OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

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I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM05) Unit FM2 Mechanics

Wednesday 18 January 2023 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to two significant figures, unless stated otherwise.
- Unless stated otherwise, the acceleration due to gravity, g, should be taken as 9.8 m s^{-2}

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

Answer **all** questions in the spaces provided.

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1 A particle moves on a straight horizontal line.

At time t seconds the velocity of the particle is v m s^{-1}

The particle moves so that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{v}{2}$$

When $t = 0$ it is given that $v = 10$

Find v in terms of t

[4 marks]

$v =$ _____

4

Do not write outside the box

2 A ball of mass 0.3 kg moves with velocity $(3\mathbf{i} + 9\mathbf{j})$ m s^{-1}

The ball is then hit by a bat and the ball's velocity becomes $(-2\mathbf{i} - 5\mathbf{j})$ m s^{-1}

Calculate the magnitude of the impulse exerted on the ball by the bat, giving your answer to three significant figures.

[4 marks]

Answer _____

4

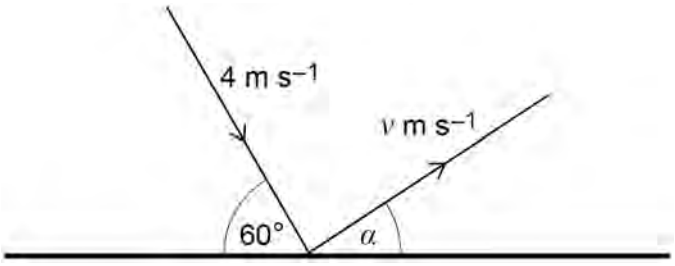
Do not write outside the box

3 A disc is sliding on a smooth horizontal surface when it hits a smooth fixed vertical wall.

When the disc hits the wall, it has a speed of 4 m s^{-1} and is moving at an angle of 60° to the wall.

When the disc leaves the wall, it has a speed of v m s^{-1} and is moving at an angle of α degrees to the wall.

The motion of the disc is shown in the diagram below.



The coefficient of restitution between the disc and the wall is e

3 (a) Find v in terms of e

[4 marks]

$v =$ _____

Do not write outside the box

3 (b) Determine the range of possible values of v

[2 marks]

Answer _____

6

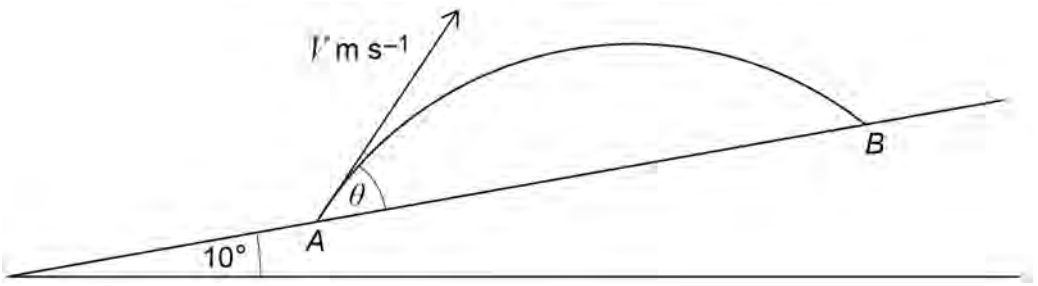
Turn over for the next question

4 A plane is inclined at an angle of 10° to the horizontal.

A ball is projected from a point A on the plane and hits the plane for the first time at a point B

The line AB is a line of greatest slope of the plane.

The initial velocity of the ball is V m s^{-1} at an angle θ above the plane, as shown in the diagram below.



The time taken for the ball to move from A to B is 1.5 seconds.

The distance between the points A and B is 15 metres.

4 (a) Find the value of θ giving your answer to the nearest degree.

[5 marks]

Do not write outside the box

Answer _____

4 (b) Calculate the value of V giving your answer to **two** significant figures.

[2 marks]

$V =$ _____

7

Do not write outside the box

5 A body moves with simple harmonic motion on a straight line between two points A and B

The period of the motion is 5π seconds.

The point C is 0.5 metres from A, between A and B

At C the body has velocity 0.8 m s^{-1} towards A

5 (a) Show that $AB = 8.5$ metres.

[6 marks]

Do not write outside the box

5 (b) Find the time that it takes for the body to move directly from C to A, giving your answer to three significant figures.

[4 marks]

Answer _____

10

Turn over for the next question

Do not write outside the box

6 An elastic string has natural length 0.8 metres and modulus of elasticity 2.8 newtons.

One end of the string is attached to a fixed point O and the other end is attached to a small sphere of mass 0.2 kg

The sphere is released from rest at O and falls vertically.

6 (a) Find the maximum length of the string in the subsequent motion.

[4 marks]

Answer _____

6 (b) Find the maximum tension in the string.

[3 marks]

Do not write outside the box

Answer _____

6 (c) Find the maximum speed of the sphere.

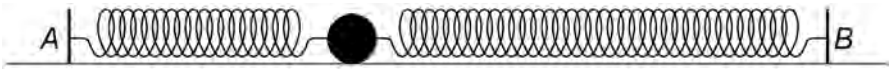
[5 marks]

Answer _____

12

7 A particle of mass m kg is on a smooth horizontal surface.

Two springs are attached to the particle, with the other ends of the springs attached to fixed points A and B, as shown in the diagram.



The spring attached to A has stiffness $4k$ N m^{-1} and natural length $2d$ metres.

The spring attached to B has stiffness $3k$ N m^{-1} and natural length $3d$ metres.

The distance between A and B is $7d$ metres.

7 (a) Find in terms of d the distance of the particle from A when it is in equilibrium.

[3 marks]

Answer _____

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7 (b) The particle is moved a distance $\frac{d}{2}$ metres towards B from its equilibrium position and released from rest.

In the subsequent motion the displacement of the particle to the right of its equilibrium position is x metres at time t seconds.

7 (b) (i) Find in terms of d, k and x the tension in the spring attached to B

[2 marks]

Answer _____

7 (b) (ii) Show that the motion of the particle is simple harmonic.

[5 marks]

Question 7 continues on the next page

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7 (b) (iii) Find in terms of k, m and π the period of the motion.

[2 marks]

Answer _____

7 (b) (iv) Determine the maximum speed of the particle in terms of d, k and m

[2 marks]

Answer _____

14

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Turn over for the next question

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8 A block of mass 1.5 kg is placed on a rough horizontal surface.

The coefficient of friction between the block and the surface is 0.3

A light inextensible string is attached to the block and passes over a smooth peg.

The smooth peg is positioned on the edge of the horizontal surface.

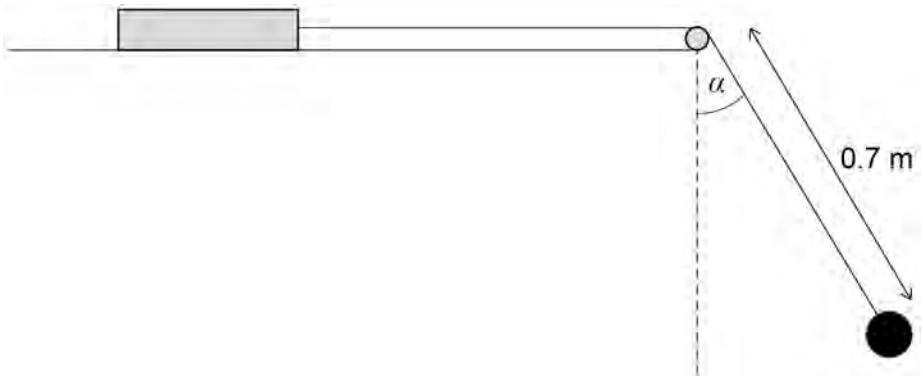
The string between the block and the peg is horizontal.

The other end of the string is attached to a particle of mass 0.4 kg

The particle is released from rest with the string at an angle α degrees to the downward vertical.

The initial path of the particle is part of a vertical circle of radius 0.7 metres.

The diagram below shows the initial situation.



8 (a) In a first case $\alpha = 30$

Find the angle between the string and the vertical when the block is on the point of sliding for the first time, giving your answer to the nearest degree.

[7 marks]

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Answer _____

8 (b) In a second case the block does not move.

Find the maximum possible value of α giving your answer to the nearest degree.

[5 marks]

Answer _____

12

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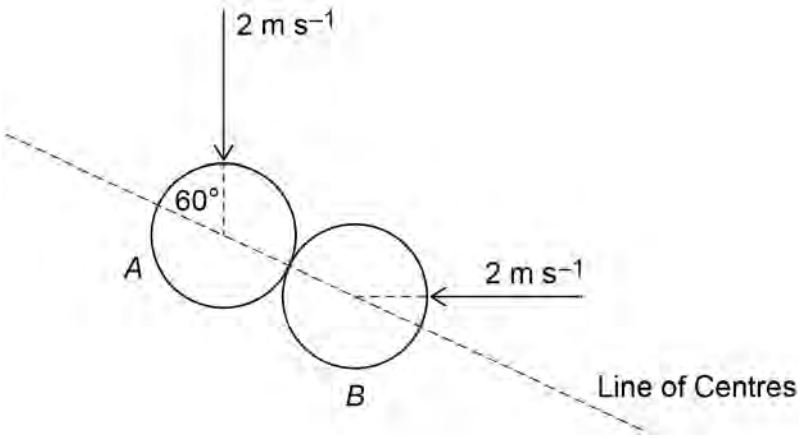
9 Two smooth spheres, A and B, have the same radius of 1 cm and are moving on a horizontal surface when they collide.

The velocities of the spheres are perpendicular when they collide.

Before the collision, the speeds of both A and B are 2 m s^{-1}

On impact, the velocity of A makes an angle of 60° to the line of centres.

The diagram shows the directions of the velocities of the spheres before the collision.



The mass of A is 4 kg and the mass of B is 5 kg.

The coefficient of restitution between the spheres is $\frac{3}{4}$

Find the distance between the centres of the spheres 2 seconds after the collision, giving your answer to three significant figures.

[11 marks]

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Answer _____

11

END OF QUESTIONS

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