OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

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Candidate signature ______________________
I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM03) Unit FP2 Pure Mathematics

Tuesday 11 January 2022 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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11	
12	
13	
TOTAL	

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Answer **all** questions in the spaces provided.

1 (a) The matrix $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Describe fully the **single** transformation represented by the matrix **A**

[2 marks]

1 (b) The matrix $\mathbf{B} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

State the line of invariant points for the transformation represented by the matrix **B**

[1 mark]

Answer ______________________

3

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2 The vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are such that

$$\mathbf{v} \times \mathbf{w} = 5\mathbf{i} \quad \text{and} \quad \mathbf{u} \times \mathbf{v} = 2\mathbf{j}$$

Simplify

$$(4\mathbf{u} + 3\mathbf{v} + 6\mathbf{w}) \times (2\mathbf{u} - 4\mathbf{v} + 3\mathbf{w})$$

giving your answer in the form $a\mathbf{i} + b\mathbf{j}$ where a and b are integers.

[5 marks]

Answer ____________________

5

Turn over ►

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3 The sequence $u_1,\ u_2,\ u_3,\ \ldots$ is defined by

$$u_1 = 3 \quad \text{and} \quad u_{n+1} = \frac{9u_n - 5}{5u_n - 1}$$

Prove by induction that for all integers $n \geq 1$

$$u_n = \frac{5n+1}{5n-3}$$

[6 marks]

6

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4 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=2\sin 2x+14\cos 2x$$

[7 marks]

$y =$

7

Turn over ►

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5 (a) Use the trigonometric identity

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

to show that

$$\frac{1}{2}\left[\sin(2r+1)x - \sin(2r-1)x\right] = \cos 2rx \sin x$$

[1 mark]

5 (b) Hence use the method of differences to show that

$$\sum_{r=1}^{n} \sin^2 rx = \frac{n}{2} - \frac{\sin nx \cos(n+1)x}{2\sin x}$$

[6 marks]

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7

Turn over for the next question

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6 Evaluate the improper integral

$$\int_0^{\infty} \left(x^2 + 1\right) \mathrm{e}^{-x} \, \mathrm{d}x$$

showing the limiting process used.

[8 marks]

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Answer ______

8

Turn over for the next question

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7 A curve C is defined for $x > 0$

All points (x, y) on the curve satisfy the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \left(\frac{1}{x} - \frac{1}{x+2}\right)y = x$$

7 (a) Use an integrating factor to find the general solution of this differential equation.

[7 marks]

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Answer ____________________

7 (b) The curve C has a stationary point when $x = 2$

Find the equation of the curve C giving your answer in the form $y = \mathrm{f}(x)$

[3 marks]

$y =$ ____________________

10

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8 The plane Π_1 has vector equation

$$\mathbf{r} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 14$$

8 (a) Find the shortest distance from the origin to the plane Π_1 giving your answer in an exact form.

[2 marks]

Answer ______________________________

8 (b) The line L has Cartesian equations

$$\frac{x-2}{3} = \frac{y+1}{2} = 2z-4$$

The line L intersects the plane Π_1 at the point P

8 (b) (i) Find the coordinates of P

[3 marks]

Answer ______________________________

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8 (b) (ii) Calculate the acute angle between the line L and the plane Π_1 giving your answer to the nearest 0.1°

[4 marks]

Answer

8 (c) The plane Π_2 has vector equation $\mathbf{r} \cdot \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = 4$

Find direction cosines for the line of intersection of the planes Π_1 and Π_2

[3 marks]

Answer

12

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9 The matrix **M** is defined as

$$\mathbf{M} = \begin{bmatrix} 4 & 3 & k \\ 5 & 4 & k+1 \\ 1 & 1 & 3 \end{bmatrix}$$

where k is a constant.

9 (a) Show that **M** is a non-singular matrix.

[2 marks]

9 (b) Find $\mathbf{M}^{-1}$ in terms of k

[5 marks]

Answer

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9 (c) The transformation represented by the matrix $\begin{bmatrix} 4 & 3 & 5 \\ 5 & 4 & 6 \\ 1 & 1 & 3 \end{bmatrix}$ maps the straight line L

onto the straight line whose vector equation is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

Find the vector equation of the line L giving your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$

[5 marks]

Answer ____________________

12

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10 (a) Use the Maclaurin series for $\ln(1+x)$ to show that the first three non-zero terms in the Maclaurin series expansion in ascending powers of x of

$$\tanh^{-1}x \quad \text{are} \quad x+\frac{x^3}{3}+\frac{x^5}{5}$$

[3 marks]

10 (b) It is given that $y = \tan x$

10 (b) (i) Show that when $x = 0$

$$\frac{\mathrm{d}^5 y}{\mathrm{d}x^5} = 16$$

[4 marks]

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10 (b) (ii) Show that the first non-zero term in the Maclaurin series expansion in ascending powers of x of

$$\tanh^{-1}x - \tan x \quad \text{is} \quad \frac{x^5}{15}$$

[3 marks]

10 (c) Hence show that

$$\lim_{x \to 0}\left[\frac{\tan x + \tanh^{-1}x - 2x}{x(1-\cos 2x)}\right]$$

exists and find its value.

[4 marks]

Answer

14

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11 A curve C is given parametrically by the equations

$$x = t^2 \qquad y = 2t \quad \text{where} \quad t \geq 0$$

The origin O and the point P lie on the curve C

The x-coordinate of P is 3

11 (a) The arc OP of the curve C is rotated through 2π radians about the x-axis.

Show that the area of the curved surface generated is $\frac{56}{3}\pi$

[5 marks]

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11 (b) Show that the length of the arc *OP* of the curve *C* is $2\sqrt{3}+\sinh^{-1}\left(\sqrt{3}\right)$

[7 marks]

12

Turn over ►

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12 (a) (i) Use de Moivre's theorem to show that if $z = \cos\theta + \mathrm{i}\sin\theta$ then

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

[3 marks]

12 (a) (ii) Given that

$$(2\,\mathrm{i}\sin\theta)^6 (2\cos\theta)^2 = \left(z - \frac{1}{z}\right)^4 \left(z^2 - \frac{1}{z^2}\right)^2$$

use the result in **part (a)(i)** to show that

$$128\sin^6\theta\cos^2\theta = 5 - 4\cos 2\theta - 4\cos 4\theta + 4\cos 6\theta - \cos 8\theta$$

[4 marks]

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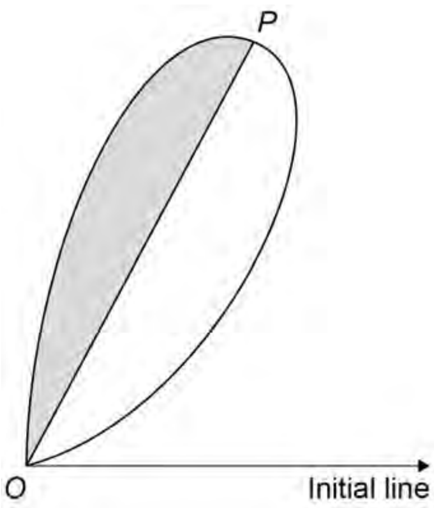
12 (b) The diagram shows a curve, the pole O, the initial line and a point P which lies on the curve.



P is the point on the curve that is furthest from the pole O

The curve has polar equation $r = 32\sin^3\theta\cos\theta$ where $0 \le \theta \le \frac{\pi}{2}$

12 (b) (i) By differentiating r with respect to θ find the polar coordinates of the point P

[4 marks]

Answer ____________________

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12 (b) (ii) Find the area of the shaded region bounded by the line *OP* and the upper part of the curve.

Give your answer in the form $a\pi + b\sqrt{n}$ where a and b are rational and n is a prime number.

[4 marks]

Answer ______________________

15

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13 The cubic equation

$$tx^3 + ux^2 + vx + w = 0$$

has coefficients t, u, v and w which are all real constants.

The three roots of this cubic equation can be arranged as successive terms of an arithmetic sequence.

13 (a) Show that

$$2u^3 - 9tuv + 27t^2w = 0$$

[3 marks]

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13 (b) It is given that the roots of the cubic equation

$$kx^3 - 36x^2 + mx - 3 = 0$$

where k and m are real constants, can be arranged as three successive terms of an arithmetic sequence with common difference d

13 (b) (i) Find an expression for d^2 in terms of k

[2 marks]

Answer ____________________

13 (b) (ii) Given that $m = 38$ find the possible values for d giving your values in an exact form.

[4 marks]

Turn over ►

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Answer

9

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