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Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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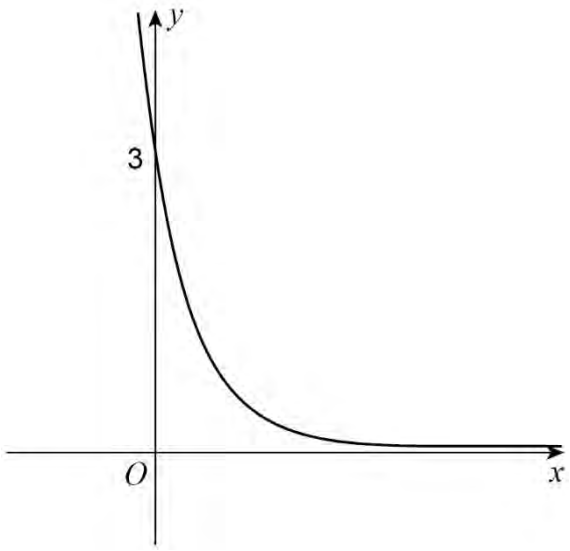
Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	See artwork below	B1 B1	Decreasing exponential curve of the correct form in the first and second quadrants asymptotic to the positive x-axis. Correct value of y-intercept indicated. Allow correct coordinates instead of value.
			
		2	

Q	Answer	Marks	Comments
1(b)	$\left[y = \frac{1}{9^{(2\log_9 a - 0.5)}} = \frac{1}{9^{2\log_9 a} \times 9^{-0.5}} \right]$ $\left[9^{2\log_9 a} = \right] a^2$ or $\left[9^{-2\log_9 a} = \right] a^{-2}$ $\frac{3}{a^2}$	M1 A1ft	**PI** Expressing $9^{2\log_9 a}$ or $9^{-2\log_9 a}$ as the correct power of a Correct y-coordinate of *P* in the correct form. **ft** their value for the y-intercept from **part (a)** for '3' provided it is positive. Allow $3a^{-2}$ for $\frac{3}{a^2}$
		2	

	Question 1 Total	**4**	

Q	Answer	Marks	Comments
2(a)	$[\text{Arc Length} =]\ 10\theta$ or $AB = 6$ [cm] $10\theta + 10 + 10 = 26$ or $10\theta + 20 = 26$ and $\theta = 0.6$	**M1** **A1**	Correct length of arc AB in terms of θ or identifies that arc AB is 6 [cm] Possibly embedded in later working. Sets expression for perimeter equal to 26. **PI** by $10\theta = 6$ and **AG** Must be convincingly shown
		2	

Q	Answer	Marks	Comments
2(b)	$\dfrac{\sin(\angle BOC)}{14}=\dfrac{\sin(0.7)}{10}$	**M1**	**PI** **oe** Use of sine rule with correct values substituted. Allow **AWRT** 40.1° for 0.7 radians.
	$\left[\angle BOC=\sin^{-1}\left(\dfrac{14\sin(0.7)}{10}\right)=\right]$ $1.12[415...]$ [radians]	**A1**	Correct $\angle BOC$ **AWRT** 1.12 Allow **AWRT** 64.4°
	$[\angle OBC=]\ \pi-0.7-1.12[415...]$	**m1**	**oe** **ft** their $\angle BOC$ provided **M1** scored. Allow angles in degrees. **PI** by final answer of $1.317[43...]$ rounded **or** truncated to at least 3dp or **AWRT** 75.5°
	$[\angle OBC=]$ 1.32 [radians]	**A1**	**AWRT** 1.32 Must be in radians.
2(b) **ALT**	[Length of $OC=x$] $10^2=14^2+x^2-2\times14\times x\times\cos(0.7)$ or $\cos(0.7)=\dfrac{14^2+x^2-10^2}{2\times14\times x}$	**M1**	**oe** Correct use of cosine rule with values substituted. Allow **AWRT** 40.1° for 0.7 radians.
	$[x^2-21.4[1558...]x+96=0\Rightarrow]$ $[x=]\ 15.0[2714...]$	**A1**	Correct length of *OC* **AWRT** 15 Condone 6.38844… seen as well.
	$\dfrac{\sin(\angle OBC)}{15.0[2714...]}=\dfrac{\sin(0.7)}{10}$ or $\cos(\angle OBC)=\dfrac{10^2+14^2-(15.0[2714...])^2}{2\times10\times14}$	**m1**	**oe** **ft** their length of *OC* provided **M1** scored. Correct use of sine rule or cosine rule with values substituted. Allow **AWRT** 40.1° for 0.7 radians. **PI** by correct final answer or anything that truncates to 75°
	$[\angle OBC=]$ 1.32 [radians]	**A1**	**AWRT** 1.32 Must be in radians. Accept **AWRT** 1.31
		4	

Q	Answer	Marks	Comments
2(c)	$\left[\frac{1}{2}\times 10\times 10\times 0.6 =\right]\quad 30\left[\text{cm}^2\right]$	B1	**PI** Correct area of sector *OAB*
	$\frac{1}{2}\times 10\times 14\times \sin\left(1.31[743...]\right)$	M1	**oe** Correct method for calculating the area of triangle *OBC* with values substituted. **ft** their $\angle OBC$
	$67.76[524...]\left[\text{cm}^2\right]$	A1	Correct area of triangle *OBC* **PI** by correct final answer. **AWFW** 67.55 to 67.85
	$97.8\left[\text{cm}^2\right]$	A1	**CAO** **AWFW** 97.55 to 97.85
		4	

	Question 2 Total	10	

Q	Answer	Marks	Comments
3(a)	$\left(\frac{8+11}{2}, \frac{5+(-10)}{2}\right)$ or $\left(\frac{19}{2}, -\frac{5}{2}\right)$	M1	**oe** Correct coordinates of, or correct method for finding, the midpoint of *PQ* Possibly embedded in later working.
	$\frac{5-(-10)}{8-11}$ or $\frac{15}{-3}$	M1	**oe** Correct method to find the gradient of *PQ*
	$\left(y-\left(-\frac{5}{2}\right)\right)=\frac{1}{5}\left(x-\frac{19}{2}\right)$ **oe** or $2x-10y=44$ **oe** and $x-5y=22$	A1	Dependent on **M1 M1** May see $y=\frac{1}{5}x+p$ and substitution of coordinates of the mid-point of *PQ* to find p but must be a complete method. **AG** Correct equation given but not in the required form or unsimplified before required result stated. Must be convincingly shown
		3	

Q	Answer	Marks	Comments
3(b)(i)	$x-5y=22$ and $y=8x-59$ $(7,-3)$	 M1 A1	 **M1**: Correct x-coordinate or y-coordinate. **A1**: Correct coordinates.
		2	

Q	Answer	Marks	Comments
3(b)(ii)	$(8-7)^2+(5-(-3))^2$ or $\sqrt{(8-7)^2+(5-(-3))^2}$ or $(11-7)^2+((-10)-(-3))^2$ or $\sqrt{(11-7)^2+((-10)-(-3))^2}$	M1	Method to find the radius or the square of the radius of *C* using either the coordinates of *P* or *Q* **ft** their centre of *C*
	$[r=]\ \sqrt{65}$ or $[r^2=]\ 65$	A1ft	**ft** their centre of *C*
	$(x-7)^2+(y+3)^2=65$	A1ft	Correct equation in the correct form. **ft** their centre and r^2 provided all values are integers and **M1** scored.
		3	

Q	Answer	Marks	Comments
3(c)	$\left[d^2=(7-2)^2+(-3-(-9))^2\right]$ $\left[d^2=\right]\ 61$ or $[d=]\ \sqrt{61}$	B1ft	Correct distance or square of distance from centre of *C* to *R* **ft** their **(b)(i)** or **(b)(ii)** Allow $7.8[1024...]$ for $\sqrt{61}$
	Since $\sqrt{61}<\sqrt{65}$ (or $61<65$) then *R* lies inside the circle.	E1ft	Compares their $\sqrt{61}$ with their $\sqrt{65}$ **oe** and gives a correct conclusion. **ft** their distance or square of distance from centre of *C* to *R*, and their *r* or r^2 provide both coordinates of the centre of *C* are integers. Allow $7.8[1024...]$ for $\sqrt{61}$ and $8[.0622...]$ for $\sqrt{65}$
		2	

	Question 3 Total	10	

Q	Answer	Marks	Comments
4(a)(i)	$25^p = Y^2$ or $\left(5^p\right)^2 = Y^2$ or $5^{2p} = Y^2$ or $5^{p+2} = 25Y$ or $5^{p+2} = \left(5^2\right)Y$	**M1**	Correctly expressing 25^p or 5^{p+2} in terms of Y Possibly seen embedded in a quadratic equation.
	$Y^2 - 25Y = 54$ or $Y^2 - 25Y - 54 = 0$ and $(Y+2)(Y-27) = 0$	**A1**	Substitutes both correct expressions for Y into the quadratic equation before **AG** Must be convincingly shown
		2	

Q	Answer	Marks	Comments
4(a)(ii)	$[Y =] -2$ $[$or $5^p = -2]$ or $[Y =]$ 27 $[$or $5^p = 27]$	**M1**	**PI** States a correct possible value of Y
	$Y = -2$ $[$or $5^p = -2]$ is not possible since $Y > 0$ and $-2 < 0$	**E1**	Rejects $Y = -2$ as a possible solution and gives a valid reason. Accept '$\log(-2)$ does not exist.', for example. Must see $[Y =] -2$
	$[p =]$ $\log_5 27$	**A1**	**CAO**, **ISW** Must have correct base.
		3	

Q	Answer	Marks	Comments
4(b)	$\log_6\left(\frac{x^3}{y^3}\right)-2=\log_6\left(6x^3y^2\right)$ or $3\log_6 x-3\log_6 y-2=\log_6\left(6x^3y^2\right)$ or $3\log_6\left(\frac{x}{y}\right)-2=\log_6 6x^3+\log_6 y^2$	**M1**	**oe** Applies one logarithm rule correctly.
	$[2=]\ \log_6 36$ or $\log_6 6^2$ or $2\log_6 6$ or $\log_6 6=1$	**B1**	**PI** in later working May be seen at any point during the working
	$\log_6\left(\frac{x^3}{36y^3}\right)=\log_6\left(6x^3y^2\right)$ or $-2=\log_6\left(\frac{6x^3y^5}{x^3}\right)$	**M1**	**oe** Forms a correct equation with a single logarithm on one or both sides. **PI** by $\frac{x^3}{36y^3}=6x^3y^2$
	$[y=]\ 6^{-\frac{3}{5}}$	**A1**	**ACF**, **ISW** Accept $[y=]\ \frac{1}{\sqrt[5]{216}}$ or $[y=]\ \frac{1}{216^{\frac{1}{5}}}$ or $[y=]\ 216^{-\frac{1}{5}}$ or $[y=]\ \sqrt[5]{6^{-3}}$ or $[y=]\ \left(6^{-3}\right)^{\frac{1}{5}}$
		4	

	Question 4 Total	**9**	

Q	Answer	Marks	Comments
5(a)	$\frac{\sin\theta}{1+\cos\theta}+\frac{1}{\frac{\sin\theta}{\cos\theta}}$ or $\frac{\sin\theta}{1+\cos\theta}+\frac{\cos\theta}{\sin\theta}$	M1	Use of $\tan\theta=\frac{\sin\theta}{\cos\theta}$
	$\frac{\sin^2\theta+(1+\cos\theta)\cos\theta}{(1+\cos\theta)\sin\theta}$ or $\frac{\sin^2\theta+\cos^2\theta+\cos\theta}{(1+\cos\theta)\sin\theta}$ or $\frac{\sin\theta(1-\cos\theta)}{\sin^2\theta}+\frac{\sin\theta\cos\theta}{\sin^2\theta}$	M1	**oe** Rearrangement to give a correct expression in terms of $\sin\theta$ and $\cos\theta$ with a common denominator. Allow $1-\cos^2\theta$ for $\sin^2\theta$
	$\frac{1+\cos\theta}{(1+\cos\theta)\sin\theta}$ or $\frac{\sin\theta}{\sin^2\theta}$ or $\frac{1-\cos\theta}{\sin\theta}+\frac{\cos\theta}{\sin\theta}$ and $\frac{1}{\sin\theta}$	A1	Uses $\sin^2\theta+\cos^2\theta=1$ **AG** Must be convincingly shown.
		3	

Q	Answer	Marks	Comments
5(b)	$\frac{2}{\sin 2x}=4\sin 2x$	B1	**oe**, condone θ for $2x$ throughout.
	$2x=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$	M1	**PI** by 45° or 135° or one correct final answer. Ignore $2x=\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$
	$[x=]$ 22.5°, 67.5°	A2,1	**A1**: At least one correct answer. **A2**: Both correct answers with no others seen.
		4	

	Question 5 Total	**7**	

Q	Answer	Marks	Comments
6(a)	[Variance $= 40 \times 0.4(1 - 0.4)] = 9.6$	**B1**	**oe**
		1	

Q	Answer	Marks	Comments
6(b)	$P(L = 19) = \binom{40}{19} \times 0.4^{19} \times (1-0.4)^{40-19}$ or $0.8702 - 0.7911$ $= 0.079$	**M1** **A1**	**oe**, **PI AWRT** 0.079 Uses correct formula for $P(L = 19)$ or uses $P(L \leq 19) - P(L \leq 18)$ **CAO**
		2	

Q	Answer	Marks	Comments
6(c)	$[P(L > 13) = 1 - P(L \leq 13)]$ $= 1 - 0.2112$ $= 0.789$	**M1** **A1**	**PI** Uses formula. **AWRT** 0.789
		2	

	Question 6 Total	**5**	

Q	Answer	Marks	Comments
7(a)(i)	$[a =]$ 1	B1	
	$[b =]$ 4	B1	
		2	

Q	Answer	Marks	Comments
7(a)(ii)	$0.4\times1+0.3\times4+0.3c=3.4$	M1	Applies expectation formula for their a and b and sets equal to 3.4
	$[c =]$ 6	A1	
		2	

Q	Answer	Marks	Comments
7(b)	$\left[\mathrm{E}\left(X^2\right)=\right]$ $0.4\times1^2+0.3\times4^2+0.3\times6^2 \quad [=16]$	M1	Applies formula for $\mathrm{E}\left(X^2\right)$ for their values of a, b and c **PI** by correct variance for their values of a, b and c
	$\left[\mathrm{Var}\left(X\right)=16-3.4^2=\right]$ 4.44 or $\frac{111}{25}$	A1ft	Correctly finds variance for their values of a, b and c Must use $\mathrm{E}\left(X\right)=3.4$
	$\mathrm{Var}\left(X+Y\right)=17.44$ or $\frac{436}{25}$	A1ft	**ft** their $\mathrm{Var}\left(X\right)+13$ Dependent on at least **M1** awarded
		3	

	Question 7 Total	**7**	

Q	Answer	Marks	Comments
8(a)	0.24×0.74 $= 0.1776$ or $\frac{111}{625}$	M1 A1	
		2	

Q	Answer	Marks	Comments
8(b)	$0.24 + 0.61 - 0.1776$ $= 0.6724$ or $\frac{1681}{2500}$	M1 A1ft	Applies the Addition rule with their **part (a)** [0.1776] **ft** their **part (a)** [0.1776] provided final answer is between 0 and 1
		2	

Q	Answer	Marks	Comments
8(c)	$0.24 - 0.1776$ or $0.6724 - 0.61$ or 0.24×0.26 $= 0.0624$ or $\frac{39}{625}$	M1 A1	**ft** 0.24 – their **part (a)** or their **part (b)** – 0.61 **CAO**
		2	

Q	Answer	Marks	Comments
8(d)	$\frac{0.0624}{1-0.61}$ $= 0.16$ or $\frac{4}{25}$	M1 A1	**oe**, **ft** their **part (c)** **CAO**
		2	

	Question 8 Total	8	

Q	Answer	Marks	Comments
9(a)	$2-2\times0.6-k=0$ or $[k=]\ 2-2\times0.6$ $[k=]\ 0.8$	M1 A1	**oe** **PI** by correct answer
		2	

Q	Answer	Marks	Comments
9(b)	0.6 N	B1	Condone omission of units
		1	

	Question 9 Total	3	

Q	Answer	Marks	Comments
10(a)	$\left[v = \int (0.72 - 0.18t)\,\mathrm{d}t\right]$		
	$[v =]\ 0.72t - 0.09t^2$	**B1**	**oe** Correct expression for v Allow '$+c$'
	$0 = 0.72 - 0.18t$	**M1**	Uses $a = 0$ **oe**, **PI**
	$[t =]\ 4$ [seconds]	**A1**	
	$[v =]\ 0.72t - 0.09t^2$		
	$[v_{\max} =]\ 0.72 \times 4 - 0.09 \times 4^2$	**m1**	**oe** Substitutes their 4 into their integrated expression for v
	$[v_{\max} =]\ 1.44\ \mathrm{m\,s^{-1}}$	**A1**	**CAO** Condone omission of units
		5	

Q	Answer	Marks	Comments
10(b)(i)	$\left[v^2 = u^2 + 2as\right]$		
	$v^2 = 0^2 + 2\times 9.8(7.68 - 2)$	M1	**oe** If more than one constant equation formula used it must be a complete method. **PI** by **AWRT** 111.3 Condone one sign error
	$\left[v =\right]$ 10.55	A1	**AWRT** 11 Exact answer is $\frac{14\sqrt{355}}{25}$
	m s^{-1}	B1	Correct units.
		3	

Q	Answer	Marks	Comments
10(b)(ii)	$\left[s = \frac{1}{2}(u+v)t\right]$		
	$\left[s =\right] \frac{1}{2}\times(0+10.55)\times 0.3$	M1	**oe PI ft** their answer to **part (b)(i)** If more than one constant equation formula used it must be a complete method.
	$\left[s =\right]$ 1.58 [metres]	A1	**AWRT** 1.58, allow 1.59 **PI** by correct final answer
	$\left[\text{Height} = 2 - 1.58 =\right]$ 0.42 metres	B1ft	**AWRT** 0.42, allow 0.41 **ft** $0 <$ their $1.58 < 2$ Condone omission of units
		3	

	Question 10 Total	11	

Q	Answer	Marks	Comments
11(a)	Conservation of Momentum $4\times4.8=4v+3m$ $[4v=19.2-3m]$ $v=4.8-0.75m$	 M1 A1	 **oe** Correct unsimplified equation. Allow sign error. **AG** Must be convincingly shown
		2	

Q	Answer	Marks	Comments
11(b)	$0<4.8-0.75m \quad [\Rightarrow m<6.4]$	M1	**oe** Considers inequality or equality for $v=0$ **PI** by 6.4 Condone equality or weak inequality
	$3\geq 4.8-0.75m \quad [\Rightarrow m\geq 2.4]$	M1	**oe** Considers inequality or equality for $v=3$ **PI** by 2.4 Condone equality or strict inequality
	$2.4\leq m$ or $m<6.4$	A1	**oe** At least one inequality correct For one of $2.4\leq m$ or $m<6.4$ Accept $2.4<m$ for $2.4\leq m$ but not $m\leq 6.4$ for $m<6.4$
	$2.4\leq m<6.4$	A1	**oe** Both inequalities correct Accept $2.4<m<6.4$ but not $2.4\leq m\leq 6.4$
		4	

	Question 11 Total	**6**	