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Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
- there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
 - use this as a guide as to how much time to spend on each guestion.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶







1. The continuous random variable X has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < 0 \\ k(6x^2 - x^4) & 0 \leqslant x \leqslant 1 \\ 1 & x > 1 \end{cases}$$

where k is a positive constant.

(a) Show that $k = \frac{1}{5}$

(1)

(b) Find P(X > 0.6)

(2)

(c) (i) Show that the median, m, of X satisfies

$$am^4 + bm^2 + c = 0$$

where a, b and c are integers to be found.

(ii) Hence, find the value of m

(3)

(d) Find, using calculus, the mode of X

(3)

Given that E(X) = 0.64

(e) state, giving a reason, the skewness of the distribution.

(2)

Question 1 continued



Question 1 continued

Question 1 continued
(Total for Question 1 is 11 marks)
(Total for Question 1 is 11 marks)



- **2.** On Friday evenings, a shop opens from 7 pm to 11 pm. During this time customers are known to enter the shop at a mean rate of 20 per hour.
 - (a) State a suitable distribution to model the number of customers that enter this shop in a 30-minute interval on Friday evenings.

(1)

(b) State a necessary assumption for the model in part (a) to be valid.

(1)

The manager makes alterations to the shop's layout.

Following these alterations, the manager wants to find out whether the mean rate of customers entering the shop has changed.

To test this, the manager decides to monitor the number of customers entering the shop the following Friday. The manager randomly selects a 30-minute interval between 7 pm and 11 pm.

(c) Write down suitable null and alternative hypotheses that the manager should use.

(1)

(d) Using a 3% level of significance, find the critical region for the manager's test.

(3)

(e) Find the actual significance level of this test based on your critical region from part (d)

(2)

During the 30-minute interval that the manager monitored, 16 customers entered the shop.

(f) Comment on this finding in the light of your critical region found in part (d)

(2)

Question 2 continued



Question 2 continued

Question 2 continued
(Total for Question 2 is 10 marks)
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3. A sweet shop produces different coloured sweets and mixes them to sell in small, medium or large bags.

The proportion of sweets that are blue is p

Each small bag is filled with a random sample of n sweets.

The mean number of blue sweets in a small bag is 3 and the variance of the number of blue sweets in a small bag is 2.55

(a) Find the value of p and the value of p

(3)

The proportion of sweets that are green is 0.2

Each medium bag contains a random sample of 40 sweets.

For a medium bag,

(b) find the probability that the number of green sweets exceeds the **expected** number of green sweets.

(3)

The proportion of sweets that are yellow is 0.1

Each large bag contains a random sample of 80 sweets.

- (c) (i) Use a Poisson approximation to find the probability that a large bag of sweets contains at least 12 yellow sweets.
 - (ii) Justify why a Poisson approximation is suitable in this case.

(4)



Question 3 continued



Question 3 continued

Question 3 continued
(Total for Question 3 is 10 marks)



4. A bag contains a large number of beads of the same size and shape. The beads are either black or white.

Black beads and white beads occur in the ratio 2:5 respectively.

In a game a player takes a random sample of 3 beads from the bag.

The player scores

- 4 points for each black bead taken
- 1 point for each white bead taken

The random variable *X* represents the total number of points for the 3 beads.

(a) Find the sampling distribution of X

(6)

A random sample of n sets of 3 beads is taken. The random variable Y represents the number of these n sets that have a total of exactly 9 points.

(b) Calculate the minimum value of *n* such that $P(Y \ge 1) > 0.95$

(3)



Question 4 continued



Question 4 continued

Question 4 continued	
(Tata	l for Question 4 is 9 marks)
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5. In this question solutions relying entirely on calculator technology are not acceptable.

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{2}{21}x & 0 \le x \le k \\ \frac{2}{15}(6-x) & k < x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that k = 3.5

You must make your method clear and show all stages of your working.

(5)

(b) Use algebraic integration to find E(X) You must show all stages of your working.

(4)

Given that $E(X^2) = \frac{277}{24}$

(c) find Var(X)

(2)

Question 5 continued



Question 5 continued

Question 5 continued	
	(Total for Ougstion 5 is 11 sules)
	(Total for Question 5 is 11 marks)



6. The continuous random variable X is uniformly distributed over the interval [a, b]

Given that

- $P(2 < X < 5) = \frac{3}{16}$
- $\bullet \quad E(X) = 6$
- (a) find the value of a and the value of b

(3)

(b) find the value of the constant c such that E(cX+1)=3

(2)

(c) find the exact value of Var(X)

(1)

(d) find the exact value of $E(X^2)$

(2)

(e) find $P\left(\frac{3}{2}X - b > a\right)$

(2)

Question 6 continued	
(Total for Question 6 is 10 marks)	—



7. When purchasing a pair of glasses from an optician, customers are offered insurance to cover accidental damage.

Past records from the optician show that 30% of customers buy insurance when they purchase a pair of glasses.

In a random sample of 40 customers who have purchased a pair of glasses, X represents the number who buy insurance.

(a) State a suitable model for the distribution of X

(1)

(b) State a necessary assumption for the model in part (a) to be valid.

(1)

The probability that fewer than r customers who have purchased a pair of glasses buy insurance is less than 0.05

(c) Find the largest possible value of r

(2)

A second random sample of 200 customers who have purchased a pair of glasses is taken.

Using a normal approximation, the probability that at least *t* of these 200 customers buy insurance is 0.9474, correct to 4 decimal places.

(d) Find the value of *t* You must show your working.

(5)

The optician decided to introduce a special offer in which customers who purchase a pair of glasses get the first three months of insurance free if they buy insurance.

Following this, a random sample of 25 customers who purchase a pair of glasses is taken and 11 of them buy insurance.

(e) Test, at the 10% level of significance, whether or not there is evidence that the proportion of customers who buy insurance when purchasing a pair of glasses has increased. State your hypotheses clearly.

(5)



Question 7 continued



Question 7 continued

Question 7 continued



Question 7 continued
(Total for Question 7 is 14 marks)
TOTAL FOR PAPER IS 75 MARKS