# Thermal Diffusivity of Tortured Rubber

PHY324

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**Abstract**

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## Introduction

In thermal physics, the heat transfer between two bodies is the amount of heat energy conducted from one region of high temperature to a region of low temperature. This is studied as a means of energy transfer, which is conserved when no external elements contribute by adding more heat into the system. The thermal diffusivity of a solid is a direct measure of the rate of heat transfer over two regions in a solid.

Thermal diffusivity has some interesting effects. For example, let's say that a thermometer is placed inside of a tube of tortured rubber and and placed an extreme hot bath and an extreme cold bath, and switched between them at some constant interval (i.e. making a square wave of exposed temperature). Something strange happens as this thermometer was transfered between the hot and cold baths: for a short while after a transition, say from cold to hot water, the internal temperature of the tortured rubber continues to decrease! (see for results that show this)

This phase delay of internal temperature is a consequence of thermal diffusivity. The higher a material's coefficient of thermal diffusivity $m$, the larger $m$ is, the larger the phase delay. The purpose of this experiment was to determine the value of the thermal diffusivity constant of tortured rubber.

### Theory of Heat Conduction

How can we extract a value of $m$? Using the physics of head conduction of course!

For an object with a volume $V$ bounded by a surface $S$, the amount of heat entering or exiting the volume over a time interval may be written in terms of the heat flux vector $\mathbf{q}$:

$$\frac{d}{dt}\int_V \rho e \, dV = -\int_S \mathbf{q}\cdot\mathbf{n}\, dA, \tag{1}$$

where $e$ is the energy per unit mass, $\rho$ the density of the body, and $\mathbf{n}$ the outward unit normal of the surface. Due to the time-independence of the body's density, the time derivative to be taken within the integral:

$$\int_V \rho\frac{\partial e}{\partial t}\, dV = -\int_V \nabla\cdot\mathbf{q}\, dV, \tag{2}$$

with Gauss's divergence theorem having been applied on the right hand side to re-write the flux of the heat vector.

Since the internal heat $e$ and respective temperature $T$ are only related by the specific heat proportionality, we may in turn write, with the exception of the integral,

$$\rho\gamma\frac{dT}{dt} = -\nabla\cdot\mathbf{q}. \tag{3}$$

Now, consider Fourier's contribution. For thermal conduction, it was experimentally shown that the rate of heat flow over a surface is directly proportional to the temperature gradient applied over the body, hence allowing Fourier to arrive at the thermal conductivity equation

$$\mathbf{q} = -\kappa\nabla T, \tag{5}$$

where $\kappa$ is the proportionality constant, denoted the thermal conductivity. Taking equations (3) and (4) and substituting, we arrive at the thermal diffusion (or heat) equation in three dimensions:

$$\frac{dT}{dt} = -\frac{\kappa}{\rho\gamma}\nabla^2 T = -m\nabla^2 T \tag{5}$$

with $m = \dfrac{\kappa}{\rho\gamma}$ being the thermal diffusivity constant.

In one dimension, this equation takes the simple form

$$\frac{\partial T}{\partial t} = -m\frac{\partial^2 T}{\partial x^2}. \tag{6}$$

In two dimensions, a change of variables to polar coordinates $x = r\cos\theta$,
$y = r\sin\theta$ yields the form of the Laplacian operator $\nabla^2 = \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2}{\partial\theta^2}$. Furthermore, due to cylindrical symmetry, only the radial component of the temperature needs to be considered. Thus the diffusion equation takes the form

$$-m\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] = \frac{\partial T}{\partial t}. \tag{7}$$

(7) can be solved by a seperation of variables $T(r,t) = R(r)T(t)$. Hence, by diving out $-m$,

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} = -\frac{1}{m}\frac{T'}{T} = -\lambda^2 = \text{constant} \tag{8}$$

which implies firstoff that $T'(t) = \lambda^2 mT(t)$, or $T(t) = e^{i\lambda^2 m t}$.

The purpose of this experiment, however, is to determine the thermal diffusivity $m$ by applying an external temperature on the solid of a frequency rate $\omega$, which yields the expectation that $T(t) = e^{i\omega t}$, implying $\lambda^2 = -\dfrac{i\omega}{m}$. Secondly, (8) yields the Bessel equation:

$$\frac{\partial^2 R}{\partial r^2} + \frac{1}{r}\frac{\partial R}{\partial r} + \lambda^2 R = 0. \tag{9}$$

Since equation (9) is the Bessel equation of zeroth order, executing a series solution in terms of $\lambda r$ yields the order zero Bessel function $AJ_0(\lambda r)$

$$AJ_0(\lambda r) = A\sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^2}\left(\frac{\lambda r}{2}\right)^{2k} \tag{10.1}$$

$$= A\sum_{k=0}^{\infty}\frac{1}{(k!)^2}\left(\frac{i\omega r^2}{4m}\right)^{k} \tag{10.2}$$

which may be expanded in terms of the order zero Kelvin functions $\mathrm{ber}_0$ and $\mathrm{bei}_0$,

$$J_0(-ix) = \mathrm{ber}_0(x) + i\,\mathrm{bei}_0(x) \tag{11.1}$$

$$= \sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}\right)}{(k!)^2}\left(\frac{x}{2}\right)^2 + i\sum_{k=0}^{\infty}\frac{\sin\left(\frac{k\pi}{2}\right)}{(k!)^2}\left(\frac{x}{2}\right)^2 \tag{11.2}$$

where $x = \sqrt{\dfrac{\omega}{m}}r$. Upon resubstitution of $R(r)$ into $T(r,t) = R(r)e^{i\omega t}$, the expression for the temperature is obtained:

$$\begin{aligned} T(r,t) &= A\,\mathbb{R}\mathrm{e}\Big\{\Big[\mathrm{ber}_0(\sqrt{\omega/mr}) + i\,\mathrm{bei}_0(\sqrt{\omega/mr})\Big]e^{i\omega t}\Big\} \\ &= A\mathrm{ber}_0(\sqrt{\omega/mr})\cos(\omega t) + \mathrm{bei}_0(\sqrt{\omega/mr})\sin(\omega t), \end{aligned} \tag{12}$$

where the real part is taken, since (12) is the measured value acquired from taking data at the inner radius of the rubber tube.

## Materials and Methods

This experiments revolves around two beakers. One is filled with ice and water and placed on an adjustable stand, the other is placed on a hot plate and brought to a boil. A thermometer is held in each of these by a retort stand. A third thermometer is encased in a cylinder of tortured rubber (not only on its sides, but also below). This third, insulated thermometer is transferred between these two beakers at regular intervals. These intervals were measured by an iphone stopwatch. A video of the experiment was taken where all three thermometers where clearly visible at all times. The temperatures of all three of the thermometers were taken at 10 second intervals.

Three trials were performed. The first trial had the insulated thermometer start at it a room-temperature of 27 °C and was transferred from one beaker to another in 60 second intervals. The second also had the insulated thermometer start at it a room-temperature of 27 °C, but was transferred at 45 second intervals. The insulated thermometer was placed in the hot bath first for both of these trials and experienced 5 cycles of hot-cold baths. For the third and final trial, the insulated thermometer was placed in the hot beaker and allowed to come up to an extreme temperature before being transferred between the beakers at 60 second intervals for 6 cycles.