

Mark Scheme (Results)

Summer 2016

Pearson Edexcel IAL in Further Pure Mathematics 1 (WFM01/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

- 1. The total number of marks for the paper is 75
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- d... or dep dependent
- indep independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper or ag- answer given
- L or d... The second mark is dependent on gaining the first mark

- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.
- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2+bx+c)=(x+p)(x+q)$$
, where $|pq|=|c|$, leading to $x=...$
 $(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to $x=...$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving
$$x^2 + bx + c = 0$$
: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number		Scheme	Notes	Marks			
1.	$\sum_{r=1}^{n} r(r^2 -$	$-3) = \sum_{r=1}^{n} r^3 - 3 \sum_{r=1}^{n} r$					
		$= \frac{1}{4}n^2(n+1)^2 - 3\left(\frac{1}{2}n(n+1)\right)$	Attempts to expand $r(r^2-3)$ and attempts to substitute at least one correct standard formula into their resulting expression.	M1			
			Correct expression (or equivalent)	A1			
			dependent on the previous M mark Attempt to factorise at least $n(n+1)$ having attempted to substitute both the standard formulae	dM1			
		$=\frac{1}{4}n(n+1)\left[n^2+n-6\right]$	{this step does not have to be written]				
		$= \frac{1}{4}n(n+1)\left[n^2 + n - 6\right]$ $= \frac{1}{4}n(n+1)(n+3)(n-2)$	Correct completion with no errors	A1 cso			
				(4)			
			0 4 1 1 1 4	4			
1.	Note		Question 1 Notes of the printed equation without applying the standard	d formulae			
1.	Note		other combination of these numbers is M0A0M0A0				
	Alt	Alternative Method: Obtains	$\sum_{r=1}^{n} r(r^2 - 3) = \frac{1}{4}n(n+1) \Big[n(n+1) - 6 \Big] = \frac{1}{4}n(n+a)e^{-2}$ $2)(1+b)(1+c) \text{ and } n = 2 \implies 0 = \frac{1}{4}(2)(3)(2+b)(2-a)e^{-2}$	(n+b)(n+c)			
				. •)			
	dM1	leading to either $b=-2$, $c=3$ of dependent on the previous M n					
	ulvii	Substitutes in values of n and sol					
	A1		ther combination of these numbers.				
	Note						
	Note	Allow final dM1A1 for $\frac{1}{4}n^4 + \frac{1}{2}n^3 - \frac{5}{4}n^2 - \frac{3}{2}n$ or $\frac{1}{4}n(n^3 + 2n^2 - 5n - 6)$					
		or $\frac{1}{4}(n^4 + 2n^3 - 5n^2 - 6n) \rightarrow \frac{1}{4}$	n(n+1)(n+3)(n-2), from no incorrect working.				
	Note	Give final A0 for eg. $\frac{1}{4}n(n+1)$	$[n^2 + n - 6] \rightarrow = \frac{1}{4}n(n+1)(x+3)(x-2)$ unless red	covered.			

Question Number		Scheme	Notes	Marks		
2.	$P: y^2 = 2$	$8x \text{ or } P(7t^2, 14t)$				
(a)	$(y^2 = 4ax)$	$\Rightarrow a = 7) \Rightarrow S(7,0)$	Accept $(7,0)$ or $x = 7$, $y = 0$ or 7 marked on the <i>x</i> -axis in a sketch	B1 (1)		
(b)	{ <i>A</i> and <i>B</i> l	have x coordinate} $\frac{7}{2}$	Divides their <i>x</i> coordinate from (a) by 2 and substitutes this into the parabola	(1)		
	So $y^2 = 2$	$8\left(\frac{7}{2}\right) \Rightarrow y^2 = 98 \Rightarrow y = \dots$	equation and takes the square root to find $y =$ or applies	MI		
	or $y = \sqrt{(2)^n}$ or	$(7) - 3.5)^2 - (3.5)^2 \left\{ = \sqrt{(10.5)^2 - (3.5)^2} \right\}$	$y = \sqrt{\left(2("7") - \left(\frac{"7"}{2}\right)\right)^2 - \left(\frac{"7"}{2}\right)^2}$	M1		
	-	$\Rightarrow t = \sqrt{0.5} \Rightarrow y = 2(7)\sqrt{0.5}$	or solves $7t^2 = 3.5$ and finds $y = 2(7)$ "their t "			
	$y = (\pm)7$	$\sqrt{2}$	At least one correct exact value of y. Can be un-simplified or simplified.	A1		
	A, B have	coordinates $\left(\frac{7}{2}, 7\sqrt{2}\right)$ and $\left(\frac{7}{2}, -7\sqrt{2}\right)$				
	Area trian	()				
	$\bullet \frac{1}{2}$	$\cdot \left(2(7\sqrt{2})\right)\left(\frac{7}{2}\right)$	dependent on the previous M mark			
	1	7 3.5 3.5 7	A full method for finding	dM1		
	$\frac{1}{2}$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	he area of triangle ABS.			
		$=\frac{49}{2}\sqrt{2}$	Correct exact answer.	A1		
				(4)		
		Question	1 2 Notes			
2. (a)	Note	You can give B1 for part (a) for correct re	elevant work seen in either part (a) or part ((b)		
(b)	1 st M1	Allow a slip when candidates find the x c $0 < \text{their midpoint} < \text{their } a$	oordinate of their midpoint as long as			
	Note	Give 1 st M0 if a candidate finds and uses	y = 98			
	1 st A1	Allow any exact value of either $7\sqrt{2}$, $-7\sqrt{2}$, $\sqrt{98}$, $-\sqrt{98}$, $14\sqrt{0.5}$, awrt 9.9 or awrt -9.9				
	2 nd dM1	Either $\frac{1}{2} \left(2 \times \text{their "} 7\sqrt{2} \text{ "} \right) \left(\text{their } x_{\text{midpoint}} \right)$ or $\frac{1}{2} \left(2 \times \text{their "} 7\sqrt{2} \text{ "} \right) \left(\text{their "} 7 \text{ "} - x_{\text{midpoint}} \right)$				
	Note	Condone area triangle $ABS = \left(7\sqrt{2}\right)\left(\frac{7}{2}\right)$, i.e. $\left(\text{their "}7\sqrt{2}\right)\left(\frac{\text{their "}7}{2}\right)$				
	2 nd A1	Allow exact answers such as $\frac{49}{2}\sqrt{2}$, $\frac{49}{\sqrt{2}}$, $24.5\sqrt{2}$, $\frac{\sqrt{4802}}{2}$, $\sqrt{\frac{4802}{4}}$, $3.5\sqrt{2}$, $49\sqrt{\frac{1}{2}}$				
		or $\frac{7}{2}\sqrt{98}$ but do not allow $\frac{1}{2}(3.5)(2\sqrt{98})$	8 seen by itself			
	Note	Give final A0 for finding 34.64823228	without reference to a correct exact value.			

Question Number	Scheme			Notes	Marks
3.	$f(x) = x^2 + \frac{3}{x} - 1, x < 0$				
(a)	$f'(x) = 2x - 3x^{-2}$	A		ither $x^2 \to \pm Ax$ or $\frac{3}{x} \to \pm Bx^{-2}$ e A and B are non-zero constants.	M1
	$f(-1.5) = -0.75$, $f'(-1.5) = -\frac{13}{3}$		-4.33 or	Correct differentiation $f'(-1.5) = -\frac{13}{3}$ or a correct numerical expression for either $f(-1.5)$ or $f'(-1.5)$ an be implied by later working	A1 B1
	$\left\{\alpha \simeq -1.5 - \frac{f(-1.5)}{f'(-1.5)}\right\} \Rightarrow \alpha \simeq -1.5 - \frac{-0.5}{-4.333}$.75	depe Valid a	endent on the previous M mark ttempt at Newton-Raphson using values of $f(-1.5)$ and $f'(-1.5)$	dM1
	$\left\{ \alpha = -1.67307692 \text{ or } -\frac{87}{52} \right\} \Rightarrow \alpha = -1.67307692$	7		endent on all 4 previous marks -1.67 on their first iteration [gnore any subsequent iterations)	A1 cso cao
	Correct differentiation followed by		ct answer	scores full marks in (a)	
	Correct answer with <u>no</u> v	working 	scores no	marks in (a)	(5)
(b) Way 1	f(-1.675) = 0.01458022 f(-1.665) = -0.0295768	,	within ±0	a suitable interval for x , which is .005 of their answer to (a) and at ast one attempt to evaluate $f(x)$.	(5) M1
	Sign change (positive, negative) (and $f(x)$ is continuous) therefore (a root) $\alpha = -1.67$ (2 dp	p)		values correct awrt (or truncated) 1 sf, sign change and conclusion.	A1 cso
				Q7	(2)
(b)	Alt 1: Applying Newton-Raphson again E	g. Using	$\alpha = -1.6$	$7, -1.673 \text{ or } -\frac{87}{52}$	
Way 2	• $\alpha \simeq -1.67 - \frac{-0.007507185629}{-4.415692926} \left\{ = -0.007507185629 \right\}$ • $\alpha \simeq -1.673 - \frac{0.005743106396}{-4.41783855} \left\{ = -0.006082942257 \right\}$ • $\alpha \simeq -\frac{87}{52} - \frac{0.006082942257}{-4.417893838} \left\{ = -1.006082942257 \right\}$	-1.67170	0019}	Evidence of applying Newton- Raphson for a second time on their answer to part (a)	M1
	So $\alpha = -1.67 (2 dp)$			$\alpha = -1.67$	A1
		1			(2)
					7

		Question 3 Notes	
3. (a)	Note	Incorrect differentiation followed by their estimate of α with no evid	ence of applying the
		NR formula is final dM0A0.	
	B1	B1 can be given for a correct numerical expression for either $f(-1.5)$	or $f'(-1.5)$
		Eg. either $(-1.5)^2 + \frac{3}{(-1.5)} - 1$ or $2(-1.5) - \frac{3}{(-1.5)^2}$ are fine for B1.	
	Final	This mark can be implied by applying at least one correct value of eit	ner $f(-1.5)$ or $f'(-1.5)$
	dM1	in $-1.5 - \frac{f(-1.5)}{f'(-1.5)}$. So just $-1.5 - \frac{f(-1.5)}{f'(-1.5)}$ with an incorrect answer	er and no other evidence
		scores final dM0A0.	
	Note	Give final dM0 for applying $1.5 - \frac{f(-1.5)}{f'(-1.5)}$ without first quoting the	correct N-R formula.
3. (b)	A1	Way 1: correct solution only	. 1) 1 6 1 21
		Candidate needs to state both of their values for $f(x)$ to awrt (or trun	=
		a reason and conclusion. Reference to change of sign or eg. f(-1.6	
		or a diagram or < 0 and > 0 or one positive, one negative are suffi- be a (minimal, not incorrect) conclusion, eg. $\alpha = -1.67$, root (or α or	
		and a square are all acceptable. Ignore the presence or absence of any	•
			•
		A minimal acceptable reason and conclusion is "change of sign, hence No explicit reference to 2 decimal places is required.	e 100t .
	Note	Stating "root is in between -1.675 and -1.665" without some refere	$\alpha = -1.67 \text{ is not}$
	11000	sufficient for A1	1.07 15 1100
	Note	Accept 0.015 as a correct evaluation of $f(-1.675)$	
	A1	Way 2: correct solution only	
		Their conclusion in Way 2 needs to convey that they understand that	$\alpha = -1.67$ to 2 decimal
		places. Eg. "therefore my answer to part (a) [which must be -1.67] is	s correct" is fine for A1.
	Note	$-1.67 - \frac{f(-1.67)}{f'(1.67)} = -1.67(2 \text{ dp})$ is sufficient for M1A1 in part (b).	
	Note	The root of $f(x) = 0$ is -1.67169988 , so candidates can also choose	x_1 which is less than
		-1.67169988 and choose x_2 which is greater than -1.67169988	with both x_1 and x_2 lying
		in the interval $\begin{bmatrix} -1.675, -1.665 \end{bmatrix}$ and evaluate $f(x_1)$ and $f(x_2)$.	1 2
3. (b)	Note	Helpful Table	
3. (6)	11000	x $f(x)$	
		-1.675 0.014580224	
		-1.674 0.010161305	
		-1.673 0.005743106	
		-1.672 0.001325627	
		-1.671 -0.003091136	
		-1.670 -0.007507186	
		-1.669 -0.011922523	
		-1.668 -0.016337151	
		-1.667 -0.020751072	
		-1.666 -0.025164288	
		-1.665 -0.029576802	

Question Number		Scheme		Notes	Marks
4.	$\mathbf{A} = \begin{pmatrix} k \\ -1 \end{pmatrix}$	$\binom{3}{k+2}$, where <i>k</i> is a constant and let §	$g(k) = k^2 + 2k +$	3	
(a)	$\left\{ \det(\mathbf{A}) = \right.$	$=$ $k(k+2)+3$ or k^2+2k+3	Correct det(A), un-simplified or simplified	B1
Way 1	=	$(k+1)^2 - 1 + 3$	Att	tempts to complete the square [usual rules apply]	M1
	=	$(k+1)^2 + 2 > 0$		$(k+1)^2 + 2$ and > 0	A1 cso
(a)	∫det(∧) =	$= \begin{cases} k(k+2) + 3 \text{ or } k^2 + 2k + 3 \end{cases}$	Correct det(A), un-simplified or simplified	(3) B1
Way 2		$= \begin{cases} 2^2 - 4(1)(3) \end{cases}$			M1
	All of	- \(\frac{2}{2} \)	Аррис	es " $b^2 - 4ac$ " to their $\det(\mathbf{A})$	IVII
		$a^2 - 4ac = -8 < 0$			
		ome reference to $k^2 + 2k + 3$ being above	ve the <i>x</i> -axis	Committee on locking	A 1
	• so	$o \det(\mathbf{A}) > 0$	T	Complete solution	A1 cso
(a)	g(k) = d	$et(\mathbf{A}) = \begin{cases} k(k+2) + 3 \text{ or } k^2 + 2k + 3 \end{cases}$	Correct det(A), un-simplified or simplified	(3) B1
Way 3		$k + 2 = 0 \Rightarrow k = -1$		alue of k for which $g'(k) = 0$	
	$g_{\min} = (-1)$	$(1)^2 + 2(-1) + 3$		tutes this value of k into $g(k)$	M1
		so $det(\mathbf{A}) > 0$	8	$g_{\min} = 2$ and states $\det(\mathbf{A}) > 0$	A1 cso
					(3)
(b)	$\mathbf{A}^{-1} = \frac{1}{k^2}$	$\frac{1}{\binom{2}{2}+2k+3}\binom{k+2}{1}\binom{k+2}{k}$		$\frac{1}{\text{their det}(\mathbf{A})} \begin{pmatrix} k+2 & -3 \\ 1 & k \end{pmatrix}$	M1
				Correct answer in terms of k	A1
					(2)
		Ques	stion 4 Notes		
4. (a)	B1	Also allow $k(k+2)-3$			
	Note	Way 2: Proving $b^2 - 4ac = -8 < 0$	-		
	Note	To gain the final A1 mark for Way 2,	candidates need	1 to show $b^2 - 4ac = -8 < 0$ a	nd make
		some reference to $k^2 + 2k + 3$ being a positive or evaluates $det(\mathbf{A})$ for any quadratic curve that is above the x-ax	value of k to giv	e a positive result or sketches	
	Note	Attempting to solve $det(\mathbf{A}) = 0$ by approximation by the solution \mathbf{A} and \mathbf{A} by approximation \mathbf{A} by a proximation \mathbf{A} by a proximati		_	$\sqrt{2}$ i
	1,000	is enough to score the M1 mark for W			
		some reference to $k^2 + 2k + 3$ being a positive or evaluates $det(\mathbf{A})$ for any			
		quadratic curve that is above the x -ax	•	•	
(b)	A1	Allow either $\frac{1}{(k+1)^2 + 2} \begin{pmatrix} k+2 & -3\\ 1 & k \end{pmatrix}$	(·····	t.

Question Number		Scheme	Notes	Marks
5.	$2z + z^* =$	$=\frac{3+4i}{7+i}$		
Way 1	$\left\{2z+z^*=\right.$	$= \begin{cases} 2(a+ib) + (a-ib) \end{cases}$	Left hand side = $2(a+ib) + (a-ib)$ Can be implied by eg. $3a + ib$ Note: This can be seen anywhere in their solution	B1
	= -	$\frac{(3+4i)}{(7+i)}\frac{(7-i)}{(7-i)}$	Multiplies numerator and denominator of the right hand side by $7 - i$ or $-7 + i$	M1
	=	25 + 25i 50	Applies $i^2 = -1$ to and collects like terms to give right hand side = $\frac{25 + 25i}{50}$ or equivalent	A1
		$\mathbf{i}b = \frac{1}{2} + \frac{1}{2}\mathbf{i}$	dependent on the previous B and M marks Equates either real parts or imaginary parts to give at least one of $a =$ or $b =$	ddM1
	$\Rightarrow a = \frac{1}{6}$	$b = \frac{1}{2}$ or $z = \frac{1}{6} + \frac{1}{2}i$	Either $a = \frac{1}{6}$ and $b = \frac{1}{2}$ or $z = \frac{1}{6} + \frac{1}{2}i$	A1
				(5)
Way 2	`	$= \begin{cases} 2(a+ib) + (a-ib) \end{cases}$	Left hand side = $2(a+ib) + (a-ib)$ Can be implied by eg. $3a + ib$	B1
	(3a + ib)	$(7 + i) = \dots$	Multiplies their $(3a + ib)$ by $(7 + i)$	M1
	21 <i>a</i> + 3 <i>a</i> i	$i + 7bi - b = \dots$	Applies $i^2 = -1$ to give left hand side = $21a + 3ai + 7bi - b$	A1
		(a-b) + (3a+7b) = 3 + 4i (a-b) = 3, 3a+7b=4	dependent on the previous B and M marks Equates both real parts and imaginary parts to give at least one of $a =$ or $b =$	ddM1
	$\Rightarrow a = \frac{1}{6}$	$b = \frac{1}{2}$ or $z = \frac{1}{6} + \frac{1}{2}i$	Either $a = \frac{1}{6}$ and $b = \frac{1}{2}$ or $z = \frac{1}{6} + \frac{1}{2}i$	A1
				(5)
				5
			Question 5 Notes	
5.	Note	Some candidates may let $z = x$	$z + iy$ and $z^* = x - iy$.	
		So apply the mark scheme with	$h x \equiv a \text{ and } y \equiv b.$	
	Note	For the final A1 mark, you can	accept exact equivalents for a, b .	

Question Number	Scheme		Notes	Marks
6.	$H: xy = 25$, $P\left(5t, \frac{5}{t}\right)$ is a general point on			
(a)	Either $5t \left(\frac{5}{t} \right) = 25$ or $y = \frac{25}{x} = \frac{25}{5t} = \frac{25}{5t}$	$=\frac{5}{t}$ or	$x = \frac{25}{y} = \frac{25}{\frac{5}{t}} = 5t$ or states $c = 5$	B1
				(1)
(b)	$y = \frac{25}{x} = 25x^{-1} \Rightarrow \frac{dy}{dx} = -25x^{-2} = -\frac{25}{x^2}$		$\frac{dy}{dx} = \pm k x^{-2}$ where <i>k</i> is a numerical value	
	$xy = 25 \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$		Correct use of product rule. The sum of two terms, one of which is correct.	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \cdot \frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{5}{t^2} \left(\frac{1}{5}\right)$		$\frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{1}{\mathrm{their}} \frac{\mathrm{d}x}{\mathrm{d}t}$	
	$\left\{ \text{At } A, \ t = \frac{1}{2}, \ x = \frac{5}{2}, \ y = 10 \right\} \Rightarrow \frac{\text{d}y}{\text{d}x} = -4$		Correct numerical gradient at A, which is found using calculus. Can be implied by later working	A1
	So, $m_N = \frac{1}{4}$	Арр	polies $m_N = \frac{-1}{m_T}$, to find a numerical m_N , where m_T is found from using calculus. Can be implied by later working	M1
	$\bullet y - 10 = \frac{1}{4} \left(x - \frac{5}{2} \right)$		Correct line method for a normal where a numerical $m_N (\neq m_T)$ is found	M1
	• $10 = \frac{1}{4} \left(\frac{5}{2} \right) + c \Rightarrow c = \frac{75}{8} \Rightarrow y = \frac{1}{4}$	$x + \frac{75}{8}$	from using calculus. Can be implied by later working	IVII
	leading to $8y - 2x - 75 = 0$ (*)		Correct solution only	A1
				(5)
(c)	$y = \frac{25}{x} \implies 8\left(\frac{25}{x}\right) - 2x - 75 = 0$			
	or $x = 5t, y = \frac{5}{t} = \frac{5}{t}$	$\Rightarrow 8(5t)$	$-2\left(\frac{5}{t}\right) - 75 = 0$	M1
	Substitutes $y = \frac{25}{x}$ or $x = \frac{25}{y}$ or x		·	
	or their normal equation to obtain an			
			or $2t^2 + 15t - 8 = 0$ or $10t^2 + 75t - 40 = 0$	
	$(2x-5)(x+40) = 0 \Rightarrow x = \dots$ or $(y-10)(8y+5) = 0 \Rightarrow y = \dots$ or $(2t-1)(t+8) = 0 \Rightarrow t = \dots$ dependent on the previous M mark Correct attempt of solving a 3TQ to find either $x = \dots$, $y = \dots$ or $t = \dots$			
	Finds at least one of e	ither x	$=-40 \text{ or } y = -\frac{5}{8}$	A1
	$B\left(-40, -\frac{5}{8}\right)$ sta		correct coordinates (If coordinates are not y can be paired together as $x =, y =$)	A1
		· 		(4)
				10

		Question 6 Notes
6. (a)	Note	A conclusion is not required on this occasion in part (a).
	B1	Condone reference to $c = 5$ (as $xy = c^2$ and $\left(ct, \frac{c}{t}\right)$ are referred in the Formula book.)
(b)	Note	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \cdot \frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{5}{t^2} \left(\frac{1}{5}\right) = -\frac{1}{t^2} \Rightarrow m_N = t^2 \Rightarrow y - 10 = t^2 \left(x - \frac{5}{2}\right)$ scores only the first M1.
		When $t = \frac{1}{2}$ is substituted giving $y - 10 = \frac{1}{4} \left(x - \frac{5}{2} \right)$
		the response then automatically gets A1(implied) M1(implied) M1
(c)	Note	You can imply the final three marks (dM1A1A1) for either
		$\bullet 8\left(\frac{25}{x}\right) - 2x - 75 = 0 \longrightarrow \left(-40, -\frac{5}{8}\right)$
		$\bullet 8y - 2\left(\frac{25}{y}\right) - 75 = 0 \rightarrow \left(-40, -\frac{5}{8}\right)$
		$\bullet 8(5t) - 2\left(\frac{5}{t}\right) - 75 = 0 \rightarrow \left(-40, -\frac{5}{8}\right)$
		with no intermediate working.
		You can also imply the middle dM1A1 marks for either
		• $8\left(\frac{25}{x}\right) - 2x - 75 = 0 \to x = -40$
		• $8y - 2\left(\frac{25}{y}\right) - 75 = 0 \to y = -\frac{5}{8}$
		• $8(5t) - 2(\frac{5}{t}) - 75 = 0 \rightarrow x = -40 \text{ or } y = -\frac{5}{8}$
		with no intermediate working.
	Note	Writing $x = -40$, $y = -\frac{5}{8}$ followed by $B\left(40, \frac{5}{8}\right)$ or $B\left(-\frac{5}{8}, -40\right)$ is final A0.
	Note	Ignore stating $B\left(\frac{5}{2}, 10\right)$ in addition to $B\left(-40, -\frac{5}{8}\right)$

Question Number	Scheme		Notes	Marks
7. (a)	Rotation		Rotation	B1
	67 degrees (anticlockwise) awrt 67 degrees, awrt 1.2, truncated 1.1 (a		er $\arctan\left(\frac{12}{5}\right)$, $\tan^{-1}\left(\frac{12}{5}\right)$, $\sin^{-1}\left(\frac{12}{13}\right)$, $\cos^{-1}\left(\frac{5}{13}\right)$, grees, awrt 1.2, truncated 1.1 (anticlockwise), 293 degrees clockwise or awrt 5.1 clockwise	B1 o.e.
	about (0,0)	Т	The mark is dependent on at least one of the previous B marks being awarded. About $(0,0)$ or about O or about the origin	dB1
	Note: Give 2 nd B0 for 67 degrees	clockwise o.e.		(3)
(b)	$\left\{\mathbf{Q} = \right\} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$		Correct matrix	B1
				(1)
(c)	$\left\{ \mathbf{R} = \mathbf{PQ} = \right\} \begin{pmatrix} \frac{5}{13} & -\frac{12}{13} \\ \frac{12}{12} & \frac{5}{12} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; =$	$\begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{13}{13} \end{pmatrix}$	Multiplies P by their Q in the correct order and finds at least one element	M1
` ,	$\left(\begin{array}{ccc} \frac{12}{13} & \frac{3}{13} \right) \left(\begin{array}{ccc} 1 & 0 \end{array}\right)$	$\left(\begin{array}{cc} \frac{3}{13} & \frac{12}{13} \right)$	Correct matrix	A1
				(2)
(d) Way 1	$\begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix} \begin{pmatrix} x \\ kx \end{pmatrix} = \begin{pmatrix} x \\ kx \end{pmatrix}$	Allow x bei	ing the equation "their matrix \mathbf{R} " $\begin{bmatrix} x \\ kx \end{bmatrix} = \begin{bmatrix} x \\ kx \end{bmatrix}$ ing replaced by any non-zero number eg. 1. lied by at least one correct ft equations below.	M1
	$-\frac{12}{13}x + \frac{5kx}{13} = x \text{ or } \frac{5}{13}x + \frac{12kx}{13}$		Uses their matrix equation to form an	M1
	So <i>k</i> = 5		dependent on only the previous M mark $k = 5$	A1 cao
	Dependent on all previous marks	s being scored	d in this part. Either	
	• Solves both $-\frac{12}{13}x + \frac{5kx}{13}$ • Finds $k = 5$ and checks that	at it is true for	13	A1 cso
	• Confirms that $ \begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix} $	5x = 5x		
	12			(4)
(d)	Either $\cos 2\theta = -\frac{12}{13}$, $\sin 2\theta = \frac{5}{13}$			M1
Way 2			hod of finding 2θ , then θ and applying $\tan \theta$	M1
	$\left\{k = \right\} \tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$		$\tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ or $\tan\left(\operatorname{awrt} 78.7^{\circ}\right)$ or	A1
			tan(awrt 1.37). Can be implied.	
	So $k = 5$		k = 5 by a correct solution only	A1
				(4)
				10

		Question 7 Notes
7. (a)	Note	Condone "Turn" for the 1st B1 mark.
	Note	Penalise the first B1 mark for candidates giving a combination of transformations.
(c)	Note	Allow 1 st M1 for eg. "their matrix \mathbf{R} " $\begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} 1 \\ k \end{pmatrix}$ or "their matrix \mathbf{R} " $\begin{pmatrix} k \\ k^2 \end{pmatrix} = \begin{pmatrix} k \\ k^2 \end{pmatrix}$
		or "their matrix \mathbf{R} " $\begin{pmatrix} \frac{1}{k} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{k} \\ 1 \end{pmatrix}$ or equivalent
	Note	$y = (\tan \theta)x : \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} = \begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix}$

Question Number	Scheme		Notes	Marks		
8.	$f(z) = z^4 + 6z^3 + 76z^2 + az + b$, a, b a	$+6z^3 + 76z^2 + az + b$, a, b are real constants. $z_1 = -3 + 8i$ is given.				
(a)	-3-8i		-3-8i	B1		
(b)	or any va		tempt to expand $(z-(-3+8i))(z-(-3-8i))$ valid method to establish a quadratic factor $-3\pm 8i \Rightarrow z+3=\pm 8i \Rightarrow z^2+6z+9=-64$ or sum of roots -6 , product of roots 73 to give $z^2 \pm (\text{sum})z + \text{product}$	M1	(1)	
			$\frac{z^2 + 6z + 73}{\text{Attempts to find the other quadratic factor.}}$			
	$f(z) = (z^2 + 6z + 73)(z^2 + 3)$	e	g. using long division to get as far as $z^2 +$ or eg. $f(z) = (z^2 + 6z + 73)(z^2 +)$	M1		
			z^2+3	A1		
	$\left\{z^2 + 3 = 0 \Rightarrow z = \right\} \pm \sqrt{3}i$	Corre	dependent on only the previous M mark ct method of solving the 2 nd quadratic factor	dM1		
	($\sqrt{3}i$ and $-\sqrt{3}i$	A1		
					(6)	
(c)	8		 Criteria -3±8i plotted correctly in quadrants 2 and 3 with some evidence of symmetry Their other two <i>complex roots</i> (which are found from solving their 2nd quadratic in (b)) are plotted correctly with some evidence of symmetry about the <i>x</i>-axis 			
	-3 $-\sqrt{3}$ Re -8		Satisfies at least one of the two criteria Satisfies both criteria with some indication of scale or coordinates stated. All points (arrows) must be in the correct positions relative to each other.	B1 ft		
	1				(2)	
		Опе	stion 8 Notes		9	
8. (b)	Note Give 3^{rd} M1 for $z^2 + k = 0$.		at least one of either $z = \sqrt{k}$ i or $z = -\sqrt{k}$	i		
` '	Note Give 3^{rd} M0 for $z^2 + k = 0$,					
	Note Give 3^{rd} M0 for $z^2 + k = 0$,		· · · · · · · · · · · · · · · · · · ·			
	Note Candidates do not need to fi					

Question Number	Scheme		Notes	Marks
9.	$2x^2 + 4$	4x - 3	= 0 has roots α , β	
(a)	$\alpha + \beta = -\frac{4}{2} \text{ or } -2, \ \alpha\beta = -\frac{3}{2}$		Both $\alpha + \beta = -\frac{4}{2}$ and $\alpha\beta = -\frac{3}{2}$. This may be seen or implied anywhere in this question.	B1
(i)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \dots$		Use of a correct identity for $\alpha^2 + \beta^2$ (May be implied by their work)	M1
	$= (-2)^2 - 2\left(-\frac{3}{2}\right) = 7$		7 from correct working	A1 cso
(ii)	$\alpha^{3} + \beta^{3} = (\alpha + \beta)^{3} - 3\alpha\beta(\alpha + \beta) = \dots$	•••	Use of an appropriate and correct	
	or $= (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) = \dots$		identity for $\alpha^3 + \beta^3$ (May be implied by their work)	M1
	$= (-2)^3 - 3(-\frac{3}{2})(-2) = -17$ or $= (-2)(7\frac{3}{2}) = -17$		-17 from correct working	A1 cso
				(5)
(b)	$Sum = \alpha^{2} + \beta + \beta^{2} + \alpha$ $= \alpha^{2} + \beta^{2} + \alpha + \beta$ $= 7 + (-2) = 5$		Uses at least one of their $\alpha^2 + \beta^2$ or $\alpha + \beta$ in an attempt to find a numerical value for the sum of $(\alpha^2 + \beta)$ and $(\beta^2 + \alpha)$	M1
	Product = $(\alpha^2 + \beta)(\beta^2 + \alpha)$]	Expands $(\alpha^2 + \beta)(\beta^2 + \alpha)$ and uses at least one of	
	$= (\alpha\beta)^2 + \alpha^3 + \beta^3 + \alpha\beta$	thei	r $\alpha\beta$ or $\alpha^3 + \beta^3$ in an attempt to find a numerical	M1
	$= \left(-\frac{3}{2}\right)^2 - 17 - \frac{3}{2} = -\frac{65}{4}$		value for the product of $(\alpha^2 + \beta)$ and $(\beta^2 + \alpha)$	
	$x^2 - 5x - \frac{65}{4} = 0$		Applies $x^2 - (\text{sum})x + \text{product (Can be implied)}$ ("= 0" not required)	M1
	$4x^2 - 20x - 65 = 0$		Any integer multiple of $4x^2 - 20x - 65 = 0$, including the "= 0"	A1
				(4)
			$\alpha^2 + \beta$ and $\beta^2 + \alpha$ explicitly	
(b)	Eg. Let $\alpha = \frac{-4 + \sqrt{40}}{4}$, $\beta = \frac{-4 + \sqrt{40}}{4}$	40 and	d so $\alpha^2 + \beta = \frac{5 - 3\sqrt{10}}{2}$, $\beta^2 + \alpha = \frac{5 + 3\sqrt{10}}{2}$	
	$\left \left(x - \left(\frac{5 - 3\sqrt{10}}{2} \right) \right) \left(x - \left(\frac{5 + 3\sqrt{10}}{2} \right) \right) \right $		Uses $(x - (\alpha^2 + \beta))(x - (\beta^2 + \alpha))$	M1
	7,(7,		with exact numerical values. (May expand first) Attempts to expand	
	$= x^2 - \left(\frac{5 + 3\sqrt{10}}{2}\right)x - \left(\frac{5 - 3\sqrt{10}}{2}\right)x + \left(\frac{5 - 3\sqrt{10}}{2}\right)\left(\frac{5 + 3\sqrt{10}}{2}\right)$ using exact numerical values for $\alpha^2 + \beta$ and		M1	
	$\Rightarrow x^2 - 5x - \frac{65}{4} = 0$		$\beta^2 + \alpha$ Collect terms to give a 3TQ.	M1
			("=0" not required)	1111
	$4x^2 - 20x - 65 = 0$		Any integer multiple of $4x^2 - 20x - 65 = 0$, including the "= 0"	A1
				(4)
<u> </u>				9

		Question 9 Notes			
9. (a)	1st A1	$\alpha + \beta = 2$, $\alpha\beta = -\frac{3}{2} \Rightarrow \alpha^2 + \beta^2 = 4 - 2\left(-\frac{3}{2}\right) = 7$ is M1A0 cso			
(a)	Note	Finding $\alpha + \beta = -2$, $\alpha\beta = -\frac{3}{2}$ by writing down or applying $\frac{-4 + \sqrt{40}}{4}$, $\frac{-4 + \sqrt{40}}{4}$ but then			
		writing $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4 + 3 = 7$ and $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = -8 - 9 = -17$			
		scores B0M1A0M1A0 in part (a).			
	Note Applying $\frac{-4 + \sqrt{40}}{4}$, $\frac{-4 + \sqrt{40}}{4}$ explicitly in part (a) will score B0M0A0M0A0 Eg: Give no credit for $\left(\frac{-4 + \sqrt{40}}{4}\right)^2 + \left(\frac{-4 + \sqrt{40}}{4}\right)^2 = 7$				
		or for $\left(\frac{-4 + \sqrt{40}}{4}\right)^3 + \left(\frac{-4 + \sqrt{40}}{4}\right)^3 = -17$			
(b)	Note	Candidates are allowed to apply $\frac{-4 + \sqrt{40}}{4}$, $\frac{-4 + \sqrt{40}}{4}$ explicitly in part (b).			
	Note	A correct method leading to a candidate stating $a = 4$, $b = -20$, $c = -65$ without writing a final answer of $4x^2 - 20x - 65 = 0$ is final M1A0			

Question Number	Scheme	Notes	Marks		
10.	$u_1 = 5, \ u_{n+1} = 3u_n + 2, \ n \ge 1$. Required to prove the result, $u_n = 2 \times (3)^n - 1, \ n \in \square^+$				
(i)	$n=1$: $u_1=2(3)-1=5$	$u_1 = 2(3) - 1 = 5$ or $u_1 = 6 - 1 = 5$	B1		
	(Assume the result is true for $n = k$)				
	$u_{k+1} = 3(2(3)^k - 1) + 2$	Substitutes $u_k = 2(3)^k - 1$ into $u_{k+1} = 3u_k + 2$	M1		
		dependent on the previous M mark	dM1		
	$=2(3)^{k+1}-1$	Expresses u_{k+1} in term of 3^{k+1}	divii		
		$u_{k+1} = 2(3)^{k+1} - 1$ by correct solution only	A1		
	If the result is <u>true for $n = k$</u> , then it is <u>true for $n = k + 1$</u> . As the result has been shown to be				
	true for $n = 1$, then the result is true for all n				
			5		
	Required to prove the result	$\sum_{r=1}^{n} \frac{4r}{3^r} = 3 - \frac{(3+2n)}{3^n} , n \in \square^+$			
(ii)	$n=1: LHS = \frac{4}{3}, RHS = 3 - \frac{5}{3} = \frac{4}{3}$	Shows or states both LHS = $\frac{4}{3}$ and RHS = $\frac{4}{3}$	B1		
	3 3	or states LHS = RHS = $\frac{4}{3}$			
	(Assume the result is true for $n = k$)				
	$\sum_{r=1}^{k+1} \frac{4r}{3^r} = 3 - \frac{(3+2k)}{3^k} + \frac{4(k+1)}{3^{k+1}}$	Adds the $(k+1)^{th}$ term to the sum of k terms	M1		
		dependent on the previous M mark			
	3(3+2k) $4(k+1)$	Makes 3^{k+1} or $(3)3^k$	dM1		
	$=3-\frac{3(3+2k)}{3^{k+1}}+\frac{4(k+1)}{3^{k+1}}$	a common denominator for their fractions.			
	3	Correct expression with common denominator 3^{k+1} or $(3)3^k$ for their fractions.	A1		
	$= 3 - \left(\frac{3(3+2k) - 4(k+1)}{3^{k+1}}\right) = 3 - \left(\frac{5+2k}{3^{k+1}}\right)$				
	$= 3 - \frac{(3+2(k+1))}{3^{k+1}}$ 3 - $\frac{(3+2(k+1))}{3^{k+1}}$ by correct solution only				
	If the result is <u>true</u> for $n = k$, then it is <u>true</u> for $n = k + 1$. As the result has been shown to be				
	true for $n = 1$, then the result is true for all n				
			11		
(1) 0 (11)	Question 10 Notes				
(i) & (ii)	Note Final A1 for parts (i) and (ii) is dependent on all previous marks being scored in the				
	It is gained by candidates conveying the ideas of all four underlined points either at the end of their solution or as a narrative in their solution.				
(i)	Note $u_1 = 5$ by itself is not sufficient for the 1 st B1 mark in part (i).				
(1)	· · · · · · · · · · · · · · · · · ·				
(ii)	Note $u_1 = 3 + 2$ without stating $u_1 = 2(3) - 1 = 5$ or $u_1 = 6 - 1 = 5$ is B0 Note LHS = RHS by itself is not sufficient for the 1 st B1 mark in part (ii).				
(11)	110te 2115 1415 by itself is not suffic	noncior die 1 Bi mark in part (ii).			