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INTERNATIONAL A-LEVEL
MATHEMATICS
MA03

(9660/MA03) Unit P2 Pure Mathematics

Mark scheme

June 2024

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246XMA03/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	$\left[2\cos\theta + \sqrt{5}\sin\theta =\right]$		
	$R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$	**M1**	**PI**
	$R = 3$	**B1**	
	$\alpha = 48°$	**A1**	Condone **AWRT** 48
	$3\cos(\theta - 48°)$		
		3	

Q	Answer	Marks	Comments
1(b)(i)	[Min value =] –3	**B1ft**	
		1	

Q	Answer	Marks	Comments
1(b)(ii)	228°	**B1**	Condone **AWRT** 228
		1	

	Question 1 Total	**5**	

Q	Answer	Marks	Comments
2(a)	$0 \le \mathrm{f}(x) \le \sqrt{18}$ or $[0, \sqrt{18}]$	**B1**	**oe** Accept f and y for $\mathrm{f}(x)$
		1	

Q	Answer	Marks	Comments
2(b)(i)	$[\mathrm{fg}(x) =] \sqrt{18 - 2\left(\dfrac{3}{2x-1}\right)^2}$	**B1**	**oe ISW**
		1	

Q	Answer	Marks	Comments
2(b)(ii)	$[0 \le] \left(\dfrac{3}{2x-1}\right) \le 3$	**M1**	or better
	$x \ge 1$ only	**A1**	
		2	

Q	Answer	Marks	Comments
2(b)(iii)	$\sqrt{18 - 2\left(\dfrac{3}{2x-1}\right)^2} = 4$	**M1**	*their* $\sqrt{18 - 2\left(\dfrac{3}{2x-1}\right)^2} = 4$
	$2\left(\dfrac{3}{2x-1}\right)^2 = 2$	**A1**	**PI** by seeing either of $x = 2$ or $x = -1$
	$x = 2$ and $x = -1$ (rejected)	**A1**	Only this value of x included in final answer
		3	

Q	Answer	Marks	Comments
2(c)	$x=\sqrt{18-2\left(\frac{3}{2y-1}\right)^2}$	**M1**	$x = their\ \sqrt{18-2\left(\frac{3}{2y-1}\right)^2}$
	$x^2=18-2\left(\frac{3}{2y-1}\right)^2$		
	$\left(\frac{3}{2y-1}\right)^2=9-\frac{1}{2}x^2$	**M1**	Correctly isolating *their* term in 'y' Must have scored **B1** in part (b)(i)
	$\mathrm{h}(x)=0.5\left(1+\sqrt{\frac{18}{18-x^2}}\right)$	**A1**	**oe ISW**
		3	

	Question 2 Total	**10**	

Q	Answer	Marks	Comments
3	$4x^2 + 12x + 9 = (2x+3)^2$ or $(2(x+1.5))^2$ or $4x^2 + 12x + 9 = 4\left(x+\frac{3}{2}\right)^2$	**B1**	For either factorisation
	Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$	**M1**	Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$
	$\begin{bmatrix} -1.5 \\ 0 \end{bmatrix}$	**A1**	$\begin{bmatrix} -3 \\ 0 \end{bmatrix}$
	Stretch	**M1**	Stretch
	SF 4, parallel to y-axis	**A1**	SF 0.5, parallel to x-axis
	OR		
	Stretch	**[M1]**	Stretch
	SF 0.5, parallel to x-axis	**[A1]**	SF 4, parallel to y-axis
	Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$	**[M1]**	Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$
	$\begin{bmatrix} -1.5 \\ 0 \end{bmatrix}$	**[A1]**	$\begin{bmatrix} -1.5 \\ 0 \end{bmatrix}$

	Question 3 Total	**5**	

Q	Answer	Marks	Comments
4(a)	$X = Ae^{-kt}$		
	$\dfrac{dX}{dt} = -kAe^{-kt} = -kX$	**B1**	**AG** Must be convincingly shown
		1	

Q	Answer	Marks	Comments
4(b)	$X = Ae^{-kt}$		
	$t = 0,\ X = 90,\ A = 90$	**B1**	
	$t = 5,\ X = 80,\ 80 = 90e^{-5k}$	**M1**	
	$e^{-5k} = \dfrac{8}{9}$ $\left[k = \dfrac{1}{5}\ln\left(\dfrac{9}{8}\right) = 0.02355...\right]$	**A1**	Allow $k = 0.024$ AWRT
	$X = 22.5$		
	$22.5 = 90e^{-kt}$		**oe**
	$t = -\dfrac{1}{k}\ln\left(\dfrac{22.5}{90}\right)$ $\left[= -\dfrac{5}{\ln\left(\dfrac{9}{8}\right)}\ln\left(\dfrac{1}{4}\right)\right]$	**m1**	**PI**
	$t = 58.8$ [mins]	**A1**	Condone 58.7 to 58.9
		5	

	Question 4 Total	**6**	

<table>
<tr><th>Q</th><th>Answer</th><th>Marks</th><th>Comments</th></tr>
<tr><td>5(a)</td><td>
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>0</td><td>$3-\tan^{-1}(-0.5) = 3.4636476$</td></tr>
<tr><td>0.5</td><td>$3-\tan^{-1}(0.5) = 2.5363524$</td></tr>
<tr><td>1</td><td>$3-\tan^{-1}(1.5) = 2.0172063$</td></tr>
<tr><td>1.5</td><td>$3-\tan^{-1}(2.5) = 1.8097101$</td></tr>
<tr><td>2</td><td>$3-\tan^{-1}(3.5) = 1.7075033$</td></tr>
</table>
</td><td>B1

M1</td><td>All five correct x values (and no extra used) PI by five correct y values

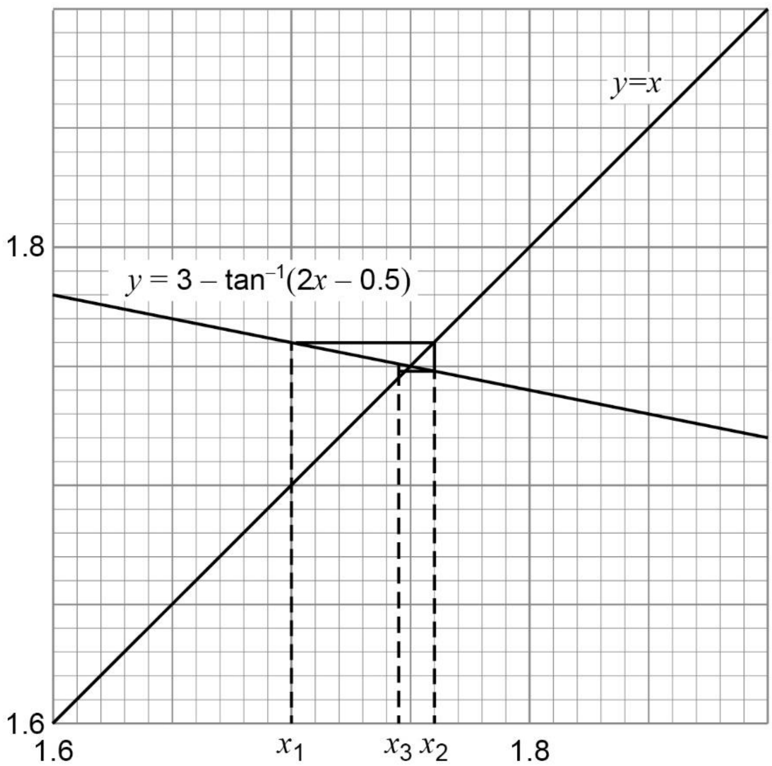
At least four correct y values in exact form or decimals, rounded or truncated to two dp or better (in table or formula)</td></tr>
<tr><td></td><td>$\frac{1}{3}\times 0.5\left(3.46...+1.707...+4\left(2.536...+1.809...\right)+2\times 2.017...\right)$</td><td>m1</td><td>Correct sub into formula with $h = 0.5$ oe and at least four correct y values either listed, with + signs, or totalled
(PI by AWRT correct answer)</td></tr>
<tr><td></td><td>$\int_0^2 3-\tan^{-1}(2x-0.5)\,dx = 4.432$</td><td>A1</td><td>CAO, must see this value exactly</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
</table>

<table>
<tr><th>Q</th><th>Answer</th><th>Marks</th><th>Comments</th></tr>
<tr><td>5(b)(i)</td><td>$f(x) = 0.5\tan(3-x)+0.25-x$</td><td></td><td>Or reverse</td></tr>
<tr><td></td><td>$f(1.7) = 0.35...$

$f(1.8) = -0.26...$</td><td>M1</td><td>Both values rounded or truncated to at least 1 sf</td></tr>
<tr><td></td><td>Change of sign, $1.7 < \alpha < 1.8$</td><td>A1</td><td>Must have both statement and interval in words or symbols
or comparing 2 sides:
M1 $y(1.7) = 2.05 > 1.7$
and $y(1.8) = 1.54 < 1.8$
A1: Conclusion as before</td></tr>
<tr><td></td><td></td><td>2</td><td></td></tr>
</table>

Q	Answer	Marks	Comments
5(b)(ii)	$0.5\tan(3-x) = x - 0.25$		
	$\tan(3-x) = 2x - 0.5$	**M1**	**oe**
	$3 - x = \tan^{-1}(2x - 0.5)$		
	$x = 3 - \tan^{-1}(2x - 0.5)$	**A1**	**AG** Must be convincingly shown
		2	

Q	Answer	Marks	Comments
5(b)(iii)	$x_2 = 1.761$	**B1**	
	$x_3 = 1.749$	**B1**	
		2	

Q	Answer	Marks	Comments
5(b)(iv)	 x_2 and x_3 in (approx.) correct position on axis	M1 A1	Cobweb diagram
		2	

	Question 5 Total	**12**	

Q	Answer	Marks	Comments
6(a)	$\left(\sin\theta - \operatorname{cosec}\theta\right)\left(\cos\theta - \sec\theta\right)$		
	$= \left(\sin\theta - \dfrac{1}{\sin\theta}\right)\left(\cos\theta - \dfrac{1}{\cos\theta}\right)$		
	$= \dfrac{\sin^2\theta - 1}{\sin\theta} \times \dfrac{\cos^2\theta - 1}{\cos\theta}$	**B1**	All in terms of sin/cos
	$= \dfrac{-\cos^2\theta}{\sin\theta} \times \dfrac{-\sin^2\theta}{\cos\theta}$	**M1**	Correct use of trig identity
	$= \cos\theta\sin\theta$ $= 0.5\sin 2\theta$	**A1**	**AG** Must be convincingly shown
	OR		
	$\left(\sin\theta - \operatorname{cosec}\theta\right)\left(\cos\theta - \sec\theta\right)$		
	$= \left(\dfrac{1}{\operatorname{cosec}\theta} - \operatorname{cosec}\theta\right)\left(\dfrac{1}{\sec\theta} - \sec\theta\right)$		
	$= \dfrac{1 - \operatorname{cosec}^2\theta}{\operatorname{cosec}\theta} \times \dfrac{1 - \sec^2\theta}{\sec\theta}$	**[B1]**	All in terms of sec/cosec
	$= \dfrac{-\cot^2\theta}{\operatorname{cosec}\theta} \times \dfrac{-\tan^2\theta}{\sec\theta}$	**[M1]**	Correct use of trig identity
	$= \cos\theta\sin\theta$ $= 0.5\sin 2\theta$	**[A1]**	**AG** Must be convincingly shown
		3	

Q	Answer	Marks	Comments
6(a)	**OR** $\left(\sin\theta - \operatorname{cosec}\theta\right)\left(\cos\theta - \sec\theta\right)$ $= \sin\theta\cos\theta - \dfrac{\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\sin\theta} + \dfrac{1}{\sin\theta\cos\theta}$		
	$= 0.5\sin 2\theta + \dfrac{-\sin^2\theta - \cos^2\theta + 1}{\sin\theta\cos\theta}$	[B1]	All in terms of sin/cos
	$= 0.5\sin 2\theta + \dfrac{-1+1}{\sin\theta\cos\theta}$	[M1]	Correct use of trig identity
	$= 0.5\sin 2\theta$	[A1]	**AG** Must be convincingly shown
		3	

Q	Answer	Marks	Comments
6(b)	$\left(\sin(1.5x+0.1)-\operatorname{cosec}(1.5x+0.1)\right)$ $\times\left(\cos(1.5x+0.1)-\sec(1.5x+0.1)\right)=0.4$ $0.5\sin(3x+0.2)=0.4$		
	$\sin(3x+0.2)=0.8$	**B1**	PI by later work
	$3x+0.2=0.927...,\ \pi-0.927...,$ $2\pi+0.927...,\ 3\pi-0.927...$		
	$x=0.242,\ 0.671,\ 2.337,\ 2.766$ AWRT	**B2,1**	**B1** for at least one correct value **B2** for all four correct values and no others
		3	

	Question 6 Total	**6**	

Q	Answer	Marks	Comments
7(a)	$\left[\int \frac{x-2}{2x^2-8x+3}\,\mathrm{d}x=\right]$		
	$a\ln\left(2x^2-8x+3\right)\ \left[+c\right]$	M1	
	$=0.25\ln\left(2x^2-8x+3\right)\ \left[+c\right]$	A1	**ISW** Accept $0.25\ln\left(x^2-4x+1.5\right)\ \left[+c\right]$
		2	

Q	Answer	Marks	Comments
7(b)	$\frac{5x+1}{5x-1}=\frac{5x-1+2}{5x-1}\ \left[=1+\frac{2}{5x-1}\right]$	B1	**PI**
	$\left[\int \frac{5x+1}{5x-1}\,\mathrm{d}x=\right]\ x+\frac{2}{5}\ln\left(5x-1\right)\ \left[+c\right]$	M1 A1	**M1**: $x+k\ln\left(5x-1\right)$ **A1**: **ACF** eg $x-\frac{1}{5}+\frac{2}{5}\ln\left(5x-1\right)\ \left[+c\right]$
		3	

Q	Answer	Marks	Comments
7(c)	$\cos 6x=1-2\sin^2 3x$	B1	
	$\sin^2 3x=\frac{1}{2}\left(1-\cos 6x\right)$		
	$\int \sin^2 3x\,\mathrm{d}x=\frac{1}{2}\left(x-\frac{1}{6}\sin 6x\right)\ \left[+c\right]$	M1 A1	**M1**: $ax+b\sin 6x$ **A1**: **ACF** **eg** $\frac{x}{2}-\frac{\sin 3x\cos 3x}{6}\ \left[+c\right]$
		3	

	Question 7 Total	**8**	

Q	Answer	Marks	Comments
8(a)	$A(0, 5)$ $B(1.25, 0)$	**B1**	
		1	

Q	Answer	Marks	Comments
8(b)(i)	$y=(5-4x)\mathrm{e}^{-0.5x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=(5-4x)\times -0.5\mathrm{e}^{-0.5x}+(-4)\times \mathrm{e}^{-0.5x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=(2x-6.5)\mathrm{e}^{-0.5x}$	 **M1 A1** **A1**	May use $y=5\mathrm{e}^{-0.5x}-4x\mathrm{e}^{-0.5x}$ **M1**: At least one exponential term differentiated correctly **A1**: All terms correct, unsimplified **CAO** (no ISW)
		3	

Q	Answer	Marks	Comments
8(b)(ii)	$(2x-6.5)\mathrm{e}^{-0.5x}=0$ $2x-6.5=0$ $x=3.25$ $y=(5-4\times 3.25)\mathrm{e}^{-1.625}$ $y=-8\mathrm{e}^{-1.625}$ $(3.25,\ -8\mathrm{e}^{-1.625})$	 **M1** **A1** **A1**	 Sets *their* derivative equal to zero and attempts to solve **ACF** **ISW**
		3	

Q	Answer	Marks	Comments
8(c)	$\text{Area} = 0.5\times5\times1.25 - \int_0^{1.25}(5-4x)\,\mathrm{e}^{-0.5x}\mathrm{d}x$	B1	**PI (**by later work)
	$\int(5-4x)\,\mathrm{e}^{-0.5x}\ \mathrm{d}x$		$\int\ 5\mathrm{e}^{-0.5x}\ -\ 4x\,\mathrm{e}^{-0.5x}\mathrm{d}x$
	$u = 5-4x \qquad \frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^{-0.5x}$ $\frac{\mathrm{d}u}{\mathrm{d}x} = -4 \qquad v = -2\mathrm{e}^{-0.5x}$	M1	$u = -4x \qquad \frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^{-0.5x}$ $\frac{\mathrm{d}u}{\mathrm{d}x} = -4 \qquad v = -2\mathrm{e}^{-0.5x}$ Correct use of parts formula
	$\int(5-4x)\,\mathrm{e}^{-0.5x}\ \mathrm{d}x = (5-4x)\times\left(-2\mathrm{e}^{-0.5x}\right) - \int -2\,\mathrm{e}^{-0.5x}\ \times(-4)\ \mathrm{d}x$	m1	$\int\ 5\mathrm{e}^{-0.5x}\ -4x\,\mathrm{e}^{-0.5x}\mathrm{d}x\ =\ -10\,\mathrm{e}^{-0.5x}\ +(-4x)\times(-2\,\mathrm{e}^{-0.5x}\ )$ $-\int -2\,\mathrm{e}^{-0.5x}\ \times(-4)\ \mathrm{d}x$
	$= (5-4x)\left(-2\mathrm{e}^{-0.5x}\right) + 16\mathrm{e}^{-0.5x} \quad [+\ c]$	A1	$= -10\mathrm{e}^{-0.5x} + 8x\mathrm{e}^{-0.5x} + 16\mathrm{e}^{-0.5x} \quad [+\ c]$
	$[\text{Area} = 0.5\times5\times1.25 -]$ $\left(\left(0+16\mathrm{e}^{-0.625}\right)-\left(-10+16\right)\right)$	M1	Correct substitution of limits into $(5-4x)\left(p\mathrm{e}^{-0.5x}\right)+q\mathrm{e}^{-0.5x}$ oe PI by
	$= 9.125 - 16\mathrm{e}^{-0.625}$	A1	**ACF**
		6	

	Question 8 Total	**13**	

Q	Answer	Marks	Comments
9(a)	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2a - 2a\cos 2\theta \qquad \frac{\mathrm{d}y}{\mathrm{d}\theta} = 2a\sin 2\theta$	M1 A1	**M1**: At least one correct **A1**: Both correct
	$\left[\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2a\sin 2\theta}{2a - 2a\cos 2\theta}\right]$		
	$\theta = \frac{\pi}{4}$		
	$x = a\left(2\times\frac{\pi}{4} - \sin\left(\frac{\pi}{2}\right)\right) = a\left(\frac{\pi}{2} - 1\right)$		Condone $0.571a$
	$y = a\left(1 - \cos\left(\frac{\pi}{2}\right)\right) = a$	B1	Both x and y correct
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2a\sin\left(\frac{\pi}{2}\right)}{2a - 2a\cos\left(\frac{\pi}{2}\right)}\ [=1]$	B1	
	$y - a = x - a\left(\frac{\pi}{2} - 1\right)$	m1 A1	**m1**: Substitutes in their coordinates and their gradient into the equation of a straight line **A1**: **ACF**
	$\left[y = x - \frac{\pi a}{2} + 2a\right]$		Condone $y = x + 0.429a$
		6	

Q	Answer	Marks	Comments
9(b)	$y - a = -\left(x - a\left(\frac{\pi}{2} - 1\right)\right)$	M1	Uses *their* $-\left(\frac{2a - 2a\cos\left(\frac{\pi}{2}\right)}{2a\sin\left(\frac{\pi}{2}\right)}\right)$ and *their* x and y
	$y = -x + \frac{\pi a}{2}$	A1	Condone $y = -x + 1.57a$
		2	

Q	Answer	Marks	Comments
9(c)	Area $= \frac{1}{2}(\pi a - 2a) \times a\left(\frac{\pi}{2} - 1\right)$	**M1**	Uses *their* exact intercepts
	$= \frac{1}{4}a^2(\pi - 2)^2$	**A1**	**AG** Must be convincingly shown
		2	

	Question 9 Total	**10**	

Q	Answer	Marks	Comments
10(a)	$y = 4\times4-(4-y)^2$ $y=16-16+8y-y^2$ $y^2-7y=0 \quad\Rightarrow\quad y=0,\ 7$ $AB=7$	 **M1** **A1**	 Attempt to solve for y **PI**
		2	

Q	Answer	Marks	Comments
10(b)	$\dfrac{dy}{dx}=4-2(x-y)\left(1-\dfrac{dy}{dx}\right)$ $\left[\dfrac{dy}{dx}=\dfrac{2x-2y-4}{2x-2y-1}\right]$ $\dfrac{dy}{dx}=0,\quad x-y=2$ $\left[\begin{aligned}(x-y)^2&=4x-y\\&=(x-y)+3x\end{aligned}\right]$ $2^2=2+3x$ $x=\dfrac{2}{3}$ $\dfrac{2}{3}-y=2 \quad\Rightarrow\quad y=-\dfrac{4}{3}$ $\left(\dfrac{2}{3},-\dfrac{4}{3}\right)$	**M1 A1** **m1** **A1**	**M1**: Attempt at implicit differentiation $y\dfrac{dy}{dx}$ or $x\dfrac{dy}{dx}$ seen **A1**: All correct oe Substitutes *their* $ax+by=c$ in original equation oe $x=\dfrac{2}{3},\ y=-\dfrac{4}{3}$
		4	

	Question 10 Total	**6**	

Q	Answer	Marks	Comments
11(a)	$\frac{75}{(5-x)(5+2x)^2} = \frac{A}{5-x} + \frac{B}{5+2x} + \frac{C}{(5+2x)^2}$		
	$75 = A(5+2x)^2 + B(5-x)(5+2x) + C(5-x)$	**M1**	Correctly eliminating fractions **PI** by at least one correct value for *A*, *B* or *C*
	$x = 5: \quad 75 = A(15)^2 \quad \Rightarrow \quad A = \frac{1}{3}$	**A1**	At least one constant correct
	$x = -2.5: \quad 75 = C(7.5) \quad \Rightarrow \quad C = 10$	**A1**	At least two constants correct
	$x^2: \quad 0 = 4A - 2B \quad \Rightarrow \quad B = \frac{2}{3}$		
	$\left[\frac{75}{(5-x)(5+2x)^2} = \right]$		
	$\frac{1/3}{5-x} + \frac{2/3}{5+2x} + \frac{10}{(5+2x)^2}$	**A1**	All correct
		4	

Q	Answer	Marks	Comments
11(b)	$(5+2x)^{-1} = 5^{-1}\left(1+\frac{2x}{5}\right)^{-1}$	**M1**	
	$= \frac{1}{5}\left(1+(-1)\left(\frac{2x}{5}\right)+\frac{(-1)\times(-2)\times\left(\frac{2x}{5}\right)^2}{2} \quad [+\ldots]\right)$		
	$= \frac{1}{5} - \frac{2}{25}x + \frac{4}{125}x^2$	**A1**	All correct Condone $\frac{1}{5}\left(1-\frac{2}{5}x+\frac{4}{25}x^2\right)$
		2	

Q	Answer	Marks	Comments
11(c)	$(5-x)^{-1}=\frac{1}{5}\left(1+\frac{1}{5}x+\frac{1}{25}x^2\right)$	**M1**	Either expansion correct
	$(5+2x)^{-2}=\frac{1}{25}\left(1-\frac{4}{5}x+\frac{12}{25}x^2\right)$	**A1**	Both correct
	$\mathrm{f}(x)=$		
	$\frac{1}{3}\left(\frac{1}{5}+\frac{1}{25}x+\frac{1}{125}x^2\right)+\frac{2}{3}\left(\frac{1}{5}-\frac{2}{25}x+\frac{4}{125}x^2\right)$	**M1**	Correct substitution of at least two of their expressions with their A, B, C
	$+10\left(\frac{1}{25}-\frac{4}{125}x+\frac{12}{625}x^2\right)$	**A1ft**	All correct
	$=\frac{3}{5}-\frac{9}{25}x+\frac{27}{125}x^2$	**A1**	
		5	

	Question 11 Total	**11**	

Q	Answer	Marks	Comments
12	$x\left(16-y^2\right)=2\left(x^2+5\right)\frac{\mathrm{d}y}{\mathrm{d}x}$		
	$\int \frac{x}{x^2+5}\,\mathrm{d}x=\int \frac{2}{16-y^2}\,\mathrm{d}y$	**M1**	Separates variables Condone omission of integral signs
	$\frac{2}{16-y^2}=\frac{A}{4+y}+\frac{B}{4-y}$	**m1**	Attempt to use partial fractions **PI** by correct integration of $\int \frac{k}{16-y^2}\,\mathrm{d}y$
	$2=A(4-y)+B(4+y)$		
	$A=\frac{1}{4}\qquad B=\frac{1}{4}$	**A1**	Both correct **PI** by correct integration of $\int \frac{k}{16-y^2}\,\mathrm{d}y$
	$\left[\int \frac{x}{x^2+5}\,\mathrm{d}x=\int \frac{1/4}{4+y}+\frac{1/4}{4-y}\,\mathrm{d}y\right]$		
	$\frac{1}{2}\ln\left(x^2+5\right)=\frac{1}{4}\ln(4+y)-\frac{1}{4}\ln(4-y)\ [+c]$	**m1 A1**	**m1**: Attempt to integrate both sides **A1**: Both sides integrated correctly
	At $(0,1)\qquad 2\ln(5)=\ln\left(\frac{5}{3}\right)+c$	**m1**	Attempt to find c, or to use limits for x and y
	$c=\ln 15$		or $c=\ln\frac{1}{15}$
	$2\ln\left(x^2+5\right)=\ln(4+y)-\ln(4-y)+\ln 15$	**A1**	**oe eg** $\ln\left(\frac{\left(x^2+5\right)^2}{15}\right)=\ln\left(\frac{4+y}{4-y}\right)$
	$\left(x^2+5\right)^2=\frac{15(4+y)}{4-y}$		
	$\left(x^2+5\right)^2(4-y)-15y=60$	**M1**	Attempt to isolate y from $\left(x^2+5\right)^2=\frac{k(4\pm y)}{4\pm y},\ k\neq 0,1$
	$y=\frac{4\left(x^2+5\right)^2-60}{\left(x^2+5\right)^2+15}$	**A1**	**ACF eg** $y=4-\frac{120}{x^4+10x^2+40}$

	Question 12 Total	**9**	

Q	Answer	Marks	Comments
13(a)	$[AB: \ \mathbf{r} =] \begin{bmatrix} 2 \\ -2 \\ 6 \end{bmatrix} + \lambda \begin{bmatrix} 6 \\ 8 \\ -10 \end{bmatrix}$	**B1**	**oe** $\begin{bmatrix} 8 \\ 6 \\ -4 \end{bmatrix} + \lambda \begin{bmatrix} 6 \\ 8 \\ -10 \end{bmatrix}$
		1	

Q	Answer	Marks	Comments
13(b)(i)	$2+6\lambda = -2+4\mu$ or $8+6\lambda = -2+4\mu$		
	$-2+8\lambda = 3-5\mu$ or $6+8\lambda = 3-5\mu$	**M1**	Equating x and y coordinates
	$\mu = 1, \ \lambda = 0$ or $\mu = 1, \ \lambda = -1$	**A1**	
	$6-10\lambda = c+3\mu$ or $-4-10\lambda = c+3\mu$		
	$c = 3$	**A1**	**AG** Must be convincingly shown
		3	

Q	Answer	Marks	Comments
13(b)(ii)	Coords of P $(4p-2, -5p+3, 3p+3)$		
	$\overrightarrow{XP} = \begin{bmatrix} 4p-4 \\ -5p+1 \\ 3p-3 \end{bmatrix}$	**M1**	**oe**
	$\begin{bmatrix} 4p-4 \\ -5p+1 \\ 3p-3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -5 \\ 3 \end{bmatrix}$ $[=0]$	**m1**	Correct dot product seen
	$-16+16p-5+25p-9+9p=0$		
	$[50p=30]$ $\quad p=\frac{3}{5}$	**A1**	
	$QP=\sqrt{(2-0.4)^2+(2-0)^2+(6-4.8)^2}$	**m1**	Attempt at distance, **m1** dependent on **M1m1** scored
	$QP\left[=\sqrt{8}\right]=2\sqrt{2}$	**A1**	
		5	

OR	Answer	Marks	Comments
	$\sqrt{(4p-4)^2+(-5p+1)^2+(3p-3)^2}=\sqrt{D}$	**M1**	
	$D=16p^2-32p+16+25p^2-10p+1+9p^2-18p+9$	**m1**	
	$=50p^2-60p+26$	**A1**	
	$=50\left(p^2-\frac{6}{5}p+\frac{9}{25}\right)+8$		
	$=50\left(p-\frac{3}{5}\right)^2+8$	**m1**	
	Shortest distance $\left[=\sqrt{8}\right]=2\sqrt{2}$	**A1**	
		5	

	Question 13 Total	**9**	

Q	Answer	Marks	Comments
14(a)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sin x(-\sin x)-\cos x(\cos x)}{\sin^2 x}$	**M1**	Attempt at quotient rule
	$=\frac{-\sin^2 x-\cos^2 x}{\sin^2 x}$		Must see a ‘middle’ line
	$=-\operatorname{cosec}^2 x$	**A1**	**AG** Must be convincingly shown
		2	

Q	Answer	Marks	Comments
14(a)(ii)	$u=1+\cot x \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}x}=-\operatorname{cosec}^2 x$		
	$\int \frac{\operatorname{cosec}^2 x}{(1+\cot x)^2}\,\mathrm{d}x = -\int \frac{1}{u^2}\,\mathrm{d}u$	**M1**	Everything in terms of u
	$=u^{-1}\ [+c]$	**A1**	
	$\left[\frac{1}{u}\right]_{1+\sqrt{3}}^{2}=\left[\frac{1}{1+\cot x}\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$	**B1**	Change limits or change back to x
	$\left[\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\operatorname{cosec}^2 x}{(1+\cot x)^2}\,\mathrm{d}x=\right]\ \frac{1}{2}-\frac{1}{1+\sqrt{3}}$	**m1**	Substitute *their* limits into *their* expression of the form ku^{-1}
	$=1-\frac{\sqrt{3}}{2}$	**A1**	
		5	

Q	Answer	Marks	Comments
14(b)	$\dfrac{1}{\sqrt{(2x+1)}+\sqrt{(2x-1)}}\times\dfrac{\sqrt{(2x+1)}-\sqrt{(2x-1)}}{\sqrt{(2x+1)}-\sqrt{(2x-1)}}$ $=\dfrac{\sqrt{(2x+1)}-\sqrt{(2x-1)}}{2x+1-2x+1}=\dfrac{\sqrt{(2x+1)}}{2}-\dfrac{\sqrt{(2x-1)}}{2}$ $\displaystyle\int\frac{1}{\sqrt{(2x+1)}+\sqrt{(2x-1)}}\,\mathrm{d}x$ $=\dfrac{(2x+1)^{1.5}}{6}-\dfrac{(2x-1)^{1.5}}{6}\quad[+c]$	**B1** **M1 A1**	Correctly rationalising **M1**: $a(2x+1)^{1.5}+b(2x-1)^{1.5}$ **A1**: Fully correct
		3	

	Question 14 Total	**10**	