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(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

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2 2 6 X F M 0 2 / M S

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Key to mark scheme abbreviations

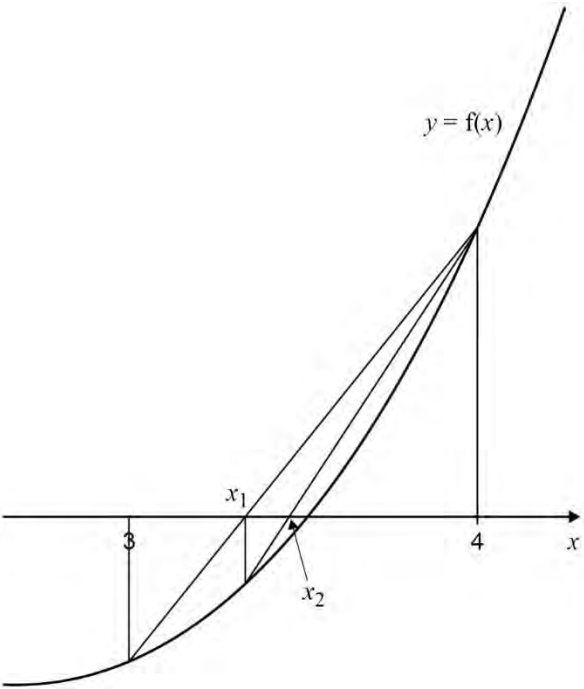
M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$hf(x, y) = 0.2 \left(\frac{\sqrt{3+2}}{4.8 \times (3+1)} \right)$ $[= 0.023292375]$ $y_2 = 4.8 + 0.023...$ $y_2 = 4.823292375$ $y_3 = 4.823292375 + 0.2 \left(\frac{\sqrt{3.2+2}}{4.823292375 \times (3.2+1)} \right)$ $y_3 = 4.846$	<p>M1</p> <p>m1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>PI</p> <p>4.8 + their $hf(x, y)$</p> <p>AWRT 4.82</p> <p>ft their y_2</p> <p>CAO</p>
	Total	5	

[illegible]

Q	Answer	Marks	Comments
2(d) ALT	<p>CAB =</p> $\begin{bmatrix} \left(\frac{1}{2} + \frac{\sqrt{3}}{2}p\right)\cos 2\theta + \left(\frac{1}{2}p - \frac{\sqrt{3}}{2}\right)\sin 2\theta & \frac{\sqrt{3}}{2}\cos 2\theta + \frac{1}{2}\sin 2\theta \\ \left(\frac{\sqrt{3}}{2} - \frac{1}{2}p\right)\cos 2\theta + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}p\right)\sin 2\theta & -\frac{1}{2}\cos 2\theta + \frac{\sqrt{3}}{2}\sin 2\theta \end{bmatrix}$ <p>det CAB =</p> $\left(\left(\frac{1}{2} + \frac{\sqrt{3}}{2}p\right)\cos 2\theta + \left(\frac{1}{2}p - \frac{\sqrt{3}}{2}\right)\sin 2\theta\right)\left(-\frac{1}{2}\cos 2\theta + \frac{\sqrt{3}}{2}\sin 2\theta\right) - \left(\left(\frac{\sqrt{3}}{2} - \frac{1}{2}p\right)\cos 2\theta + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}p\right)\sin 2\theta\right)\left(\frac{\sqrt{3}}{2}\cos 2\theta + \frac{1}{2}\sin 2\theta\right)$ <p>det CAB = $-\frac{1}{4}(\cos^2 2\theta + \sin^2 2\theta) - \frac{3}{4}(\cos^2 2\theta + \sin^2 2\theta)$</p> <p>det CAB = -1</p>	<p>B1</p> <p>M1</p> <p>A1</p>	<p>CSO</p>
		3	
	Total	10	

Q	Answer	Marks	Comments												
3(a)	$\log_{10} y = n \log_{10} x + b$ $\log_{10} y = \log_{10} x^n + \log_{10} 10^b$ $\log_{10} y = \log_{10} 10^b x^n$ $y = 10^b x^n$	M1 A1	Apply at least one law of logarithms correctly CSO												
		2													
3(b)	<table border="1"><tr><td>X</td><td>0.30</td><td>0.60</td><td>1</td><td>1.30</td><td>1.60</td></tr><tr><td>Y</td><td>0.66</td><td>0.48</td><td>0.26</td><td>0.08</td><td>−0.10</td></tr></table>	X	0.30	0.60	1	1.30	1.60	Y	0.66	0.48	0.26	0.08	−0.10	B1 B1	X values – at least 1 dp or exact value Y values – at least 2 dp
X	0.30	0.60	1	1.30	1.60										
Y	0.66	0.48	0.26	0.08	−0.10										
		2													
3(c)(i)	All their points plotted ± 1 square Line of best fit drawn	B1 B1	Must have linear scales												
		2													
3(c)(ii)	$a = 10^{(\text{their } y\text{-intercept})}$ $a = 6.8$ $n = \text{their gradient}$ $n = -0.58$	M1 A1 M1 A1	PI Between 6 and 8 Between −0.7 and −0.5												
		4													
	Total	10													

Q	Answer	Marks	Comments
4(a)	$f(3) = -3$ and $f(4) = 6$ Change of sign and f is continuous on the interval so α is in the interval (3, 4)	M1 A1	Correct evaluation of a suitable interval Must state that there is a change of sign and that the curve is continuous (condone unbroken) and concludes a root is present in the interval
		2	
4(b)	Vertical lines to curve at 3 and 4 Correct points on graph joined by line that meets x -axis at x_1 Vertical line to curve at their x_1 Correct points on graph joined by line that meets x -axis at x_2	M1 A1 M1 A1	See diagram
			
		4	
4(c)	$x_1 = \frac{3 \times 4 + 6 \times 3}{3 + 6}$ $x_1 = \frac{10}{3}$ and $f(x_1) = -\frac{38}{27}$ $x_2 = 3.46$ and $f(x_2) = -0.464664$ $x_3 = 3.50$	M1 A1 A1 A1	Uses values from (a) to find x_1 AWRT 3.33 and -1.41 AWRT 3.46 (if earlier values rounded) and -0.465 CSO , accept 3.5
		4	
	Total	10	

Q	Answer	Marks	Comments
5	$\mathbf{P}(4\mathbf{I} + \mathbf{Q}) = \mathbf{R}$ $4\mathbf{I} + \mathbf{Q} = \begin{bmatrix} -2 & 4 \\ -2 & 5 \end{bmatrix} \text{ and } \det(4\mathbf{I} + \mathbf{Q}) = -2$ $(4\mathbf{I} + \mathbf{Q})^{-1} = \begin{bmatrix} -2.5 & 2 \\ -1 & 1 \end{bmatrix}$ $\mathbf{P} = \mathbf{R}(4\mathbf{I} + \mathbf{Q})^{-1}$ $\mathbf{P} = \begin{bmatrix} -2 & 3 \\ -16 & 37 \end{bmatrix} \begin{bmatrix} -2.5 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>Condone 4 for 4I if I implied by later work PI</p> <p>PI</p>
5 ALT	$\mathbf{P} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $4\mathbf{P} + \mathbf{PQ} = \begin{bmatrix} -2a - 2b & 4a + 5b \\ -2c - 2d & 4c + 5d \end{bmatrix}$ $-2a - 2b = -2$ $4a + 5b = 3$ $-2c - 2d = -16$ $4c + 5d = 37$ $\mathbf{P} = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>PI At least two correct elements</p> <p>PI Fully correct</p> <p>Form simultaneous equations in a and b</p> <p>Form simultaneous equations in c and d</p>
	Total	5	

Q	Answer	Marks	Comments
6(a)	0.121032	B1	CAO
		1	
6(b)	$P(R > 6) = (1 - 0.18)^6$ 0.304	M1 A1	PI Applies formula (oe) or finds $P(R \leq 6)$ [= 0.6959...] AWRT
		2	
6(c)	$E(R) = 5.56$	B1	AWRT
		1	
6(d)	$\text{Var}(R) = 25.3$	B1	AWRT
		1	
6(e)	$\text{Var}(R - S) = 28.3$	B1ft	AWRT ft their $\text{Var}(R) + 3$
		1	
	Total	6	

Q	Answer	Marks	Comments
7(a)	$G_Y(t) = (0.22 + 0.78t)^2$	B1	Applies formula for $G_Y(t)$
	$G_Y(t) = 0.6084t^2 + 0.3432t + 0.0484$	M1	Multiplies out
	$G'_Y(t) = 1.2168t + 0.3432$	M1	Differentiates $G'_Y(t) = 2 \times 0.78(0.22 + 0.78t)$ gains both M marks
	$G''_Y(t) = 1.2168$ or $\frac{1521}{1250}$	A1	CAO
		4	
7(b)	$G'_Y(1) = 1.2168 \times 1 + 0.3432$	M1	PI Accept sight of 1.56 Their $G'_Y(t)$ must have non-constant terms
	$\text{Var}(Y) = G''_Y(1) + G'_Y(1) - (G'_Y(1))^2$ $= 1.2168 + 1.56 - 1.56^2$	M1	Uses variance formula and substitutes in their values
	$= 0.3432$ or $\frac{429}{1250}$	A1	AWFW [0.34, 0.35]
		3	
7(b) ALT	$Y \sim B(2, 0.78)$	M1	PI Identifies binomial distribution
	$\text{Var}(Y) = 2 \times 0.78 \times 0.22$	M1	Applies variance formula
	$= 0.3432$ or $\frac{429}{1250}$	A1	
		3	
	Total	7	

Q	Answer	Marks	Comments
8(a)	$\frac{1}{5}$	B1	oe such as $\frac{7}{35}$
		1	
8(b)(i)	<p>First ball</p> <p>Second ball</p> <p>$\frac{1}{5}$ A $\frac{6}{34}$ B</p> <p>$\frac{28}{34}$ B'</p> <p>$\frac{4}{5}$ A' $\frac{7}{34}$ B</p> <p>$\frac{27}{34}$ B'</p>	<p>B1ft</p> <p>B1ft</p> <p>B1ft</p>	<p>1/5 and 4/5 oe ft their part (a)</p> <p>6/34 and 28/34 oe ft their 1/5 and 4/5</p> <p>7/34 and 27/34 oe ft their 1/5 and 4/5</p>
		3	
8(b)(ii)	$\frac{1}{5} \times \frac{28}{34}$ $\frac{1}{5} \times \frac{28}{34} + \frac{4}{5} \times \frac{27}{34}$ $\frac{14}{85} = \frac{7}{34}$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>Finds numerator using Bayes Theorem Sight of 14/85 oe</p> <p>Find denominator using Bayes Theorem Sight of 4/5 oe</p> <p>oe AWRT 0.206</p>
		3	
	Total	7	

Q	Answer	Marks	Comments
9	$I = 0.3 \times 7 - 0.3 \times (-5)$	M1	Applies impulse equation. Condone sign errors for the velocities.
	$I = 3.6 \text{ N s}$	A1	If units stated, must be correct.
	Total	2	

Q	Answer	Marks	Comments
10	$(LT^{-1})^2 = [\omega]^2 L^2$	M1	Forms correct dimensional analysis equation. Condone $[\omega]^2 (L^2 - L^2)$ for RHS
	$[\omega]^2 = \frac{L^2 T^{-2}}{L^2} = T^{-2}$		
	$[\omega] = T^{-1}$	A1	
	Units of ω are s^{-1}	B1	
	Total	3	

Q	Answer	Marks	Comments
11	$v^2 = 2^2 + 6^2 - 2 \times 2 \times 6 \cos 45^\circ$	M1	Uses cosine rule for a valid triangle oe
	$v = 4.7989 \left[= 4.80 \text{ m s}^{-1} \text{ to 3sf} \right]$	A1	Finds correct resultant velocity
	$\frac{\sin \alpha}{2} = \frac{\sin 45^\circ}{4.7989} \text{ or } \frac{\sin \beta}{6} = \frac{\sin 45^\circ}{4.7989}$	M1	Uses sine rule with correct sides oe Condone 135 instead of 45
	$\alpha = 17.14 \text{ or } \beta = 62.14$	A1	AWRT 17 or 62
	Bearing = 107° to the nearest degree	A1ft	ft 90 + their α or 45 + their β
	Total	5	

Q	Answer	Marks	Comments
12(a)	$3 \times 5 = 3v_A + 2v_B$	B1	Correct conservation of momentum equation
	$v_B - v_A = 0.4 \times 5$	B1	Correct coefficient of restitution equation
	$v_A = \frac{11}{5} = 2.2, v_B = \frac{21}{5} = 4.2$	M1	Attempt at solving simultaneous equations to find a value for v_A or v_B
	Speed for A = 2.2 m s ⁻¹	A1	Correct speed for A or B
	Speed for B = 4.2 m s ⁻¹	A1	Correct speeds for A and B including units
		5	
12(b)	$2 \times 4.2 = 2w_B + w_C$	B1ft	Correct conservation of momentum equation, ft their v_B
	$w_C - w_B = 4.2e$	B1ft	Correct coefficient of restitution equation, ft their v_B
	$w_B = \frac{8.4 - 4.2e}{3}$	M1	Solves simultaneous equations to find an expression for w_B
	$\frac{8.4 - 4.2e}{3} \geq 2.2$	M1	Forms an inequality using their $w_B \geq v_A$ Condone $w_B > v_A$
	$e \leq \frac{3}{7}$	A1	Correct simplified inequality. Accept any equivalent fraction.
		5	
	Total	10	