INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA03) Unit P2 Pure Mathematics

Friday 17 January 2020   07:00 GMT   Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables (enclosed).
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working: otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
TOTAL	

Answer **all** questions in the spaces provided.

1 (a) (i) Express $5\sin\theta - 12\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the value of α to the nearest 0.1°

[3 marks]

Answer _____

1 (a) (ii) Hence solve the equation

$$5\sin\theta - 12\cos\theta = -1$$

in the interval $-180° < \theta < 180°$, giving all solutions to the nearest 0.1°

[3 marks]

Answer _____

1 (b) Solve the equation

$$2 \cot^2 x = 10 - 5 \operatorname{cosec} x$$

in the interval $-90° < x < 270°$, giving all solutions to the nearest degree.

[5 marks]

Answer _____

11

Turn over ▶

2 The function f is defined by

$$f(x) = \sin^{-1}(2x - 1)$$
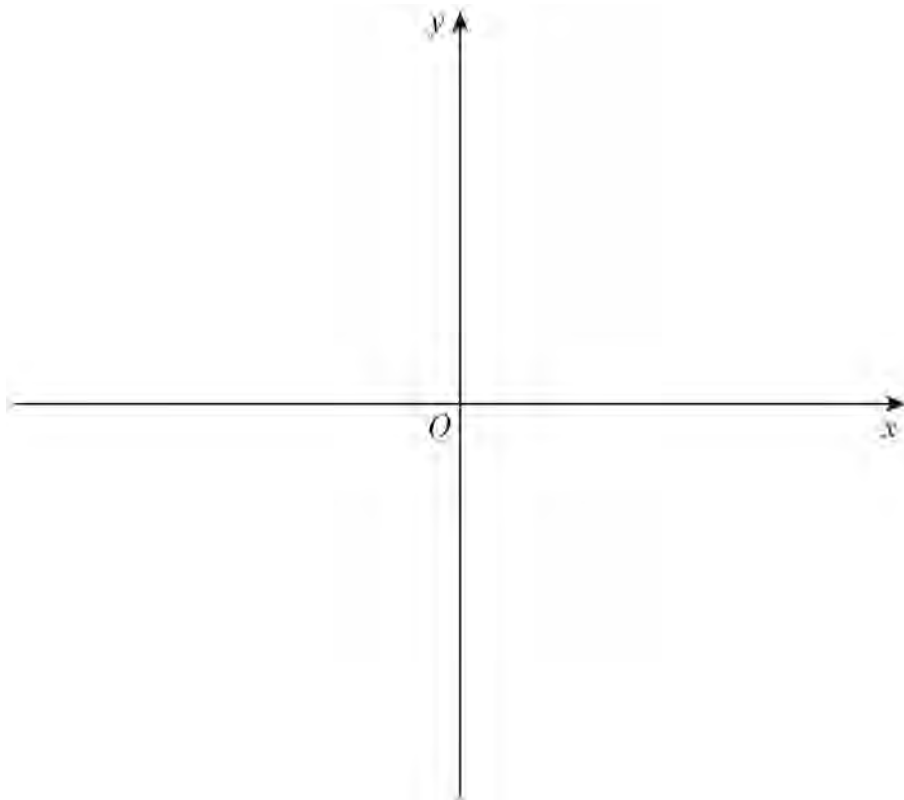
2 (a) State the largest possible domain of f.

[1 mark]

Answer _____

2 (b) On the axes below, sketch the graph of $y = f(x)$.

Show the coordinates of the end-points on the graph.

[2 marks]

2 (c) Describe a sequence of **two** geometrical transformations that maps the graph of
$y = \sin^{-1} x$ onto the graph of $y = \sin^{-1}(2x - 1)$

[4 marks]

2 (d) The root of the equation $\sin^{-1}(2x - 1) + x - 1 = 0$ is α.

Show that α lies between 0.6 and 0.7

[2 marks]

Question 2 continues on the next page

Turn over ▶

2 (e) Use the iterative formula

$$x_{n+1} = \frac{1 + \sin(1 - x_n)}{2} \quad \text{with } x_1 = 0.6$$

to find the values of x_2 and x_3, giving your answers to three decimal places.

[2 marks]

Answer _____

11

3 (a) Find $\int x \ln x \; \mathrm{d}x$

[3 marks]

Answer _____

3 (b) Find $\int \ln x \; \mathrm{d}x$

[3 marks]

Answer _____

6

Turn over ▶

4 (a) The polynomial $f(x)$ is defined by

$$f(x) = 8x^3 + bx^2 + cx + 6$$

where b and c are constants.

When $f(x)$ is divided by $(2x - 1)$ the remainder is 5.25

When $f(x)$ is divided by $(2x - 3)$ the remainder is -3.75

Find the value of b and the value of c.

[4 marks]

$b=$ _____ $c=$ _____

4 (b) Show that $\dfrac{12x^3 - 8x^2 + x + 7}{4x^2 - 1}$ can be written in the form $3x + d + \dfrac{ex + f}{4x^2 - 1}$

where d, e and f are integers.

[4 marks]

8

Turn over ▶

5 (a) Express $\dfrac{12}{9-u^2}$ in the form $\dfrac{A}{3-u}+\dfrac{B}{3+u}$

[2 marks]

Answer _____

5 (b) Use the substitution $u = \sin x$ to find the exact value of $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{12\cos x}{8+\cos^2 x}\, \mathrm{d}x$

Give your answer in the form $\ln q$, where q is a rational number.

[6 marks]

Answer _____

8

Turn over ▶

6 (a) (i) Find the binomial expansion of $(1 + 2x)^{0.5}$ in ascending powers of x up to and including the term in x^2

[2 marks]

Answer _____

6 (a) (ii) Find the binomial expansion of $(1 - 4x)^{-0.5}$ in ascending powers of x up to and including the term in x^2

[2 marks]

Answer _____

6 (b) (i) Hence find the binomial expansion of $\sqrt{\dfrac{1 + 2x}{1 - 4x}}$ in ascending powers of x up to and including the term in x^2

[2 marks]

Answer _____

6 (b) (ii) State the values of x for which the expansion of $\sqrt{\dfrac{1+2x}{1-4x}}$ is valid.

[1 mark]

Answer _____

6 (c) Use your expansion of $\sqrt{\dfrac{1+2x}{1-4x}}$ to find an estimate for $\sqrt{2}$, giving your answer to

three decimal places.

[3 marks]

Answer _____

10

Turn over ▶

7   A curve has equation $y = e^{3x} - 24x$

7 (a)   Find an equation of the tangent to the curve at (0, 1)

[3 marks]

Answer _____

7 (b)   Find the coordinates of the stationary point of the curve, giving your answer in an exact form.

[3 marks]

Answer _____

7 (c) Find $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ and hence determine the nature of this stationary point.

[3 marks]

Answer _____

9

Turn over for the next question

Turn over ▶

8 During a scientific experiment a liquid evaporates.

The rate of change of the evaporated mass, x grams, is directly proportional to the mass of the liquid which remains at time t minutes after the start of the experiment.

Initially there were 80 grams of the liquid.

8 (a) Explain briefly why this information can be represented by the differential equation

$$\frac{dx}{dt} = k(80 - x)$$

[1 mark]

8 (b) After 60 minutes, 30 grams of the liquid has evaporated.

Solve the differential equation in part **(a)** to find the value of k.

[5 marks]

Answer

8 (c) (i) Find the number of grams of the liquid that has evaporated after 120 minutes.

[2 marks]

Answer _____

8 (c) (ii) Find how many minutes it takes for there to be only 10 grams of the liquid remaining.

[3 marks]

Answer _____

11

Turn over ▶

9 The region bounded by the curve $y = \dfrac{1}{2+x}$, the lines $y = 0.2$, $y = 0.25$ and the y-axis is rotated through 2π radians about the **y-axis** to form a solid.

Find the exact value of the volume of the solid generated.

[6 marks]

Answer _____

10 (a) Use the mid-ordinate rule with six strips, to find an estimate for $\int_{1.5}^{3} x^{-x}\,\mathrm{d}x$, giving your answer to three decimal places.

[4 marks]

Answer

10 (b) By taking logarithms of both sides of $y = x^{-x}$ and then using implicit differentiation, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, giving your answer in terms of x only.

[4 marks]

Answer

8

Turn over ▶

11 The point *A* has coordinates (10, 2, −3).

The point *B* has coordinates (2, −2, 5).

11 (a) Find the vector equation of the line *AB* and hence show that it can be written as

$$\mathbf{r} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

[4 marks]

Answer

11 (b) The point *D* has coordinates (−2, 1, 7).

The point *C* lies on the line *AB*.

The line *CD* is perpendicular to the line *AB*.

11 (b) (i) Find the coordinates of *C*.

[5 marks]

Answer _____

11 (b) (ii) Show that the distance $CD = \sqrt{q}$ where q is a constant.

[2 marks]

11 (c) The point $P\,(4 + 2p,\; -1 + p,\; 3 - 2p)$ lies on the line AB such that triangle DCP is isosceles.

Find the possible exact values of p, giving your answers in the form $s + t\sqrt{q}$,

where s and t are constants.

[5 marks]

Answer _____

Turn over ▶

16

12 A curve has equation $6y^2 + 2e^{4x} = y^3 e^x$

12 (a) Given that the curve has a stationary point (p, q), show that $2e^p = q$

[5 marks]

12 (b) Find the exact value of p and the exact value of q.

[4 marks]

Answer _____

9

13 A curve *C* is defined by the parametric equations $x = at^2$, $y = 2at$, where *a* is a constant.

The tangent to the curve at the point *P* $(ap^2, 2ap)$ meets the tangent to the curve at the point *Q* $(aq^2, 2aq)$ at the point *R*.

Given that *p* and *q* vary so that $p^2 + q^2 = 1$, find the Cartesian equation of the curve on which *R* lies, giving your answer in the form $y^2 = f(x)$

[7 marks]

Answer _____

7

END OF QUESTIONS

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