OXFORD INTERNATIONAL AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________

I declare this is my own work.

INTERNATIONAL AS
MATHEMATICS

(9660/MA01) Unit P1 Pure Mathematics

Tuesday 3 January 2023 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

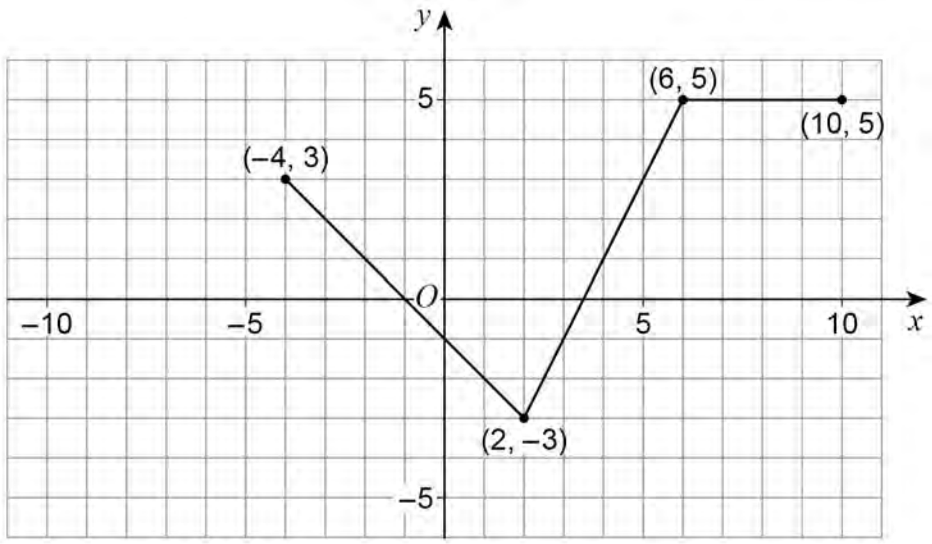
Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	

Answer **all** questions in the spaces provided.

1 The graph of a function with equation $y = \mathrm{f}(x)$ is shown in **Figure 1**

Figure 1

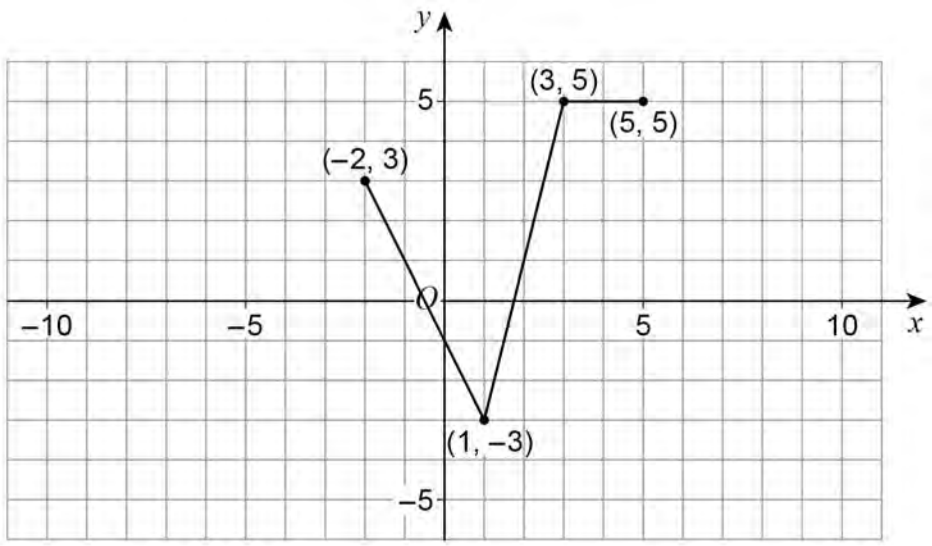


1 (a) (i) State the equation of the graph of the function shown in **Figure 2**

Circle your answer.

[1 mark]

Figure 2



$y = \mathrm{f}\left(\frac{1}{2}x\right)$ $\qquad$ $y = \mathrm{f}(2x)$ $\qquad$ $y = \frac{1}{2}\mathrm{f}(x)$ $\qquad$ $y = 2\mathrm{f}(x)$

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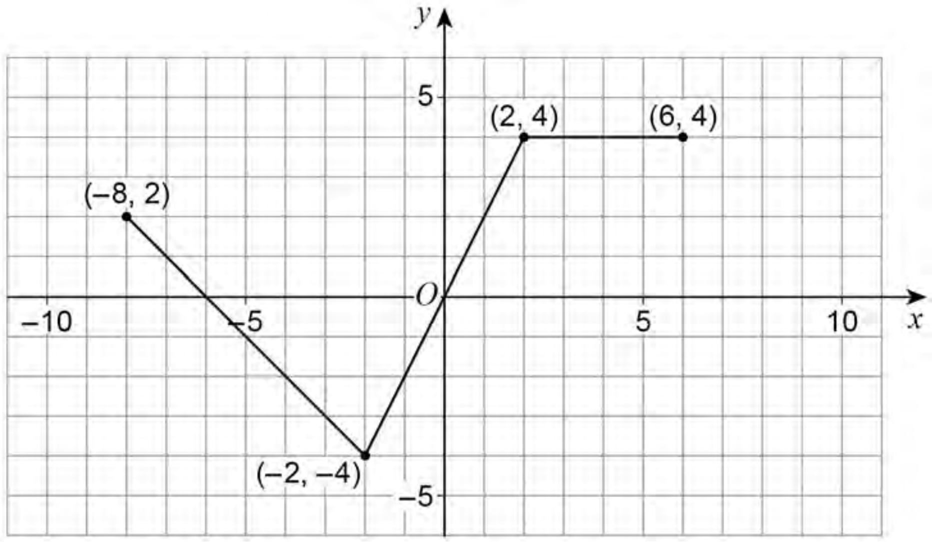
1 (a) (ii) State the equation of the graph of the function shown in **Figure 3**

Circle your answer.

[1 mark]

Figure 3

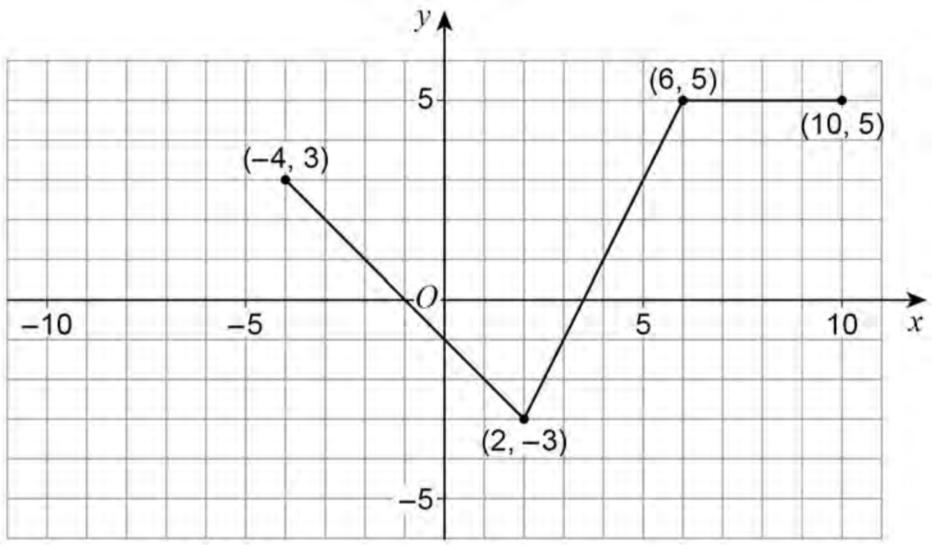


$y = \mathrm{f}(x-4)-1$ $y = \mathrm{f}(x-4)+1$ $y = \mathrm{f}(x+4)-1$ $y = \mathrm{f}(x+4)+1$

1 (b) The graph of the function with equation $y = \mathrm{f}(x)$ is shown again below.

By drawing a suitable straight line find the roots of the equation $\mathrm{f}(x) = x - 3$

[2 marks]



___

$x =$ ___

4

Turn over ►

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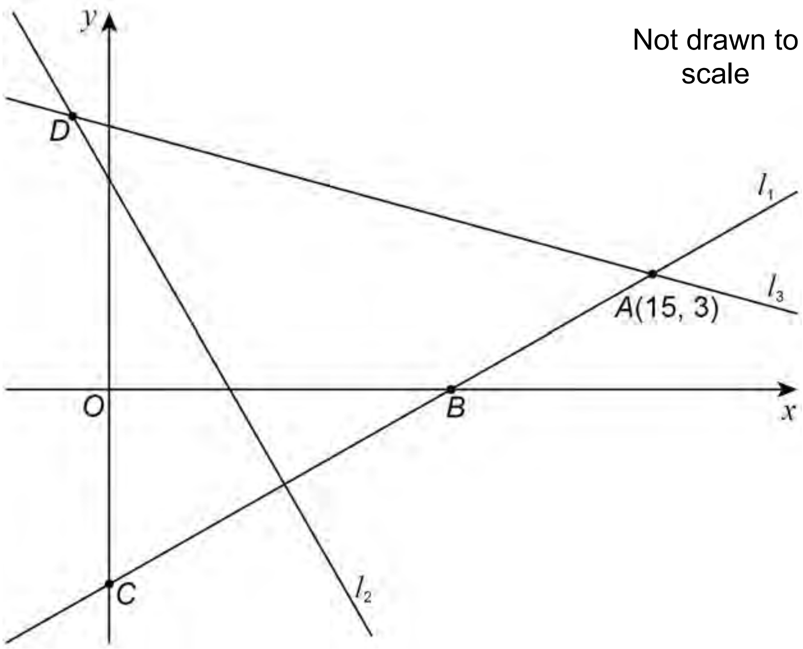
2 The points A, B, C and D, and the lines l_1, l_2 and l_3 are shown in the diagram.



The lines l_1 and l_3 intersect at $A(15, 3)$

2 (a) The line l_1 has gradient $\dfrac{3}{5}$

Show that l_1 has the equation $3x - 5y - 30 = 0$

[1 mark]

2 (b) l_1 intersects the x-axis at B and the y-axis at C

l_2 passes through the mid-point of the line segment BC

l_1 and l_2 are perpendicular.

Find the equation of l_2 giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

[5 marks]

Do not write outside the box

Answer

2 (c) l_3 has the equation $x + 4y - 27 = 0$

l_2 and l_3 intersect at D

Find the coordinates of D

[1 mark]

Answer

2 (d) Find the length of the line segment AD

Give your answer in the form $n\sqrt{p}$ where p is a prime number.

[2 marks]

Answer

9

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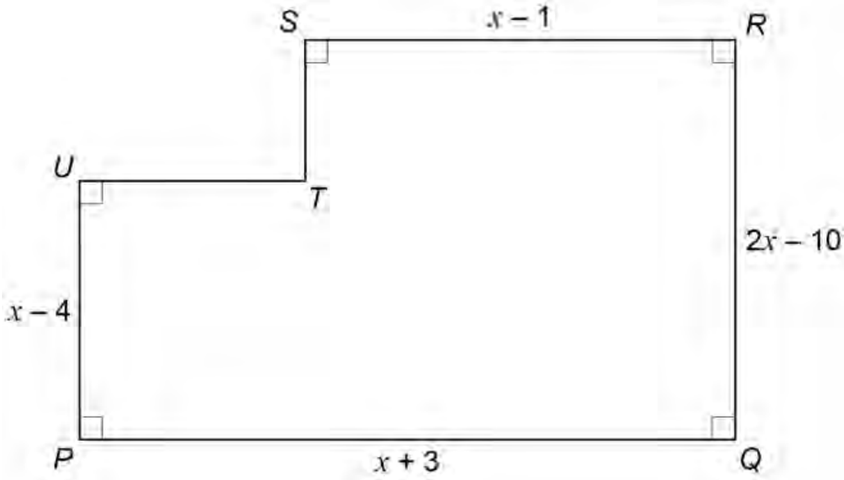
3 The diagram shows the plan of a garden.



The angle at each corner of the garden is a right-angle.

The lengths of the sides in metres are

$$PQ = x + 3\,,\ QR = 2x - 10\,,\ RS = x - 1 \ \text{ and }\ PU = x - 4$$

3 (a) The perimeter of the garden is greater than 31 metres.

Show that $x > 7.5$

[1 mark]

3 (b) The area of the garden is less than 58 m^2

Show that $x^2 - 4x - 32 < 0$

[3 marks]

Do not write outside the box

3 (c) Solve the inequality $x^2 - 4x - 32 < 0$

Show clearly each step of your working.

[2 marks]

Answer

3 (d) The length of the side ST is y metres.

Using your answers to **parts (a)** and **(c)** find the possible values of y

[2 marks]

Answer

8

4 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^2(2x-5)-48$$

4 (a) Use the Factor Theorem to show that $(x-4)$ is a factor of $\mathrm{p}(x)$

[2 marks]

4 (b) Show that $\mathrm{p}(x)$ can be written in the form

$$\mathrm{p}(x) = (x-4)\left(ax^2+bx+c\right)$$

where a, b and c are integers to be found.

[2 marks]

Do not write outside the box

4 (c) Show that $p(x) = 0$ has exactly one real root and state its value.

[3 marks]

Answer

7

Turn over for the next question

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5 The nth term of the sequence A is u_n and the sequence is defined by

$$u_{n+1} = u_n + 8\left(1 + 3^n\right)$$

The second, third and fourth terms of this sequence are

$$u_2 = 61 \qquad u_3 = 141 \qquad \text{and} \qquad u_4 = 365$$

5 (a) (i) Find the first term u_1 of sequence A

[1 mark]

Answer ______________________

5 (a) (ii) Find the fifth term u_5 of sequence A

[1 mark]

Answer ______________________

5 (b) The sequence A can be found using the formula

nth term of sequence A = nth term of sequence B + nth term of sequence C

where sequence B and sequence C are two different sequences.

5 (b) (i) Sequence B is a geometric sequence with first term $a = 12$ and common ratio $r = 3$

Find the first five terms of sequence B

[1 mark]

Answer ______________________

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5 (b) (ii) Hence find the first five terms of sequence C

[2 marks]

Answer ____________________

5 (c) (i) Sequence C is an arithmetic sequence.

Using your answer to **part (b)(ii)** write down the common difference for sequence C

[1 mark]

Answer ____________________

5 (c) (ii) Find an expression in terms of n for the nth term of sequence C

[1 mark]

Answer ____________________

7

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6 The curve C has the equation

$$y = 3x^3 + 14x^2 + 17x + 11$$

The point $P(-2, 9)$ lies on C

The line l is the normal to C at the point P

6 (a) (i) Find $\dfrac{dy}{dx}$

[2 marks]

Answer ____________________

6 (a) (ii) Show that the equation of l is $y = \dfrac{1}{3}x + \dfrac{29}{3}$

[3 marks]

6 (b) The line l intersects C at three distinct points.

Show that the x-coordinates of these points of intersection satisfy the equation

$$9x^3 + 42x^2 + 50x + 4 = 0$$

[2 marks]

Do not write outside the box

6 (c) The equation $9x^3 + 42x^2 + 50x + 4 = 0$ can be written in the form

$$(x+2)(9x^2 + 24x + 2) = 0$$

6 (c) (i) Express $9x^2 + 24x + 2$ in the form $a(x+b)^2 + c$ where a, b and c are constants.

[3 marks]

Answer ____________________

6 (c) (ii) The points of intersection of l and C are $P(-2, 9)$, Q and R

Using your answer to **part (c)(i)** find the exact x-coordinates of Q and R

Show clearly each step of your working.

[3 marks]

Answer ____________________

13

Do not write outside the box

7 A curve has equation $y = \mathrm{f}(x)$ where $x > 0$

It is given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x^{\frac{3}{2}} - 9x^{\frac{3}{4}} - 56$$

7 (a) Find $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$

[2 marks]

Answer ________________________

7 (b) By substituting $t = x^{\frac{3}{4}}$ into the given expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (at + b)(t - c)$$

where a, b and c are positive integers.

[2 marks]

Do not write outside the box

7 (c) The curve has one stationary point for $x > 0$

7 (c) (i) By writing x as a power of t and then using **part (b)** find the x-coordinate of this stationary point.

[3 marks]

Answer ______________________

7 (c) (ii) Using **part (a)** show that this stationary point is a minimum.

[1 mark]

7 (d) State the values of x for which f is a decreasing function.

[1 mark]

Answer ______________________

9

Do not write outside the box

8 (a) Show that for any positive real number a

$$\left(2+\sqrt{3}-\sqrt{a}\right)\left(2+\sqrt{3}+\sqrt{a}\right)=7+b\sqrt{3}-a$$

where b is a constant to be found.

[2 marks]

Do not write outside the box

8 (b) Hence show that

$$\frac{12}{2+\sqrt{3}-\sqrt{7}}$$

can be written in the form $p+q\sqrt{r}+\sqrt{s}$ where p, q, r and s are integers and $q>1$

[3 marks]

5

Turn over for the next question

Do not write outside the box

9 (a) The expression $\left(3-2\sqrt{x}\right)^3$ can be written in the form

$$27 - p\sqrt{x} + qx - 8x\sqrt{x}$$

where p and q are positive integers.

Show that $p = 54$ and find the value of q

[3 marks]

$q =$ ________________

9 (b) It is given that $x > 0$

9 (b) (i) Find $\displaystyle\int \left(\frac{\left(3-2\sqrt{x}\right)^3}{\sqrt{x}} + 12 \right) \mathrm{d}x$

[4 marks]

Answer ________________

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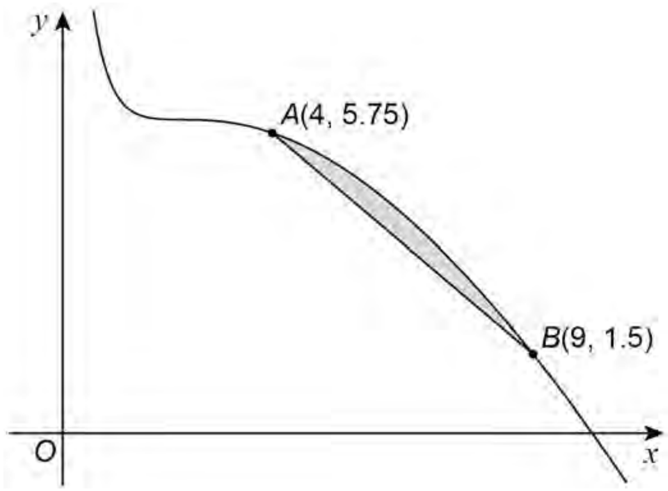
9 (b) (ii) Hence find the value of $\displaystyle\int_4^9 \left(\frac{\left(3-2\sqrt{x}\right)^3}{\sqrt{x}}+12\right)\mathrm{d}x$

[2 marks]

Answer ______________________

9 (c) A curve with equation $y=\dfrac{\left(3-2\sqrt{x}\right)^3}{2\sqrt{x}}+6$ is drawn below.



The points $A(4, 5.75)$ and $B(9, 1.5)$ lie on the curve.

Using your answer to **part (b)(ii)** find the area of the shaded region bounded by the curve and the line segment AB

[2 marks]

Answer ______________________

11

Turn over ►

10 A finite arithmetic sequence has k terms and common difference d

The first term is $a = 12$

The sum of the **first** 10 terms is 480

The sum of the **last** 10 terms is 3360

Show that $d = 8$ and hence find the sum of **all** of the terms in the sequence.

[7 marks]

Answer

7

END OF QUESTIONS

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