Polarization of Light

PHY324, February 19 2023

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Abstract

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Introduction

In introductory optics and electromagnetism, light polarization is the instrinsic property of electromagnetic waves which is given by the orientation of propogation. Generally, there are three types of light polarization: linear, circular, and elliptical. Most light is linearly polarized, and may be manually polarized by means of a polarizer, by reflection, scattering, or refraction through denser media.

Maxwell's equations predict the linear polarization of light as perpendicular electric and magnetic field components, which then are both perpendicular to the direction of the propogation. This allows various projections of these components on vertical and horizontal axes, hence polarizing the light. Any light passing through a polarizer is then polarized in the direction of the polarizer.

Today, polarizers are used in sunglasses, laser physics, photography, and other ranges of electromagnetic waves (radio, x-rays, gamma rays, etc).

Theory

In the absence of electric charge and current distributions, Maxell's equations may be rearranged and re-substituted to obtain the wave equations for typical electric and magnetic field components:

$$\Box^2\,\mathbf{E}(\mathbf{r},t)=0, \qquad \Box^2\,\mathbf{B}(\mathbf{r},t)=0, \tag{1}$$

where $\Box^2 \equiv \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is the D'Alembertian operator. Solving these equations by D'Alembert's method provides expressions for $\mathbf{E}$ and $\mathbf{B}$ as

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\left\{\boldsymbol{\mathcal{E}}_0 \exp\left(i\frac{\omega}{c}(\mathbf{n}\cdot\mathbf{r}-ct)\right)\right\} \tag{2.1}$$

$$\mathbf{B}(\mathbf{r},t) = \mathrm{Re}\left\{\boldsymbol{\mathcal{B}}_0 \exp\left(i\frac{\omega}{c}(\mathbf{n}\cdot\mathbf{r}-ct)\right)\right\}, \tag{2.2}$$

where $\boldsymbol{\mathcal{E}}_0$ and $\boldsymbol{\mathcal{B}}_0$ are the electric and magnetic wave directions, respectively, ω is the angular frequency of the wave, and $\mathbf{n}$ the direction of propogation.

Then, (2.1) and (2.2) are related by Faraday's Law,

$$\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0, \tag{3}$$

which yields the electro-magnetic wave relation

$$\mathrm{Re}\left\{\left(i\frac{\omega}{c}\mathbf{n}\times\boldsymbol{\mathcal{E}}_0 - i\omega\boldsymbol{\mathcal{B}}_0 \exp\left(i\frac{\omega}{c}(\mathbf{n}\cdot\mathbf{r}-ct)\right)\right)\right\}=0, \tag{4}$$

which is true for all space and time components. Therefore $\boldsymbol{\mathcal{B}}_0 = \frac{1}{c}\mathbf{n}\times\boldsymbol{\mathcal{E}}_0$, resulting in perpendicular electric and magnetic field components of a light wave. Polarization is the effect of projecting these vector components of (4) onto a plane, reducing the light intensity and specifying a polarized direction.

This report focuses on two interesting effects of light polarization: Malus's law, and reflectance polarization in the form of Brewster's angle. Malus's law models the effects of polarizers aligned in series to each other, and how the intensity of the light changes.

In its simplist form, Malus's law states that the intensity of light as it passes through two polarizers is proportional to $\cos^2\theta$, where θ is the angle between the two polarizers. Suppose the polarizer is oriented in the $\hat{y}$ direction, and let the transmission axis of this second polarizer be $\hat{y}'$. In this case, the components of the wave which pass through this second polarizer is

$$E_{x'} = E\sin\theta \qquad\qquad E_{y'} = E\cos\theta \tag{5}$$

with the $\hat{y}'$ component being transmitted only (since $\hat{x}'$ is orthogonal to the transmission axis of the second polarizer). By definition, since intensity is proportional to the square of the electric field component, $I_0 = E^2$ (the intensity between the polarizers), thus the intensity of light transmitted by both of them is

$$I(\theta) = E^2\cos^2\theta = I_0\cos^2\theta \tag{6}$$

This expression is called Malus's Law. For this experiment, θ is known from our measurements, and $I(\theta)$ will be extrapolated using an optimization algorithm.

If a third polarizer is placed further along the $\hat{z}$ axis (after the polarizer and the analyzer) such that its transmission axis is orthoginal to that of the polarizer, some intensity does, interstingly, transmit through. The intensity that passes through this third polaroid can be found by applying Malus's law again. If the intensity of light passing through the polarizer is I_1, the the intensity through the analyzer is

$$I_2 = I_1\cos^2\varphi$$

where φ is the angle between the transmission axes of the polarizer and analyzer. Applying Malus's law for the second time yields

$$\begin{aligned} I_3 &= I_2\cos^2(\frac{\pi}{2} - \varphi) \\ &= I_1\cos^2(\varphi)\cos^2(\frac{\pi}{2} - \varphi) \\ &= \frac{I_1}{4} sin^2(2\varphi) \end{aligned} \tag{1}$$

Much like the expression for two polaroids, φ is known and I_1 must be extrapolated using an optimization program.

Methodology

Data Analysis

The .csv files generated in tracker were analyzed by a bespoke python program. The data was extracted using the *pandas* library. This extracted data analyzed using *numpy* and *scipy.optimize* and plotted using *matplotlib*. These cartesian coordinates were then translated into polar coordinates in the way outlined above. Before moving forward with the analysis, it is important to note that much of this work was done by *Tracker*'s "autotracker" feature, measuring the mass's position in x and y. This autotracker feature uses some very simple computer vision concepts to track the object. This rudementary computer vision system introduces significant error, so an uncertainty equivalent to the angular width of the mass (± 0.1 radians) was added to the x and y values pulled out of the tracker .csv files. The first quantity calculated was a measure of the asymmetry of each pendulum. This was done by calculating the mean of the θ position for each trial, and taking the mean of this value for the 4 trial with each m, with the standard deviation of those four trials as the uncertainty. This yelds the asymmetry of $m_1 = 0.04 \pm 0.02$ radians and of $m_2 = 0.01 \pm 0.03$ radians.

The next step was to calculate the period for each trial. To do this, a program was written to find all points higher than both of its nearest neighbours, and then find the difference between each peak and its two neighbouring peaks. The overall period of any given trial is the mean of all of its individual periods with the uncertainy being the standard deviation (these results are tabulated in).

The exponential model was tested by fitting the model in (3) to the graph of $m_1L_2\theta_1$ using *scipy.optimize.curve_fit*(). *scipy.optimize.curve_fit*() also yielded an estimated value for τ The results of this fit can be seen in figure 4. A similar method was used to find the value of τ for $m_1L_2\theta_2$, $m_1L_1\theta_1$ and $m_2L_2\theta_1$ such that all of m, L, θ_0 are varied. These results can be found in Table 2.

Results and Discussion

The computed values for the thermal diffusivity m, along with the respective uncertainties, were extracted from the optimal curve_fit parameters. These values, along with the fitting data and χ^2 probabilities, are included in Table 1. The data was plotted, including the applied temperature square wave (Figure 3). The uncertainty analysis, described previously, was then carried out and plotted in Figure 4.

These values were compared with an expected value for thermal diffusivity, taken from [1], which was $m = 0.95 \pm 0.17\,\mathrm{mm}^2/\mathrm{s}$, while another source [2] yielded
$m = 0.089 - 0.13\,\mathrm{mm}^2/\mathrm{s}$. Overall, in comparison to the results, a significant overlap from expected and computed values was noted, thus concluding a successful draw of results from the data.

Lastly, from examining results, a large difference in m was noted between the 120s and 90s trials. This was attributed to the 90s trial being too short of a time interval, hence yielding a value of m higher than expected due to the shorter amount of time for energy transfer, since this assumes a denser medium. From Figure 3, it is noticeable that the 90s trial (2) has a much smaller amplitude than that of the 120s trials (1 and 3). In the future, it is recommended to perform trials with longer periods and significant patience.

Conclusions

Overall, it was concluded that the thermal diffusivity of the rubber tube was within the range of the expected experimental value specified in literature for polypropylene. Despite difficulties such as tedious data collection, curve fitting, and uncertainty analysis, the results yielded were valid within the uncertainty range.

Bibliography

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[3] Thermal Diffusivity of Tortured Rubber and Bessel Functions. University of Toronto Practicals, PHY324 Manual. https://www.physics.utoronto.ca/~phy224_324/experiments/thermal-diffusivity/labheat.pdf

Appendix I: Figures and Tables

Trial	Period (s)
$m_1L_1\theta_1$	1.442±0.612
$m_1L_1\theta_2$	1.623±0.649
$m_1L_2\theta_1$	1.275±0.48
$m_1L_2\theta_2$	1.11±0.223
$m_2L_1\theta_1$	1.386±0.644
$m_2L_1\theta_2$	1.434±0.324
$m_2L_2\theta_1$	1.181±0.293
$m_2L_2\theta_2$	1.369±0.589

[Table 1] Results obtained for the computed values of the thermal diffusivity for each of the three trials. Included is the applied angular period, the intial temperature of the rubber, the curve_fit computed value for the thermal diffusivity and uncertainty, and the quality of the χ^2 fit.

Trial	τ (s^{-1})	χ^2 (probability)
$m_1L_2\theta_1$	166±2.1	0.2
$m_1L_2\theta_2$	334±12	0.3
$m_1L_2\theta_1$	86.5±49	0.0
$m_2L_2\theta_1$	26.9±6.9	0.0

[Table 1] Results obtained for the computed values of the thermal diffusivity for each of the three trials. Included is the applied angular period, the intial temperature of the rubber, the curve_fit computed value for the thermal diffusivity and uncertainty, and the quality of the χ^2 fit.