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(9660/MA02) Unit PSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

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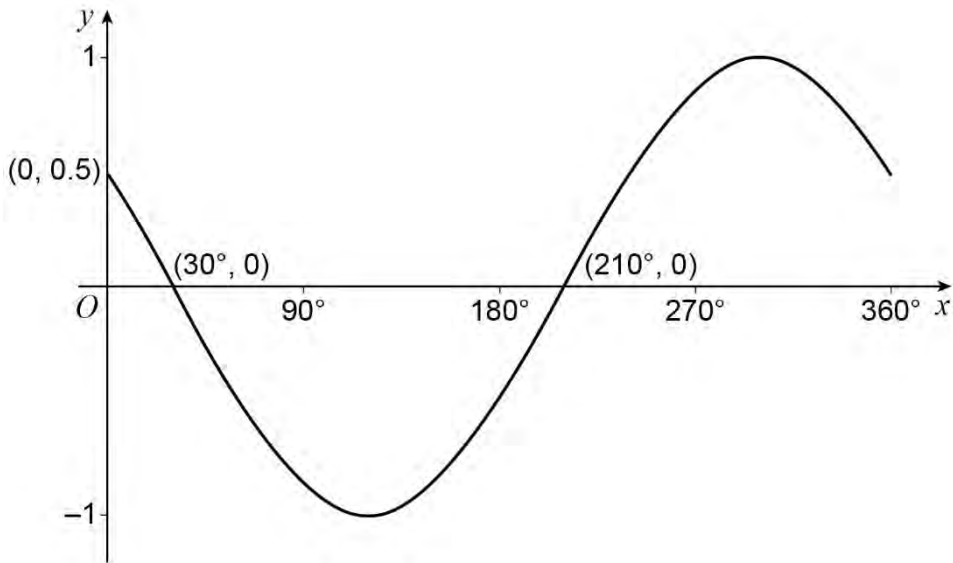
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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	See artwork below	<p>B1</p> <p>B1</p> <p>B1</p>	<p>Sinusoidal curve of correct form with one minimum below the x-axis and one maximum above, both with the correct amplitude</p> <p>Ignore any section of the curve drawn outside of $0^\circ \leq x \leq 360^\circ$</p> <p>The curve must intersect the x-axis at exactly two points; once between 0° and 90° and again between 180° and 270°</p> <p>Both correct coordinates for intercepts with the x-axis given and no others</p> <p>Condone values only indicated Correct coordinates of intercept with the y-axis</p> <p>Condone value only indicated</p>
			
		3	

Q	Answer	Marks	Comments
1(b)	$\frac{360^\circ}{4} [= 90^\circ]$ <p>or</p> $360^\circ = \frac{4 \times 180^\circ}{k} \quad \text{oe}$ $k = 2$	<p>M1</p> <p>A1</p>	<p>PI Dividing the period of f by 4 Could be implied by indicating that the graph of $y = \tan x$ is mapped onto the graph of $y = \tan(kx)$ by a stretch sf $\frac{1}{2}$ in the x-direction.</p> <p>Condone 2π in place of 360° (i.e. working in radians)</p> <p>CAO Accept $k = -2$ or $k = \pm 2$</p>
		2	

	Question 1 Total	5	
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[illegible]

Q	Answer	Marks	Comments
2(b)	$[\angle BDC =] \frac{5.1}{8.50723...} [= 0.59948...] \text{ oe}$ <p>and</p> $[\angle BDC =] 0.6 \text{ [radians]}$	B1	Condone $BD = 8.5$
		1	

Q	Answer	Marks	Comments
2(c)	$\left[\frac{1}{2}r^2\theta =\right] \frac{1}{2} \times (8.50723\dots)^2 \times 0.59948\dots$	M1	Correct expression for area of sector <i>BCD</i> Condone $BD = 8.5$ and $\angle BDC = 0.6$
	[Area of Sector =] 21.69344... [cm ²]	A1	AWRT 21.7 PI by correct final answer or 1.28
	[Area of Triangle =] $\frac{1}{2} \times (8.50723\dots)^2 \times \sin 0.59948\dots$	M1	Correct expression for area of triangle <i>BCD</i> Condone $BD = 8.5$ and $\angle BDC = 0.6$
	[Area of Triangle =] 20.41719... [cm ²]	A1	AWRT 20.4 PI by correct final answer or 1.28
	[Shaded Area =] 1.3 [cm ²]	A1	CAO
		5	
	Question 2 Total	8	

Q	Answer	Marks	Comments
3(a)	$\frac{17-5}{9-k}$ $\frac{17-5}{9-k} \times \frac{1}{3} = -1$ $3k - 27 = 12$ oe and $k = 13$	B1 M1 A1	PI Correct expression in terms of k for the gradient of QR oe Use of $m \times m' = -1$ and their gradient of QR to form an equation in k AG CAO Must be a further line of working: linear equation in k and required result stated
3(a) Alt	-3 $y = -3x + 44$ $5 = -3k + 44$ oe and $k = 13$	B1 M1 A1	Correct gradient of QR PI in later working. oe Correct equation of the line QR CSO Substitutes the coordinates of R into the equation of the line QR and AG
		3	

Q	Answer	Marks	Comments
3(b)(i)	$\sqrt{(13-8)^2 + (5-10)^2}$ $r = 5\sqrt{2}$ or $\sqrt{50}$	M1 A1	oe Correct expression for the radius unsimplified Exact value only
		2	

Q	Answer	Marks	Comments
3(b)(ii)	$(x-8)^2 + (y-10)^2 = 50$	B1ft	ft their r^2 provided it is an exact value.
		1	

Q	Answer	Marks	Comments
3(c)(i)	$(x-4)^2 + \dots + (y+3)^2 + \dots \quad [= 0]$	M1	Attempts to complete the square twice by, for example $(x \pm 4)^2$ and $(y \pm 3)^2$
	$(4, -3)$	A1	Condone not given as coordinates but must be clearly identified
		2	

Q	Answer	Marks	Comments
3(c)(ii)	Translation	E1	Independent. Correctly identifies the type of transformation
	$\begin{bmatrix} 4 \\ 13 \end{bmatrix}$	E1ft	Correct vector Must be given in vector form ft their centre from part (c)(i)
		2	

	Question 3 Total	10	
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Q	Answer	Marks	Comments
4(a)	$[\alpha =] 22^\circ$	B1	CAO
	$[\alpha =] 68^\circ$	B1	CAO SC1 Both correct values of α given but neither rounded to the nearest degree. $\alpha = 22.213502\dots^\circ$ $\alpha = 67.786498\dots^\circ$
		2	

Q	Answer	Marks	Comments
4(b)(i)	$\frac{10}{\sin x} - 3 \frac{\sin x}{\cos x} = \frac{11 \cos x}{\sin x}$	M1	PI in later working. Use of $\tan x = \frac{\sin x}{\cos x}$ obtaining a correct equation in $\sin x$ and $\cos x$
	$10 - 3 \frac{\sin^2 x}{\cos x} = 11 \cos x$ or $10 \cos x - 3 \sin^2 x = 11 \cos^2 x$	M1	Manipulation to include $\sin^2 x$ Must be seen
	$10 - 3 \frac{(1 - \cos^2 x)}{\cos x} = 11 \cos x$ or $10 \cos x - 3(1 - \cos^2 x) = 11 \cos^2 x$ or $8 \cos^2 x + 3(\cos^2 x + \sin^2 x) - 10 \cos x = 0$ and $8 \cos^2 x - 10 \cos x + 3 = 0$	A1	Clear use of $\cos^2 x + \sin^2 x = 1$ and required result stated Be convinced. AG
		3	

Q	Answer	Marks	Comments
4(b)(ii)	$[\cos x =] \frac{1}{2} \quad \text{and} \quad [\cos x =] \frac{3}{4}$ $\cos(\theta - 15^\circ) = \frac{1}{2} \quad \text{or} \quad [\theta - 15^\circ =] \cos^{-1}\left(\frac{1}{2}\right)$ or $\cos(\theta - 15^\circ) = \frac{3}{4} \quad \text{or} \quad [\theta - 15^\circ =] \cos^{-1}\left(\frac{3}{4}\right)$ $[\theta =] 56.4^\circ$ $[\theta =] 75^\circ$	B1 M1 A1 A1	PI Both correct roots PI by $41.4[09622\dots]^\circ$ or 60° or a correct angle q ft their roots. CAO CAO Condone 75.0° If both correct answers given award A1A0 if their final answer includes incorrect angle(s) in the given interval.
		4	
	Question 4 Total	9	

Q	Answer	Marks	Comments
5(a)	$\log_7 7^{p-5} = \log_7 9$	M1	PI in later working. Coordinates substituted into the equation and \log_7 of both sides taken
	or $p - 5 = \log_7 9$		
	$p - 5$ or $2\log_7 3$	M1	PI Correct application of a logarithm rule to one term
	$[p =] 5 + 2\log_7 3$	A1	Condone log instead of \log_7
		3	

Q	Answer	Marks	Comments
5(b)	$[8 =] \log_n n^8$	B1	PI Writing 8 as $\log_n n^8$
	$8 + \log_n k + \log_n (2y)^4 = \log_n (n^2 y)^6$	M1	oe One correct use of logarithm rule in an equation. Powers may be partially manipulated
	$\log_n (n^8 k (2y)^4) = \log_n (n^2 y)^6$	M1	oe One correct use of second logarithm rule in an equation. Powers may be partially manipulated. Must have single logarithm on both sides
	$\left[\log_n (n^8 k (2y)^4) = \log_n (n^2 y)^6 \right]$	m1	oe \log_n correctly eliminated and rearranged to isolate term in y Dependant on M1 M1 being awarded
	$\Rightarrow n^8 k (2y)^4 = (n^2 y)^6$		
	$\Rightarrow 16n^8 ky^4 = n^{12} y^6$		
	$\Rightarrow y^2 = \frac{16n^8 k}{n^{12}}$	A1	CAO Accept $y = 4k^{\frac{1}{2}} n^{-2}$
	$y = \frac{4\sqrt{k}}{n^2}$		
		5	

[illegible]

Q	Answer	Marks	Comments
5(b) ALT 2	$[8 =] \log_n n^8$	B1	PI Writing 8 as its equivalent in \log_n
	$8 = \log_n (n^2 y)^6 - \log_n k - \log_n (2y)^4$	M1	oe One correct use of logarithm rule in an equation. Powers may be partially manipulated
	$8 = \log_n \left[\frac{(n^2 y)^6}{k(2y)^4} \right]$	M1	oe One correct use of second logarithm rule in an equation. Powers may be partially manipulated. Must have single logarithm on the right-hand side
	$n^8 k (2y)^4 = (n^2 y)^6$ $\Rightarrow 16n^8 k y^4 = n^{12} y^6$ $\Rightarrow y^2 = \frac{16n^8 k}{n^{12}}$	m1	oe \log_n correctly eliminated and rearranged to isolate term in y Dependant on M1 M1 being awarded
	$y = \frac{4\sqrt{k}}{n^2}$	A1	CAO Accept $y = 4k^{\frac{1}{2}} n^{-2}$
		5	
	Question 5 Total	8	

Q	Answer	Marks	Comments
6(a)	$k + 4k + 9k + 16k$ $k + 4k + 9k + 16k = 1$ oe and $k = \frac{1}{30}$	M1 A1	oe Award for $30k$ seen. AG Sets the sum of the four probabilities in terms of k equal to 1, leading to correct final answer
		2	

Q	Answer	Marks	Comments
6(b)	$P(X \geq 3) = \frac{5}{6}$	B1	oe AWRT 0.833
		1	

Q	Answer	Marks	Comments
6(c)	$E\left(\frac{1}{X^3}\right) = \frac{1}{1^3} \times \frac{1}{30} + \frac{1}{2^3} \times \frac{4}{30} + \frac{1}{3^3} \times \frac{9}{30}$ $+ \frac{1}{4^3} \times \frac{16}{30}$ $= \frac{5}{72}$	<p>M1</p> <p>A1</p>	<p>Applies formula PI by final answer of AWRT 0.069</p> <p>oe Must be exact value.</p>
		2	

Q	Answer	Marks	Comments
6(d)	$E(X - Y) [= E(X) - E(Y)] = \frac{10}{3} - 10$	M1	PI by correct final answer
	$= -\frac{20}{3}$	A1	oe, AWRT -6.67
		2	

	Question 6 Total	7	
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Q	Answer	Marks	Comments
7(a)	$[P(A \cap B) = P(A) + P(B) - P(A \cup B)]$ $P(A \cap B) = 0.2 + 0.35 - 0.48$ $P(A \cap B) = 0.07$	M1 A1	oe
		2	

Q	Answer	Marks	Comments
7(b)	A and B are not mutually exclusive as $P(A \cap B) \neq 0$	E1	Condone 'no' as statement and a correct reason
		1	

Q	Answer	Marks	Comments
7(c)	$P(B A) = \frac{0.07}{0.2}$ $P(B A) = 0.35$	M1 A1	PI
		2	

Q	Answer	Marks	Comments
7(d)	A and B are independent as $P(B A) = P(B)$	E1	Condone 'yes' as statement and $P(B A) = P(B)$ or $P(A \cap B) = P(A) \times P(B)$
		1	

	Question 7 Total	6	
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Q	Answer	Marks	Comments
8(a)(i)	$E(X) = p$	B1	
		1	

Q	Answer	Marks	Comments
8(a)(ii)	$\text{Var}(X) = p - p^2$ or $p(1 - p)$	B1	
		1	

Q	Answer	Marks	Comments
8(b)	$E\left(\sum_{i=1}^n X_i\right) = np$ $\text{Var}\left(\sum_{i=1}^n X_i\right) = np(1 - p)$ $np = 6.25$ and $np(1 - p) = 4.6875$ $6.25(1 - p) = 4.6875$ $p = 0.25$ $n = 25$	B1ft B1ft M1 A1 A1	PI in later working. ft their answer from part (a)(i) oe PI in later working. ft their answer from part (a)(ii) Forms two equations in terms of n and p using their expectation and variance, and correctly eliminates p or n PI by correct value of p or n Allow q in place of $1 - p$ oe
		5	

	Question 8 Total	7	
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Q	Answer	Marks	Comments
9(a)	10 [Ns]	B1	
		1	

Q	Answer	Marks	Comments
9(b)	$(5 \times 2) - (-1.5 \times 2)$	M1	oe Condone one sign error for M1
	13 [Ns]	A1	
		2	

	Question 9 Total	3	
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Q	Answer	Marks	Comments
10	$T - 0.2 \times 9.8 = 0.2 \times 1.25$ $m \times 9.8 - T = m \times 1.25$ $9.8m - 1.96 = 0.25 + 1.25m$ or $9.8m - 2.21 = 1.25m$ $m = 0.26$	M1 M1 M1 A1	PI in later working. Correct equation of motion for P PI in later working. Correct equation of motion for Q Forms correct equation in m only, possibly by adding both equations of motion or by finding $T = 2.21$ and using it to form the equation of motion for Q AWRT
		4	

	Question 10 Total	4	
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Q	Answer	Marks	Comments
11(a)	$\left[a = \frac{dv}{dt} = \right] \frac{3t^2}{2} - 6t + 5$	M1	oe At least two correct terms.
	$\frac{3 \times 2^2}{2} - 6 \times 2 + 5$	m1	Substitutes $t = 2$ into their differentiated expression
	$-1 \text{ [ms}^{-2}\text{]}$	A1	CAO
		3	

Q	Answer	Marks	Comments
11(b)	$\frac{t^3}{2} - 3t^2 + 5t = \frac{t}{2}(t^2 - 6t + 10)$	M1	oe Factorising by t or attempt to complete the square of the quadratic. PI by $t^2 - 6t + 10$
	$\left[\frac{t}{2} \left((t-3)^2 + 1 \right) \right]$	A1	oe Correct completed square form or shows the correct discriminant is negative
	or $[\Delta =](-6)^2 - 4 \times 1 \times 10 = -4$ $\left[v \geq 0 \text{ or } (t-3)^2 + 1 > 0 \text{ or } \frac{t}{2}(t-3)^2 + \frac{t}{2} \geq 0 \right]$ Velocity is never negative, so the particle does not change direction	E1	
	or As the quadratic equation has no real roots, the velocity is never zero [for $t > 0$] Hence, the particle does not change direction.		
		3	

Q	Answer	Marks	Comments
11(c)	$\left[\int \left(\frac{t^3}{2} - 3t^2 + 5t \right) dt = \right] \frac{t^4}{8} - t^3 + \frac{5t^2}{2} [+c]$	M1	At least two terms correct.
	$\frac{4^4}{8} - 4^3 + \frac{5 \times 4^2}{2} \left[- \left(\frac{0^4}{8} - 0^3 + \frac{5 \times 0^2}{2} \right) \right]$	M1	Sight of a correct attempt to evaluate their integral between $t = 0$ and $t = 4$ PI by correct answer
	8 [m]	A1	CAO
		3	
	Question 11 Total	9	

Q	Answer	Marks	Comments
12(a)	$\left[s = ut + \frac{1}{2}at^2 \right]$ $150 = 0 \times 3.4 + \frac{1}{2} \times a \times 3.4^2$ <p>or</p> $150 = 5.78a$ 26 ms^{-2}	<p>M1</p> <p>A1</p> <p>B1</p>	<p>oe Use of one or more constant acceleration formulae to obtain a correct equation in a</p> <p>AWRT 26 Correct value for the acceleration.</p> <p>Independent. Correct units.</p>
		3	

Q	Answer	Marks	Comments
12(b)	<p>Any correct reason, such as:</p> <ul style="list-style-type: none"> Initial speed unlikely to be 0 [due to momentum of the firework or due to the explosion] The part may be moving in a direction different to the vertical 	E1	<p>Any valid reason that implies any of $u \neq 0$, $s \neq 150$ or non-linear motion</p> <p>Do not award for 'answer is rounded', 'errors in the calculation' or '9.81 should be used for acceleration due to gravity'</p>
		1	

	Question 12 Total	4	
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