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FM05

(9665/FM05) Unit FM2 Mechanics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$5\cos 50° = v\cos\alpha°$	M1	Equation for motion parallel to the wall
	$0.8\times 5\sin 50° = v\sin\alpha°$	M1 A1	Equation for motion perpendicular to the wall Both equations correct
	$v^2 = (5\cos 50°)^2 + (4\sin 50°)^2$	M1	Eliminates α
	$v = 4.44$	A1	Correct v. Must be a positive value. Condone AWRT 4.4
		5	

Q	Answer	Marks	Comments
1(b)	$\tan\alpha° = \dfrac{4\sin 50°}{5\cos 50°}$	M1	Eliminates v
	$\alpha = 44$	A1	Correct α Condone AWRT 43.6
		2	

Q	Answer	Marks	Comments
1(c)	$I = 0.3(4.4\sin 43.6°) - 0.3(-5\sin 50°)$ or $I = 0.3(0.8\times 5\sin 50°) - 0.3(-5\sin 50°)$	M1 A1	Applies impulse formula. Must resolve in some way. Correct expression
	$= 2.1\,\text{Ns}$ [to 2 sf]	A1	Correct answer with correct units. Must be two significant figures. Do not accept a negative answer.
		3	

	Question 1 Total	**10**	

Q	Answer	Marks	Comments
2(a)	$\int_{0.02}^{0.01} -\left(\frac{8}{x^2}\right) \mathrm{d}x$ $= \left[\frac{8}{x}\right]_{0.02}^{0.01}$ $= 800 - 400$ $= 400\ \mathrm{J}$	**M1** **A1** **A1** **A1**	Uses $\int \pm F\,\mathrm{d}x$ Uses limits that are consistent with their sign of F Correct integration Obtains correct work done from consistent working. Must show some numeric simplification after the integration.
		4	

Q	Answer	Marks	Comments
2(b)	$\frac{1}{2} \times 4v^2 = 400$ $v = \sqrt{200} = 14.1\ \mathrm{m\,s^{-1}}$	**M1** **A1**	Forms an energy equation to find v Do not condone use of constant acceleration equations. Correct speed AWRT 14.1
		2	

	Question 2 Total	**6**	

Q	Answer	Marks	Comments
3(a)	$\omega = \dfrac{2\pi}{5}$	**M1**	Finding ω
	$v_{\max} = \dfrac{3}{2} \times \dfrac{2\pi}{5} = \dfrac{3\pi}{5}$ m s^{-1}	**A1**	Correct speed oe
		2	

Q	Answer	Marks	Comments
3(b)	$v^2 = \left(\dfrac{2\pi}{5}\right)^2 \left(\left(\dfrac{3}{2}\right)^2 - \left(\dfrac{1}{2}\right)^2\right)$	**M1**	Uses SHM speed formula with their ω Allow one error in the distances.
	$= \dfrac{8\pi^2}{25}$	**A1**	Correct substitutions
	$v = \dfrac{2\sqrt{2}\pi}{5}$ m s^{-1}	**A1**	Correct speed, ACF, for example $\sqrt{\dfrac{8}{25}}\pi$ or $\sqrt{0.32}\pi$
		3	

Q	Answer	Marks	Comments
3(c)(i)	$x = \dfrac{3}{2}\cos\left(\dfrac{2\pi t}{5}\right)$	**B1**	Correct expression with ±
		1	

Q	Answer	Marks	Comments
3(c)(ii)	$\dot{x} = -\dfrac{3\pi}{5}\sin\left(\dfrac{2\pi t}{5}\right)$	**M1**	Differentiates to find velocity
	$\ddot{x} = -\dfrac{6\pi^2}{25}\cos\left(\dfrac{2\pi t}{5}\right)$	**M1**	Differentiates to find acceleration
	$\ddot{x}_{\max} = \dfrac{6\pi^2}{25}$ m s^{-2}	**A1**	Correct maximum acceleration Accept $0.24\pi^2$ Do not accept a negative answer.
		3	

	Question 3 Total	**9**	

Q	Answer	Marks	Comments
4(a)(i)	Let θ be the angle between the string and the vertical at time t		PI
	$m\times 0.7\ \ddot{\theta} = -m\times 9.8\sin\theta$	**M1** **A1**	Forms differential equation Correct differential equation
	$\sin\theta \approx \theta$	**M1**	Uses small angle approximation, **PI**
	$\ddot{\theta} \approx -\frac{9.8}{0.7}\theta = -14\theta$	**A1**	Simplifies to SHM form with clear use of small angle approximation Accept proof for any l in general case
	Therefore SHM as [angular] acceleration proportional to the [angular] displacement.	**E1**	Concludes that motion is SHM from correct working
		5	

Q	Answer	Marks	Comments
4(a)(ii)	Light	**B1**	Correct assumption
	Inextensible / inelastic / fixed length	**B1**	Correct assumption
		2	

Q	Answer	Marks	Comments
4(b)	Period $= \frac{2\pi}{\sqrt{14}} = \frac{\pi\sqrt{14}}{7}$ s	**B1**	Correct period, **oe** such as 0.53π [s]
		1	

	Question 4 Total	**8**	

Q	Answer	Marks	Comments
5	$2.5v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -10v^2$	**M1**	Sets up correct differential equation. Condone missing negative sign.
	$\dfrac{1}{v}\times\dfrac{\mathrm{d}v}{\mathrm{d}x} = -4$	**A1ft**	Rearranges differential equation correctly ready to integrate Follow through sign error from differential equation.
	$\ln v = -4x + c$	**A1ft**	Correct integration Follow through sign error from differential equation.
	$x = 0, v = 20 \Rightarrow c = \ln 20$ $\ln v = -4x + \ln 20$ $v = 10$	**M1**	Finds constant of integration or uses correct limits in a definite integral
	$x = \dfrac{1}{4}(\ln 20 - \ln 10) = \dfrac{1}{4}\ln 2$	**A1**	Correct distance, **ACF**
		5	

ALTERNATIVE METHOD

Q	Answer	Marks	Comments
5	$2.5\dfrac{\mathrm{d}v}{\mathrm{d}t} = -10v^2$ $\int \dfrac{1}{v^2}\,\mathrm{d}v = \int -4\,\mathrm{d}t$ $-\dfrac{1}{v} = -4t + c$ $v = 20, t = 0 \Rightarrow c = -\dfrac{1}{20}$	**M1**	Sets up correct differential equation. Condone missing negative sign.
	$v = \dfrac{20}{80t+1}$	**A1ft**	Correct expression for v in terms of t Follow through sign error from differential equation to obtain $v = \dfrac{20}{1-80t}$ oe
	$x = \dfrac{20}{80}\ln(80t+1) + d$ $x = 0, t = 20 \Rightarrow d = 0$ $x = \dfrac{1}{4}\ln(80t+1)$	**A1ft**	Correct expression for x in terms of t Follow through sign error from differential equation to obtain $x = -\dfrac{1}{4}\ln(1-80t)$ oe
	$v = 10 \Rightarrow t = \dfrac{1}{80}$	**M1**	Finds time for a velocity of 10
	$x = \dfrac{1}{4}\ln\left(\dfrac{80}{80}+1\right) = \dfrac{1}{4}\ln 2$	**A1**	Correct distance, **ACF**
		5	

	Question 5 Total	**5**	

Q	Answer	Marks	Comments
6(a)	$ke = 2g$ $e = \frac{2g}{k}$	**M1** **A1**	Equation for equilibrium Correct extension Do not accept $\frac{mg}{k}$
		2	

Q	Answer	Marks	Comments
6(b)(i)	$2\ddot{x} = 2g - T$ $2\ddot{x} = 2g - k\left(x + \frac{2g}{k}\right)$ $2\ddot{x} = 2g - kx - 2g$ $\ddot{x} = -\frac{k}{2}x$ $\therefore$ SHM as acceleration proportional to the displacement.	**M1** **M1** **A1** **A1** **E1**	Forms three term differential equation PI Uses tension with their extension from **part (a)** Allow $2\ddot{x} = \pm 2g \pm k\left(x + \frac{2g}{k}\right)$ Correct differential equation Differential equation simplified to SHM form Accept $\ddot{x} = -\frac{k}{m}x$ Concludes that motion is SHM from correct working
		5	

Q	Answer	Marks	Comments
6(b)(ii)	$x = 2a\cos\left(\sqrt{\frac{k}{2}}t\right)$	**M1**	Expression for displacement using their ω
	$\frac{3a}{2} = 2a\cos\left(\sqrt{\frac{k}{2}} \times \frac{\pi}{8}\right)$	**A1**	Correct substitution of time and displacement
	$0.7227... = \sqrt{\frac{k}{2}} \times \frac{\pi}{8}$	**M1**	Solves for k Correct k, **AWRT** 6.8
	$k = 6.8$	**A1**	**SC2** for using $x = 2a\sin\left(\sqrt{\frac{k}{2}}t\right)$ leading to $k = 9.3$ **SC1** for seeing $\frac{3}{2}a = 2a\sin\left(\sqrt{\frac{k}{2}}t\right)$
		4	

	Question 6 Total	**11**	

Q	Answer	Marks	Comments
7(a)	$v^2 = 0^2 + 2 \times g \times 2a$	**M1**	Uses constant acceleration equation to find speed with $u = 0$
	$v = 2\sqrt{ag}$	**A1**	Correct speed oe
		2	

Q	Answer	Marks	Comments
7(b)	Resolving parallel and perpendicular to *OB*		
	$v = -2\sqrt{ag}\cos 30°$	**M1**	Equation for velocity perpendicular to the string
	Speed $= 2\sqrt{ag} \times \frac{\sqrt{3}}{2}$		
	Speed $= \sqrt{3ag}$	**A1**	Correct speed, from correct working
	$I = m \times 0 - m \times -2\sqrt{ag}\sin 30°$	**M1**	Uses impulse equation with some resolving
		A1	Correct impulse equation
	$\lvert\mathbf{I}\rvert = 2m\sqrt{ag} \times \frac{1}{2} = m\sqrt{ag}$	**A1**	Correct magnitude of impulse
		5	

Q	Answer	Marks	Comments
7(c)	Resolving radially with zero tension	**M1**	Resolving radially
	$mg\cos\theta = \frac{mv^2}{2a}$	**A1**	Correct equation
	$v^2 = 2ag\cos\theta$		
	Energy Equation		
	$\frac{1}{2}mv^2 + mg \times a(1+2\cos\theta) = \frac{1}{2}m \times 3ag$	**M1**	Forms a three term energy equation
		A1	Correct energy equation
	$ag\cos\theta + ag + 2ag\cos\theta = \frac{3}{2}ag$		
	$3ag\cos\theta = \frac{1}{2}ag$		
	$\cos\theta = \frac{1}{6}$	**m1**	Solves for θ
	$\theta = 80°$	**A1**	Correct angle
		6	

	Question 7 Total	**13**	

Q	Answer	Marks	Comments
8	Speed at $B = \sqrt{6g}$	**B1**	Correct speed at B
	$\mathbf{u} = \begin{bmatrix} \sqrt{6g}\sin 20° \\ \frac{3}{5}\sqrt{6g}\cos 20° \end{bmatrix} \left[= \begin{bmatrix} 2.62 \\ 4.32 \end{bmatrix} \right]$	**M1**	Uses coefficient of restitution to find the velocity after the bounce at B
		A1	Correct velocity
	$0 = \frac{3}{5}\sqrt{6g}\cos 20° t - \frac{1}{2} g \cos 20° t^2$	**M1**	Equation for motion perpendicular to the slope
		A1	Correct equation
	$t = \frac{6\sqrt{6g}}{5g} = 0.939$	**M1**	Solves for t
		A1	Correct t AWRT 0.94
	$BC = \sqrt{6g}\sin 20° t + \frac{1}{2} g \sin 20° t^2$	**M1**	Substitutes t into equation for motion parallel to the slope
	$= 3.94$ $= 3.9$ metres $[$to 2 sf$]$	**A1**	Correct distance **AFWT** [3.9, 3.95]
		9	

	Question 8 Total	**9**	

Q	Answer	Marks	Comments
9	$2\times 2-3\times 1=2v_A+3v_B$ $1=2v_A+3v_B$	**M1**	Equation for conservation of momentum
	$v_A-v_B=-e\times(2-(-1))$ $v_A-v_B=-3e$	**M1** **A1**	Equation for coefficient of restitution Both equations correct
	$1=2v_A+3(v_A+3e)$ $1=5v_A+9e$	**M1**	Solves for v_A
	$v_A=\dfrac{1-9e}{5}$	**A1**	Correct v_A
	Let θ be the angle between the velocity of *A* and the line of centres.		
	$\tan\theta=\dfrac{5\times 2\sqrt{3}}{1-9e}=\dfrac{10\sqrt{3}}{1-9e}$	**M1**	Finds $\tan\theta$ using the perpendicular component
	$e=0\Rightarrow\tan\theta=10\sqrt{3}\Rightarrow\theta=87°$ $e=1\Rightarrow\tan\theta=\dfrac{10\sqrt{3}}{-8}\Rightarrow\theta=115°$	**M1**	Uses $e=0$ or $e=1$ in their expression for $\tan\theta$
	$87°\le\theta\le 115°$	**A1**	At least one correct value for θ
	$27\le\alpha\le 55$	**A1**	Accept $27\le\alpha\le 54$ **AWRT** 27 and 54 or 55
		9	

	Question 9 Total	**9**	