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(9665/FM02) Unit P2 − Unit FPSM1 − Pure, Statistics and Mechanics

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$hf(x,y) = 0.25\dfrac{\sqrt{7}+4.3}{4.3\sqrt{4.3}}$ $= 0.194741$	M1	PI
	$y_2 = 4.3 + 0.194741 = 4.494741$	A1	
	$y_3 = 4.494741 + 0.25\dfrac{\sqrt{7.25}+4.494741}{4.494741\sqrt{4.494741}}$	m1	
	$= 4.68330$	A1ft	ft their y_2 to at least 3 dp
	4.683	A1	CAO
	Total	**5**	

Q	Answer	Marks	Comments
2(a)	$\begin{vmatrix}1 & k\\ 3 & 2\end{vmatrix} = 0$	M1	
	$2 - 3k = 0$		
	$k = \dfrac{2}{3}$	A1	CAO
2(b)	$\mathbf{M} = \mathbf{B^{-1}BM} = \mathbf{B^{-1}}\begin{bmatrix}1 & 2 & -1\\ 0 & 1 & 4\end{bmatrix}$	B1	Alternative method: **B1** for 6 sim. equations (at least 5 correct) (PI) **M1** for attempting to solve a pair of their equations for their unknowns **A1** for correctly solving a pair of the equations. Then last two marks as for main method.
	$\mathbf{B^{-1}} = \left(\dfrac{1}{2-3k}\right)\begin{bmatrix}2 & -k\\ -3 & 1\end{bmatrix}$	M1 A1ft	**M1** for $\begin{bmatrix}2 & -k\\ -3 & 1\end{bmatrix}$ **A1ft** division by their $\lvert\mathbf{B}\rvert$
	$\mathbf{M} = \left(\dfrac{1}{2-3k}\right)\begin{bmatrix}2 & -k\\ -3 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & -1\\ 0 & 1 & 4\end{bmatrix}$	M1	For correctly calculating the 2nd or 3rd column of $\begin{bmatrix}2 & 4-k & -2-4k\\ -3 & -5 & 7\end{bmatrix}$
	$\mathbf{M} = \left(\dfrac{1}{2-3k}\right)\begin{bmatrix}2 & 4-k & -2-4k\\ -3 & -5 & 7\end{bmatrix}$	A1	oe CAO

	Total	**7**	

Q	Answer	Marks	Comments
3(a)	531 405 195 -74	**B1**	
3(b)	All points correct Line of best fit drawn	**B1** **B1**	
3(c)	X-intercept = 91 $x = \sqrt{91}$ (= 9.5)	**M1** **A1**	In the range [86, 94] Square root of their X-intercept in the range [9.3, 9.7]
3(d)	b = 650 a = gradient = -7.1	**B1** **M1** **A1**	their Y-intercept in the range [630, 670] No marks if not consistent with graph PI In [-7.5, -6.7]
	Total	**8**	

Q	Answer	Marks	Comments
4(a)	f(-1) = 1 and f(-1.5) = -19/8 Change of sign and f is continuous on the interval so α is in the interval (- 1.5, - 1).	**M1** **A1**	
4(b)	$\mathrm{f}'(x) = 3x^2 - 4x - 3$ $x_2 = -1.5 - \frac{-19/8}{39/4}$ -1.256	**B1** **B1** **B1** **B1**	PI **B1** for numerator in correct form **B1** for denominator in correct form
4(c)(i)	Tangent drawn at $x = -1.5$ to meet x- axis	**B1**	
4(c)(ii)	P correct Q correct	**B1** **B1**	
	Total	**9**	

Q	Answer	Marks	Comments
5(a)	(3,1), (11,3) and (8,2) Drawn correctly, labelled and joined up	**B1** **B1**	
5(b)	$\begin{vmatrix} 3 & -4 \\ 1 & -1 \end{vmatrix} = 1$ Area of $OABC$ = 2 Therefore, area of $OA'B'C'$ = $1\times$ area of $OABC$ = 1×2 = 2 square units	**M1** **A1**	With explanation Alt. method: $\begin{vmatrix} 3 & 11 \\ 1 & 3 \end{vmatrix} = -2$ **M1** so area = 2 **A1** Or use of determinant made of any pair of position vectors of vertices
5(c)	$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} x \\ mx \end{pmatrix}$ $3-4m=1$ so $m=\frac{1}{2}$ or $1-m=m$ so $m=\frac{1}{2}$	**M1** **E1**	Must have only one unknown
5(d)	$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ kx \end{pmatrix} = \begin{pmatrix} X \\ 3X \end{pmatrix}$ $3x-4kx=X$ $x-kx=3X$ $x-kx=3(3x-4kx)$ $1-k=9-12k$ $k=\frac{8}{11}$	**M1** **M1** **A1** **M1** **A1**	Must have two different unknowns For two equations in 2 unknowns For eliminating 1 unknown
	Total	**11**	

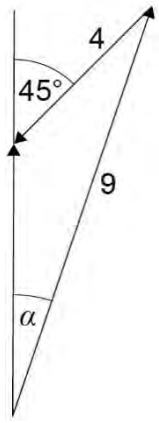
Q	Answer	Marks	Comments
6	$G_X(t) = \sum_{k=0}^{n}\binom{n}{k}p^k(1-p)^{n-k}t^k$	**M1**	Applies $G_X(t)$ formula
	$= \sum_{k=0}^{n}\binom{n}{k}(pt)^k(1-p)^{n-k}$	**M1**	Simplifies expression
	$= (1 - p + pt)^n$ (AG)	**A1**	Complete proof with no errors seen
	Total	**3**	

Q	Answer	Marks	Comments
7(a)	$E(X^2) = \sum_{x=1}^{n}\frac{x^2}{n} = \frac{1^2+2^2+...+n^2}{n}$	**M1**	Applies formula for $E(X^2)$
	$= \frac{\frac{1}{6}n(n+1)(2n+1)}{n}$	**A1**	Applies formula for $\sum n^2$
	$= \frac{(n+1)(2n+1)}{6}$		
	$Var(X) = \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2$	**M1**	Applies $Var(X) = E(X^2) - (E(X))^2$
	$= \frac{n^2-1}{12}$	**A1**	Complete proof with no errors seen Needs at least one intermediate line
7(b)(i)	$\frac{n^2-1}{12} = 33.25$	**M1**	Sets up equation using Var(X) = 33.25
	n = 20	**A1**	
7(b)(ii)	P(D > 18) = 0.1	**B1ft**	Accept 1/10 oe Follow through their (their n – 18) × 1/20
7(b)(iii)	$(1 - 0.1)^4 \times 0.1$	**M1**	(1 – their P(D > 18))^4 × their P(D > 18)
	0.06561	**A1**	CAO
	Total	**9**	

Q	Answer	Marks	Comments
8(a)	$\text{Var}(aX) = 0.25a^2$	**B1**	Uses $\text{Var}(aX) = a^2\,\text{Var}(X)$
	$\text{Var}((1-a)Y) = 0.16(1-a)^2$ or $0.16 - 0.32a + 0.16a^2$	**B1**	Uses $\text{Var}((1-a)Y) = (1-a)^2\,\text{Var}(Y)$
	$2\rho\sqrt{\text{Var}(aX)\text{Var}((1-a)Y)}$ = $2 \times 0.25 \times 0.5a \times 0.4(1-a)$ or $0.1a - 0.1a^2$	**M1**	Finds $2\rho\sqrt{\text{Var}(aX)\text{Var}((1-a)Y)}$ Or $2a(1-a)\text{Cov}(X,Y)$
	$\text{Var}(aX + (1-a)Y)$ $= 0.25a^2 + 0.16 - 0.32a + 0.16a^2 + 0.1a - 0.1a^2$ $= 0.31a^2 - 0.22a + 0.16$	**A1**	Applies $\text{Var}(aX + (1-a)Y)$ $= \text{Var}(aX) + \text{Var}((1-a)Y) + 2\rho\sqrt{\text{Var}(aX)\text{Var}((1-a)Y)}$ (or uses Or $2a(1-a)\text{Cov}(X,Y)$ here) and simplifies to given answer
8(b)(i)	$\frac{\text{d}(\text{Var})}{\text{da}} = 0.62a - 0.22$	**B1**	Correct differentiation
	$0.62a - 0.22 = 0$	**M1**	Sets $\frac{\text{d}(\text{Var})}{\text{da}} = 0$
	$a = \frac{11}{31}$	**A1**	CAO 0.35483….. is **B1M1A0**
8(b)(ii)	$\frac{226}{31}$	**B1ft**	Follow through their a × 6 + (1 – their a) × 8 Accept AWRT 7.3
	Total	**8**	

Q	Answer	Marks	Comments
9	$[a]=[r][\omega]^2$ $LT^{-2}=L[\omega]^2$ $[\omega]^2=T^{-2}$ $[\omega]=T^{-1}$	**M1** **A1ft** **A1**	ft their dimensions of acceleration (at least two terms)
	Total	**3**	

Q	Answer	Marks	Comments
10(a)	$e=\frac{5}{8}=0.625$	**B1**	
10(b)	$I=0.2\times 8-0.2\times(-5)$ $=2.6$ Ns	**M1** **A1**	-2.6 is **M1A0**
10(c)	$2.6=0.25F$ $F=\frac{2.6}{0.25}=10.4$ N	**M1** **A1**	
	Total	**5**	

Q	Answer	Marks	Comments
11(a)		**M1**	Showing all details
	$\frac{\sin\alpha}{4} = \frac{\sin 135°}{9}$	**M1A1**	
	$\sin\alpha = 0.31427$		
	$\alpha = 018$	**A1**	Condone 18º
			Alt. method $\mathbf{s} = \begin{pmatrix} 2\sqrt{2}t \\ 5000 + 2\sqrt{2}t \end{pmatrix}$ **M1** $\mathbf{p} = \begin{pmatrix} 9(\sin\alpha°)t \\ 9(\cos\alpha°)t \end{pmatrix}$ **M1** (condone switched) $\sin\alpha° = \frac{2\sqrt{2}}{9}$ **A1** $\alpha = 018$ **A1**
11(b)	$\text{Time} = \frac{5000}{9\cos\alpha - 4\sin 45°}$	**M1A1**	Use of 5 instead of 5000 is **M1A0** $\frac{v}{\sin 26.7°} = \frac{9}{\sin 135°}$ and $t = \frac{5000}{v}$ is **M1A1**
	$\text{Time} = 875 \text{ seconds}$	**A1**	oe Accept [871, 879]
	Total	**7**	

Q	Answer	Marks	Comments
12	For *A*: $-9 = 2 \times v_A - 2 \times 5$ $v_A = 0.5$ For *B*: $9 = 3 \times v_B - 3 \times (-2)$ $v_B = 1$ $0.5 - 1 = -e\big(5 - (-2)\big)$ $e = \dfrac{-0.5}{-7} = \dfrac{1}{14}$	**M1** **A1** **M1** **A1** **A1**	Or for a correct conservation of momentum equation with one unknown
	Total	**5**	