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# MATHEMATICS

# 9665

FM01 Further Pure Mathematics Unit 1

Mark scheme

January 2019

Version 1.0 Final

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

# Level of response marking instructions

Level of response mark schemes are broken down into levels, each of which has a descriptor. The descriptor for the level shows the average performance for the level. There are marks in each level.

Before you apply the mark scheme to a student's answer read through the answer and annotate it (as instructed) to show the qualities that are being looked for. You can then apply the mark scheme.

## Step 1 Determine a level

Start at the lowest level of the mark scheme and use it as a ladder to see whether the answer meets the descriptor for that level. The descriptor for the level indicates the different qualities that might be seen in the student's answer for that level. If it meets the lowest level then go to the next one and decide if it meets this level, and so on, until you have a match between the level descriptor and the answer. With practice and familiarity you will find that for better answers you will be able to quickly skip through the lower levels of the mark scheme.

When assigning a level you should look at the overall quality of the answer and not look to pick holes in small and specific parts of the answer where the student has not performed quite as well as the rest. If the answer covers different aspects of different levels of the mark scheme you should use a best fit approach for defining the level and then use the variability of the response to help decide the mark within the level, ie if the response is predominantly level 3 with a small amount of level 4 material it would be placed in level 3 but be awarded a mark near the top of the level because of the level 4 content.

## Step 2 Determine a mark

Once you have assigned a level you need to decide on the mark. The descriptors on how to allocate marks can help with this. The exemplar materials used during standardisation will help. There will be an answer in the standardising materials which will correspond with each level of the mark scheme. This answer will have been awarded a mark by the Lead Examiner. You can compare the student's answer with the example to determine if it is the same standard, better or worse than the example. You can then use this to allocate a mark for the answer based on the Lead Examiner's mark on the example.

You may well need to read back through the answer as you apply the mark scheme to clarify points and assure yourself that the level and the mark are appropriate.

Indicative content in the mark scheme is provided as a guide for examiners. It is not intended to be exhaustive and you must credit other valid points. Students do not have to cover all of the points mentioned in the Indicative content to reach the highest level of the mark scheme.

An answer which contains nothing of relevance to the question must be awarded no marks.

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 1(a) | $(-2+h)^3 = -8+12h-6h^2+h^3$ | B1 | Accept unsimplified coefficients |
| | $y(-2+h) = 16-6h^2+h^3$ | B1ft | PI; ft numerical error |
| | Use of correct formula for gradient | M1 | |
| | $-6h+h^2$ | A1 | |
| 1(b) | Limit of $-6h+h^2$ as $h \to 0$ is zero | E1ft | ft if limit is zero |
| | Hence it is a stationary point | E1dep | Depends on 1st E1 |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 2(a) | $S_n = 2\sum r^3 + 3\sum r^2 - 5\sum r$ | M1 | |
| | $= \frac{1}{2}n^2(n+1)^2 + \frac{1}{2}n(n+1)(2n+1) - \frac{5}{2}n(n+1)$ | m1 | |
| | $= n(n+1)\left\{\frac{1}{2}n^2 + \frac{1}{2}n + n + \frac{1}{2} - \frac{5}{2}\right\}$ | m1 | |
| | $= \frac{1}{2}n(n+1)(n^2+3n-4)$ | m1 | PI |
| | $= \frac{1}{2}n(n+1)(n+4)(n-1)$ | A1 | |
| 2(b) | $n-1$, $n$ and $n+1$ are consecutive numbers | E1 | |
| | The product of three consecutive integers is a multiple of 6 and therefore $S_n$ is a multiple of 3 | E1 | Accept: So one of them is a multiple of 3, and $S_n$ must be an integer |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 3 | The other root is $p - 2\mathrm{i}$ | B1 | PI |
| | $(p + 2\mathrm{i}) + (p - 2\mathrm{i}) = -3$ | M1 | |
| | $p = -\frac{3}{2}$ | A1 | oe |
| | $c = \left(-\frac{3}{2} + 2\mathrm{i}\right)\left(-\frac{3}{2} - 2\mathrm{i}\right)$ | M1 | ft<br>or $c = p^2 + 4$ |
| | $c = \frac{25}{4}$ | A1 | oe |
| | **Alternative method** | | |
| | $x = \frac{-3 \pm \sqrt{9 - 4c}}{2}$ | M1 | |
| | Taking real part | m1 | PI |
| | $p = -\frac{3}{2}$ | A1 | |
| | $(4\mathrm{i})^2 = 9 - 4c$ | M1 | |
| | $c = \frac{25}{4}$ | A1 | and no extras |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 4(a) | $\lim\limits_{h\to 0}\int_h^{12} x^{-\frac{1}{2}}\,\mathrm{d}x$ | M1 | PI |
| | $=\lim\limits_{h\to 0}\left[2x^{\frac{1}{2}}\right]_h^{12}$ | M1 | |
| | $=4\sqrt{3}$ oe | A1 | NMS 1/3<br><br>SC2 for correct integration and correct answer without use of limiting process |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 4(b) | $\lim\limits_{h\to 0}\int_h^{12} x^{-4}\,\mathrm{d}x$ | B1 | |
| | $=\lim\limits_{h\to 0}\left[\frac{x^{-3}}{-3}\right]_h^{12}$ | B1 | Condone omission of limits |
| | But $\lim\limits_{h\to 0}(h^{-3})$ is not defined, so the integral has no finite value. | E1 | Must have integrated first |

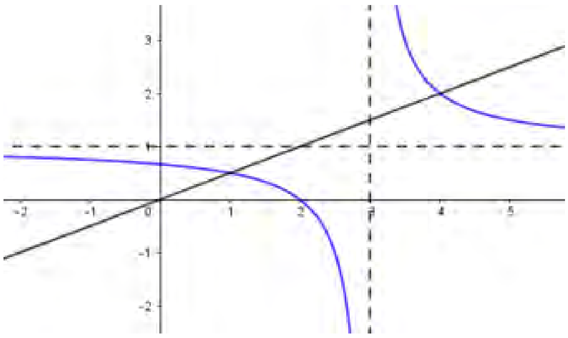
| Q | Answer | Mark | Comments |
|---|---|---|---|
| 5(a) | $\frac{x}{2}+\frac{\pi}{4}=\frac{\pi}{6}+n\pi$ | B1<br>B1 | B1 for $\frac{\pi}{6}$<br>B1 for $+n\pi$ |
| | $x+\frac{\pi}{2}=\frac{\pi}{3}+2n\pi$ | M1 | For multiplying by 2 |
| | $x=2n\pi-\frac{\pi}{6}$ | A1 | |
| 5(b) | First term $= -16\pi-\frac{\pi}{6}$ | B1ft | |
| | Last term $= 18\pi-\frac{\pi}{6}$ | B1ft | |
| | A.P. with 18 terms | M1 | or for listing and cancelling terms |
| | $\frac{18}{2}(2\pi-\frac{\pi}{3})$ | M1 | |
| | $15\pi$ | A1 | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 6 | $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}x^{-\frac{2}{3}}$ | M1 | |
| | $= \frac{1}{12}$ when $x = 8$ | A1 | |
| | $\delta y \cong \frac{\mathrm{d}y}{\mathrm{d}x} \times \delta x$ | M1 | |
| | $\delta x = 0.06$ | M1 | |
| | $\frac{1}{12} \times 0.06$ or 0.005 | M1 | PI |
| | 2.005 | A1 | Working must be seen. |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 7(a) | ${z_1}^* = x - \mathrm{iy}$ | M1 | PI |
| | $\mathrm{i}z_1 = -y + \mathrm{i}x$ | M1 | PI: both M1 marks are implied by $2(x - \mathrm{iy}) + 3\mathrm{i} = -y + \mathrm{i}x$ |
| | Re: $2x = -y$ | M1 | |
| | Im: $3 - 2y = x$ | M1 | |
| | $x = -1$ | A1 | |
| | $y = 2$ | A1 | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 7(b) | $\lvert z_1 \rvert = \sqrt{5}$ | B1ft | |
| | $\tan\theta = -2$ | M1 | |
| | $\arg(z_1) = \pi - \tan^{-1} 2$ or 2.03 | A1 | oe: accept correct answer in degrees, accept 2 with working |

| | | | |
|---|---|---|---|
| **7(c)** | **Method 1** | | |
| | $y = x$ | M1 | |
| | Perpendicular line through (-1, 2) is $y = -x + 1$ | M1 | |
| | Point of intersection $(\frac{1}{2}, \frac{1}{2})$ | M1 | |
| | Distance = $\frac{3\sqrt{2}}{2}$ | A1 | oe; accept approx. answer |
| | **Alternative method** | | |
| | $\pi - \tan^{-1} 2 - \frac{\pi}{4}$ | M1 | |
| | 1.249 radians approx. | M1 | or equivalent in degrees |
| | $\sqrt{5} \times \sin 1.249$ | M1 | |
| | 2.12 units | A1 | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 8(a) | $x = 3$ | B1 | |
| | $y = 1$ | B1 | |
| 8(b) | $\frac{1}{2}x = \frac{x-2}{x-3}$<br>$x^2 - 3x = 2x - 4$ | M1 | |
| | $x^2 - 5x + 4 = 0$ | M1 | |
| | $x = 1$ or 4 | A1 | |
| | (1, ½) and (4, 2) | A1 | |
| 8(c) | Correct shape of curve | B1 |  |
| | Asymptotes | B1 | |
| | Axis intercepts (2,0), (0, 2/3) and (0,0) clearly shown | B1 | |
| | Line correct and meets curve correctly | B1 | |
| 8(d) | $1 \leq x < 3$ | B1 | |
| | $x \geq 4$ | B1 | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 9(a) | $2\sqrt{(x-6)^2+y^2}=x$ | M1 | PI |
| | $4\{(x-6)^2+y^2\}-x^2=0$ | M1 | |
| | $3x^2-48x+144+4y^2=0$ | M1 | |
| | $3(x^2-16x+64)-192+144+4y^2=0$ | M1 | for completing the square |
| | $3(x-8)^2+4y^2=48$ | M1 | |
| | $\dfrac{(x-8)^2}{16}+\dfrac{y^2}{12}=1$ | A1 | |
| | This is a translation of $E_1$ by $\begin{bmatrix}8\\0\end{bmatrix}$ | E1 | |
| 9(b) | (4,0) and (12,0) | B1 | ft translation of (-4, 0) and (4, 0) by a non-zero vector |
| 9(c) | $\dfrac{(x-8)^2}{16}+\dfrac{(mx+c)^2}{12}=1$<br>or<br>$3(x-8)^2+4(mx+c)^2-48=0$ | M1 | or<br>$3x^2-48x+144+4y^2=0$<br>if seen in part (a) |
| | $3x^2-48x+192+4m^2x^2+8mcx+4c^2-48=0$ | M1 | |
| | $(3+4m^2)x^2+(8mc-48)x+(4c^2+144)=0$ | A1 | AG, no errors seen |

| | | | |
|---|---|---|---|
| **9(d)** | Substitute $c = 8$ so<br>$(3 + 4m^2)x^2 + (64m - 48)x + 400 = 0$ | M1 | |
| | Equal roots<br>$\Rightarrow (64m - 48)^2 - 1600(3 + 4m^2) = 0$ | M1 | |
| | $-2304m^2 - 6144m - 2496 = 0$<br>or $12m^2 + 32m + 13 = 0$ | M1 | oe |
| | $y = -\frac{1}{2}x + 8$ | A1 | |
| | $y = -\frac{13}{6}x + 8$ | A1 | |