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FM05

(9665/FM05) Unit FM2 Mechanics

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$-5\mathbf{i}-4\mathbf{j}=3(2\mathbf{i}-3\mathbf{j})-3\mathbf{w}$	M1	Forms equation based on the impulse equation. Condone sign errors.
		A1	Correct equation.
	$\mathbf{w}=\dfrac{11\mathbf{i}-5\mathbf{j}}{3}$	A1	Correct velocity. Solutions may be seen using column vectors.
		3	
1(b)	$2\mathbf{v}+3\left(\dfrac{11\mathbf{i}-5\mathbf{j}}{3}\right)=5(2\mathbf{i}-3\mathbf{j})$	M1	Forms equation based on the impulse equation or conservation of momentum, with their $\mathbf{w}$ from part (a).
		A1ft	Correct equation, with their $\mathbf{w}$ from part (a).
	$\mathbf{v}=\dfrac{-1\mathbf{i}-10\mathbf{j}}{2}$	A1	Correct velocity. Solutions may be seen using column vectors.
		3	
	Total	6	

Q	Answer	Marks	Comments
2(a)	$\text{WD} = \int_4^9 5\sqrt{x}\ dx$	M1	Forms correct integral. Condone incorrect / missing limits.
	$= \left[\frac{10}{3}x^{\frac{3}{2}}\right]_4^9$	A1	Correct integration, with correct limits.
	$= 90 - \frac{80}{3}$		
	$= \frac{190}{3}\ [\text{J}]$	A1	Correct WD. Accept 63 J Condone missing units.
		3	
2(b)	$\frac{1}{2}\times 6\times 4^2 + \frac{190}{3} = \frac{1}{2}\times 6\times v^2$	M1	Forms three term energy equation, with correct terms and any signs. Allow their work done from part (a).
		A1ft	Correct equation. Allow their work done from part (a).
	$3v^2 = 48 + \frac{190}{3}$		
	$v = 6.09\ \left[\text{m s}^{-1}\right]$	A1	Correct speed. Condone missing units.
		3	
	Total	**6**	

Q	Answer	Marks	Comments
3(a)	$21=\omega^2(a^2-0.2^2)$ $9=\omega^2(a^2-0.4^2)$	M1	Forms two equations connecting a and ω
		A1	Both equations correct.
	$\frac{(a^2-0.4^2)}{9}=\frac{(a^2-0.2^2)}{21}$ $7a^2-1.12=3a^2-0.12$ $a^2=0.25$	M1	Eliminates ω
	$a=0.5$ [metres]	A1	Obtains correct amplitude, must be positive. Condone missing units.
		4	
3(b)	$9=\omega^2(0.5^2-0.4^2)$ $\omega^2=\frac{9}{(0.5^2-0.4^2)}$	M1	Uses an equation to find ω with their a.
	$\omega=10$	A1	Correct ω
	Period $=\frac{2\pi}{10}=\frac{\pi}{5}$ [seconds]	A1	Correct period. Accept 0.63
		3	
3(c)	Max Speed $=0.5\times10=5\text{ m s}^{-1}$	B1	Obtains correct speed with the correct units.
		1	
	Total	**8**	

Q	Answer	Marks	Comments
4(a)	$m\frac{dv}{dt}=-kv^2$	M1	States correct differential equation.
	$\frac{1}{v^2}\frac{dv}{dt}=-\frac{k}{m}$ $\int\frac{1}{v^2}\ dv=\int-\frac{k}{m}\ dt$ $-v^{-1}=-\frac{k}{m}t+c$	M1 A1	Separates variables and integrates. Correct integration.
	$t=0, v=U \Rightarrow c=-\frac{1}{U}$	M1	Finds constant of integration or uses correct limits of integration.
	$-\frac{1}{v}=-\frac{1}{U}-\frac{kt}{m}$ $\frac{1}{v}=\frac{m+ktU}{mU}$ $v=\frac{mU}{m+ktU}$	A1	Correct constant and correct final answer from correct working.
		5	
4(b)	$0.9U=\frac{mU}{m+ktU}$	M1	Substitutes 0.9U
	$0.9mU+0.9U^2kt=mU$ $0.9Ukt=0.1m$	M1	Solves for t
	$t=\frac{m}{9kU}$	A1	Obtains correct result.
		3	
	Total	**8**	

Q	Answer	Marks	Comments
5(a)	$0.04\times 2.45\ \ddot{\theta}=-0.04\times 9.8\sin\theta$	M1	Forms differential equation.
	$\sin\theta\approx\theta$	B1	Small angle approximation seen or used.
	$\ddot{\theta}\approx -4\theta$	A1	Obtains $\ddot{\theta}\approx -4\theta$ or $\ddot{\theta}\approx -\frac{g}{l}\theta$
	[Angular] acceleration is proportional to the [angular] displacement and in the opposite direction [so therefore SHM].	E1	Concludes that motion is SHM from correct working. Condone use of = instead of ≈
		4	
5(b)		M1	Finds period using the value of their ω
	Period $=\frac{2\pi}{\sqrt{4}}=\pi$ [seconds]	A1ft	Correct period. Allow 3.1
		2	
5(c)	$\theta=\frac{\pi}{12}\cos(2t)$	M1	States or uses a formula for angle based on their ω
	$\frac{\pi}{24}=\frac{\pi}{12}\cos(2t_1)\Rightarrow t_1=0.5236$	A1	Correct time for one angle.
	$\frac{\pi}{36}=\frac{\pi}{12}\cos(2t_2)\Rightarrow t_2=0.6155$	A1	Correct time for other angle.
	$0.6155-0.5236=0.092$ seconds	A1	Correct difference from correct working. AWRT 0.09
		4	
	Total	10	

Q	Answer	Marks	Comments
6(a)	$0^2 = (25\sin 40°)^2 + 2 \times s \times \left(-9.8\cos 20°\right)$	M1 A1	Forms equation based on motion perpendicular to the slope to find the maximum distance from the plane. Correct equation.
	$s = \dfrac{(25\sin 40°)^2}{2 \times 9.8\cos 20°} = 14 \text{ metres}$	M1 A1	Solves for max height. Correct height.
		4	
6(b)	$0 = 25\sin 40° t - 4.9\cos 20° t^2$ $t = 0 \quad \text{or} \quad t = \dfrac{25\sin 40°}{4.9\cos 20°} = 3.49$	M1 A1 A1	Equation to find time of flight. Correct equation. Correct time.
	$v_x = 25\cos 40° - 9.8\sin 20°\left(\dfrac{25\sin 40°}{4.9\cos 20°}\right) = 7.453$ $v_y = 25\sin 40° - 9.8\cos 20°\left(\dfrac{25\sin 40°}{4.9\cos 20°}\right) = -16.07$	M1 A1	Finding one correct component of velocity on impact. Both components correct.
	$\tan\alpha = \dfrac{16.07}{7.453}$ $\alpha = 65°$	M1 A1	Finding angle. Correct angle. Allow ±65° Final answer must be to the nearest degree.
		7	
	Total	**11**	

Q	Answer	Marks	Comments
7(a)	$mg\cos 30° = \dfrac{mv^2}{r}$	M1	Applying Newton's second Law at the point where the particle leaves the hemisphere. Must include a trig term.
	$v^2 = \dfrac{\sqrt{3}\,gr}{2}$	A1	Correct equation.
		A1	Correct expression for v or v^2
	$\dfrac{1}{2}mv^2 = \dfrac{1}{2}mU^2 + mgr(1-\cos 30°)$	M1	Applying conservation of energy.
	$v^2 = U^2 + gr\left(2-\sqrt{3}\right)$	A1	Correct equation.
		A1	Correct expression for v or v^2
	$\dfrac{\sqrt{3}\,gr}{2} = U^2 + gr\left(2-\sqrt{3}\right)$	m1	Eliminating v
	$U^2 = gr\left(\dfrac{3\sqrt{3}}{2}-2\right)$		
	$U = \sqrt{gr\left(\dfrac{3\sqrt{3}}{2}-2\right)}$	A1	Correct expression for U
		8	
7(b)	As U as independent of mass, so U does not change.	B1	States no change and provides a reason.
		E1	Correct reason.
		2	
7(c)	U would decrease as it is directly proportional to the square root of the radius, so U does change.	B1	States that there is a change and provides a reason.
		E1	Correct reason.
		2	
	Total	**12**	

Q	Answer	Marks	Comments
8	Before Collision along line of centres:		
	$u_P = 5\cos 60° = 2.5$ $u_Q = -4$	B1	Correct components along line of centres seen.
	$3\times 2.5 + 2\times(-4) = 3v_P + 2v_Q$ $-0.5 = 3v_P + 2v_Q$	M1	Conservation of momentum along the line of centres.
		A1	Correct equation.
	$v_P - v_Q = -\frac{2}{5}(2.5-(-4))$	M1	Applies coefficient of restitution along lines of centres.
	$v_P - v_Q = -2.6$	A1	Correct equation.
	$v_P = -\frac{57}{50}$	M1	Solves their simultaneous equations.
	$v_Q = \frac{73}{50}$	A1	Correct velocity component for P.
	$\text{Speed of } Q = \frac{73}{50} = 1.46 \text{ m s}^{-1}$	A1	Recognises that Q has no perpendicular component and states the speed of *Q*
	$\text{Speed of } P = \sqrt{\left(\frac{57}{50}\right)^2 + (5\sin 60°)^2}$	M1	Uses component perpendicular to the line of centres.
	$= \sqrt{\left(\frac{57}{50}\right)^2 + \left(\frac{75}{4}\right)}$ $= 4.48 \text{ m s}^{-1}$	A1	Correct speed for *P*
	Total	**10**	

Q	Answer	Marks	Comments
9	At max speed: $T + 2T\cos 30° = mg$	M1	Uses resultant force is zero to find tension.
	$T = \dfrac{mg}{1+\sqrt{3}}$	A1	Correct tension.
	$e = \dfrac{2}{\sin 30°} - 3 = 1$	M1	Finds extension.
	$\dfrac{\lambda}{3}e = \dfrac{mg}{1+\sqrt{3}}$ $\lambda = \dfrac{3mg}{1+\sqrt{3}}$	A1	Correct modulus of elasticity or stiffness $k = \dfrac{mg}{1+\sqrt{3}}$
	At Lowest point Let x be the distance of the particle below *O*		
	Length of string attached to *A*/*B* = $\sqrt{x^2+4}$	B1	Maximum length of angled ropes.
	Length of string attached to *C* = $x + 4 - 2\sqrt{3}$	B1	Maximum length of vertical rope.
	$mgx = 2 \times \dfrac{3mg}{6(1+\sqrt{3})} \times \left(\sqrt{x^2+4} - 3\right)^2 + \dfrac{3mg}{6(1+\sqrt{3})}\left(x+1-2\sqrt{3}\right)^2$ $2\left(\sqrt{x^2+4}-3\right)^2 + \left(x+1-2\sqrt{3}\right)^2 - 2x\left(1+\sqrt{3}\right) = 0$	M1	Equation for conservation of energy with at least two terms correct. Must be based on EPE for three strings and one GPE for the particle and no other terms.
	Substituting $x = 5.85$ gives -0.244	A1	Correct equation.
	Substituting $x = 5.94$ gives 0.980 $\therefore x = 5.9$ to 2 sf.	A1	Value of 5.9 justified.
	Total	**9**	