OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM04) Unit FS2 Statistics

Monday 16 January 2023 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Answer **all** questions in the spaces provided.

Do not write outside the box

1 A random sample of size 10 is taken from a normal distribution with population standard deviation σ

The random sample is used to conduct a test at the 5% level of significance with the hypotheses

$$H_0: \ \sigma = \sigma_0$$
$$H_1: \ \sigma \neq \sigma_0$$

where σ_0 is the null hypothesis value for the standard deviation of the population.

The sample variance is 100

The test conclusion is that there is no reason to reject the null hypothesis.

1 (a) Show that the smallest possible value for σ_0 is 6.88 correct to three significant figures.

[4 marks]

Do not write outside the box

1 (b) Find the largest possible value for σ_0

Give your answer correct to three significant figures.

[2 marks]

Answer _____

6

Turn over for the next question

Do not write outside the box

2 A bag contains 5 cent and 50 cent coins only.

It is given that 70% of the coins in the bag are 5 cent coins.

A coin is selected at random and its value recorded.

The coin is returned to the bag and this process is repeated a further two times.

2 (a) Complete the sampling distribution table for V the total value (in cents) of the **three** coins selected.

[3 marks]

Total value v (cents)	15			150
P($V = v$)	0.343		0.189	

2 (b) By using your sampling distribution table in **part (a)** or otherwise

2 (b) (i) Complete the sampling distribution table of the modal value M of a coin in the sample.

[2 marks]

Modal value m (cents)	5	50
P($M = m$)		

Do not write outside the box

2 (b) (ii) Determine $\text{Var}(M)$ giving your answer correct to three significant figures.

[3 marks]

Answer _____

8

Turn over for the next question

Do not write outside the box

3 Raj, a business analyst, is monitoring the times of conference calls at his company.

The times of these conference calls are known to come from a normal distribution with variance 6.6 minutes^2

He randomly selects a sample of the conference calls and correctly calculates a 95% confidence interval as (27.8, 30.4) minutes.

3 (a) Find the mean of the sample used.

[1 mark]

Answer _____

3 (b) Show that the sample size used is 15

[3 marks]

Do not write outside the box

3 (c) The company target is for a mean conference call time of 30 minutes.

State with a reason whether or not Raj's sample provides evidence that the company target has been met.

[2 marks]

3 (d) Another business analyst thinks Raj's statistical test is invalid as the sample size is too small.

Identify **two** features of the distribution that justify Raj's calculation.

[2 marks]

8

Turn over for the next question

Do not write outside the box

4 The standard normal variable $Z \sim \mathrm{N}(0,1)$ has moment generating function

$$M_Z(t) = \mathrm{e}^{\frac{1}{2}t^2}$$

4 (a) Use $M_Z(t)$ to verify that the mean of Z is 0 and the variance of Z is 1

[5 marks]

Do not write outside the box

4 (b) The random variable X is given by

$$X = a + bZ$$

where a and b are real numbers.

Determine the moment generating function $M_X(t)$ giving your answer in the form $e^{f(t)}$

[2 marks]

Answer _____

4 (c) State $E(X)$ and $\text{Var}(X)$ in terms of a and b

[1 mark]

$E(X) =$ _____ $\text{Var}(X) =$ _____

4 (d) Use your results in **parts (b) and (c)** to find the moment generating function $M_Y(t)$ for

$$Y \sim N(\mu, \sigma^2)$$

giving your answer in terms of μ, σ and t

[2 marks]

Answer _____

10

Do not write outside the box

5 Two independent random variables X and Y have Poisson distributions with parameters λ and 2λ respectively.

Samples of n observations are taken from each population and sample means $\overline{X}$ and $\overline{Y}$ are determined.

Two estimators S and T for λ are proposed where

$$S = \frac{\overline{X} + \overline{Y}}{3} \quad \text{and} \quad T = \overline{Y} - \overline{X}$$

5 (a) Show that S and T are unbiased estimators of λ

[3 marks]

5 (b) Show that the relative efficiency of S to T is independent of the sample size n

[5 marks]

Do not write outside the box

5 (c) Show that S and T are both consistent estimators.

[2 marks]

10

Do not write outside the box

6 An athletics trainer claims that the reaction time of world-class sprinters can be modelled by the random variable T milliseconds, which has probability density function

$$f(t) = \begin{cases} -\frac{\pi}{200}\sin\left(\frac{\pi t}{100}\right) & 100 \le t \le 200 \\ 0 & \text{otherwise} \end{cases}$$

6 (a) Show that the cumulative distribution function F is given by

$$F(t) = \begin{cases} 0 & t < 100 \\ \frac{1}{2}\cos\left(\frac{\pi t}{100}\right) + \frac{1}{2} & 100 \le t \le 200 \\ 1 & t > 200 \end{cases}$$

[3 marks]

6 (b) Measurements of 1720 reaction times from competing athletes in 100 metre sprints at world class championship events were recorded and are grouped in the table below.

Interval (milliseconds)	100–120	120–140	140–160	160–180	180–200
Number of sprints	145	390	561	470	154

Do not write outside the box

6 (b) (i) Use the result given in **part (a)** to complete the expected frequency table below for 1720 reaction times, giving your values to three significant figures.

[3 marks]

Interval (milliseconds)	100–120	120–140	140–160	160–180	180–200
Expected number of sprints	164	430			

6 (b) (ii) Use a χ^2 test at the 1% level of significance to investigate the athletics trainer's claim.

[7 marks]

13

Do not write outside the box

7 A researcher is undertaking a study comparing the lengths of two species of penguins: Galapagos penguins and Fairy penguins.

The researcher collects data from two independent random samples, one for each species of penguin.

The table below shows the summary of her results.

Penguin species	Sample size	Sample mean length (cm)
Galapagos	60	53.4
Fairy	80	45.0

The distributions of lengths for each species of penguin are assumed to come from populations that are normally distributed.

The standard deviation for Galapagos penguins is 6 cm

The standard deviation for Fairy penguins is 4 cm

The researcher believes that the population mean length of Galapagos penguins is 10 cm longer than the population mean length of Fairy penguins.

7 (a) (i) The researcher begins to test her belief by using the hypotheses

$$H_0: \quad \mu_G - \mu_F = 10$$
$$H_1: \quad \mu_G - \mu_F \neq 10$$

where μ_G is the population mean length for the Galapagos penguins

and μ_F is the population mean length for the Fairy penguins.

State whether the researcher is performing a one-tailed test or a two-tailed test.

[1 mark]

Do not write outside the box

7 (a) (ii) Complete the researcher's test at the 5% level of significance.

[6 marks]

7 (b) The researcher subsequently thinks that an Exponential distribution more accurately fits the distribution of lengths for Galapagos penguins.

Comment on the validity of the result in **part (a)(ii)** if this is the case.

[1 mark]

8

Do not write outside the box

8 A test is used to investigate the population mean of a normal distribution with standard deviation 10

A sample size of 30 is used.

The hypotheses are

$$H_0: \quad \mu = 100$$
$$H_1: \quad \mu > 100$$

The level of significance used is 5%

It is given that P(Type I error) $>$ P(Type II error)

Show that the population mean must be greater than 106.0 correct to **four** significant figures.

[5 marks]

Do not write outside the box

5

Turn over for the next question

Do not write outside the box

9 A new version of a computer operating system is designed to reduce the start-up times of the system. The operating system is tested on 10 computers.

The table below shows the start-up times in seconds of the old version and the new version of the operating system, with the exception of one missing value a

	Computer start-up time (seconds)									
Operating system	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Old version	52.3	29.2	113.5	92.6	37.5	95.7	47.8	130.1	62.3	72.8
New version	50.1	37.3	a	86.0	35.0	88.1	47.9	134.1	58.1	74.0

A test is conducted for evidence that the start-up times with the new version have reduced.

9 (a) Show that the test statistic to be used has value

$$\frac{\sqrt{10}\,(0.1a - 12.32)}{\sqrt{0.1a^2 - 22.48a + 1286}}$$

where numbers shown in the denominator are given correct to **four** significant figures.

[6 marks]

Do not write outside the box

9 (b) It is given that $a = 91.8$

Test at the 10% level of significance whether there is evidence that the start-up times with the new version of the operating system have reduced.

[6 marks]

12

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED