OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number						Candidate number				

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I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM03) Unit FP2 Pure Mathematics

Thursday 12 January 2023 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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14	
TOTAL	

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Answer **all** questions in the spaces provided.

1 A curve C_1 has polar equation

$$r = 6 + 2\sin\theta \quad \text{where} \quad 0 \le \theta \le 2\pi$$

A circle C_2 has polar equation $r = 3$

1 (a) Show that C_1 and C_2 do not intersect.

[1 mark]

1 (b) Show that the area of the region bounded by C_1 and C_2 is 29π

[4 marks]

5

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2 Prove by induction that, for all integers $n \geq 1$

$$\begin{bmatrix} -3 & 1 \\ -16 & 5 \end{bmatrix}^n = \begin{bmatrix} 1-4n & n \\ -16n & 4n+1 \end{bmatrix}$$

[5 marks]

5

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3 Use integration by parts to show that

$$\int_{1}^{\sqrt{3}} 2x \tan^{-1} x \, dx = \frac{5\pi}{6} + 1 - \sqrt{3}$$

[5 marks]

5

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4 The position vectors of three points are

$$\mathbf{u} = \begin{bmatrix} 4 \\ 3 \\ 8 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} -1 \\ n \\ n \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} 5 \\ -1 \\ n \end{bmatrix}$$

where n is a constant.

The vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are coplanar.

4 (a) Use a scalar triple product to find the two values of n

[3 marks]

Answer ________________________________

4 (b) For each value of n found in **part (a)**, express $\mathbf{u}$ in terms of $\mathbf{v}$ and $\mathbf{w}$

[2 marks]

Answer ________________________________

5

Turn over ►

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5 A curve has Cartesian equation

$$4y^2 = (2+x)(2-3x)$$

Find the polar equation of the curve in the form

$$r = \frac{k}{\mathrm{f}(\cos\theta)}$$

where k is a constant and $r > 0$

[4 marks]

Answer

4

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6 By using an integrating factor, find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + (\tan x)y = \tan^3 x \quad \text{where} \quad 0 \le x < \frac{\pi}{2}$$

[6 marks]

Answer

6

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7 The quartic equation

$$z^4 + pz + q = 0$$

where p and q are constants, has roots α, β, γ and δ

7 (a) Write down the value of $\alpha + \beta + \gamma + \delta$

[1 mark]

Answer ____________________

7 (b) It is given that $\alpha + \beta + \gamma = 2 - \mathrm{i}$ and that both p and q are real.

7 (b) (i) Find the value of p

[4 marks]

$p =$ ____________________

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7 (b) (ii) Show that $\alpha^4 + \beta^4 + \gamma^4 + \delta^4 = -220$

[3 marks]

8

Turn over ►

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8 The matrix $\mathbf{M}$ is defined as

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & -c \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

where c is a constant.

8 (a) (i) Find the value of c for which $\mathbf{M}$ is a singular matrix.

[2 marks]

$c =$ ________________

8 (a) (ii) Given that $\mathbf{M}$ is a non-singular matrix, find $\mathbf{M}^{-1}$ in terms of c

[5 marks]

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Answer

8 (b) Given that $\lambda = 1$ is the only real eigenvalue of **M** find all the possible values of c

[5 marks]

Answer

12

Turn over ►

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9 The differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 2x$$

such that $y = -2$ and $\frac{dy}{dx} = 2$ when $x = 0$ has the solution $y = f(x)$

9 (a) Find $f(x)$

[8 marks]

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$f(x) =$ ______

9 (b) Hence, or otherwise, find the Maclaurin series expansion of $f(x)$ in ascending powers of x up to and including the term in x^4

[3 marks]

Answer ______

11

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10 (a) Use the definitions of $\cosh\theta$ and $\sinh\theta$ in terms of e^{θ} to show that

$$\cosh x \cosh y + \sinh x \sinh y = \cosh(x+y)$$

[4 marks]

10 (b) A curve has equation

$$y = 8\sinh(x + \ln 4) + 4\cosh x - 7x$$

Prove that the curve has exactly one stationary point P and show that the y-coordinate of P can be expressed in the form $u + v\ln w$ where u, v and w are prime numbers.

[9 marks]

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13

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11 The plane Π_1 has vector equation $\mathbf{r} \, . \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix} = 5$

The plane Π_2 has Cartesian equation $x - 3y + 3z = 3$

11 (a) (i) Write down a vector equation of Π_2 in the form $\mathbf{r} \, . \, \mathbf{n} = d$

[1 mark]

Answer ______________________________

11 (a) (ii) Find the acute angle between the planes Π_1 and Π_2 giving your answer to the nearest 0.1°

[4 marks]

Answer ______________________________

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11 (b) The line of intersection of Π_1 and Π_2 is L

11 (b) (i) Find the direction ratios of the line L

[2 marks]

Answer ______________________

11 (b) (ii) Find Cartesian equations for the line L

[3 marks]

Answer ______________________

11 (c) The plane Π_3 has Cartesian equation $x + y = 5$

Using your answer to **part (b)(ii)** or otherwise, find the coordinates of the point of intersection of Π_1, Π_2 and Π_3

[2 marks]

Answer ______________________

12

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12 (a) For real constants m and n given that, in exponential form

$$m + \mathrm{i}n = r\mathrm{e}^{\mathrm{i}\theta} \quad \text{and} \quad -n + \mathrm{i}m = r\mathrm{e}^{\mathrm{i}\phi}$$

express ϕ in terms of θ and π

[2 marks]

$\phi =$ ________________

12 (b) In the Argand diagram opposite, the points P, Q and R represent the roots of the equation

$$z^3 = a + \mathrm{i}b$$

where a and b are real constants.

12 (b) (i) Find, in terms of a and b, the radius of the circle on which P, Q and R lie.

[2 marks]

Answer ________________

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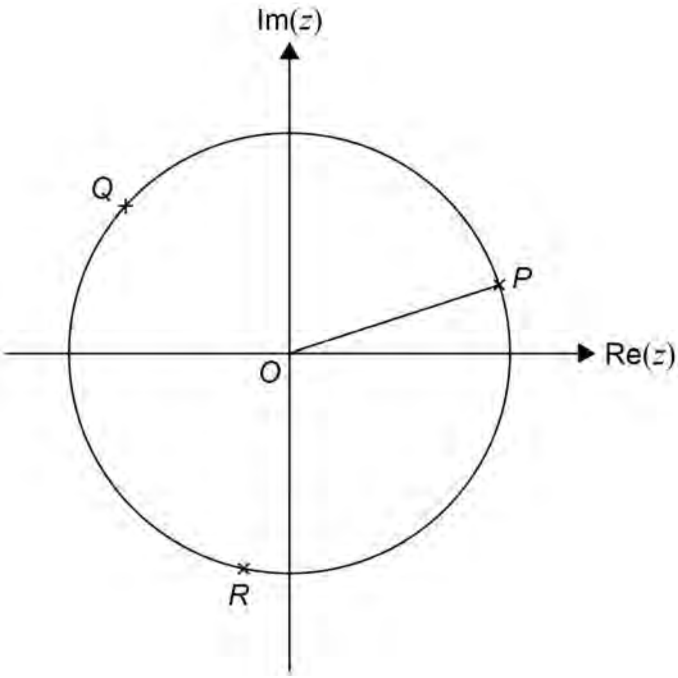
12 (b) (ii) On the Argand diagram below, mark and label the approximate position of the point T which represents the root of the equation

$$z^3 = -b + \mathrm{i}a$$

that is closest to the point P

[1 mark]



12 (c) In the case where

area of triangle $OTP = 16$

and $b = a\sqrt{3}$ where $a > 0$

find, in exponential form, the complex number which represents the midpoint of the **chord** TP

[6 marks]

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Answer

11

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13 (a) Given that $u = \sinh^{-1}\left(\frac{1}{x}\right)$ show that $\frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{1}{x\sqrt{1+x^2}}$

[3 marks]

13 (b) A curve C has equation $y = \ln x$ where $x > 0$

The length of the arc of C between the points on the curve where $x = \frac{7}{24}$ and $x = \frac{12}{5}$ is equal to s

Using the result in **part (a)** show that $s = p + \ln q$ where p and q are rational.

[7 marks]

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10

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14 (a) By applying de Moivre's theorem to $(\cos\theta + \mathrm{i}\sin\theta)^4$, express $\cos 4\theta$ in terms of $\sin\theta$

[4 marks]

Answer ____

14 (b) Hence, show that the equation $\cos 4\theta = \cos\left(\frac{\pi}{2} - 3\theta\right)$ can be written in the form

$$8\sin^4\theta + 4\sin^3\theta + a\sin^2\theta + b\sin\theta + c = 0$$

where a, b and c are integers.

[4 marks]

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14 (c) Hence, prove that

$$\sin\left(\frac{\pi}{14}\right)+\sin\left(\frac{5\pi}{14}\right)=\frac{1}{2}+\sin\left(\frac{3\pi}{14}\right)$$

[5 marks]

13

END OF QUESTIONS

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