Lab 2: Linear Regression

An Island Never Cries

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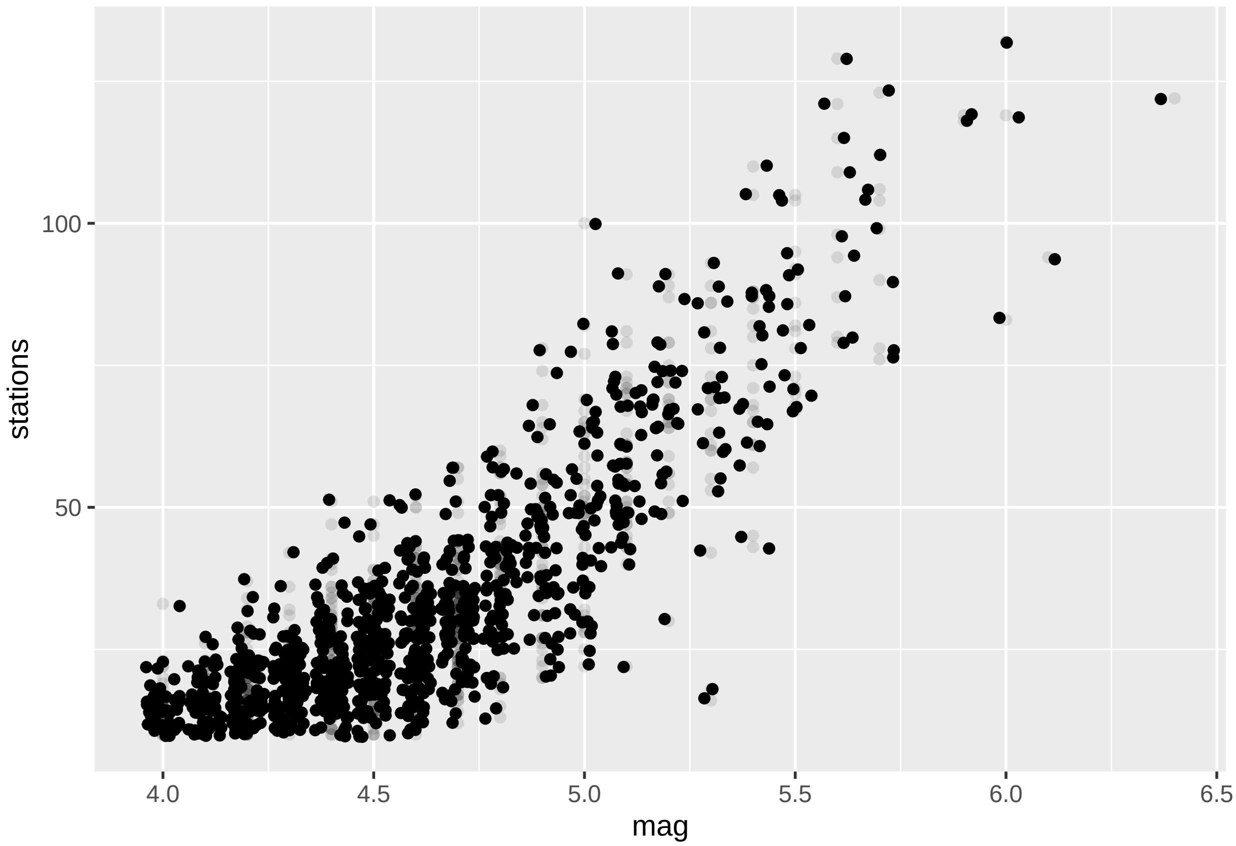
Exercise 1

```
library(MASS)
library("ggplot2")
library("tidyverse")
```

```
## -- Attaching packages --------------------------------- tidyverse 1.2.1 --
## v tibble  2.1.1       v purrr   0.3.2
## v tidyr   0.8.3       v dplyr   0.8.0.1
## v readr   1.3.1       v stringr 1.4.0
## v tibble  2.1.1       v forcats 0.4.0
## -- Conflicts ------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## x dplyr::select() masks MASS::select()
```

```
data(quakes)

ggplot(quakes, aes(mag, stations)) + geom_point(alpha = 0.1) + geom_jitter()
```



There is a strong positive linear relationship between the magnitude of an earthquake and the number o stations that detect the earthquake. In general, as the magnitude of the earthquake increases, the number of reporting stations increases as wel There is a strong positive linear relationship between the magnitude of an earthquake and the number o stations that detect the earthquake. In general, as the magnitude of the earthquake increases, the number of reporting stations increases as well.

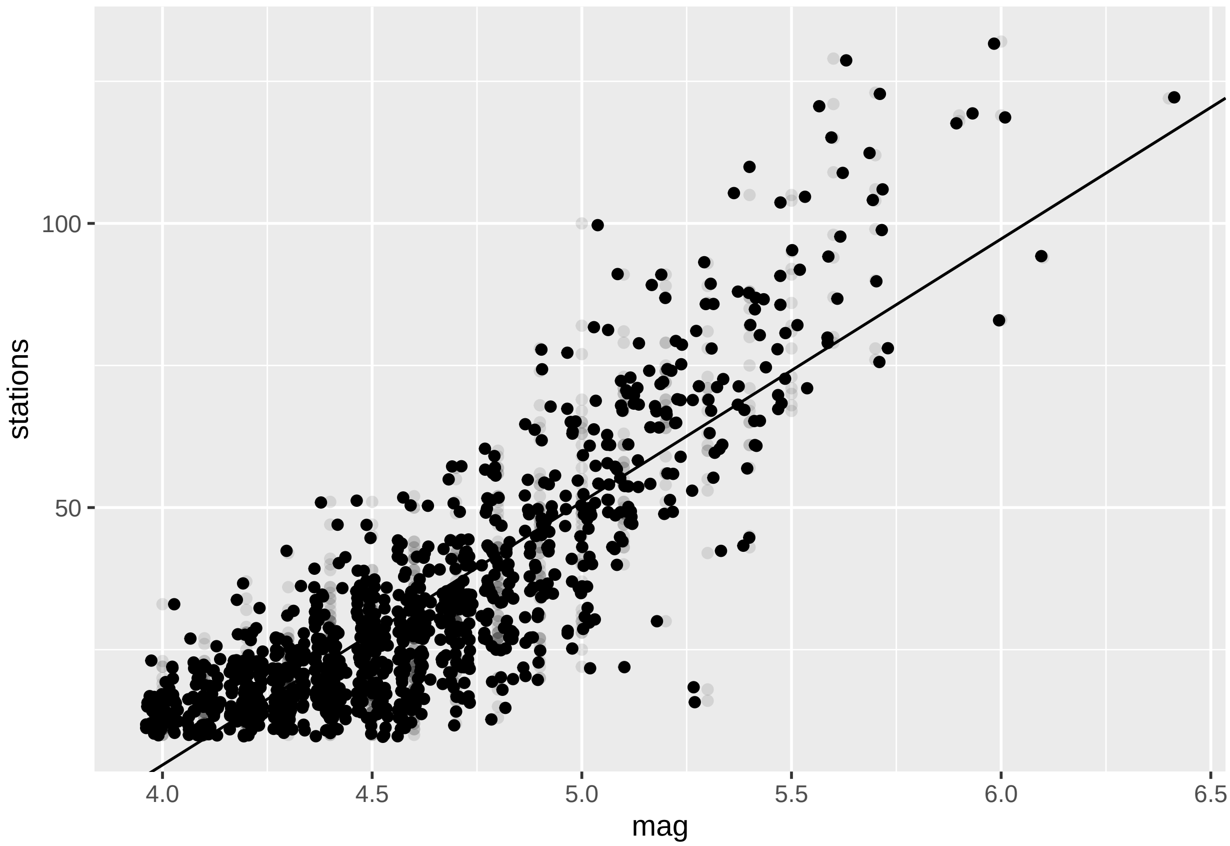
Exercise 2

```
mean(quakes$stations)
```

```
## [1] 33.418
```

The slope would be zero. It follows that the intercept would be 33.418, the mean of Y, since $\hat{B}_0 = \overline{y}$ - $\hat{B}_1$ $\overline{x}$ #### Exercise 3

```
m1 <- lm(stations ~ mag, data = quakes)
ggplot(quakes, aes(mag, stations)) + geom_point(alpha = 0.1) +  geom_abline(intercept = m1$coef[1], slop
```




```r
summary(m1)
```

```
##
## Call:
## lm(formula = stations ~ mag, data = quakes)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -48.871  -7.102  -0.474   6.783  50.244
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -180.4243     4.1899  -43.06   <2e-16 ***
## mag           46.2822     0.9034   51.23   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.5 on 998 degrees of freedom
## Multiple R-squared:  0.7245, Adjusted R-squared:  0.7242
## F-statistic:  2625 on 1 and 998 DF,  p-value: < 2.2e-16
```

The slope is 46.2822 and the intercept is -180.4243. The model predicts that as the magnitude of an earthquake increases by one unit on the Richter Magnitude scale, the number of stations that detect the earthquake will increase by approaximately 46.2822. The model also predicts that if the magnitude of an earthquake on the Richter Magnitude scale is 0, the number of stations that can detect it will be -180.4243. That is to say, no stations will detect an earthquake or event that registers as zero on the Richter scale.

Exercise 4

```
r <- cor(quakes$stations, quakes$mag)

y_diff <- quakes$stations - mean(quakes$stations)
y_diff_squared <- (y_diff)^2
std_dev_y <- sqrt(sum(y_diff_squared/999))
std_dev_y
```

```
## [1] 21.90039
```

```
x_diff <- quakes$mag - mean(quakes$mag)
x_diff_squared <- (x_diff)^2
std_dev_x <- sqrt(sum(x_diff_squared/999))
std_dev_x
```

```
## [1] 0.402773
```

```
slope <- r*(std_dev_y/std_dev_x)
slope
```

```
## [1] 46.28221
```

Using the equation

$$\hat{\beta}_1 = r * (s_y/s_x)$$

, I calculated a slope of 46.28221 between the magnitude of the earthquakes and the number of reporting stations. This corresponds with the slope computed by the fitted linear model.

Exercise 5

```
summary(m1)$coef[[2]]+ qt(1-(0.05/2), 998) * summary(m1)$coef[[4]]
```

```
## [1] 48.05498
```

```
summary(m1)$coef[[2]] - qt(1-(0.05/2), 998) * summary(m1)$coef[[4]]
```

```
## [1] 44.50944
```

```
confint(m1,'mag', level = 0.95)
```

```
##        2.5 %   97.5 %
## mag 44.50944 48.05498
```

We are 95% confident that the true slope between the magnitude of the earthquakes and the number of stations that detect them lies between 44.50944 and 48.05498.

Exercise 6

```
predict(m1, data.frame(mag = 7.0))
```

```
##        1
## 143.5511
```

I predict that around 144 (143.5511) stations would be able to detect an earthquake with a magnitude of 7.0.

Exercise 7

Question 1 - Data description Question 2 - Inference Question 3 - Inference Question 4 - Inference Question 5 - Inference Question 6 - Prediction

SIMULATION

Exercise 9

```
dat <- seq(from = 4.0, to = 6.0, by = 0.1)
sim_dat <- sample(dat, size = 1000, replace = TRUE)
sim_dat
```

```
##    [1] 5.6 4.6 4.1 4.0 4.3 5.3 4.0 6.0 4.8 4.3 4.3 5.2 5.9 5.4 6.0 4.4 4.5
##   [18] 4.0 5.2 4.5 5.3 5.0 4.7 5.4 4.3 4.7 4.2 5.0 4.1 4.8 5.1 5.7 5.2 5.0
##   [35] 5.1 4.5 4.6 4.3 5.1 5.0 5.3 5.9 4.1 6.0 5.0 4.1 5.1 4.0 4.6 5.8 4.0
##   [52] 5.4 5.9 5.2 5.2 4.4 4.0 4.6 4.9 4.6 4.5 5.3 4.8 4.5 4.0 4.5 5.2 5.0
##   [69] 5.4 4.4 4.1 5.5 5.2 4.7 5.5 5.0 4.6 5.5 4.9 4.5 4.9 4.6 4.2 4.5 5.0
##   [86] 4.9 5.8 5.5 5.4 5.6 5.7 5.6 5.0 5.2 5.3 5.7 5.6 5.8 4.7 5.4 5.8 4.2
##  [103] 5.3 5.1 4.2 4.4 4.9 5.0 5.0 5.4 5.1 4.1 4.4 5.4 5.2 4.8 5.3 4.3 4.7
##  [120] 4.8 5.9 5.1 5.0 4.7 5.0 5.5 5.2 4.4 4.8 4.0 5.2 4.1 4.3 4.1 5.4 4.0
##  [137] 4.4 5.8 4.1 4.4 4.5 4.4 5.4 5.9 5.1 4.9 4.3 5.8 4.5 4.3 4.8 5.1 5.3
##  [154] 4.0 5.9 5.8 4.8 5.9 6.0 4.0 4.8 4.3 5.1 5.9 4.2 5.1 4.2 4.0 4.8 5.7
##  [171] 5.6 4.3 5.3 4.2 4.0 4.2 5.7 5.2 5.7 5.6 4.8 5.8 5.0 5.7 5.3 5.5 4.2
##  [188] 4.4 5.1 5.9 4.4 5.8 4.9 6.0 5.9 5.8 5.3 5.5 5.2 4.9 4.2 6.0 4.5 4.5
##  [205] 5.9 5.1 5.3 5.3 6.0 4.8 5.4 4.4 5.2 5.3 5.1 4.4 4.9 4.1 4.0 5.9 5.8
##  [222] 4.6 5.5 5.2 4.4 5.0 5.9 5.8 4.0 4.6 5.1 6.0 4.7 4.3 5.6 4.3 4.2 5.3
##  [239] 5.9 4.6 5.1 4.2 4.2 4.3 4.9 4.7 5.0 4.1 4.0 5.5 4.8 4.9 5.9 4.4 5.0
##  [256] 5.3 4.5 5.3 5.2 4.1 4.1 4.8 5.0 5.7 5.9 5.2 5.3 5.7 5.9 5.2 4.2 4.4
##  [273] 4.2 4.0 4.9 4.7 5.9 4.6 5.9 4.9 5.0 4.0 5.6 5.3 4.4 5.2 5.3 5.0 5.6
##  [290] 5.9 5.7 5.7 4.2 4.2 5.9 5.4 4.5 4.8 5.2 5.1 4.4 5.1 5.2 4.9 5.6 5.9
##  [307] 5.3 4.1 5.8 4.4 4.2 5.8 4.2 4.2 4.3 4.1 5.9 4.9 4.9 4.1 5.4 4.7 4.2
##  [324] 4.4 4.0 5.9 5.7 4.3 5.4 5.7 4.7 4.0 6.0 5.7 4.0 5.5 5.6 5.5 4.3 4.5
##  [341] 4.9 5.2 4.4 5.7 5.2 5.5 5.1 4.9 4.4 5.6 5.7 4.8 5.4 4.1 4.4 4.8 5.9
##  [358] 4.5 4.7 4.4 4.5 4.9 5.1 5.7 4.0 5.6 4.5 5.7 5.3 4.1 5.5 5.5 4.1 5.4
##  [375] 5.9 4.8 4.2 4.1 4.8 5.8 5.4 4.8 5.3 4.4 5.4 4.5 4.1 5.3 4.9 4.6 5.6
##  [392] 4.3 5.3 5.6 4.5 5.9 4.9 4.3 4.6 5.8 5.1 5.4 5.5 4.9 4.5 5.7 5.7 5.0
##  [409] 5.7 5.7 4.9 4.2 4.8 5.0 5.4 5.0 5.8 5.4 4.0 4.5 4.8 5.6 4.0 4.5 4.9
##  [426] 5.7 5.3 6.0 6.0 4.8 5.1 4.4 5.4 4.5 5.9 5.9 6.0 4.0 4.8 4.1 4.2 4.3
##  [443] 5.9 6.0 5.8 4.2 4.6 5.4 5.1 5.2 5.1 5.6 4.8 4.2 5.7 5.6 4.3 6.0 5.3
##  [460] 5.1 4.5 5.2 5.4 5.2 5.8 5.6 5.5 4.6 5.0 6.0 4.2 4.0 4.0 5.5 5.9 4.9
##  [477] 4.4 5.4 4.1 4.7 5.3 4.4 4.1 5.5 5.4 4.2 4.5 5.2 4.9 4.1 4.9 5.5 4.9
##  [494] 4.2 5.0 5.8 5.8 4.3 5.5 5.7 4.2 5.8 6.0 4.9 5.8 4.2 4.1 5.7 4.0 4.9
##  [511] 4.4 5.9 5.0 5.0 5.4 5.1 6.0 5.3 5.0 4.9 4.1 4.5 5.0 5.4 4.8 5.1 5.0
##  [528] 4.9 5.4 5.7 5.8 4.9 4.8 5.9 4.3 5.1 4.3 4.2 5.4 4.6 4.4 4.0 4.8 4.1
##  [545] 5.3 5.0 4.1 4.6 4.9 5.9 5.8 5.3 4.9 5.0 5.3 4.3 5.9 4.3 4.7 4.5 4.1
##  [562] 4.9 5.8 4.5 5.8 5.4 6.0 4.2 5.7 5.5 5.7 4.2 5.6 4.4 5.2 5.0 6.0 5.6
##  [579] 5.7 4.7 6.0 4.5 5.7 4.5 4.7 5.2 5.0 5.4 5.1 4.8 6.0 5.7 5.0 4.9 4.6
##  [596] 5.3 4.3 5.0 5.0 5.5 5.3 4.0 4.2 5.3 4.4 5.4 5.5 4.2 5.4 4.6 4.5 6.0
##  [613] 4.6 4.0 4.5 5.3 5.6 5.2 5.1 5.9 5.2 5.5 4.0 4.8 4.6 4.6 5.0 5.8 4.9
##  [630] 4.5 5.9 4.9 6.0 5.8 4.4 4.2 5.7 5.0 4.6 6.0 5.9 4.8 5.8 4.1 4.5 5.1
##  [647] 4.8 4.8 5.7 5.3 5.7 4.5 4.1 5.2 4.5 5.9 4.2 4.4 4.7 4.7 5.9 5.4 4.8
##  [664] 4.8 5.6 4.1 5.3 6.0 4.4 5.9 5.1 4.0 5.2 5.1 4.9 5.7 5.5 5.8 5.4 5.3
##  [681] 5.1 5.7 4.4 4.7 6.0 5.0 5.5 4.9 5.2 5.2 5.3 4.6 4.2 5.5 5.5 5.8 4.6
##  [698] 5.1 4.8 5.7 4.7 4.2 5.4 4.6 4.3 5.0 5.5 5.6 4.5 4.6 5.0 5.0 6.0 5.0
##  [715] 5.0 5.4 5.2 4.9 4.3 5.7 4.0 4.8 4.5 4.6 5.6 5.4 6.0 4.1 5.3 4.3 5.1
##  [732] 4.5 5.0 5.3 4.8 4.2 5.5 4.3 4.0 5.0 4.1 5.6 5.5 4.6 5.8 5.4 5.7 5.2
##  [749] 4.2 5.2 5.6 5.8 5.3 4.7 4.5 5.6 4.3 4.2 4.6 5.9 4.8 4.8 5.3 5.4 5.5
##  [766] 5.9 4.5 5.7 5.3 4.4 4.0 5.8 4.2 5.2 4.9 5.1 5.3 4.5 5.7 5.0 5.9 4.9
```

```
## [783] 5.2 5.7 4.1 4.6 4.5 4.3 5.7 6.0 5.4 4.7 4.7 5.3 4.9 6.0 5.6 4.6 5.7
## [800] 5.4 4.1 5.7 5.6 4.0 5.2 4.6 5.6 5.1 4.0 5.6 4.2 4.4 4.0 5.6 5.6 5.2
## [817] 5.0 4.0 4.2 4.3 5.9 5.0 4.6 6.0 5.0 5.1 4.6 4.8 5.8 4.8 4.0 4.5 4.4
## [834] 4.6 4.4 4.1 4.7 4.2 4.5 4.0 4.2 5.7 4.4 5.3 5.2 5.5 4.0 4.7 5.7 5.7
## [851] 5.3 4.7 4.9 5.9 5.4 5.1 4.6 4.8 4.6 4.6 5.8 5.6 4.5 5.0 6.0 5.4 4.9
## [868] 4.4 5.5 4.6 4.1 6.0 4.4 4.3 4.5 4.4 5.4 5.5 5.3 4.4 4.2 5.6 4.2 5.4
## [885] 4.4 4.8 4.0 5.5 5.7 4.9 4.7 4.4 6.0 4.8 5.6 5.8 4.0 5.7 4.1 5.9 5.8
## [902] 5.1 5.4 4.1 4.5 4.9 5.6 5.5 5.5 4.0 4.3 5.2 5.0 4.2 5.7 4.9 4.0 5.7
## [919] 5.8 4.2 5.8 4.3 4.9 5.9 6.0 4.8 4.7 4.9 6.0 5.4 5.1 5.7 5.1 5.5 5.0
## [936] 5.7 4.7 5.8 5.8 4.5 5.2 4.5 5.2 4.8 4.7 5.7 4.3 5.0 4.2 5.7 4.4 5.8
## [953] 5.4 4.4 6.0 5.2 4.7 4.2 4.2 4.1 4.5 5.4 4.4 5.3 4.9 4.2 5.8 5.3 5.5
## [970] 5.6 5.1 4.4 5.8 4.3 5.5 4.1 5.3 4.1 4.1 5.4 4.7 5.5 5.2 5.7 5.2 5.2
## [987] 5.3 4.2 4.2 5.3 5.7 4.7 5.1 4.9 4.9 5.2 5.2 4.7 5.7 4.1
```

Exercise 10

```r
f_hat <- function(x) {-180.4243 + 46.2822*x}
y_hat <- f_hat(sim_dat)
y_hat
```

```
##    [1] 78.75602 32.47382  9.33272  4.70450 18.58916 64.87136  4.70450
##    [8] 97.26890 41.73026 18.58916 18.58916 60.24314 92.64068 69.49958
##   [15] 97.26890 23.21738 27.84560  4.70450 60.24314 27.84560 64.87136
##   [22] 50.98670 37.10204 69.49958 18.58916 37.10204 13.96094 50.98670
##   [29]  9.33272 41.73026 55.61492 83.38424 60.24314 50.98670 55.61492
##   [36] 27.84560 32.47382 18.58916 55.61492 50.98670 64.87136 92.64068
##   [43]  9.33272 97.26890 50.98670  9.33272 55.61492  4.70450 32.47382
##   [50] 88.01246  4.70450 69.49958 92.64068 60.24314 60.24314 23.21738
##   [57]  4.70450 32.47382 46.35848 32.47382 27.84560 64.87136 41.73026
##   [64] 27.84560  4.70450 27.84560 60.24314 50.98670 69.49958 23.21738
##   [71]  9.33272 74.12780 60.24314 37.10204 74.12780 50.98670 32.47382
##   [78] 74.12780 46.35848 27.84560 46.35848 32.47382 13.96094 27.84560
##   [85] 50.98670 46.35848 88.01246 74.12780 69.49958 78.75602 83.38424
##   [92] 78.75602 50.98670 60.24314 64.87136 83.38424 78.75602 88.01246
##   [99] 37.10204 69.49958 88.01246 13.96094 64.87136 55.61492 13.96094
##  [106] 23.21738 46.35848 50.98670 50.98670 69.49958 55.61492  9.33272
##  [113] 23.21738 69.49958 60.24314 41.73026 64.87136 18.58916 37.10204
##  [120] 41.73026 92.64068 55.61492 50.98670 37.10204 50.98670 74.12780
##  [127] 60.24314 23.21738 41.73026  4.70450 60.24314  9.33272 18.58916
##  [134]  9.33272 69.49958  4.70450 23.21738 88.01246  9.33272 23.21738
##  [141] 27.84560 23.21738 69.49958 92.64068 55.61492 46.35848 18.58916
##  [148] 88.01246 27.84560 18.58916 41.73026 55.61492 64.87136  4.70450
##  [155] 92.64068 88.01246 41.73026 92.64068 97.26890  4.70450 41.73026
##  [162] 18.58916 55.61492 92.64068 13.96094 55.61492 13.96094  4.70450
##  [169] 41.73026 83.38424 78.75602 18.58916 64.87136 13.96094  4.70450
##  [176] 13.96094 83.38424 60.24314 83.38424 78.75602 41.73026 88.01246
##  [183] 50.98670 83.38424 64.87136 74.12780 13.96094 23.21738 55.61492
##  [190] 92.64068 23.21738 88.01246 46.35848 97.26890 92.64068 88.01246
##  [197] 64.87136 74.12780 60.24314 46.35848 13.96094 97.26890 27.84560
##  [204] 27.84560 92.64068 55.61492 64.87136 64.87136 97.26890 41.73026
##  [211] 69.49958 23.21738 60.24314 64.87136 55.61492 23.21738 46.35848
##  [218]  9.33272  4.70450 92.64068 88.01246 32.47382 74.12780 60.24314
##  [225] 23.21738 50.98670 92.64068 88.01246  4.70450 32.47382 55.61492
##  [232] 97.26890 37.10204 18.58916 78.75602 18.58916 13.96094 64.87136
```

```
## [239] 92.64068 32.47382 55.61492 13.96094 13.96094 18.58916 46.35848
## [246] 37.10204 50.98670  9.33272  4.70450 74.12780 41.73026 46.35848
## [253] 92.64068 23.21738 50.98670 64.87136 27.84560 64.87136 60.24314
## [260]  9.33272  9.33272 41.73026 50.98670 83.38424 92.64068 60.24314
## [267] 64.87136 83.38424 92.64068 60.24314 13.96094 23.21738 13.96094
## [274]  4.70450 46.35848 37.10204 92.64068 32.47382 92.64068 46.35848
## [281] 50.98670  4.70450 78.75602 64.87136 23.21738 60.24314 64.87136
## [288] 50.98670 78.75602 92.64068 83.38424 83.38424 13.96094 13.96094
## [295] 92.64068 69.49958 27.84560 41.73026 60.24314 55.61492 23.21738
## [302] 55.61492 60.24314 46.35848 78.75602 92.64068 64.87136  9.33272
## [309] 88.01246 23.21738 13.96094 88.01246 13.96094 13.96094 18.58916
## [316]  9.33272 92.64068 46.35848 46.35848  9.33272 69.49958 37.10204
## [323] 13.96094 23.21738  4.70450 92.64068 83.38424 18.58916 69.49958
## [330] 83.38424 37.10204  4.70450 97.26890 83.38424  4.70450 74.12780
## [337] 78.75602 74.12780 18.58916 27.84560 46.35848 60.24314 23.21738
## [344] 83.38424 60.24314 74.12780 55.61492 46.35848 23.21738 78.75602
## [351] 83.38424 41.73026 69.49958  9.33272 23.21738 41.73026 92.64068
## [358] 27.84560 37.10204 23.21738 27.84560 46.35848 55.61492 83.38424
## [365]  4.70450 78.75602 27.84560 83.38424 64.87136  9.33272 74.12780
## [372] 74.12780  9.33272 69.49958 92.64068 41.73026 13.96094  9.33272
## [379] 41.73026 88.01246 69.49958 41.73026 64.87136 23.21738 69.49958
## [386] 27.84560  9.33272 64.87136 46.35848 32.47382 78.75602 18.58916
## [393] 64.87136 78.75602 27.84560 92.64068 46.35848 18.58916 32.47382
## [400] 88.01246 55.61492 69.49958 74.12780 46.35848 27.84560 83.38424
## [407] 83.38424 50.98670 83.38424 83.38424 46.35848 13.96094 41.73026
## [414] 50.98670 69.49958 50.98670 88.01246 69.49958  4.70450 27.84560
## [421] 41.73026 78.75602  4.70450 27.84560 46.35848 83.38424 64.87136
## [428] 97.26890 97.26890 41.73026 55.61492 23.21738 69.49958 27.84560
## [435] 92.64068 92.64068 97.26890  4.70450 41.73026  9.33272 13.96094
## [442] 18.58916 92.64068 97.26890 88.01246 13.96094 32.47382 69.49958
## [449] 55.61492 60.24314 55.61492 78.75602 41.73026 13.96094 83.38424
## [456] 78.75602 18.58916 97.26890 64.87136 55.61492 27.84560 60.24314
## [463] 69.49958 60.24314 88.01246 78.75602 74.12780 32.47382 50.98670
## [470] 97.26890 13.96094  4.70450  4.70450 74.12780 92.64068 46.35848
## [477] 23.21738 69.49958  9.33272 37.10204 64.87136 23.21738  9.33272
## [484] 74.12780 69.49958 13.96094 27.84560 60.24314 46.35848  9.33272
## [491] 46.35848 74.12780 46.35848 13.96094 50.98670 88.01246 88.01246
## [498] 18.58916 74.12780 83.38424 13.96094 88.01246 97.26890 46.35848
## [505] 88.01246 13.96094  9.33272 83.38424  4.70450 46.35848 23.21738
## [512] 92.64068 50.98670 50.98670 69.49958 55.61492 97.26890 64.87136
## [519] 50.98670 46.35848  9.33272 27.84560 50.98670 69.49958 41.73026
## [526] 55.61492 50.98670 46.35848 69.49958 83.38424 88.01246 46.35848
## [533] 41.73026 92.64068 18.58916 55.61492 18.58916 13.96094 69.49958
## [540] 32.47382 23.21738  4.70450 41.73026  9.33272 64.87136 50.98670
## [547]  9.33272 32.47382 46.35848 92.64068 88.01246 64.87136 46.35848
## [554] 50.98670 64.87136 18.58916 92.64068 18.58916 37.10204 27.84560
## [561]  9.33272 46.35848 88.01246 27.84560 88.01246 69.49958 97.26890
## [568] 13.96094 83.38424 74.12780 83.38424 13.96094 78.75602 23.21738
## [575] 60.24314 50.98670 97.26890 78.75602 83.38424 37.10204 97.26890
## [582] 27.84560 83.38424 27.84560 37.10204 60.24314 50.98670 69.49958
## [589] 55.61492 41.73026 97.26890 83.38424 50.98670 46.35848 32.47382
## [596] 64.87136 18.58916 50.98670 50.98670 74.12780 64.87136  4.70450
## [603] 13.96094 64.87136 23.21738 69.49958 74.12780 13.96094 69.49958
## [610] 32.47382 27.84560 97.26890 32.47382  4.70450 27.84560 64.87136
```

```
## [617] 78.75602 60.24314 55.61492 92.64068 60.24314 74.12780  4.70450
## [624] 41.73026 32.47382 32.47382 50.98670 88.01246 46.35848 27.84560
## [631] 92.64068 46.35848 97.26890 88.01246 23.21738 13.96094 83.38424
## [638] 50.98670 32.47382 97.26890 92.64068 41.73026 88.01246  9.33272
## [645] 27.84560 55.61492 41.73026 41.73026 83.38424 64.87136 83.38424
## [652] 27.84560  9.33272 60.24314 27.84560 92.64068 13.96094 23.21738
## [659] 37.10204 37.10204 92.64068 69.49958 41.73026 41.73026 78.75602
## [666]  9.33272 64.87136 97.26890 23.21738 92.64068 55.61492  4.70450
## [673] 60.24314 55.61492 46.35848 83.38424 74.12780 88.01246 69.49958
## [680] 64.87136 55.61492 83.38424 23.21738 37.10204 97.26890 50.98670
## [687] 74.12780 46.35848 60.24314 60.24314 64.87136 32.47382 13.96094
## [694] 74.12780 74.12780 88.01246 32.47382 55.61492 41.73026 83.38424
## [701] 37.10204 13.96094 69.49958 32.47382 18.58916 50.98670 74.12780
## [708] 78.75602 27.84560 32.47382 50.98670 50.98670 97.26890 50.98670
## [715] 50.98670 69.49958 60.24314 46.35848 18.58916 83.38424  4.70450
## [722] 41.73026 27.84560 32.47382 78.75602 69.49958 97.26890  9.33272
## [729] 64.87136 18.58916 55.61492 27.84560 50.98670 64.87136 41.73026
## [736] 13.96094 74.12780 18.58916  4.70450 50.98670  9.33272 78.75602
## [743] 74.12780 32.47382 88.01246 69.49958 83.38424 60.24314 13.96094
## [750] 60.24314 78.75602 88.01246 64.87136 37.10204 27.84560 78.75602
## [757] 18.58916 13.96094 32.47382 92.64068 41.73026 41.73026 64.87136
## [764] 69.49958 74.12780 92.64068 27.84560 83.38424 64.87136 23.21738
## [771]  4.70450 88.01246 13.96094 60.24314 46.35848 55.61492 64.87136
## [778] 27.84560 83.38424 50.98670 92.64068 46.35848 60.24314 83.38424
## [785]  9.33272 32.47382 27.84560 18.58916 83.38424 97.26890 69.49958
## [792] 37.10204 37.10204 64.87136 46.35848 97.26890 78.75602 32.47382
## [799] 83.38424 69.49958  9.33272 83.38424 78.75602  4.70450 60.24314
## [806] 32.47382 78.75602 55.61492  4.70450 78.75602 13.96094 23.21738
## [813]  4.70450 78.75602 78.75602 60.24314 50.98670  4.70450 13.96094
## [820] 18.58916 92.64068 50.98670 32.47382 97.26890 50.98670 55.61492
## [827] 32.47382 41.73026 88.01246 41.73026  4.70450 27.84560 23.21738
## [834] 32.47382 23.21738  9.33272 37.10204 13.96094 27.84560  4.70450
## [841] 13.96094 83.38424 23.21738 64.87136 60.24314 74.12780  4.70450
## [848] 37.10204 83.38424 83.38424 64.87136 37.10204 46.35848 92.64068
## [855] 69.49958 55.61492 32.47382 41.73026 32.47382 32.47382 88.01246
## [862] 78.75602 27.84560 50.98670 97.26890 69.49958 46.35848 23.21738
## [869] 74.12780 32.47382  9.33272 97.26890 23.21738 18.58916 27.84560
## [876] 23.21738 69.49958 74.12780 64.87136 23.21738 13.96094 78.75602
## [883] 13.96094 69.49958 23.21738 41.73026  4.70450 74.12780 83.38424
## [890] 46.35848 37.10204 23.21738 97.26890 41.73026 78.75602 88.01246
## [897]  4.70450 83.38424  9.33272 92.64068 88.01246 55.61492 69.49958
## [904]  9.33272 27.84560 46.35848 78.75602 74.12780 74.12780  4.70450
## [911] 18.58916 60.24314 50.98670 13.96094 83.38424 46.35848  4.70450
## [918] 83.38424 88.01246 13.96094 88.01246 18.58916 46.35848 92.64068
## [925] 97.26890 41.73026 37.10204 46.35848 97.26890 69.49958 55.61492
## [932] 83.38424 55.61492 74.12780 50.98670 83.38424 37.10204 88.01246
## [939] 88.01246 27.84560 60.24314 27.84560 60.24314 41.73026 37.10204
## [946] 83.38424 18.58916 50.98670 13.96094 83.38424 23.21738 88.01246
## [953] 69.49958 23.21738 97.26890 60.24314 37.10204 13.96094 13.96094
## [960]  9.33272 27.84560 69.49958 23.21738 64.87136 46.35848 13.96094
## [967] 88.01246 64.87136 74.12780 78.75602 55.61492 23.21738 88.01246
## [974] 18.58916 74.12780  9.33272 64.87136  9.33272  9.33272 69.49958
## [981] 37.10204 74.12780 60.24314 83.38424 60.24314 60.24314 64.87136
## [988] 13.96094 13.96094 64.87136 83.38424 37.10204 55.61492 46.35848
```

```
## [995] 46.35848 60.24314 60.24314 37.10204 83.38424  9.33272
```

Exercise 11

```r
res <- m1$res
y_pop <- res + y_hat
y_pop
```

```
##           1           2           3           4           5           6 
##  78.0257349  33.5128614 -17.1668915  14.3717625  24.8846436  72.1668436 
##           7           8           9          10          11          12 
##   5.9742149  89.0514993  39.6281960  18.9999803   8.3717593  43.7692971 
##          13          14          15          16          17          18 
##  88.4232793  56.2821793  89.3717410  15.6282003  13.5766620  -2.1411218 
##          19          20          21          22          23          24 
##  50.0257393  22.6281993  54.0257382  49.0257414  31.8846393  54.3975160 
##          25          26          27          28          29          30 
##   6.0895485  47.3975236  -0.5129029  69.7435306   6.4870982  35.5128593 
##          31          32          33          34          35          36 
##  44.1410771  76.2821760  60.5128549  56.2821836  37.7692982  22.2564203 
##          37          38          39          40          41          42 
##  21.6281982   3.1153171  63.2821825  38.7692993  61.7692960  92.1668371 
##          43          44          45          46          47          48 
##   9.1153193 104.6797203  45.5128571  -3.0257861  51.7692982  -0.5129007 
##          49          50          51          52          53          54 
##  34.8846403 105.3975117   3.7435414  79.7950636  86.1668371  54.6539603 
##          55          56          57          58          59          60 
##  62.2821814  30.9999793  -6.1411218  38.7693036  46.1410793  25.8846403 
##          61          62          63          64          65          66 
##  25.7435360  67.5386225  51.7435328  27.3717571  -1.6540061  15.7435360 
##          67          68          69          70          71          72 
##  63.9104025  44.9999728  68.6539582  40.0895474  15.6282036  76.2821782 
##          73          74          75          76          77          78 
##  55.6539603  51.8588706  85.9103993  47.3975203  34.1410825  69.2821782 
##          79          80          81          82          83          84 
##  47.3975214  22.9742096  51.1153106  31.6281982  19.4870971  24.2564203 
##          85          86          87          88          89          90 
##  57.2821836  42.7693003  82.9103960  72.2821782  71.5386214 102.1668403 
##          91          92          93          94          95          96 
##  88.7692917  63.6539560  54.7435306  60.2821814  71.9104014  96.6797236 
##          97          98          99         100         101         102 
##  71.9103982  81.7692906  26.4870917  65.3975160  90.6797225   6.4870971 
##         103         104         105         106         107         108 
##  57.7692960  40.5128560  10.4870971  24.2564214  48.1410793  40.5128571 
##         109         110         111         112         113         114 
##  43.6024253  53.5128528  55.7692982   9.3717614  34.5128636  66.7692949 
##         115         116         117         118         119         120 
##  58.0257393  52.7693014  66.9999696  -4.5129040  47.3717549  40.7693014 
##         121         122         123         124         125         126 
##  78.9103949  48.0257403  40.8846360  32.2564182  45.7692993  61.5128517 
##         127         128         129         130         131         132 
##  67.2821814  41.2306528  58.9999749   5.1153203  54.3975182  19.3717614 
##         133         134         135         136         137         138 
##  11.7435382   6.8588771  64.9104003 -20.3975640  35.6024317  69.5386171
```

```
##        139        140        141        142        143        144 
## -6.0257861 26.2564214 38.3717571 32.5128636 64.5128528 103.4232793 
##        145        146        147        148        149        150 
## 64.6539614 42.3975214 17.3717593 83.6539539 25.9742096 32.8846436 
##        151        152        153        154        155        156 
## 64.3459853 61.8330977 70.2821803 10.7435414 71.5386160 89.9103960 
##        157        158        159        160        161        162 
## 41.7693014 93.0515003 81.9103939  2.2306571 51.3975225 10.8588749 
##        163        164        165        166        167        168 
## 53.1410771 92.1668371  7.2306549 41.6281928 33.2048863 12.8331096 
##        169        170        171        172        173        174 
## 45.6281960 82.5386182 84.1668403 27.1153171 66.5386225 18.6282025 
##        175        176        177        178        179        180 
##  5.3717625 20.5766653 82.3975128 54.3975182 94.0515025 73.2821771 
##        181        182        183        184        185        186 
## 60.8846382 94.4232803 44.7692993 84.4232814 69.5386225 89.7692939 
##        187        188        189        190        191        192 
##  6.3717603 30.8588739 47.1410771 89.1668371 29.3459896 85.9103960 
##        193        194        195        196        197        198 
## 25.8846371 108.9361625 101.1668371 97.0515014 59.3975171 60.2821782 
##        199        200        201        202        203        204 
## 72.6539603 60.1153106 18.3717603 104.5643836 25.7435360 35.9999782 
##        205        206        207        208        209        210 
## 83.7950582 50.0257403 70.6281906 51.0257382 92.1668360 38.7693014 
##        211        212        213        214        215        216 
## 74.3975160 17.6282003 69.9104025 80.3717485 59.0257403 25.6282003 
##        217        218        219        220        221        222 
## 49.3975214 17.8588771 10.4870993 92.6797214 80.9103960 38.9999771 
##        223        224        225        226        227        228 
## 75.7692939 68.2821814 21.3717582 56.6281939 96.6797214 82.1668382 
##        229        230        231        232        233        234 
##  8.7177728 24.4870928 52.5128560 102.6797203 29.8846393  3.2306539 
##        235        236        237        238        239        240 
## 65.9103982 25.8846436 10.7435393 76.8846328 82.5386160 31.7435349 
##        241        242        243        244        245        246 
## 60.7692982 20.6282025 -34.9104504 28.8588749 47.3717528 32.6281971 
##        247        248        249        250        251        252 
## 58.3975203 -9.7693440  0.8331096 72.1668414 30.6281960 44.5128582 
##        253        254        255        256        257        258 
## 84.0257317 14.8588739 46.7692993 72.9104014 29.3717571 62.7692960 
##        259        260        261        262        263        264 
## 69.2821814  5.9742139 25.7177717 36.6281960 37.7692993 87.1668393 
##        265        266        267        268        269        270 
## 94.6797214 50.8846339 48.3975171 74.1668393 97.1668371 55.3975182 
##        271        272        273        274        275        276 
##  5.7435393 11.8588739 16.6282025  1.7435414 66.9742053 29.6281971 
##        277        278        279        280        281        282 
## 100.6539528 29.8846403 83.7950582 42.1153106 49.7692993  2.7435414 
##        283        284        285        286        287        288 
## 63.2821771 70.1668436 20.9999793 52.1410760 66.9104014 49.8846360 
##        289        290        291        292        293        294 
## 76.9103982 91.6539528 81.3975128 59.6539549 12.4870971 16.4870971 
##        295        296        297        298        299        300 
## 98.6539528 65.5128528 29.0895463 53.0257436 71.5386236 46.1410771 
```

```
##         301         302         303         304         305         306 
##  21.9999793  53.8846349  53.1410760  28.8846371  81.1668403  84.5386160 
##         307         308         309         310         311         312 
##  74.5386225 -17.0257861  91.5386171   7.4870949  35.6024339  89.3975117 
##         313         314         315         316         317         318 
##  26.4613285   7.1153182  26.2306539   0.1153193  96.3079425  34.4870896 
##         319         320         321         322         323         324 
##  43.5128582 -11.0257861  55.7692949  45.8588706  13.7435393  13.9999793 
##         325         326         327         328         329         330 
##  13.3459939  80.4232793  79.4232814   6.1153171  52.0257371  73.5128496 
##         331         332         333         334         335         336 
##  14.2306496 -12.1411218  85.0514993  60.3975128   7.0895517  60.2821782 
##         337         338         339         340         341         342 
##  76.9103982  72.6281885   6.7435382   7.7435360  46.3975214  31.5128549 
##         343         344         345         346         347         348 
##   8.3717582  76.7950603  55.6539603  66.5386203  34.1410771  30.5128582 
##         349         350         351         352         353         354 
##   1.2306528  56.2821771  65.9103971  22.3717539  77.1668425   3.2048874 
##         355         356         357         358         359         360 
##   7.1153160  39.7435328  86.0257317  35.7177674  25.6281971   5.1153160 
##         361         362         363         364         365         366 
##  27.8846414  63.6539636  53.1153085  83.7950603  -2.5129007  68.2821771 
##         367         368         369         370         371         372 
##  20.2306517  69.5386182  71.9104014  19.9999825  67.5128517 109.6281885 
##         373         374         375         376         377         378 
##  14.7177717  67.8846317  90.4232793  81.3459853   4.7435393  10.7177717 
##         379         380         381         382         383         384 
##  40.2564171 108.8846274  81.8846317  44.5128593  38.8846328  21.2306528 
##         385         386         387         388         389         390 
##  62.8846317  35.2306517   9.3717614  76.0257382  55.6539636  22.1153139 
##         391         392         393         394         395         396 
##  81.1668403   3.8588749  62.6539593  78.7950614  26.4870939  99.6797214 
##         397         398         399         400         401         402 
##  70.4870896  -2.3975672  61.0895453 116.1410696  42.5128560  73.0257371 
##         403         404         405         406         407         408 
##  72.5386203  41.8588685  34.6281993  80.7950603  87.7950603  41.8846360 
##         409         410         411         412         413         414 
##  86.1668393  90.4232814  53.5128582  29.2306549  36.9999749  47.3975203 
##         415         416         417         418         419         420 
##  61.2821793  46.3717517  81.7950593  86.2821793   9.1153203   5.1153149 
##         421         422         423         424         425         426 
##  50.1410803  79.9103982  17.3717625  30.2306517  53.2564160  78.9103971 
##         427         428         429         430         431         432 
##  71.7692960  76.7950571  89.0514993  50.7693014  56.2821825   8.1153160 
##         433         434         435         436         437         438 
##  81.7950636  19.1153149  88.4232793  88.0515003  95.0514993 -13.1411218 
##         439         440         441         442         443         444 
##  42.8846382   2.2306560  13.3717603  10.4870960  83.9103949  78.7950571 
##         445         446         447         448         449         450 
## 100.3975117  -4.1411240  34.8846403 104.8846317  85.4870874  57.3975182 
##         451         452         453         454         455         456 
##  48.0257403  81.1668403  27.6281960  13.3717603  84.7950603  66.2821771 
##         457         458         459         460         461         462 
##  13.7435382  84.4232782  82.3717485  62.1410771  15.6281993 109.2564128 
```

```
##         463         464         465         466         467         468 
##  81.2564106  52.0257393  83.3975117  72.5386193  71.9103993  15.7435349 
##         469         470         471         472         473         474 
##  49.7692993  99.1668360   7.8588760  10.4870993  -1.8846797  63.1410728 
##         475         476         477         478         479         480 
##  83.7950582  40.8846371  18.7177685  60.0257371  -5.5129018  26.8846393 
##         481         482         483         484         485         486 
##  55.0257382  23.8846425  17.6282036  66.7692939  73.1668425  32.7177706 
##         487         488         489         490         491         492 
##  45.6281993  61.9104025  42.9999739  -3.1411229  35.5128582  62.3975149 
##         493         494         495         496         497         498 
##  51.3975214  22.9999814  56.6539625  84.8846274  83.7950593  -3.1411251 
##         499         500         501         502         503         504 
##  83.1668414  77.6539549  13.6024339  74.5386171 103.4232782  47.1410793 
##         505         506         507         508         509         510 
##  83.5386171  18.9999814   0.2306560 111.1668393  27.8588782  47.9999739 
##         511         512         513         514         515         516 
##  20.1153160 110.5128474  43.8846360  50.5128571  74.1668425  46.5128560 
##         517         518         519         520         521         522 
##  85.7950571  71.2821803  47.7692993  53.2564160  15.7435403  40.5128625 
##         523         524         525         526         527         528 
##  43.8846360  57.6539582  48.6024274  44.7692982  62.5128571  60.7435317 
##         529         530         531         532         533         534 
##  74.2821793  81.2821760  86.8846274  52.8846371  53.0257436  84.1668371 
##         535         536         537         538         539         540 
##  10.8588749  54.5128560  14.4870960  19.9999814  86.9999685  33.9999771 
##         541         542         543         544         545         546 
##  35.0895474   5.7435414  35.8846382   2.4870982  43.1410749  36.5128571 
##         547         548         549         550         551         552 
##  19.8331085  33.9999771  25.3717528  91.0515003  93.4232803  61.1410749 
##         553         554         555         556         557         558 
##  32.2564160  49.8846360  50.1410749  21.6282014  94.6797214  43.9484431 
##         559         560         561         562         563         564 
##  32.6281971  24.9999782  11.9742139  26.2564160  66.6539539  40.9742096 
##         565         566         567         568         569         570 
##  95.0515014  78.5386214  87.4232782  20.7177706  78.5386182  74.3717463 
##         571         572         573         574         575         576 
## 110.1410706   7.2306549  79.9103982   0.2306528  63.0257393  41.1410782 
##         577         578         579         580         581         582 
## 106.7950571  74.9103982  97.1410706  35.4870917  97.5386149  27.2564203 
##         583         584         585         586         587         588 
##  82.7692917   9.7435360  24.6281971  69.5128549  41.5128571  71.2821793 
##         589         590         591         592         593         594 
##  50.1410771  35.1153117  98.6797203  79.1668393  43.8846360  51.6281949 
##         595         596         597         598         599         600 
##  38.1410825  65.3975171   7.2306539  63.2821836  45.8846360  69.0257360 
##         601         602         603         604         605         606 
##  93.6281906  -9.7693429  15.4870971  41.5128539  38.8331053  58.3975160 
##         607         608         609         610         611         612 
##  68.9103993   5.6024339  70.7692949  26.9999771  22.6281993  90.5386149 
##         613         614         615         616         617         618 
##  23.3717560   8.7435414  26.2306517  79.1410749  79.3975139  53.6281917 
##         619         620         621         622         623         624 
##  65.0257403  99.4232793  56.1410760  61.0257360  25.8331096  57.1153117 
```

```
##         625         626         627         628         629         630 
##  24.1153139  27.6281982  54.8846360  97.0515014  40.7435317  43.1153149 
##         631         632         633         634         635         636 
##  96.6797214  60.6281949  94.0514993  86.1668382  22.6282003  64.2048863 
##         637         638         639         640         641         642 
##  94.6797236  35.9999728  46.5128614  95.7950571  78.1668371  41.8846382 
##         643         644         645         646         647         648 
##  69.0257328  14.2306560  31.3717571  63.8846349  53.2564171  35.5128593 
##         649         650         651         652         653         654 
##  91.6281863  63.9104014 101.8846285  29.1153149  24.5766663  64.0257393 
##         655         656         657         658         659         660 
##  19.7435360  87.1668371  37.3459917  12.7435371  38.2564182  46.1410814 
##         661         662         663         664         665         666 
##  93.9103949  71.1410739  72.6024274  31.7435328  85.7950614  18.0895506 
##         667         668         669         670         671         672 
##  59.1410749 104.0514993  21.4870949 100.4232793  58.2821825   3.3459939 
##         673         674         675         676         677         678 
##  51.7692971  59.7692982  -0.5129104  81.6539549  82.6539571  83.2821749 
##         679         680         681         682         683         684 
##  49.3975160  68.5128539  50.7435296  87.1668393  23.7435371  35.5128603 
##         685         686         687         688         689         690 
##  91.0514993  64.0257414  78.7950625  47.3975214  73.2564128  60.9104025 
##         691         692         693         694         695         696 
##  66.2821803  39.2306506   9.9999814  82.2821782  79.9103993  80.7950593 
##         697         698         699         700         701         702 
##  46.4870928  61.9104036  24.9999749  83.3975128  44.1410814  44.7177706 
##         703         704         705         706         707         708 
##  77.2564106  14.6281982  14.9999803  42.1410782  52.6539571  94.1410717 
##         709         710         711         712         713         714 
##  40.6281993  29.8846403  50.8846360  81.2306463  96.9103939  17.3717517 
##         715         716         717         718         719         720 
##  31.5128571  64.2821793  67.5128549  49.3975214  38.4870960  85.0515025 
##         721         722         723         724         725         726 
##   0.6024360  54.0257436  24.4870939  31.4870928  66.9103982  69.2821793 
##         727         728         729         730         731         732 
## 106.5643836  11.4870982  50.7692960  15.3717593  61.5128560  22.7435360 
##         733         734         735         736         737         738 
##  60.2821836  64.9104014  31.8846382  20.8588760  63.2821782  24.2564225 
##         739         740         741         742         743         744 
## -14.7693429  57.3975203   6.8588771  83.5128506  63.6539571  31.3717560 
##         745         746         747         748         749         750 
## 102.0257328  58.2564106  80.1668393  45.7692971   4.1153182  67.5386236 
##         751         752         753         754         755         756 
##  84.7950614  92.1410696  91.2306431  32.7435339  33.9999782  69.5386193 
##         757         758         759         760         761         762 
##  24.0895485  35.0895496  23.8588717 103.3079425  40.1410803  31.8846382 
##         763         764         765         766         767         768 
##  65.5386225  83.8846317  80.2564096  93.7950582  34.5128625  86.1668393 
##         769         770         771         772         773         774 
##  51.7692960  28.5128636  10.0895517  99.3079436  16.3717603  62.6539603 
##         775         776         777         778         779         780 
##  63.6539636  56.2821825  58.6539593  19.2564203  81.4232814  58.2821836 
##         781         782         783         784         785         786 
##  89.7950582  41.2564160  57.2564128  86.7950603   7.7177717  29.3717560 
```

```
##         787         788         789         790         791         792 
##  31.9742096  30.6024328  84.5386182 123.2821728  76.1668425  17.7435339 
##         793         794         795         796         797         798 
##  43.7693025  76.1668436  50.1410793  88.4232782  88.5386193  41.6281982 
##         799         800         801         802         803         804 
##  83.7950603  86.9104003  -5.6540072  92.5386182  63.9103982  11.4870993 
##         805         806         807         808         809         810 
##  67.3975182  45.1410825 110.3975139  45.5128560   4.1153203  78.2821771 
##         811         812         813         814         815         816 
##  24.4870971  11.9742106  -4.1411218  71.1668403  98.2821771  69.5386236 
##         817         818         819         820         821         822 
##  62.1410782  -7.0257851  11.4870971  20.2564225  96.2821739  53.8846360 
##         823         824         825         826         827         828 
##  46.1410825  98.6797203  44.7692993  62.9104036   5.7435349  43.5128593 
##         829         830         831         832         833         834 
##  98.7950593  30.2564171  18.7435414  33.5128625  27.2564214  39.7693036 
##         835         836         837         838         839         840 
##  28.8846425  16.9999825  28.9999760  16.2306549  16.2306517  -5.2822272 
##         841         842         843         844         845         846 
##  10.2306549  99.1668393   8.2306528  60.6281906  65.2821814  80.7950625 
##         847         848         849         850         851         852 
##  22.9742149  36.2564182  86.3975128 108.7692917  71.5386225  40.9999760 
##         853         854         855         856         857         858 
##  45.1153106  98.9103949  74.6539582  61.9104036  44.3717560  47.3975225 
##         859         860         861         862         863         864 
##  30.8846403  30.8846403  97.3079436  92.6539560  45.5128625  50.0257414 
##         865         866         867         868         869         870 
##  91.5386149  58.7692949  47.3975214  38.2564214  68.7435253  67.2048820 
##         871         872         873         874         875         876 
##  15.3717614 101.4232782  12.8588739  17.3717593  38.1410836  24.6282003 
##         877         878         879         880         881         882 
##  78.5386214  76.2821782  54.1410749  27.9999793  17.9999814  76.7950614 
##         883         884         885         886         887         888 
##  -6.0257872  70.5386214  26.9742106  43.8846382   7.2306571  45.1410728 
##         889         890         891         892         893         894 
##  84.3975128  47.8588685  23.3717549  30.2564214  93.1410674  48.1410803 
##         895         896         897         898         899         900 
##  85.4232825  86.7950593  32.3459939 100.5386182  18.9742139 120.9361636 
##         901         902         903         904         905         906 
##  87.1668382  44.6281928  70.1410739  12.4870982  29.8846414  69.3975214 
##         907         908         909         910         911         912 
##  98.7950614  43.8846306  76.5386203  -7.9104483  36.1153171  41.7692971 
##         913         914         915         916         917         918 
##  63.7692993  22.7435393 101.0257339  30.7435317  18.7435414  76.2821760 
##         919         920         921         922         923         924 
##  97.3079436  50.2048863 104.1410696  49.6024328  54.7693003  94.3079425 
##         925         926         927         928         929         930 
## 108.6539517  49.8846382  12.7435339  37.8588685 100.1668360  56.7692949 
##         931         932         933         934         935         936 
##  55.7692982  92.4232814  73.1410771  69.6539571  93.2306463 123.8846285 
##         937         938         939         940         941         942 
##  52.3975236  99.7692906  86.0515014  47.7435360  56.6539603  13.1153149 
##         943         944         945         946         947         948 
##  57.7692971  60.2306485  22.3717549  72.9103971  26.6282014  43.8588674 
```

```
##         949         950         951         952         953         954
##  -7.1411240  81.2821760  27.3717582  91.8846274  75.6539582  34.8846425
##         955         956         957         958         959         960
## 115.5643836  73.6539603  26.9999760  17.6024339  17.1153182  27.2306560
##         961         962         963         964         965         966
##  21.9999782  69.7692949  24.6282003  78.5386225  37.8588685  30.6282025
##         967         968         969         970         971         972
##  86.2821749  70.9104014  57.3975149  54.2564085  46.3975193  36.9742106
##         973         974         975         976         977         978
##  73.9103960   7.2306539  54.7692939  27.3717614  88.6539593   3.1153193
##         979         980         981         982         983         984
##   1.7435403  43.1410739  53.1153128  58.0257360  62.8846339  86.7950603
##         985         986         987         988         989         990
##  49.3975182  62.2821814  55.6281906  -0.7693451  21.2564236  60.7692960
##         991         992         993         994         995         996
##  82.7950603  45.5128603  74.2564139  58.6539636  42.3975214  59.0257393
##         997         998         999        1000
##  58.1410760  43.2564182  69.5386182  31.0637820
```

```r
res_squared = (m1$res)^2
sum(res_squared)
```

```
## [1] 131999.6
```

```r
sum(res_squared)/998
```

```
## [1] 132.2641
```

$$\hat{\sigma}^2 = RSS/n-2$$

=

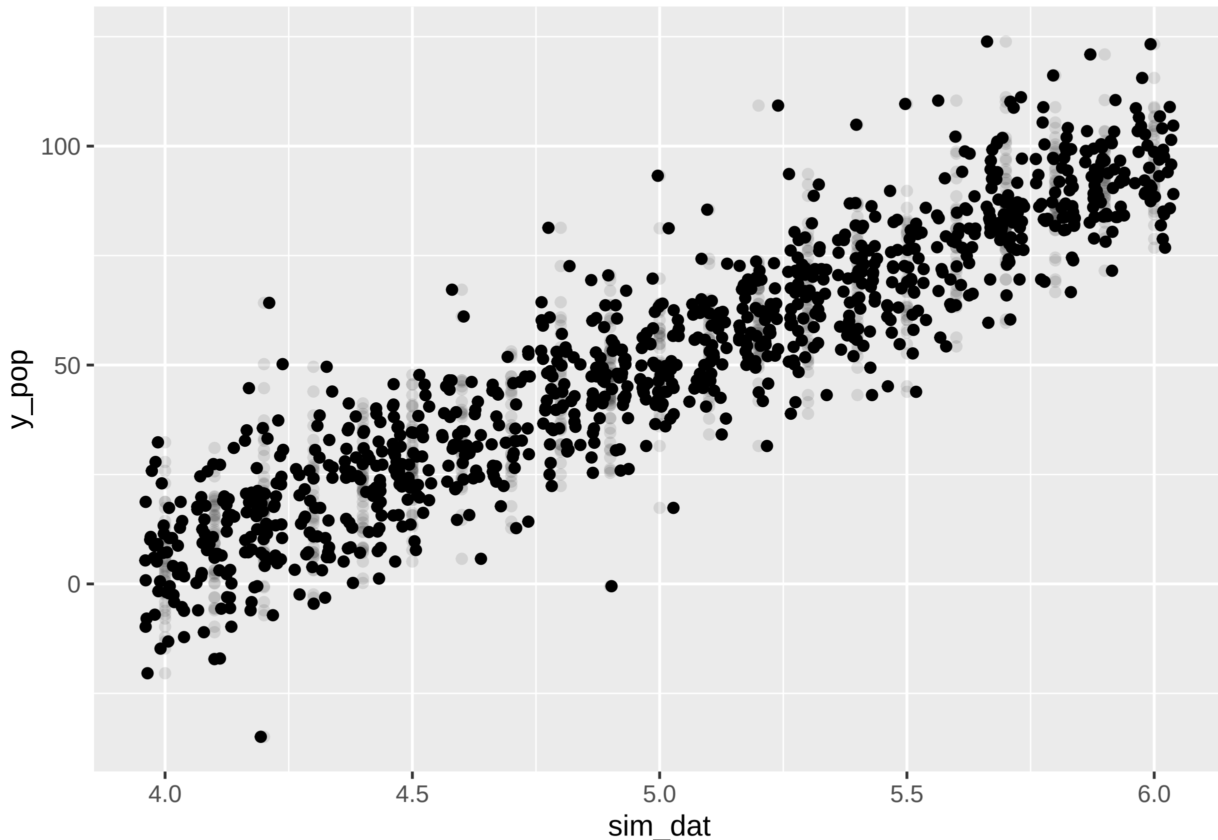
$$\hat{\sigma}^2 = 131999.6/998$$

= =

$$\hat{\sigma}^2 = 132.2641$$

Exercise 12

```r
ggplot(quakes, aes(sim_dat, y_pop)) + geom_point(alpha = 0.1) + geom_jitter()
```




```r
m2 = lm(y_pop ~ sim_dat, data = quakes)
summary(m2)
```

```
##
## Call:
## lm(formula = y_pop ~ sim_dat, data = quakes)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -49.032  -7.101  -0.388   6.754  50.084
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -179.4188     3.0548  -58.73   <2e-16 ***
## sim_dat       46.0809     0.6071   75.90   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.5 on 998 degrees of freedom
## Multiple R-squared:  0.8523, Adjusted R-squared:  0.8522
## F-statistic:  5761 on 1 and 998 DF,  p-value: < 2.2e-16
```

As with the original data, there is a strong, positive, and linear relationship between the magnitude of an earthquake and number of stations detecting it in the simulated data. However, the intercept is higher, and the slope is a little bit lower, suggesting that for every unit increase in the magnitude of an earthquake on the Richter scale, the number of stations that detect it increases by 45.6. To increase my model, I might find an intercept that lies around the mean of -180.4243 and -177.0066 and a slope that lies betwen mean of 45.6 (the slope of the simulated data) and the original slope of 46.2822. #### PROBLEM SET 2: #### Exercise 1

1)

$$H_0$$

: While holding all other predictors constant, the amount of spending on TV advertising has no effect on sales.

The model suggests that the relationship between the amount of spending on TV advertising and sales is statistically significant, allowing us to reject the null hypthesis ($p < 0.0001$). Spending an additional $1000 on TV advertising leads to an increase in sales by approzimately 46 units.

2)

$$H_0$$

: While holding all other predictors constant, the amount of spending on radio advertising has no effect on sales.

The model also suggests that relationship between the amount of spending on radio advertising and sales is statistically significant, allowing us to reject the null hypothesis ($p < 0.0001$). Spending an additional $1000 on radio advertising leads to an increases in sales by approximately 189 units.

3)

$$H_0$$

: While holding all other predictors constant, the amount of spending on newspaper advertising has no effect on sales.

While the model reports that spending an additional $1000 on newspaper advertising may lead to a decrease in sales by one unit, this relationship is not statistically significant ($p = 0.8599$).

When there is no spending on either TV, radio, or newspaper advertising the model makes the statistically significant prediction that the mean value of sales would be 2.939 units ($p < 0.0001$).

Exercise 4

a) Despite the fact that the true relationship is linear, I would expect the training RSS of the cubic regression to be lower because its greater flexibility gives it the capacity to fit greater variance in the training data.

b) In this case, the greater flexibility cubic regression model will overfit the training data whereas the greater bias of the linear regression model is mediated by the true linear relationship. Consequently I would expect the test RSS of the linear regression to be lower.

c) As in a), I would expect the training RSS of the cubic regression to be lower because its greater flexibility allows it to encompass the greater variance in training data regardless of the true relationship between X and Y.

d) There is not enough information to tell because we do not know the linearity of the true relationship between X and Y. If the true relationship is closer to linear, the linear regression model would be a better fit with a lower test RSS. However, if the relationship is extremely non-linear then it is possible that the greater flexibility of the cubic regression model wll be a better fit and yield a lower RSS. #### Exercise 5

1. If we are given the form of the ith fitted value as:

$$\hat{y} = x_i\hat{B}$$

where:

$$\hat{B} = (\sum_{i=1}^{n} x_i y_i)/(\sum_{i'=1}^{n} x_{i'}^2)$$

2. Then, after substitution:

$$\hat{y} = (x_i \sum_{i=1}^{n} x_{i'} y_{i'})/(\sum_{i"=1}^{n} x_{i"}^2)$$

3. We were also given that:

$$\hat{y} = \sum_{i'=1}^{n} a_{i'} y_{i'}$$

,

So:

$$\hat{y} = \sum_{i'=1}^{n} a_{i'} y_{i'} = (x_i \sum_{i=1}^{n} x_{i'} y_{i'})/(\sum_{i"=1}^{n} x_{i"}^2)$$

=

$$a_{i'} y_{i'} = x_i x_{i'} y_{i'} / \sum_{i"=1}^{n} x_{i"}^2$$

=

$$a_i = x_i x_{i'} / \sum_{i"=1}^{n} x_{i"}^2$$

ADDITIONAL EXERCISES

Exercise 1

1) Variance:

a) We know that k-nearest regression is defined as:

$$[\hat{f}(x) = \frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} y_i]$$

b) To find the variance:

$$Var(\hat{f}(x)) = Var\frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} y_i$$

=

$$Var(\hat{f}(x)) = \frac{1}{k^2} Var(\sum_{x_i \in \mathcal{N}(x)} y_i)$$

=

$$Var(\hat{f}(x)) = \frac{1}{k^2} \sum_{x_i \in \mathcal{N}(x)} Var(y_i)$$

=

$$Var(\hat{f}(x)) = \frac{1}{k^2} \sum_{x_i \in \mathcal{N}(x)} (\sigma^2)$$

=

$$Var(\hat{f}(x)) = \frac{1}{k^2} k(\sigma^2)$$

=

$$Var(\hat{f}(x)) = \frac{k}{k^2} (\sigma^2)$$

=

$$Var(\hat{f}(x)) = \frac{\sigma^2}{k}$$

2) Bias

a) We know that k-nearest regression is defined as:

$$[\hat{f}(x) = \frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} y_i]$$

b) To find bias:

$$Bias(\hat{f}(x)) = E[f(x)] - E[\hat{f}(x)]$$

=

$$Bias(\hat{f}(x)) = E[f(x)] - E[\frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} y_i]$$

=

$$Bias(\hat{f}(x))^2 = (E[f(x)] - E[\frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} y_i])^2$$

=

$$Bias(\hat{f}(x))^2 = (f(x) - (\frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} y_i))^2$$

c) Decomposition of MSE:

$$MSE = Var(f(x)) + Bias(\hat{f}(x))^2 + Var(\hat{f}x)$$

=

$$MSE = Var(f(x)) + (f(x) - (\frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} y_i))^2 + \frac{\sigma^2}{k}$$

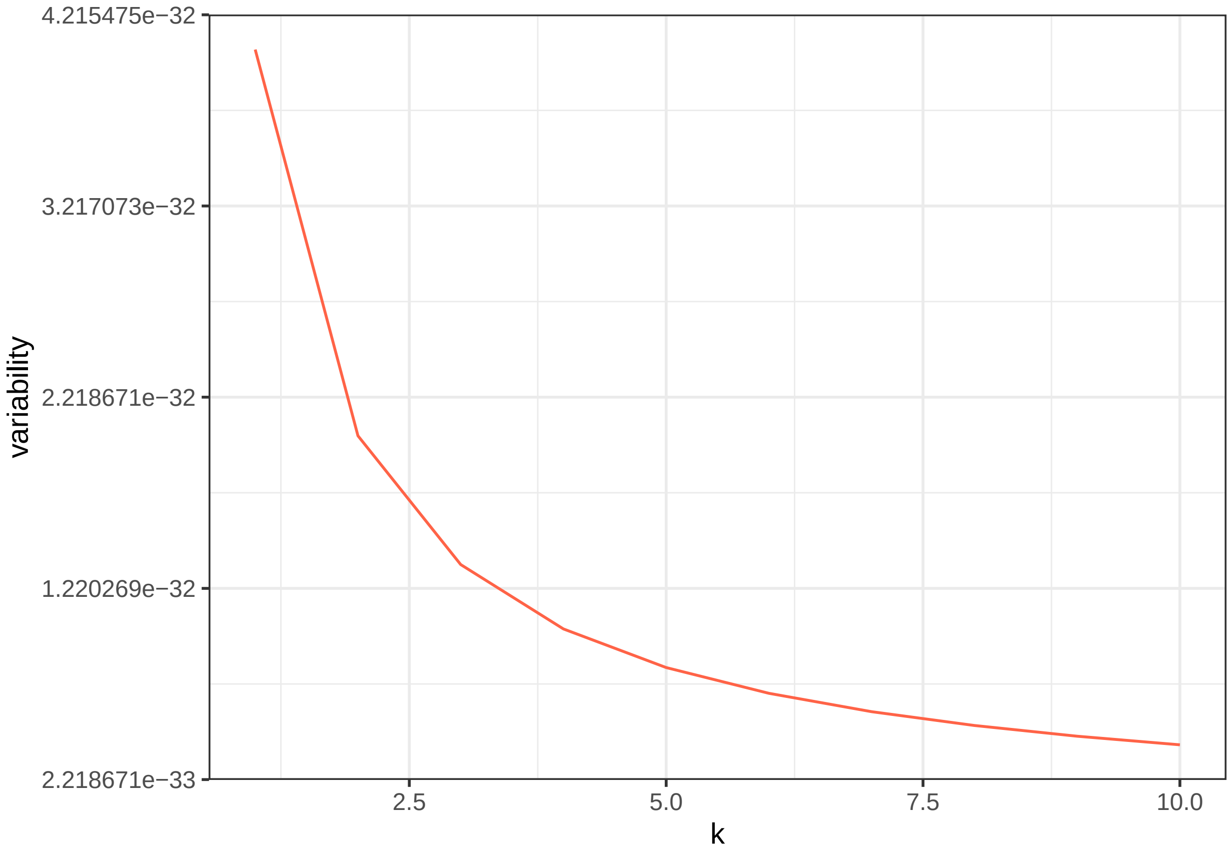
=

$$MSE = \sigma^2 + (f(x) - (\frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} y_i))^2 + \frac{\sigma^2}{k}$$

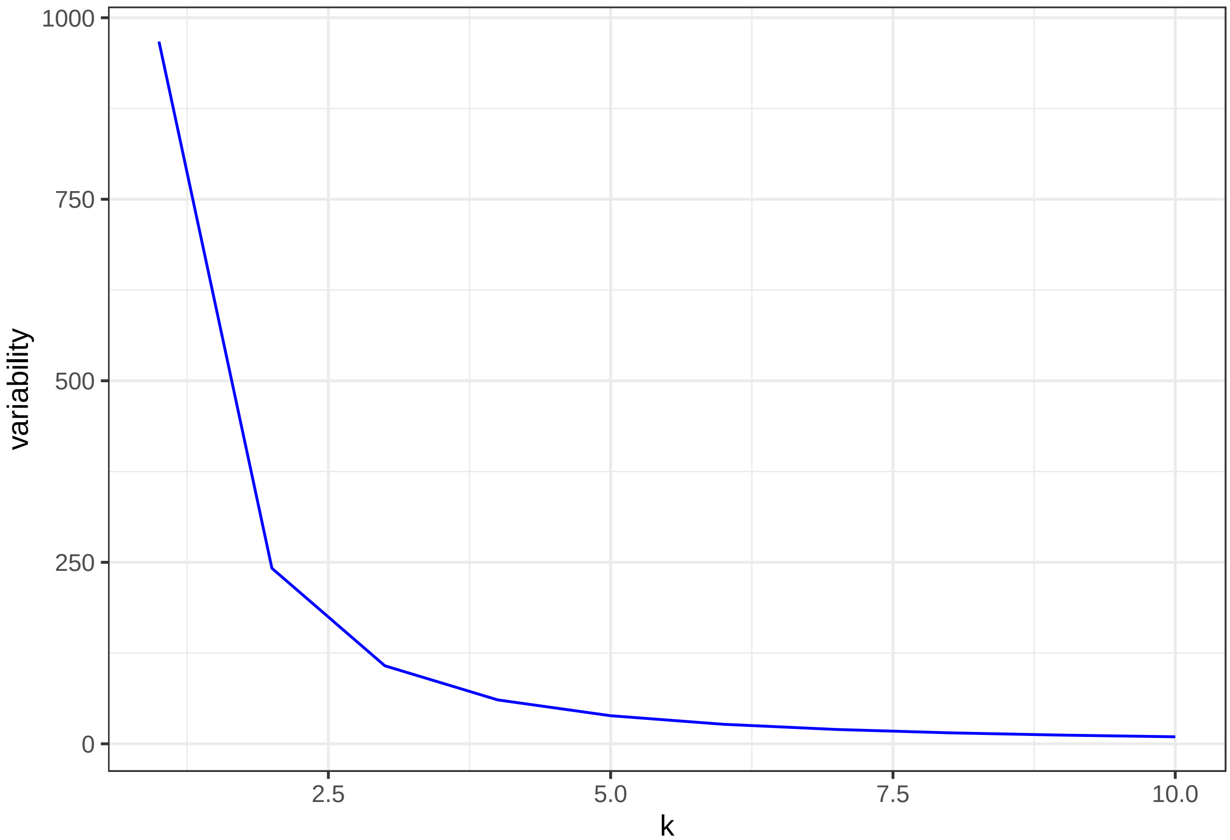
Exercise 2

```
library(tidyverse)
x <- c(1:3, 5:12)
y <- c(-7.1, -7.1, .5, -3.6, -2, -1.7, -4, -.2, -1.2, -1.2, -3.5)
y_mean = mean(y)
std_dev <- function(y) {(sum(y-y_mean)^2)/11}
std_dev <- std_dev(y)



my_fun <- function(k, x, y) {
  f_k <- rep(NA, length(k))
  for (i in 1:length(k)) {
    f_k[i] <- std_dev/k[i]
  }
  f_k
}
y_sum <- sum(y)
my_fun_2 <- function(k, x, y) {
  m_k <- rep(NA, length(k))
  for (i in 1:length(k)) {
```

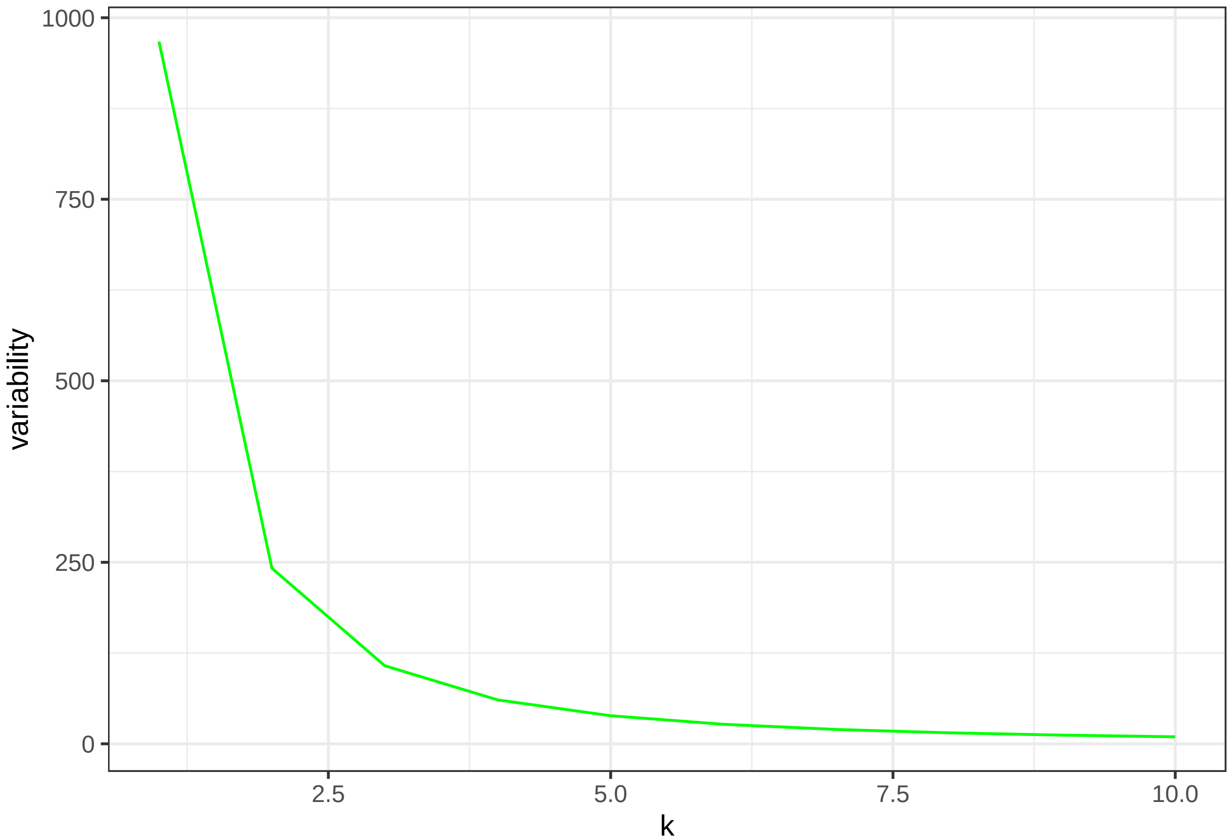
```
    m_k[i] <- std_dev + (std_dev/k[i]) + (y_sum/k[i])^2
  }
  m_k
}


my_fun_3 <- function(k, x, y) {
  b_k <- rep(NA, length(k))
  for (i in 1:length(k)) {
    b_k[i] <- (y_sum/k[i])^2
  }
  b_k
}

k <- 1:10
f_k <- my_fun(k, x, y)

m_k <- my_fun_2(k,x,y)


k <- 1:10
b_k <- my_fun_3(k, x, y)

df <- tibble(k = k, f_k = f_k)
ggplot(df, aes(x = k, y = f_k)) + geom_line (col = "tomato") +
  theme_bw() +
  ylab("variability")
```




```
df <- tibble(k = k, m_k = m_k)
ggplot(df, aes(x = k, y = m_k)) + geom_line (col = "blue") +
  theme_bw() +
  ylab("variability")
```




```
df <- tibble(k = k, b_k = b_k)
ggplot(df, aes(x = k, y = b_k)) + geom_line (col = "green") +
  theme_bw() +
  ylab("variability")
```



Exercise 7

Exercise 7

Exercise 7

Exercise 7

Exercise 7

Exercise 7

Exercise 7