

Please write clearly in block capitals.

Centre number

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Candidate signature

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I declare this is my own work.

# INTERNATIONAL A-LEVEL MATHEMATICS

(9660/MA03) Unit P2 Pure Mathematics

Friday 2 June 2023

07:00 GMT

Time allowed: 2 hours 30 minutes

## Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

## Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

## Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
<b>TOTAL</b>	



J U N 2 3 M A 0 3 0 1

Answer **all** questions in the spaces provided.

**1 (a)** For each of the following find  $\frac{dy}{dx}$

**1 (a) (i)**  $y = (3x^5 - 4)^8$

**[2 marks]**


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Answer \_\_\_\_\_

**1 (a) (ii)**  $y = \frac{3x^4}{7x-5}$

**[2 marks]**


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Answer \_\_\_\_\_

**1 (a) (iii)**  $\ln(x^2) + y^2 = xy$

**[3 marks]**


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Answer \_\_\_\_\_



1 (b) (i) Find  $\int \frac{x-6}{x^2-12x+5} dx$

[2 marks]

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Answer \_\_\_\_\_

1 (b) (ii) Find  $\int \frac{8x+6}{(2x^2+3x-1)^3} dx$

[3 marks]

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Answer \_\_\_\_\_

Turn over ►



- 2 (a)** The curve with equation  $y = 2^{-x}$  intersects the line  $y = 4 - 2x$  at a point where  $x = \alpha$  and  $\alpha > 0$

- 2 (a) (i)** Show that  $\alpha$  lies between 1.8 and 1.9

**[2 marks]**

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- 2 (a) (ii)** Show that the equation  $4 - 2x = 2^{-x}$  can be rearranged into the form  $x = 2 - 2^{-(x+1)}$

**[1 mark]**

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- 2 (a) (iii)** Use the iterative formula

$$x_{n+1} = 2 - 2^{-(x_n+1)}$$

with  $x_1 = 1.8$  to find the values of  $x_2$  and  $x_3$  giving your answers to three decimal places.

**[2 marks]**

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$$x_2 = \underline{\hspace{2cm}} \quad x_3 = \underline{\hspace{2cm}}$$



**2 (b)**

giving your answer to four significant figures.

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**9**

- 3 (a)** Describe a sequence of **two** geometrical transformations that maps the graph of  $y = \sec \theta$  onto the graph of  $y = 0.5\sec(\theta + 1)$

[4 marks]

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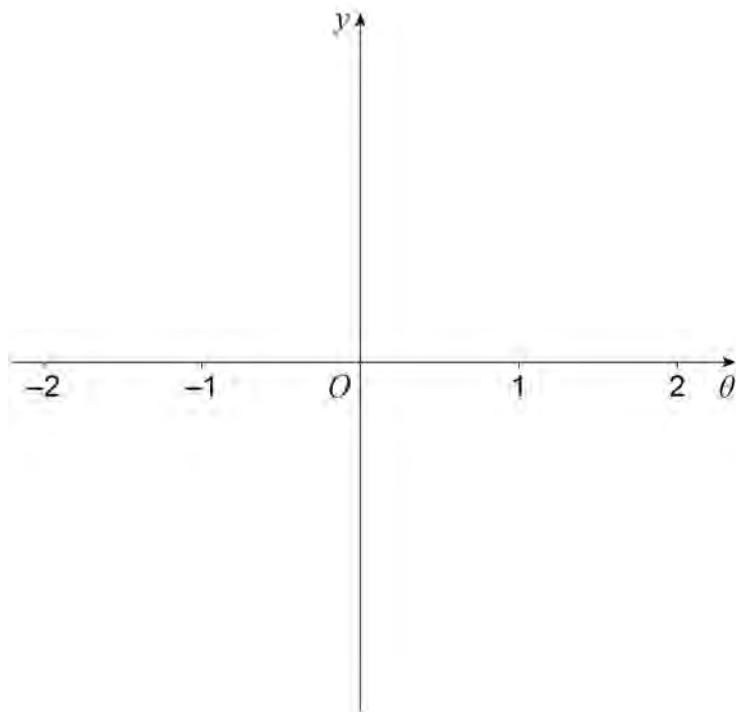
- 3 (b)** Sketch on the axes below the curve with equation

$$y = 0.5\sec(\theta + 1) \quad \text{for } -2 < \theta < 2$$

where  $\theta$  is in radians.

State the coordinates of the stationary point and the coordinates of any intercept with the axes.

[3 marks]




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3 (c) It is given that  $x = \frac{1}{2\cos y}$

3 (c) (i) Find  $\frac{dx}{dy}$

[1 mark]

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Answer \_\_\_\_\_

3 (c) (ii) Hence find  $\frac{dy}{dx}$  in terms of  $\sin y$

[2 marks]

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Answer \_\_\_\_\_

10

Turn over ►



- 4** The functions  $f$  and  $g$  are defined by

$$f(x) = \sqrt{1 - 0.25x} \quad \text{for } x \leq 4$$

$$g(x) = \frac{8}{x-1} \quad \text{for all real values of } x, x \neq 1$$

- 4 (a)** State the range of the function  $f$

**[1 mark]**

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Answer \_\_\_\_\_

- 4 (b) (i)** Find  $fg(x)$

**[1 mark]**

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Answer \_\_\_\_\_

- 4 (b) (ii)** Find the domain of the function  $fg$

**[2 marks]**

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Answer \_\_\_\_\_

- 4 (b) (iii)** Solve the equation  $fg(x) = 3$

**[2 marks]**

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Answer \_\_\_\_\_





**4 (c) (i)** Find  $h(x)$  giving your answer in the form  $h(x) = \frac{p-x^2}{q-x^2}$  where  $p$  and  $q$  are integers.

[illegible]

**4 (c) (ii)** Solve the equation  $h(x) = \frac{11}{3}$

[illegible]

11

**[3 marks]**

Answer

$$2\operatorname{cosec} 4x + 2\cot 4x = \cot x - \tan x$$

**[4 marks]**



$$5\tan^2(2y+3) = 7 - 4\sec(2y+3)$$

**[5 marks]**

[illegible]

Answer

**Turn over ►**



6

$$f(x) = 4x^3 + ax^2 + bx + c$$

where  $a$ ,  $b$  and  $c$  are constants.

It is given that  $(2x - 1)$  is a factor of  $f(x)$

When  $f(x)$  is divided by  $(2x - 3)$  the remainder is 15

**6 (a)** Show that  $2a + b = 2$

**[3 marks]**

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

**[4 marks]**

[illegible]

Answer \_\_\_\_\_

7

**7 (a)** A curve is defined by the parametric equations

$$x = \frac{t}{2t-1} \quad \text{and} \quad y = 2t - \sqrt{t}$$

Find an equation of the normal to the curve at the point where  $t = 1$

Give your answer in the form  $py + qx = r$  where  $p$ ,  $q$  and  $r$  are integers.

**[6 marks]**

[illegible]

Answer

**7 (b)** A second curve is defined by the parametric equations

$$x = e^{2m} + e^{-2m} \quad \text{and} \quad y = e^{2m} - e^{-2m}$$

**7 (b) (i)** Find a Cartesian equation of this curve, giving your answer in the form  $x^2 = f(y)$ .

**[3 marks]**

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Answer \_\_\_\_\_

- 7 (b) (ii)** Hence write down a Cartesian equation of the curve defined by the parametric equations

$$x = e^{kn} + e^{-kn} \quad \text{and} \quad y = e^{kn} - e^{-kn}$$

where  $n$  is the parameter and  $k$  is a positive integer.

**[1 mark]**

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Answer \_\_\_\_\_

- 7 (c)** A third curve is defined by the parametric equations

$$x = a \sin \theta + b \cos \theta \quad \text{and} \quad y = a \cos \theta - b \sin \theta$$

where  $\theta$  is the parameter and  $a$  and  $b$  are constants.

Find a Cartesian equation of this curve.

**[3 marks]**

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Answer \_\_\_\_\_



- 8 (a)** The rate at which the population of a colony of ants is increasing is directly proportional to the population  $P$  of the colony.

Currently the population is 1 000 000 and is increasing at a rate of 3000 per month.

Construct a differential equation for the population of the colony.

**[2 marks]**

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Answer \_\_\_\_\_

- 8 (b)** It is given that  $y = f(x)$  satisfies the differential equation

$$2x \frac{dy}{dx} + y^2 = 4$$

Find the solution of the differential equation for which  $y = 1$  when  $x = 1$

Give your answer in the form  $y = f(x)$

**[6 marks]**

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8

**Turn over ►**



**9 (a)**

$$y = x^{0.5} e^{-0.5x}$$

Give the coordinates in an exact form.

**[4 marks]**

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Answer

**9 (b)**

Find the exact value of the volume of this solid.

Give your answer in the form  $\pi\left(\frac{a}{e} + \frac{b}{e^2}\right)$  where  $a$  and  $b$  are integers.

**[5 marks]**

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Answer \_\_\_\_\_

Use the substitution  $u = 1 + \sin \theta$  to find the value of

giving your answer in the form  $\frac{2}{3}(p\sqrt{2}+q)$  where  $p$  and  $q$  are integers.

**[8 marks]**

[illegible]

Answer \_\_\_\_\_



**[4 marks]**

[illegible]

**[2 marks]**

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Answer



**11 (c)**

$$\frac{7x^2 - 17x + 12}{(3-x)^2(1-3x)} = D + Ex + Fx^2$$

**[4 marks]**

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Answer



- 12 (a)** Find a vector equation of the line  $l_1$  which passes through the points  $A(2, -1, 3)$  and  $B(5, -2, -1)$

[1 mark]

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Answer \_\_\_\_\_

- 12 (b)** The line  $l_2$  has equation  $\mathbf{r} = \begin{bmatrix} -2 \\ 5 \\ 7 \end{bmatrix} + \mu \begin{bmatrix} -1 \\ -2 \\ k \end{bmatrix}$

The lines  $l_1$  and  $l_2$  intersect.

- 12 (b) (i)** Find the value of  $k$

[3 marks]

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Answer \_\_\_\_\_

- 12 (b) (ii)** Find the coordinates of the point of intersection.

[1 mark]

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Answer \_\_\_\_\_





The perpendicular from  $C$  to the line  $l_2$  meets  $l_2$  at the point  $D$

**[6 marks]**

[illegible]

Answer

11

**END OF QUESTIONS**



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[illegible]

Question number	<p style="text-align: center;"><b>Additional page, if required.</b> <b>Write the question numbers in the left-hand margin.</b></p>
	<div style="border: 1px solid black; height: 550px; width: 100%;"></div>
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