



# Mark Scheme (Results)

January 2022

Pearson Edexcel International Advanced Level  
In Decision Mathematics (WDM11/01) Paper 01

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

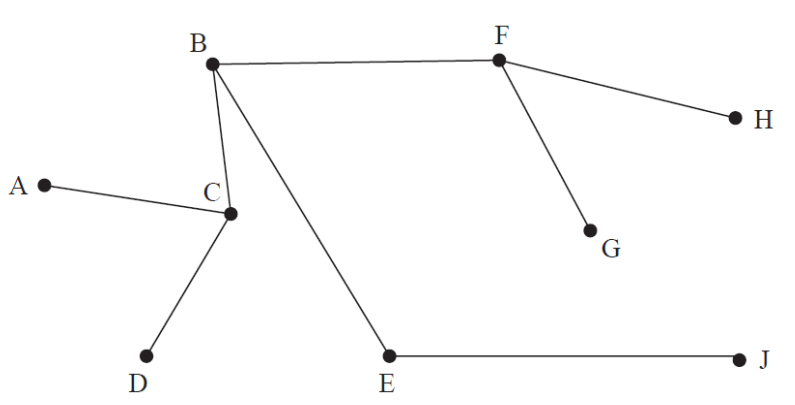
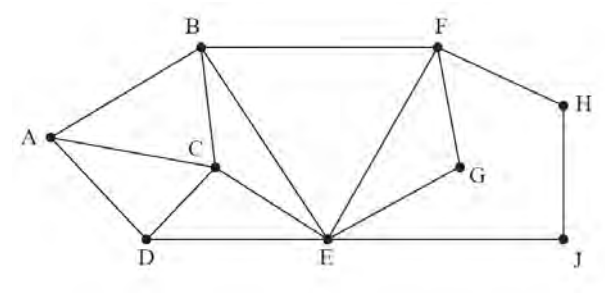
1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\surd$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
  5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
  6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
  7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks																																																																													
1.(a)	$\frac{131}{40} = 3.275$ so lower bound is 4	M1 A1 (2)																																																																													
(b)	Container 1: <u>17</u> <u>9</u> <u>8</u> 4 Container 2: <u>15</u> <u>20</u> 5 Container 3: <u>13</u> 12 Container 4: <u>28</u>	<u>M1</u> <u>A1</u> A1 (3)																																																																													
(c)	middle right <table><tr><td>17</td><td>9</td><td>15</td><td>8</td><td>20</td><td><b>13</b></td><td>28</td><td>4</td><td>12</td><td>5</td><td>Pivots</td></tr><tr><td>9</td><td>8</td><td><b>4</b></td><td>12</td><td>5</td><td><u>13</u></td><td>17</td><td>15</td><td><b>20</b></td><td>28</td><td>13</td></tr><tr><td><u>4</u></td><td>9</td><td>8</td><td><b>12</b></td><td>5</td><td><u>13</u></td><td>17</td><td><b>15</b></td><td><u>20</u></td><td>28</td><td>4, 20</td></tr><tr><td><u>4</u></td><td>9</td><td><b>8</b></td><td>5</td><td><u>12</u></td><td><u>13</u></td><td><u>15</u></td><td>17</td><td><u>20</u></td><td>28</td><td>12, 15, (28)</td></tr><tr><td><u>4</u></td><td>5</td><td><u>8</u></td><td>9</td><td><u>12</u></td><td><u>13</u></td><td><u>15</u></td><td>17</td><td><u>20</u></td><td>28</td><td>8, (17)</td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>Sort complete</td></tr></table>	17	9	15	8	20	<b>13</b>	28	4	12	5	Pivots	9	8	<b>4</b>	12	5	<u>13</u>	17	15	<b>20</b>	28	13	<u>4</u>	9	8	<b>12</b>	5	<u>13</u>	17	<b>15</b>	<u>20</u>	28	4, 20	<u>4</u>	9	<b>8</b>	5	<u>12</u>	<u>13</u>	<u>15</u>	17	<u>20</u>	28	12, 15, (28)	<u>4</u>	5	<u>8</u>	9	<u>12</u>	<u>13</u>	<u>15</u>	17	<u>20</u>	28	8, (17)											Sort complete	M1 A1 A1 (3)											
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(d)	$\left\lceil \frac{1+10}{2} \right\rceil = 6$ 13 – reject 13 – 28 $\left\lceil \frac{1+5}{2} \right\rceil = 3$ 8 – reject 4 – 8 $\left\lceil \frac{4+5}{2} \right\rceil = 5$ 12 – reject 12 $[4] = 4$ 9 – found	M1 A1 A1 (3)																																																																													
		11 marks																																																																													
Notes for Question 1																																																																															
<p><b>a1M1:</b> Attempt to find the lower bound <math>(131 \pm 28) / 40</math>. A value of 3.275 seen with no corresponding calculation can imply this mark. Allow this mark for a clear intention of adding <b>all</b> ten values and dividing by 40, for example,</p> $\frac{17 + 9 + 15 + 8 + 20 + 13 + 28 + 4 + 12 + 5}{40}$ <p>scores M1</p> <p><b>a1A1:</b> cao - correct calculation seen followed by 4 <b>or</b> the correct value of 3.275 followed by 4. However, an answer of 4 with no working scores no marks in (a)</p> <p><b>b1M1:</b> First four values placed correctly and at least eight values placed in containers. The first four values are those that are boxed. Condone cumulative totals for M1 only (e.g. for Container 1: 17 26 34 38)</p>																																																																															

Question Number	Scheme	Marks
<p><b>b1A1:</b> First eight values placed correctly (the boxed <b>and</b> underlined values). This mark cannot be awarded if you see <b>any</b> repeated values or more than ten values placed in containers (even if the first eight items have been placed correctly)</p> <p><b>b2A1:</b> cso – no additional or repeated values (dependent on both previous marks)</p>		
<p><b>c1M1:</b> Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values less than the pivot), pivot, (values greater than the pivot). If sorting into descending order then M1 only (even if the list is reversed at the end of the sort). This mark can be scored if one number only is either missing or incorrect or an additional number is added to the list</p> <p><b>c1A1:</b> First two passes correct and next pivots chosen correctly for third pass (but third pass does not need to be attempted or correct)</p> <p><b>c2A1:</b> cso (correct solution only – all previous marks in this part must have been awarded) – if middle right then either a fifth pass <b>or</b> a 'sort complete' statement (e.g. 'sorted', 'complete', etc. but not just underlining the fourth pass) is required after the fourth pass. If middle left then a fifth pass (with the value of 9) is required (but no 'sort complete' statement is required)</p>		
<p><b>d1M1:</b> Choosing middle right pivot (choosing middle left '12' is M0) <b>and</b> an attempt at discarding/retaining half the list (condone if retaining the wrong half of the list or if retaining 1 – 6)</p> <p><b>d1A1:</b> First and second passes correct i.e. selecting the 6<sup>th</sup> item in the first pass <b>and</b> using 1<sup>st</sup> to 5<sup>th</sup> items in the second pass (must not be using the 6<sup>th</sup> item for the second pass) and then correctly selecting the 3<sup>rd</sup> item (the 8) in the second pass and rejecting the 1<sup>st</sup> to 3<sup>rd</sup> items</p> <p><b>d2A1:</b> cao - search completed correctly (so rejecting the 12 in the third pass) together with 'found'. Condone candidates who say that after the 12 in the third pass has been rejected the only value left is the 9 so it has been found. It must be clear that the 9 has been 'found' and not just stated as the final value</p>		
<p>In (d) candidates must be using a correct ordered list (4 5 8 9 12 13 15 17 20 28) – if it is clear that the candidates are not using this list then M0. With regards to using the original (unsorted) list the 6<sup>th</sup> value is 13 too so what the candidates do next will most likely indicate if the correct list is being used (e.g. if the next pivot is 15 then M0)</p> <p>Candidates who have sorted the list into descending order can earn full marks in (d) – scheme above applies in the same way for descending – must be choosing middle right (the 12) not middle left (the 13) for M1</p> <p>28 20 17 15 13 12 9 8 5 4</p> $\left\lceil \frac{1+10}{2} \right\rceil = 6 \quad 12 - \text{reject } 28 - 12$ $\left\lceil \frac{7+10}{2} \right\rceil = 9 \quad 5 - \text{reject } 5 - 4$ $\left\lceil \frac{7+8}{2} \right\rceil = 8 \quad 8 - \text{reject } 8$ $\lceil 7 \rceil = 7 \quad 9 - \text{found}$		
<b>2.(a)</b>	e.g. A – B – F – H – J	B1 (1)
<b>(b)</b>	A – B – C – D – E – G – F – H – J is not an example of a tour on T as although it contains every vertex it does not return to A	B1 (1)

Question Number	Scheme	Marks
(c)	Kruskal: AC(9), BE(11), BF(12), not EF(14), FG(15), FH(17), not EG(18), EJ(20), $\left\{ \begin{array}{l} \text{not HJ (21)} \\ \text{BC (21)} \end{array} \right\}$ , not CE(23), not AB(24), CD(25) (not DE, AD)	M1 A1 A1 (3)
(d)		B1 (1)
(e)	130 (km)	B1 (1)
7 marks		
Notes for Question 2		
<p><b>a1B1:</b> Any correct example of a path (so no vertex appearing more than once) from A to J</p> <p><b>b1B1:</b> No with a correct reason i.e. must mention that a tour must begin and start at the same vertex, e.g. the route given does not finish at A scores B1</p> <p><b>c1M1:</b> Kruskal's algorithm - first four arcs (AC, BE, BF, FG) correctly chosen <b>and</b> at least one rejection seen at some point (the rejection need not be correct or at the correct time)</p> <p><b>c1A1:</b> All arcs in tree selected correctly in the correct order (AC, BE, BF, FG, FH, EJ, BC, CD) with no additional arcs included in MST</p> <p><b>c2A1:</b> cso - including all rejections correct and at the correct time (do not need to see DE and/or AD rejected but if they are rejected then must be after CD has been selected). Note that BC can be included before HJ rejected</p> <p><b>d1B1:</b> cao</p> <p><b>e1B1:</b> cao (130) – no units required/ignore units even if incorrect</p>		
For reference:		
		
3.	e.g.	M1  A1

Question Number	Scheme	Marks
	<p><b>Note that this solution is not unique e.g. A and B could be interchanged</b></p>	A1 A1 A1 <b>(5)</b>
		<b>5 marks</b>

### Notes for Question 3

Condone lack of, or incorrect, numbered events throughout. ‘Dealt with correctly’ means that the activity starts from the correct event but need not necessarily finish at the correct event, e.g. ‘D dealt with correctly’ requires the correct immediate precedences for this activity, i.e. A, B and C labelled correctly and leading into the same node and D starting from that node but do not consider the end event for D. **Activity on node is M0**

If one arc is not labelled, for example if the arc for activity F is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod.

**If two or more arcs are not labelled then mark strictly according to the scheme below and therefore no bod as mentioned above.** Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be)

**Ignore lack of arrows on the activities for the first four marks only**

**a1M1:** Seven activities (labelled on arc), one start and at least two dummies placed

**a1A1:** Activities A, B, C, 1<sup>st</sup> two dummies (including correct arrows on these two dummies) and F dealt with correctly. The first two dummies are those at the end of activities A (or possibly B if A and B are interchanged in their network) and C

**a2A1:** D, E, G and H dealt with correctly – this mark can be scored on the bod if the arrows are missing off the first two dummies provided the dummies are in the correct place

**a3A1:** I and J dealt with correctly – so this requires the third and fourth dummies (including correct arrows)

**a4A1:** cso – all arrows correctly placed for each activity with one finish and at most four dummies (so must have scored the first four marks)

**Please check all arcs carefully for arrows – if no dummy has an arrow then M1 only**

**Note that additional (but unnecessary) ‘correct’ dummies that still maintain precedence for the network should only be penalised with the final A mark if earned**

**For reference in checking immediately preceding activities (for ‘dealt with correctly’)**

A	B	C	D	E	F	G	H	I	J
-	-	-	A, B, C	A, B, C	C	F	D	D, E, G	D, E

<b>4. (a)</b>	Float on F is twice float on D $\Rightarrow 22 - 8 - y = 2(8 - 3 - x)$ (oe)	B1
	BFDM is 10 less than critical path	B1



Question Number	Scheme	Marks
	$\Rightarrow 3 + x + y + 3 = 26 - 10$ (oe)	
	$x + y = 10$ $-2x + y = 4$ and solve simultaneously to find both $x$ and $y$	M1
	$x = 2, y = 8$	A1 (4)
(b)		M1 A1 A1 A1 (4)
(c)	Lower bound is 4 workers e.g. activities F, I, J and K together with $15 < \text{time} < 16$	M1 A1 (2)
		<b>10 marks</b>

#### Notes for Question 4

**a1B1:** cao (any equivalent form) – allow  $14 - y = 2(5 - x)$  or  $14 - y = 10 - 2x$

**a2B1:** cao (any equivalent form) – allow  $x + y + 6 = 16$

**a1M1:** Setting up two equations both including  $x$  and  $y$  (dependent on one correct equation) and an attempt to solve for both  $x$  and  $y$  leading to a value for  $x$  and a value for  $y$  – this mark can be implied if the candidate has two equations, both in  $x$  and  $y$  (with at least one correct), followed by values for  $x$  and  $y$

**a1A1:** cao ( $x = 2, y = 8$ ) – if both correct values are stated without any working/justification please send to review

**b1M1:** At least nine activities labelled including at least five floats. A scheduling diagram scores M0

**b1A1:** The critical activities dealt with correctly and appearing just once (A, G, I and L) and three non-critical activities dealt with correctly

**b2A1:** Any six non-critical activities correct (this mark is not dependent on the previous A mark)

**b3A1:** Completely correct Gantt chart (exactly thirteen activities appearing just once)

**c1M1:** Either a statement with the correct number of workers (4) and the correct activities (F, I, J and K) with **any** numerical time (or time interval) stated **or** the correct number of workers (4) and a time in the interval  $15 \leq t \leq 16$  – mark the numerical value only not their use of day/time. In either case they must state the correct number of workers. M0 for ‘F, J, K and the critical activity’ (they must explicitly state activity I in this case) unless a time in the interval stated above is given too

**c1A1:** A completely correct statement with details of both time and activities. Candidates must give a time within the correct interval of  $15 < t < 16$ . Please note the strict inequalities for the time interval (e.g. implying a time of 15 is incorrect). Answers given as an interval of time are acceptable provided the time interval stated is correct for all its possible values (e.g. time 15 – 16 is A0). Note that ‘on day 16’ is correct but ‘on day 15’ is not correct. A completely correct statement with an additional incorrect statement scores A0 (so no isw)

Question Number	Scheme	Marks
<b>5. (a)</b>	Pair the odd nodes: C, D <b>or</b> repeated arcs are CF, FG, DG	B1
	Time = $82 + 7 = 89$	B1
	e.g. route GDGJHEADCABEFBCFCGFG	B1 <b>(3)</b>
<b>(b)</b>	$BC + DG = B(F)C + DG = 6 + 3 = 9^*$	M1 A1
	$BD + CG = B(FG)D + C(F)G = 11 + 4 = 15$	A1
	$BG + CD = B(F)G + C(FG)D = 8 + 7 = 15$	A1
	Repeat arcs: BF, CF, DG	A1 <b>(5)</b>
<b>(c)</b>	Route starting from G is quicker	B1
	e.g. difference = $(82 + 9) - 89 = 2$ or $9 - 7 = 2$	B1 <b>(2)</b>
		<b>10 marks</b>

#### Notes for Question 5

**a1B1:** cao (correctly stating the two odd nodes **or** correct repeated arcs stated) – so must either state that C and D are odd **or** state the arcs CF, FG, DG only, but B0 if only stating C and D or CD (without mention of ‘odd’)

**a2B1:** cao (89)

**a3B1:** Correct route: checks – starts and finishes at G, 20 nodes, CF, FG and DG repeated, A(2), B(2), C(3), D(2), E(2), F(3), G(4), H(1), J(1)

**b1M1:** Correct three distinct pairings of the correct four odd nodes B, C, D and G

**b1A1:** Any one row correct including pairing **and** total

**b2A1:** Any two rows correct including pairings **and** totals

**b3A1:** All three rows correct including pairings **and** totals

**b4A1:** cao - correct arcs clearly stated and not just in their working as BF, CF and DG – must be these arcs. Do not accept BC, BFC or BC via F

**c1B1:** cao (oe e.g. B to G is slower) – dependent on the correct repeats arcs (possibly implied) in (a) and (b) **or** clearly implied in (c) (e.g. correct values compared in this part) – must be clear that it is the route starting at G which is quicker

**c2B1:** cao (difference of 2 **or** comparing 89 and 91 **or** comparing 7 with 9)

<b>6.(a)</b>		M1 A1 (ABCD) A1 (EF) A1ft (GH)  dM1 A1 <b>(6)</b>
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Question Number	Scheme									Marks																																																																																	
	<table><tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td></tr><tr><td>A</td><td>-</td><td>7</td><td>8</td><td>9</td><td>11</td><td>18</td><td>25</td><td>26</td></tr><tr><td>B</td><td>7</td><td>-</td><td>14</td><td>2</td><td>4</td><td>11</td><td>18</td><td>19</td></tr><tr><td>C</td><td>8</td><td>14</td><td>-</td><td>12</td><td>10</td><td>15</td><td>22</td><td>23</td></tr><tr><td>D</td><td>9</td><td>2</td><td>12</td><td>-</td><td>2</td><td>9</td><td>16</td><td>17</td></tr><tr><td>E</td><td>11</td><td>4</td><td>10</td><td>2</td><td>-</td><td>7</td><td>14</td><td>15</td></tr><tr><td>F</td><td>18</td><td>11</td><td>15</td><td>9</td><td>7</td><td>-</td><td>7</td><td>8</td></tr><tr><td>G</td><td>25</td><td>18</td><td>22</td><td>16</td><td>14</td><td>7</td><td>-</td><td>1</td></tr><tr><td>H</td><td>26</td><td>19</td><td>23</td><td>17</td><td>15</td><td>8</td><td>1</td><td>-</td></tr></table>										A	B	C	D	E	F	G	H	A	-	7	8	9	11	18	25	26	B	7	-	14	2	4	11	18	19	C	8	14	-	12	10	15	22	23	D	9	2	12	-	2	9	16	17	E	11	4	10	2	-	7	14	15	F	18	11	15	9	7	-	7	8	G	25	18	22	16	14	7	-	1	H	26	19	23	17	15	8	1	-	
	A	B	C	D	E	F	G	H																																																																																			
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G	25	18	22	16	14	7	-	1																																																																																			
H	26	19	23	17	15	8	1	-																																																																																			
(b)	NNA: A – B – D – E – F – G – H – C – A									B1																																																																																	
	$7 + 2 + 2 + 7 + 7 + 1 + 23 + 8 = 57$ (km)									B1 (2)																																																																																	
(c)(i)	Prim (starting at C): CE, DE, BD, EF, FG, GH									M1 A1																																																																																	
	RMST weight = $10 + 2 + 2 + 7 + 7 + 1 = 29$																																																																																										
(c)(ii)	$29 + 7(AB) + 8(AC) = 44$ (km)									M1 A1 (4)																																																																																	
(d)	$44 \leq \text{optimal distance} \leq 57$									B2, 1, 0 (2)																																																																																	
										14 marks																																																																																	
Notes for Question 6																																																																																											
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 23 21 18 in that order (so 23 18 21 is incorrect). It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling																																																																																											

Question Number	Scheme	Marks
	<p><b>a1M1:</b> Any larger working value replaced by any smaller working value at at <b>least two nodes</b> except A, B, C and D</p> <p><b>a1A1:</b> All values at A, B, C and D correct and the working values in the correct order. Condone lack of a zero as a working value at A</p> <p><b>a2A1:</b> All values at E and F correct and the working values in the correct order. Penalise order of labelling only once per question (so E and F must be labelled in that order and E must be labelled after D)</p> <p><b>a3A1ft:</b> All values at G and H correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through G check that the working values at G follow from the candidate's final values for the nodes that are directly attached to G (which are E, F and H). For example, if correct then the order of labelling of nodes E, F and H are 5, 6 and 8 respectively so the working values at G should come from E and F in that order. The first working value at G should be 18 (the Final value at F) + 7 (the weight of the arc FG) and the second working value at G should be 11 (the Final value at E) + 16 (the weight of the arc EG). Repeat the process for H (which will have working values from D, E and G with the order of these nodes determined by the candidate's order of labelling at D, E and G)</p> <p><b>a2M1:</b> Correct entries in the table following through their final values – dependent on the previous M mark (need only fill in either the A row or A column)</p> <p><b>a4A1:</b> cao</p> <p><b>b1B1:</b> Correct nearest neighbour route starting and finishing at A (A – B – D – E – F – G – H – C – A)</p> <p><b>b2B1:</b> cao (57) on length of route</p> <p><b>ci1M1:</b> First three arcs (CE, DE, BD) <b>or</b> all 7 nodes {C, E, D, B, F, G, H} correctly chosen in order. If any explicit rejections seen then M1 only in (c)(i). Order of nodes may be seen at the top of the matrix/table {4, 1, 3, 2, 5, 6, 7}. Award M0 for a correct tree with no working. Award M1 only for the first three arcs (oe) selected correctly if starting at a different node than C. If correct values circled in the table but no indication of order of selection then M0</p> <p><b>ci1A1:</b> cso – all <b>arcs</b> correctly <b>stated</b> and chosen in the correct order (with no additional arcs). They must be considering arcs for this mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen). If AB and AC added explicitly in (c)(i) then A0 but can score both marks in (c)(ii)</p> <p><b>cii2M1:</b> Weight of RMST + 7 + 8 (two smallest arcs incident to A) with <math>19 \leq \text{RMST} \leq 39</math> (if clearly not six arcs in RMST then M0). Give bod if 15 is added to the total of six values circled in the table provided those six values sum to a value between 19 and 39 inclusive</p> <p><b>cii2A1:</b> cao (44) – if correct RMST stated in (c)(i) followed by 44 (with no additional working) then award M1A1 in (c)(ii). This mark is dependent on Prim's algorithm being used to find the RMST (allow this mark if rejections seen in (c)(i) when applying Prim). So in (c) M1A0M1A1 is possible e.g. if only stating the node (instead of the arc) selection in order when applying Prim. If the correct six values are circled in the table and added to 15 to give 44 but the order of arc/node selection is not stated (so no indication that Prim has been applied) then A0 (as the qu. says, 'Hence...')</p> <p><b>d1B1:</b> Any indication of an interval from their answer to (c)(ii) to their answer to (b) with one value correct (e.g. <math>44 \sim 57</math> scores B1 but <math>57 \sim 44</math> or <math>57 \leq \text{optimal distance} \leq 44</math> scores B0). <b>If correct route seen in (b) but the upper bound not stated in (b) allow recovery in part (d) it stated here (but still withhold the second mark in (b)).</b></p> <p><b>d2B1:</b> cao (<math>44 \leq \text{optimal distance} \leq 57</math>) including correct inequalities (allow <math>44 &lt; \text{optimal distance} \leq 57</math>) – allow interval notation e.g., [44, 57] or (44, 57]</p>	
7.(a)	Minimise $(P=)9x+12y+16z$	B1
	$x+y+z \geq 40$	B1
	$z \geq 2y$	B1
	$\frac{3}{5}(x+y+z) \geq x$ simplifies to $2x \leq 3y+3z$	M1 A1
	$x+1.5y+2.5z \leq 75$ simplifies to $2x+3y+5z \leq 150$	M1 A1
	$(x \geq 0, y \geq 0, z \geq 0)$	(7)
(b)	$9x+12y+16(45-x-y)$	M1

Question Number	Scheme	Marks
	which leads to $-7x - 4y + 720$	A1
	$+720$ is a constant so the total value is minimised when $-7x - 4y$ is <b>and</b> $-7x - 4y = -(7x + 4y)$ , as $-(7x + 4y)$ is negative (for all positive values of $x$ and $y$ ) and so minimising an expression which is negative is equivalent to maximising the corresponding positive expression $7x + 4y$	A1 (3)
(c)		B1 B1 B1 B1 (4)
(d)	Correct objective line	B1
	V correctly labelled	B1 (2)
(e)	27 small containers, 6 medium containers and 12 large containers	B1
	Total cost is (£)507	B1 (2)
		<b>18 marks</b>

#### Notes for Question 7

**a1B1:** cao – expression correct and ‘minimise’ or ‘min’ but not ‘minimum’

**a2B1:** cao ( $x + y + z \geq 40$  oe but must be four terms only with integer coefficients e.g.  $x + y + z - 40 \geq 0$ )

**a3B1:** cao ( $z \geq 2y$  oe (e.g.  $4y - 2z \leq 0$ ) but must be two terms only with integer coefficients)

**a1M1:** correct method - must see  $\frac{3}{5}(x + y + z) \bullet x$  where  $\bullet$  is any inequality or  $=$ . The bracket must be present or implied by later working. Allow 0.6 but not 60% (unless implied by later working)

**a1A1:** cao – simplified (one term only in  $x, y$  and  $z$ ) – answer must have integer coefficients (e.g.  $4x - 6y - 6z \leq 0$ )  
- the correct simplified inequality with either no working **or** working with % sign implies M1A1

**a2M1:** correct method – must see  $x + 1.5y + 2.5z \bullet 75$  (oe) where  $\bullet$  is any inequality or  $=$

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<p><b>a2A1:</b> cao – simplified (one term only in <math>x, y</math> and <math>z</math>) – answer must have integer coefficients ( e.g. <math>4x + 6y \leq 300 - 10z</math> ) – the correct simplified inequality with no working implies M1A1</p> <p><b>b1M1:</b> substituting <math>z = 45 - x - y</math> into <math>9x + 12y + 16z</math></p> <p><b>b1A1:</b> cao of <math>-7x - 4y + 720</math> <b>and</b> <u>any</u> attempt at <u>explaining</u> why the minimum total cost is achieved when <math>7x + 4y</math> is maximised</p> <p><b>b2A1:</b> stating that 720 is a constant (and so doesn't impact on the optimal values of <math>x, y</math> and <math>z</math>) <b>and</b> a correct deduction that minimising a negative expression is equivalent to maximising the corresponding positive expression (so just stating that <math>-7x - 4y</math> is minimised when <math>7x + 4y</math> is maximised is A0)</p> <p>The lines in (c) must define the correct FR <b>and if extended</b> pass within a small square of the points stated:</p> <p><math>x + 3y = 45</math> with points (0, 15) and (45, 0)</p> <p><math>3x + 2y = 75</math> with points (0, 37.5) and (25, 0)</p> <p><math>x = 27</math> with points (27, 0) and (27, 40) – a common wrong response is to draw either <math>y = 27</math> or <math>x = 28</math> – these are both B0</p> <p><b>c1B1:</b> Any one line drawn correctly</p> <p><b>c2B1:</b> Any two lines drawn correctly</p> <p><b>c3B1:</b> Any three lines drawn correctly</p> <p><b>c4B1:</b> Correct <math>R</math> labelled – not just implied by shading – dependent on scoring the first three marks in this part (condone if no shading below the <math>x</math>-axis)</p> <p><b>d1B1:</b> A correct objective line drawn on the graph with a gradient of <math>-1.75</math> – line must be at least the length of (2, 0) to (0, 3.5) and within one small square - for reference common intersections points with each axes are given below</p> <table><tr><th><math>x</math></th><th><math>y</math></th><th><math>x</math></th><th><math>y</math></th></tr><tr><td>4</td><td>7</td><td>2.85...</td><td>5</td></tr><tr><td>5</td><td>8.75</td><td>5.71...</td><td>10</td></tr><tr><td>8</td><td>14</td><td>8.57...</td><td>15</td></tr><tr><td>10</td><td>17.5</td><td>11.4...</td><td>20</td></tr><tr><td>12</td><td>21</td><td>14.3...</td><td>25</td></tr><tr><td>15</td><td>26.25</td><td>17.1...</td><td>30</td></tr><tr><td>16</td><td>28</td><td>20</td><td>35</td></tr><tr><td>20</td><td>35</td><td>22.9...</td><td>40</td></tr></table> <p><b>d2B1:</b> <math>V</math> labelled clearly on their graph. This mark is dependent on</p> <ul style="list-style-type: none"><li>the first three marks in (c)</li><li>not labelling or implying that any other region is the FR</li><li>the first B mark in (d)</li></ul> <p>By clearly labelled the vertex should either be labelled '<math>V</math>' or circled or clearly distinguishable from any other vertex (but B0 if not clear e.g. another vertex circled too) (note that <math>V(27, 6)</math>)</p> <p><b>e1B1:</b> cao (must be in context – so not in terms of <math>x, y</math> and <math>z</math>) – dependent on first three marks in (c) and the first mark in (d) (27 small, 6 medium and 12 large)</p> <p><b>e2B1:</b> cao (507) - dependent on first three marks in (c) and the first mark in (d) – units not required. Condone incorrect units e.g. \$</p>			$x$	$y$	$x$	$y$	4	7	2.85...	5	5	8.75	5.71...	10	8	14	8.57...	15	10	17.5	11.4...	20	12	21	14.3...	25	15	26.25	17.1...	30	16	28	20	35	20	35	22.9...	40
$x$	$y$	$x$	$y$																																			
4	7	2.85...	5																																			
5	8.75	5.71...	10																																			
8	14	8.57...	15																																			
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