Write your name here		
Surname	Other r	names
Pearson Edexcel International Advanced Level	Centre Number	Candidate Number
Further Pu Mathema Advanced/Advance	tics F3	
Monday 26 June 2017 – Aft Time: 1 hour 30 minutes	ternoon	Paper Reference WFM03/01

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for each question are shown in brackets
 use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶



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1. Solve the equation	
$18\cosh x + 14\sinh x = 11 + e^x$	
Give your answers in the form $\ln a$, where a is rational.	
	(5)

Question 1 continued		blank
Question 1 continued		
		Q1
	(Total 5 marks)	



2. $\mathbf{A} = \begin{pmatrix} -1 & 3 & a \\ 2 & 0 & 1 \\ 1 & -2 & 1 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 2 & 0 & 4 \\ 3 & -2 & 3 \\ 1 & 2 & b \end{pmatrix}$

where a and b are constants.

(a) Write down \mathbf{A}^{T} in terms of a.

(1)

(b) Calculate AB, giving your answer in terms of a and b.

(2)

(c) Hence show that

$$(\mathbf{A}\mathbf{B})^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$$

(3)

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uestion 2 continued	



Question 2 continued		

Question 2 continued		b
		Q2
	(Total 6 marks)	



3. Given that

$$y = x - \operatorname{artanh}\left(\frac{2x}{1+x^2}\right)$$

(a) show that

$$1 - \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{1 - x^2}$$

where k is a constant to be found.

(4)

(b) Hence, or otherwise, show that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + x \left(1 - \frac{\mathrm{d}y}{\mathrm{d}x} \right)^2 = 0$$

(4)

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Question 3 continued	



Question 3 continue	ed		

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Question 3 continued	
	Q3
(Total 8 marks)	



- 4.
 - (a) Show that 6 is an eigenvalue of the matrix **M** and find the other two eigenvalues of M.
- **(4)**
- (b) Find a normalised eigenvector corresponding to the eigenvalue 6
- **(4)**



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Question 4 continued		

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	(Total 8 marks)	



5. $I_n = \int \csc^n x dx, \quad 0 < x < \frac{\pi}{2}, \quad n \geqslant 0$

(a) Show that, for $n \ge 2$

$$I_n = \frac{n-2}{n-1} I_{n-2} - \frac{1}{n-1} \cot x \csc^{n-2} x$$

(b) Hence, or otherwise, find

$$\int \csc^4 x \, \mathrm{d}x$$

giving your answer in terms of $\cot x$.

(5)

(4)



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		Q5
	(Total 9 marks)	



6. The hyperbola *H* has equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

and the ellipse E has equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

where a > b > 0

The line *l* is a tangent to **hyperbola** *H* at the point $P(a \sec \theta, b \tan \theta)$, where $0 < \theta < \frac{\pi}{2}$

(a) Using calculus, show that an equation for l is

$$bx \sec \theta - ay \tan \theta = ab$$

(4)

Given that the point F is the focus of **ellipse** E for which x > 0 and that the line l passes through F,

(b) show that l is parallel to the line y = x

(5)



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Question 6 continued	
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Question 6 continued	

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		Q6
(Total	9 marks)	



7. (a) Find

$$\int \frac{5+x}{\sqrt{4-3x^2}} \, \mathrm{d}x$$

(b) Hence find the exact value of

$$\int_0^1 \frac{5+x}{\sqrt{4-3x^2}} \, \mathrm{d}x$$

giving your answer in the form $p\pi\sqrt{3}+q$, where p and q are rational numbers to be found.

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(31)

(5)

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Question 7 continued	
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Question 7 continued	

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	Q7	
(Total 8 mar		



8. The curve C has parametric equations

$$x = \theta - \sin \theta$$
, $y = 1 - \cos \theta$, $0 \le \theta \le 2\pi$

The curve C is rotated through 2π radians about the x-axis. The area of the curved surface generated is given by S.

(a) Show that

$$S = 2\pi\sqrt{2} \int_0^{2\pi} \left(1 - \cos\theta\right)^{\frac{3}{2}} d\theta$$
 (4)

(b) Hence find the exact value of S.

6)
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Question 8 continued	



Question 8 continued	

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	Q8
(Total 10 marks)	



9	With respect to a fixed origin O , the points $A(-1, 5, 1)$, $B(1, 0, 3)$, $C(2, -1, 2)$ and $D(3, 6, -1)$ are the vertices of a tetrahedron.
	(a) Find the volume of the tetrahedron <i>ABCD</i> . (4)
	The plane Π contains the points A , B and C .
	(b) Find a cartesian equation of Π . (4)
	The point T lies on the plane Π .
	The line DT is perpendicular to Π .
	(c) Find the exact coordinates of the point T. (4)



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Question 9 continued	
Question 5 continued	



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Question 9 continued	
Question 9 Continued	



 END	TOTAL FOR PAP	ER: 75 MARKS
 	(Total 12 marks)