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FURTHER MATHEMATICS

FM01

(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

January 2020

Version: 1.0 Final Mark Scheme

201Xfm01/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√**or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$z^* = a - b\mathrm{i}$	**B1**	PI
	$4a + 4b\mathrm{i} - \mathrm{i}(a - b\mathrm{i}) = 7 + 3\mathrm{i}$ $4a - b + 4b\mathrm{i} - \mathrm{i}a = 7 + 3\mathrm{i}$	**M1**	For either line
	$4a - b = 7$ $4b - a = 3$	**M1**	For two sim. eqns. each with at least three non-zero terms
	$4a - b = 7$ $-4a + 16b = 12$		
	$15b = 19$ so $b = \frac{19}{15}$	**A1**	For a or b
	$a = 4b - 3$ so $a = \frac{31}{15}$	**M1**	
	$z = \dfrac{31}{15} + \dfrac{19}{15}\mathrm{i}$	**A1**	Must be seen in this form
	Total	**6**	

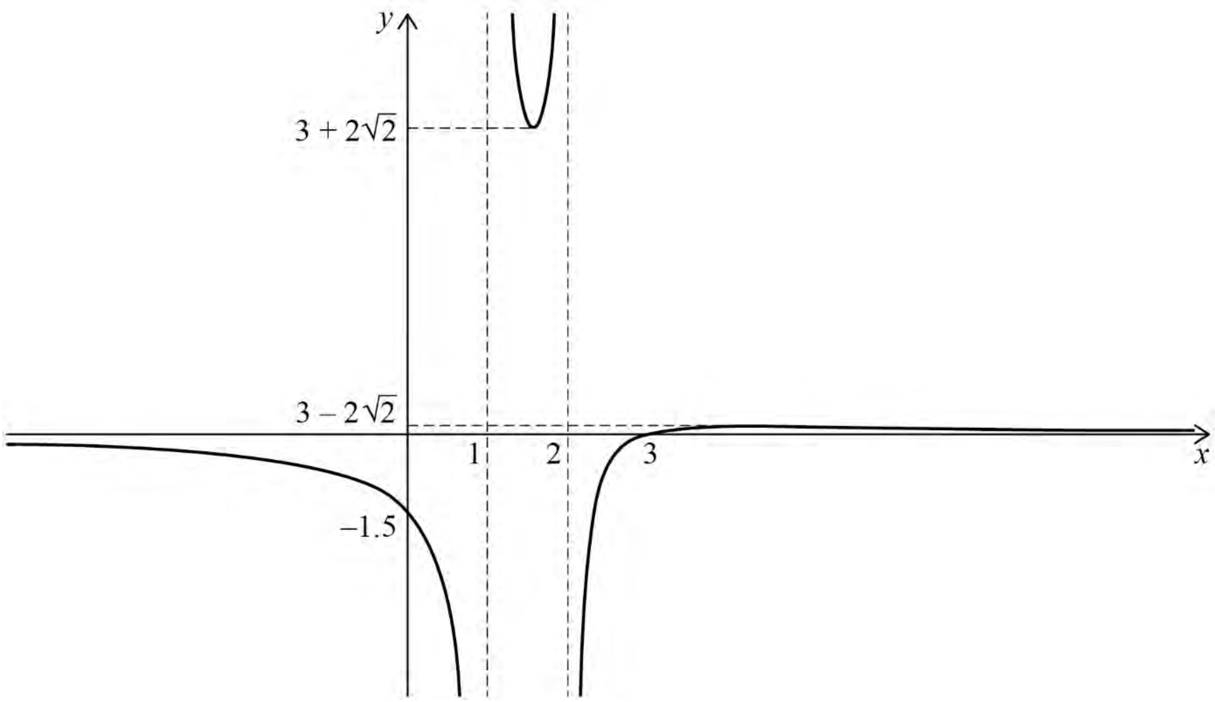
Q	Answer	Marks	Comments
2(a)	$\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$	**B1**	oe
	Use of $2n\pi$ and use of second solution to $\cos x = -\frac{\sqrt{3}}{2}$	**M1**	(or $n\pi$) at any stage
	Going from $\left(2x - \frac{\pi}{4}\right)$ to x	**m1**	including division of all terms by 2
	$x = n\pi + \frac{13\pi}{24}$ or $x = n\pi - \frac{7\pi}{24}$	**A1 A1**	oe
2(b)	Sum of admissible solutions = $2k\pi + \dfrac{13\pi}{24} + (2k+1)\pi - \dfrac{7\pi}{24}$ $+(2k+1)\pi + \dfrac{13\pi}{24} + (2k+2)\pi - \dfrac{7\pi}{24}$	**M1**	For adding at least two terms consistent with their answer to part (a)
	$= (8k+4)\pi + \dfrac{\pi}{2}$	**A1**	
	Mean $= \left\{(8k+4)\pi + \frac{\pi}{2}\right\} \div 4$ $= 2k\pi + \frac{9\pi}{8}$ as required	**A1**	
	Total	**8**	

Q	Answer	Marks	Comments
3(a)	Gradient $= \dfrac{5+h+\frac{1}{5+h}-\left(5+\frac{1}{5}\right)}{5+h-5}$	M1	
	$= \dfrac{h+\frac{1}{5+h}-\frac{1}{5}}{h}$	M1	
	$= \dfrac{h+\frac{-h}{5(5+h)}}{h}$	M1	
	$= 1-\dfrac{1}{5(5+h)}$	A1	
3(b)	Gradient of curve $= \lim\limits_{h\to 0}\left[1-\dfrac{1}{5(5+h)}\right]$	B1	Limit of their expression from part (a)
	$= 1-\dfrac{1}{25}=\dfrac{24}{25}$	B1	
	Total	**6**	

Q	Answer	Marks	Comments
4(a)	$\alpha+\beta=\dfrac{7}{2}$	B1	
	$\alpha\beta=5$	B1	
4(b)	Sum of roots $= \alpha^3+\beta^3$ $= (\alpha+\beta)^3-3\alpha^2\beta-3\alpha\beta^2$	M1	PI
	$= (\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$	M1	PI
	$= \left(\dfrac{7}{2}\right)^3-3\times 5\times\dfrac{7}{2}$ $= -\dfrac{77}{8}$	A1	
	Product of roots $= (\alpha\beta)^3$	M1	PI
	$= 125$	A1	
	$8x^2+77x+1000=0$	A1	oe (integer coefficients)
	Total	**8**	

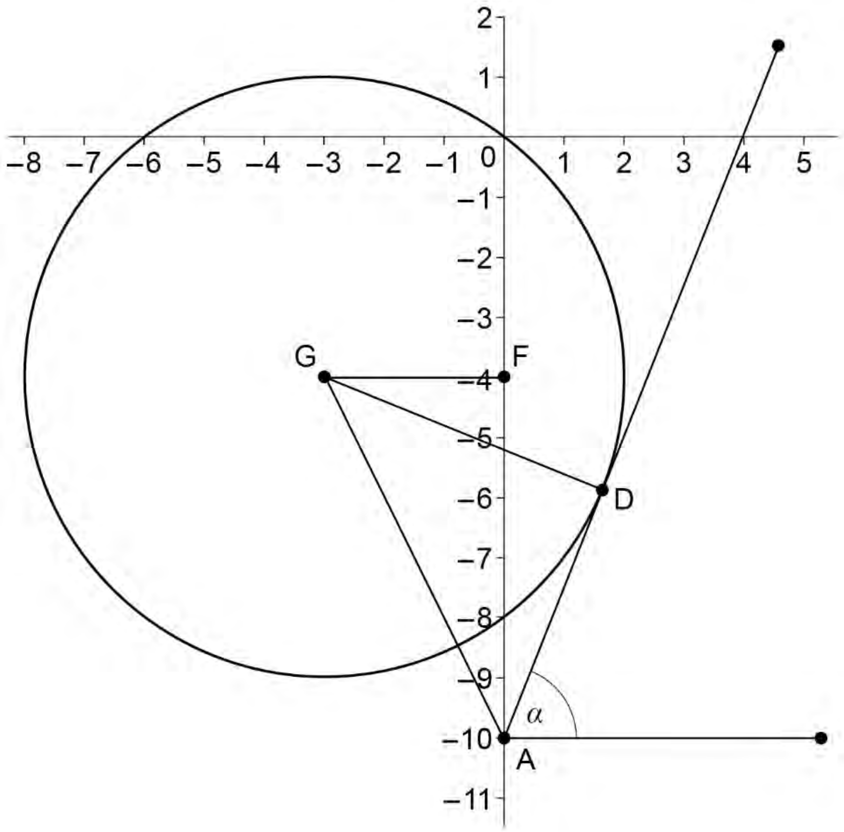
Q	Answer	Marks	Comments
5	$\frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$	B1	
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t}$	M1	
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{4\pi r^2} \times 50$	M1	
	When $V = \frac{500\pi}{3}$, $r = 5$	M1	
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{4\pi(5^2)} \times 50$	M1	
	$= \frac{1}{2\pi}$	A1	With all steps clearly shown in a logical sequence
	Total	**6**	

Q	Answer	Marks	Comments
6(a)	$x = 1 \quad x = 2$ $y = 0$	B1 B1	For both
6(b)	Let $\mathrm{f}(x) = k$ $\frac{x-3}{(x-2)(x-1)} = k$ $k(x-2)(x-1) = x-3$ $kx^2 - (3k+1)x + 2k + 3 = 0$ For real roots $(3k+1)^2 - 4k(2k+3) \geq 0$ $k^2 - 6k + 1 \geq 0$ $k^2 - 6k + 1 = 0$ has roots $k = 3 \pm 2\sqrt{2}$ $\mathrm{f}(x) \leq 3 - 2\sqrt{2}$ or $\mathrm{f}(x) \geq 3 + 2\sqrt{2}$	M1 A1 M1 A1 A1A1	A1A0 if < and/or > used
6(c)	Three branches with correct shape Asymptotes shown Both values at axis intercepts shown Both y-coordinates of stationary points shown	B1 B1 B1 B1F	PI for horizontal asymptote FT their answers to part **(b)**



	Total	**12**	

Q	Answer	Marks	Comments
7(a)	$(x+1)^4-(x-1)^4=$ $x^4+4x^3+6x^2+4x+1$ $-(x^4-4x^3+6x^2-4x+1)$	M1	
	$=8(x^3+x)$	A1	
7(b)	$8\sum_{r=1}^{n}(r^3+r)=\sum_{r=1}^{n}\{(r+1)^4-(r-1)^4\}$	M1	Must use method of differences to gain any marks
	$=\cancel{2^4}-0^4$ $+\cancel{3^4}-1^4$ $+\cancel{4^4}-\cancel{2^4}$ $+\cdots$ $+\cancel{(n-1)^4}-\cancel{(n-3)^4}$ $+\quad n^4\quad-\cancel{(n-2)^4}$	M1	
	$+(n+1)^4-\cancel{(n-1)^4}$	M1	Must have at least the first three terms and last two terms (or first two and last three)
	$=n^4+(n+1)^4-1$	A1	
	$\therefore\sum_{r=1}^{n}(r^3+r)=\frac{1}{8}(n^4+(n+1)^4-1)$ as required	A1	
7(c)	$\sum_{r=1}^{n}(r^3+r)$ is an even integer.	E1	
	$\therefore\frac{1}{8}(n^4+(n+1)^4-1)$ is an even integer and $n^4+(n+1)^4-1$ is a multiple of 16	E1	Second **E1** can only be earned if the first **E1** is awarded
	Total	**9**	

Q	Answer	Marks	Comments
8(a)	Circle with centre (-3, -4) Passing through origin Line with positive gradient touching circle Starts at (0, -10)	B1 B1 B1 B1	 condone extra bit in 3rd quadrant
8(b)	Given points A(0, -10), G(-3,-4), F(0,-4) and D where L touches C: $AG^2 = 6^2 + 3^2$ so $AG = 3\sqrt{5}$	B1	
	$\widehat{GAD} = \sin^{-1}\frac{5}{3\sqrt{5}}$	M1	
	$= 0.84107$	A1	
	$\widehat{GAF} = \tan^{-1}\frac{1}{2} = 0.46365$	B1	
	$\alpha = \frac{\pi}{2} + 0.46365 - 0.84107$	M1	
	= 1.19	A1	Must be 3 sig. fig.
			
	Total	10	

Q	Answer	Marks	Comments
9(a)	$xy = 8 \Rightarrow x = {}^{8}/_{y}$ $y^2 = 8({}^{8}/_{y}) = \frac{64}{y}$	M1	
	$y = 4, x = 2$ so (2, 4)	A1	And no other solution
9(b)	H has branches in 1st and 3rd quadrants	B1	And roughly correct shape
	P passes through origin and is symmetrical about the positive x-axis	B1	
9(c)	$y = mx + c$ and $xy = 8$ so $x(mx + c\ ) = 8$	M1	Solves simultaneously
	$mx^2 + cx - 8 = 0$	A1	
	For tangency, $\Delta = 0$ so $c^2 - 4(m)(-8) = 0$	M1	This must be stated in some form
	$c^2 + 32m = 0$ as required	A1	
9(d)	For P: $y = mx + c$ and $y^2 = 8x$ so $(mx + c)^2 = 8x$	M1	
	$m^2x^2 + (2mc - 8)x + c^2 = 0$	A1	
	$\Delta = 0$ so $(2mc - 8)^2 - 4m^2c^2 = 0$	M1	
	Giving $m = \frac{2}{c}$	A1	
	Solving $m = \frac{2}{c}$ and $c^2 + 32m = 0$ simultaneously,	M1	
	$c = -4$ or $m = -\frac{1}{2}$	A1	
	so $y = -\frac{1}{2}x - 4$	A1	
	Total	**15**	