The onset of ΛΛ hypernuclear binding

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Binding energies of light, $A \leq 6$, ΛΛ hypernuclei are calculated using the stochastic variational method in a pionless effective field theory ($\not\pi$EFT) approach at leading order with the purpose of assessing critically the onset of binding in the strangeness $\mathcal{S} = -2$ hadronic sector. The $\not\pi$EFT input in this sector consists of (i) a ΛΛ contact term constrained by the ΛΛ scattering length $a_{\Lambda\Lambda}$, using a range of values compatible with ΛΛ correlations observed in relativistic heavy ion collisions, and (ii) a $\Lambda\Lambda N$ contact term constrained by the only available $A \leq 6$ ΛΛ hypernucler binding energy datum of ${}_{\Lambda\Lambda}^{\ 6}$He. The recently debated neutral three-body and four-body systems ${}_{\Lambda\Lambda}^{\ 3}$n and ${}_{\Lambda\Lambda}^{\ 4}$n are found unbound by a wide margin. A relatively large value of $|a_{\Lambda\Lambda}| \gtrsim 1.5$ fm is needed to bind ${}_{\Lambda\Lambda}^{\ 4}$H, thereby questioning its particle stability. In contrast, the particle stability of the $A = 5$ ΛΛ hypernuclear isodoublet ${}_{\Lambda\Lambda}^{\ 5}$H–${}_{\Lambda\Lambda}^{\ 5}$He is robust, with Λ separation energy of order 1 MeV.

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Introduction. Single-Λ and double-Λ (ΛΛ) hypernuclei provide a unique extension of nuclear physics into strange hadronic matter [1]. Whereas the behavior of a single Λ hyperon in atomic nuclei has been deduced quantitatively by studying Λ hypernuclei (${}_{\Lambda}^{A}$Z) from A=3 to 208 [2], only three ΛΛ hypernuclei (${}_{\Lambda\Lambda}^{\ A}$Z) are firmly established: the lightest known ${}_{\Lambda\Lambda}^{\ 6}$He Nagara event [3] and two heavier ones, ${}_{\Lambda\Lambda}^{10}$Be and ${}_{\Lambda\Lambda}^{13}$B [4]. Remarkably, their binding energies come out consistently in shell-model calculations [5]. Few ambiguous emulsion events from KEK [6] and J-PARC [7] have also been reported. However, and perhaps more significant is the absence of any good data on the onset of ΛΛ hypernuclear binding for $A < 6$. In distinction from the heavier species, these very light s-shell species, if bound, could be more affected by microscopic strangeness $\mathcal{S} = -2$ dynamics. An obvious issue is the effect of a possible ΞN dominated H dibaryon resonance some 20–30 MeV above the ΛΛ threshold [8, 9] on ΛΛ hypernuclear binding in general.

Several calculations of light $A < 6$ s-shell ΛΛ hypernuclei using ΛΛ interactions fitted to ${}_{\Lambda\Lambda}^{\ 6}$He suggest a fairly weak ΛΛ interaction, with the onset of ΛΛ hypernuclear binding defered to $A = 4$. Indeed, a slightly bound $I = 0$ ${}_{\Lambda\Lambda}^{\ 4}$H($1^+$) was found in $\Lambda\Lambda pn$ four-body calculations by Nemura *et al.* [10, 11] but not in a four-body calculation by Filikhin and Gal [12] who nonetheless got it bound as a $\Lambda\Lambda d$ cluster. Unfortunately, the AGS-E906 counter experiment [13] searching for light ΛΛ hypernuclei failed to provide conclusive evidence for the particle stability of ${}_{\Lambda\Lambda}^{\ 4}$H [14, 15]. Interestingly, the neutral four-body system ${}_{\Lambda\Lambda}^{\ 4}$n has been assigned in Ref. [15] to the main yet unexplained signal observed by AGS-E906. Recent few-body calculations of ${}_{\Lambda\Lambda}^{\ 4}$n [16, 17] diverge on its particle stability, but since none was constrained by the ${}_{\Lambda\Lambda}^{\ 6}$He binding energy datum, no firm conclusion can be drawn yet.

In the present work we study the light $A \leq 6$ s-shell ΛΛ hypernuclei together with their nuclear and Λ hypernuclear cores at leading-order (LO) $\not\pi$EFT, extending our recent stochastic variational method (SVM) calculations of the s-shell Λ hypernuclei [18]. The $\not\pi$EFT approach was first applied to few-nucleon atomic nuclei in Refs. [19, 20] and recently also in lattice calculations of nuclei [21–24]. Focusing on $\not\pi$EFT applications to $\mathcal{S} = -2$ light systems, we note Λ-Λ-core LO calculations done for $A = 4$ [25] and separately for $A = 6$ [26], which therefore limits their predictive power. Among past non-EFT studies, the only work that covers *all* s-shell ΛΛ hypernuclei is by Nemura *et al.* [11] who used simulated forms of outdated hard-core YN and YY Nijmegen potentials [27]. No chiral EFT (χEFT) calculations of ΛΛ hypernuclei have been reported, although χEFT representations of the ΛΛ interaction at LO [28] and NLO [29] do exist. Hence, the present LO $\not\pi$EFT work is the first comprehensive EFT application to ΛΛ hypernuclei, and could be extended to study multi-Λ hypernuclei and strange hadronic matter.

Extending the $\not\pi$EFT baryonic Lagrangian from nuclei and single-Λ hypernuclei to multi-Λ hypernuclei requires one ΛΛ and one $\Lambda\Lambda N$ new interaction terms. Here we fit the needed ΛΛ contact term to a ΛΛ scattering length value spanning a range of values suggested by recent analyses of ΛΛ correlations observed in relativistic heavy-ion collisions [30–32]. For each choice we fix a $\Lambda\Lambda N$ three-body contact term promoted to LO by fitting to $\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{\ 6}\text{He}) = B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{\ 6}\text{He}) - 2B_{\Lambda}({}_{\Lambda}^{5}\text{He}) = 0.67 \pm 0.17$ MeV. We then show that unless $|a_{\Lambda\Lambda}| \gtrsim 1.5$ fm, ${}_{\Lambda\Lambda}^{\ 4}$H is unlikely to be particle stable. The neutral systems ${}_{\Lambda\Lambda}^{\ 3}$n and ${}_{\Lambda\Lambda}^{\ 4}$n are found unstable by a wide margin. A robust particle stability is established for the ${}_{\Lambda\Lambda}^{\ 5}$H–${}_{\Lambda\Lambda}^{\ 5}$He $A = 5$ isodoublet, with Λ separation energy of order 1 MeV, providing further support for a recent J-PARC proposal [33] to produce ${}_{\Lambda\Lambda}^{\ 5}$H. Possible extensions of our work are briefly discussed in the concluding section.

Extension of $\not\pi$EFT to $\Lambda\Lambda$ hypernuclei. With $\Lambda\Lambda$ one-pion exchange (OPE) forbidden by isospin invariance, the lowest mass pseudoscalar meson exchange is provided by a short range ($\approx$0.4 fm) η exchange which is rather weak in SU(3) flavor. Pions appear in the $\Lambda\Lambda$ dynamics through excitation to fairly high-lying $\Sigma\Sigma$ intermediate states. Therefore, a reasonable choice of a $\not\pi$EFT breakup scale is $2m_\pi$, same as argued for in our recent work on Λ hypernuclei [18]. Excitation from $\Lambda\Lambda$ states to the considerably lower mass ΞN intermediate states requires a shorter range K meson exchange which, together with other short-range exchanges, is accounted for implicitly by the chosen $\not\pi$EFT contact interactions. To provide a meaningful $\not\pi$EFT expansion parameter we note that since $\Delta B_{\Lambda\Lambda}({}^{\;\,6}_{\Lambda\Lambda}\text{He})$ is less than 1 MeV, considerably smaller than $B_\Lambda({}^5_\Lambda\text{He})$, a Λ momentum scale Q in ${}^{\;\,6}_{\Lambda\Lambda}$He may be approximated by that in ${}^5_\Lambda$He [18], namely $p_\Lambda \approx \sqrt{2M_\Lambda B_\Lambda} = 83$ MeV/c, yielding a $\not\pi$EFT expansion parameter $(Q/2m_\pi) \approx 0.3$ and LO accuracy of order $(Q/2m_\pi)^2 \approx 0.09$.

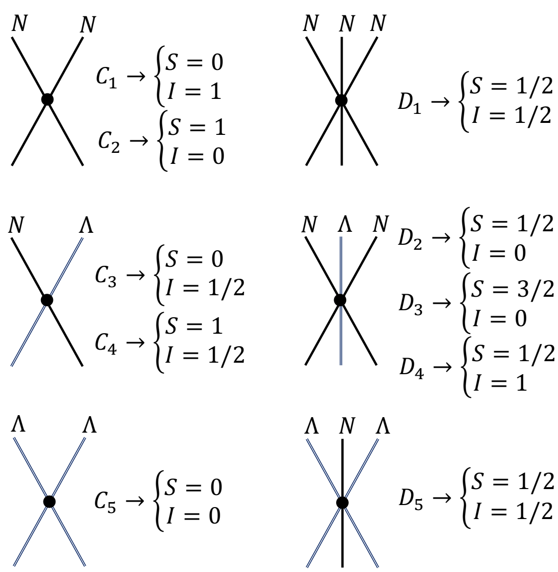

FIG. 1: Diagrammatic presentation of two-body (left) and three-body (right) contact terms, and their associated LEC input $(C_1,\ldots,C_5)$ and $(D_1,\ldots,D_5)$ to a LO $\not\pi$EFT calculation of light nuclei (upper) Λ hypernuclei (middle) and $\Lambda\Lambda$ hypernuclei (lower), with values of spin S and isospin I corresponding to s-wave configurations.

To construct the appropriate $\not\pi$EFT Lagrangian density at LO we follow our previous work on single-Λ hypernuclei [18]:

$$\mathcal{L}^{(\text{LO})} = \sum_B B^\dagger(i\partial_0 + \frac{\nabla^2}{2M_B})B - \mathcal{V}_2 - \mathcal{V}_3, \qquad (1)$$

where $B = (N, \Lambda)$ and $\mathcal{V}_2, \mathcal{V}_3$ consist of two-body and three-body s-wave contact interaction terms, each of which is associated with its own low-energy constant (LEC). These contact terms are shown diagrammatically in Fig. 1 and the corresponding LECs are listed alongside. Going from single-Λ hypernuclei to multi-Λ hypernuclei brings in one new $\Lambda\Lambda$ two-body LEC, C_5, and one new $\Lambda\Lambda N$ three-body LEC, D_5, each one labelled by the total Pauli-spin and isospin involved. This completes the set of LECs required to describe single-, double- and in general multi-Λ hypernuclei at LO. Further contact terms, such as a three-body $\Lambda\Lambda\Lambda$ term, appear only at subleading orders.

Following the procedure applied in Ref. [19], the two-body contact interaction term $\mathcal{V}_2$ gives rise to a two-body potential

$$V_2 = \sum_{IS} C_\lambda^{IS} \sum_{i<j} \mathcal{P}_{IS}(ij)\delta_\lambda(\boldsymbol{r}_{ij}), \qquad (2)$$

where $\mathcal{P}_{IS}$ are projection operators on s-wave $NN, \Lambda N, \Lambda\Lambda$ pairs with isospin I and spin S values associated in Fig. 1 with two-body LECs. These LECs are fitted to low-energy two-body observables, e.g., to the corresponding $NN, \Lambda N, \Lambda\Lambda$ scattering lengths. The subscript λ attached to C^{IS} in Eq. (2) stands for a momentum cutoff introduced in a Gaussian form to regularize the zero-range contact terms:

$$\delta_\lambda(\boldsymbol{r}) = \left(\frac{\lambda}{2\sqrt{\pi}}\right)^3 \exp\left(-\frac{\lambda^2}{4}\boldsymbol{r}^2\right), \qquad (3)$$

thereby smearing a zero-range (in the limit $\lambda \to \infty$) Dirac $\delta^{(3)}(\boldsymbol{r})$ contact term over distances $\sim \lambda^{-1}$. The cutoff parameter λ may be viewed as a scale parameter with respect to typical values of momenta Q. To make observables cutoff independent, the LECs must be properly renormalized. Truncating $\not\pi$EFT at LO and using values of λ higher than the breakup scale of the theory (here $\approx 2m_\pi$), observables acquire a residual dependence $O(Q/\lambda)$ which diminishes with increasing λ.

The three-body contact interaction, promoted to LO, gives rise to a three-body potential

$$V_3 = \sum_{\alpha IS} D_{\alpha\lambda}^{IS} \sum_{i<j<k} \mathcal{Q}_{IS}(ijk) \left(\sum_{\text{cyc}} \delta_\lambda(\boldsymbol{r}_{ij})\delta_\lambda(\boldsymbol{r}_{jk})\right), \qquad (4)$$

where $\mathcal{Q}_{IS}$ projects on NNN, $NN\Lambda$ and $\Lambda\Lambda N$ s-wave triplets with isospin I and spin S values associated in Fig. 1 with three-body LECs which are fitted to given binding energies. The subscript α distinguishes between the two $IS = \frac{1}{2}\frac{1}{2}$ NNN and $\Lambda\Lambda N$ triplets marked in the figure.

Using two-body V_2 and three-body V_3 regularized contact interaction terms as described above, we solved the A-body Schrödinger equation variationally by expanding

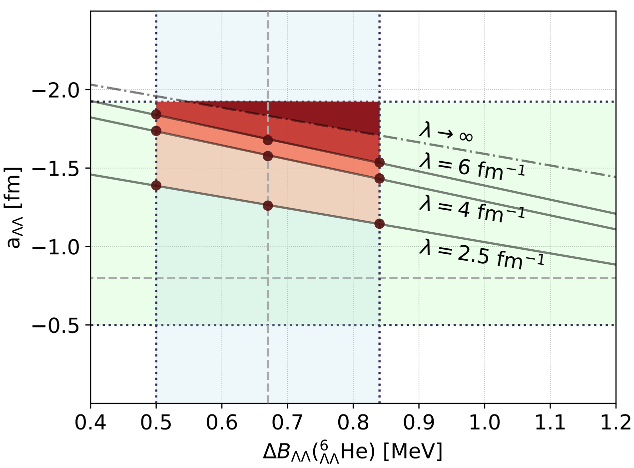

FIG. 2: Minimum values of $|a_{\Lambda\Lambda}|$ for which $^{4}_{\Lambda\Lambda}$H becomes bound are plotted, for given values of cutoff λ, as a function of $\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\mathrm{He})$. The vertical dotted lines mark the experimental uncertainty of $\Delta B_{\Lambda\Lambda}$. The horizontal dotted lines mark the range of $a_{\Lambda\Lambda}$ values $[-0.5, -1.9]$ fm suggested by studies of $\Lambda\Lambda$ correlations [30–32]. The $\lambda \to \infty$ limit is reached assuming a Q/λ asymptotic behavior, similar to the discussion around Eq. (5) below.

TABLE I: Λ separation energies $B_\Lambda(^{A}_{\Lambda\Lambda}\mathrm{Z})$ for A=3–6, calculated using $a_{\Lambda\Lambda}$=−0.8 fm, cutoff λ=4 fm^{-1} and the Alexander[B] ΛN interaction model [18]. In each row a $\Lambda\Lambda N$ LEC was fitted to the underlined binding energy constraint.

Constraint (MeV)	$^{3}_{\Lambda\Lambda}$n	$^{4}_{\Lambda\Lambda}$n	$^{4}_{\Lambda\Lambda}$H	$^{5}_{\Lambda\Lambda}$H	$^{6}_{\Lambda\Lambda}$He
$\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\mathrm{He})=\underline{0.67}$	–	–	–	1.21	3.28
$B_\Lambda(^{4}_{\Lambda\Lambda}\mathrm{H})=\underline{0.05}$	–	–	0.05	2.28	4.76
$B(^{4}_{\Lambda\Lambda}\mathrm{n})=\underline{0.10}$	–	0.10	0.86	4.89	7.89
$B(^{3}_{\Lambda\Lambda}\mathrm{n})=\underline{0.10}$	0.10	15.15	18.40	22.13	25.66

the wave function Ψ in a correlated Gaussian basis using the SVM. For a comprehensive review of this method, see Ref. [35]. For a specific calculation of the three-body interaction matrix elements, see Ref. [36]. The SVM was used in our recent single-Λ hypernuclear work [18] and its extension here is straightforward.

Results and discussion. We first discuss the case of $^{4}_{\Lambda\Lambda}$H, with $I = 0$ and $J^\pi = 1^+$, which following the brief discussion in the Introduction could signal the onset of $\Lambda\Lambda$ hypernuclear binding. For each of several given cutoff values λ we searched for minimum values of $|a_{\Lambda\Lambda}|$, as a function of $\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\mathrm{He})$, that would make $^{4}_{\Lambda\Lambda}$H particle stable. The choice of a specific value for this $\Delta B_{\Lambda\Lambda}$ determines the $\Lambda\Lambda N$ LEC necessary for the $^{4}_{\Lambda\Lambda}$H calculation, in addition to the $\Lambda\Lambda$ LEC determined by $a_{\Lambda\Lambda}$. The resulting values of $|a_{\Lambda\Lambda}|$ above which $^{4}_{\Lambda\Lambda}$H is particle stable are plotted in Fig. 2 as a function of $\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\mathrm{He})$. Choosing sufficiently large values of the cutoff λ, say $\lambda \gtrsim 4$ fm^{-1}, for which convergence to the renormalization scale invariance limit $\lambda \to \infty$ is seen explicitly in the figure, one concludes that $|a_{\Lambda\Lambda}|$ needs to be larger than ≈1.5 fm to bind $^{4}_{\Lambda\Lambda}$H. A $\Lambda\Lambda$ scattering length of such size would make the $\Lambda\Lambda$ interaction almost as strong as the ΛN interaction, whereas most theoretical constructions, e.g. recent Nijmegen models, suggest that it is considerably weaker, say $|a_{\Lambda\Lambda}| \approx 0.8$ fm [37]. For this reason we argue that $^{4}_{\Lambda\Lambda}$H is unlikely to be particle stable.

Using representative values $a_{\Lambda\Lambda} = -0.8$ fm and cutoff λ=4 fm^{-1}, values for which according to Fig. 2 $^{4}_{\Lambda\Lambda}$H is particle unstable, one may reduce the repulsive $\Lambda\Lambda N$ LEC in order to make it particle stable. According to the first two rows in Table I, this will overbind $^{6}_{\Lambda\Lambda}$He by ≈1.5 MeV. Reducing further the $\Lambda\Lambda N$ LEC one binds the neutral systems, first $^{4}_{\Lambda\Lambda}$n (third row) and then $^{3}_{\Lambda\Lambda}$n (fourth row), at a price of overbinding further $^{6}_{\Lambda\Lambda}$He. In fact, the particle stability of these $A = 3, 4$ neutral $\Lambda\Lambda$ systems is incompatible with the $^{6}_{\Lambda\Lambda}$He Nagara event binding energy datum for all values of cutoff λ and scattering length $a_{\Lambda\Lambda}$ tested in Fig. 2. These results suggest quantitatively that the $A = 3, 4$ light neutral $\Lambda\Lambda$ hypernuclei are unbound within a large margin.

Calculated values of the Λ separation energy $B_\Lambda(^{5}_{\Lambda\Lambda}\mathrm{H})$ are shown in Fig. 3. Several representative values of the $\Lambda\Lambda$ scattering length were used: $a_{\Lambda\Lambda}$=−0.5, −0.8, −1.9 fm, spanning a broad range of values suggested by analyses of $\Lambda\Lambda$ correlations observed recently in relativistic heavy-ion collisions [30–32] and by analyzing the KEK-PS E522 [38] invariant mass spectrum in the reaction $^{12}\mathrm{C}(K^-, K^+)\Lambda\Lambda X$ near the $\Lambda\Lambda$ threshold [39]. Again, the choice of $a_{\Lambda\Lambda}$ determines the one $\Lambda\Lambda$ LEC required at LO, while the $\Lambda\Lambda N$ LEC was fitted to the $\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\mathrm{He})$=0.67±0.17 MeV datum. For the ΛN scattering lengths we generally used the Alexander[B] ΛN model (a_s=−1.8 fm, a_t=−1.6 fm); for cutoff λ=4 fm^{-1} we also used three other ΛN interaction models from Ref. [18], demonstrating that the ΛN model dependence is rather weak when it comes to double-Λ hypernuclei, provided B_Λ values of single-Λ hypernuclei for $A < 5$ are fitted to generate the necessary ΛNN LECs. Calculated values of $B_\Lambda(^{5}_{\Lambda}\mathrm{He})$, compatible with those from Ref. [18] are also shown in the figure, demonstrating the suitability of the input ΛN model. One observes that $^{5}_{\Lambda\Lambda}$H comes out particle stable over a broad range of finite cutoff values used in the calculations. This is not the case for $^{4}_{\Lambda\Lambda}$H which, as discussed above, is unbound with respect to $^{3}_{\Lambda}$H for most of the permissible parameter space.

The calculated B_Λ values shown in Fig. 3 exhibit renormalization scale invariance in the limit of $\lambda \to \infty$. To figure out the associated $B_\Lambda(\lambda \to \infty)$ values, we extrapolated $B_\Lambda(\lambda)$ for $\lambda \geq 4$ fm^{-1} using a power series in the small parameter Q/λ:

$$\frac{B_\Lambda(\lambda)}{B_\Lambda(\infty)} = \left[1 + \alpha\frac{Q}{\lambda} + \beta\left(\frac{Q}{\lambda}\right)^2 + \ldots\right]. \qquad (5)$$

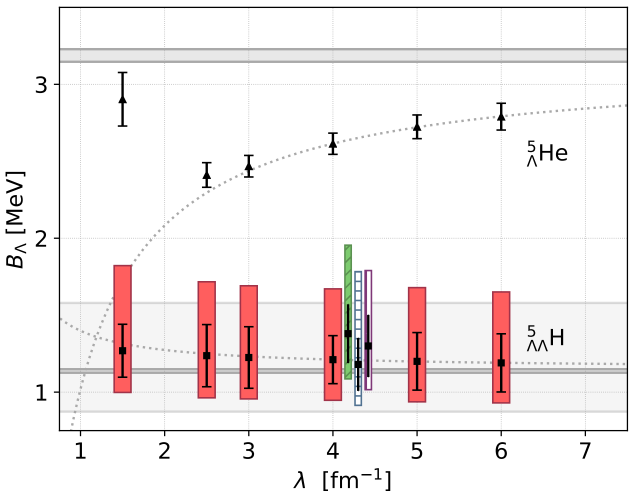

FIG. 3: Λ separation energies $B_\Lambda({}^{\ 5}_{\Lambda\Lambda}\mathrm{H})$ and $B_\Lambda({}^5_\Lambda\mathrm{He})$ from SVM calculations that use $\not\pi$EFT LO two-body (2) and three-body (4) regularized contact interactions, constrained by requiring $\Delta B_{\Lambda\Lambda}({}^{\ 6}_{\Lambda\Lambda}\mathrm{He})$=0.67±0.17 MeV, are plotted as a function of the cutoff λ. Error bars (in black) reflect the experimental uncertainty inherent in the ${}^3_\Lambda$H, ${}^4_\Lambda$H, ${}^4_\Lambda$H* and ${}^{\ 6}_{\Lambda\Lambda}$He binding-energy input data, and (red) rectangles include also varying $a_{\Lambda\Lambda}$ between -0.5 to -1.9 fm. The ΛN interaction model used is Alexander[B] [18], with results for models χLO, χNLO and NSC97f shown from left to right in this order for λ=4 fm^{-1}. Dotted lines show extrapolations, as $\lambda \to \infty$, to the respective scale renormalization invariance limits marked by gray horizontal bands. The wider ${}^{\ 5}_{\Lambda\Lambda}$H band accounts for uncertainties in the experimental values of binding energies used in extrapolating to $\lambda \to \infty$.

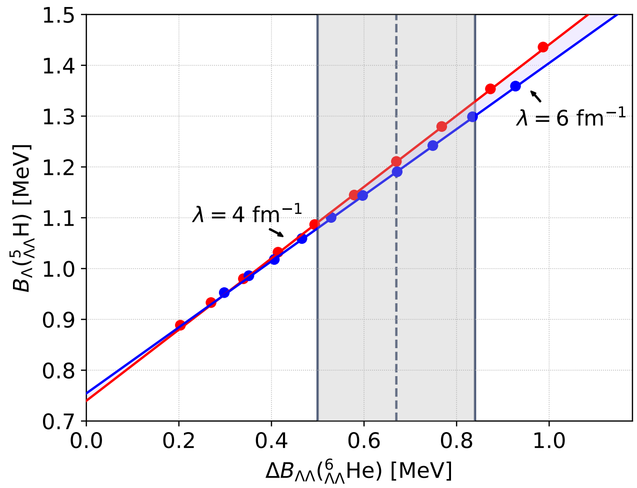

FIG. 4: Hypernuclear Tjon lines: calculated Λ separation energies $B_\Lambda({}^{\ 5}_{\Lambda\Lambda}\mathrm{H})$ are plotted as a function of the constrained value assumed for $\Delta B_{\Lambda\Lambda}({}^{\ 6}_{\Lambda\Lambda}\mathrm{He})$ for two cutoff values, using $a_{\Lambda\Lambda}$=-0.8 fm. The shaded vertical area marks the observed value $\Delta B_{\Lambda\Lambda}({}^{\ 6}_{\Lambda\Lambda}\mathrm{He})$=0.67±0.17 MeV. The ΛN interaction model used is Alexander[B] [18].

The corresponding extrapolation curves are shown by dashed lines in Fig. 3, converging at asymptotic values $B_\Lambda(\infty)$ given with their extrapolated uncertainties by the gray horizontal bands in the figure. ${}^{\ 5}_{\Lambda\Lambda}$H remains particle stable in this limit with Λ separation energy $B_\Lambda(\infty) = 1.14 \pm 0.01^{+0.44}_{-0.26}$ MeV, where the first uncertainty is due to extrapolating by use of Eq. (5) and the second one is due to the $a_{\Lambda\Lambda}$ and B_Λ uncertainties.

The Λ separation energies $B_\Lambda({}^{\ 5}_{\Lambda\Lambda}\mathrm{H})$ studied above are correlated with those of ${}^{\ 6}_{\Lambda\Lambda}$He in a way reminiscent of the Tjon line correlation between binding energies calculated for ^{3}H and ^{4}He [40]. This is shown in Fig. 4 by the linear dependence of $B_\Lambda({}^{\ 5}_{\Lambda\Lambda}\mathrm{H})$, for two given values of the cutoff λ, on the value assumed for $\Delta B_{\Lambda\Lambda}({}^{\ 6}_{\Lambda\Lambda}\mathrm{He})$, which was varied for this purpose around the 'physical' value 0.67±0.17 MeV. We note that the cutoff dependence of this correlation is very weak. The hypernuclear correlation noted here is generated by variation of the $\Lambda\Lambda N$ LEC which is derived from $\Delta B_{\Lambda\Lambda}({}^{\ 6}_{\Lambda\Lambda}\mathrm{He})$. This is similar to the origin and realization of Tjon-line correlations in nuclear physics, where many-body contact interaction terms beyond three-body terms do not appear at LO [44]. However unlike other physics applications where Tjon lines were shown to hold, its appearance here does not require proximity to the unitary limit.

We note that $a_{\Lambda\Lambda}$ includes implicitly the coupling of the $\Lambda\Lambda$ channel to the higher mass I=S=0 ΞN and $\Sigma\Sigma$ channels. However, beginning with ${}^{\ 6}_{\Lambda\Lambda}$He the coupling to the relatively low-lying ΞN channel is partially Pauli blocked (with the formed nucleon excluded from the s shell). It could be argued then that the reference value of $\Delta B_{\Lambda\Lambda}({}^{\ 6}_{\Lambda\Lambda}\mathrm{He})$ used in this work has to be somewhat increased in order to account for the blocked states which are included effectively in the present LO application of $\not\pi$EFT to $\Lambda\Lambda$ hypernuclei. The coupled-channel calculations by Vidaña *et al.* [45] suggest an increase of ≈0.25 MeV which according to Fig. 4 would increase $B_\Lambda({}^{\ 5}_{\Lambda\Lambda}\mathrm{H})$ by roughly 0.15 MeV and ${}^{\ 4}_{\Lambda\Lambda}$H, had it been bound, by no more than 0.03 MeV.

Summary and outlook. The focus in this first comprehensive EFT application to light $\Lambda\Lambda$ hypernuclei was to study the onset of binding in the $\mathcal{S} = -2$ hadronic sector by constraining $\Delta B_{\Lambda\Lambda}({}^{\ 6}_{\Lambda\Lambda}\mathrm{He})$ to the most recent value 0.67±0.17 MeV [4] assigned to the Nagara event [3]. We varied the value assumed for $a_{\Lambda\Lambda}$ over a range of values compatible with those deduced from $\Lambda\Lambda$ correlations observed in relativistic heavy-ion collisions at the RHIC and LHCb facilities [30]. Our results suggest with little model dependence that both members of the $A = 5$ isodoublet pair, ${}^{\ 5}_{\Lambda\Lambda}$H and ${}^{\ 5}_{\Lambda\Lambda}$He, are particle stable. Of the $A = 4$ $\Lambda\Lambda$ hypernuclei, the particle stability of the $I = 0$ ${}^{\ 4}_{\Lambda\Lambda}\mathrm{H}(1^+)$ requires values of $|a_{\Lambda\Lambda}| \gtrsim 1.5$ fm, which are unlikely in our opinion. The $I = 1$ excited state ${}^{\ 4}_{\Lambda\Lambda}\mathrm{H}(0^+)$, or its isospin analog state ${}^{\ 4}_{\Lambda\Lambda}$n are far from being bound; if any of these were established experimentally, the soundness of the Nagara event would have suffered a serious setback.

Extensions of the present LO work should consider explicit $\Lambda\Lambda$-ΞN-$\Sigma\Sigma$ coupling in the $^1S_0$ channel or, at least, address momentum dependent $\Lambda\Lambda$ interaction components generated in NLO EFT through effective-range ($r_{\Lambda\Lambda}$) contributions. We note that no conclusive determination of $r_{\Lambda\Lambda}$ exists yet because of the scarce and inaccurate hyperon-hyperon (mostly $\Xi^- p$) scattering and reaction data available in the ≈25 MeV interval between the $\Lambda\Lambda$ and ΞN thresholds. For example, small values of $r_{\Lambda\Lambda}$ between 0.3 to 0.8 fm were derived from such data in the LO χEFT work of the Jülich-Bonn group [28] using values of $a_{\Lambda\Lambda}$ about -1.5 fm. In contrast, large values of $r_{\Lambda\Lambda}$ between 5 to 7 fm were derived from the same data in the NLO χEFT work of the Jülich-Bonn-Munich group [29] using values of $a_{\Lambda\Lambda}$ about -0.65 fm. This dichotomy is apparent also for the Nijmegen soft core potentials listed in Table I of Ref. [39] and would have to be considered in any quantitative future study of $\Lambda\Lambda$ hypernuclei.

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