OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL A-LEVEL
MATHEMATICS
MA03

(9660/MA03) Unit P2 − Pure Mathematics

Mark scheme

June 2022

Version: 1.0 Final

2 0 6 X M A 0 3 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	<table><tr><th>x</th><th>y</th></tr><tr><td>0.1</td><td>$\sin\left(e^{0.1}\right) = 0.8935409$</td></tr><tr><td>$0.3$</td><td>$\sin\left(e^{0.3}\right) = 0.9756924$</td></tr><tr><td>$0.5$</td><td>$\sin\left(e^{0.5}\right) = 0.9969654$</td></tr><tr><td>$0.7$</td><td>$\sin\left(e^{0.7}\right) = 0.9034885$</td></tr></table>	B1 M1	All 4 correct x values (and no extra used) **PI** by 4 correct y values At least 3 correct y values in exact form or decimals, rounded or truncated to 2 dp or better (in table or formula) (**PI** by AWRT correct answer)
	$0.2\times[0.89...+0.97...+0.99...+0.90...]$	m1	Correct sub into formula with $h=0.2$ **OE** and at least 3 correct y values either listed, with + signs, or totalled. (**PI** by AWRT correct answer)
	$=0.754$	A1	**CAO**, must see this value exactly and no error seen
		4	
1(b)(i)	$f(x)=\sin\left(e^{x}\right)-3x+2$ $f(0.8)=0.39...$ $f(0.9)=-0.069...$	M1	Or reverse Both values rounded or truncated to at least 1sf
	Change of sign, $0.8<\alpha<0.9$	A1	Must have both statement and interval in words or symbols **or** comparing 2 sides: at 0.8, $\sin\left(e^{0.8}\right)>3\times0.8-2$; at 0.9, $\sin\left(e^{0.9}\right)<3\times0.9-2$ **(M1)** Conclusion as before **(A1)**
		2	
1(b)(ii)	$x_2=0.931$ $x_3=0.856$	B1 B1	
		2	
	Total	8	

Q	Answer	Marks	Comments
2(a)	$\left[\frac{dy}{dx}=\right]\frac{(2x+5)\times(-3)-(1-3x)\times 2}{(2x+5)^2}$	M1	or use of product rule **PI** by correct answer
	$=\frac{-17}{(2x+5)^2}$	A1	
		2	
2(b)	$\left[\frac{dy}{dx}=\right]\ \frac{-17}{(2x+5)^2}\times\frac{(2x+5)}{(1-3x)}$	M1	their $(\mathbf{a})\times\frac{2x+5}{1-3x}$ **oe** such as $\frac{A}{1-3x}-\frac{B}{5+2x}$ with $A<0$ and $B>0$
	$=\frac{-17}{(2x+5)(1-3x)}$	A1	**ft** their **(a)** **ACF** such as $\frac{-3}{1-3x}-\frac{2}{2x+5}$
		2	
	Total	4	

Q	Answer	Marks	Comments
3(a)	$[16\sin\theta + 30\cos\theta =]$ $R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$	M1	Implied by $16 = R\cos\alpha$ and $30 = R\sin\alpha$
	$\alpha = 1.08$	A1	
	$R = 34$	B1	
	$\left[\ 34\sin\left(\theta + 1.08\right)\right]$		
		3	
3(b)(i)	[Min value =] –34	B1ft	
		1	
3(b)(ii)	$[\theta =]$ 3.63	B1	**oe** such as –2.65 Accept 3.6 Accept $1.5\pi - 1.08$ $[\pm 2n\pi]$
		1	
	Total	5	

Q	Answer	Marks	Comments
4(a)(i)	$18(-0.5)^3 + b(-0.5)^2 + c(-0.5) - 4 = 0$	M1	At least one correct substitution or **M1** use of long division $9x^2 + \left(\frac{b-9}{2}\right)x$
	$18\left(\frac{1}{3}\right)^3 + b\left(\frac{1}{3}\right)^2 + c\left(\frac{1}{3}\right) - 4 = -5$	A1	Both substitutions correct or **A1** for $9x^2 + \left(\frac{b-9}{2}\right)x + \frac{1}{2}\left(c - \frac{b-9}{2}\right)$
	$b - 2c = 25$ $b + 3c = -15$	m1	Attempt to solve their simultaneous equations
	$b = 9, \quad c = -8$	A1	Both values correct
		4	
4(a)(ii)	$\mathrm{f}(x) = (2x+1)(9x^2 - 4)$	M1	**PI**
	$= (2x+1)(3x+2)(3x-2)$	A1	Condone p = 3 and q = 2
		2	
4(b)	$\frac{\mathrm{f}(x)}{(3x+2)(x^2-2)} = \frac{(2x+1)(3x-2)}{(x^2-2)}$	M1	Substitutes their $\mathrm{f}(x)$ and correctly cancels the factor of $3x + 2$ in numerator and denominator
	$= \frac{6x^2 - x - 2}{x^2 - 2}$		
	$= \frac{6x^2 - 12 - x + 10}{x^2 - 2}$		
	$= 6 + \frac{10 - x}{x^2 - 2}$	A1	Be convinced
		2	
	Total	**8**	

Q	Answer	Marks	Comments
5(a)	$3\sec^2 Y = 2 - 4\tan Y$ $3\left(1+\tan^2 Y\right) = 2 - 4\tan Y$	M1	Correct use of trig identity
	$3\tan^2 Y + 4\tan Y + 1 = 0$		
	$\tan Y = -1, -\frac{1}{3}$	m1	Attempt to solve their quadratic
	$Y = -0.785 \left(\text{or } -\frac{\pi}{4}\right), \quad -0.322$	A1	At least one correct Y value **PI** by a correct value of x Condone Y value(s) given in degrees, e.g. –45°, –18.4…°
	$x = 0.11,\ 1.68;\ 0.34,\ 1.91$	B2,1	2 dp or better **B1**: at least 3 correct values for x **B2**: all 4 correct values for x and no others Allow use of X, $2x - 1$ etc in place of Y throughout
		5	
5(b)	$\frac{\sin 4x(1-\cos 2x)}{\cos 2x(1-\cos 4x)} = \frac{2\sin 2x(1-\cos 2x)}{1-\cos 4x}$	M1	Use of $\sin 4x = 2\sin 2x \cos 2x$
	$\frac{2\sin 2x(1-\cos 2x)}{1-\cos 4x}$ $= \frac{2\sin 2x\left(1-1+2\sin^2 x\right)}{1-1+2\sin^2 2x}$	m1	Use of cos4x trig identity
	$\frac{2\sin 2x\left(1-1+2\sin^2 x\right)}{1-1+2\sin^2 2x} = \frac{2\sin^2 x}{\sin 2x}$	m1	Cancelling of $\sin 2x$ **oe**
	$\frac{2\sin^2 x}{\sin 2x} = \frac{\sin x}{\cos x} = \tan x$	A1	**AG** Be convinced
		4	
	Total	9	

Q	Answer	Marks	Comments
6(a)	$=1+\left(-\frac{1}{3}\right)\times(-x)+\frac{\left(-\frac{1}{3}\right)\times\left(-\frac{4}{3}\right)\times(-x)^2}{2}+\frac{\left(-\frac{1}{3}\right)\times\left(-\frac{4}{3}\right)\times\left(-\frac{7}{3}\right)\times(-x)^3}{6}$	M1 A1	**M1**: At least 3 terms correct (unsimplified) **A1**: All terms correct (unsimplified)
	$=1+\frac{1}{3}x+\frac{2}{9}x^2+\frac{14}{81}x^3$	A1	
		3	
6(b)(i)	$\sqrt[3]{\frac{1}{1-2x}}=(1-2x)^{-\frac{1}{3}}$ $=1+\frac{1}{3}\times 2x+\frac{2}{9}\times(2x)^2+\frac{14}{81}\times(2x)^3$	M1	Substitutes $2x$ in to their **(a)**
	$=1+\frac{2}{3}x+\frac{8}{9}x^2+\frac{112}{81}x^3$	A1	
		2	
6(b)(ii)	$-0.5<x<0.5$	B2	**oe** such as $\lvert x\rvert<0.5$ **B1** for $-0.5\le x\le 0.5$
		2	
6(c)	$[x=0.1]\quad 1+\frac{2}{3}\times0.1+\frac{8}{9}\times0.1^2+\frac{112}{81}\times0.1^3$ $[=1.0769...]$	B1ft	Substitutes $x=0.1$ into their **(b)(i)**
	$[x=0.1]\quad \frac{1}{\sqrt[3]{1-0.2}}=\frac{1}{\sqrt[3]{0.8}}=\sqrt[3]{\frac{10}{8}}$	M1	**oe**
	$\sqrt[3]{10}\quad[=2\times1.0769...]=2.154$	A1	**AWRT** 2.154 from correct use of binomial expansion Value calculated using this binomial expansion is 2.153876543
		3	
	Total	**10**	

Q	Answer	Marks	Comments
7(a)	Translation $\begin{bmatrix}0\\k\end{bmatrix}$ or Stretch + either I or II	M1	
	$k=1$ / I: Parallel to y-axis II: SF $\frac{1}{3}$	A1	
	[Followed by] Stretch + either I or II / [Followed by] Translation $\begin{bmatrix}0\\k\end{bmatrix}$	M1	
	I: Parallel to y-axis II: SF $\frac{1}{3}$ / $k=\dfrac{1}{3}$	A1	
		4	
7(b)	$x=\dfrac{1+\cos y}{3}$	M1	Interchanging x and y
	$3x-1=\cos y$		
	$\left[\mathrm{f}^{-1}(x)=\right]\ \cos^{-1}(3x-1)$	A1	
		2	
	Total	6	

Q	Answer	Marks	Comments
8(a)(i)	$6 = a\left(x^2+1\right) + x(bx)$	M1	
	$a = 6,\ b = -6$	A1	
	$\dfrac{6}{x^3+x} = \dfrac{6}{x} - \dfrac{6x}{x^2+1}$		
		2	
8(a)(ii)	$\int \dfrac{6}{x^3+x}\ [\mathrm{d}x] = \int \dfrac{6}{x} - \dfrac{6x}{x^2+1}\ [\mathrm{d}x]$	M1	**ft** their a and b from **(a)(i)**
	$= 6\ln x - 3\ln\left(x^2+1\right)$	A1	
	$\left[\int_1^2 \dfrac{6}{x^3+x}\ \mathrm{d}x\right]$		
	$= (6\ln 2 - 3\ln 5) - (0 - 3\ln 2)$	M1	Substitutes $x = 1$ and $x = 2$ into their integration, provided it is in the form $= a\ln x + b\ln\left(x^2+1\right)$
	$= \ln\left(\dfrac{512}{125}\right)$	A1	**ACF** such as $\ln\left(\dfrac{2^9}{5^3}\right)$
		4	

8(b)(i)	$\frac{\mathrm{d}}{\mathrm{d}y}(\cos y)^{-1} = -1(\cos y)^{-2} \times (-\sin y)$	**M1**	
	$= \frac{\sin y}{\cos^2 y}$		Need to see an intermediate line of working
	$= \sec y \tan y$	**A1**	
		2	
8(b)(ii)	$\left[\frac{\mathrm{d}u}{\mathrm{d}x} =\right] \cos x$	**B1**	
	$\left[\int \frac{u}{(1-u^2)^{1.5}}\,\mathrm{d}u =\right] \int \frac{\sin x \cos x}{(1-\sin^2 x)^{1.5}}\,\mathrm{d}x$	**M1**	All in terms of x condone omission of $\mathrm{d}x$
	$\left[= \int \frac{\sin x \cos x}{\cos^3 x}\,\mathrm{d}x\right]$		
	$= \int \sec x \tan x\,\mathrm{d}x$	**A1**	
	$[= \sec x]$		
	$\int_0^{0.5} \mathrm{d}u = \int_0^{\frac{\pi}{6}} \mathrm{d}x$	**B1**	Change of limits, maybe seen earlier (may change back to u and not change limits)
	$\left[\int_0^{0.5} \frac{u}{(1-u^2)^{1.5}}\,\mathrm{d}u =\right] \frac{2}{\sqrt{3}} - 1$	**A1**	**oe** such as $\frac{1}{3}(2\sqrt{3}-3)$
		5	
	Total	**13**	

Q	Answer	Marks	Comments
9(a)	$\dfrac{1}{(30-x)(10-x)}=\dfrac{A}{30-x}+\dfrac{B}{10-x}$		
	$1=A(10-x)+B(30-x)$	M1	
	$A=-\dfrac{1}{20},\ B=\dfrac{1}{20}$	A1	
	$\dfrac{1}{(30-x)(10-x)}=-\dfrac{1}{20(30-x)}+\dfrac{1}{20(10-x)}$		
		2	

9(b)	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = k(30-x)(10-x)$	**B1**	
	$\displaystyle\int \frac{1}{(30-x)(10-x)}\mathrm{d}x = \int \frac{-1}{20(30-x)} + \frac{1}{20(10-x)}\mathrm{d}x$	**M1**	Uses their partial fractions to separate variables
	$= \dfrac{1}{20}\ln(30-x) - \dfrac{1}{20}\ln(10-x)$	**m1**	Attempt to integrate
	$\dfrac{1}{20}\ln\left(\dfrac{30-x}{10-x}\right) = kt + c$	**A1ft**	**ft** their A and B from **(a)**
	$[t=0,\ x=0 \quad \Rightarrow] \quad c = \dfrac{1}{20}\ln 3$	**m1**	Attempt to find c
	$\dfrac{1}{20}\ln\left(\dfrac{30-x}{10-x}\right) = kt + \dfrac{1}{20}\ln 3$		
	$[t=2,\ x=6 \quad \Rightarrow] \quad \dfrac{1}{20}\ln\left(\dfrac{24}{4}\right) = 2k + \dfrac{1}{20}\ln 3$	**M1**	Attempt to find k
	$k = \dfrac{1}{40}\ln 2$	**A1**	Both c and k correct
	$\dfrac{1}{20}\ln\left(\dfrac{30-x}{10-x}\right) = \left(\dfrac{1}{40}\ln 2\right)t + \dfrac{1}{20}\ln 3$		
	$\ln\left(\dfrac{30-x}{10-x}\right) = \dfrac{t}{2}\ln 2 + \ln 3$	**m1**	Attempt to solve
	$\left(\dfrac{30-x}{10-x}\right) = 3\times 2^{0.5t}$		
	$x = \dfrac{30\times\left(2^{0.5t}-1\right)}{3\times 2^{0.5t}-1}$	**A1**	**ACF**
		9	
	Total	**11**	

Q	Answer	Marks	Comments
10(a)	$\dfrac{dx}{dt} = -3\cos^2 t\sin t$	M1	At least one derivative correct
	$\dfrac{dy}{dt} = -2\cos t\sin^2 t + \left(2+\cos^2 t\right)\cos t$ $\left[= -2\cos t\left(1-\cos^2 t\right) + 2\cos t + \cos^3 t = 3\cos^3 t\right]$	A1	Both derivatives correct
	$\dfrac{dy}{dx} = \dfrac{-3\cos^3 t}{3\cos^2 t\sin t}$	M1	Correct use of trig identity
	$\dfrac{dy}{dx} = -\dfrac{\cos t}{\sin t} = -\cot t$	A1	**AG** Be convinced
		4	
10(b)	gradient at $t = p$ is $-\dfrac{1}{-\cot p}\left[= \dfrac{\sin p}{\cos p} = \tan p\right]$	M1	Must be in terms of p
	$y - \left(2+\cos^2 p\right)\sin p = \dfrac{\sin p}{\cos p}\left(x - \cos^3 p\right)$	A1	**ACF** $y = (\tan p)x + 2\sin p$
		2	
10(c)	$[x = 0 \;\Rightarrow]\; y - \left(2+\cos^2 p\right)\sin p = \dfrac{\sin p}{\cos p}\left(-\cos^3 p\right)$	M1	Maybe seen in **(b)**
	$y = 2\sin p + \sin p\cos^2 p - \sin p\cos^2 p \; = 2\sin p$	A1	
	$[y = 0 \;\Rightarrow]\; -\left(2+\cos^2 p\right)\sin p = \dfrac{\sin p}{\cos p}\left(x - \cos^3 p\right)$	M1	Maybe seen in **(b)**
	$x = -2\cos p - \cos^3 p + \cos^3 p = -2\cos p$	A1	
	$[AB =]\; \sqrt{4\sin^2 p + 4\cos^2 p}$	M1	
	$[AB =]\; 2$	A1	**CAO**
		6	
	Total	**12**	

Q	Answer	Marks	Comments
11(a)	$A(-2.5,\ 0),\quad B(0,\ 5)$	**B1**	
		1	
11(b)(i)	$\left[\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right]\ 2\mathrm{e}^{-x}-(5+2x)\mathrm{e}^{-x}$ $\left[\dfrac{\mathrm{d}y}{\mathrm{d}x}=-3\mathrm{e}^{-x}-2x\mathrm{e}^{-x}\right]$	**M1 A1**	**M1**: $a\mathrm{e}^{-x}+bx\mathrm{e}^{-x}$ **A1**: $-3\mathrm{e}^{-x}-2x\mathrm{e}^{-x}$ **ACF**
		2	
11(b)(ii)	$-3\mathrm{e}^{-x}-2x\mathrm{e}^{-x}=0$ $-\mathrm{e}^{-x}(3+2x)=0$ $\left[\mathrm{e}^{-x}\neq 0\ \ \Rightarrow\right]\ 3+2x=0$ $\left(-1.5,\ 2\mathrm{e}^{1.5}\right)$	 **M1** **A1**	 **ft** their [simplified] derivative **oe**
		2	
11(b)(iii)	$\left[\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=3\mathrm{e}^{-x}+2x\mathrm{e}^{-x}-2\mathrm{e}^{-x}\right]$ $x=-1.5,\ \ \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-2\mathrm{e}^{1.5}\ [=-8.96...]\ <0$ Hence [local] maximum	 **B1** **E1**	or considers first derivative either side of $x=-1.5$, e.g. $x<-1.5\ \Rightarrow\ \dfrac{\mathrm{d}y}{\mathrm{d}x}>0$ $x>-1.5\ \Rightarrow\ \dfrac{\mathrm{d}y}{\mathrm{d}x}<0$ Must have been awarded **B1**
		2	

11(c)	$\int(5+2x)\mathrm{e}^{-x}\mathrm{d}x=-(5+2x)\mathrm{e}^{-x}+\int 2\mathrm{e}^{-x}\ \mathrm{d}x$	**M1**	**oe** Use of integration by parts Condone omission of $\mathrm{d}x$ throughout
	$=-5\mathrm{e}^{-x}-2x\mathrm{e}^{-x}-2\mathrm{e}^{-x}$	**m1**	Complete use of integration by parts
	$=-7\mathrm{e}^{-x}-2x\mathrm{e}^{-x}$	**A1**	
	$\int_{-2.5}^{0}(5+2x)\mathrm{e}^{-x}\mathrm{d}x=\left[-7\mathrm{e}^{-x}-2x\mathrm{e}^{-x}\right]_{-2.5}^{0}$		
	$=(-7)-\left(-7\mathrm{e}^{2.5}+5\mathrm{e}^{2.5}\right)$	**M1**	Their integral of the form $a\mathrm{e}^{-x}+bx\mathrm{e}^{-x}$ evaluated between their –2.5 and 0
	$=-7+2\mathrm{e}^{2.5}$		
	[Area of Triangle =] $0.5\times5\times2.5\ \left[=\frac{25}{4}\right]$	**B1**	May be seen at any point in their solution
	[Area =] $-7+2\mathrm{e}^{2.5}-0.5\times5\times2.5$		
	[Area =] $-13.25+2\mathrm{e}^{2.5}$	**A1**	**oe** such as $2\mathrm{e}^{2.5}-\frac{53}{4}$
		6	
	Total	**13**	

Q	Answer	Marks	Comments
12	$y = x\ln(x+y)$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \ln(x+y) + \dfrac{x}{x+y}\left(1+\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)$	**M1 A1**	**M1**: Attempt at implicit differentiation **A1**: All correct
	$(x+y)\dfrac{\mathrm{d}y}{\mathrm{d}x} = (x+y)\ln(x+y) + x\left(1+\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)$	**m1**	Eliminates the fraction
	$x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y\dfrac{\mathrm{d}y}{\mathrm{d}x} = (x+y)\dfrac{y}{x} + x + x\dfrac{\mathrm{d}y}{\mathrm{d}x}$	**m1**	Expands & eliminates logarithm or correctly isolates $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ term
	$y\dfrac{\mathrm{d}y}{\mathrm{d}x} = (x+y)\dfrac{y}{x} + x$	**A1**	Expands & eliminates logarithm and correctly isolates $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ term
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = (x+y)\dfrac{1}{x} + \dfrac{x}{y}$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 + \dfrac{y}{x} + \dfrac{x}{y}$	**A1**	**AG** Be convinced
	Total	**6**	

12 ALT 1	$\left[\frac{y}{x}=\ln(x+y)\ \Rightarrow\right]\quad e^{\frac{y}{x}}=x+y$		
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\frac{y}{x}}\right)=\frac{\mathrm{d}}{\mathrm{d}x}(x+y)$	**M1**	**PI**
	$e^{\frac{y}{x}}\frac{\left(x\frac{\mathrm{d}y}{\mathrm{d}x}-y\right)}{x^2}=1+\frac{\mathrm{d}y}{\mathrm{d}x}$	**m1 A1**	**m1**: Attempt at implicit differentiation **A1**: All correct
	$(x+y)\left(x\frac{\mathrm{d}y}{\mathrm{d}x}-y\right)=x^2\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)$	**m1**	Eliminates exponential term or correctly isolates $\frac{\mathrm{d}y}{\mathrm{d}x}$ term
	$x^2\frac{\mathrm{d}y}{\mathrm{d}x}-xy+yx\frac{\mathrm{d}y}{\mathrm{d}x}-y^2=x^2+x^2\frac{\mathrm{d}y}{\mathrm{d}x}$		
	$yx\frac{\mathrm{d}y}{\mathrm{d}x}=y^2+x^2+xy$	**A1**	Eliminates exponential term and correctly isolates $\frac{\mathrm{d}y}{\mathrm{d}x}$ term
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y^2+x^2+xy}{yx}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y}{x}+\frac{x}{y}+1$	**A1**	**AG** Be convinced
	Total	**6**	
12 ALT 2	$\frac{y}{x}=\ln(x+y)$		
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{y}{x}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(x+y)\right)$	**M1**	**PI**
	$\frac{x\frac{\mathrm{d}y}{\mathrm{d}x}-y}{x^2}=\frac{1}{x+y}\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)$	**m1 A1**	**m1**: Attempt at implicit differentiation **A1**: All correct
	$(x+y)\left(x\frac{\mathrm{d}y}{\mathrm{d}x}-y\right)=x^2\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)$	**m1**	Eliminates fractions or correctly isolates $\frac{\mathrm{d}y}{\mathrm{d}x}$ term
	$\frac{\mathrm{d}y}{\mathrm{d}x}\left(x(x+y)-x^2\right)=xy+y^2+x^2$	**A1**	Eliminates fractions and correctly isolates $\frac{\mathrm{d}y}{\mathrm{d}x}$ term
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{xy+y^2+x^2}{xy}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=1+\frac{y}{x}+\frac{x}{y}$	**A1**	**AG** Be convinced
	Total	**6**	

Q	Answer	Marks	Comments
13(a)	$[AB=]\sqrt{(16-2)^2+(-1-(-3))^2+(-1-7)^2}$ $[AB=]\sqrt{14^2+2^2+(-8)^2}$ $AB=\sqrt{264}$	M1 A1	**oe** **oe** such as $2\sqrt{66}$ Condone 16.2[48…]
		2	
13(b)(i)	$2+14\lambda=9+5\mu$ $-3+2\lambda=-2-4\mu$ $\lambda=0.5, \quad \mu=0$ $7-8\lambda=q+5\mu$ $q=3$	 M1 A1 A1	May use *B* instead Equating x and y
		3	
13(b)(ii)	$\begin{bmatrix}14\\2\\-8\end{bmatrix}.\begin{bmatrix}5\\-4\\5\end{bmatrix}=22$ $\cos\theta=\frac{\pm 22}{\sqrt{66}\sqrt{264}}\quad\left[=\pm\frac{1}{6}\right]$ $[\theta=]\ 80.4^{[\circ]}$	M1 A1 M1 A1	**M1**: Use of scalar product with direction vectors of l and AB **A1**: Correctly finds 22 **ft** their 22 from the scalar product between the two correct vectors
		4	

13(c)	$C(9+5c,\ -2-4c,\ 3+5c)$ or $\overrightarrow{OC}=\begin{bmatrix}9+5c\\-2-4c\\3+5c\end{bmatrix}$	**B1**	
	$\overrightarrow{CD}=\begin{bmatrix}10+5c\\-4-4c\\5c\end{bmatrix}$	**M1**	**oe**
	$\begin{bmatrix}10+5c\\-4-4c\\5c\end{bmatrix}.\begin{bmatrix}5\\-4\\5\end{bmatrix}=0$	**m1**	
	$50+25c+16+16c+25c=0$		$66c=-66,\quad c=-1$
	$C(4,\ 2,-2)$ or $\overrightarrow{OC}=\begin{bmatrix}4\\2\\-2\end{bmatrix}$	**A1**	
	$BC^2=(4-16)^2+(2-(-1))^2+(-2-(-1))^2=\sqrt{154}$	**m1**	or finding $\overrightarrow{BC}\cdot\overrightarrow{AC}=-24+15+9$ Note $\overrightarrow{AC}=\begin{bmatrix}2\\5\\-9\end{bmatrix}$ and $\overrightarrow{BC}=\begin{bmatrix}-12\\3\\-1\end{bmatrix}$
	$AC^2=(4-2)^2+(2-(-3))^2+(-2-7)^2=\sqrt{110}$		
	$AB^2=AC^2+BC^2$ [so right-angled triangle]	**A1**	$\overrightarrow{BC}\cdot\overrightarrow{AC}=0$ [so right-angled triangle]
		6	
	Total	**15**	