OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS
FM05

(9665/FM05) Unit FM2 Mechanics

Mark scheme

January 2023

Version: 1.0 Final

231XFM05/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from oxfordaqaexams.org.uk

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{v}{2}$		
	$\int \frac{1}{v}\mathrm{d}v = \int -\frac{1}{2}\mathrm{d}t$	**M1**	Separates variables.
	$\ln v = -\frac{t}{2} + c$	**A1**	Correct integration, with or without a constant of integration.
	$t = 0,\ v = 10$ $c = \ln 10$	**m1**	Finds constant of integration.
	$\ln v = -\frac{t}{2} + \ln 10$		
	$v = 10\mathrm{e}^{-\frac{t}{2}}$	**A1**	Correct result **oe**, logarithms must be simplified.
		4	

	Question 1 Total	**4**	

Q	Answer	Marks	Comments
2	$\mathbf{I} = 0.3\times(-2\mathbf{i}-5\mathbf{j})-0.3\times(3\mathbf{i}+9\mathbf{j})$	**M1**	Applies impulse equation in vector or component form.
	$\mathbf{I} = -1.5\mathbf{i}-4.2\mathbf{j}$	**A1**	Correct impulse **oe**
	$\lvert\mathbf{I}\rvert = \sqrt{1.5^2+4.2^2}$	**M1**	Finds magnitude
	$\lvert\mathbf{I}\rvert = 4.46$ Ns	**A1ft**	Correct magnitude Condone more than 3sf. Condone incorrect units.
		4	

	Question 2 Total	**4**	

Q	Answer	Marks	Comments
3(a)	$v\sin\alpha = 4e\sin 60°$ $v\cos\alpha = 4\cos 60°$	**M1**	Equations for velocities parallel and perpendicular to the wall
		A1	Both correct
	$v = \sqrt{(v\sin\alpha)^2 + (v\cos\alpha)^2}$		
	$v = \sqrt{(4e\sin 60°)^2 + (4\cos 60°)^2}$	**M1**	Eliminates α
	$v = \sqrt{12e^2 + 4} = 2\sqrt{3e^2 + 1}$	**A1**	Correct expression for v
		4	

Q	Answer	Marks	Comments
3(b)	$0 < e \leq 1$	**M1**	Uses range of values for e
	$2 < v \leq 4$	**A1ft**	Correct range for values of v Condone $2 \leq v \leq 4$
		2	

	Question 3 Total	**6**	

Q	Answer	Marks	Comments
4(a)	$0 = V\sin\theta \times 1.5 - 4.9\cos 10° \times 1.5^2$	M1	Equation for motion perpendicular to plane, with at least one correct term.
	$15 = V\cos\theta \times 1.5 - 4.9\sin 10° \times 1.5^2$	M1 A1	Equation for motion parallel to plane, with at least one correct term. Both equations correct May be in terms of g
	$\tan\theta = \frac{V\sin\theta}{V\cos\theta} = \frac{4.9\cos 10° \times 1.5}{10 + 4.9\sin 10° \times 1.5}$ $= \frac{7.238}{11.27} = 0.642$	M1	Forms equation for $\tan\theta$ or finds V
	$\theta = 32.696.. = 33°$ to the nearest degree	A1	Correct angle to nearest degree
		5	

Q	Answer	Marks	Comments
4(b)	$V = \frac{4.9\cos 10° \times 1.5}{\sin 32.696°} = 13.39..$ $V = 13$ (to 2sf)	M1 A1	Equation to find V Correct V Must be given to 2 sf
		2	

	Question 4 Total	**7**	

Q	Answer	Marks	Comments
5(a)	$\omega = \dfrac{2\pi}{5\pi} = \dfrac{2}{5}$	**B1**	Correct ω
	$0.8^2 = \left(\dfrac{2}{5}\right)^2 \left(a^2 - (a-0.5)^2\right)$	**M1** **A1**	Applies SHM formula Condone use of 0.5 in place of $a-0.5$ Correct equation
	$\dfrac{64}{100} = \dfrac{4}{25}\left(a - \dfrac{1}{4}\right)$	**M1**	Solves equation that is linear in a
	$16 = 4a - 1$		
	$a = \dfrac{17}{4}$	**A1**	Correct amplitude
	$AB = 8.5$ metres	**A1**	Correct distance
		6	

Q	Answer	Marks	Comments
5(b)	$x = 4.25\cos\left(\dfrac{2t}{5}\right)$ or $x = 4.25\sin\left(\dfrac{2t}{5}\right)$	**M1**	Expression for displacement at time t
	$3.75 = 4.25\cos\left(\dfrac{2t}{5}\right)$ or $3.75 = 4.25\sin\left(\dfrac{2t}{5}\right)$	**M1** **A1**	Equation to find time at C Correct equation to find time at C
	$t = 1.2248\ldots$ or $t = 2.702...$		
	Time $= 1.22$ seconds to (3sf) or Time = $\dfrac{1}{4} \times 5\pi - 2.702 = 1.22$ seconds	**A1**	Correct time Condone 1.23
		4	

	Question 5 Total	**10**	

Q	Answer	Marks	Comments
6(a)	$0.2\times 9.8\times l=\frac{1}{2}\times\frac{2.8}{0.8}(l-0.8)^2$	**M1**	Energy equation
		A1	Correct energy equation
	$1.75l^2-4.76l+1.12=0$	**A1**	Correct quadratic
	$l=2.4598\ldots=2.5$ metres to 2 sf	**A1**	Correct length to at least 2 sf
		4	

Q	Answer	Marks	Comments
6(b)	$T=\frac{2.8}{0.8}\times(2.4598-0.8)$	**M1**	Uses Hooke's law to find tension with their answer to **6(a)**
		A1ft	Correct expression for the tension
	$T=5.8093\ldots=5.8$ N to 2 sf	**A1**	Correct tension to at least 2 sf Condone 5.9 N or 6.0 N from use of 2.5 metres
		3	

Q	Answer	Marks	Comments
6(c)	Max speed when:		
	$0.2\times 9.8=\frac{2.8}{0.8}e$	**M1**	Equation to find extension at max speed
	$e=0.56$	**A1**	Correct extension
	$\frac{1}{2}\times 0.2v^2=0.2\times 9.8\times 1.36-\frac{1}{2}\times\frac{2.8}{0.8}\times 0.56^2$	**m1**	Energy equation to find the max speed
		A1	Correct equation
	$v=4.600\ldots=4.6$ m s^{-1} to 2 sf	**A1**	Correct max speed to at least 2 sf
		5	

	Question 6 Total	**12**	

Q	Answer	Marks	Comments
7(a)	$4ke = 3k\left(7d - 5d - e\right)$	**M1** **A1**	Equation for equilibrium Correct equation
	$e = \frac{6d}{7}$		
	$\text{distance} = 2d + \frac{6d}{7} = \frac{20d}{7}$	**A1**	Correct distance
		3	

Q	Answer	Marks	Comments
7(b)(i)	$T_B = 3k\left(7d - \frac{20d}{7} - x - 3d\right)$	**M1**	Uses Hooke's law to find tension with their answer to **7(a)**
	$T_B = 3k\left(\frac{8d}{7} - x\right)$	**A1**	Correct tension
		2	

Q	Answer	Marks	Comments
7(b)(ii)	$T_A = 4k\left(\frac{20d}{7} + x - 2d\right)$	**M1**	Uses Hooke's law to find tension with their answer to **7(a)**
	$T_A = 4k\left(x + \frac{6d}{7}\right)$	**A1**	Correct tension
	$m\ddot{x} = T_B - T_A$	**M1**	Applies Newton's Second Law with their tensions
	$m\ddot{x} = 3k\left(\frac{8d}{7} - x\right) - 4k\left(x + \frac{6d}{7}\right)$		
	$m\ddot{x} = -7kx$	**A1**	Correct simplified differential equation
	As the acceleration is proportional to the displacement and in the opposite direction so the motion is SHM.	**E1**	Correct conclusion following correct working.
		5	

Q	Answer	Marks	Comments
7(b)(iii)	$\text{Period} = 2\pi\sqrt{\frac{m}{7k}}$	**M1** **A1ft**	Uses their ω to find period Correct period, consistent with their SHM equation in part (b)(ii). **oe**
		2	

Q	Answer	Marks	Comments
7(b)(iv)	$\text{Max Speed} = \frac{d}{2}\times\sqrt{\frac{7k}{m}}$	**M1** **A1ft**	Uses their ω to find max speed Correct max speed, consistent with their SHM equation in part (b)(ii). **oe**
		2	

	Question 7 Total	**14**	

Q	Answer	Marks	Comments
8(a)	$F = 0.3\times1.5\times9.8 = 0.45g = 4.41$	B1	Correct friction
	$\frac{1}{2}\times0.4v^2 = 0.4\times9.8\times0.7(\cos\theta - \cos30°)$	M1	Energy equation
	$v^2 = 1.4g(\cos\theta - \cos30°)$	A1	Correct energy equation
	$T - 0.4g\cos\theta = \frac{0.4v^2}{0.7}$	M1	Apply Newton's Second Law radially
		A1	Correct equation
	$0.45g - 0.4g\cos\theta = 0.8g(\cos\theta - \cos30°)$		
	$\cos\theta = \frac{0.45 + 0.8\cos30°}{1.2} = 0.95235...$	M1	Eliminating T to find $\cos\theta$
	$\theta = 17.76... = 18°$	A1	Correct angle AWRT 18
		7	

Q	Answer	Marks	Comments
8(b)	$\frac{1}{2}\times 0.4v^2 = 0.4\times 9.8\times 0.7(1-\cos\alpha°)$	**M1**	Energy equation for lowest point
	$v^2 = 13.72(1-\cos\alpha°)$	**A1**	Correct energy equation
	$T - 3.92 = \frac{0.4v^2}{0.7}$	**B1**	Correct application of Newton's Second Law at lowest point
	$T - 3.92 = \frac{0.4v^2}{0.7}$	**M1**	Eliminates T
	$\cos\alpha = \frac{7.35}{7.84} = 0.9375$		
	$\alpha = 20.36... = 20$	**A1**	Correct value of α AWRT 20
		5	

	Question 8 Total	**12**	

Q	Answer	Marks	Comments
9	Before collision along line of centres:		
	$u_A = 2\cos 60° = 1$		
	$u_B = -2\cos 30° = -\sqrt{3}$	**B1**	Correct components along line of centres seen
	$4\times 1 - 5\sqrt{3} = 4v_A + 5v_B$	**M1**	Conservation of momentum along the line of centres
	$4 - 5\sqrt{3} = 4v_A + 5v_B$	**A1**	Correct equation
	$v_A - v_B = -\frac{3}{4}\left(1+\sqrt{3}\right)$	**M1** **A1**	Applies coefficient of restitution along lines of centres Correct equation
	$v_A = \frac{1-35\sqrt{3}}{36}$	**A1**	One correct velocity
	$v_B = \frac{7-2\sqrt{3}}{9}$	**A1**	Other correct velocity
	$\mathbf{s}_A = \left(\frac{2-70\sqrt{3}}{36}\right)\mathbf{i} - 2\sqrt{3}\mathbf{j}$ $\mathbf{s}_B = \left(\frac{14-4\sqrt{3}}{9} + \frac{2}{100}\right)\mathbf{i} - 2\mathbf{j}$	**M1**	Uses both components to find displacements
	$\mathbf{s}_A - \mathbf{s}_B = \left(\frac{-3-3\sqrt{3}}{2} - \frac{1}{50}\right)\mathbf{i} + \left(2-2\sqrt{3}\right)\mathbf{j}$	**A1**	Correct difference of displacements
	$d^2 = \left(\frac{-3-3\sqrt{3}}{2} - \frac{1}{50}\right)^2 + \left(2-2\sqrt{3}\right)^2$	**M1**	Finds distance for their difference of displacements
	$d = 4.37$ to 3 sf	**A1**	Correct distance
		11	

	Question 9 Total	**11**	