin\left(5^\circ\right)}$	**A1**	**oe**, e.g. $T = 112.398...$
	$T = \dfrac{15\times 9.8\times\left(\sin\left(35^\circ\right) + 0.25\cos\left(35^\circ\right)\right)}{\cos\left(5^\circ\right) + 0.25\sin\left(5^\circ\right)}$ $T = 112$	**A1**	**CAO** to 3 sf
		6	

Q	Answer	Marks	Comments
9(c)(i)	$W = 112.398... \cos\left(5^\circ\right) \times 25$ $W = 2800\,[\text{J}]$	 **B1**	 **AG** Must be convincingly shown 2810 J or 112 × 25 scores **B0**
		1	

Q	Answer	Marks	Comments
9(c)(ii)	Time taken to move 25 m up the slope $\left[= \frac{25}{4}\right] = 6.25\,\text{s}$ $\left[\text{Power} = \frac{2800}{6.25} =\right]\ 450\left[\text{J}\,\text{s}^{-1}\right]$	**M1** **A1ft**	or use of $P = Fv$ with their tension from **part (b)** **AWRT** to 450 [J s^{-1}]
		2	

	Question 9 Total	**12**	