

OXFORD

INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL AS FURTHER MATHEMATICS FM02

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

June 2023

Version: 1.0 Final



Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from oxfordaqaexams.org.uk

Copyright information

OxfordAQA retains the copyright on all its publications. However, registered schools/colleges for OxfordAQA are permitted to copy material from this booklet for their own internal use, with the following important exception: OxfordAQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Copyright © 2023 Oxford International AQA Examinations and its licensors. All rights reserved.

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$hf(2,1) = 0.2 \times \frac{1}{1+\sqrt{2}}$ $= 0.0828427$ $y_2 = 1 + 0.0828427 = 1.0828427$ $y_3 = 1.0828427 + 0.2 \times \frac{1}{1.0828427 + \sqrt{2.2}}$ $[= 1.0828427 + 0.2 \times 0.3896991 = 1.1607825]$ 1.161	<p>M1</p> <p>A1</p> <p>m1</p> <p>M1</p> <p>A1</p>	<p>Correct substitution into RHS of this expression</p> <p>AWRT 0.0828 PI</p> <p>1 + their value of $hf(2,1)$</p> <p>Correct substitution using their y_2 into the second term.</p> <p>Correct answer given to 3 dp</p>
		5	
	Question 1 Total	5	

Q	Answer	Marks	Comments
2(a)	$= \begin{bmatrix} 2p & -1 \\ -2 & 3p \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 2p \times 3 + (-1) \times (-2) \\ (-2) \times 3 + 3p \times (-2) \\ 3 \times 3 + 0 \times (-2) \end{bmatrix}$ $= \begin{bmatrix} 6p+2 \\ -6-6p \\ 9 \end{bmatrix}$	<p>M1</p> <p>A1</p>	<p>transpose A and correct calculation for at least one row</p> <p>PI by correct answer</p> <p>Condone multiplying by $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ as a misread</p>
		2	

Q	Answer	Marks	Comments
2(b)(i)	$\mathbf{C}^2 = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$ $k = -2$	<p>M1</p> <p>A1</p>	<p>Finds \mathbf{C}^2</p> <p>May be seen within incorrect working</p> <p>Clearly stated</p>
		2	

Q	Answer	Marks	Comments
2(b)(ii)	$\mathbf{C}^{12} = (-2)^6 \mathbf{I} \text{ or } \mathbf{C}^{12} = \begin{bmatrix} 64 & 0 \\ 0 & 64 \end{bmatrix}$ $\text{so } \mathbf{C}^{13} = \begin{bmatrix} 0 & 128 \\ -64 & 0 \end{bmatrix}$	<p>M1</p> <p>A1ft</p>	<p>Use of $\mathbf{C}^2 = k\mathbf{I}$</p> <p>PI by $\mathbf{C}^{13} = \begin{bmatrix} 64 & 0 \\ 0 & 64 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$,</p> <p>$\mathbf{C}^{13} = \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 64 & 0 \\ 0 & 64 \end{bmatrix}$ or</p> <p>$\mathbf{C}^{13} = 64 \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$</p> <p>ACF</p> <p>ft from their k</p> <p>NMS scores zero</p>
		2	

	Question 2 Total	6	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
3(a)(i)		B1	line of best fit seen
		1	

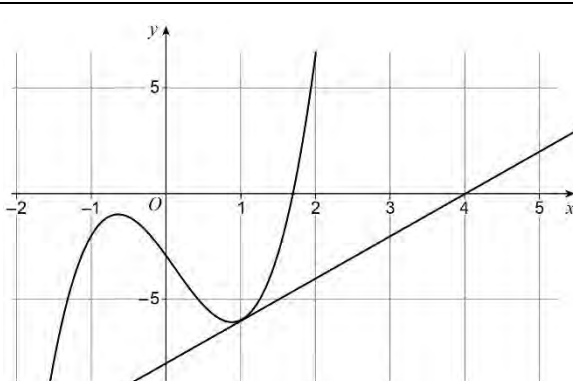
Q	Answer	Marks	Comments
3(a)(ii)	$\log_{10} S = \log_{10} a + t \log_{10} b$ y -intercept = 0.27 and gradient = 0.13 $0.27 = \log_{10} a$ or $0.13 = \log_{10} b$ $[a = 10^{0.27} \text{ or } b = 10^{0.13}]$ $a = 1.9$ $b = 1.3$	M1 M1 M1 A1	Takes logs of both sides to reduce to linear form PI Sight of y -intercept and gradient values Must be from a line of best fit At least one logarithm for a or b and set equal to their y -intercept or gradient AWFW [1.8, 2.0] for a AWFW [1.3, 1.4] for b All M marks must be scored
		4	

Q	Answer	Marks	Comments
3(b)	$[11.2 = ab^t]$ $t = \frac{\log_{10} \left(\frac{11.2}{a} \right)}{\log_{10} b}$ $t = 6.8$	<p>M1</p> <p>A1ft</p>	<p>with their values of a and b</p> <p>oe, eg $t = \log_b \left(\frac{11.2}{a} \right)$ or $t = \frac{\log_{10} 11.2 - \text{y-intercept}}{\text{gradient}}$</p> <p>ft their a and b or their y-intercept and gradient</p>
		2	

	Question 3 Total	7	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
4(a)	$f(1) = -6$ and $f(2) = 7$ sign change & continuous function, so the root α lies in the interval $1 < x < 2$	M1 A1	Correct evaluation of a suitable interval Must state that there is a change of sign and that the curve is continuous (condone unbroken) and concludes a root is present in the interval
		2	

Q	Answer	Marks	Comments
4(b)	$f(1.5) = -2.625$; negative so $1.5 < \alpha < 2$ $f(1.75) = 1.265625$ so $1.5 < \alpha < 1.75$	M1 m1 A1	range PI by subsequent calculation of $f(1.75)$ CSO but accept rounded or truncated values of $f(1.5) = -2.625$ and $f(1.75) = 1.265625$
		3	

Q	Answer	Marks	Comments
4(c)(i)	 <p>tangent meets x-axis further from root [than $x = 1$]</p>	B1 E1	tangent at $x = 1$ drawn, crossing x -axis may include reference to root in interval $1.5 < x < 1.75$ from part (b)
		2	

Q	Answer	Marks	Comments
4(c)(ii)	$f'(x) = 9x^2 - 2x - 5$	B1	Correct first derivative PI
	$f'(1.75) = 9(1.75)^2 - 2(1.75) - 5$ [= 19.0625...]	M1	Substitution of $x = 1.75$ into their first derivative PI
	$1.75 - \frac{1.265625}{19.0625}$	m1	Use of Newton-Raphson formula PI
	= 1.6836	A1	CAO , must be 4 dp
		4	

	Question 4 Total	11	
--	-------------------------	-----------	--

Q	Answer	Marks	Comments
5(a)	$\det(\mathbf{A}) = (1 - 0.4k)(1 + 0.4k) - (-0.8k)(0.2k)$	M1	Correct expression for determinant
	$= 1 - 0.16k^2 + 0.16k^2 = 1 \neq 0$ So non-singular for all values of k	A1	Finds determinant to be 1 and non-zero and gives conclusion
		2	

Q	Answer	Marks	Comments
5(b)(i)	$\begin{bmatrix} 1.4 & 0.8 \\ -0.2 & 0.6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$	M1	LHS correct and attempt at multiplication resulting in a 2×1 vector
	(3, 1)	A1	CAO must be coordinates
		2	

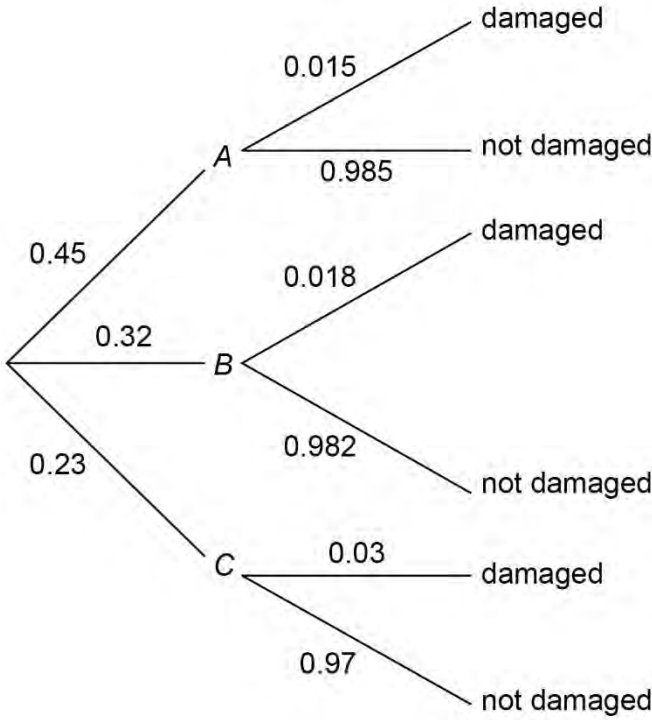
Q	Answer	Marks	Comments
5(b)(ii)	$x' = 1.4x + 0.8(mx + c)$	M1	Valid attempt to find x', y'
	$y' = -0.2x + 0.6(mx + c)$		Condone $x = 1.4x + 0.8(mx + c)$ and $mx + c = -0.2x + 0.6(mx + c)$
	$-0.2x + 0.6(mx + c)$	m1	ft their $y' = m(\text{their } x') + c$
	$= m(1.4x + 0.8(mx + c)) + c$	m1	Attempt to find m and c by comparing coefficients or setting coefficients = 0
	$0.8m^2 + 0.8m + 0.2 = 0$ $0.6c - 0.8mc - c = 0$ $m = -\frac{1}{2}$	B1	correct value of m
	lines are $y = -\frac{1}{2}x + c$ [where c is real]	A1	no restrictions on c
		5	

Q	Answer	Marks	Comments
5(b)(iii)	The determinant of A is equal to 1	E1	
	All the invariant lines of A are parallel	E1	
		2	
	Question 5 Total	11	

Q	Answer	Marks	Comments
6(a)	$G'_w(t) = p + 3(0.9 - p)t^2$	B1	oe
		1	

Q	Answer	Marks	Comments
6(b)	$G'_w(1) = p + 3(0.9 - p) = 2.5$ $p = 0.1$ $P(W \leq 1) = 0.1 + p$ $P(W \leq 1) = 0.2$	M1 A1 M1 A1	Forms an equation in p using $G'_w(1) = 2.5$ PI
		4	

	Question 6 Total	5	
--	------------------	---	--

Q	Answer	Marks	Comments
7(a)		<p>M1</p> <p>A1</p>	<p>Correct structure – branches and events Allow use of any letter for damaged or not damaged</p> <p>Fully correct</p>
		2	

Q	Answer	Marks	Comments
7(b)	$P(C \mid \text{not damaged}) = \frac{0.23 \times 0.97}{0.45 \times 0.985 + 0.32 \times 0.982 + 0.23 \times 0.97}$ $= 0.2275[160873]$ $= 0.228$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>Numerator calculation seen</p> <p>Denominator calculation seen</p> <p>AG Must see an answer given to four or more significant figures before final answer</p>
		3	

	Question 7 Total	5	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
8(a)	$\frac{n+1}{2} = 15 \Rightarrow n = 29$	M1	Forms correct equation and attempts to solve to find n Condone slips on rearranging to find n
	$\text{Var}(X) = \frac{29^2 - 1}{12} = 70$	A1	
		2	

Q	Answer	Marks	Comments
8(b)(i)	$p(1-p) = 0.1824$	M1	Forms correct equation
	$p = 0.76$ and $p = 0.24$	A1	Finds both solutions oe
	$p = 0.24 \Rightarrow E(Y) = \frac{1}{0.24}$ or $p = 0.76 \Rightarrow E(Y) = \frac{1}{0.76}$	M1	Attempts to find $E(Y)$ for one of their values of p where $0 < p < 1$
	$\text{Var}(Y) = \frac{1-0.24}{0.24^2}$ or $\text{Var}(Y) = \frac{1-0.76}{0.76^2}$	M1	Attempts to find $\text{Var}(Y)$ for one of their values of p where $0 < p < 1$
	$p = 0.24$ gives $E(Y) > 2$ but $p = 0.76$ gives $E(Y) < 2$ so $p = 0.24$ so $\text{Var}(Y) = \frac{1-0.24}{0.24^2} = \frac{475}{36}$	A1	AG States that $p = 0.24$ gives $E(Y) > 2$ but $p = 0.76$ gives $E(Y) < 2$ so $p = 0.24$ and so $\text{Var}(Y) = \frac{1-0.24}{0.24^2} = \frac{475}{36}$
		5	

Q	Answer	Marks	Comments
8(b)(ii)	$\text{Var}(X) + 6^2\text{Var}(Y)$ $= 70 + 6^2 \times \frac{475}{36}$ $= 545$	M1	Substitutes their value of $\text{Var}(X)$ and $\frac{475}{36}$ into $\text{Var}(X) + 6^2\text{Var}(Y)$
	X and Y are dependent as $\text{Var}(X - 6Y) \neq \text{Var}(X) + 6^2\text{Var}(Y)$	A1ft	ft their $\text{Var}(X)$ from part (a) Implied by $\text{Cov}(X, Y) = -0.5$
		E1	CSO , oe eg $\text{Cov}(X, Y)$ is non-zero Conclusion with justification
		3	

	Question 8 Total	10	
Q	Answer	Marks	Comments
9(a)	$[I] = [Ft]$ $= MLT^{-2} \times T$ $= MLT^{-1}$	M1 A1	Uses the formula $I = Ft$ [or $I = mv - mu$] and attempts to find the dimensions of impulse Condone use of units Correct dimensions of impulse.
		2	

Q	Answer	Marks	Comments
9(b)	$v = eu$ $[e] = \frac{LT^{-1}}{LT^{-1}} = L^0T^0 = 1$ $[\therefore \text{dimensionless}]$	B1	Correct argument Condone only seeing L^0T^0
		1	

Q	Answer	Marks	Comments
9(c)	$[mu(1+e)] = M \times LT^{-1} \times 1$ $= MLT^{-1}$ $[I] = MLT^{-1}$ $\therefore \text{dimensionally consistent}$	M1 A1 A1	Finds dimensions of RHS Condone use of units Correct dimensions of RHS Compares with dimensions of impulse and reaches correct conclusion.
		3	

	Question 9 Total	6	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
10(a)	$I = 6 \times 0.08$ $= 0.48 \text{ N s}$	B1	Correct impulse. Condone omission of units.
		1	

Q	Answer	Marks	Comments
10(b)	$-0.48 = 0.1 \times v_p - 0.1 \times 12$ $v_p = \frac{1.2 - 0.48}{0.1} \Rightarrow v_p = 7.2 \text{ m s}^{-1}$	M1 A1	Forms an equation using $I = mv - mu$ Condone sign errors AG Correct speed, with intermediate working shown.
		2	

Q	Answer	Marks	Comments
10(c)	$0.48 = 0.4 \times v_Q - 0.4 \times 8$ or $0.1 \times 12 + 0.4 \times 8 = 0.1 \times 7.2 + 0.4 \times v_Q$ $\Rightarrow v_Q = 9.2 \text{ m s}^{-1}$ $7.2 - 9.2 = -e(12 - 8)$ $e = \frac{2}{4} = 0.5$	M1 M1 A1	Attempts to find velocity of Q after the collision Condone sign errors Uses Newton's Experimental Law with their velocities Condone sign errors Correct coefficient of restitution.
		3	

	Question 10 Total	6	
--	-------------------	---	--

Q	Answer	Marks	Comments
11(a)	$\mathbf{r}_A = \begin{bmatrix} 80 \\ 100 \end{bmatrix} + t \begin{bmatrix} 5 \\ p \end{bmatrix}$ $\mathbf{r}_B = \begin{bmatrix} 200 \\ 40 \end{bmatrix} + t \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ $80 + 5t = 200 + 4t$ $t = 120$ $100 + 120p = 40 + 5 \times 120$ $p = \frac{540}{120} = \frac{9}{2} = 4.5$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Forms correct expressions for position vectors of A and B PI</p> <p>Forms a correct equation to find the time of interception oe</p> <p>Correct time for interception.</p> <p>Correct value for p</p>
		4	

[illegible]