Please check the examination details below	before entering your candidate information
Candidate surname	Other names
Pearson Edexcel nternational Advanced Level	e Number Candidate Number
Friday 8 January	2021
Afternoon (Time: 1 hour 30 minutes)	Paper Reference WFM01/01
Mathematics International Advanced Sub Further Pure Mathematics F	•
You must have: Mathematical Formulae and Statistical	Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit. Inexact answers should be given to three significant figures unless
- otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
 - use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶







1.	(a)	Show that the equation $4x - 2\sin x - 1 = 0$, where x is in radians, has a root α in the interval [0.2, 0.6]
		(2)
	(b)	Starting with the interval [0.2, 0.6], use interval bisection twice to find an interval of width 0.1 in which α lies.
		(3)

Question 1 continued	blank
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	Q1
(Total 5 marks)	



2. Given that $x = \frac{3}{8} + \frac{\sqrt{71}}{8}i$ is a root of the equation

$$4x^3 - 19x^2 + px + q = 0$$

(a) write down the other complex root of the equation.

(1)

Given that x = 4 is also a root of the equation,

(b) find the value of p and the value of q.

(4)

uestion 2 continued	



3. The matrix M is defined by

$$\mathbf{M} = \begin{pmatrix} k+5 & -2 \\ -3 & k \end{pmatrix}$$

(a) Determine the values of k for which M is singular.

(2)

Given that **M** is non-singular,

(b) find \mathbf{M}^{-1} in terms of k.

(2)

Question 3 continued	blank
	Q3
(Total 4 marks)	



4. The equation $2x^2 + 5x + 7 = 0$ has roots α and β

Without solving the equation

(a) determine the exact value of $\alpha^3 + \beta^3$

(3)

(b) form a quadratic equation, with integer coefficients, which has roots

$$\frac{\alpha^2}{\beta}$$
 and $\frac{\beta^2}{\alpha}$

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uestion 4 continued	



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Question 4 continued	
	Q4
(Total 8 marks)	



5. (a) Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that

$$\sum_{r=1}^{n} (r+1)(r+5) = \frac{n}{6}(n+7)(2n+7)$$

for all positive integers n.

(5)

(b) Hence show that

$$\sum_{r=n+1}^{2n} (r+1)(r+5) = \frac{7n}{6}(n+1)(an+b)$$

where a and b are integers to be determined.

(2)

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Question 5 continued		

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Question 5 continued	
	Q5
(Total 7 marks)	



6. The complex number z is defined by

$$z = -\lambda + 3i$$
 where λ is a positive real constant

Given that the modulus of z is 5

(a) write down the value of λ

(1)

(b) determine the argument of z, giving your answer in radians to one decimal place.

(2)

In part (c) you must show detailed reasoning.

Solutions relying on calculator technology are not acceptable.

- (c) Express in the form a + ib where a and b are real,
 - (i) $\frac{z + 3i}{2 4i}$
 - (ii) z^2

(5)

(d) Show on a single Argand diagram the points *A*, *B*, *C* and *D* that represent the complex numbers

$$z, z^*, \frac{z+3i}{2-4i} \text{ and } z^2$$
 (3)



estion 6 continued		



Question 6 continued		

Question 6 continued	blank
	Q6
(Total 11 marks)	
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7. The matrix A is defined by

$$\mathbf{A} = \begin{pmatrix} 4 & -5 \\ -3 & 2 \end{pmatrix}$$

The transformation represented by A maps triangle T onto triangle T'

Given that the area of triangle T is 23 cm²

(a) determine the area of triangle T'

(2)

The point P has coordinates (3p + 2, 2p - 1) where p is a constant. The transformation represented by A maps P onto the point P' with coordinates (17, -18)

(b) Determine the value of p.

(2)

Given that

$$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(c) describe fully the single geometrical transformation represented by matrix **B**

(2)

The transformation represented by matrix A followed by the transformation represented by matrix C is equivalent to the transformation represented by matrix B

(d) Determine C

(3)





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Question 7 continued		

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8. The hyperbola H has Cartesian equation xy = 25

The parabola *P* has parametric equations $x = 10t^2$, y = 20t

The hyperbola H intersects the parabola P at the point A

(a) Use algebra to determine the coordinates of A

(3)

The point B with coordinates (10,20) lies on P

(b) Find an equation for the normal to P at B

Give your answer in the form ax + by + c = 0, where a, b and c are integers to be determined.

(5)

(c) Use algebra to determine, in simplest form, the exact coordinates of the points where this normal intersects the hyperbola $\cal H$

(6)

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uestion 8 continued	



Question 8 continued		

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	Q8
(Total 14 marks)	



9. (i) A sequence of numbers $u_1, u_2, u_3,...$ is defined by

$$u_{n+1} = \frac{1}{3}(2u_n - 1) \qquad u_1 = 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3\left(\frac{2}{3}\right)^n - 1\tag{6}$$

(ii)
$$f(n) = 2^{n+2} + 3^{2n+1}$$

Prove by induction that, for $n \in \mathbb{Z}^+$, f(n) is a multiple of 7

(6)

uestion 9 continued	



Question 9 continued		

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Question 9 continued		blank
		Q9
	(Total 12 marks)	
	TOTAL FOR PAPER: 75 MARKS	•