INTERNATIONAL A-LEVEL

MATHEMATICS

MA05

(9660/MA05) Unit M2 Mechanics

Mark scheme

January 2024

Version: 1.0 Final

241XMA05/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from oxfordaqa.com

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)(i)	Taking moments about *B*		
	Anticlockwise moments		
	$= (3.60 - x) \times 800 \times 9.8$	M1	**oe**
	Clockwise moments		
	$= 3.60 \times 6100 \;\; [= 21960]$	M1	**oe**
	Principle of moments		
	$3.60 \times 6100 = (3.60 - x) \times 800 \times 9.8$		
	$x = 3.60 - \dfrac{3.60 \times 6100}{800 \times 9.8}$		
	$x = 0.80$	A1	**AWRT** 0.80 Note: unrounded answer is 0.798…
		3	

Q	Answer	Marks	Comments
1(a)(ii)	[Let R_B be the reaction force on the front wheels]		
	Forces in equilibrium gives		
	$R_B + 6100 = 800 \times 9.8$		
	$R_B = 1700 \;\; [\text{N, to 2 sf}]$	B1	**AWRT** 1700 [N] from correct working Note: unrounded answer is 1740 [N]
		1	

Q	Answer	Marks	Comments
1(b)(i)	$W = Fd$ $W = \left(0.95 \times 20^2\right) \times \left(20 \times 7.5\right)$ $W = 57{,}000\ [\text{J}]$	 **M1** **A1**	 **oe**
		2	

Q	Answer	Marks	Comments
1(b)(ii)	$F = \dfrac{P}{v} - 0.95v^2$ $F = \dfrac{780 \times 10^3}{50} - 0.95 \times 50^2$ $F = 13{,}000\ [\text{N, to 2 sf}]$	 **M1** **A1**	 **oe** Note: unrounded answer is 13,225 N
		2	

Q	Answer	Marks	Comments
1(b)(iii)	$\dfrac{P}{v} - 0.95v^2 = 0$ $v = \sqrt[3]{\dfrac{P}{0.95}}$ $v = \sqrt[3]{\dfrac{780 \times 10^3}{0.95}}$ $v = 94\ \left[\text{m s}^{-1}\text{, to 2 sf}\right]$	 **M1** **A1**	 Note: unrounded answer is 93.639… [m s^{-1}]
		2	

	Question 1 Total	**10**	

Q	Answer	Marks	Comments
2(a)	[Equilibrium perpendicular to slope]		
	$[N =]P + mg\cos(30°)$	M1	N = normal reaction force
	[Equilibrium parallel to slope]		
	$[f =]mg\sin(30°)$	M1	f = friction **PI** by use or sight of 83.3 [N]
	$mg\sin(30°) \le \mu(P + mg\cos(30°))$	m1	Use of $f \le \mu N$ or $f = \mu N$
	$mg\left(\dfrac{\sin(30°)}{\mu} - \cos(30°)\right) \le P$		
	$17 \times 9.8 \times \left(\dfrac{\sin(30°)}{0.4} - \cos(30°)\right) \le P$		$208.25 - 144.279... \le P$
	$63.97... \le P$		
	Least value of P is 64 [to 2 sf]	A1	**AWRT** 64
		4	

Q	Answer	Marks	Comments
2(b)(i)	The block moves in a direction that is perpendicular to the force P newtons [so no work is done on the block by the force P newtons]	E1	**oe**
		1	

Q	Answer	Marks	Comments
2(b)(ii)	Work is done against friction	E1	**oe**
		1	

Q	Answer	Marks	Comments
2(b)(iii)	$ma = mg\sin(30°) - \mu(P + mg\cos(30°))$	M1	Forms equation for the resultant force acting on the block, or better
	$a = 9.8 \times \sin(30°) - 0.4 \times \left(\frac{40}{17} + 9.8 \times \cos(30°)\right)$		
	$a = 0.56\ [\text{m s}^{-2}]$	A1	**AWRT** 0.56 Note: unrounded answer is 0.5640… [m s^{-2}]
		2	

Q	Answer	Marks	Comments
2(b)(iv)	$s = ut + \frac{1}{2}at^2$		
	$s = 0.5 \times 0.564... \times 6^2$	M1	Use of $s = ut + \frac{1}{2}at^2$ with $u = 0$ and their $a \neq 9.8$ from **(b)(ii)**
	$s = 10\ [\text{m}]$	A1ft	**AWRT** 10 **ft** their acceleration from **(b)(iii)** Note: unrounded answer is 10.15… [m]
		2	

	Question 2 Total	**10**	

Q	Answer	Marks	Comments
3(a)	$[\mathbf{r} =]\ (1.5\cos(4t) + c_1)\mathbf{i} + (1.5\sin(4t) + c_2)\mathbf{j}$	M1 A1	**M1**: Uses integration to find at least one correct component **A1**: Finds both correct components Condone no constants of integration for **M1 A1**
	When $t = 0$ $[\mathbf{r} =]\ (1.5 + c_1)\mathbf{i} + c_2\mathbf{j} = 1.5\mathbf{i}$ $\Rightarrow c_1 = 0$ and $c_2 = 0$		
	$[\mathbf{r} =]\ 1.5\cos(4t)\mathbf{i} + 1.5\sin(4t)\mathbf{j}$	B1	Finds correct position vector at time t by explicitly showing both constants of integration are zero/the constant of integration vector is zero
	$[\lvert\mathbf{r}\rvert =]\ \sqrt{(1.5\cos(4t))^2 + (1.5\sin(4t))^2}$	m1	
	$[\lvert\mathbf{r}\rvert =]\ 1.5\ [\text{m}]$ so *A* is a constant distance away from *O*, meaning it moves on a circular path	A1	Must have reference to constant distance, not just 1.5 [m]
		5	

Q	Answer	Marks	Comments
3(b)(i)	$[\omega =]$ 4 rad s^{-1}	**B1 B1**	**B1**: Correct value **B1**: Correct units
		2	

Q	Answer	Marks	Comments
3(b)(ii)	$m\omega^2 r = 4.9 \times 4^2 \times 1.5$ or $\frac{mv^2}{r} = \frac{4.9 \times 6^2}{1.5}$	**M1**	Use of $m\omega^2 r$ with their ω and $r = 1.5$ or use of $\frac{mv^2}{r}$ with $r = 1.5$ or use of **F** = m**a** and differentiation of **v** **PI** By correct magnitude of force
	120 [N, to 2 sf]	**A1**	**CAO, AWRT** 120 Exact answer is 117.6 [N]
	Towards *O*	**B1**	**oe**, such as $-\cos(4t)\mathbf{i} - \sin(4t)\mathbf{j}$
		3	

Q	Answer	Marks	Comments
3(c)	[Tension in string = 117.6 N]		
	$117.6 = mg$	**M1**	
	$m = 12$	**A1**	**AWRT** 12 Note: m = 12.244… if using T = 120 N
		2	

	Question 3 Total	**12**	

Q	Answer	Marks	Comments
4(a)	$s = ut + \frac{1}{2}at^2$		
	$t = \sqrt{\frac{2d}{g}}$ or $t = \frac{d}{u}$	B1	Time to travel from O to A
	$d[= ut] = u\sqrt{\frac{2d}{g}}$ or $d = \frac{1}{2}g\left(\frac{d}{u}\right)^2$	M1	
	$u = \sqrt{\frac{gd}{2}}$	A1	oe
		3	

Q	Answer	Marks	Comments
4(b)	[Vertical component of velocity immediately before colliding with ground at A]		
	$v = u + at$		
	$\left[v = 0 + g \times \sqrt{\frac{2d}{g}}\right]$		
	$v = \sqrt{2gd}$	B1	or $v = 2u$
	Speed immediately before colliding with ground at A		
	$= \sqrt{(2u)^2 + u^2}$	M1	or $= \sqrt{(2gd) + u^2}$ or $= \sqrt{(2gd) + \left(\frac{gd}{2}\right)}$
	$= \sqrt{5}\ u$	A1	
		3	

	Question 4 Total	**6**	

Q	Answer	Marks	Comments
5(a)	The logo is symmetric in the line through *C*, *B* and *M*	**E1**	Allow any mention of symmetry or that the centres of mass of the circle and the triangle lie on *CBM*
		1	

Q	Answer	Marks	Comments
5(b)	[C.O.M. of the circle from *M* is]		
	$2d\sin(60°)+\frac{d}{2}\quad\left[=d\left(\sqrt{3}+\frac{1}{2}\right)\right]$	**B1**	**PI**
	[C.O.M. of the triangle from *M* is]		
	$\frac{1}{3}\times 2d\sin(60°)\quad\left[=d\frac{\sqrt{3}}{3}\right]$	**B1**	**PI**
	$\rho\times\left(\frac{1}{2}\times 4d^2\sin(60°)+\frac{\pi d^2}{4}\right)\bar{Y}$ $=\rho\times\frac{1}{2}\times 4d^2\sin(60°)\times\frac{2d\sin(60°)}{3}$ $+\rho\times\frac{\pi d^2}{4}\times\left(2d\sin(60°)+\frac{d}{2}\right)$	**M1 m1 A1**	**M1**: At least one of the three terms correct **m1**: At least two of the three terms correct **A1**: Fully correct equation Condone lack of ρ
	$\left(\sqrt{3}\,d^2+\frac{\pi d^2}{4}\right)\bar{Y}=\sqrt{3}\,d^2\times\frac{\sqrt{3}}{3}d$ $+\frac{\pi d^2}{4}\times d\left(\sqrt{3}+\frac{1}{2}\right)$		
	$d^2\left(\sqrt{3}+\frac{\pi}{4}\right)\bar{Y}=d^3\left(1+\frac{\pi}{4}\left(\sqrt{3}+\frac{1}{2}\right)\right)$		
	$\bar{Y}=\frac{4+\pi\left(\sqrt{3}+\frac{1}{2}\right)}{4\sqrt{3}+\pi}\times d$		
	$k=1.09$ [3 sf]	**A1**	Allow $k=\frac{4+\pi\left(\sqrt{3}+\frac{1}{2}\right)}{4\sqrt{3}+\pi}$
		6	

Q	Answer	Marks	Comments
5(c)	[Let α be the angle *OM* makes with the vertical]		
	$\tan\alpha = \dfrac{1.094...\times d}{d}$	M1	$\tan\alpha = \dfrac{\overline{Y}}{d}$ using their $\overline{Y}$
	$\alpha = 48°$	A1	Allow $47°$ as final answer if rounded answer of $1.09d$ used
		2	

	Question 5 Total	9	

Q	Answer	Marks	Comments
6(a)	Loss in GPE for *A* $mg\Delta h = 3 \times 9.8 \times 5$		
	$= 147$ [J]	B1	Sight or use of 147 or $15g$ or use of 29.4 **PI** by $v = 7\sqrt{2}$ [m s^{-1}]
	Loss in GPE = Gain in KE for *A* $147 = \frac{1}{2}mv^2$		
	$v = \sqrt{\frac{2 \times 147}{3}} = 7\sqrt{2}$ [m s^{-1}]	B1	**oe,** eg $\sqrt{10g}$ **AWRT** 9.9
	Total momentum of system before collision		
	[$p =$] $3 \times 7\sqrt{2} = 21\sqrt{2}$ [kg m s^{-1}]	M1	**oe,** eg $3\sqrt{10g}$ **PI** by sight or use of **AWRT** 30 Total momentum before collision **ft** their speed of *A* before the collision
	Conservation of momentum $21\sqrt{2} = 3 \times (\pm 2) + 10v_B$	M1	**oe,** eg $3\sqrt{10g} = 3 \times (\pm 2) + 10v_B$ Condone + or – instead of ± Total momentum after collision
	If *A* moves in the <u>same</u> direction after the collision, then speed of *B* is		
	$v_B = 2.37$ [m s^{-1}]	A1ft	Answer given to 3 sf **ft** their speed of *A* before the collision
	If *A* moves in the <u>opposite</u> direction after the collision, then speed of *B* is		
	$v_B = 3.57$ [m s^{-1}]	A1ft	Answer given to 3 sf **ft** their speed of *A* before the collision
		6	

Q	Answer	Marks	Comments
6(b)	$\left[\Delta p_B =\right] 10 \times 2.37 = 23.7 \left[\text{kg m s}^{-1}\right]$	**M1**	**PI** by correct answer
	$\left[F = \dfrac{\Delta p}{\Delta t} =\right] \dfrac{23.7}{0.20} = 120 \left[\text{N, to 2 sf}\right]$	**A1ft**	**AWRT** 120 N
		2	

Q	Answer	Marks	Comments
6(c)	Total KE of system after collision $0.5 \times 3 \times 2^2 + 0.5 \times 10 \times 3.57^2$ $= 69.7245\ [\text{J}]$	**M1**	**AWRT** 70 **PI** by correct answer
	KE lost during the collision $147 - 69.7245$ $= 77\ [\text{J, to 2 sf}]$	**A1**	**AWRT** 77 **CAO**
		2	

	Question 6 Total	**10**	

Q	Answer	Marks	Comments
7(a)	Resultant force acting on the particle $\left[\begin{bmatrix} 10\cos^2 t \\ 30t \\ 50e^{-2t} \end{bmatrix} + \begin{bmatrix} 10\sin^2 t \\ 90t^2 \\ -31 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -49 \end{bmatrix} = \right]$ $\begin{bmatrix} 10 \\ 30t + 90t^2 \\ 50e^{-2t} - 80 \end{bmatrix}$ [N]	**M1**	**oe** At least two components correct Condone unsimplified
	Acceleration of the particle $\mathbf{a} = \begin{bmatrix} 2 \\ 6t + 18t^2 \\ 10e^{-2t} - 16 \end{bmatrix}$ [m s^{-2}]	**A1**	Condone unsimplified Condone –6.2 instead of –16 in **k** component
	Velocity of the particle $\mathbf{v} = \begin{bmatrix} 2t + c_1 \\ 3t^2 + 6t^3 + c_2 \\ -5e^{-2t} - 16t + c_3 \end{bmatrix}$	**M1 A1**	**M1**: At least one correct component **A1**: All three components correct Condone no constants of integration for **M1 A1**
	$\left[\text{When } t = 0,\ \mathbf{v} = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ -5 + c_3 \end{bmatrix}\right]$		
	$\mathbf{v} = \begin{bmatrix} 2t + 3 \\ 3t^2 + 6t^3 - 1 \\ -5e^{-2t} - 16t + 10 \end{bmatrix}$ [m s^{-1}]	**A1**	**CAO**
		5	

Q	Answer	Marks	Comments
7(b)	$\mathbf{r} = \begin{bmatrix} t^2 + 3t + c_4 \\ t^3 + \frac{3}{2}t^4 - t + c_5 \\ \frac{5}{2}\mathrm{e}^{-2t} - 8t^2 + 10t + c_6 \end{bmatrix}$	**M1 A1**	**M1**: At least one correct component **A1**: All three components correct Condone no constants of integration for **M1 A1**
	$\mathbf{r} = \begin{bmatrix} t^2 + 3t \\ t^3 + \frac{3}{2}t^4 - t \\ \frac{5}{2}\mathrm{e}^{-2t} - 8t^2 + 10t - \frac{5}{2} \end{bmatrix}$	**A1**	Fully correct position vector
	$\lvert\mathbf{r}\rvert^2 = \left(t^2 + 3t\right)^2 + \left(t^3 + \frac{3}{2}t^4 - t\right)^2 + \left(\frac{5}{2}\mathrm{e}^{-2t} - 8t^2 + 10t - \frac{5}{2}\right)^2$	**M1**	Writes down an unsimplified expression for the distance or distance-squared in terms of t or evaluates their position vector at $t = 0.8$ **PI**
	When $t = 0.8$ $\lvert\mathbf{r}\rvert^2 = (3.04)^2 + (0.3264)^2 + (0.88474...)^2$ $[= 10.13090412]$	**m1**	Substitutes $t = 0.8$ into their expression for $\lvert\mathbf{r}\rvert^2$ or $\lvert\mathbf{r}\rvert$ **PI** by at least two terms correct or correct final answer Dependent on both previous **M1** marks
	$\lvert\mathbf{r}\rvert = 3.18$ [m, to 3 sf]	**A1**	**CAO** to 3 sf
		6	

	Question 7 Total	**11**	

Q	Answer	Marks	Comments
8(a)	[Time taken for the golf ball to cover the 90 m horizontal displacement]		
	$T = \dfrac{90}{30\cos\theta}$	B1	Any subject
	$[s =]\ 30\sin\theta\, T - 0.5 \times 9.8 \times T^2$	M1 A1	**M1**: Use of $s = ut + \frac{1}{2}at^2$ with $u = 30\sin\theta$ and $a = \pm 9.8$ **PI** **A1**: Fully correct
	$[s =]\ 30\sin\theta \times \dfrac{90}{30\cos\theta} - 0.5 \times 9.8 \times \left(\dfrac{90}{30\cos\theta}\right)^2$	m1	Eliminating T **PI**
	$[s =]\ 90\tan\theta - \dfrac{44.1}{\cos^2\theta}$		Correct simplification with second term in $\cos^2\theta$
	$[s =]\ 90\tan\theta - 44.1 \times \left(1 + \tan^2\theta\right)$	m1	Use of $\sec^2\theta = 1 + \tan^2\theta$ **PI**
	$-2.4 = 90\tan\theta - 44.1 - 44.1\tan^2\theta$	B1	Use of $s = -2.4$
	$\left[44.1\tan^2\theta - 90\tan\theta + 41.7 = 0\right]$		
	$\tan\theta = 1.3296...,\ 0.7111...$	m1	**PI**
	$\theta = 53.054...,\ 35.417...$	A1	At least one unrounded value of θ
	$\theta = 53.1$, or $\theta = 35.4$	A1	Both values of θ to 3 sf and no others
		9	

Q	Answer	Marks	Comments
8(b)	$T = \dfrac{90}{30\cos\theta}$		
	$T = \dfrac{90}{30\cos(53.054...^\circ)}$	M1	or use of their largest angle with $s = ut + \frac{1}{2}at^2$, $u = 30\sin\theta$ and $a = \pm 9.8$ **PI**
	$T = 5.0$ [s, to 2 sf]	A1ft	**ft** their larger angle from **part (a)** Note: unrounded answer is 4.99… [s]
		2	

Q	Answer	Marks	Comments
8(c)	The assumption is unlikely to be true [as the golf ball is moving quickly through the air]	E1	Allow any sensible comment
		1	

	Question 8 Total	**12**	