

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

I declare this is my own work.

INTERNATIONAL AS FURTHER MATHEMATICS

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Thursday 16 January 2020 07:00 GMT Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables (enclosed).
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
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8	
9	
10	
11	
12	
TOTAL	



J A N 2 0 F M 0 2 0 1

Answer **all** questions in the spaces provided.

$$\frac{dy}{dx} = \frac{\sqrt{x+y}}{y\sqrt{y}}$$

[5 marks]

[illegible]

Answer _____



[2 marks]

Answer _____

$$\mathbf{BM} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 4 \end{bmatrix}$$

[5 marks]

Answer

- 3** The variables x and y are related by an equation of the form

$$y^3 = ax^2 + b$$

where a and b are constants.

The following experimental values of x and y are recorded.

x	2	4	6	8	10
y	8.6	8.1	7.4	5.8	-4.2

- 3 (a)** Complete the table below for $X = x^2$ and $Y = y^3$

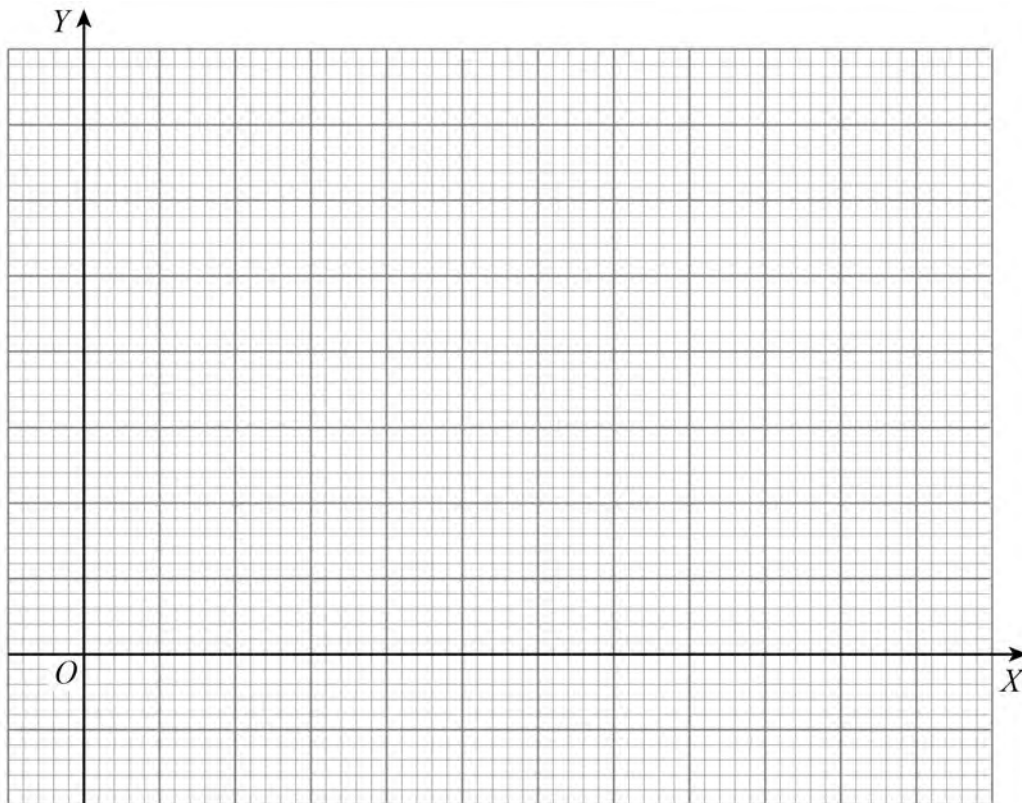
X	4	16	36	64	100
Y	636				

[1 mark]

- 3 (b)** Plot the values of Y against X on the grid below.

Draw a line of best fit.

[2 marks]



3 (c) Use your graph to estimate the value of x when $y = 0$

[2 marks]

Answer _____

3 (d) Use your graph to estimate the value of a and the value of b .

[3 marks]

$a =$ _____

$b =$ _____

8

Turn over ►



4 The function f is defined by $f(x) = x^3 - 2x^2 - 3x + 1$

The equation $f(x) = 0$ has one negative root, α .

4 (a) Show that α lies in the interval $-1.5 < \alpha < -1$

[2 marks]

4 (b) Taking $x_1 = -1.5$ as a first approximation to α , use the Newton–Raphson method to find a second approximation, x_2 , to α .

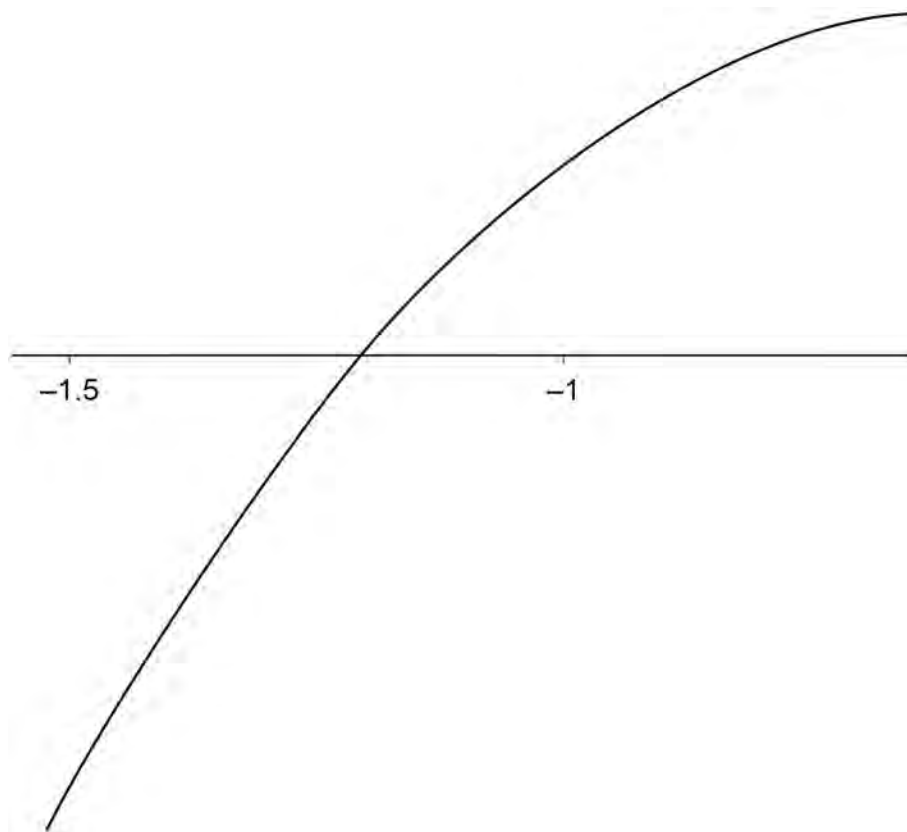
Give your answer to three decimal places.

[4 marks]

Answer _____



- 4 (c) The diagram shows part of the graph of $y = f(x)$



- 4 (c) (i) Draw a line on the diagram above which illustrates the Newton–Raphson method as used in part (b).

[1 mark]

- 4 (c) (ii) The point P is $(x_2, 0)$

The point Q is $(\alpha, 0)$

Label the points P and Q on the diagram above.

[2 marks]



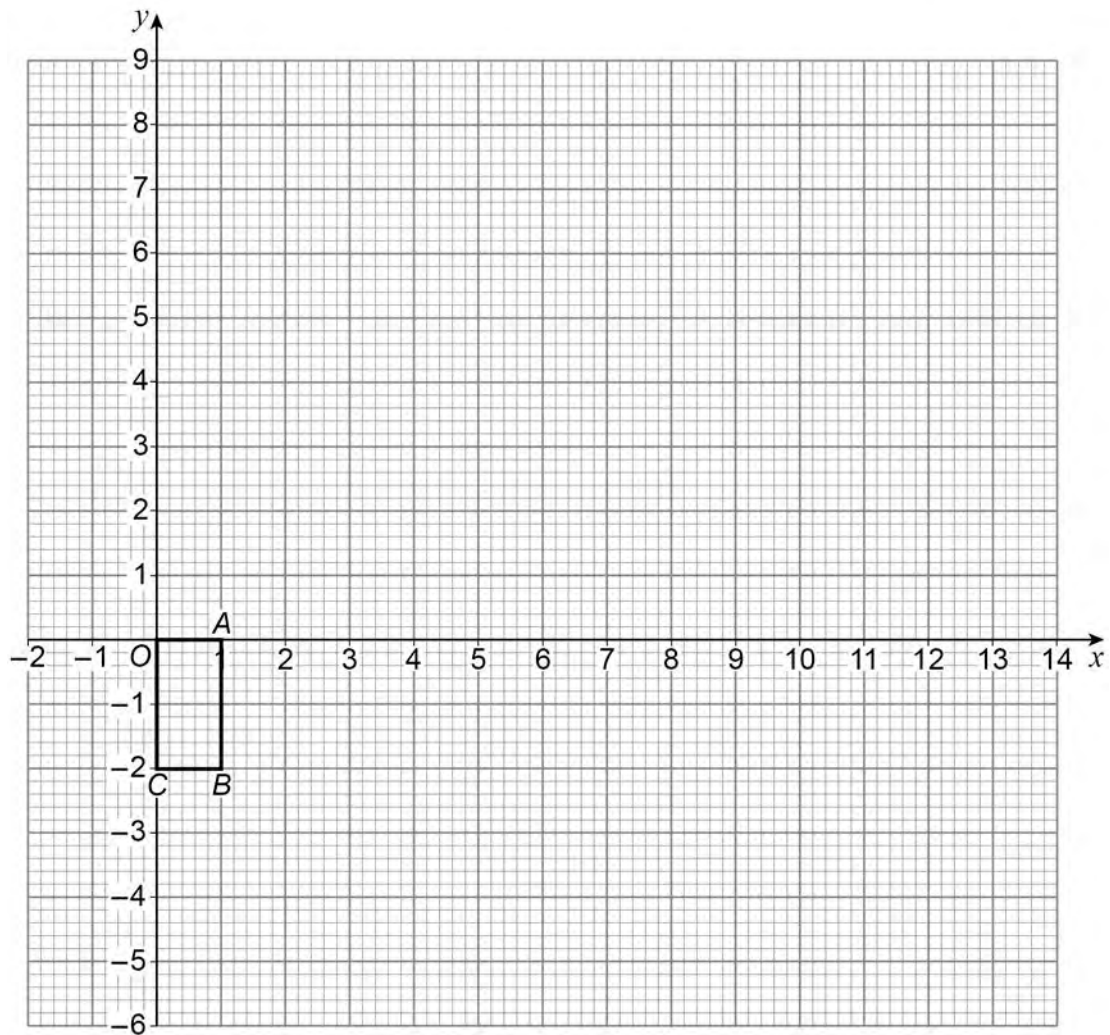
- 5 The transformation T is a shear which is not parallel to the x -axis or the y -axis.

T is represented by the matrix $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

- 5 (a) T transforms the rectangle $OABC$ shown on the diagram to $OA'B'C'$

Draw $OA'B'C'$ on the diagram, labelling each of the points A' , B' and C' clearly.

[2 marks]



- 5 (b)** Show that the area of the quadrilateral $OA'B'C'$ is 2 square units.

[2 marks]

- 5 (c)** The line $y = mx$ is a line of invariant points of T.

Show that $m = \frac{1}{2}$

[2 marks]

Question 5 continues on the next page

Turn over ►



Find the value of k .

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Answer _____



Section B**Statistics**

Answer **all** questions in the spaces provided.

- 6** The discrete random variable X is binomially distributed such that $X \sim B(n, p)$

Show that the probability generating function of X is given by

$$G_x(t) = (1 - p + pt)^n$$

[3 marks]

3

Turn over for the next question

Turn over ►



7 (a) Given that $E(D) = \frac{n+1}{2}$, prove that

$$\text{Var}(D) = \frac{n^2 - 1}{12}$$

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- 7 (b)** The random variable D represents the value on the upper face of a dice after it is rolled.

The dice has n faces which are labelled with the integers 1, 2, 3, ..., n

The variance of D is 33.25

- 7 (b) (i)** Find the value of n .

[2 marks]

Answer _____

- 7 (b) (ii)** Find $P(D > 18)$.

[1 mark]

Answer _____

- 7 (b) (iii)** The dice is rolled until a value greater than 18 is obtained.

Find the probability that the dice is rolled exactly 5 times.

[2 marks]

Answer _____



8 (a) Show that

[4 marks]

[illegible]

8 (b) (i) Find the exact value of a .

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8 (b) (ii) Find the value of $E(aX + (1-a)Y)$.

8



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Mechanics

9 For a body moving around a circle at constant speed, the magnitude of its acceleration, a , is given by

where r is the radius of the circle.

Find the dimensions of ω .

[3 marks]

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3

Turn over ►



- 10** A circular disc, of mass 0.2 kg , is sliding on a smooth horizontal surface when it hits a vertical wall and rebounds.

The wall is perpendicular to the path of the disc.

The disc hits the wall with speed 8 m s^{-1} and rebounds with speed 5 m s^{-1}

- 10 (a)** Find the coefficient of restitution between the disc and the wall.

[1 mark]

Answer _____

- 10 (b)** Find the magnitude of the impulse on the disc due to the wall.

[2 marks]

Answer _____



10 (c) The disc is in contact with the wall for 0.25 seconds.

A simple model assumes that the wall exerts a constant force on the disc.

Find the magnitude of this force.

[2 marks]

Answer

5

Turn over for the next question

Turn over ►



The sailing boat is travelling north-east at a speed of 4 m s^{-1}

The patrol boat travels at 9 m s^{-1} on a bearing of α degrees, so that it intercepts the sailing boat in the shortest possible time.

11 (a) Find α , giving your answer to the nearest integer.

[4 marks]

[illegible]

Answer



[3 marks]

[illegible]

$\frac{\quad}{7}$

Turn over ►



Two smooth spheres, A and B , are moving directly towards each other and collide.

B has mass 3 kg and is moving at 2 m s^{-1} before the collision.

During the collision B exerts an impulse of magnitude 9 N s on A .

Find the coefficient of restitution between A and B , giving your answer as a fraction.

[5 marks]

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5

END OF QUESTIONS



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