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abstract  versions	id-1524932712398135317  The magnetohydrodynamics (MHD) equations are generally known to be difficult to solve numerically, due to their highly nonlinear structure and the strong coupling between the electromagnetic and hydrodynamic variables, especially for high Reynolds and coupling numbers. In the first part of this work, we present a scalable augmented Lagrangian preconditioner for a finite element discretisation of the \$\mathbf{B}\\$-\$\mathbf{E}\\$ formulation of the incompressible viscoresistive MHD equations. For stationary problems, our solver achieves robust performance with respect to the Reynolds and coupling numbers in two dimensions and good results in three dimensions. Our approach relies on specialised parameter-robust multigrid methods for the hydrodynamic and electromagnetic blocks. The scheme ensures exactly divergence-free approximations of both the velocity and the magnetic field up to solver tolerances. In the second part, we focus on incompressible, resistive Hall MHD models and derive structure-preserving finite element methods for these equations. We present a variational formulation of Hall MHD that enforces the magnetic Gauss's law precisely (up to solver tolerances) and prove the well-posedness of a Picard linearisation. For the transient problem, we present time discretisations that preserve the energy and magnetic and hybrid helicity precisely in the ideal limit for two types of boundary conditions. In the third part, we investigate anisothermal MHD models. We start by performing a bifurcation analysis for a magnetic Rayleigh-B\end{e}-B\end{e}-enard problem at a high coupling number \$S=1 {,}000\$ by choosing the Rayleigh number in the range between 0 and \$100 {,}000\$ as the bifurcation parameter. We study the effect of the coupling number on the bifurcation diagram and outline how we create initial guesses to obtain complex solution patterns and disconnected branches for high coupling numbers.	abstract	The magnetohydrodynamics (MHD) equations are generally known to be difficult to solve numerically, due to their highly nonlinear structure and the strong coupling between the electromagnetic and hydrodynamic variables, especially for high Reynolds and coupling numbers. In the first part of this work, we present a scalable augmented Lagrangian preconditioner for a finite element discretisation of the \$\mathbf{B}\$\\$-\$\mathbf{E}\$\\$ formulation of the incompressible viscoresistive MHD equations. For stationary problems, our solver achieves robust performance with respect to the Reynolds and coupling numbers in two dimensions and good results in three dimensions. Our approach relies on specialised parameter-robust multigrid methods for the hydrodynamic and electromagnetic blocks. The scheme ensures exactly divergence-free approximations of both the velocity and the magnetic field up to solver tolerances. In the second part, we focus on incompressible, resistive Hall MHD models and derive structure-preserving finite element methods for these equations. We present a variational formulation of Hall MHD that enforces the magnetic Gauss's law precisely (up to solver tolerances) and prove the well-posedness of a Picard linearisation. For the transient problem, we present time discretisations that preserve the energy and magnetic and hybrid helicity precisely in the ideal limit for two types of boundary conditions. In the third part, we investigate anisothermal MHD models. We start by performing a bifurcation analysis for a magnetic Rayleigh-B\end{e}-B\end{e}-enard problem at a high coupling number \$\$S=1{,}000\$ by choosing the Rayleigh number in the range between 0 and \$100{,}000\$ as the bifurcation parameter. We study the effect of the coupling number on the bifurcation diagram and outline how we create initial guesses to obtain complex solution patterns and disconnected branches for high coupling numbers. Comment: Doctoral thesis, Mathematical Institute, University of Oxford. 174 page	DUPLICATES	)666