Mark Scheme (Results)

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Pearson Edexcel International Advanced Level
In Statistics S2 (WST02) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

<u>'M' marks</u>
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

<u>'A' marks</u>
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

<u>'B' marks</u>
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through

- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp   decimal places
- sf   significant figures
- ✱   The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Special notes for marking Statistics exams (for AAs only)

- Any correct method should gain credit. If you cannot see how to apply the mark scheme but believe the method to be correct then please send to review.
- For method marks, we generally allow or condone a slip or transcription error if these are seen in an expression. We do not, however, condone or allow these errors in accuracy marks.

Question Number	Scheme	Marks
1 (a)	Po(isson) with $(\lambda =)4$	B1
		(1)
(b)	Pairs of **shoes** (are **sold**) singly/randomly/independently/at a **constant** (average) **rate**	B1
		(1)
(c) (i)	X = number of sales per hour $\Rightarrow X \sim$ Po(4)	
	$P(X > 4) = 1 - P(X \leqslant 4)$	M1
	$= 0.3712$ \qquad\qquad awrt 0.371	A1
(ii)	$('0.371...')^3$	M1
	$= 0.051147\ldots$ \qquad\qquad 0.05115 or awrt 0.0511	A1
		(4)
(d)	$H_0 : \lambda = '4'$ \qquad $H_1 : \lambda > '4'$	B1ft
	$P(X \geqslant 7) = 1 - P(X \leqslant 6)$ \quad or \quad $P(X \geqslant 9) = 1 - P(X \leqslant 8) = 0.0214$	M1
	$= 0.1107$ \qquad\qquad or \qquad CR $X \geqslant 9$ \qquad\qquad awrt 0.111	A1
	Not significant/Do not reject H_0 /Not in the critical region	M1
	There is insufficient evidence of an <u>increase</u> in <u>sales</u> following the appearance of the advert/<u>manager's</u> <u>belief</u> is not supported.	dA1
		(5)
	Notes	**Total 11**
(a)	**B1**	For Po or Poisson and 4 must be seen in part (a). Do not allow P(4)
(b)	**B1**	For one of the given assumptions in context (must have context of **shoes** or **sales**). Ignore extraneous non-contradictory comments.
(c) (i)	**M1**	For writing or using $P(X > 4) = 1 - P(X \leqslant 4)$
	A1	awrt 0.371
(ii)	**M1**	'part (i)'^{3}
	A1	0.05115 or awrt 0.0511 (Calculator gives 0.051132…)
(d)	**B1ft**	Both hypotheses correct. Must be in terms of λ or μ ft their λ from part (a) Must be attached to H_0 and H_1
	M1	For writing or using $P(X \geqslant 7) = 1 - P(X \leqslant 6)$ If a CR approach is taken then award M1 for $P(X \geqslant 9) = 1 - P(X \leqslant 8)$ written or used This mark may be implied by a correct p-value or CR
	A1	awrt 0.111 or CR $X \geqslant 9$
	M1	Any correct ft statement consistent with their p-value and 0.05 **or** their CR and 7 – no context needed but do not allow contradicting non contextual comments. The comparison of their p-value and the significance level is not counted as a non contextual statement. May be implied by a correct ft conclusion in context.
	dA1	Dependent on 1st M1 - Correct conclusion in context which must be **not rejecting** H_0. Must include the underlined words (oe).

Question Number	Scheme	Marks
2 (a)	20, 20, 20 20, 20, 50 (×3) 20, 50, 50 (×3) 50, 50, 50	B2
		(2)
(b)	$a = 30$ and $b = 40$	B1
		(1)
(c)	$p^3 = \dfrac{4913}{8000}$ or $q^3 = \dfrac{27}{8000}$	M1
	$p = \dfrac{17}{20}$ (0.85) and $q = \dfrac{3}{20}$ (0.15)	A1
		(2)
(d)	$[\mathrm{P}(30)] = 3 \times "p^2" \times "q"$ $[\mathrm{P}(40)] = 3 \times "p" \times "q^2"$	M1 M1
	$c = \dfrac{2601}{8000}$ $d = \dfrac{459}{8000}$	A1
		(3)

(e)			
	M	20	50
	P($M = m$)	$\dfrac{3757}{4000}$	$\dfrac{243}{4000}$

B1 M1 A1ft

(3)

Total 11

		Notes
(a)	**B2**	For all 4 correct combinations (B1 for 3 correct combinations) Ignore extraneous repetitions of any of the given combinations
(b)	**B1**	For $a = 30$ and $b = 40$
(c)	**M1**	Either $p^3 = \dfrac{4913}{8000}$ or $q^3 = \dfrac{27}{8000}$
	A1	$p = 0.85$oe and $q = 0.15$oe
(d)	**M1**	$[\mathrm{P}(30)] = 3 \times (\text{their } p)^2 \times (\text{their } q)$ or $[\mathrm{P}(40)] = 3 \times (\text{their } p) \times (\text{their } q)^2$ must see values substituted and must be using their values from part (c)
	M1	$[\mathrm{P}(30)] = 3 \times (\text{their } p)^2 \times (\text{their } q)$ and $[\mathrm{P}(40)] = 3 \times (\text{their } p) \times (\text{their } q)^2$ or use of sum of probabilities $= 1$ i.e. $c + d = \dfrac{153}{400}$
	A1	For $c = \dfrac{2601}{8000} (= 0.325125)$ and $d = \dfrac{459}{8000} (= 0.057375)$
(e)	**B1**	For 20 and 50 only (ignore notation used for M)
	M1	Either $\dfrac{4913}{8000} + \text{their } c$ or $\dfrac{27}{8000} + \text{their } d$ for ft answers only values will need to be checked
	A1ft	For $\dfrac{3757}{4000}$ oe and $\dfrac{243}{4000}$ oe Follow through their values for c and d but $\mathrm{P}(M = 20) + \mathrm{P}(M = 50)$ must sum to 1 (A table is not required).
	NB	If a and b are reversed then allow $a = 40$ and $b = 30$ – this will mean $p = 0.15$ and $q = 0.85$, $c = \dfrac{459}{8000}$ $d = \dfrac{2601}{8000}$

Question Number	Scheme		Marks
3 (a) (i)	$X \sim \mathrm{B}(10, 0.1)$		
	$\mathrm{P}(X \geqslant 4) = 1 - \mathrm{P}(X \leqslant 3) = 1 - 0.9872$		M1
	$= 0.0128$	awrt 0.0128	A1
(ii)	$\mathrm{P}(1 < X < 5) = \mathrm{P}(X \leqslant 4) - \mathrm{P}(X \leqslant 1) = 0.9984 - 0.7361$ or $\mathrm{P}(X = 2) + \mathrm{P}(X = 3) + \mathrm{P}(X = 4) = 0.1937 + 0.0574 + 0.0112$		M1
	$= 0.2623$	awrt 0.262	A1
			(4)
(b)	$\mathrm{H}_0 : p = 0.1 \qquad \mathrm{H}_1 : p < 0.1$		B1
	$X \sim \mathrm{B}(50, 0.1)$		
	$\mathrm{P}(X \leqslant 2) = 0.1117 \quad$ or $\quad$ CR $X \leqslant 1$		B1
	Do not reject H_0/Not in the critical region		M1
	There is insufficient evidence to suggest that this result supports the managing <u>director's claim</u>/not enough evidence to suggest a <u>reduction</u> in the probability of a tennis ball <u>failing</u> the bounce <u>test</u>		A1
			(4)
(c)	$X \sim \mathrm{B}(n, 0.1)$ and we reject H_0 if $\mathrm{P}(X = 0) < 0.01$		
	$\mathrm{P}(X = 0) = \left[{}^{n}C_0 \times 0.1^0 \right] \times 0.9^n [< 0.01]$		M1
	$0.9^{44} = 0.00969...[< 0.01]$	$n > \dfrac{\ln 0.01}{\ln 0.9} \Rightarrow n > 43.7$	M1
	$n = 44$		A1
			(3)

		Notes	Total 11
(a) (i)	**M1**	for writing or using $\mathrm{P}(X \geqslant 4) = 1 - \mathrm{P}(X \leqslant 3)$	
	A1	awrt 0.0128	
(ii)	**M1**	for writing or using $\mathrm{P}(X \leqslant 4) - \mathrm{P}(X \leqslant 1)$ or for writing or using $\mathrm{P}(X = 2) + \mathrm{P}(X = 3) + \mathrm{P}(X = 4)$	
	A1	awrt 0.262	
(b)	**B1**	Both hypotheses correct. Must be in terms of p or π Must be attached to H_0 and H_1	
	B1	awrt 0.112 or CR $\leqslant 1$	
	M1	A correct ft statement consistent with their p–value and 0.05 **or** their CR and 2– no context needed but do not allow contradicting non contextual comments. The comparison of their p-value and the significance level is not counted as a non contextual statement. May be implied by a correct ft conclusion in context. Must have a p-value or CR to access this mark.	
	A1	Correct conclusion in context which must be **not rejecting** H_0. Must use underlined words (oe). No hypotheses then A0	
(c)	**M1**	For recognising $\mathrm{P}(X=0)=0.9^n$	
	M1	For $0.9^{44} (= 0.00969...)$ or $0.9^{43} (= 0.01077...)$ or rearranging to $n > \dfrac{\ln 0.01}{\ln 0.9}$ … (Allow =) $n >$ awrt 43.7 implies M1M1 (Allow $n = $ awrt 43.7 for M1M1)	
	A1	Cao	
	SC	Use of tables only, $n = 40$, $p = 0.0148$ **and** $n = 50$, $p = 0.0052$ scores M1M0A0	

Question Number		Scheme	Marks
4 (a)		$\dfrac{9}{20}$	B1
			(1)
(b)		$\left(21k - k\right) \times \dfrac{\pi}{20} = 1$	M1
		$k = \dfrac{1}{\pi}$ *	A1*
			(2)
(c) (i)		$\left[\mathrm{E}(X) = \dfrac{1}{2}\left(k + 21k\right) \right] = \dfrac{11}{\pi}$	B1
(ii)		$\mathrm{Var}(X) = \dfrac{1}{12}\left(21k - k\right)^2$ $\qquad$ or $\quad \mathrm{Var}(X) = \displaystyle\int_{\frac{1}{\pi}}^{\frac{21}{\pi}} \dfrac{\pi}{20} x^2 \, \mathrm{d}x - \left(\dfrac{11}{\pi}\right)^2$	M1
		$= \dfrac{100}{3\pi^2}$	A1
			(3)
(d)		$\mathrm{E}(A) = \pi \mathrm{E}(X^2) + 4\mathrm{E}(X) + \dfrac{4}{\pi}$ $\quad \Bigg|$ $\mathrm{E}(A) = \displaystyle\int_{k}^{21k} \mathrm{f}(x)(A)\,\mathrm{d}x = \int_{k}^{21k} \tfrac{\pi}{20}\left(\pi\right)\left(x^2 + \tfrac{4}{\pi} x + \tfrac{4}{\pi^2}\right)\mathrm{d}x$	M1
		$\mathrm{E}(X^2) = \dfrac{100}{3\pi^2} + \left(\dfrac{11}{\pi}\right)^2 = \dfrac{463}{3\pi^2}$ $\quad \Bigg|$ $\mathrm{E}(A) = \tfrac{\pi}{20}\left(\pi\right)\left(\dfrac{x^3}{3} + \left(\tfrac{4}{\pi}\right)\dfrac{x^2}{2} + \tfrac{4}{\pi^2} x\right)$	M1
		$\mathrm{E}(A) = \dfrac{463}{3\pi} + \dfrac{44}{\pi} + \dfrac{4}{\pi}$ $\qquad \Bigg|$ sub limits $\dfrac{21}{\pi}$ and $\dfrac{1}{\pi}$	M1
		$= \dfrac{607}{3\pi}$ $\qquad\qquad\qquad$ = awrt 64.4	A1
			(4)
Notes			**Total 10**
(a)	**B1**	0.45oe cao	
(b)	**M1**	use of the area of the rectangle = 1 Any equivalent rearrangement, allow $20k$ instead of $(21k - k)$	
	A1*	answer is given so a fully correct solution must be seen	
(c)(i)	**B1**	oe must be in terms of π (isw after correct answer seen)	
(ii)	**M1**	use of $\dfrac{(b-a)^2}{12}$ or $\mathrm{Var}(X) = \displaystyle\int_{\frac{1}{\pi}}^{\frac{21}{\pi}} \dfrac{\pi}{20} x^2 \, \mathrm{d}x - \left(\dfrac{11}{\pi}\right)^2$	
	A1	for $\dfrac{100}{3\pi^2}$ oe must be in terms of π (isw after correct answer seen)	
	SC	If both final answers are given in terms of k, score B1M1A0 for (c)(i) $11k$ and (c)(ii) $\dfrac{100}{3}k^2$	
(d)	**M1**	for expanding $\mathrm{E}(A) = \mathrm{E}\left(\pi X^2 + 4X + \dfrac{4}{\pi}\right)$ **or** for setting up correct integral (ignore limits)	
	M1	Valid method for finding $\mathrm{E}(X^2)$ i.e. use of $\mathrm{Var}(X) + \mathrm{E}(X)^2$ or integration of $x^2\mathrm{f}(x)$ **or** for integration of their $\mathrm{f}(x)A$ with at least one $x^n \to x^{n+1}$	
	M1	substitution of their $\mathrm{E}(X)$ and their $\mathrm{E}(X^2)$ into their $\mathrm{E}(A)$ **or** for use of correct limits	
	A1	for $\dfrac{607}{3\pi}$ or awrt 64.4	

Question Number		Scheme		Marks
5 (a)		$X \sim \text{Po}(5)$		
		$\text{P}(X \leqslant 5) = 0.6160$	awrt 0.616	M1 A1
				(2)
(b)		$X \sim \text{B}(4,"0.616")$		B1ft
		$\text{P}(X < 2) = \text{P}(X \leqslant 1)$		M1
		$= 0.384^4 + 4 \times 0.616 \times 0.384^3$		M1
		$= 0.16126...$	awrt 0.161	A1
				(4)
(c)		$X = $ The number of defects per x meters		
		$X \sim \text{N}\left(\dfrac{x}{16}, \dfrac{x}{16}\right)$		B1
		$\text{P}(X < 26) = \text{P}\left(Z < \dfrac{25.5 - \dfrac{x}{16}}{\sqrt{\dfrac{x}{16}}}\right) = 0.5398$		M1
		$\dfrac{25.5 - \dfrac{x}{16}}{\dfrac{1}{4}\sqrt{x}} = 0.1$		B1 M1 A1ft
		$\dfrac{1}{16}x + \dfrac{1}{40}\sqrt{x} - 25.5 = 0 \;\rightarrow\; \sqrt{x} = 20 \quad (\text{or } \sqrt{x} = -20.4)$		M1
		$(\sqrt{x})^2 = 20^2$		M1
		$x = 400$		A1
				(8)
		Notes		**Total 14**
(a)	M1	For writing or using $\text{P}(X \leqslant 5)$		
	A1	awrt 0.616		
(b)	B1ft	For $X \sim \text{B}(4, 0.616)$ Follow through their part (a). May be implied by a correct ft expression for the 2nd M1		
	M1	For writing or using $\text{P}(X \leqslant 1)$ (May be implied by 2nd M1)		
	M1	For $= [^4C_0](1-p)^4 + {}^4C_1 \times p \times (1-p)^3 \qquad 0 < p < 1$		
	A1	awrt 0.161 correct answer on its own scores 4 out of 4		
(c)	B1	For $X \sim \text{N}\left(\dfrac{x}{16}, \dfrac{x}{16}\right)$ May be implied by values in standardisation.		
	M1	For use of a continuity correction either 25.5 or 26.5 (Allow 24.5)		
	B1	$z = \pm 0.1$ Allow calculator value if seen $\pm 0.0999(2986\ldots)$		
	M1	Standardising using either 24.5 or 25 or 25.5 or 26 or 26.5 and equate to a z value. Follow through their mean and variance		
	A1ft	A correct equation with compatible signs ft their mean and variance provided mean = variance		
	M1	For solving their 3 term equation by factorising, completing the square or use of formula. May be implied by -20.4, otherwise if answer is incorrect working must be shown.		
	M1	For correct squaring of both sides. May be implied by 416[.16] from correct equation This mark may be scored prior to solving a 3TQ, e.g. $\left(25.5 - \dfrac{x}{16}\right)^2 = \left(\dfrac{1}{40}\sqrt{x}\right)^2$. Do not award if squaring each individual term		
	A1	$x = 400$ only. This is dependent upon all previous marks in (c).		
	SC	Use of $X \sim \text{N}\left(\dfrac{x}{16}, \dfrac{15x}{256}\right)$ leading to $x = 400$ scores max B0M1B1M1A0M1M1A0		

Question Number	Scheme	Marks
6 (a)	$[\mathrm{F}(k)=1\Rightarrow]ak+bk^2=1\Rightarrow ak=1-bk^2\;*$	B1*
		(1)
(b)	$\mathrm{f}(x)=a+2bx$	B1
	$\mathrm{E}(X)=\int_0^k\left(ax+2bx^2\right)\,\mathrm{d}x\left[=\dfrac{6}{5}\right]\Rightarrow\left[\dfrac{ax^2}{2}+\dfrac{2bx^3}{3}\right]_0^k\left[=\dfrac{6}{5}\right]$	M1
	$\dfrac{ak^2}{2}+\dfrac{2bk^3}{3}=\dfrac{6}{5}$	dM1, A1
	$15ak^2+20bk^3=36$	
	$15k\left(1-bk^2\right)+20bk^3=36$	M1
	$5bk^3=36-15k\;*$	A1*
		(6)
(c)	$\mathrm{E}(X^2)=\int_0^k\left(ax^2+2bx^3\right)\,\mathrm{d}x\Rightarrow\left[\dfrac{ax^3}{3}+\dfrac{bx^4}{2}\right]_0^k$	M1
	$\mathrm{Var}(X)=\dfrac{ak^3}{3}+\dfrac{bk^4}{2}-\dfrac{36}{25}=\dfrac{22}{75}$	dM1 A1
	$10ak^3+15bk^4=52$	
	$10k^2\left(1-bk^2\right)+15bk^4=52$	M1
	$5bk^4=52-10k^2\;*$	A1*
		(5)
(d)	$\dfrac{1}{k}=\dfrac{36-15k}{52-10k^2}$	M1
	$5k^2-36k+52=0$	A1
	$(k-2)(5k-26)=0$	M1
	$k=2$	A1
		(4)
(e)	$'40'b=36-'30'\Rightarrow b=\dfrac{3}{20}\qquad$ or $\qquad '80'b=52-'40'\Rightarrow b=\dfrac{3}{20}$	B1ft
	$2a+\dfrac{3}{5}=1\Rightarrow a=\dfrac{1}{5}$	B1ft
		(2)
Notes		**Total 18**
(a)	**B1*** Answer is given so no incorrect working can be seen	
(b)	**B1** For a correct expression for f(x) (may be implied by a correct expression for E(X))	
	M1 For an attempt to integrate $x\,\mathrm{f}(x)$ (Ignore limits) at least one $(x^n\rightarrow x^{n+1})$. F.t. their f(x) f(x) must be a changed expression from F(x) so integrating $x\mathrm{F}(x)$ is M0	
	dM1 Dependent on the previous M mark. For equating to $\frac{6}{5}$ and substitution of k (no need to see substitution of lower limit 0).	
	A1 For a correct equation any form	
	M1 For substitution of $ak=1-bk^2$ oe into their equation	
	A1* Answer is given so no incorrect working can be seen	

(c)	**M1**	For an attempt to integrate x^2 f(x) (Ignore limits) at least one $(x^n \rightarrow x^{n+1})$ F.t. their f(x) x^2 F(x) is M0
	dM1	Dependent on previous M mark. For substitution of correct limits and subtraction of $\frac{36}{25} = \frac{22}{75}$
	A1	For a correct equation any form
	M1	For substitution of $ak = 1 - bk^2$ oe into their equation
	A1*	Answer is given so no incorrect working can be seen
(d)	**M1**	For solving simultaneously to set up an equation in k only
	A1	For a correct 3 term quadratic
	M1	For solving their 3 term quadratic by factorising, completing the square or using formula. $k = 5.2$ implies M1A1M1
	A1	2 only cao. Correct answer on its own scores 4 out of 4
(e)	**B1ft**	For $b = \dfrac{3}{20}$ ft their k $\qquad b = \dfrac{36 - 15k}{5k^3}$ Common ft answer is $b = \dfrac{-525}{8788} = $ awrt -0.0597 coming from choosing $k = 5.2$
	B1ft	For $a = \dfrac{1}{5}$ ft their k and their b $\qquad a = \dfrac{1 - bk^2}{k}$ Common ft answer is $a = \dfrac{85}{169} = $ awrt 0.503 coming from choosing $k = 5.2$