

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

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Forename(s)

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Candidate signature

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I declare this is my own work.

# INTERNATIONAL AS FURTHER MATHEMATICS

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Monday 9 January 2023    07:00 GMT    Time allowed: 1 hour 30 minutes

## Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

## Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

## Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

### For Examiner's Use

Question	Mark
1	
2	
3	
4	
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6	
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8	
9	
10	
11	
12	
<b>TOTAL</b>	



J A N 2 3 F M 0 2 0 1

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ANSWER IN THE SPACES PROVIDED**



Answer **all** questions in the spaces provided.

$$\frac{dy}{dx} = 3x + \frac{2x^3}{y} \quad \text{for } y \neq 0$$

Give your answer to four decimal places.

**[4 marks]**

[illegible]

Answer

4

- 2 The variables  $P$  and  $Q$  are related by the equation

$$Q = aP + b \quad \text{where } a \text{ and } b \text{ are constants.}$$

It is given that  $P = \frac{1}{x}$  and  $Q = \frac{1}{y}$  where  $x \neq 0$  and  $y \neq 0$

During an experiment the following pairs of values for  $x$  and  $y$  were obtained.

$x$	1	2	3	4
$y$	1.10	0.88	0.83	0.80

- 2 (a) Complete the table below giving the values of  $Q$  to **two** decimal places.

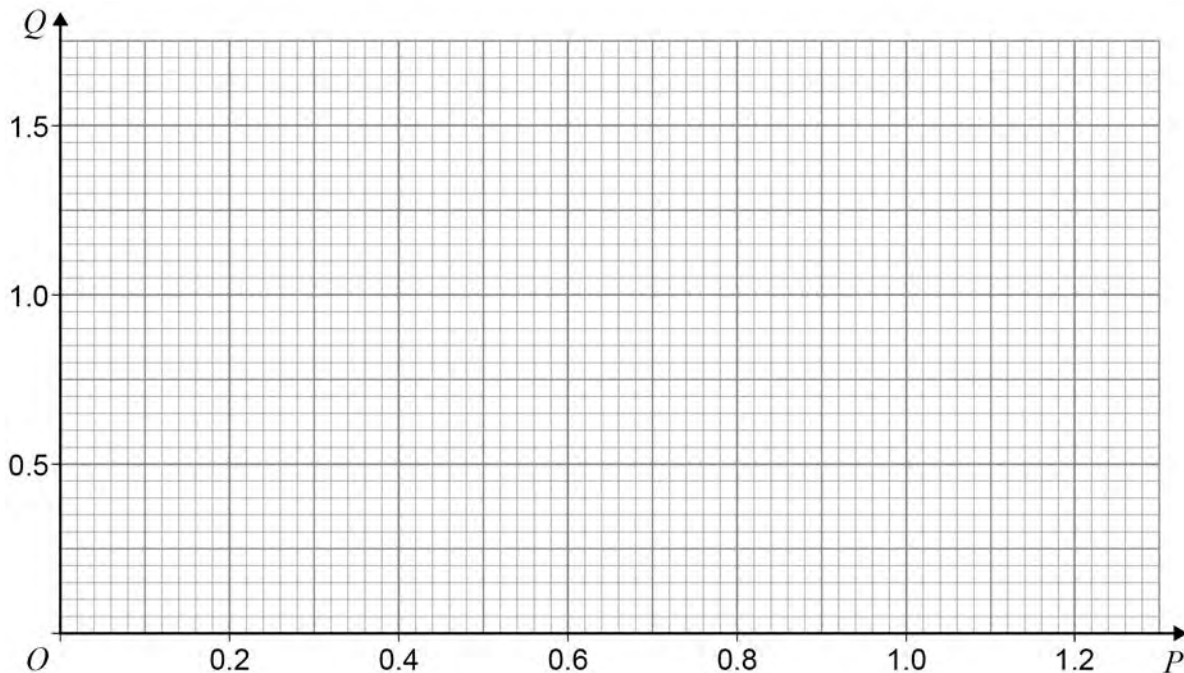
[1 mark]

$P$	1	0.5	0.33	0.25
$Q$				

- 2 (b) On the grid plot  $Q$  against  $P$

Draw a line of best fit for the points you have plotted.

[2 marks]



**2 (c) (i)** Use your line of best fit to find estimates for  $a$  and  $b$  to two significant figures.

**[2 marks]**

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$a =$  \_\_\_\_\_  $b =$  \_\_\_\_\_

**2 (c) (ii)** Use your values for  $a$  and  $b$  to write down an equation relating  $x$  and  $y$

**[1 mark]**

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Answer \_\_\_\_\_

**2 (d)** Use your equation to estimate the value of  $y$  when  $x = 1.6$

Give your answer to two decimal places.

**[1 mark]**

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Answer \_\_\_\_\_

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Turn over ►



3

The matrix  $\mathbf{B}$  is defined by  $\mathbf{B} = \begin{bmatrix} 0.5 & 8 & 3.5 \end{bmatrix}$  where  $k$  is a constant.

**3 (a)**

[illegible]

$$k = \underline{\hspace{10cm}}$$



3 (b) (i) Using your value of  $k$  find  $\mathbf{AB}$

[2 marks]

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Answer \_\_\_\_\_

3 (b) (ii) Hence find the matrix  $\mathbf{C}$  such that  $\mathbf{C} \begin{bmatrix} 1 & 6 & 3 \\ 2 & 32 & 14 \\ 0 & 8 & 4 \end{bmatrix} = \mathbf{AB}$

[2 marks]

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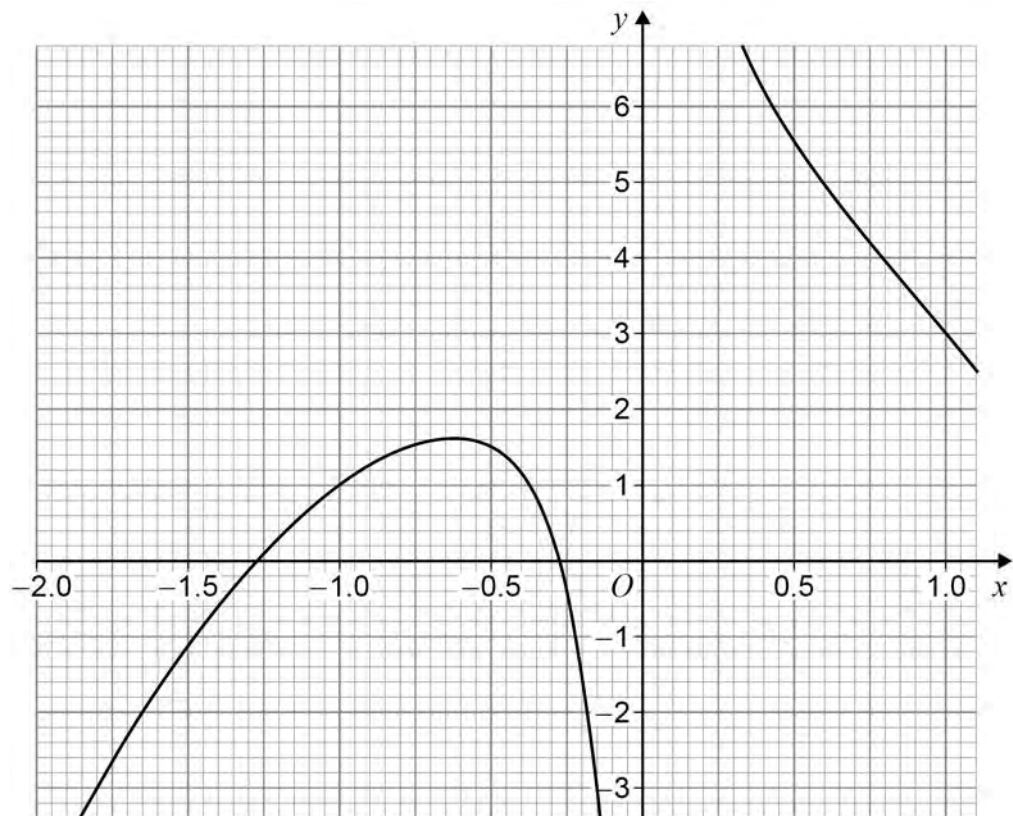
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Answer \_\_\_\_\_

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- 4 The graph shows part of the curve  $y=f(x)$  where  $f(x)=\frac{1}{x}-2x^2+4$



The equation  $f(x)=0$  has three roots  $\alpha$ ,  $\beta$  and  $\gamma$

The roots  $\alpha$  and  $\beta$  are such that  $-2 < \alpha < -1$  and  $-1 < \beta < 0$

- 4 (a) Show that  $1 < \gamma < 2$

[2 marks]

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**[2 marks]**

[illegible]

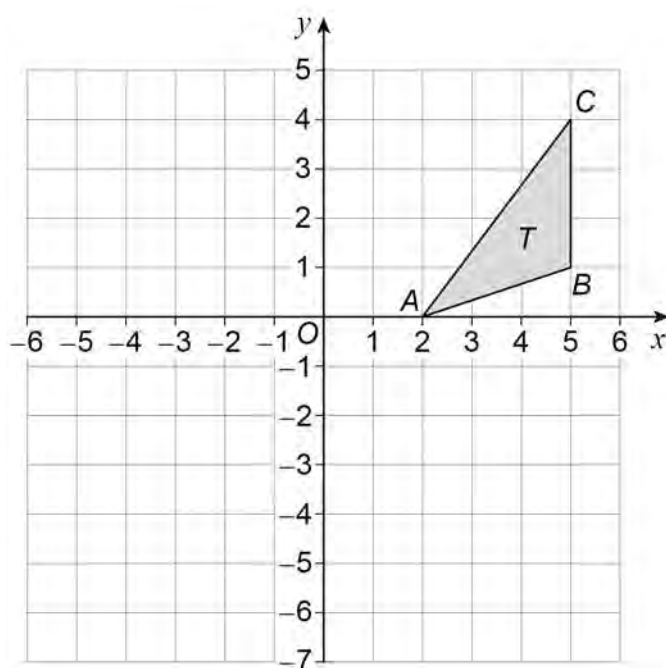
**[4 marks]**

[illegible]

Answer



- 5 The triangle  $T$  with vertices  $A(2, 0)$ ,  $B(5, 1)$  and  $C(5, 4)$  is shown in the diagram.



The matrix  $\mathbf{M} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$  is used to transform triangle  $T$

The transformation represented by matrix  $\mathbf{M}$  maps  $ABC$  onto  $A'B'C'$  where  $A'$ ,  $B'$  and  $C'$  are the vertices of triangle  $R$

The point  $A'$  has coordinates  $(-1, p)$  where  $p$  is a constant.

It is given that  $p$  is negative ( $p < 0$ )

- 5 (a) (i) Find the exact value of  $p$

[3 marks]

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**[1 mark]**

**[3 marks]**

**Turn over ►**



**5 (b)**

The point  $A'$  is invariant under this transformation.

**5 (b) (i)**

**[3 marks]**

[illegible]

$c =$  \_\_\_\_\_  $d =$  \_\_\_\_\_



- 5 (b) (ii)** Using answers from **parts (a)(ii)** and **(b)(i)**, find the matrix representing the single transformation which maps triangle  $T$  onto triangle  $S$

[2 marks]

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Answer \_\_\_\_\_

- 5 (b) (iii)** Describe fully the **single** transformation which maps triangle  $T$  onto triangle  $S$

[3 marks]

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**Section B****Statistics**Answer **all** questions in the spaces provided.

- 6** The discrete random variable  $X$  has probability generating function

$$G_X(t) = (0.2 + 0.8t)^3$$

- 6 (a)** Using differentiation find  $E(X)$

**[4 marks]**


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Answer \_\_\_\_\_

- 6 (b)** Find  $P(X \geq 2)$

**[1 mark]**


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Answer \_\_\_\_\_



**7** The probability that Imran is late for school on a randomly chosen day is  $p$

The discrete random variable  $X$  represents the number of days up to and including the first day Imran is late for school.

The variance of  $X$  is 3.75

**7 (a)** Find the value of  $p$

**[3 marks]**

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$p =$  \_\_\_\_\_

**7 (b)** Find the mean of  $X$

**[1 mark]**

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Answer \_\_\_\_\_

**7 (c)** Find the probability that the first day Imran is late for school is during the next 5 days.

**[2 marks]**

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Answer \_\_\_\_\_

6
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Turn over ►



**8** The discrete random variable  $X$  has a uniform distribution and takes values  
1, 2, 3, ..., 7

The discrete random variable  $Y$  has a uniform distribution and takes values  
1, 2, 3, ..., 10

**8 (a)** Find  $E(4X - 3Y)$

**[4 marks]**

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Answer \_\_\_\_\_

**8 (b)** Given that  $\text{Var}(X + Y) = 10$  find the value of the correlation between  $X$  and  $Y$ , giving your answer to three decimal places.

**[5 marks]**

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Answer \_\_\_\_\_





## Mechanics

9 A student is creating a model for the magnitude of the air resistance force,  $F$  newtons, acting on a sphere as it moves at a speed of  $v \text{ m s}^{-1}$

$$F = k\nu^n$$

Find the dimensions of  $k$  in terms of  $n$

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Answer

3



- 10** A disc of mass  $0.3 \text{ kg}$  is sliding at a speed of  $6 \text{ m s}^{-1}$  on a smooth horizontal surface when it collides with a vertical wall that is perpendicular to its path.

The disc rebounds at a speed of  $4 \text{ m s}^{-1}$  after the collision with the wall.

- 10 (a)** Calculate the magnitude of the impulse exerted by the wall on the disc.

**[2 marks]**

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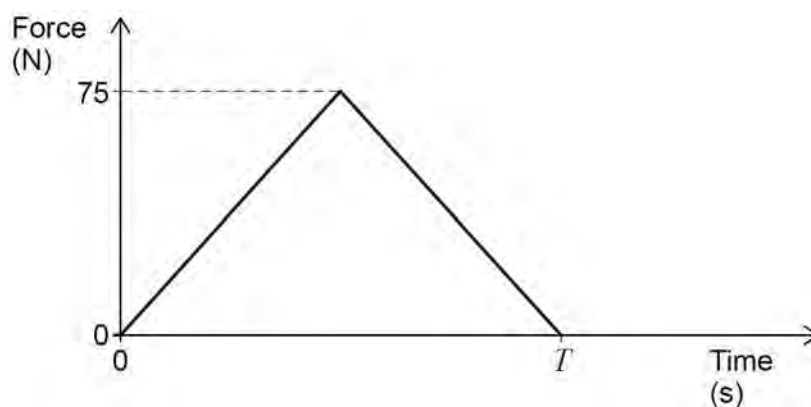


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Answer \_\_\_\_\_

- 10 (b)** The graph shows how the magnitude of the force exerted by the wall on the disc varies with time.

The disc is in contact with the wall for  $T$  seconds.



Find the value of  $T$

**[2 marks]**

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$T =$  \_\_\_\_\_



Hannah and Rachel start jogging at the same time from different positions and move with constant velocities on a horizontal field.

	Initial Position Vector (m)	Constant Velocity (m s <sup>-1</sup> )
Hannah	$-20\mathbf{i} + 450\mathbf{j}$	$2\mathbf{i} - 0.5\mathbf{j}$
Rachel	$80\mathbf{i} + 150\mathbf{j}$	$\mathbf{i} + U\mathbf{j}$

Hannah and Rachel meet at a position on the field and then stop.

**[4 marks]**

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$$U =$$

4







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	<div style="border: 1px solid black; height: 550px; width: 100%;"></div>
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