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doc_1

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abstract	We investigate regular elliptic boundary-value problems in bounded domains and show the Fredholm property for the related operators in an extended scale formed by inner product Sobolev spaces (of arbitrary real orders) and corresponding interpolation Hilbert spaces. In particular, we can deal with boundary data with arbitrary low regularity. In addition, we show interpolation properties for the extended scale, embedding results, and global and local a priori estimates for solutions to the problems under investigation. The results are applied to elliptic problems with homogeneous right-hand side and to elliptic problems with rough boundary data in Nikolskii spaces, which allows us to treat some cases of white noise on the boundary. 2020 Mathematics Subject Classification. Primary: 35J40; Secondary: 35R60, 46E35, 60H40. Key words and phrases. Elliptic boundary-value problem, generalized Sobolev space, rough boundary data, Fredholm property, a priory estimate of solution, boundary white noise. 697 698 ANNA ANOP, ROBERT DENK AND ALEKSANDR MURACH They are of the form H Î± (Î©), where Î± âˆˆ OR is an O-regularly varying function (see, e.g., [11, Section 2.0.2]). Note that the smoothness parameter Î± is a function, in contrast to the classical Sobolev spaces, where the smoothness is measured by some real number. The Hilbert spaces H Î± (Î©) are special cases of distribution spaces introduced by HÃ¶rmander [21, 22] for a wide class of weight functions and based on the L p -norm. In the situation considered here, the weight function is radially symmetric, and we restrict ourselves to the Hilbert space case of p = 2. We remark that for p = 2 the HÃ¶rmander spaces coincide with the spaces introduced by Volevich and Paneah in [55, Section 2], which were recently also studied by Faierman in [14] . The class {H Î± (Î©) : Î± âˆˆ OR} contains the classical Sobolev spaces H r (Î©) with r âˆˆ R and can be seen as a finer scale of regularity, which allows for more precise embedding and trace theorems. On the other hand, the space H Î± (Î©) can be obtained from the classical Sobolev spaces by interpolation with a function parameter, see Section 5 below. Recently, Mikhailets and Murach developed a general theory of solvability of elliptic boundary-value problems in a class of HÃ¶rmander Hilbert spaces called the refined Sobolev scale (see [34, 35, 36, 37] , and the monograph [39]). The (larger) extended Sobolev scale was considered in [4], for a parabolic version we refer to [30] . In these publications, the boundary data had sufficient regularity to guarantee the existence of boundary traces. More precisely, if Î±(t) â‰¡ Ï•(t)t 2q , then the lower Matuszewska index of Ï• was assumed to be larger than âˆ’1/2 (see Section 3 and Proposition 4.1 below for details). Motivated by applications with rough boundary data, in this paper we consider the situation where this condition on the Matuszewska index does not hold. Even for Sobolev spaces, the case of rough boundary data is quite sophisticated. One approach is the modification of the Sobolev spaces with low regularity as developed by Roitberg [47, 48, 49] . Another way to treat this problem is to include the norm of Au in the norm of the Sobolev space, see Lions and Magenes [29, Chapter 2, Section 6]. Both approaches are applicable to L p -Sobolev spaces with 1 < p < âˆž [27, 28, 48, 49] . In connection with negative order boundary spaces, we also refer to [16] for recent results on weak and very weak traces and to [10, Chapter 5] for the theory of boundary triplets. This paper has the following structure: Section 2 contains the precise formulation of the boundary-value problem (A, B); in Section 3 we introduce the extended Sobolev scales over R n , Î©, and Î“. The main results are formulated in Section 4. We show here that (A, B) induces a Fredholm operator in the extended Sobolev scale (Theorem 4.2). We obtain global and local (up to the boundary) elliptic regularity in the extended scale (see Theorems 4.7 and 4.8, resp.) and elliptic a priori estimates (see Theorem 4.13 for the global and Theorem 4.14 for the local version). Theorems 4.8 and 4.14 are new even in the case of Sobolev spaces. In Section 5, we discuss interpolation properties of the extended Sobolev scale, which will also be used in the proof of the main results in Section 6. In Section 7, we study semihomogeneous boundary value problems, namely the case of f = 0. Defining the space H Î± A (Î©) := {u âˆˆ H Î± (Î©) : Au = 0}, we obtain, e.g., conditions for uniform convergence of sequences of solutions to the homogeneous elliptic equation (Theorems 7.5 and 7.6) and interpolation properties for H Î± A (Î©) (Theorems 7.8 and 7.9). Finally, in Section 8 we apply the results to elliptic boundary-value problems whose boundary data belong to some Nikolskii space B s 2,âˆž (Î“). Based on an embedding result (Proposition 8.1), we show that the solution belongs pathwise to the space ELLIPTIC PROBLEMS WITH ROUGH BOUNDARY DATA 699
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doc_2

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abstract	We investigate regular elliptic boundary-value problems in bounded domains and show the Fredholm property for the related operators in an extended scale formed by inner product Sobolev spaces (of arbitrary real orders) and corresponding interpolation Hilbert spaces. In particular, we can deal with boundary data with arbitrary low regularity. In addition, we show interpolation properties for the extended scale, embedding results, and global and local a priori estimates for solutions to the problems under investigation. The results are applied to elliptic problems with homogeneous right-hand side and to elliptic problems with rough boundary data in Nikoskii spaces, which allows us to treat some cases of white noise on the boundary.
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