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INTERNATIONAL AS FURTHER MATHEMATICS

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphic calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

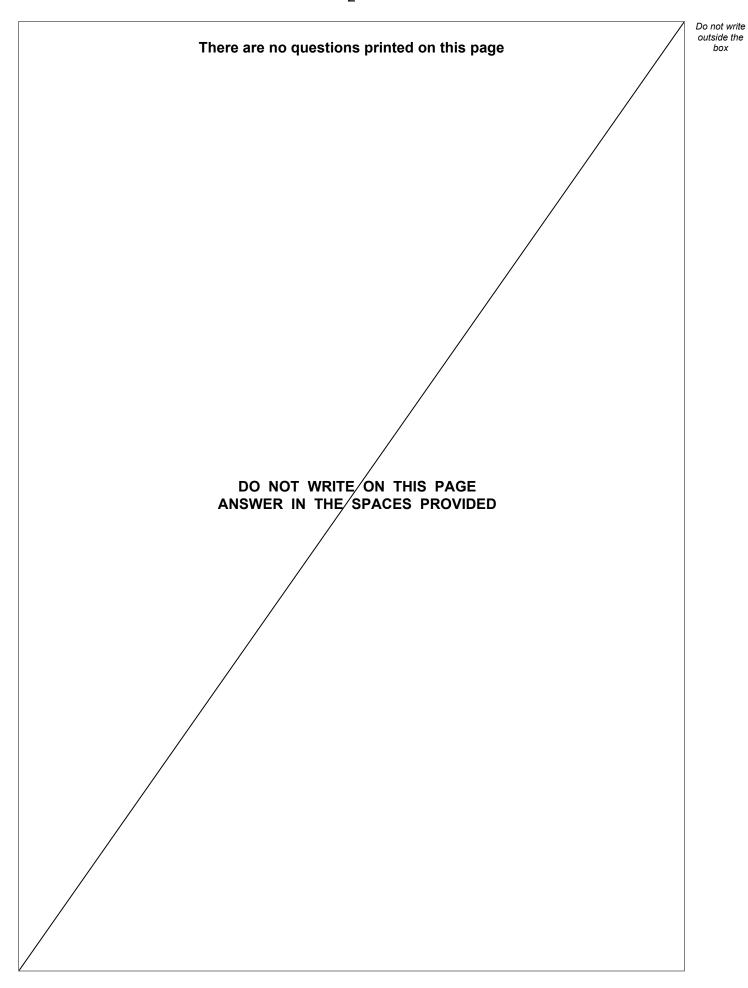
Information

- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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2 3 4 5 6 7 8 9 10 11	Question	Mark				
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Section A

Pure Mathematics

Answer all questions in the spaces provided.

A curve passes through the point (3, 4.8) and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{x+2}}{y(x+1)}$$

Use Euler's step-by-step method with a step length of 0.2 to estimate the value of y when x=3.4

Give your	answer	to three	decimal	places.
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Answer

Turn over ▶

[5 marks]



2	The matrix A is defined by A =	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
2	The matrix A is defined by A =	$\left[-\frac{\sqrt{3}}{2}\right]$	$\frac{1}{2}$

The matrix **B** is defined by $\mathbf{B} = \begin{bmatrix} 1 & 0 \\ p & 1 \end{bmatrix}$ where p is a constant and p > 1

2	(a)	Describe fully the transformation represented by the matrix A	
			[2 marks]

2	(b)	State the type of transformation represented by the matrix B	[1 mark]



2	(c)	Show that $AB - BA = k \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$ where k is a constant.	
			[4 marks]
2	(d)	The matrix C represents a reflection in the line $y = (\tan \theta)x$	
		Find the determinant of CAB	[3 marks]
		Answer	

Turn over ▶



3	The variables	X	and	Y	are related b	ov an	equation	of the	form
•	THE VARIABLES	2 L	arra	-	are related a	<i>,</i> ,	oquation	01 1110	

$$Y = nX + b$$

where n and b are constants.

3 (a) Given that
$$X = \log_{10} x$$
 and $Y = \log_{10} y$

show that $y = a x^n$ where a is a constant.

[2	m	arl	(S]
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3 (b) An experiment gives the following values of x and y

X	2	4	10	20	40
y	4.6	3.0	1.8	1.2	0.8

Complete the table below.

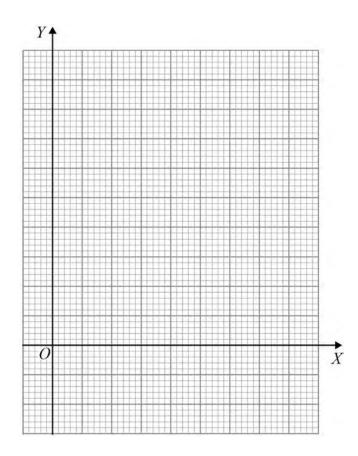
X	0.30		
Y	0.66		

[2 marks]

3 (c) (i) On the grid, plot Y against X

Draw a line of best fit for the points you have plotted.

[2 marks]



3 (c) (ii) Use your line of best fit to find estimates for a and n

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a = _____

n =

10



4 The function f is defined by $f(x) = x^3 - 4x^2 + 6$

4 (a) Show that the equation f(x) = 0 has a root, α , in the interval $3 < \alpha < 4$

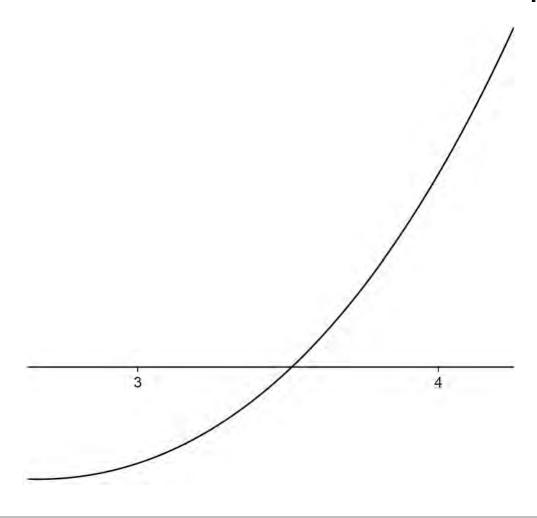
[2 marks]

4 (b) The diagram below shows part of the graph of y = f(x)

Draw suitable lines on the diagram to show how linear interpolation can be used to find the first two approximations, x_1 and x_2 , to α

Mark the positions of x_1 and x_2 on the x-axis.

[4 marks]



10

4 (c)	Use linear interpolation three times to find an estimate for α	
	Give your answer to two decimal places.	[4 marks]
	Answer	
	Turn over for the next question	



The matrix \mathbf{Q} is defined by $\mathbf{Q} = \begin{bmatrix} -6 & 4 \\ -2 & 1 \end{bmatrix}$	
The matrix R is defined by $\mathbf{R} = \begin{bmatrix} -2 & 3 \\ -16 & 37 \end{bmatrix}$	
The matrix P is such that $4P + PQ = R$	
Find P	[5 mark



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Section B

	Statistics				
		Answer all questions in the spaces provided.			
6		Customers at a tennis club may rent a racket or use their own racket. The probability that a randomly selected customer rents a racket is 0.18 . The random variable $\it R$ represents the number of customers on a particular day up to and including the first customer who rents a racket.			
6	(a)	Find the exact probability that the third customer is the first to rent a racket. [1 mark]			
		Answer			
6	(b)	Find the probability that the first six customers do not rent a racket, giving your answer to three significant figures. [2 marks]			
		Answer			



6	(c)	Find $\mathrm{E}(R)$ giving your answer to three significant figures.	[1 mark]
6	(d)	Answer	
			[1 mark]
		Answer	
6	(e)	The random variable S represents the number of customers on a particular day including the first customer who rents a pair of shorts. The variance of S is 3 R and S are independent. Find $\mathrm{Var}(R-S)$ giving your answer to three significant figures.	
			[1 mark]
		Answer	

Turn over ►



7		The probability generating function of a Bernoulli random variable X_i is given by		
		$G_{X_i}(t) = 0.22 + 0.78t$		
		X_1 and X_2 are independent.		
		Let Y be the random variable such that $Y = X_1 + X_2$		
7	(a)	Find $G_Y''(t)$ [4 marks]		
		Answer		



7 (b)	Find $Var(Y)$		Do not write outside the box
		[3 marks]	
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	Answer		7

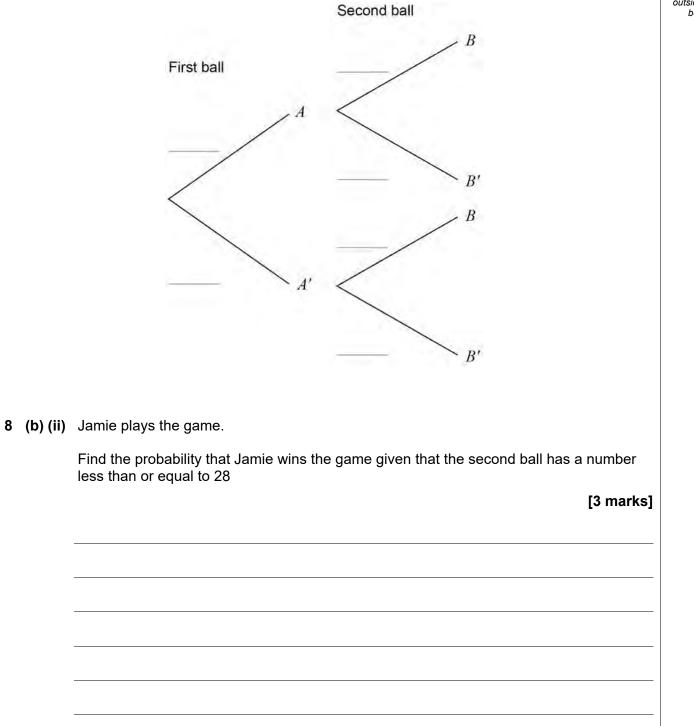
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8		In a bag there are 35 balls.
		Each ball has a unique whole number on it from 1 to 35
8	(a)	A ball is randomly selected from the bag. Find the probability that the number on the ball is greater than 28 [1 mark]
		Answer
8	(b)	In a game, a ball is randomly selected from the bag and its number is recorded.
		The ball is not replaced in the bag.
		A second ball is then randomly selected from the bag and its number is recorded.
		Event A is 'the first ball selected from the bag has a number greater than 28'
		Event B is 'the second ball selected from the bag has a number greater than 28'
		The game is won if the first ball has a number greater than 28 and the second ball has a number less than or equal to 28
8	(b) (i)	Complete the tree diagram by giving the probability associated with each branch. [3 marks]



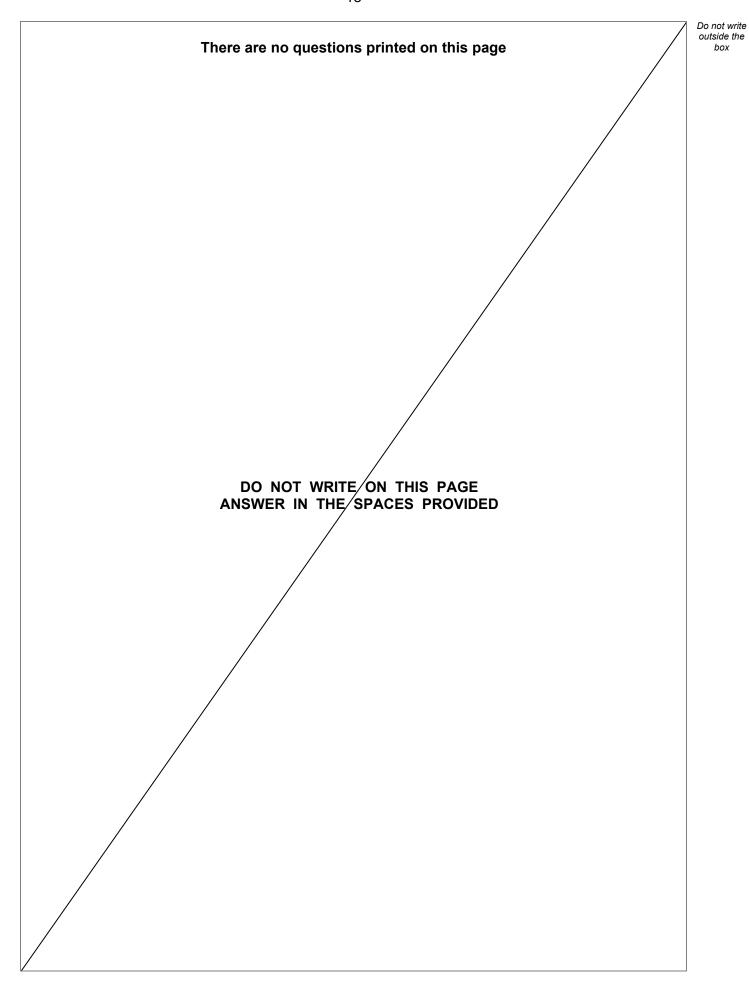
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Answer			









Section C

Mechanics

Answer all questions in the spaces provided.

9	A ball of mass $0.3~{\rm kg}$ is moving at a speed of $7~{\rm m~s^{-1}}$ when it collides with a vertical wall.
	The velocity of the ball is perpendicular to the wall when the ball collides with the wall.
	The ball rebounds with speed $5~{\rm m~s^{-1}}$
	Calculate the magnitude of the impulse exerted by the wall on the ball. [2 marks]
	Answer

Turn over for the next question

10	A particle is moving between two points, A and B, on a line.	
	The speed of the particle can be found using the formula	
	$v^2 = \omega^2 \left(\left(\frac{d}{2} \right)^2 - x^2 \right)$	
	where	
	v is the speed of the particle in m s ⁻¹	
	ω is a constant	
	d is the distance between the points A and B in metres	
	x is the distance of the particle from the midpoint of AB in metres.	
	Use dimensional analysis to find the units of ω	[3 marks]
		[3 marks]
	Answer	



F	A boat is travelling at a speed of 6 m s^{-1} relative to the water.
T	The direction of the boat's motion relative to the water is due east.
T	The water is flowing towards the south west at a speed of 2 m s ⁻¹
	Find the resultant velocity of the boat, giving the direction as a bearing to the nearest legree. [5 n
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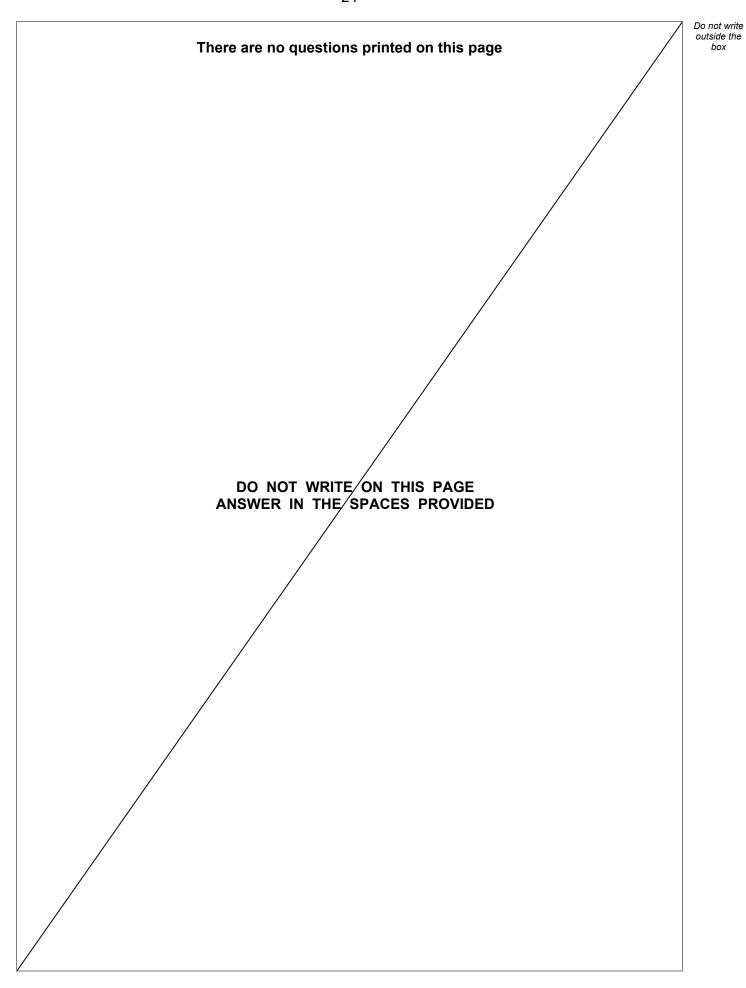
12		Three spheres A, B and C of equal radius are at rest on a straight horizontal line.
		The mass of A is 3 kg
		The mass of B is 2 kg
		The mass of C is 1 kg
		Sphere A is set in motion so that it moves with speed 5 m s ⁻¹ towards sphere B
		The two spheres collide directly.
		The coefficient of restitution between \emph{A} and \emph{B} is 0.4
12	(a)	Find the speed of A and the speed of B after their collision. [5 marks]
		Speed of <i>A</i> = Speed of <i>B</i> =



)	The sphere B then collides directly with sphere C	
	The coefficient of restitution between B and C is e	
	The spheres A and B do not collide again.	
	Show that $e \le \frac{m}{n}$ where m and n are integers.	[5 mar

END OF QUESTIONS







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Question number	Additional page, if required. Write the question numbers in the left-hand margin.



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