OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL A-LEVEL

MATHEMATICS

MA04

(9660/MA04) Unit S2 Statistics

Mark scheme

January 2023

Version: 1.0 Final

231XMA04/MS

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$\mathrm{E}(3Y-2X) = 3\times15-2\times12$ $= 21$	M1 A1	
		2	

Q	Answer	Marks	Comments
1(b)	$\mathrm{Var}(3Y-2X) = 3^2\times2.5+2^2\times5$ $= 42.5$	M1 A1	Condone a sign error **oe eg** 85/2 $85\div2$ $\frac{85}{2}$
		2	

	Question 1 Total	**4**	

Q	Answer	Marks	Comments
2(a)	Stages 1, 4 and 8 contain errors	**B1**	At least two stages correctly identified
	Stage 1 – Hypothesis are wrong should be $H_0 : \mu = 18.3$ $H_1 : \mu \neq 18.3$	**E1**	Clear comment regarding population parameter μ not the sample $\overline{x}$
	Stage 4 – $[z_{critical} = 1.2816]$ should be $z_{critical} = (\pm)1.6449$	**E1**	Allow $\pm$ but not –
	Stage 8 – wrong conclusion: should not reject H_0 : Insufficient evidence to suggest a change in mean has occurred at the 10% level of significance	**E1**	**oe**
		4	

Q	Answer	Marks	Comments
2(b)	The sample was taken from a normal distribution so the sample is also normally distributed.	**E1**	**oe** for example 'a sample inherits the characteristics of the parent population'
		1	

Q	Answer	Marks	Comments
2(c)	A Critical Region	**B1**	
		1	

	Question 2 Total	**6**	

Q	Answer	Marks	Comments
3(a)	$\frac{\mathrm{d}\mathrm{F}(t)}{\mathrm{d}t}=\frac{3t^2}{64}$	**M1**	Obtains kt^2
	$\mathrm{f}(t)=\frac{3t^2}{64}, \quad 0\le t\le 4$	**A1**	Requires both parts
	0 , otherwise	**B1**	
		3	

Q	Answer	Marks	Comments
3(b)(i)	$\left[\mu=\mathrm{E}(T)=\right]\ \int_0^4 t\times\frac{3t^2}{64}\mathrm{d}t=\left[\frac{3t^4}{256}\right]_0^4$	**M1**	Attempt to integrate their $t\,\mathrm{f}(t)$ with correct limits **PI**
	$=3$	**A1**	
		2	

Q	Answer	Marks	Comments
3(b)(ii)	$\left[\mathrm{E}\left(T^2\right)=\right]\ \int_0^4 t^2\times\frac{3t^2}{64}\mathrm{d}t=\left[\frac{3t^5}{320}\right]_0^4$	**M1**	Attempt to integrate their $t^2\mathrm{f}(t)$ with correct limits **PI**
	$=9.6$	**A1ft**	**PI**, **oe** **ft** their $\mathrm{E}\left(t^2\right)-\mathrm{E}(t)^2$
	$\sigma^2=\left[\mathrm{E}\left(T^2\right)-\mathrm{E}(T)^2=\right]\ 9.6-3^2$	**M1**	**PI** Condone $\sigma=0.6$
	$\sigma=\sqrt{0.6}=0.775\ (3\text{ sf})$	**A1**	**CAO** to 3 sf
		4	

Q	Answer	Marks	Comments
3(c)	$\mathrm{P}\left(3-2\sqrt{0.6} \le T \le 3+2\sqrt{0.6}\right)$	**M1**	Allow their μ & σ **PI**
	$=\mathrm{P}\left(1.45\left[0806662\right] \le T \le 4.54\left[9193338\right]\right)$		
	$\mathrm{F}\left(4.54\right)-\mathrm{F}\left(1.45\left[0806662\right]\right)$	**M1**	
	$\mathrm{F}\left(4.54\right)=\left[\mathrm{F}\left(4\right)=\right]\ 1$	**B1**	**PI** by correct answer
	$1-0.04771431013=0.952$ (3 sf)	**A1**	**AWRT** 0.952
		4	

	Question 3 Total	**13**	

Q	Answer	Marks	Comments
4	$H_0 : \mu = 25$ $H_1 : \mu < 25$	**B1**	Both hypotheses
	$\overline{x} = 24.86$	**B1**	
	$s^2 = \frac{1}{500-1} \times \left(310000 - \frac{12430^2}{500} \right)$	**M1**	Attempt at variance formula Allow one slip Implied by correct answer
	$= 1.98\left[4368737...\right]$	**A1**	**AWRT** 1.98 Accept $s = 1.40\left[8676236\right]$
	$\overline{X} \sim N\left(25, \frac{1.98[\ldots]}{500}\right)$	**M1**	**PI**, $\overline{X} \sim N\left(25, \frac{s^2}{500}\right)$
	$z = \frac{24.86 - 25}{\sqrt{\frac{1.98[...]}{500}}}$	**M1**	Calculates z with their s^2
	$= -2.2\left[22295719...\right]$	**A1**	**AWRT** –2.2 or $\lvert z \rvert = 2.2$ or probability of 0.013
	$z_{\text{critical}} = -2.3263$	**B1**	**AWRT** –2.3 Allow $z_{\text{critical}} = \pm 2.3263$
	Do not reject H_0 as $-2.2\left[222...\right] > -2.3\left[263...\right]$ or $z > z_{\text{crit}}$	**A1ft**	**ft** their z and z_{crit} provided signs are consistent Allow a correct comparison of z_{crit} and $\lvert z \rvert$ or compares their 0.013 to 0.01 and correct statement
	Evidence to suggest that the hand-drying time does not have a mean lower than 25 [at the 1% level of significance]	**E1ft**	Must be consistent with their conclusion on whether or not to reject H_0, or their z and z_{crit} if not explicitly stated Condone definite statement
		10	

	Question 4 Total	**10**	

Q	Answer	Marks	Comments
5(a)(i)	Condition 1: The number of events/trials, n must be large; and Condition 2: The probability of a successful outcome, p, must be small	**B1**	n must be large **and** the p value must be small Allow **eg** $n \geq 50$ and $p \leq 0.1$
		1	

Q	Answer	Marks	Comments
5(a)(ii)	[Mean $= np =$] $\lambda = 500 \times 0.02 = 10$	**B1**	
		1	

Q	Answer	Marks	Comments
5(a)(iii)	$\mathrm{P}(X < 10) = \mathrm{P}(X \leq 9)$ [= 0.4579297391…] = 0.458	**M1** **A1**	**PI** **AWRT** 0.458
		2	

Q	Answer	Marks	Comments
5(b)(i)	$\mathrm{H}_0 : \lambda = 8$ $\mathrm{H}_1 : \lambda \neq 8$ $X = 5$ [per hectare] $\mathrm{P}(X \leq 5) = 0.191 [2360621...]$ $0.191 > 0.05$ Do not reject H_0 Insufficient evidence to suggest the mean number of ground nests [per hectare] has changed from 8 [at the 10% level of significance]	**B1** **M1** **A1** **A1** **B1ft** **E1**	Allow $\mathrm{H}_0 : \lambda = 240$ $\mathrm{H}_1 : \lambda \neq 240$ **PI** Comparison of their probabilities or a lower CR of $X \leq 3$ Correct conclusion on H_0 based on their comparison of test statistic with critical value Must be in context, must not be definite and all the previous 5 marks must have been awarded
		6	

Q	Answer	Marks	Comments
5(b)(ii)	$P(X \leq c) < 0.05$ $P(X \leq 3) = 0.0424$ [or $P(X \geq 4) = 0.9576$] and $P(X \leq 4) = 0.0996$ [or $P(X \geq 5) = 0.9004$]	**M1**	**PI** by correct CR, could be written as a single CR later
	$P(X \geq c) < 0.05$ $P(X \geq 14) = 0.0342$ [or $P(X \leq 13) = 0.9658$] and $P(X \geq 13) = 0.0638$ [or $P(X \leq 12) = 0.9362$]	**M1**	**PI** by correct CR, could be written as a single CR later
	Both included CR = {0, 1, 2, 3, 14, 15, … }	**A1**	Allow $X \leq 3$ and $X \geq 14$ **oe**
		3	

Q	Answer	Marks	Comments
5(c)(i)	Concluding that the mean number of ground nests [per hectare] has changed [from 8] even though it hasn't.	**B1**	
		1	

Q	Answer	Marks	Comments
5(c)(ii)	[0.0424 + 0.0342 =] 0.0766	**B1**	**CAO**
		1	

	Question 5 Total	**15**	

Q	Answer	Marks	Comments
6(a)	$\mathrm{P}(X_1 < 10) = \mathrm{P}\left(Z < \frac{10-12}{1.5}\right)$	**M1**	Attempt to standardise **PI** **oe, e.g.** $\mathrm{P}(Z < -1.33[3...])$ or $1-\mathrm{P}(Z < 1.33[3...])$
	$= 0.0912[1121973...]$	**A1**	**AWFW** 0.0912 to 0.0918
		2	

Q	Answer	Marks	Comments
6(b)	$\mathrm{P}(X_2 < a) = 0.9$		
	$z = 1.28[1551638...]$	**B1**	**Allow** $\pm 1.28[1551638...]$
	$\frac{a-8}{0.8} = 1.28$	**M1**	Forms an equation with their $1.2 < \lvert z \rvert < 1.3$
	9.03 [mins] (to 3 sf)	**A1**	$9.025[241311...]$ **AWRT** 9.03 **oe**, e.g. 9 minutes, 2 seconds
		3	

Q	Answer	Marks	Comments
6(c)	$z = 0.915[3645453]$ or $z = [-]0.075[26985986]$	B1	**PI** by correct standardisation 0.9154 or 0.0753 from tables
	$0.915[3645453] = \frac{16-\mu}{\sigma}$ or $-0.075[26985986] = \frac{10-\mu}{\sigma}$	M1	**oe** At least one correct standardisation **PI** by at least one correct value for μ or σ
	$0.915[3645453] = \frac{16-\mu}{\sigma}$ and $-0.075[26985986] = \frac{10-\mu}{\sigma}$	A1	**oe** Both correct standardisations **PI** by at least one correct value for μ or σ
	$(0.915[3645453] + 0.075[26985986])\sigma = 16 - 10$	M1	Attempt to eliminate one variable simultaneously **PI** by at least one correct value for μ or σ
	$\sigma = 6.06 \ \ (3\text{ sf})$	A1	**AWFW** 6.05 to 6.06
	$\mu = 10.5 \ \ (3\text{ sf})$	A1	**AWRT** 10.5
		6	

Q	Answer	Marks	Comments
6(d)	$\mu - 2\sigma < 0$ or $\mu - 3\sigma < 0$ or time cannot be less than zero in this context	B1	A clear comment about the lower tail having $t < 0$
		1	

Q	Answer	Marks	Comments
6(e)	$T = X_1 + X_2 + X_3 + X_4$ $T \sim \text{N}(12+8+7+14, 1.5^2 + 0.8^2 + 1^2 + 2^2)$ $T \sim \text{N}(41, 7.89)$	M1	
	$\text{P}(T < 35) = \text{P}\left(Z < \frac{35-41}{\sqrt{7.89}}\right)$ $[= \text{P}(Z < -2.136056564\ldots)]$	M1	**PI**
	$= 0.0163[3740189...]$	A1	**AWFW** 0.0162 to 0.0164
		3	

	Question 6 Total	**15**	

Q	Answer	Marks	Comments
7(a)	$\mathrm{P}(X=4)=\dfrac{\mathrm{e}^{-2.5}\times 2.5^4}{4!}$	M1	PI
	$=0.1336[018858]$	A1	
		2	

Q	Answer	Marks	Comments
7(b)	$\lambda = 6\times 2.5 = 15$	B1	PI
	$\mathrm{P}(X>18)=1-\mathrm{P}(X\leq 18)$	M1	
	$=1-0.8194[717351]$		
	$=0.1805$ (to 4 sf)	A1	**AWRT** 0.1805
		3	

Q	Answer	Marks	Comments
7(c)(i)	$\left[\dfrac{1}{\lambda}=\right]$ 0.4	B1	
		1	

Q	Answer	Marks	Comments
7(c)(ii)	$\left[\left(\dfrac{1}{\lambda}\right)^2=0.4^2=\right]$ 0.16	B1	
		1	

Q	Answer	Marks	Comments
7(d)	$1-\mathrm{e}^{-2.5c}=0.9$	M1	$1-\mathrm{e}^{-kc}=0.9$ for $k>0$ or correct integration with correct limits
	$\mathrm{e}^{-2.5c}=0.1$		
	$-2.5c=\ln 0.1$		
	$c=\dfrac{2}{5}\ln 10$	M1	**oe** Finds their c in exact form or as a value for the number of weeks
	$c=0.9210[340372...]$ [weeks]		
	$c=6$ [days]	A1	
		3	

	Question 7 Total	**10**	

Q	Answer	Marks	Comments
8(a)	$\left[\mathrm{F}(x)=\int_2^x \mathrm{f}(x)\,\mathrm{d}x=\right]\ \int_2^x \frac{96}{(5x+k)^2}\,\mathrm{d}x$		
	$=\left[\frac{96}{-5(5x+k)}\right]_2^x$	**M1**	Attempt at integration to obtain $a(5x+k)^{-1}$ with $a \neq 96$
	$=-\frac{96}{5}\left(\frac{1}{5x+k}-\frac{1}{10+k}\right)$	**M1**	**oe** Substitute limits into their integration of the form $a(5x+k)^{-1}$ with $a \neq 96$
	$=-\frac{96}{5}\left(\frac{(10+k)-(5x+k)}{(10+k)(5x+k)}\right)$		
	or $=\frac{96}{5}\left(\frac{5x-10}{(10+k)(5x+k)}\right)$	**A1**	**oe** Correct single fraction
	or $=\frac{96}{5}\left(\frac{5(x-2)}{(10+k)(5x+k)}\right)$		
	$\mathrm{F}(x)=\begin{cases}0 & x<2\\ \frac{96(x-2)}{(10+k)(5x+k)} & 2\le x\le d\\ 1 & x>d\end{cases}$	**A1**	**AG** Must be convincingly shown **SC2** for correctly differentiating $\mathrm{F}(x)$ w.r.t. to x to show $\mathrm{f}(x)=\frac{96}{(5x+k)^2}$
		4	

Q	Answer	Marks	Comments
8(b)	$\left[\mathrm{F}(4.4)=0.8\Rightarrow\right]\ \ 0.8=\dfrac{96(4.4-2)}{(10+k)(5\times4.4+k)}$	M1	oe
	$220+32k+k^2=288$		
	$k^2+32k-68=0$		$0.8k^2+25.6k-54.4=0$
	$(k-2)(k+34)=0$	m1	Attempts to solve the correct quadratic equation **PI** by $k=2$
	$k=2$	A1	Must not include $k=-34$
		3	

	Question 8 Total	**7**	