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MA04

(9660/MA04) Unit S2 Statistics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	$\left[E\left[3X_1+2X_2-X_3\right]=3E\left[X_1\right]+2E\left[X_2\right]-3E\left[X_3\right]\right]$ $=3f+2h-g$	B1	
		1	

Q	Answer	Marks	Comments
1(a)(ii)	$\left[\text{Var}\left[4X_1-3X_2\right]=4^2\text{Var}\left[X_1\right]+3^2\text{Var}\left[X_2\right]\right]$ $=25g$	B1	
		1	

Q	Answer	Marks	Comments
1(b)	$3h=f$ and $f+g+h=7.1$	B1	**PI**
	$2g+2=9 \Rightarrow g=3.5$	B1	**oe**
	$3h=f$, $f+h=3.6$	M1	**PI** by correct answer. For a clear attempt to find f **and** h
	$f=2.7$ and $h=0.9$	A1	**oe**
		4	

	Question 1 Total	**6**	

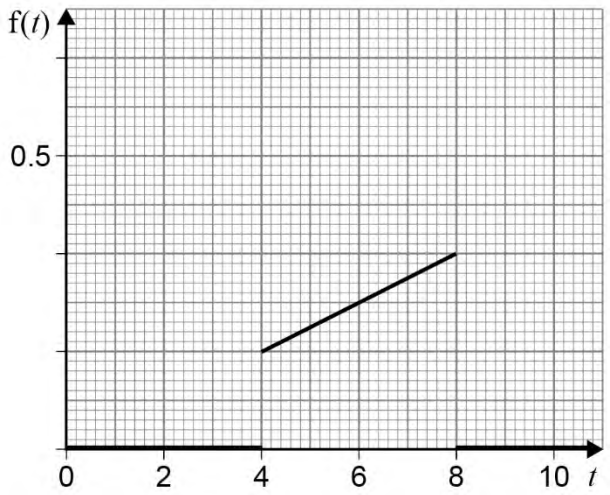
Q	Answer	Marks	Comments
2(a)	$X \sim \mathrm{B}(25,\ 0.9)$	B1	PI [or $Y \sim \mathrm{B}(25,\ 0.1)$]
	$\mathrm{H}_0 : p = 0.9$ $\mathrm{H}_1 : p > 0.9$	B1	$\mathrm{H}_0 : p = 0.1$ $\mathrm{H}_1 : p < 0.1$
	$\left[1 - P(X \leq 23) =\right] \quad 1 - 0.729 = 0.271$	M1	0.271
	$P(X \geq 24) = 0.271$	A1	$P(Y \leq 1) = 0.271$
	$0.271 > 0.1$	m1	Comparison of probability with 0.1 Allow **CR** for Y as {0} for **M1 A1** as is a **CR** for X is {25}
	Do not reject H_0	A1ft	Allow 'accept H_0'
	No evidence to suggest that Steve has a higher 'at least satisfactory' rating	E1	Correct statement must be in context and must follow from fully correct solution.
		7	

Q	Answer	Marks	Comments
2(b)(i)	0.0718	B1	
		1	

Q	Answer	Marks	Comments
2(b)(ii)	Accepting that the proportion of Steve's deliveries that are 'at least satisfactory' is greater than 90% when it is 90%	B1	Must be in context.
		1	

	Question 2 Total	**9**	

Q	Answer	Marks	Comments
3(a)	$F(8)=1$ so $64k-\frac{1}{3}=1$ **oe** **or** $F(4)=0$ so $16k-\frac{1}{3}=0$ **oe**	**M1**	Correct substitution of upper or lower limit
	$k=\frac{1}{48}$	**A1**	**oe** $k=0.02083\dot{}$
		2	

Q	Answer	Marks	Comments
3(b)		**B1**	Use of a straight line with positive gradient between $t=4$ **and** $t=8$
		B1	Correct end points for the correct straight line between $t=4$ **and** $t=8$ $\left[\left(4,\frac{1}{6}\right)\text{ and }\left(8,\frac{1}{3}\right)\right]$
		B1	Correct graph between for $0\le t\le 4$ **and** $8\le t\le 10$
		3	

	Question 3 Total	**5**	

Q	Answer	Marks	Comments
4	$H_0 : \mu = 70$ [days] $H_1 : \mu > 70$	B1	
	$\bar{x} = 70.6$	B1	Seen or used
	$s^2 = \frac{1}{99} \times \left(499000 - \frac{7060^2}{100} \right)$	M1	Attempt at variance formula Allow one slip
	$= 5.\dot{6}\dot{9}$	A1	Implied by correct answer (**oe** 188/33) **ARWT** 5.70 accept s = 2.38[6832566]
	$\bar{X} \sim \mathrm{N}\left(70, \frac{5.\dot{6}\dot{9}}{100} \right)$	M1	$\bar{X} \sim \mathrm{N}\left(\mu, \frac{s^2}{100} \right)$
	$z = \frac{70.6 - 70}{\sqrt{\frac{5.\dot{6}\dot{9}}{100}}}$	M1	Calculates z with their s^2, their $\bar{x}$ and their μ **or** for se = 0.238…. **or** $P\left(\bar{X} \geq 70.6\right) = 0.006$
	= 2.5[13791745]	A1	**or** sight of 70.56 for $\bar{x}$ **or** 0.00597
	z_{crit} = 2.3[26347931]	B1	**AWRT** 2.3 **or** for CR is $\bar{x} > 70.56$
	Reject H_0 as 2.5[137…] > 2.3[263…] **or** $z > z_{\text{crit}}$	A1ft	Allow 'accept H_1' Follow through their z and z_{crit} provided signs are consistent or comparison of 70.6 > 70.56 **or** comparison of their '0.00597' to 0.01
	Evidence to suggest that weed treatment has an increase in the 'mean number of days' claimed [at the 1% level]	E1	Must be consistent with their conclusion on whether to accept H_1 or not based on their z and z_{crit} if not explicitly stated
		10	

	Question 4 Total	10	

Q	Answer	Marks	Comments
5(a)	$\mathrm{P}(X=7)=\dfrac{\mathrm{e}^{-4}\times 4^7}{7!}$ or $\mathrm{P}(X=5)=\dfrac{\mathrm{e}^{-4}\times 4^5}{5!}$	B1	**or** correct use of $\mathrm{P}(X=n)$ **and** $\mathrm{P}(X=n+2)$
	$\mathrm{P}(X=7)=k\mathrm{P}(X=5)$ $\dfrac{\mathrm{e}^{-4}\times 4^7}{7!}=k\dfrac{\mathrm{e}^{-4}\times 4^5}{5!}$ $k=\dfrac{4^2\times 5!}{7!}$	M1	**PI** by sight of $k=\dfrac{16}{42}$ **eg** $\mathrm{P}(X=7)=\dfrac{\lambda^2}{7.6}\mathrm{P}(X=5)$
	$k=\dfrac{8}{21}$ or $k=0.\dot{3}80952\dot{}$	A1	If **B0 M0** scored then **SC2** for use of statistical tables which lead to a value for k as 0.381 or better
		3	

Q	Answer	Marks	Comments
5(b)(i)	$\mathrm{P}(X<3)=\mathrm{P}(X\leq 2)$ = 0.6767	M1 A1	**oe** **CAO**
		2	

Q	Answer	Marks	Comments
5(b)(ii)	$\lambda=14$	B1	Seen or used
	$\left[\mathrm{P}(8<X<17)=\right]\quad \mathrm{P}(X\leq 16)-\mathrm{P}(X\leq 8)$	M1	**PI** by correct final answer Condone one slip **oe** $0.7559-0.0621$
	= 0.6938 or 0.6939	A1	**CAO**
		3	

Q	Answer	Marks	Comments
5(c)(i)	$\sum fx = 446$ **or** $\sum fx^2 = 3554$	**M1**	Summary statistics found **PI** by at least one correct answer
	$\bar{x} = 6.97 \quad (3 \text{ sf})$	**A1**	**AWRT** 6.97
	$s^2 = 7.08 \quad (3 \text{ sf})$	**A1**	**AWRT** 7.08 Note: $s^2 = 6.97$ is **A0** [using n rather than $n - 1$ as denominator]
		3	

Q	Answer	Marks	Comments
5(c)(ii)	$\bar{x} \approx s^2$ [so this supports Dave's claim]	**B1ft**	Comment directly comparing their mean and variance
	comparison of their values $\bar{x}$ **or** s^2 to (5 + 2) = 7 [so this supports Dave's claim]	**B1ft**	**B0** for conflicting statements
		2	

	Question 5 Total	**13**	

Q	Answer	Marks	Comments
6(a)	$\left[\text{Variance} = \frac{1}{0.2^2} = \right] \quad 25 \ [\text{mins}]$	B1	
		1	

Q	Answer	Marks	Comments
6(b)	$\text{P}(T < 15) = 1 - \text{e}^{-0.2\times 15}$ [= 0.9502129316] = 0.950	M1 A1	Attempts to find correct probability using cdf of exponential or integration of pdf
		2	

Q	Answer	Marks	Comments
6(c)	$\text{P}(T > 30 \mid T > 20) = \text{P}(T > 10)$ $\text{P}(T > 10) = \text{e}^{-10\times 0.2}$ [= 0.1353352832] = 0.135	M1 M1 A1	A clear attempt at the no memory rule or $\left(\frac{\text{e}^{-30\times 0.2}}{\text{e}^{-20\times 0.2}}\right)$ or use of $\text{P}(T > 10)$ or $\text{P}(T < 10)$ Calculates their probability
		3	

Q	Answer	Marks	Comments
6(d)	$\text{e}^{-0.2t} = 0.6$ $-0.2t = \ln(0.6)$ $t = -5\ln(0.6)$ t = 2.55 [mins]	M1 A1	
		2	

	Question 6 Total	**8**	

Q	Answer	Marks	Comments
7	$\left[\int \frac{1}{x}\mathrm{f}(x)\mathrm{d}x =\right] \quad \frac{1}{60}\int_1^3 \frac{1}{x}\times\left(10x+x^3\right)\mathrm{d}x$	M1	PI
	$=\frac{1}{60}\left[10x+\frac{x^3}{3}\right]_1^3$	m1	correct integration with correct limits
	$=\frac{1}{60}\left(39-\left(10+\frac{1}{3}\right)\right)$		
	$=\frac{43}{90}$	A1	oe
		3	

	Question 7 Total	**3**	

Q	Answer	Marks	Comments
8	$H_0 : \mu = 200$ $H_1 : \mu \neq 200$	B1	Both hypotheses
	$[\text{dof}] \quad \nu = 9$	M1	**PI** by correct t_{crit}
	$t_{crit} = \pm 2.262$	A1	Seen or used
	$t = [\pm]\dfrac{200.3 - 200}{\left(\dfrac{0.55}{\sqrt{10}}\right)}$	M1	Seen or used must be +ve
	$= 1.72[487872...]$	A1	Follow through their t and t_{crit} Implied by correct conclusion in context
	Do not reject H_0 as $[-t_{crit}<] \, t < t_{crit}$	A1ft	Allow 'accept H_0' Must be consistent with their conclusion on whether to accept H_0 or not **or** their t and t_{crit} if not explicitly stated
	No evidence to suggest that the mean mass of the spheres produced by the machine has changed (at the 5% level)	E1	Must be in context and must not be definite.
		7	

	Question 8 Total	**7**	

Q	Answer	Marks	Comments
9(a)(i)	The midpoint of 170 and 200 is 185 and the normal distribution is symmetric	E1 E1	Allow calculation from 2 standardised equations
		2	

Q	Answer	Marks	Comments
9(a)(ii)	$\mathrm{P}(Z<2)-(1-\mathrm{P}(Z<2))$ = 0.9545	M1 A1	**PI** by a correct sketch. Allow standardising with their 'μ' or $2\mathrm{P}(Z<2)-1$ **CAO**
		2	

Q	Answer	Marks	Comments
9(b)(i)	$\left[\Phi^{-1}(0.45)=z=\right]-0.1257$ $\mathrm{P}\left(Z<\frac{190-200}{\sigma}\right)=0.45$ $\frac{190-200}{\sigma}=-0.1257$ $\sigma=79.6$	B1 M1 A1	Seen or used. Condone +0.1256613431 Attempt to standardise with a z-value
		3	

Q	Answer	Marks	Comments
9(b)(ii)	1	B1	
		1	

	Question 9 Total	8	

Q	Answer	Marks	Comments
10(a)	[from symmetry] $x = 5$	**B1**	
		1	

Q	Answer	Marks	Comments
10(b)	$a = \frac{0.1 - 0.05}{4 - 0}$ or $b = \frac{0.05 - 0.1}{10 - 6}$	**M1**	**PI**. Attempt to find gradient of a or b Allow one slip
	$a = 0.0125$	**A1**	For a or b correct, **oe**
	$b = -0.0125$	**B1**	**ft** for '–their a' or '–their b'
		3	

Q	Answer	Marks	Comments
10(c)	$\int_4^6 \left(q - n\left(x-5\right)^2\right)\mathrm{d}x = \left[qx - \frac{n\left(x-5\right)^3}{3}\right]_4^6$	M1	
	$\left[\text{Area beneath pdf between 0 and 4} = 0.3\right]$		
	$\left[qx - \frac{n\left(x-5\right)^3}{3}\right]_4^6 = 1 - 2\times 0.3$	M1	or $\left[qx - \frac{n\left(x-5\right)^3}{3}\right]_4^5 = 0.2$
	$2q - \frac{2n}{3} = 0.4$		
	$\left[\mathrm{f}\left(4\right) =\right] q - n = 0.1$	M1	sub in $x = 4$ **or** $x = 6$ into $\mathrm{f}\left(x\right)$
	$q = 0.25$ $n = 0.15$	A1 A1	
		5	

Q	Answer	Marks	Comments
10(d)	$\int_0^4 \left(\alpha x + 0.05\right)\mathrm{d}x$ $+\int_4^{4.5} \left(q - n\left(x-5\right)^2\right)\mathrm{d}x = 0.3 + \left[qx - \frac{n\left(x-5\right)^3}{3}\right]_4^{4.5}$	M1	Attempt to add two correct probabilities can be a trapezium for area **or** sight of 0.3 and $\frac{13}{160}$
	$= \frac{61}{160}$	A1	**oe** 0.38125
		2	
	Question 10 Total	11	