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MATHEMATICS

MA05

(9660/MA05) Unit M2 Mechanics

Mark scheme

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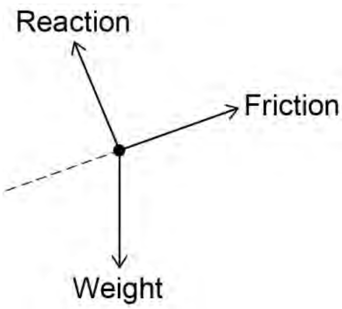
Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$\mathbf{v} = \begin{bmatrix} 6\cos\left(3 \times \frac{2\pi}{3}\right) \\ 2 - 8\sin\left(4 \times \frac{2\pi}{3}\right) \\ 2 \times \frac{2\pi}{3} + 4\mathrm{e}^{-2\times\frac{2\pi}{3}} \end{bmatrix}$	**M1**	PI, OE, condone poor use of brackets
	$\lvert\mathbf{v}\rvert = \sqrt{\left(6\cos(2\pi)\right)^2 + \left(2 - 8\sin\left(\frac{8\pi}{3}\right)\right)^2 + \left(\frac{4\pi}{3} + 4\mathrm{e}^{-\frac{4\pi}{3}}\right)^2}$	**M1**	PI, OE, explicit calculation of the magnitude of their vector (all three components non-zero)
	$\lvert\mathbf{v}\rvert = 8.9\ \mathrm{m\ s^{-1}}$	**A1**	$8.85\ \mathrm{m\ s^{-1}}$ to 3 sf. Condone lack of units
1(b)	$\mathbf{r} = \begin{bmatrix} 2\sin(3t) \\ 2t + 2\cos(4t) \\ t^2 - 2\mathrm{e}^{-2t} \end{bmatrix} + \mathbf{c}$	**M1** **A1**	**M1**: At least one correct component **A1**: All 3 components correct (Condone lack of $+\mathbf{c}$)
	When t = 0, $\mathbf{r} = \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix}$	**m1**	Use of $+\mathbf{c}$ and initial condition ($\mathbf{c}$ not the zero vector)
	$\mathbf{c} = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$	**A1**	OE, Correct vector
	$\mathbf{r} = \begin{bmatrix} 2\sin(3t) + 4 \\ 2t + 2\cos(4t) + 3 \\ t^2 - 2\mathrm{e}^{-2t} + 5 \end{bmatrix}$	**A1**	OE stated as final answer, CAO
1(c)	$\mathbf{a} = \begin{bmatrix} -18\sin(3t) \\ -32\cos(4t) \\ 2 - 8\mathrm{e}^{-2t} \end{bmatrix}$	**M1** **A1**	**M1**: At least one correct component **A1**: All 3 components correct
	$\mathbf{a} = \begin{bmatrix} 0 \\ -32 \\ -6 \end{bmatrix}$ or $\mathbf{F} = 7.5\begin{bmatrix} -18\sin(3t) \\ -32\cos(4t) \\ 2 - 8\mathrm{e}^{-2t} \end{bmatrix}$	**A1F**	Their acceleration vector evaluated at t = 0 or their resultant force vector
	$\mathbf{F} = \begin{bmatrix} 0 \\ -240 \\ -45 \end{bmatrix}$	**A1**	CAO, OE stated as sole final answer Final answer of 244 N is **A0**
	Total	**12**	

Q	Answer	Marks	Comments
2(a)	Vertically component of velocity is initially zero, so $h = \frac{1}{2}gt^2$	M1	Explicit use of $s = ut + \frac{1}{2}at^2$ with $u = 0$
	[Time to fall to the ground] $t = \sqrt{\frac{2h}{g}}$	A1	
	$x = ut$ $x = u \times \sqrt{\frac{2h}{g}}$	A1	Use of $s = ut + \frac{1}{2}at^2$ with $a = 0$ or explanation that there is no horizontal acceleration AG
2(b)	[Vertical component of velocity immediately before hitting the ground] $V^2 = 2gh$	M1	Use of $v^2 = u^2 + 2as$ with $u = 0$ Use of $v^2 = u^2 + 2as$ with $u = v$ scores 0/3 PI
	Speed^2 of particle immediately before hitting the ground $u^2 + V^2 = u^2 + 2gh$	m1	PI
	Kinetic energy of particle immediately before hitting the ground $E_K = \frac{1}{2}mu^2 + mgh$	A1	OE, stated as final answer
2(c)	The particle will do work against air resistance, which will decrease the final kinetic energy	E1	An explanation must be given, not simply ‘air resistance’
	Total	**7**	

Q	Answer	Marks	Comments
3(a)		B1	Must have clear labels of the three forces near the arrows Do not accept symbols (N, W, mg etc) as labels unless the labels are defined elsewhere Do not accept ‘gravity’ in place of ‘weight’
3(b)	Component of weight down the slope $= 12 \times 9.8 \sin(35°)$ $= 67.4[526]$ N	B1	AWRT 67 N (PI)
	Normal reaction force on block $= 12 \times 9.8 \cos(35°)$ $= 96.3[323]$ N	M1	AWRT 96 N (PI)
	Friction on block $= 0.65 \times 96.3323\ldots$ $= 62.6[160]$ N	A1	
	Resultant force $= 67.4526 - 62.6160$ $= 4.8[4]$ N down the slope	A1F	FT their friction AWRT 4.8 N Condone lack of direction
3(c)	[Resultant force parallel to the slope = 0] $12 \times 9.8 \sin(\theta) = 0.65 \times 12 \times 9.8 \cos(\theta)$	M1	PI, either side correct
	$\tan(\theta) = 0.65$	m1	Forms an equation in $\tan(\theta)$
	$\theta = \tan^{-1}(0.65)$ $\theta = 33.0°$	A1	
	Total	**8**	

Q	Answer	Marks	Comments
4(a)	The logo is symmetric in the line *AB*	**E1**	Any mention of symmetry or that both centres of mass of the two circles lie on *AB*
4(b)	C.O.M. of complete larger circle is R from *B*	**B1**	PI
	[Use of the C.O.M. of circular hole is] $2R - r$ [from *B*]	**B1**	PI
	$\rho\pi(R^2 - r^2)\bar{Y} = \rho\pi R^2 \times R$ $-\rho\pi r^2 \times (2R - r)$	**M1** **m1**	**M1**: At least one correct term **m1**: At least two correct terms (unsimplified or better) Condone lack of ρ
	$[\bar{Y} =]\dfrac{R^3 - 2Rr^2 + r^3}{R^2 - r^2}$	**A1**	ISW, ACF, e.g. $[\bar{Y} =]\dfrac{R^2 + Rr - r^2}{R + r}$ or $R - \dfrac{r^2}{R + r}$
	Total	**6**	

Q	Answer	Marks	Comments
5(a)	[Anticlockwise moment about *B* (or *A*)] $0.4 \times 40 \times 9.8 = 156.8$ N m	**B1**	OE Condone use of 9.81 for g leading to 156.96 N m
	[Clockwise moment about *B* (or *A*) due to child's weight] $0.85 \times 25 \times 9.8 = 208.25$ N m	**B1**	OE Condone use of 9.81 for g leading to 208.4625 N m
	As 208.25 > 156.8, the diving board tips	**E1F**	Dependent on comparison of their two moment calculations
5(b)	[Clockwise moment about *B* due to child's weight] $0.85 \times m \times 9.8$ or $8.33m$	**M1**	
	$156.8 = 8.33m$ $m = 18.8$	**A1**	Stated as final answer Condone inclusion of units
5(c)	Make the distance between *B* and the end of the diving board shorter	**E1**	Any plausible statement
	This will decrease the clockwise moment a weight would have at the right-hand end of the diving board	**E1**	Corresponding plausible reason, dependent on previous E1 having been awarded
	Total	**7**	

Q	Answer	Marks	Comments
6(a)	$\dfrac{180\,000}{36} - c \times \sqrt{36} = 250 \times 5.0$	**M1** **m1** **A1**	Use of driving force = P/v Finding the correct resultant force (LHS) in terms of c Setting the correct resultant force equal to ma
	$c = 625$	**A1**	
6(b)	$F - 625 \times \sqrt{25} - 250 \times 9.8 \times \sin(6.0°) = 0$ $[F = 3381 \text{ N}]$	**M1 A1**	**M1**: Use of resultant force = 0. Condone lack of component of weight for **M1** **A1**: Correct equation
	$P = 3381 \times 25 = 84527$ W	**M1**	Their driving force multiplied by 25
	$P = 85$ kW	**A1**	CAO in kW and to 2 sf
	Total	**8**	

Q	Answer	Marks	Comments
7(a)(i)	$L\cos(40°) = mg$ or W $L = \dfrac{12000 \times 9.8}{\cos(40°)}$ $L = 1.535 \times 10^5$ $L = 1.54 \times 10^5$	**M1** **A1**	Use of vertical equilibrium AG Value of L shown to more than 3 sf or correct unsimplified equation for L (with L as subject)
7(a)(ii)	The lift force is perpendicular to the velocity of the aeroplane	**E1**	
7(b)	$L\sin(40°) = mr\omega^2$ $\omega^2 = \dfrac{1.54 \times 10^5 \times \sin(40°)}{12000 \times 2.7 \times 10^3}$ $\omega = 0.0553$ [rad s^{-1}]	**M1** **m1** **A1**	PI, use of resultant force formula for circular motion 0.003055… or 0.0030456… if exact value from (a)(i) used 0.0552 [rad s^{-1}] if exact value from (a)(i) used
7(c)(i)	[**i** component directly proportional to] $\cos(0.0553\,t)$ [**j** component directly proportional to] $\sin(0.0553\,t)$ [$\boldsymbol{r}$ =]$2700\cos(0.0553\,t)$ **i** $+ 2700\sin(0.0553\,t)$ **j**	**B1F** **B1F** **B1**	FT their ω from (b) FT their ω from (b) CAO, accept 2.7 [km] as coefficients
7(c)(ii)	[$\boldsymbol{v}$ =]$- 149\sin(0.0553\,t)$ **i** $+ 149\cos(0.0553\,t)$ **j** [$\boldsymbol{a}$ =]$- 8.26\cos(0.0553\,t)$ **i** $- 8.26\sin(0.0553\,t)$ **j**	**M1** **A1F**	PI, allow unsimplified FT on (c)(i) AWFW [–8.2, –8.3] for coefficients Accept AWFW [–0.0082, –0.0083] for coefficients [in km s^{-2}]
	Total	**11**	

Q	Answer	Marks	Comments
8(a)	[Change in GPE of the cart and driver between the start and finish lines] = 120 × 9.8 × 150 = 176 400 J	**B1**	PI
	[Total energy of the cart and driver at the start line] = 176 400 + ½ × 120 × 8.0^2 = 180 240 J	**M1** **A1**	**M1**: Initial KE (3 840 J) **A1**: Total Energy (180 240 J)
	[By the conservation of energy] ½ × 120 × v^2 = 180 240 v = 55 m s^{-1}	**A1**	AWRT 55 m s^{-1}, accept $2\sqrt{751}$ Condone missing units **SC1** for answer of 54.2 m s^{-1} [initial KE not included]
8(b)(i)	[Kinetic energy of cart and driver at the finish line] = ½ × 120 × 30^2 = 54 000 J	**B1**	PI
	[Total work done by the cart and driver against the resistive forces] = 180 240 – 54 000 = 126 240 J	**M1**	
	[Average resistive force] $F = \dfrac{126\,240}{1600}$ F = 79 N	**M1** **A1**	**M1**: Use of $W = Fd\ [\cos\theta]$ with $d = 1600$ or 150 **A1**: Correct answer, AWRT 79 N Condone missing units
8(b)(ii)	Air resistance on the cart and driver Friction in the wheel bearings	**E1** **E1**	Any plausible resistive force A second plausible resistive force
	Total	**10**	

Q	Answer	Marks	Comments
9(a)	[Time taken for the basketball to cover the 4.0 m]		
	$T = \dfrac{4.0}{9.5\cos(\theta)}$	B1	Any subject
	$[s =]9.5\sin(\theta)T - 0.5 \times 9.8 \times T^2$	M1 A1	**M1**: Use of $s = ut + \frac{1}{2}at^2$ with $u = 9.5\sin(\theta)$ and $a = \pm 9.8$ (PI) **A1**: Correct
	$[s =]9.5\sin(\theta) \times \dfrac{4.0}{9.5\cos(\theta)} - 0.5 \times 9.8 \times \left(\frac{4.0}{9.5\cos(\theta)}\right)^2$	m1	Eliminating T (PI)
	$[s =]4.0\tan(\theta) - \dfrac{1568}{1805\cos^2(\theta)}$	A1	Correct and second term in $\cos^2\theta$
	Use of $\sec^2(\theta) \equiv 1 + \tan^2(\theta)$	M1	
	$1.3 = 4.0\tan(\theta) - \dfrac{1568}{1805}\tan^2(\theta) - \dfrac{1568}{1805}$ $\left[\dfrac{1568}{1805}\tan^2(\theta) - 4.0\tan(\theta) + \dfrac{7829}{3610} = 0\right]$	M1	PI. Use of s = 1.3 at the hoop. Can be seen at any point in the solution for the mark
	$\tan(\theta) = 0.627758923, 3.976832914$	m1	PI, solving correct quadratic equation to find two values of $\tan(\theta)$
	θ = [32.1°], 75.9°	A1	Condone decimal places
	θ = 76°	E1	Rejects lower angle as basketball passes through hoop from above
9(b)	Basketball has zero volume/is assumed to be a particle	E1	Any plausible assumption
	Total	**11**	