

Lab 6

Hands off my stack

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Inventing a variable

```
#load data
library(MASS)
library(dplyr)

##
## Attaching package: 'dplyr'

## The following object is masked from 'package:MASS':
##
##      select

## The following objects are masked from 'package:stats':
##
##      filter, lag

## The following objects are masked from 'package:base':
##
##      intersect, setdiff, setequal, union

library(ggplot2)

d <- read.csv("https://bit.ly/36kibHZ")
#add column MAPE to dataframe
MAPE <- d$Price / d$Earnings_10MA_back
d$MAPE <- MAPE
summary(MAPE)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.     NA's
##      4.785  11.708  15.947  16.554  19.959  44.196     120

#Remove all rows with missing data
d <- na.omit(d)

#build linear model with MAPE
m1 <- lm(Return_10_fwd ~ MAPE, data = d)
summary(m1)

##
## Call:
## lm(formula = Return_10_fwd ~ MAPE, data = d)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.116777 -0.029650  0.004347  0.028478  0.093157
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)  0.1383475  0.0029889  46.29  <2e-16 ***
## MAPE        -0.0045885  0.0001727 -26.57  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.04321 on 1482 degrees of freedom
## Multiple R-squared:  0.3226, Adjusted R-squared:  0.3221
## F-statistic: 705.8 on 1 and 1482 DF,  p-value: < 2.2e-16

#Calculate MSE for five-fold CV
set.seed(42)
d <- d[-sample(1:nrow(d), 4), ]
k <- 5
partition_index <- rep(1:k, each = nrow(d)/k) %>%
sample()
MSE_i <- rep(NA, k)
d$partition_index <- partition_index
for(i in 1:k) {
  test <- d[d$partition_index == i, ]
  train <- d[d$partition_index != i,]
  lm1 <- lm(Return_10_fwd ~ MAPE, data = train)
  pd <- predict(lm1, newdata = test, type = "response")
  MSE_i <- mean((test$Return_10_fwd-pd)^2)
}
```

1. There are exactly 120 NAs because “Earnings_10_MA_back” measures the ten-year moving average of earnings looking back from the current date. We do not have any earnings data for the previous ten years before 1871 that allows us to have information for the ten-year moving average for the first ten years. There is no data available for the ten-year moving average of earnings until 10 years later in 1871. Each row in the dataset represents one month of each of the 12 months in the year, and there are 10 years of missing data. Accordingly, $12 \times 10 = 120$, so there are exactly 120 NAs in both “Earnings_10MA_back” and “MAPE.”
2. As evident in the linear model, the coefficient of MAPE is -0.004602 and its standard error is 0.000173 . This means that for every unit increase in “MAPE” (the ratio of “Price,” the index of U.S. stocks, to “Earnings_10MA_back,” the ten year moving average of earnings), the average rate of return over the next ten years from the current data, “Return_10_fwd,” should decrease by 0.004602 units. Moreover, this rate of return can vary by 0.000173. Since $P < 2e-16$, the effect of MAPE is statistically significant.
3. The MSE of the model under a five-fold CV is 0.001907637.

Inverting a variable

```
#Create inverted variable
Inv_MAPE <- 1 / d$MAPE
d$Inv_MAPE <- Inv_MAPE

#Fit model with Inv_MAPE
m2 <- lm(Return_10_fwd ~ Inv_MAPE, data = d)
summary(m2)

##
## Call:
## lm(formula = Return_10_fwd ~ Inv_MAPE, data = d)
```

```
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.106266 -0.030816  0.002843  0.028174  0.103910
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.007815   0.002883  -2.711  0.00679 **
## Inv_MAPE      0.997627   0.036567  27.282 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.04286 on 1478 degrees of freedom
## Multiple R-squared:  0.3349, Adjusted R-squared:  0.3345
## F-statistic: 744.3 on 1 and 1478 DF,  p-value: < 2.2e-16
```

```
#Find five-fold CV MSE
```

```
for(i in 1:k) {
  test <- d[d$partition_index == i, ]
  train <- d[d$partition_index != i,]
  lm2 <- lm(Return_10_fwd ~ Inv_MAPE, data = train)
  pd2 <- predict(lm2, newdata = test, type = "response")
  MSE_i <- mean((test$Return_10_fwd-pd2)^2)

}
```

```
MSE = mean(MSE_i)
```

```
MSE
```

```
## [1] 0.001876906
```

1. As evident in the linear model, the coefficient of Inv_MAPE is 0.997627 and its standard error is 0.036567 . This means that for every unit increase in “Inv_MAPE” (1 divided by the ratio of “Price,” the index of U.S. stocks, to “Earnings_10MA_back,” the ten year moving average of earnings), the average rate of return over the next ten years from the current data, “Return_10_fwd,” should increase by 0.997627 units. Moreover, this rate of return can vary by 0.036567. Since $P < 2e-16$, the effect of Inv_MAPE is statistically significant.
2. The CV MSE for this model is 0.001876906 which is only slightly smaller than the CV MSE of the previous model.

A simple model

```
for(i in 1:k) {
  test <- d[d$partition_index == i, ]
  train <- d[d$partition_index != i,]
  MSE_i <- mean((train$Return_10_fwd-train$Inv_MAPE)^2)
}
```

```
MSE = mean(MSE_i)
```

```
MSE
```

```
## [1] 0.001892635
```

```
for(i in 1:k) {
  test <- d[d$partition_index == i, ]
  train <- d[d$partition_index != i,]
```

```

MSE_i <- mean((test$Return_10_fwd-test$Inv_MAPE)^2)

}

MSE = mean(MSE_i)
MSE

```

```
## [1] 0.001922588
```

1. The training MSE for this model is 0.001892635. This is exactly the same as the CV test MSE for this model.
2. The test and training MSE for this model are practically equivalent because they both end up evaluating the difference between 1/MAPE and the actual returns in the dataset. In other words, 1/MAPE serves as the predicted observations for “Returns” in both the test and training data.

Is simple sufficient?

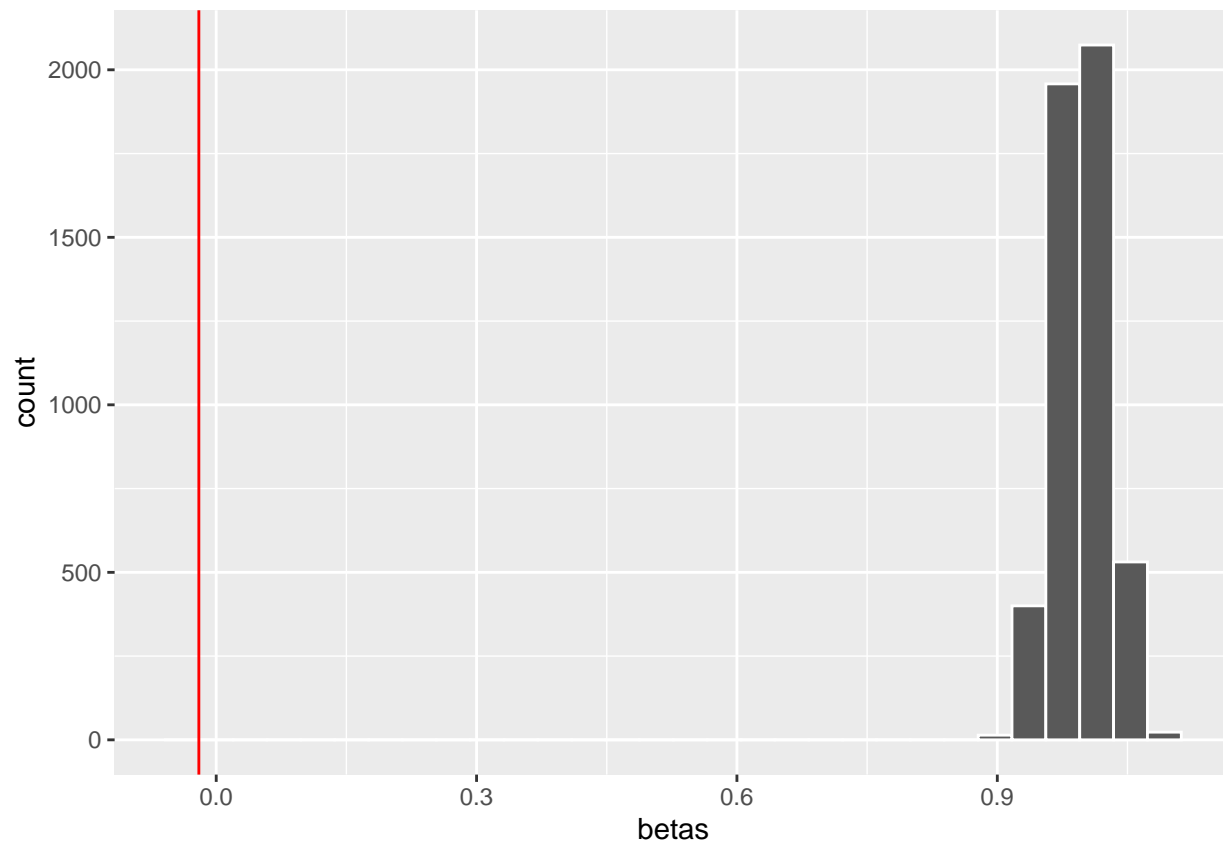
```

betas <- rep(NA, 5000)
for(i in 1:5000) {
  boot_ind <- sample(1:nrow(d),
                    size = nrow(d),
                    replace = TRUE)
  d_boot <- d[boot_ind, ]
  betas[i] <- coef(lm(Return_10_fwd ~ Inv_MAPE, data = d_boot))[2]
}

#Gao's results
df <- data.frame(betas)
ggplot(df, aes(betas)) +
  geom_histogram(col = "white") + geom_vline(xintercept= (-0.02), color="red")

## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.

```



```
theme_bw()
```

```
## List of 65
## $ line :List of 6
## ..$ colour : chr "black"
## ..$ size : num 0.5
## ..$ linetype : num 1
## ..$ lineend : chr "butt"
## ..$ arrow : logi FALSE
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_line" "element"
## $ rect :List of 5
## ..$ fill : chr "white"
## ..$ colour : chr "black"
## ..$ size : num 0.5
## ..$ linetype : num 1
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_rect" "element"
## $ text :List of 11
## ..$ family : chr ""
## ..$ face : chr "plain"
## ..$ colour : chr "black"
## ..$ size : num 11
## ..$ hjust : num 0.5
## ..$ vjust : num 0.5
## ..$ angle : num 0
## ..$ lineheight : num 0.9
```

```

## ..$ margin      : 'margin' num [1:4] Opt Opt Opt Opt
## .. ..- attr(*, "valid.unit")= int 8
## .. ..- attr(*, "unit")= chr "pt"
## ..$ debug       : logi FALSE
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ axis.title.x      :List of 11
## ..$ family       : NULL
## ..$ face         : NULL
## ..$ colour       : NULL
## ..$ size         : NULL
## ..$ hjust        : NULL
## ..$ vjust        : num 1
## ..$ angle        : NULL
## ..$ lineheight    : NULL
## ..$ margin      : 'margin' num [1:4] 2.75pt Opt Opt Opt
## .. ..- attr(*, "valid.unit")= int 8
## .. ..- attr(*, "unit")= chr "pt"
## ..$ debug       : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ axis.title.x.top   :List of 11
## ..$ family       : NULL
## ..$ face         : NULL
## ..$ colour       : NULL
## ..$ size         : NULL
## ..$ hjust        : NULL
## ..$ vjust        : num 0
## ..$ angle        : NULL
## ..$ lineheight    : NULL
## ..$ margin      : 'margin' num [1:4] Opt Opt 2.75pt Opt
## .. ..- attr(*, "valid.unit")= int 8
## .. ..- attr(*, "unit")= chr "pt"
## ..$ debug       : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ axis.title.y      :List of 11
## ..$ family       : NULL
## ..$ face         : NULL
## ..$ colour       : NULL
## ..$ size         : NULL
## ..$ hjust        : NULL
## ..$ vjust        : num 1
## ..$ angle        : num 90
## ..$ lineheight    : NULL
## ..$ margin      : 'margin' num [1:4] Opt 2.75pt Opt Opt
## .. ..- attr(*, "valid.unit")= int 8
## .. ..- attr(*, "unit")= chr "pt"
## ..$ debug       : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ axis.title.y.right :List of 11
## ..$ family       : NULL
## ..$ face         : NULL

```

```

## ..$ colour      : NULL
## ..$ size        : NULL
## ..$ hjust       : NULL
## ..$ vjust       : num 0
## ..$ angle       : num -90
## ..$ lineheight  : NULL
## ..$ margin      : 'margin' num [1:4] 0pt 0pt 0pt 2.75pt
## ..- attr(*, "valid.unit")= int 8
## ..- attr(*, "unit")= chr "pt"
## ..$ debug       : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ axis.text      :List of 11
## ..$ family      : NULL
## ..$ face        : NULL
## ..$ colour      : chr "grey30"
## ..$ size        : 'rel' num 0.8
## ..$ hjust       : NULL
## ..$ vjust       : NULL
## ..$ angle       : NULL
## ..$ lineheight  : NULL
## ..$ margin      : NULL
## ..$ debug       : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ axis.text.x    :List of 11
## ..$ family      : NULL
## ..$ face        : NULL
## ..$ colour      : NULL
## ..$ size        : NULL
## ..$ hjust       : NULL
## ..$ vjust       : num 1
## ..$ angle       : NULL
## ..$ lineheight  : NULL
## ..$ margin      : 'margin' num [1:4] 2.2pt 0pt 0pt 0pt
## ..- attr(*, "valid.unit")= int 8
## ..- attr(*, "unit")= chr "pt"
## ..$ debug       : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ axis.text.x.top :List of 11
## ..$ family      : NULL
## ..$ face        : NULL
## ..$ colour      : NULL
## ..$ size        : NULL
## ..$ hjust       : NULL
## ..$ vjust       : num 0
## ..$ angle       : NULL
## ..$ lineheight  : NULL
## ..$ margin      : 'margin' num [1:4] 0pt 0pt 2.2pt 0pt
## ..- attr(*, "valid.unit")= int 8
## ..- attr(*, "unit")= chr "pt"
## ..$ debug       : NULL
## ..$ inherit.blank: logi TRUE

```

```

##   .- attr(*, "class")= chr [1:2] "element_text" "element"
## $ axis.text.y                :List of 11
##   ..$ family                : NULL
##   ..$ face                  : NULL
##   ..$ colour                 : NULL
##   ..$ size                   : NULL
##   ..$ hjust                  : num 1
##   ..$ vjust                  : NULL
##   ..$ angle                  : NULL
##   ..$ lineheight             : NULL
##   ..$ margin                 : 'margin' num [1:4] Opt 2.2pt Opt Opt
##   .. .- attr(*, "valid.unit")= int 8
##   .. .- attr(*, "unit")= chr "pt"
##   ..$ debug                  : NULL
##   ..$ inherit.blank: logi TRUE
##   .- attr(*, "class")= chr [1:2] "element_text" "element"
## $ axis.text.y.right          :List of 11
##   ..$ family                : NULL
##   ..$ face                  : NULL
##   ..$ colour                 : NULL
##   ..$ size                   : NULL
##   ..$ hjust                  : num 0
##   ..$ vjust                  : NULL
##   ..$ angle                  : NULL
##   ..$ lineheight             : NULL
##   ..$ margin                 : 'margin' num [1:4] Opt Opt Opt 2.2pt
##   .. .- attr(*, "valid.unit")= int 8
##   .. .- attr(*, "unit")= chr "pt"
##   ..$ debug                  : NULL
##   ..$ inherit.blank: logi TRUE
##   .- attr(*, "class")= chr [1:2] "element_text" "element"
## $ axis.ticks                  :List of 6
##   ..$ colour                 : chr "grey20"
##   ..$ size                   : NULL
##   ..$ linetype               : NULL
##   ..$ lineend                : NULL
##   ..$ arrow                  : logi FALSE
##   ..$ inherit.blank: logi TRUE
##   .- attr(*, "class")= chr [1:2] "element_line" "element"
## $ axis.ticks.length           : 'unit' num 2.75pt
##   .- attr(*, "valid.unit")= int 8
##   .- attr(*, "unit")= chr "pt"
## $ axis.ticks.length.x         : NULL
## $ axis.ticks.length.x.top     : NULL
## $ axis.ticks.length.x.bottom : NULL
## $ axis.ticks.length.y         : NULL
## $ axis.ticks.length.y.left   : NULL
## $ axis.ticks.length.y.right  : NULL
## $ axis.line                   : list()
##   .- attr(*, "class")= chr [1:2] "element_blank" "element"
## $ axis.line.x                 : NULL
## $ axis.line.y                 : NULL
## $ legend.background           :List of 5
##   ..$ fill                    : NULL

```



```

## ..$ colour      : logi NA
## ..$ size        : NULL
## ..$ linetype    : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_rect" "element"
## $ legend.margin      : 'margin' num [1:4] 5.5pt 5.5pt 5.5pt 5.5pt
## ..- attr(*, "valid.unit")= int 8
## ..- attr(*, "unit")= chr "pt"
## $ legend.spacing     : 'unit' num 11pt
## ..- attr(*, "valid.unit")= int 8
## ..- attr(*, "unit")= chr "pt"
## $ legend.spacing.x   : NULL
## $ legend.spacing.y   : NULL
## $ legend.key          :List of 5
## ..$ fill             : chr "white"
## ..$ colour          : logi NA
## ..$ size             : NULL
## ..$ linetype        : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_rect" "element"
## $ legend.key.size     : 'unit' num 1.2lines
## ..- attr(*, "valid.unit")= int 3
## ..- attr(*, "unit")= chr "lines"
## $ legend.key.height   : NULL
## $ legend.key.width    : NULL
## $ legend.text         :List of 11
## ..$ family          : NULL
## ..$ face            : NULL
## ..$ colour          : NULL
## ..$ size            : 'rel' num 0.8
## ..$ hjust           : NULL
## ..$ vjust           : NULL
## ..$ angle           : NULL
## ..$ lineheight      : NULL
## ..$ margin          : NULL
## ..$ debug           : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ legend.text.align   : NULL
## $ legend.title        :List of 11
## ..$ family          : NULL
## ..$ face            : NULL
## ..$ colour          : NULL
## ..$ size            : NULL
## ..$ hjust           : num 0
## ..$ vjust           : NULL
## ..$ angle           : NULL
## ..$ lineheight      : NULL
## ..$ margin          : NULL
## ..$ debug           : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ legend.title.align  : NULL
## $ legend.position     : chr "right"

```

```

## $ legend.direction      : NULL
## $ legend.justification  : chr "center"
## $ legend.box            : NULL
## $ legend.box.margin     : 'margin' num [1:4] 0cm 0cm 0cm 0cm
##   .. attr(*, "valid.unit")= int 1
##   .. attr(*, "unit")= chr "cm"
## $ legend.box.background : list()
##   .. attr(*, "class")= chr [1:2] "element_blank" "element"
## $ legend.box.spacing    : 'unit' num 11pt
##   .. attr(*, "valid.unit")= int 8
##   .. attr(*, "unit")= chr "pt"
## $ panel.background      :List of 5
##   ..$ fill              : chr "white"
##   ..$ colour            : logi NA
##   ..$ size              : NULL
##   ..$ linetype          : NULL
##   ..$ inherit.blank: logi TRUE
##   .. attr(*, "class")= chr [1:2] "element_rect" "element"
## $ panel.border          :List of 5
##   ..$ fill              : logi NA
##   ..$ colour            : chr "grey20"
##   ..$ size              : NULL
##   ..$ linetype          : NULL
##   ..$ inherit.blank: logi TRUE
##   .. attr(*, "class")= chr [1:2] "element_rect" "element"
## $ panel.spacing         : 'unit' num 5.5pt
##   .. attr(*, "valid.unit")= int 8
##   .. attr(*, "unit")= chr "pt"
## $ panel.spacing.x       : NULL
## $ panel.spacing.y       : NULL
## $ panel.grid             :List of 6
##   ..$ colour            : chr "grey92"
##   ..$ size              : NULL
##   ..$ linetype          : NULL
##   ..$ lineend           : NULL
##   ..$ arrow             : logi FALSE
##   ..$ inherit.blank: logi TRUE
##   .. attr(*, "class")= chr [1:2] "element_line" "element"
## $ panel.grid.minor      :List of 6
##   ..$ colour            : NULL
##   ..$ size              : 'rel' num 0.5
##   ..$ linetype          : NULL
##   ..$ lineend           : NULL
##   ..$ arrow             : logi FALSE
##   ..$ inherit.blank: logi TRUE
##   .. attr(*, "class")= chr [1:2] "element_line" "element"
## $ panel.ontop           : logi FALSE
## $ plot.background       :List of 5
##   ..$ fill              : NULL
##   ..$ colour            : chr "white"
##   ..$ size              : NULL
##   ..$ linetype          : NULL
##   ..$ inherit.blank: logi TRUE
##   .. attr(*, "class")= chr [1:2] "element_rect" "element"

```

```

## $ plot.title :List of 11
## ..$ family : NULL
## ..$ face : NULL
## ..$ colour : NULL
## ..$ size : 'rel' num 1.2
## ..$ hjust : num 0
## ..$ vjust : num 1
## ..$ angle : NULL
## ..$ lineheight : NULL
## ..$ margin : 'margin' num [1:4] Opt Opt 5.5pt Opt
## ..- attr(*, "valid.unit")= int 8
## ..- attr(*, "unit")= chr "pt"
## ..$ debug : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ plot.subtitle :List of 11
## ..$ family : NULL
## ..$ face : NULL
## ..$ colour : NULL
## ..$ size : NULL
## ..$ hjust : num 0
## ..$ vjust : num 1
## ..$ angle : NULL
## ..$ lineheight : NULL
## ..$ margin : 'margin' num [1:4] Opt Opt 5.5pt Opt
## ..- attr(*, "valid.unit")= int 8
## ..- attr(*, "unit")= chr "pt"
## ..$ debug : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ plot.caption :List of 11
## ..$ family : NULL
## ..$ face : NULL
## ..$ colour : NULL
## ..$ size : 'rel' num 0.8
## ..$ hjust : num 1
## ..$ vjust : num 1
## ..$ angle : NULL
## ..$ lineheight : NULL
## ..$ margin : 'margin' num [1:4] 5.5pt Opt Opt Opt
## ..- attr(*, "valid.unit")= int 8
## ..- attr(*, "unit")= chr "pt"
## ..$ debug : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ plot.tag :List of 11
## ..$ family : NULL
## ..$ face : NULL
## ..$ colour : NULL
## ..$ size : 'rel' num 1.2
## ..$ hjust : num 0.5
## ..$ vjust : num 0.5
## ..$ angle : NULL
## ..$ lineheight : NULL

```

```

## ..$ margin      : NULL
## ..$ debug       : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ plot.tag.position      : chr "topleft"
## $ plot.margin            : 'margin' num [1:4] 5.5pt 5.5pt 5.5pt 5.5pt
## ..- attr(*, "valid.unit")= int 8
## ..- attr(*, "unit")= chr "pt"
## $ strip.background      :List of 5
## ..$ fill                : chr "grey85"
## ..$ colour              : chr "grey20"
## ..$ size                : NULL
## ..$ linetype            : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_rect" "element"
## $ strip.placement       : chr "inside"
## $ strip.text            :List of 11
## ..$ family              : NULL
## ..$ face                 : NULL
## ..$ colour              : chr "grey10"
## ..$ size                : 'rel' num 0.8
## ..$ hjust               : NULL
## ..$ vjust               : NULL
## ..$ angle               : NULL
## ..$ lineheight          : NULL
## ..$ margin              : 'margin' num [1:4] 4.4pt 4.4pt 4.4pt 4.4pt
## .. ..- attr(*, "valid.unit")= int 8
## .. ..- attr(*, "unit")= chr "pt"
## ..$ debug               : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ strip.text.x          : NULL
## $ strip.text.y          :List of 11
## ..$ family              : NULL
## ..$ face                 : NULL
## ..$ colour              : NULL
## ..$ size                : NULL
## ..$ hjust               : NULL
## ..$ vjust               : NULL
## ..$ angle               : num -90
## ..$ lineheight          : NULL
## ..$ margin              : NULL
## ..$ debug               : NULL
## ..$ inherit.blank: logi TRUE
## ..- attr(*, "class")= chr [1:2] "element_text" "element"
## $ strip.switch.pad.grid : 'unit' num 2.75pt
## ..- attr(*, "valid.unit")= int 8
## ..- attr(*, "unit")= chr "pt"
## $ strip.switch.pad.wrap : 'unit' num 2.75pt
## ..- attr(*, "valid.unit")= int 8
## ..- attr(*, "unit")= chr "pt"
## - attr(*, "class")= chr [1:2] "theme" "gg"
## - attr(*, "complete")= logi TRUE
## - attr(*, "validate")= logi TRUE

```

```
#Obtain 95% bootstrap confidence interval
m_boot = lm(Return_10_fwd ~ Inv_MAPE, data = d_boot)
confint(m_boot, level = 0.95)
```

```
##                2.5 %          97.5 %
## (Intercept) -0.0114152 -0.0002200273
## Inv_MAPE      0.8998514  1.0408814627
```

The 95% bootstrap confidence interval for the slope of $1/\text{MAPE}$ is between 0.93862423 and 1.068285499. In comparison, through directly running `confint()` on the function in question 2, we are 95% certain that the slope for $1/\text{MAPE}$ falls between 0.92589823 and 1.069356445. The lower limit of the 95% bootstrap confidence interval is only slightly higher than the interval obtained through the second method. However, the upper limit of the 95% bootstrap confidence interval is very slightly lower than the interval obtained through the second method. Consequently the confidence intervals are extremely for the slope of $1/\text{MAPE}$ are extremely similar.

```
#m1 <- lm(Return_10_fwd ~ MAPE, data = d)
#m2 <- lm(d$Return_10_fwd ~ d$Inv_MAPE, data = d)
predict(m1, newdata = data.frame(MAPE = 50), type = "response")
```

```
##          1
## -0.09107925
```

```
predict(m1, newdata = data.frame(MAPE =5), type = "response")
```

```
##          1
## 0.1154048
```

```
m2 <- lm(Return_10_fwd ~ Inv_MAPE, data = d)
```

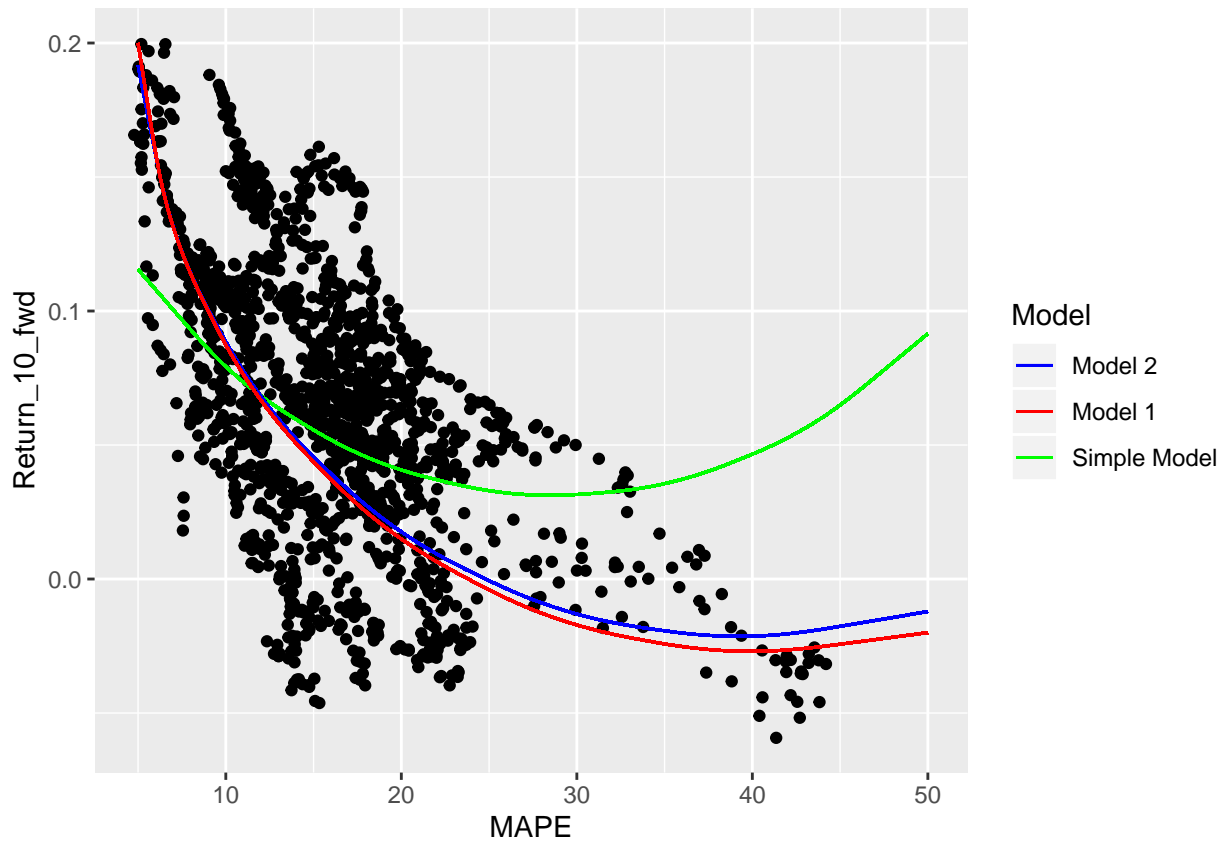
```
predict(m2, newdata = data.frame(Inv_MAPE = 1/50), type = "response")
```

```
##          1
## 0.01213715
```

```
predict(m2, newdata = data.frame(Inv_MAPE =1/5), type = "response")
```

```
##          1
## 0.1917101
```

```
ggplot(d, aes(x = MAPE, y = Return_10_fwd)) + geom_point() + geom_curve(aes(x = 5, y = 0.1155154, xend =
```



```
summary(m1)
```

```
##
## Call:
## lm(formula = Return_10_fwd ~ MAPE, data = d)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.116777 -0.029650  0.004347  0.028478  0.093157
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.1383475   0.0029889   46.29  <2e-16 ***
## MAPE        -0.0045885   0.0001727  -26.57  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.04321 on 1482 degrees of freedom
## Multiple R-squared:  0.3226, Adjusted R-squared:  0.3221
## F-statistic: 705.8 on 1 and 1482 DF, p-value: < 2.2e-16
```

```
summary(m2)
```

```
##
## Call:
## lm(formula = Return_10_fwd ~ Inv_MAPE, data = d)
##
## Residuals:
```

```
##           Min           1Q       Median           3Q           Max
## -0.106266 -0.030816  0.002843  0.028174  0.103910
##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.007815   0.002883  -2.711  0.00679 **
## Inv_MAPE     0.997627   0.036567  27.282  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.04286 on 1478 degrees of freedom
## Multiple R-squared:  0.3349, Adjusted R-squared:  0.3345
## F-statistic: 744.3 on 1 and 1478 DF,  p-value: < 2.2e-16
```

Model 1 and Model 2 seem to fit the trend of the original dataset most closely. Comparatively the simple model seems to slightly underestimate returns when MAPE is lower and overestimate returns when MAPE is higher. The predictions from Model 1 and Model 2 are very close to each other and follow the negatively correlated relationship between MAPE and returns, demonstrating how returns tend to decrease as the ratio between price and earnings increases.

The big picture

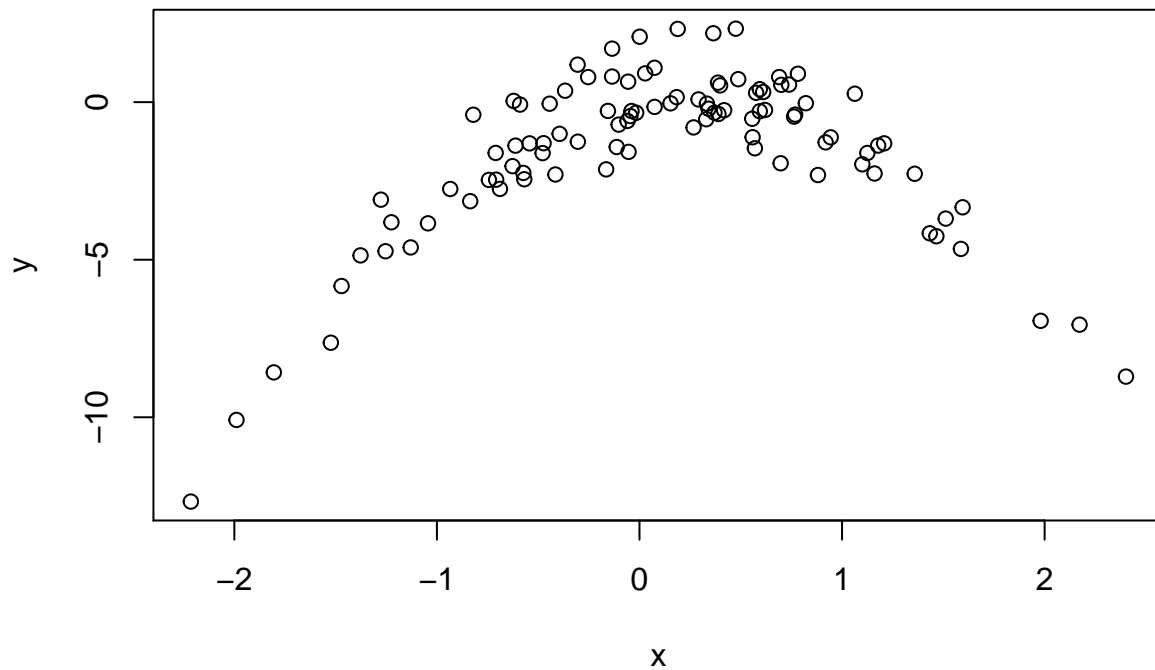
1. Since Model 2 has the lowest CV MSE of 0.0018769, I would select the model to make predictions on returns. Furthermore, looking at the plot from question 5, this seems to be a good model. This model considers the ratio of the ten year moving average of earnings to the index of U.S. stocks. However, it seems to have a slightly larger standard error than the slope of MAPE in model 1 and there is more variation in slopes for $1/\text{MAPE}$.
2. Based off my bootstrapping procedure for the slope of the linear model using $1/\text{MAPE}$ as a predictor, the simple-minded model does not seem very plausible given the data. The approximate slope of the simple-minded model seems to lie a little below 0. This is very far from the slope estimates determined from the bootstrap distribution.

Chapter 5 exercises

4. The standard deviation of the predicted observation can be estimated through the bootstrap approach. Correspondingly, we would generate many random samples from our original dataset, sampling for replacement each time, and record the estimates for the standard deviation each time. Afterwards, we would find the mean of the estimates from each time and the standard deviation across all estimates.
- 5.

```
set.seed(1)
x = rnorm(100)
y = x^2 * x^2 + rnorm(100)

plot(x,y)
```



```
#c
library(boot)
set.seed(2)
Data <- data.frame(x, y)
fit1 <- glm(y ~ x)
cv.glm(Data, fit1)$delta[1]
```

```
## [1] 7.288162
```

```
fit2 <- glm(y ~ poly(x, 2))
cv.glm(Data, fit2)$delta[1]
```

```
## [1] 0.9374236
```

```
fit3 <- glm(y ~ poly(x, 3))
cv.glm(Data, fit3)$delta[1]
```

```
## [1] 0.9566218
```

```
fit4 <- glm(y ~ poly(x, 4))
cv.glm(Data, fit4)$delta[1]
```

```
## [1] 0.9539049
```

```
set.seed(19)
fit5 <- glm(y ~ x)
cv.glm(Data, fit5)$delta[1]
```

```
## [1] 7.288162
```

```
fit6 <- glm(y ~ poly(x, 2))
cv.glm(Data, fit6)$delta[1]
```

```
## [1] 0.9374236
```

```
fit7 <- glm(y ~ poly(x, 3))
cv.glm(Data, fit7)$delta[1]
```



```
## [1] 0.9566218
```

```
fit8 <- glm(y ~ poly(x, 4))  
cv.glm(Data, fit8)$delta[1]
```

```
## [1] 0.9539049
```

```
summary(fit1)
```

```
##  
## Call:  
## glm(formula = y ~ x)  
##  
## Deviance Residuals:  
##      Min       1Q   Median       3Q      Max   
## -9.5161  -0.6800   0.6812   1.5491   3.8183   
##  
## Coefficients:  
##              Estimate Std. Error t value Pr(>|t|)      
## (Intercept)  -1.6254     0.2619  -6.205 1.31e-08 ***  
## x              0.6925     0.2909   2.380  0.0192 *     
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## (Dispersion parameter for gaussian family taken to be 6.760719)  
##  
##      Null deviance: 700.85  on 99  degrees of freedom  
## Residual deviance: 662.55  on 98  degrees of freedom  
## AIC: 478.88  
##  
## Number of Fisher Scoring iterations: 2
```

```
summary(fit2)
```

```
##  
## Call:  
## glm(formula = y ~ poly(x, 2))  
##  
## Deviance Residuals:  
##      Min       1Q   Median       3Q      Max   
## -1.9650  -0.6254  -0.1288   0.5803   2.2700   
##  
## Coefficients:  
##              Estimate Std. Error t value Pr(>|t|)      
## (Intercept)  -1.5500     0.0958  -16.18 < 2e-16 ***  
## poly(x, 2)1    6.1888     0.9580   6.46 4.18e-09 ***  
## poly(x, 2)2  -23.9483     0.9580  -25.00 < 2e-16 ***  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## (Dispersion parameter for gaussian family taken to be 0.9178258)  
##  
##      Null deviance: 700.852  on 99  degrees of freedom  
## Residual deviance:   89.029  on 97  degrees of freedom  
## AIC: 280.17  
##
```

```
## Number of Fisher Scoring iterations: 2
```

```
summary(fit3)
```

```
##
## Call:
## glm(formula = y ~ poly(x, 3))
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.9765  -0.6302  -0.1227   0.5545   2.2843
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -1.55002     0.09626  -16.102 < 2e-16 ***
## poly(x, 3)1    6.18883     0.96263   6.429 4.97e-09 ***
## poly(x, 3)2  -23.94830     0.96263 -24.878 < 2e-16 ***
## poly(x, 3)3    0.26411     0.96263   0.274  0.784
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 0.9266599)
##
##      Null deviance: 700.852  on 99  degrees of freedom
## Residual deviance:  88.959  on 96  degrees of freedom
## AIC: 282.09
##
## Number of Fisher Scoring iterations: 2
```

```
summary(fit4)
```

```
##
## Call:
## glm(formula = y ~ poly(x, 4))
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -2.0550  -0.6212  -0.1567   0.5952   2.2267
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -1.55002     0.09591  -16.162 < 2e-16 ***
## poly(x, 4)1    6.18883     0.95905   6.453 4.59e-09 ***
## poly(x, 4)2  -23.94830     0.95905 -24.971 < 2e-16 ***
## poly(x, 4)3    0.26411     0.95905   0.275  0.784
## poly(x, 4)4    1.25710     0.95905   1.311  0.193
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 0.9197797)
##
##      Null deviance: 700.852  on 99  degrees of freedom
## Residual deviance:  87.379  on 95  degrees of freedom
## AIC: 282.3
##
```

Number of Fisher Scoring iterations: 2

$n = 100$ and $p = 2$ $Y = X - 2X^2 + \epsilon$

b) There is a curved relationship between x and y . Initially, there is a positively correlated relationship between x and y as y increases as x increases. However, once x is greater than 0, there is a negatively correlated relationship between x and y as an increase in x is associated with a decrease in y .

c) Compute LOOCV errors:

- i. 7.288162
- ii. 0.9374236 iii. 0.9566218
- iii. 0.9539049

d) Compute LOOCV errors:

- i. 7.288162
- ii. 0.9374236 iii. 0.9566218
- iii. 0.9539049

The results are exactly the same as the results from (c). This is because LOOCV uses a single observation as a validation set.

- e) “Fit2” had the smallest LOOCV error. This is not surprising because the scatterplot from b demonstrates that the relationship between x and y is quadratic.
- f) After fitting each of the models, it is clear that the linear and quadratic terms are consistently statistically significant. However, the cubic and 4th degree terms are not. Consequently, the results agree with our cross-validation results in which the quadratic (“fit2”) model had the least LOOCV error.