Analyzing the Period and Amplitude Decay of a Pendulum

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Abstract

The purpose of this experiment was to find the thermal diffusivity coefficient m of tortured rubber. This was done by placing a thermometer incased in tortured rubber in extreme hot and extreme cold at various intervals. The internal and external temperature of the thermometer was measured at regular intervals. The temperature-versus-time graph was modelled by a bessel function [3] and an m value was extrapolated from this model. From our three trials, we found $m = 0.092 \pm 0.002$ mm^2/s for 60 second intervals starting at room temperature, $m = 0.14 \pm 0.01$ mm^2/s for 45 second intervals starting at room temperature, $m = 0.096 \pm 0.001$ mm^2/s for 60 second intervals starting at 97 °C. When compared to literature values of $m = 0.095 \pm 0.17$ mm^2/s [1] and $m = 0.089 \pm 0.013$ mm^2/s for rubber, the results for 60 second trials are very reasonable. The results for the 45 second trial is not. This is likely due to a lack of energy saturation in such a short trial.

Introduction

NOTE: cover that polarizers allow only the componant of light that is polarized along a particular axis through it

Theory

This report is focused on two interesting effects of the polarization of light as it passes through a polaroid.

In both cases, there is a light beam moving in the $\hat{z}$ direction. This beam hits a polaroid with the transmission axis $\hat{y}$. Since, on average, half of the light in this beam is polarized in the $\hat{x}$ direction and half in the $\hat{y}$ direction, half of the light passes through the polaroid. Malus's law describes what happens when a second, or third polarizer is added, and how that changes intensity. Brewster's angle Each of these phenomena will be discussed individually.

Malus's Law

Malus's law, in its simplist form, is a statement about the intensity of light as it passes through two polaroids. Let the transmission axis of this second polarizer be $\hat{y}'$ and the angle made by $\hat{y}$ and $\hat{y}'$ be θ. In this case, the componants of E that pass through this second polarizer is

$$E_{x'} = E \sin\theta$$
$$E_{y'} = E \cos\theta$$

but, of course, only the $\hat{y}'$ componant is transmitted (since $\hat{x}'$ is orthoginal to the transmission axis of the second polaroid). Therefore, if we set $I_0 = E^2$ (the intensity between the polaroids), the intensity of light transmitted by both of them is

$$I(\theta) = E^2 \cos^2\theta = I_0 \cos^2\theta \tag{1}$$

This expression is Malus's Law. The first polaroid is called the *polarizer* and the second is called the *analyzer*.

If a third polaroid is placed further along the $\hat{z}$ axis (after the polarizer and the analyzer) such that its transmission axis is orthoginal to that of the polarizer, some intensity does, interstingly, transmit through. The intensity that passes through this third polaroid can be found by applying Malus's law again. If the intensity of light passing through the polarizer is I_1, the the intensity through the analyzer is

$$I_2 = I_1 \cos^2\varphi$$

where φ is the angle between the transmission axes of the polarizer and analyzer. Applying Malus's law for the second time yields

$$\begin{aligned} I_3 &= I_2 \cos^2(\frac{\pi}{2} - \varphi) \\ &= I_1 \cos^2(\varphi)\cos^2(\frac{\pi}{2} - \varphi) \\ &= \frac{I_1}{4} sin^2(2\varphi) \end{aligned} \tag{2}$$

Materials and Methods

This at home pendulum is comprised of a butter knife, craft twine, a couple of textbooks, a Masterlock$^{\text{TM}}$ combination lock, a metal puzzle game in the shape of an 8 (the lock and the puzzle act as weights), and two heavy textbooks (in this case Peter Giffith's *Introduction to Electrodynamics* and Claude Cohen-Tannoudji's *Quantum Mechanics Vol. 1*), and a table. The butter knife was placed on top of a table, with its blundt end hanging 5.0 ± 0.2 cm off the edge. The sharp end of the butter knife was weighed down with the texbooks. The craft string had one end tied to the blundt end of the butter knife with a slip knot, with the knot pushed against the edge of the table. The other end of the craft string was tied to the shackle of the one of or both of the combination locks using another slip knot (see Figure 1 for reference). The string was then wrapped around the knife, pushing it away from the table (as to remove friction against the table from disrupting the pendulum). This wrapping also gave an easy way to vary the length of the string.

The main way this experiment's trials were divided was based on the mass that it used. There were four trials which had both of the masses tied to the end of the string (further denoted as mass= m_1), and four trials where only the master lock was tied to the end of the string (mass= m_2). Due to a lack of sufficiently sensitive equipment, the masses of these combination locks were not measured. The centre of mass of m_1 was taken to be $D = 4.0 \pm 1.0$ and the centre of mass for m_2 was taken to be $D = 4.8 \pm 1.0$ cm below the knot. This large uncertainty is a function of a lack of good measuring equipment to find the centre of mass. For each mass, there were two trials at with a longer string (denoted L_1) and a shorter string (deonoted L_2): for m_1, $L_1 = 38.7 \pm 0.4$ cm and $L_2 = 22.7 \pm 0.2$ cm, for m_2, $L_1 = 40.3 \pm 0.3$ cm and $L_2 = 24.3 \pm 0.3$ cm. Further, each mass, at each length, had a trial with both a large and small θ_0. Again, due to a lack of good measurement equipment, these initial angles were not measured. The small angles explore the areas around which the small angle approximation breaks down, and the large angles were very large, hoviering around ≈ 1 radians.

The form of these trials give rise to a nice and natural notation for them: $m_1L_1\theta_1$ represents the trial with the larger mass, the longer pendulum, and the larger θ_0. Similarly, $m_2L_2\theta_2$ is the trial with the smaller mass, shorter pendulum, and smaller θ_0. This is the notation of the trials that will be used in all further tables and figures.

Each of these trials were recorded on an Iphone^{TM}'s camera and were allowed to run until there was a significant decrease in energy (though never a total or near total stop of motion). These videos were exported into the open source *Tracker* software, where the mass's position in x and y was tracked, and the data tabulated into .csv files for further analysis.

Data Analysis

The .csv files generated in tracker were analyzed by a bespoke python program. The data was extracted using the *pandas* library. This extracted data analyzed using *numpy* and *scipy.optimize* and plotted using *matplotlib*. These cartesian coordinates were then translated into polar coordinates in the way outlined above. Before moving forward with the analysis, it is important to note that much of this work was done by *Tracker*'s "autotracker" feature, measuring the mass's position in x and y. This autotracker feature uses some very simple computer vision concepts to track the object. This rudementary computer vision system introduces significant error, so an uncertainty equivalent to the angular width of the mass (± 0.1 radians) was added to the x and y values pulled out of the tracker .csv files. The first quantity calculated was a measure of the asymmetry of each pendulum. This was done by calculating the mean of the θ position for each trial, and taking the mean of this value for the 4 trial with each m, with the standard deviation of those four trials as the uncertainty. This yelds the asymmetry of $m_1 = 0.04 \pm 0.02$ radians and of $m_2 = 0.01 \pm 0.03$ radians.

The next step was to calculate the period for each trial. To do this, a program was written to find all points higher than both of its nearest neighbours, and then find the difference between each peak and its two neighbouring peaks. The overall period of any given trial is the mean of all of its individual periods with the uncertainy being the standard deviation (these results are tabulated in).

The exponential model was tested by fitting the model in (3) to the graph of $m_1L_2\theta_1$ using *scipy.optimize.curve_fit*(). *scipy.optimize.curve_fit*() also yielded an estimated value for τ The results of this fit can be seen in figure 4. A similar method was used to find the value of τ for $m_1L_2\theta_2$, $m_1L_1\theta_1$ and $m_2L_2\theta_1$ such that all of m, L, θ_0 are varied. These results can be found in Table 2.

Results and Discussion

The computed values for the thermal diffusivity m, along with the respective uncertainties, were extracted from the optimal curve_fit parameters. These values, along with the fitting data and χ^2 probabilities, are included in Table 1. The data was plotted, including the applied temperature square wave (Figure 3). The uncertainty analysis, described previously, was then carried out and plotted in Figure 4.

These values were compared with an expected value for thermal diffusivity, taken from [1], which was $m = 0.95 \pm 0.17\,\mathrm{mm}^2/\mathrm{s}$, while another source [2] yielded
$m = 0.089 - 0.13\,\mathrm{mm}^2/\mathrm{s}$. Overall, in comparison to the results, a significant overlap from expected and computed values was noted, thus concluding a successful draw of results from the data.

Lastly, from examining results, a large difference in m was noted between the 120s and 90s trials. This was attributed to the 90s trial being too short of a time interval, hence yielding a value of m higher than expected due to the shorter amount of time for energy transfer, since this assumes a denser medium. From Figure 3, it is noticeable that the 90s trial (2) has a much smaller amplitude than that of the 120s trials (1 and 3). In the future, it is recommended to perform trials with longer periods and significant patience.

Conclusions

Overall, it was concluded that the thermal diffusivity of the rubber tube was within the range of the expected experimental value specified in literature for polypropylene. Despite difficulties such as tedious data collection, curve fitting, and uncertainty analysis, the results yielded were valid within the uncertainty range.

Bibliography

[1] Martínez, K., Marín, E., Glorieux, C., Lara-Bernal, A., Calderón, A., Rodríguez, G. P., & Ivanov, R. (2015). Thermal diffusivity measurements in solids by photothermal infrared radiometry: Influence of convection–radiation heat losses. International Journal of Thermal Sciences, 98, 202-207. https://doi.org/10.1016/j.ijthermalsci.2015.07.019

[2] Edge, E. (n.d.). Thermal diffusivity table. Engineers Edge - Engineering, Design and Manufacturing Solutions. Retrieved February 9, 2023, from https://www.engineersedge.com/heat_transfer/thermal_diffusivity_table_13953.htm

[3] Thermal Diffusivity of Tortured Rubber and Bessel Functions. University of Toronto Practicals, PHY324 Manual. https://www.physics.utoronto.ca/~phy224_324/experiments/thermal-diffusivity/labheat.pdf

Appendix I: Figures and Tables

Trial	Period (s)
$m_1L_1\theta_1$	1.442±0.612
$m_1L_1\theta_2$	1.623±0.649
$m_1L_2\theta_1$	1.275±0.48
$m_1L_2\theta_2$	1.11±0.223
$m_2L_1\theta_1$	1.386±0.644
$m_2L_1\theta_2$	1.434±0.324
$m_2L_2\theta_1$	1.181±0.293
$m_2L_2\theta_2$	1.369±0.589

[Table 1] Results obtained for the computed values of the thermal diffusivity for each of the three trials. Included is the applied angular period, the intial temperature of the rubber, the curve_fit computed value for the thermal diffusivity and uncertainty, and the quality of the χ^2 fit.

Trial	τ (s^{-1})	χ^2 (probability)
$m_1L_2\theta_1$	166±2.1	0.2
$m_1L_2\theta_2$	334±12	0.3
$m_1L_2\theta_1$	86.5±49	0.0
$m_2L_2\theta_1$	26.9±6.9	0.0

[Table 1] Results obtained for the computed values of the thermal diffusivity for each of the three trials. Included is the applied angular period, the intial temperature of the rubber, the curve_fit computed value for the thermal diffusivity and uncertainty, and the quality of the χ^2 fit.