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FURTHER MATHEMATICS

FM01

(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

January 2021

Version: 1.0 Final

211XFM01/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$6\times\left(\frac{2}{3}+h\right)^2-8\times\left(\frac{2}{3}+h\right)+5$ $=6\left(\frac{4}{9}+\frac{4}{3}h+h^2\right)-\frac{16}{3}-8h+5$ $=\frac{7}{3}+6h^2$	**M1**	**PI** Allow one slip
	Gradient $=\dfrac{\frac{7}{3}+6h^2-\frac{7}{3}}{h}$	**M1**	**FT** their $\frac{7}{3}+6h^2$ minus $\frac{7}{3}$
	$=6h$	**A1**	**CAO** Must score **M1 M1**
		3	

Q	Answer	Marks	Comments
1(b)	Gradient of curve $=\lim\limits_{h\to 0}[6h][=0]$	**B1ft**	**FT** their '$6h$' with correct limiting process
	So the curve has a stationary point at $x=\frac{2}{3}$	**E1**	**FT** correct conclusion based upon their '$6h$' and the gradient being zero
		2	

	Question 1 Total	**5**	

Q	Answer	Marks	Comments
2	Let $z = x + \mathrm{i}y$ $x + \mathrm{i}y - 4 = a\mathrm{i}(x + \mathrm{i}y + 5)$ $x - 4 + \mathrm{i}y = -ay + \mathrm{i}ax + 5\mathrm{i}a$		
	$x - 4 = -ay$ $y = a(x+5)$	**M1**	Equating real and imaginary parts Allow one slip
	$x - 4 = -a^2(x+5)$ $x + a^2x = 4 - 5a^2$	**M1**	Eliminating x or y from both equations
	$x = \dfrac{4-5a^2}{1+a^2}$	**A1**	
	$x + 5 = \dfrac{4 - 5a^2 + 5 + 5a^2}{1+a^2} = \dfrac{9}{1+a^2}$ $y = \dfrac{9a}{1+a^2}$	**M1**	
	$z = \dfrac{4-5a^2}{1+a^2} + \mathrm{i}\left(\dfrac{9a}{1+a^2}\right)$	**A1**	
2 ALT	$z - 4 = a\mathrm{i}z + 5a\mathrm{i}$	**M1**	
	$z(1 - a\mathrm{i}) = 4 + 5a\mathrm{i}$	**A1**	
	$z = \dfrac{4+5a\mathrm{i}}{1-a\mathrm{i}} \times \dfrac{1+a\mathrm{i}}{1+a\mathrm{i}}$	**M1**	
	$z = \dfrac{4 + 5a\mathrm{i} + 4a\mathrm{i} - 5a^2}{1+a^2}$	**M1**	
	$z = \dfrac{4-5a^2}{1+a^2} + \mathrm{i}\left(\dfrac{9a}{1+a^2}\right)$	**A1**	
		5	

	Question 2 Total	**5**	

Q	Answer	Marks	Comments
3	$\frac{dy}{dx} = -\frac{3}{2}x^{-\frac{5}{2}}$	**M1**	
	$\frac{dy}{dx} = -\frac{1}{162}$ when $x = 9$	**A1**	**PI**
	$\delta y \approx \frac{dy}{dx} \times \delta x$	**M1**	**PI** Condone use of = sign
	[Estimate =] $0.02 \times \left(-\frac{1}{162}\right)$ or $-\frac{1}{8100}$ **oe**	**A1F**	**FT** $0.02 \times \left(\text{their} -\frac{1}{162}\right)$
	[Estimate =] $\frac{1}{27} +$ their $-\frac{1}{8100}$	**M1**	**PI**
	[Estimate =] $\frac{299}{8100}$	**A1**	**CSO** Must be $\frac{299}{8100}$
		6	

	Question 3 Total	**6**	

Q	Answer	Marks	Comments
4(a)	$\frac{x}{2}+\frac{2\pi}{3}=2n\pi\pm\frac{5\pi}{6}$	B1	oe
	Going from $\left(\frac{x}{2}+\frac{2\pi}{3}\right)$ to x	M1	Including multiplication of all terms by 2
	$x=4n\pi+\frac{\pi}{3}$	A1	
	$x=4n\pi+\pi$	A1	**A1 A1** for $x=4n\pi-\frac{4\pi}{3}\pm\frac{5\pi}{3}$ **oe**
		4	

Q	Answer	Marks	Comments
4(b)	$S_1=\frac{\pi}{3}+\frac{13\pi}{3}+\ldots+\frac{109\pi}{3}$ and $S_2=\pi+5\pi+\ldots+33\pi$	M1	For forming two series
	$S_1=\frac{550\pi}{3}$	A1	For summing one AP with correct n
	$S_2=153\pi$	A1	For summing a 2nd AP with correct n
	Sum $=\frac{550\pi}{3}+153\pi$	M1	
	$\frac{1009\pi}{3}$	A1	
		5	

	Question 4 Total	9	

Q	Answer	Marks	Comments
5	$(2\alpha+\beta)(\alpha+2\beta)=73.16$ $2(\alpha+\beta)^2+\alpha\beta=73.16$ or $(-6+\alpha)(-6+\beta)=$ $36-6(\alpha+\beta)+\alpha\beta=73.16$	**M1** **M1**	or $(5x+31)(5x+59)=0$ or $2\alpha+\beta=-\frac{31}{5}$ and $\alpha+2\beta=-\frac{59}{5}$
	Using $\alpha+\beta=-6$ and $\alpha\beta=p$	**M1**	or $\alpha=-\frac{1}{5}$ and $\beta=-\frac{29}{5}$
	$p=1.16$	**A1**	**oe** **CSO**
		4	

	Question 5 Total	**4**	

Q	Answer	Marks	Comments
6	$\sum_{r=1}^{n}(8r^3+r)=8\sum_{r=1}^{n}r^3+\sum_{r=1}^{n}r$	M1	
	$=8\left(\frac{1}{4}\right)n^2(n+1)^2+\frac{1}{2}n(n+1)$	A1	
	$=\frac{1}{2}n(n+1)\left(4n(n+1)+1\right)$	M1	Must see attempt at factorising
	$=\frac{1}{2}n(n+1)(2n+1)^2$	A1	Factorising $\left(4n(n+1)+1\right)=(2n+1)^2$ **PI** by seeing $3(2n+1)$ and no errors seen
	$\sum_{r=1}^{n}r^2=\frac{1}{6}n(n+1)(2n+1)$		
	so $\sum_{r=1}^{n}(8r^3+r)=3(2n+1)\left(\sum_{r=1}^{n}r^2\right)$	A1	Be convinced
		5	

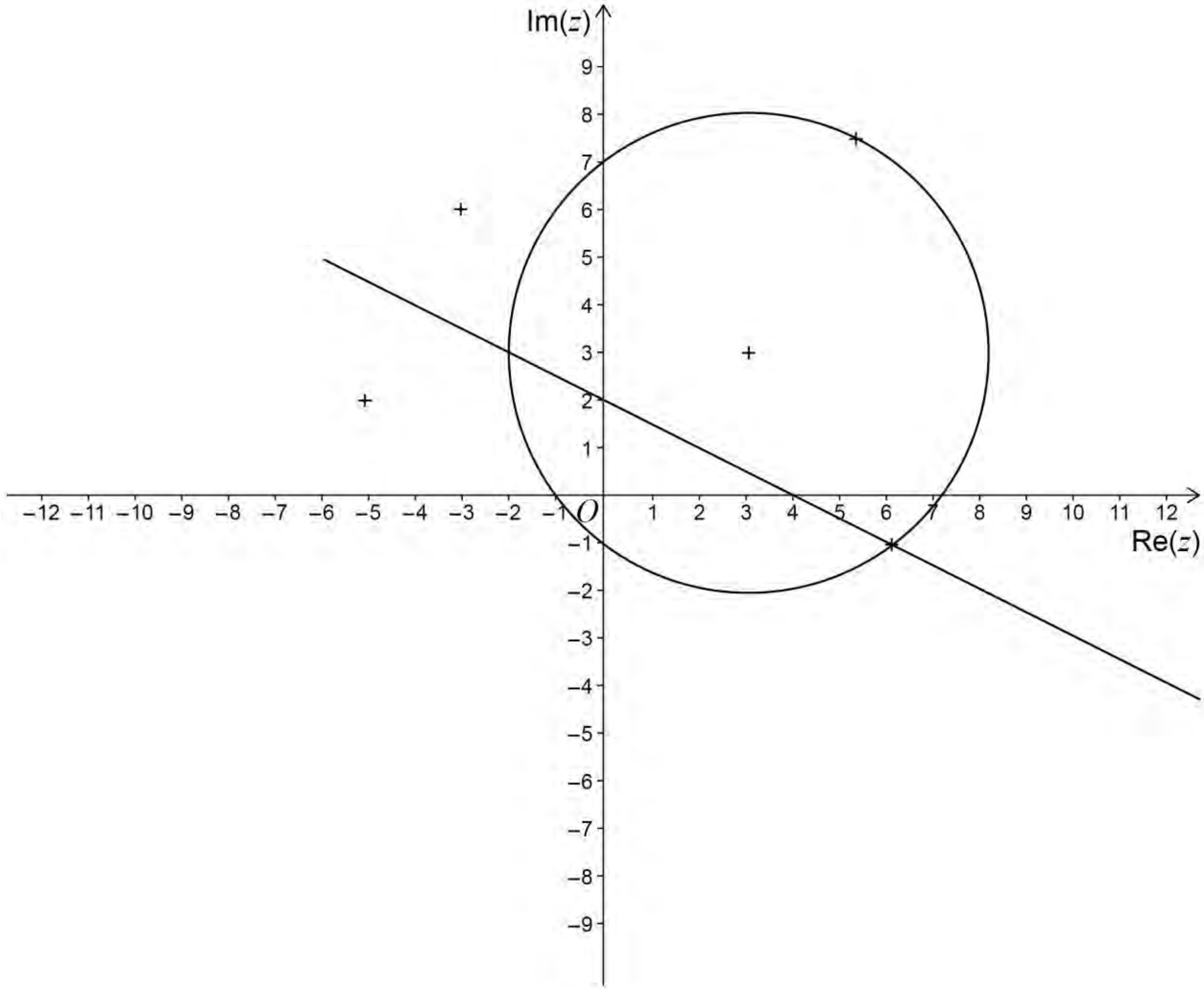
	Question 6 Total	**5**	

Q	Answer	Marks	Comments
7(a)	Ahmed $n=0:\ I_0=\int_0^9 x^{0.5}\ \mathrm{d}x$ [This is not an improper integral, as all required values of the integrand are finite] Ahmed is incorrect.	E1	
	Brian $n=-1:\ I_{-1}=\int_0^9 x^{-0.5}\ \mathrm{d}x$ This is an improper integral, because the integrand is not defined at the lower limit.	E1	or shows that $\int_0^9 x^{-0.5}\ \mathrm{d}x=6$ using a limiting process
	Brian is correct (with reason given)	E1	
	Catherine $n=-2:\ I_{-2}=\int_0^9 x^{-1.5}\ \mathrm{d}x$ $=\lim_{h\to 0}\left(\frac{9^{-0.5}}{-0.5}-\frac{h^{-0.5}}{-0.5}\right)$	B1	
	This does not have a finite value. Catherine is incorrect (with reason given)	E1	
		5	

Q	Answer	Marks	Comments
7(b)	$\left[I_{-1}=\lim_{h\to 0}\left(\frac{\sqrt{9}}{0.5}-\frac{\sqrt{h}}{0.5}\right)=\right]$ 6	B1	
		1	

	Question 7 Total	**6**	

Q	Answer	Marks	Comments
8(a)	Circle with centre $3 + 3\text{i}$, radius $= 5$ Line of negative gradient through $-4 + 4\text{i}$ Correct line	**B1** **B1** **B1** **B1**	



		4	

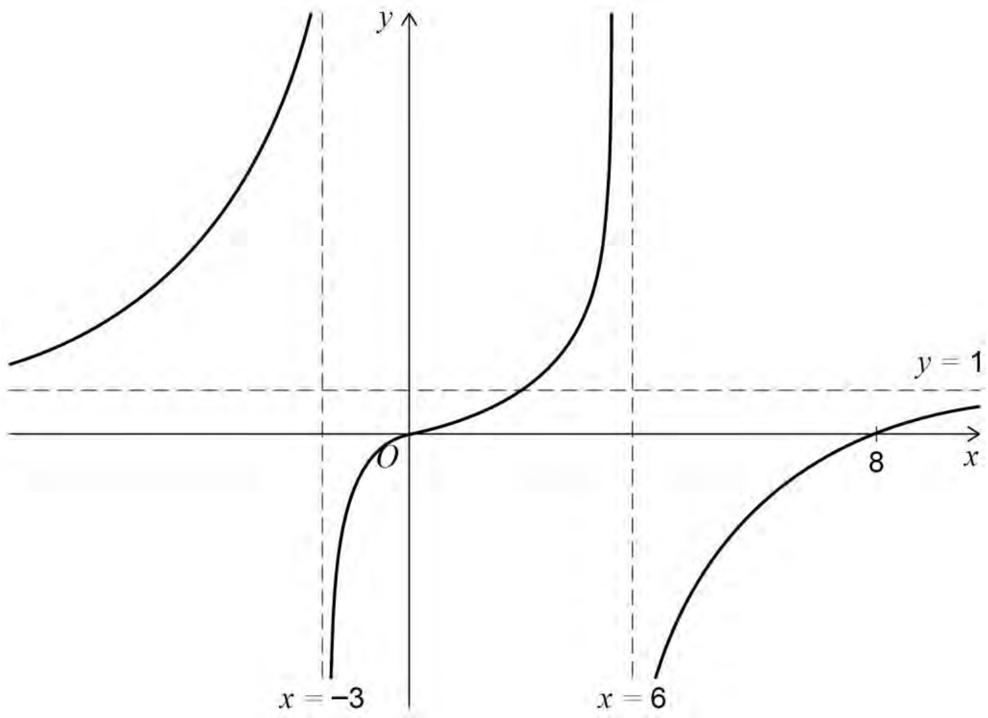
Q	Answer	Marks	Comments
8(b)	Cartesian equation of *L* $y-4=-\frac{1}{2}(x+4)$ $x=4-2y$	B1	
	Cartesian equation of *C* $(x-3)^2+(y-3)^2=25$	B1	
	$(1-2y)^2+(y-3)^2=25$ $y^2-2y-3=0$	M1	**oe** quadratic equation in x, **ie** $x^2-4x-12=0$
	Substituting $y=3$ **or** –1 into an equation to find the corresponding value for x	M1	**oe** for values of x to find y
	$z_1=-2+3\mathrm{i}$ and $z_2=6-\mathrm{i}$	A1	**or** the other way round
		5	

Q	Answer	Marks	Comments
8(c)	$\lvert z_2-z_1\rvert=\sqrt{8^2+4^2}=4\sqrt{5}$	M1	**or** $\frac{1}{2}(z_1+z_2)=2+\mathrm{i}$
	Let h = distance from (3, 3) to L Then $h^2=5^2-\left(2\sqrt{5}\right)^2$	M1	$\lvert(2+\mathrm{i})-(3+3\mathrm{i})\rvert$
	$h=\sqrt{5}$	A1	$=\sqrt{5}$
	Required distance = h + radius	M1	
	$5+\sqrt{5}$	A1	**CAO**
		5	

	Question 8 Total	**14**	

Q	Answer	Marks	Comments
9(a)	$x=-3$	B1	
	$x=6$	B1	
	$y=1$	B1	
		3	

Q	Answer	Marks	Comments
9(b)	$k\left(x^2-3x-18\right)=x^2-8x$	M1	
	$(k-1)x^2+(8-3k)x-18k=0$	A1	
	$(8-3k)^2-4(k-1)(-18k)$	M1	Discriminant in terms of k
	for real roots $(8-3k)^2-4(k-1)(-18k)\geq 0$	m1	Discriminant conditions for real roots being applied
	$81k^2-120k+64\geq 0$ $\left(9k-\frac{20}{3}\right)^2+\frac{176}{9}\geq 0$ (**or** >0) Always true so there are real roots for all real k	A1	Shows as sum of squares or Shows discriminant of $81k^2-120k+64$ is negative **and** states k^2 coefficient is positive.
		5	

9(c)	Graph of $y = \mathrm{f}(x)$ correct shape Asymptotes shown Values at axis intercepts shown	**B1** **B1** **B1**	
			
		3	

	Question 9 Total	**11**	

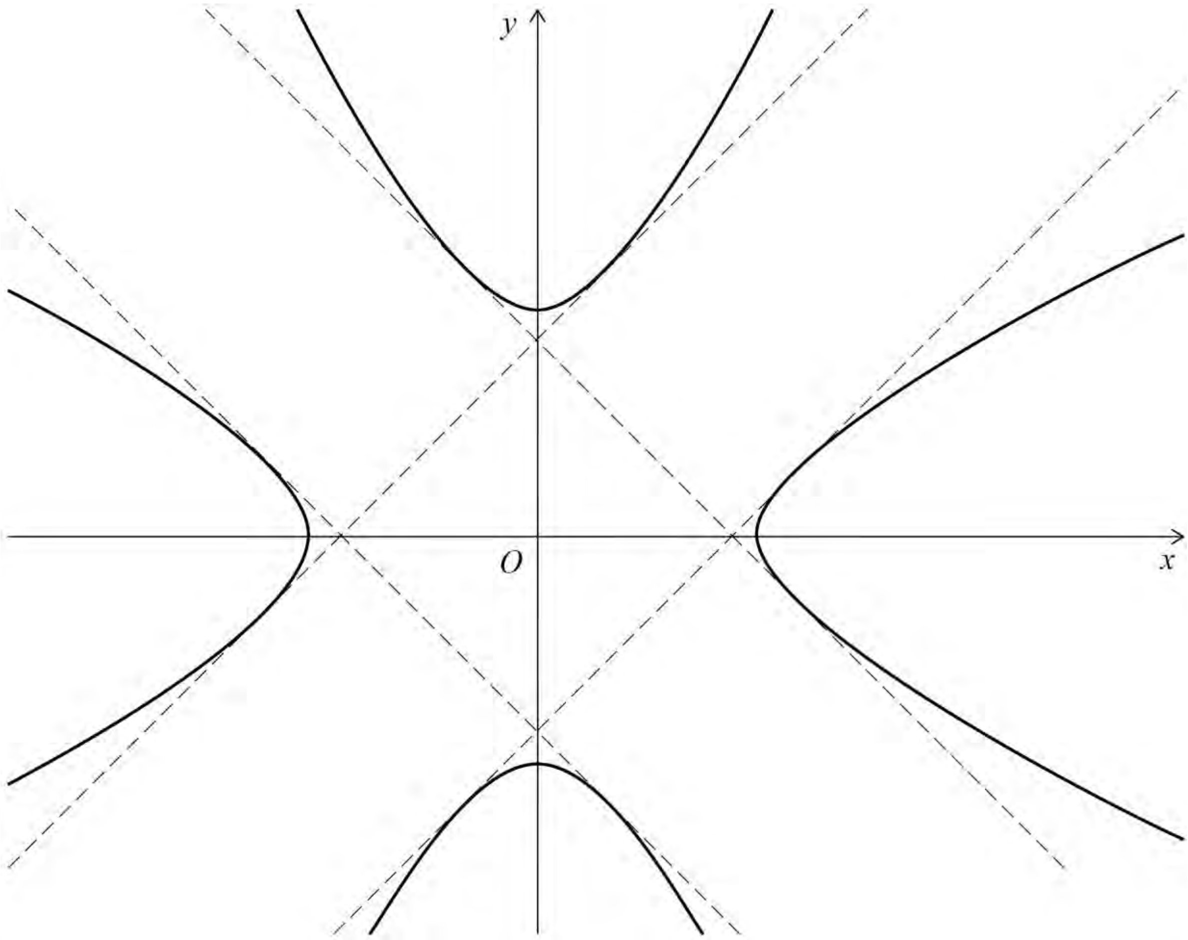
Q	Answer	Marks	Comments
10(a)	Reflection in the line $y = x$	B1	**or** reflection in the line $y = -x$
		1	

Q	Answer	Marks	Comments
10(b)	$H_1 : y = \frac{1}{2}x, \quad y = -\frac{1}{2}x$	B1	oe
	$H_2 : y = 2x, \quad y = -2x$	B1	oe
		2	

Q	Answer	Marks	Comments
10(c)	$x^2 - 4(mx + c)^2 = 1$	M1	
	$(1 - 4m^2)x^2 - 8mcx - (4c^2 + 1) = 0$	A1	
	$[\triangle = 0]$ $(-8mc)^2 + 4(1 - 4m^2)(1 + 4c^2) = 0$	M1	Their discriminant set equal to zero
	$64m^2c^2 + 4(1 - 4m^2 + 4c^2 - 16m^2c^2) = 0$	m1	Correct expansion of their discriminant
	$4 - 16m^2 + 16c^2 = 0$ $c^2 = \dfrac{4m^2 - 1}{4}$ as required	A1	
		5	

Q	Answer	Marks	Comments
10(d)	$c^2 \geq 0 \Rightarrow 4 - m^2 \geq 0$	M1	
	$-2 \leq m \leq 2$ $m = 2$ or $m = -2 \Rightarrow c = 0$ and $y = \pm 2x$ These lines are asymptotes, not tangents. So $-2 < m < 2$	A1	Allow $-2 \leq m \leq 2$
		2	

Q	Answer	Marks	Comments
10(e)	$4 - m^2 = 4m^2 - 1$ $5m^2 = 5$ $m^2 = 1$	M1	
	$c^2 = \frac{4-1}{4} = \frac{3}{4}$ $\left[\Rightarrow c = \pm\frac{\sqrt{3}}{2}\right]$	M1	
	$y = x + \frac{\sqrt{3}}{2}$, $y = x - \frac{\sqrt{3}}{2}$, $y = -x + \frac{\sqrt{3}}{2}$, $y = -x - \frac{\sqrt{3}}{2}$	A1	**oe**
		3	

Q	Answer	Marks	Comments
10(f)	Area in 1st quadrant $=\frac{1}{2}\times\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}=\frac{3}{8}$	M1	**or** for distance between adjacent vertices followed by squaring **or** other valid method **or** for finding axis intercepts and using them to calculate an area
	Total area $=\frac{3}{2}$	A1	
			
		2	
	Total	**15**	