
INTERNATIONAL A-LEVEL MATHEMATICS

MA03

(9660/MA03) Unit P2 Pure Mathematics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$h(x) = \frac{25}{x+4} - 5$	M1	
	$h(x) = \frac{5-5x}{x+4}$	A1	
		2	

Q	Answer	Marks	Comments
1(b)(i)	$x = \frac{5-5y}{y+4}$	M1	'Swap' x and y Attempt to rearrange <i>their</i> (a) oe
	$xy + 4x = 5 - 5y$	M1	
	$[y = h^{-1}(x) =] \frac{5-4x}{x+5}$	A1	
		3	

Q	Answer	Marks	Comments
1(b)(ii)	[All values of $h^{-1}(x)$], $[h^{-1}(x)] \neq -4$	B1	
		1	

	Question 1 Total	6	
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Q	Answer	Marks	Comments
2(a)(i)	$4 - \lambda = -1 - \mu$ $-2 + 5\lambda = 5 - 4\mu$ $\lambda = 3,$ $\mu = -2$ $-3 + 2\lambda = 11 + \mu c$ $c = 4$	M1 A1 A1	
		3	

Q	Answer	Marks	Comments
2(a)(ii)	(1, 13, 3)	B1F	Must be coordinates – not a column vector
		1	

Q	Answer	Marks	Comments
2(b)	$\begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -4 \\ c \end{pmatrix} = 0$ $1 - 20 + 2c = 0$ $c = 9.5$	M1 A1 A1	oe
		3	

	Question 2 Total	7	
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Q	Answer	Marks	Comments
3(a)	$[3 \sin \theta - 3 \cos \theta =]$	M1	Seen or used
	$R \sin \theta \cos \alpha - R \cos \theta \sin \alpha$	A1	Both correct
	$R = \sqrt{18}$ $\alpha = 45^\circ$		
		2	

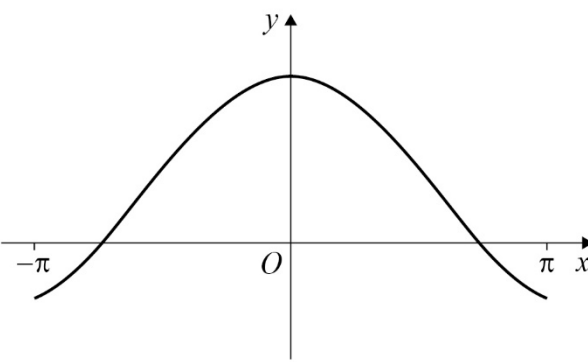
Q	Answer	Marks	Comments
3(b)(i)	$[y_{\max}^2 =] 18$	B1	
		1	

Q	Answer	Marks	Comments
3(b)(ii)	$[y_{\min}^2 =] 0$	B1	
		1	

Q	Answer	Marks	Comments
3(b)(iii)	$\sqrt{18} \sin(\theta - 45) = -\frac{3\sqrt{6}}{2}$	M1	PI
	$\sin(\theta - 45) = -\frac{\sqrt{3}}{2}$		
	$\theta - 45 = -60, -120$ $\theta = -15, -75$	A1+A1	
		3	

	Question 3 Total	7	
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Q	Answer	Marks	Comments
4(a)	Stretch + either I or II	M1	
	Parallel to y -axis I	A1	Including correct terminology
	SF 2 II	M1	
	Followed by Translation $\begin{bmatrix} 0 \\ k \end{bmatrix}$ $k = 1$	A1	Including correct terminology
4(a) ALT	Translation $\begin{bmatrix} 0 \\ k \end{bmatrix}$	(M1)	Including correct terminology
	$k = 0.5$	(A1)	
	Followed by Stretch in y -direction SF 2	(M1)	
		(A1)	Including correct terminology
		4	

Q	Answer	Marks	Comments
4(b)		B1	Correct shape, symmetric about y -axis (ignore graph outside the given range)
		B1	$y = 3$ indicated or stated
		2	

Q	Answer	Marks	Comments
4(c)	$[\text{Vol}] = \pi \int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} (1+2\cos x)^2 dx$ $(1+2\cos x)^2 = 1+4\cos x+4\cos^2 x$ $[V = \pi \int 1+4\cos x+2\cos 2x+2 dx]$ $= [\pi] (3x+4\sin x+\sin 2x)$ $= [\pi] \left\{ (2\pi+2\sqrt{3}-\frac{\sqrt{3}}{2}) - (-2\pi-2\sqrt{3}+\frac{\sqrt{3}}{2}) \right\}$ $[V =] \pi(4\pi+3\sqrt{3})$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>Including π, correct limits and dx</p> <p>PI</p> <p>Attempt at integration, must be in form $ax+b\sin x+c\sin 2x$</p> <p>Attempt at subst correct limits</p>
		6	

	Question 4 Total	12	
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Q	Answer	Marks	Comments
5(a)	$(1+x^2)^{0.5} = 1 + 0.5(x^2) + \frac{0.5 \times -0.5}{2}(x^2)^2$ $= 1 + 0.5x^2 - 0.125x^4$	M1	$1 + ax^2 + bx^4$
		A1	oe
		2	

Q	Answer	Marks	Comments
5(b)	$\int_0^{0.5} \sqrt{1+x^2} \, dx = \int 1 + 0.5x^2 - 0.125x^4 \, dx$ $= x + \frac{1}{6}x^3 - \frac{1}{40}x^5$ $\int_0^{0.5} = 0.5 + \frac{1}{6} \times 0.5^3 - \frac{1}{40} \times 0.5^5$ $= 0.52005$	M1	$x + cx^3 + dx^5$
		A1	
		m1	Attempt at subst correct limits
		A1	
		4	

Q	Answer	Marks	Comments												
5(c)	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>$\sqrt{1+0^2} = 1$</td></tr><tr><td>0.125</td><td>$\sqrt{1+0.125^2} = 1.007782$</td></tr><tr><td>0.25</td><td>$\sqrt{1+0.25^2} = 1.0307764$</td></tr><tr><td>0.375</td><td>$\sqrt{1+0.375^2} = 1.068000$</td></tr><tr><td>0.5</td><td>$\sqrt{1+0.5^2} = 1.118034$</td></tr></table>	x	y	0	$\sqrt{1+0^2} = 1$	0.125	$\sqrt{1+0.125^2} = 1.007782$	0.25	$\sqrt{1+0.25^2} = 1.0307764$	0.375	$\sqrt{1+0.375^2} = 1.068000$	0.5	$\sqrt{1+0.5^2} = 1.118034$	B1	All five correct x values (and no extra used) PI by five correct y values
	x	y													
	0	$\sqrt{1+0^2} = 1$													
	0.125	$\sqrt{1+0.125^2} = 1.007782$													
	0.25	$\sqrt{1+0.25^2} = 1.0307764$													
0.375	$\sqrt{1+0.375^2} = 1.068000$														
0.5	$\sqrt{1+0.5^2} = 1.118034$														
		M1	At least four correct y values in exact form or decimals, rounded or truncated to 3 dp or better (in table or formula)												
	$\frac{1}{3} \times 0.125 \times [1 + 1.118034 + 2(1.0307764) + 4(1.007782 + 1.068000)]$	m1	Correct sub into formula with $h = 0.125$ OE and at least four correct y values either listed, with + signs, or totalled												
	$= 0.52011$	A1	CAO , must see this value exactly and no error seen												
		4													
Question 5 Total		10													

Q	Answer	Marks	Comments
6(a)	$\tan \beta = \tan(45 - \alpha)$ $= \frac{\tan 45 - \tan \alpha}{1 + \tan 45 \tan \alpha}$ $= \frac{1 - \tan \alpha}{1 + \tan \alpha}$	M1 A1	PI
		2	

Q	Answer	Marks	Comments
6(b)	$(1 + \tan \alpha)(1 + \tan \beta)$ $= (1 + \tan \alpha)(1 + \frac{1 - \tan \alpha}{1 + \tan \alpha})$ $= (1 + \tan \alpha)(\frac{1 + \tan \alpha + 1 - \tan \alpha}{1 + \tan \alpha})$ $= (1 + \tan \alpha)(\frac{2}{1 + \tan \alpha})$ $= 2$	M1 A1	
		2	

Q	Answer	Marks	Comments
6(c)	$\tan \beta = \tan(45 - \alpha), \quad \alpha = \beta$ $x = \tan \beta$ $x = \frac{1 - x}{1 + x}$ $x^2 + 2x - 1 = 0$ $x = \frac{-2 \pm \sqrt{4 + 4}}{2}$ $x = -1 + \sqrt{2} \quad \text{only}$	M1 m1 A1	PI No errors seen
		3	

	Question 6 Total	7	
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Q	Answer	Marks	Comments
7(a)(i)	$f(x) = \sin(\ln(2x)) + 4x - 3$ $f(0.6) = -0.419\dots$ $f(0.7) = 0.130\dots$ Change of sign, $0.6 < x < 0.7$	M1 A1 (M1) (A1)	or reverse Must have both statement and interval in words or symbols or comparing 2 sides: at 0.6, $\sin(\ln(1.2)) = 0.18 < 3 - 2.4 = 0.6$; at 0.7, $\sin(\ln(1.4)) = 0.33 > 3 - 2.8 = 0.2$ Conclusion as before
		2	

Q	Answer	Marks	Comments
7(a)(ii)	$x_2 = 0.705$ $x_3 = 0.666$	B1 B1	
		2	

Q	Answer	Marks	Comments
7(b)	$\frac{dy}{dx} = -\frac{1}{x} \sin(\ln(3x))$ $\frac{dy}{dx} = 0, \quad \ln(3x) = 0$ $3x = 1, \quad x = \frac{1}{3}, \quad y = 1$ $\left(\frac{1}{3}, 1\right)$	M1 M1 A1	$\frac{k}{x} \sin(\ln(3x))$
		3	

Q	Answer	Marks	Comments
7(c)	$\frac{dy}{dx} = \frac{1}{x} A \cos(\ln(2x)) - \frac{1}{x} B \sin(\ln(3x))$ $\frac{d^2 y}{dx^2} = -\frac{1}{x^2} A \cos(\ln(2x)) - \frac{1}{x^2} A \sin(\ln(2x))$ $+ \frac{1}{x^2} B \sin(\ln(3x)) - \frac{1}{x^2} B \cos(\ln(3x))$ $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y =$ $-A \cos(\ln(2x)) - A \sin(\ln(2x)) + B \sin(\ln(3x)) - B \cos(\ln(3x))$ $+ A \cos(\ln(2x)) - B \sin(\ln(3x)) + A \sin(\ln(2x)) + B \cos(\ln(3x))$ $= 0$	M1 A1 M1 A1	$\frac{dy}{dx} = \frac{p}{x} \cos(\ln(2x)) + \frac{q}{x} \sin(\ln(3x))$ $\frac{d^2 y}{dx^2} = \frac{r}{x^2} \cos(\ln(2x)) + \frac{s}{x^2} \sin(\ln(2x))$ $+ \frac{t}{x^2} \cos(\ln(3x)) + \frac{u}{x^2} \sin(\ln(3x))$ Be convinced
		4	
	Question 7 Total	11	

Q	Answer	Marks	Comments
8(a)	$\frac{d}{dx}(\cot x) = \frac{\sin x \times -\sin x - \cos x \times \cos x}{\sin^2 x}$	M1	$\frac{d}{dx}(\cot x) = \frac{A \sin x \times \sin x + B \cos x \times \cos x}{\sin^2 x}$
	$= \frac{-1}{\sin^2 x}$	A1	Must see a middle line
	$= -\operatorname{cosec}^2 x$	2	

Q	Answer	Marks	Comments
8(b)(i)	$\left[\frac{dx}{dy}\right] = -\frac{3}{4} \operatorname{cosec}^2\left(2y - \frac{\pi}{2}\right) \times 2$	M1	$\left[\frac{dx}{dy}\right] = k \operatorname{cosec}^2\left(2y - \frac{\pi}{2}\right)$
	$= -\frac{3}{2} \operatorname{cosec}^2\left(2y - \frac{\pi}{2}\right)$	A1	All correct
		2	

Q	Answer	Marks	Comments
8(b)(ii)	At $\left(\frac{3}{4}, \frac{3\pi}{8}\right)$		
	$\frac{dx}{dy} = -\frac{3}{2} \operatorname{cosec}^2\left(2 \times \frac{3\pi}{8} - \frac{\pi}{2}\right) = -3$	M1	PI (must have scored M1 in (i))
	$\frac{dy}{dx} = -\frac{1}{3}$	M1	
	Gradient of normal = 3	A1	
		3	

	Question 8 Total	7	
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Q	Answer	Marks	Comments
9(a)	$3x^2 + 3y^2 \frac{dy}{dx} = 3x \frac{dy}{dx} + 3y$ $\frac{dy}{dx}(y^2 - x) = y - x^2$ $\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$	M1	Either LHS or RHS correct
		A1	AG must see a middle line
		2	

Q	Answer	Marks	Comments
9(b)(i)	$\frac{dy}{dx} = 0, \quad y = x^2$ $x^3 + x^6 = 3x^3$ $x^3(x^3 - 2) = 0$ $x = 2^{\frac{1}{3}}, \quad y = 2^{\frac{2}{3}} \quad \text{or} \quad (2^{\frac{1}{3}}, 2^{\frac{2}{3}})$	M1	Attempt to solve
		m1	
		A1	
		3	

Q	Answer	Marks	Comments
9(b)(ii)	$\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$ $\frac{d^2y}{dx^2} =$ $\frac{(y^2 - x)\left(\frac{dy}{dx} - 2x\right) - (y - x^2)\left(2y\frac{dy}{dx} - 1\right)}{(y^2 - x)^2}$ $\left[\begin{array}{l} \text{As } \frac{dy}{dx} = 0 \text{ at stationary points,} \\ \frac{d^2y}{dx^2} = \frac{-2xy^2 + x^2 + y}{(y^2 - x)^2} \end{array} \right]$ $\text{Num} = (2^{\frac{4}{3}} - 2^{\frac{1}{3}})(0 - 2^{\frac{4}{3}}) - (2^{\frac{2}{3}} - 2^{\frac{2}{3}})(0 - 1)$ $\frac{d^2y}{dx^2} [-2] < 0, \text{ MAX}$	M1 A1	
		m1	
		A1	
		4	

Q	Answer	Marks	Comments
11(a)	$u = x \quad dv = e^{-0.5x}$ $du = 1 \quad v = -2e^{-0.5x}$ $\int = -2xe^{-0.5x} + 2 \int e^{-0.5x} dx$ $= -2xe^{-0.5x} - 4e^{-0.5x}$ $\int_0^6 = (-12e^{-3} - 4e^{-3}) - (-4)$ $= 4 - 16e^{-3}$	M1 m1 A1 m1 A1	Correct form PI Correct subst into parts formula Subst limits into $axe^{-0.5x} + be^{-0.5x}$ ISW
		5	

Q	Answer	Marks	Comments
11(b)	$2u du = dx \quad \text{oe}$ $[\int \frac{\sqrt{x+1}}{x-3} dx] = \int \frac{u}{u^2-4} \times 2u du$ $= 2 \int 1 + \frac{4}{u^2-4} du$ $\frac{4}{u^2-4} = \frac{A}{u-2} + \frac{B}{u+2}$ $A=1, B=-1$ $\int = 2 \int 1 + \frac{1}{u-2} - \frac{1}{u+2}$ $= 2(u + \ln(\frac{u-2}{u+2}))$ $[x]_8^{15} = [u]_3^4$ $\int = 2[(4 + \ln \frac{1}{3}) - (3 + \ln \frac{1}{5})]$ $= 2(1 + \ln \frac{5}{3})$ $= 2 \ln(\frac{5e}{3})$	B1 M1 A1 m1 A1 B1 M1 A1 A1	All in terms of u Use of partial fractions Or changing back into x Correct subst into $au + b \ln(u-2) - b \ln(u+2) \quad \text{oe}$
		9	

	Question 11 Total	14	
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Q	Answer	Marks	Comments
12(a)	$f(x) = \frac{A}{1-x} + \frac{B}{2-x} + \frac{C}{5-2x}$ $4x^2 + 5 =$ $A(2-x)(5-2x) + B(1-x)(5-2x) + C(1-x)(2-x)$ $x = 1: \quad A = 3$ $x = 2: \quad B = -21$ $x = 2.5: \quad C = 40$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Correctly eliminating fractions</p> <p>Attempt at finding one constant</p> <p>At least one constant correct</p> <p>All three constants correct</p>
		4	

Q	Answer	Marks	Comments
12(b)	$(2-x)^{-1} = 2^{-1} \left(1 - \frac{x}{2} \right)^{-1}$ $(2-x)^{-1} = \frac{1}{2} + \frac{1}{4}x + \frac{1}{8}x^2$	<p>M1</p> <p>A1</p>	
		2	

Q	Answer	Marks	Comments
12(c)	$f(x):$ $(1-x)^{-1} = 1+x+x^2$ $[(2-x)^{-1} = 2^{-1}\left(1+\frac{1}{2}x+\frac{1}{4}x^2\right)]$ $(5-2x)^{-1} = 5^{-1}\left(1+\frac{2}{5}x+\frac{4}{25}x^2\right)$ oe	B1	
		B1	
	$f(x) =$ $3(1+x+x^2) - \frac{21}{2}\left(1+\frac{1}{2}x+\frac{1}{4}x^2\right) + \frac{40}{5}\left(1+\frac{2}{5}x+\frac{4}{25}x^2\right)$	M1	
	$f(x) = \frac{1}{2} + \frac{19}{20}x + \frac{331}{200}x^2$	A1	
		4	
	Question 12 Total	10	

Q	Answer	Marks	Comments
13(a)	$t = \frac{x}{c}, \quad y = c \div \frac{x}{c}$ $xy = c^2$	B1	oe
		1	

Q	Answer	Marks	Comments
13(b)	$\frac{dx}{dt} = c \quad \frac{dy}{dt} = -\frac{c}{t^2}$ $\frac{dy}{dx} = -\frac{1}{t^2}$ $y - \frac{c}{p} = -\frac{1}{p^2}(x - cp)$ $[p^2y - cp = -x + cp]$ $\text{At } A, x = 2cp \quad \text{At } B, y = \frac{2c}{p}$ $\text{Midpoint } AB \left(cp, \frac{c}{p} \right)$ Normal $y - \frac{c}{p} = p^2(x - cp)$ $\text{At } C, y = x, \quad y - \frac{c}{p} = p^2(y - cp)$ $y = \frac{c(1-p^4)}{p(1-p^2)} = \frac{c(1-p^2)(1+p^2)}{p(1-p^2)} = \frac{c(1+p^2)}{p}$ $\text{At } D, y = -x, \quad y - \frac{c}{p} = p^2(-y - cp)$ $y = \frac{c(1-p^4)}{p(1+p^2)} = \frac{c(1-p^2)(1+p^2)}{p(1+p^2)} = \frac{c(1-p^2)}{p}$ $\text{Midpoint } CD$ $x = 0.5 \left(\frac{c(1+p^2)}{p} + \frac{c(-1+p^2)}{p} \right) = cp$ $y = 0.5 \left(\frac{c(1+p^2)}{p} + -\frac{c(-1+p^2)}{p} \right) = \frac{c}{p}$ $\text{Both } AB \text{ and } CD \text{ have same midpoint}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>m1</p> <p>A1</p> <p>A1</p>	<p>Attempt to find equ of tgt</p> <p>Attempt to find equ of normal</p> <p>Attempt to find C or D</p> <p>Both correct</p>
		7	

Q	Answer	Marks	Comments
13(c)	$ AB = CD = \frac{2c}{p} \times \sqrt{1+p^4}$ As AB and CD are perp, equilateral and P is the midpoint then $ABCD$ form a square	B1 E2,1	Either AB or CD
		3	
	Question 13 Total	11	