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FURTHER MATHEMATICS
FM04

(9665/FM04) Unit FS2 Statistics

Mark scheme

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241XFM04/MS

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1	$H_0 : \mu_\alpha = \mu_\beta$ $H_1 : \mu_\alpha > \mu_\beta$	B1	Both hypotheses **oe**
	$z = \dfrac{87.5 - 75.9}{\sqrt{\dfrac{36.2^2}{150} + \dfrac{27.4^2}{120}}}$	M1	Condone use of $\dfrac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p^2\left(\dfrac{1}{n_x} + \dfrac{1}{n_y}\right)}}$ **oe**
	$= 3.00$	A1	**AWRT** 3.00 [$z = 2.9958...$] **oe**
	z critical value = 2.3263	B1	**AWRT** 2.33 **oe**
	$3.00 > 2.3263$ Reject H_0	A1ft	Correctly compares their z or t test statistic and critical value and rejects null hypothesis
	Sufficient evidence to suggest that on average, the Beta computer uses less energy per hour compared to the Alpha computer	E1	Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value Condone definite conclusion

	Question 1 Total	6	

Q	Answer	Marks	Comments
2(a)	$M_X(t) = 0.1e^{-t} + 0.2e^{2t} + 0.6e^{5t} + 0.1e^{7t}$	**M1**	Applies mgf formula Condone one slip
	$M'_X(t) = -0.1e^{-t} + 0.4e^{2t} + 3e^{5t} + 0.7e^{7t}$	**M1**	Differentiates their $M_X(t)$
	$M'_X(0) = 4$	**A1**	**CSO**
		3	

Q	Answer	Marks	Comments
2(b)	$M''_X(t) = 0.1e^{-t} + 0.8e^{2t} + 15e^{5t} + 4.9e^{7t}$	**M1**	Differentiates their $M'_X(t)$
	$M''_X(0) = 20.8$	**A1**	**CSO** **oe**
		2	

Q	Answer	Marks	Comments
2(c)	$\text{Var}(X) = M''_X(0) - (M'_X(0))^2 = 20.8 - 4^2$	**M1**	Applies formula to find variance with their $M''_X(0)$ and $M'_X(0)$
	$\text{Var}(X) = 4.8$	**A1ft**	**ft** their $M''_X(0)$ and $M'_X(0)$ **oe**
		2	

	Question 2 Total	**7**	

Q	Answer	Marks	Comments
3	$\overline{x} = 28$ and $s^2 = \dfrac{440}{3}$	**B1**	For s^2 **AWRT** $s^2 = 147$ or $s =$ **AWRT** 12.1
	$t_3 = 4.541$	**B1**	**AWRT** 4.54
	$28 \pm 4.541 \times \sqrt{\dfrac{\left(\dfrac{440}{3}\right)}{4}}$	**M1**	Applies correct formula for upper or lower limit of confidence interval with their values
	(0.5, 55.5)	**A1**	**AWRT** 0.5 for lower limit **AWRT** 55.5 for upper limit

	Question 3 Total	**4**	

Q	Answer	Marks	Comments
4(a)	Lifetimes of the moths have a normal distribution	**E1**	Condone just normal distribution stated
		1	

Q	Answer	Marks	Comments
4(b)	$H_0 : \sigma = 5$ $H_1 : \sigma \neq 5$	**B1**	Both hypotheses, **oe**
	$\frac{(n-1)s^2}{\sigma^2} = \frac{(101-1)\times 5.6^2}{5^2}$	**M1**	**PI** Condone one error
	$= 125.44$	**A1**	**AWRT** 125
	$\chi^2_{100}(0.975) = 129.561$ $\left[\text{and } \chi^2_{100}(0.025) = 74.222\right]$	**B1**	Finds correct critical value or correct probability, **AWRT** 0.043 or 0.0435
	$\left[74.222 <\right] 125.44 < 129.561$ Do not reject H_0	**A1ft**	Correctly compares their χ^2 test statistic and their critical value or their probability and 0.025 and does not reject the null hypothesis
	Sufficient evidence to suggest that the population standard deviation of the lifetimes of the moths is 5 days	**E1**	Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value Condone definite conclusion
		6	

Q	Answer	Marks	Comments
4(c)	$H_0 : \sigma_M = \sigma_B$ $H_1 : \sigma_M < \sigma_B$	**B1**	Both hypotheses **oe**
	$\frac{s_B^2}{s_M^2} = \frac{5.9^2}{5.6^2}$	**M1**	or $\frac{s_M^2}{s_B^2} = \frac{5.6^2}{5.9^2}$ **PI**
	$= 1.11$	**A1**	**AWRT** 1.11 or **AWRT** 0.90
	$F_{50,100} = 1.477$	**B1**	Finds correct critical value 1.477 or $\frac{1}{1.477}$ = **AWRT** 0.68 or correct probability **AWRT** 0.32 or 0.325
	$1.11 < 1.477$ Do not reject H_0	**A1ft**	Correctly compares their F test statistic and their critical value or their probability and 0.05 and does not reject null hypothesis
	Insufficient evidence to suggest that the population variance of the lifetimes of the butterflies is greater than the population variance of the lifetimes of the moths	**E1**	Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value Condone definite conclusion
		6	

	Question 4 Total	**13**	

Q	Answer	Marks	Comments
5	$H_0 : \mu_B = \mu_A$ $H_1 : \mu_B > \mu_A$	**B1**	Both hypotheses If use μ_D, H_1 must be consistent with their differences **oe**
	Employee / Difference: 1: 8 or –8; 2: 16 or –16; 3: 0 or 0; 4: 1 or –1; 5: –3 or 3; 6: 1 or –1; 7: 3 or –3	**B1**	All differences
	$\bar{x} = \frac{26}{7}$ or $-\frac{26}{7}$	**B1**	Sight of **AWRT** 3.7 or –3.7 Must be consistent with their differences
	$s^2 = \frac{284}{7}$	**B1**	**AWRT** 40.6 Accept s = **AWRT** 6.4
	$t = \frac{\frac{26}{7}}{\sqrt{\frac{\left(\frac{284}{7}\right)}{7}}}$ or $\frac{-\frac{26}{7}}{\sqrt{\frac{\left(\frac{284}{7}\right)}{7}}}$	**M1**	Using their mean and variance Condone use of $\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{S_p^2\left(\frac{1}{n_x}+\frac{1}{n_y}\right)}}$
	= 1.54 or –1.54	**A1**	**AWRT** Must be consistent with their differences
	t_6 critical value = 1.94 or –1.94	**B1**	**AWRT** Must be consistent with their differences unless changes –1.54 to 1.54 first
	$1.54 < 1.94$ Do not reject H_0	**A1ft**	Correctly compares their t test statistic and their critical value and does not reject null hypothesis
	Sufficient evidence to suggest that average number of errors made each day has not reduced following the training course	**E1ft**	Gives a conclusion in context based on a comparison using the t-distribution Conclusion must not be definite
	Question 5 Total	**9**	

Q	Answer	Marks	Comments
6(a)	$z = 1.96$	B1	**AWRT** 1.96
	$1.96 \times \frac{300}{\sqrt{n}} = 29.4$	M1	Set up an equation with their $z \times \frac{300}{\sqrt{n}}$ **oe**
	$n = 400$	A1	400, 399 or 401
		3	

Q	Answer	Marks	Comments
6(b)	$z = 2.5758$	B1	**AWRT** 2.58 **PI**
	Width $= 2 \times 2.5758 \times \frac{300}{\sqrt{400}}$	M1	Calculates either the width or half-width using their z and their n **PI**
	$= 77.3$	A1	If 400 final answer in **(a)** **AWRT** 77.3 If 399 final answer in **(a)** **AWRT** 77.4 If 401 final answer in **(a)** **AWRT** 77.2
		3	

Q	Answer	Marks	Comments
6(c)	Upper limit $= 4450 + 0.5 \times 77.3 = 4488.65$	B1ft	Calculates upper limit of confidence interval **ft** their width
	4500 is not in the confidence interval so Rashida will reject the null hypothesis	E1ft	Correct conclusion **ft** their confidence interval
		2	

	Question 6 Total	**8**	

Q	Answer	Marks	Comments
7(a)	H_0 : There is no association between time of day and number of snacks eaten H_1 : There is an association between time of day and number of snacks eaten	B1	Both hypotheses, variables must be stated in at least the null hypothesis **oe**
		1	

Q	Answer	Marks	Comments
7(b)	There are expected frequencies less than 5 so two columns need to be merged	E1	Explains that columns need to be merged because there are expected frequencies less than 5 If particular expected frequencies are identified, they need to be correct (1.76 and 2.24)
	So degrees of freedom $= (2-1)(2-1) = 1$	B1	Shows correct calculation of degrees of freedom
		2	

Q	Answer	Marks	Comments
7(c)	$\sum \frac{\left(\lvert O-E\rvert - 0.5\right)^2}{E}$ or $\sum \frac{\left(\lvert O_i-E_i\rvert - 0.5\right)^2}{E_i}$	B1	Correct test statistic
		1	

Q	Answer	Marks	Comments
7(d)	Critical value = 3.841	B1	**AWRT** 3.8 or correct probability **AWRT** 0.025
	$5.05 > 3.841$	M1	Correctly compares χ^2 test statistic and their critical value or their probability and 0.05
	So null hypothesis is rejected	A1	Correct conclusion
		3	

	Question 7 Total	**7**	

Q	Answer	Marks	Comments
8(a)	$\mathrm{E}(R) = \mathrm{E}\left(\frac{1}{n+2}\left(A + B + \sum_{i=1}^{n} X_i\right)\right)$		
	$= \frac{\mathrm{E}(A) + \mathrm{E}(B) + \sum_{i=1}^{n} \mathrm{E}(X_i)}{n+2}$	**M1**	Finds $\mathrm{E}(R)$ in terms of $\mathrm{E}(A)$, $\mathrm{E}(B)$, $\mathrm{E}(X_i)$ and n
	$= \frac{\mu + \mu + n\mu}{n+2}$	**A1**	Find $\mathrm{E}(R)$ in terms of μ and n
	$= \frac{\mu(n+2)}{n+2} = \mu$ therefore unbiased	**A1**	Must see $n+2$ cancelled to give μ and conclusion
		3	

Q	Answer	Marks	Comments
8(b)	$\text{Var}(R) = \text{Var}\left(\frac{1}{n+2}\left(A+B+\sum_{i=1}^{n} X_i\right)\right)$ $= \frac{\text{Var}(A)+\text{Var}(B)+\sum_{i=1}^{n}\text{Var}(X_i)}{(n+2)^2}$	M1	Finds $\text{Var}(R)$ in terms of $\text{Var}(A)$, $\text{Var}(B)$, $\text{Var}(X_i)$ and n Condoning not squaring $n+2$ May be seen in part **(c)** if not attempted in this part
	$= \frac{\sigma^2+\sigma^2+n\sigma^2}{(n+2)^2}$	A1	Find $\text{Var}(R)$ in terms of σ^2 and n May be seen in part **(c)** if not attempted in this part
	$= \frac{(n+2)\sigma^2}{(n+2)^2} = \frac{\sigma^2}{(n+2)}$ As $n \to \infty$, $\text{Var}(R) \to 0$ therefore consistent	A1	Correctly finds $\text{Var}(R) = \frac{\sigma^2}{(n+2)}$, applies limiting process and gives conclusion
		3	

Q	Answer	Marks	Comments
8(c)	$\text{Var}\left(\bar{X}\right) = \frac{\sigma^2}{n}$	B1	Finds $\text{Var}\left(\bar{X}\right) = \frac{\sigma^2}{n}$ or $\frac{n\sigma^2}{n^2}$
	Relative Efficiency $= \frac{\frac{1}{\text{Var}(R)}}{\frac{1}{\text{Var}(\bar{X})}} = \frac{\frac{n+2}{\sigma^2}}{\frac{n}{\sigma^2}}$	M1	Applies relative efficiency formula either way round with either the correct $\text{Var}(R)$ or their $\text{Var}(R)$ from part **(b)** and their $\text{Var}\left(\bar{X}\right)$
	$= \frac{n+2}{n}$	A1	Correct simplification, if calculates relative efficiency the other way round will achieve $\frac{n}{n+2}$
	The student's claim is not true as estimator R is more efficient than estimator $\bar{X}$ as $\frac{n+2}{n} > 1$	E1	Correct conclusion and justification **CSO** If calculates relative efficiency the other way round justification will be $\frac{n}{n+2} < 1$
		4	

	Question 8 Total	**10**	

Q	Answer	Marks	Comments
9(a)	$z = 2.3263$	B1	**AWRT** 2.33
	$8 \pm 2.3263 \times \sqrt{\frac{10.24}{6}}$	M1	Attempts to calculate one of the limits
	$\bar{X} < 4.961, \bar{X} > 11.039$	A1	Correct critical region **AWRT** 4.961 and **AWRT** 11.039 Condone $\bar{X} < 4.961$ and $\bar{X} > 11.039$ Do not ignore subsequent working
		3	

Q	Answer	Marks	Comments
9(b)	Power $=$ $\mathrm{P}\left(Z < \frac{4.961 - 11.4}{\sqrt{\frac{10.24}{6}}}\right) + \mathrm{P}\left(Z > \frac{11.039 - 11.4}{\sqrt{\frac{10.24}{6}}}\right)$	M1	Identifies correct probabilities corresponding to their critical region **PI**
	$= 0.61$	A1	**AWRT** 0.61
		2	

	Question 9 Total	**5**	

Q	Answer	Marks	Comments
10	H_0 : Lifetime of the star has a normal distribution H_1 : Lifetime of the star does not have a normal distribution	**B1**	Both hypotheses Variable should be stated in at least the null hypothesis
	$\overline{t} = 9.5$ and $s^2 = 0.04$	**B1**	
	t | Probability | Expected Frequency $t \leq 9.25$ | 0.10565 | 5.2825 $9.25 < t \leq 9.5$ | 0.39435 | 19.7175 $9.5 < t \leq 9.75$ | 0.39435 | 19.7175 $t > 9.75$ | 0.10565 | 5.2825	**M1** **A1ft** **A1**	**M1:** Uses $T \sim N$(their 9.5, their 0.04) to find a probability **A1:** Correct probabilities to 2 decimal places **PI** **ft** their 9.5 and 0.04 **A1:** Correct expected frequencies to 2 decimal places
	$\sum \frac{(O-E)^2}{E} = \frac{(6-5.2825)^2}{5.2825} + \frac{(22-19.7175)^2}{19.7175} + \frac{(13-19.7175)^2}{19.7175} + \frac{(9-5.2825)^2}{5.2825}$	**M1**	Attempts to calculate test statistic
	$= 5.3$	**A1**	**AWRT** 5.3
	$\nu = 4-2-1 = 1$	**B1**	**PI** by correct critical value
	$\chi^2_1(0.99) = 6.635$	**B1ft**	Finds correct critical value or correct probability, **AWRT** 0.02 **ft** their degrees of freedom
	$5.3 < 6.635$ Do not reject H_0	**A1ft**	Correctly compares their χ^2 test statistic and their critical value or their probability and 0.01 and does not reject null hypothesis
	Sufficient evidence to suggest that the lifetime of the star can be modelled by a normal distribution	**E1**	Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value Condone definite conclusion

	Question 10 Total	**11**	