Lab 2: Linear Regression

An Island Never Cries

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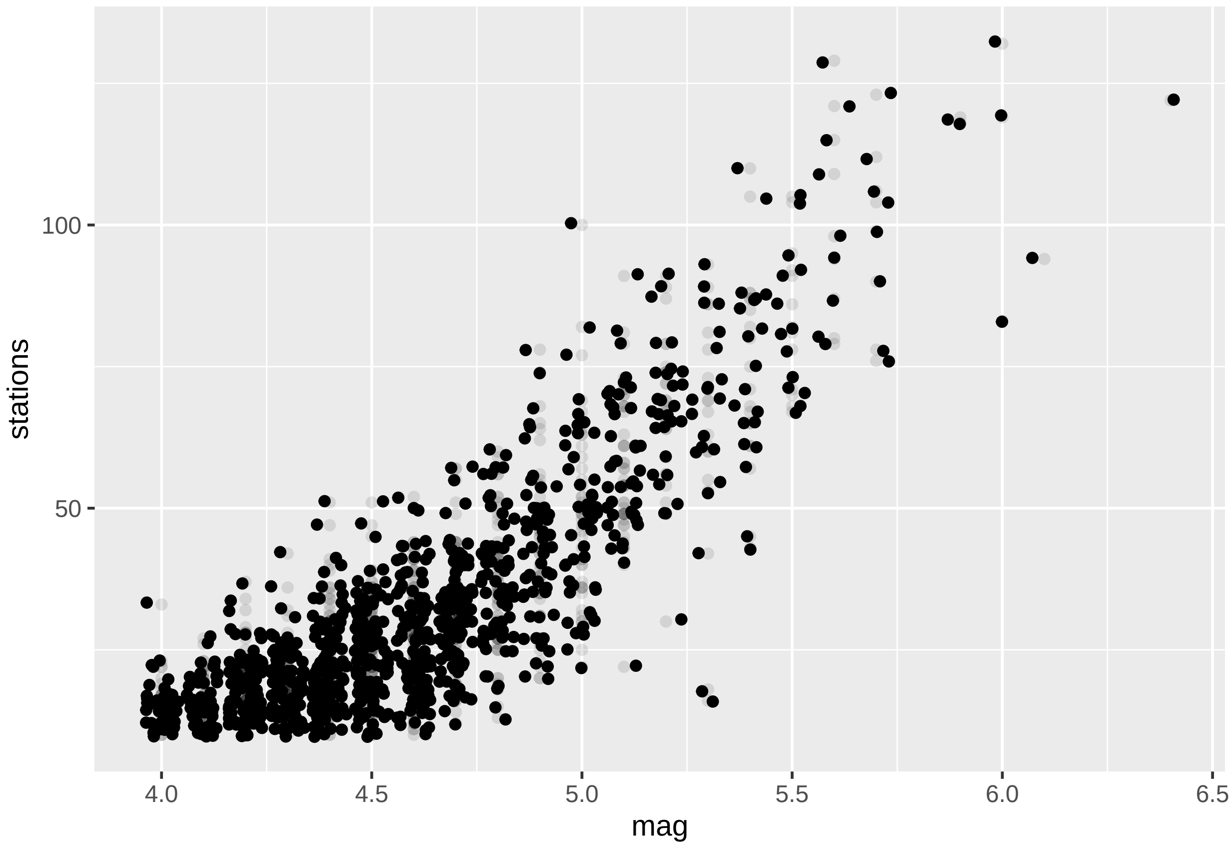
Exercise 1

```
library(MASS)
library("ggplot2")
library("tidyverse")
```

```
## -- Attaching packages -------------------- tidyverse 1.2.1 --
## v tibble  2.1.1       v purrr   0.3.2
## v tidyr   0.8.3       v dplyr   0.8.0.1
## v readr   1.3.1       v stringr 1.4.0
## v tibble  2.1.1       v forcats 0.4.0
## -- Conflicts ----------------------- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## x dplyr::select() masks MASS::select()
```

```
data(quakes)

ggplot(quakes, aes(mag, stations)) + geom_point(alpha = 0.1) + geom_jitter()
```



There is a strong positive linear relationship between the magnitude of an earthquake and the number o stations that detect the earthquake. In general, as the magnitude of the earthquake increases, the number of reporting stations increases as wel There is a strong positive linear relationship between the magnitude of an earthquake and the number o stations that detect the earthquake. In general, as the magnitude of the earthquake increases, the number of reporting stations increases as well.

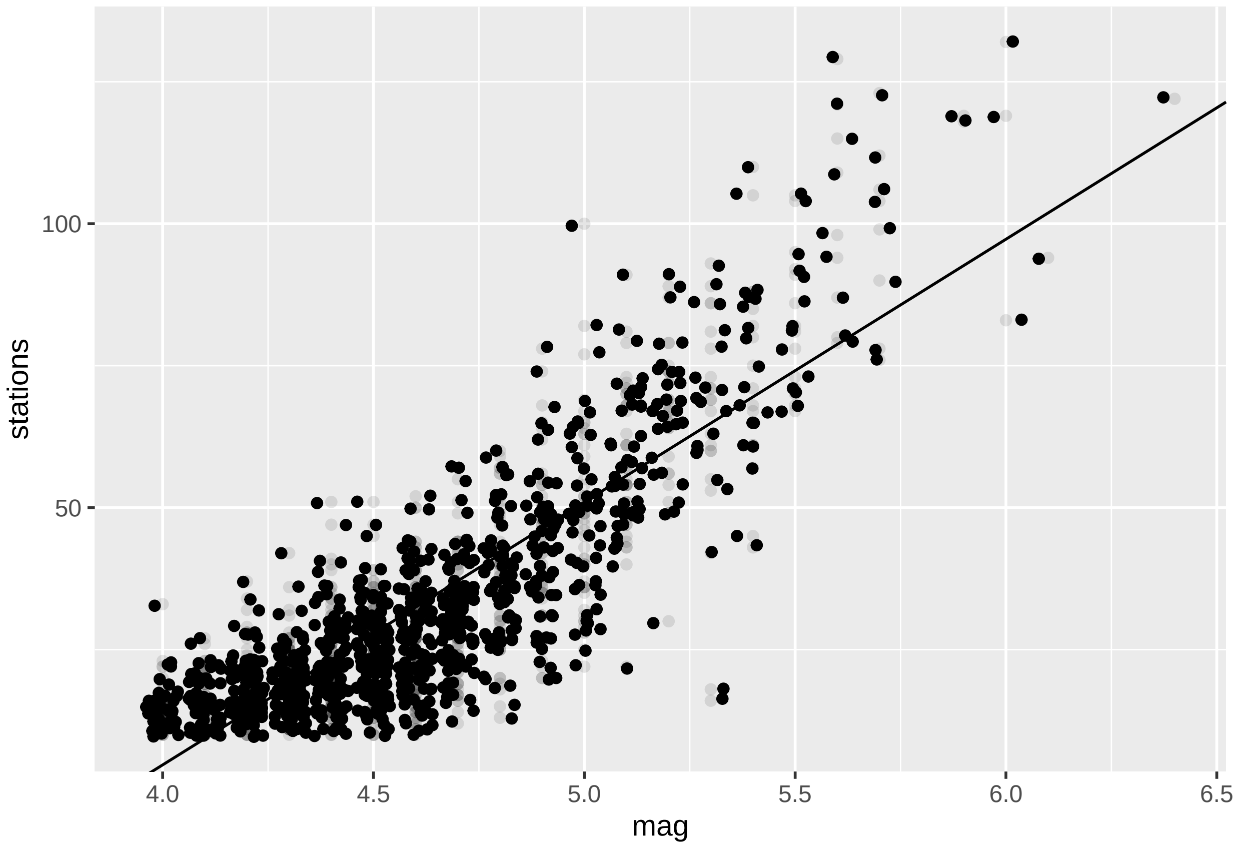
Exercise 2

```
mean(quakes$stations)
```

```
## [1] 33.418
```

The slope would be zero. It follows that the intercept would be 33.418, the mean of Y, since $\hat{B}_0 = \overline{y}$ - $\hat{B}_1$ $\overline{x}$ #### Exercise 3

```
m1 <- lm(stations ~ mag, data = quakes)
ggplot(quakes, aes(mag, stations)) + geom_point(alpha = 0.1) +  geom_abline(intercept = m1$coef[1], slop
```




```r
summary(m1)
```

```
##
## Call:
## lm(formula = stations ~ mag, data = quakes)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -48.871  -7.102  -0.474   6.783  50.244
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -180.4243     4.1899  -43.06   <2e-16 ***
## mag           46.2822     0.9034   51.23   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.5 on 998 degrees of freedom
## Multiple R-squared:  0.7245, Adjusted R-squared:  0.7242
## F-statistic:  2625 on 1 and 998 DF,  p-value: < 2.2e-16
```

The slope is 46.2822 and the intercept is -180.4243. The model predicts that as the magnitude of an earthquake increases by one unit on the Richter Magnitude scale, the number of stations that detect the earthquake will increase by approaximately 46.2822. The model also predicts that if the magnitude of an earthquake on the Richter Magnitude scale is 0, the number of stations that can detect it will be -180.4243. That is to say, no stations will detect an earthquake or event that registers as zero on the Richter scale.

Exercise 4

```
r <- cor(quakes$stations, quakes$mag)

y_diff <- quakes$stations - mean(quakes$stations)
y_diff_squared <- (y_diff)^2
std_dev_y <- sqrt(sum(y_diff_squared/999))
std_dev_y
```

```
## [1] 21.90039
```

```
x_diff <- quakes$mag - mean(quakes$mag)
x_diff_squared <- (x_diff)^2
std_dev_x <- sqrt(sum(x_diff_squared/999))
std_dev_x
```

```
## [1] 0.402773
```

```
slope <- r*(std_dev_y/std_dev_x)
slope
```

```
## [1] 46.28221
```

Using the equation

$$\hat{\beta}_1 = r * (s_y/s_x)$$

, I calculated a slope of 46.28221 between the magnitude of the earthquakes and the number of reporting stations. This corresponds with the slope computed by the fitted linear model.

Exercise 5

```
summary(m1)$coef[[2]]+ qt(1-(0.05/2), 998) * summary(m1)$coef[[4]]
```

```
## [1] 48.05498
```

```
summary(m1)$coef[[2]] - qt(1-(0.05/2), 998) * summary(m1)$coef[[4]]
```

```
## [1] 44.50944
```

```
confint(m1,'mag', level = 0.95)
```

```
##        2.5 %   97.5 %
## mag 44.50944 48.05498
```

We are 95% confident that the true slope between the magnitude of the earthquakes and the number of stations that detect them lies between 44.50944 and 48.05498.

Exercise 6

```
predict(m1, data.frame(mag = 7.0))
```

```
##        1
## 143.5511
```

I predict that around 144 (143.5511) stations would be able to detect an earthquake with a magnitude of 7.0.

Exercise 7

Question 1 - Data description Question 2 - Inference Question 3 - Inference Question 4 - Inference Question 5 - Inference Question 6 - Prediction

SIMULATION

Exercise 9

```
dat <- seq(from = 4.0, to = 6.0, by = 0.1)
sim_dat <- sample(dat, size = 1000, replace = TRUE)
sim_dat
```

```
##    [1] 5.2 5.3 4.2 4.6 4.0 5.2 5.3 4.7 4.7 5.3 6.0 5.3 5.8 5.5 5.3 4.7 4.3
##   [18] 5.7 4.8 5.9 4.5 4.6 5.5 5.7 5.4 4.4 4.3 4.5 4.2 4.2 5.9 5.5 5.6 4.1
##   [35] 5.7 4.8 5.4 5.1 5.9 5.8 4.1 5.2 5.0 5.9 4.1 4.8 5.3 5.7 4.9 4.7 4.9
##   [52] 4.2 5.4 4.1 4.2 4.1 6.0 4.8 5.8 5.4 4.4 5.2 5.5 4.2 5.6 4.9 4.2 5.4
##   [69] 4.4 4.9 5.3 4.8 5.4 5.8 4.7 4.7 4.8 4.3 4.3 4.7 5.0 5.6 5.6 4.3 4.8
##   [86] 4.0 5.7 5.0 4.9 4.9 4.1 4.9 5.2 4.6 4.7 4.0 6.0 5.4 4.5 5.6 4.9 5.0
##  [103] 5.0 5.2 4.4 4.3 4.6 4.9 5.5 5.6 4.0 5.3 5.7 4.2 5.3 4.1 4.3 4.4 4.6
##  [120] 5.3 4.0 4.8 5.7 5.9 4.4 4.6 5.7 5.8 6.0 4.5 6.0 5.2 4.6 6.0 5.2 5.4
##  [137] 4.0 5.2 5.7 4.6 5.7 4.0 4.3 4.5 4.8 5.4 5.9 4.3 5.3 6.0 5.8 4.3 5.4
##  [154] 5.8 5.7 5.9 4.1 5.4 4.1 4.2 5.7 5.5 4.8 4.2 5.8 5.8 4.3 5.6 5.5 6.0
##  [171] 5.6 4.0 5.1 5.5 5.6 5.7 5.6 4.1 5.3 4.5 5.8 5.0 5.5 5.9 4.9 4.1 6.0
##  [188] 5.6 4.7 4.5 5.8 4.1 5.3 4.1 4.0 5.5 4.7 5.6 4.8 5.6 5.0 4.6 5.0 5.9
##  [205] 5.6 6.0 5.7 6.0 4.0 4.7 4.5 4.5 4.6 5.1 5.8 5.8 5.2 4.1 5.3 5.5 4.7
##  [222] 4.4 5.3 5.4 5.7 5.9 5.3 5.6 5.1 5.4 4.8 4.0 4.0 5.7 4.9 4.3 5.0 4.2
##  [239] 4.6 4.7 5.8 5.8 4.8 5.6 5.4 4.8 5.0 4.9 4.3 5.8 5.5 4.6 5.5 5.3 4.7
##  [256] 6.0 4.5 5.3 5.9 4.6 5.1 4.0 4.9 5.1 5.0 5.9 5.5 4.0 5.9 4.8 4.6 5.3
##  [273] 4.4 5.8 4.8 4.2 4.9 4.0 5.5 4.9 4.3 5.7 4.3 4.1 5.1 5.9 4.8 4.7 5.5
##  [290] 4.9 5.1 4.4 4.0 4.6 4.9 5.9 4.1 4.1 5.3 5.9 4.1 4.2 4.1 5.3 5.7 4.1
##  [307] 6.0 5.7 4.5 4.0 5.5 5.6 4.2 5.8 5.4 5.4 5.9 5.2 5.2 4.4 5.3 4.4 5.7
##  [324] 5.5 5.3 4.0 4.6 5.1 4.7 4.8 5.1 4.6 4.4 5.0 4.6 4.0 5.2 4.1 4.0 5.0
##  [341] 5.9 5.1 6.0 4.6 4.0 4.2 5.2 4.3 5.1 5.0 6.0 6.0 5.7 4.2 4.9 5.4 4.2
##  [358] 4.2 5.9 5.3 5.2 4.3 5.7 5.0 4.5 5.6 4.4 4.3 5.0 5.9 4.9 4.9 5.3 4.1
##  [375] 5.6 4.9 4.4 5.9 5.0 5.8 5.0 4.5 4.6 5.1 4.1 4.7 4.2 5.9 4.9 5.1 4.5
##  [392] 5.2 4.3 5.2 5.7 4.3 5.9 4.9 5.7 4.2 4.3 4.2 4.7 4.4 4.5 5.6 4.5 4.4
##  [409] 5.1 4.5 5.1 4.4 4.9 4.2 5.6 4.0 6.0 5.4 4.4 4.5 4.2 5.9 5.5 5.3 5.2
##  [426] 4.0 5.4 5.0 5.2 5.5 4.0 6.0 5.5 4.0 5.7 6.0 4.3 5.1 6.0 4.9 5.6 5.7
##  [443] 5.4 4.2 5.9 5.2 5.7 4.9 4.6 5.7 5.1 4.5 5.0 5.2 6.0 5.1 5.2 5.6 4.9
##  [460] 4.7 5.4 6.0 4.6 5.9 5.7 4.9 5.2 5.4 4.6 6.0 5.7 5.6 4.7 4.5 5.2 4.3
##  [477] 5.4 5.9 4.6 6.0 4.8 5.6 5.1 5.4 4.8 4.7 4.1 5.2 5.1 5.3 5.6 5.4 5.5
##  [494] 4.2 4.9 5.4 4.1 4.4 5.6 5.9 4.8 6.0 4.2 4.6 5.3 4.8 4.2 5.8 5.4 4.2
##  [511] 4.2 5.6 6.0 4.4 5.5 4.0 4.5 4.6 5.9 5.1 5.4 4.7 4.1 5.9 4.7 4.2 5.6
##  [528] 5.9 4.9 4.5 5.2 4.1 6.0 5.6 4.4 5.8 4.1 5.0 5.4 4.8 5.9 5.4 4.1 4.4
##  [545] 4.3 5.5 5.2 5.2 4.3 5.7 4.3 5.1 4.9 4.1 5.6 5.1 4.9 5.6 4.1 5.7 5.5
##  [562] 5.1 4.8 4.9 4.0 4.2 4.1 4.8 4.3 4.5 4.4 4.2 4.2 5.4 5.5 5.0 5.9 4.6
##  [579] 5.1 4.5 5.6 5.4 4.3 4.6 4.7 4.9 4.5 4.2 4.2 4.5 5.3 5.2 4.7 4.8 5.5
##  [596] 5.0 5.1 4.0 4.4 5.6 4.0 4.4 4.3 5.5 4.2 4.2 4.0 4.0 5.3 4.2 5.9 4.0
##  [613] 5.0 6.0 4.8 5.8 6.0 4.6 4.4 5.9 4.7 5.9 4.0 5.2 5.7 5.0 5.3 5.4 4.8
##  [630] 4.1 4.1 5.9 4.6 4.0 4.4 5.7 5.8 4.0 4.9 5.3 5.9 4.5 4.0 5.8 5.6 4.1
##  [647] 6.0 4.5 5.6 5.8 4.9 4.9 5.3 4.5 5.9 5.1 4.1 4.7 5.5 4.2 4.3 5.6 5.6
##  [664] 5.4 4.7 4.9 5.7 4.9 4.7 5.5 4.4 4.1 4.2 5.8 4.6 4.9 5.2 5.9 4.7 6.0
##  [681] 5.6 4.7 5.7 4.6 5.3 4.0 4.5 4.3 5.7 4.6 6.0 5.2 4.7 5.7 5.1 4.9 5.0
##  [698] 5.8 4.8 5.6 5.3 4.0 4.2 5.3 5.4 5.9 5.6 4.0 4.3 5.7 5.9 5.9 4.7 4.6
##  [715] 4.9 5.8 4.7 5.9 5.4 4.4 5.2 4.3 5.4 6.0 5.5 6.0 4.5 4.6 4.8 4.7 5.9
##  [732] 4.4 5.3 5.7 5.5 4.2 4.9 5.4 4.4 4.8 4.0 5.5 5.6 4.2 5.5 5.8 5.7 4.0
##  [749] 4.6 5.5 5.1 5.6 4.6 4.3 4.6 5.3 4.4 4.4 4.2 4.6 4.0 4.6 4.1 5.3 5.0
##  [766] 4.8 5.1 4.9 4.2 5.7 5.3 4.2 4.3 4.3 5.9 5.8 4.4 5.3 4.0 4.7 5.0 4.4
```

```
## [783] 4.5 5.4 5.9 6.0 4.1 4.2 5.8 6.0 5.3 4.2 4.7 4.0 4.8 4.1 5.5 4.8 4.1
## [800] 5.3 5.9 5.8 5.1 4.7 5.3 4.7 4.1 5.2 4.7 4.0 4.0 4.8 6.0 5.4 4.2 5.3
## [817] 5.2 4.7 4.2 5.8 5.6 5.2 6.0 5.6 5.7 4.2 5.6 4.5 4.0 4.6 4.7 4.6 4.5
## [834] 5.2 4.5 4.0 4.4 5.6 4.7 4.9 5.9 4.4 4.6 5.9 5.5 4.2 4.4 5.4 5.0 5.9
## [851] 6.0 4.0 5.7 5.7 5.9 6.0 5.7 4.5 4.2 4.9 4.1 6.0 4.6 5.9 4.0 4.7 5.8
## [868] 4.7 4.5 4.9 4.1 4.5 4.4 4.1 5.8 5.7 5.4 4.4 5.3 5.5 4.1 4.4 5.9 5.5
## [885] 5.0 5.7 4.3 4.2 4.9 5.4 5.2 5.1 4.1 5.3 6.0 6.0 4.2 5.0 6.0 4.1 4.4
## [902] 4.5 4.4 4.5 5.1 4.5 4.8 4.2 5.7 4.2 4.1 5.8 4.1 5.7 6.0 5.1 4.1 4.2
## [919] 4.9 4.2 5.1 5.7 5.8 5.9 4.4 4.4 4.2 4.8 4.4 5.3 4.5 6.0 5.7 5.6 4.0
## [936] 4.3 4.2 5.9 4.8 6.0 5.2 5.7 6.0 5.0 4.7 5.3 5.1 5.2 5.7 5.5 4.2 5.6
## [953] 4.1 4.5 5.1 4.3 4.6 4.7 4.5 5.5 4.9 5.9 4.4 4.4 6.0 5.1 5.1 4.5 4.1
## [970] 5.0 5.6 4.9 5.6 4.2 4.2 4.3 4.2 5.8 4.6 4.0 5.5 5.5 6.0 4.4 5.4 4.7
## [987] 4.3 4.5 5.2 4.6 4.6 5.9 5.3 4.5 5.6 4.8 5.3 4.6 4.2 6.0
```

Exercise 10

```
f_hat <- function(x) {-180.4243 + 46.2822*x}
y_hat <- f_hat(sim_dat)
y_hat
```

```
##   [1] 60.24314 64.87136 13.96094 32.47382  4.70450 60.24314 64.87136
##   [8] 37.10204 37.10204 64.87136 97.26890 64.87136 88.01246 74.12780
##  [15] 64.87136 37.10204 18.58916 83.38424 41.73026 92.64068 27.84560
##  [22] 32.47382 74.12780 83.38424 69.49958 23.21738 18.58916 27.84560
##  [29] 13.96094 13.96094 92.64068 74.12780 78.75602  9.33272 83.38424
##  [36] 41.73026 69.49958 55.61492 92.64068 88.01246  9.33272 60.24314
##  [43] 50.98670 92.64068  9.33272 41.73026 64.87136 83.38424 46.35848
##  [50] 37.10204 46.35848 13.96094 69.49958  9.33272 13.96094  9.33272
##  [57] 97.26890 41.73026 88.01246 69.49958 23.21738 60.24314 74.12780
##  [64] 13.96094 78.75602 46.35848 13.96094 69.49958 23.21738 46.35848
##  [71] 64.87136 41.73026 69.49958 88.01246 37.10204 37.10204 41.73026
##  [78] 18.58916 18.58916 37.10204 50.98670 78.75602 78.75602 18.58916
##  [85] 41.73026  4.70450 83.38424 50.98670 46.35848 46.35848  9.33272
##  [92] 46.35848 60.24314 32.47382 37.10204  4.70450 97.26890 69.49958
##  [99] 27.84560 78.75602 46.35848 50.98670 50.98670 60.24314 23.21738
## [106] 18.58916 32.47382 46.35848 74.12780 78.75602  4.70450 64.87136
## [113] 83.38424 13.96094 64.87136  9.33272 18.58916 23.21738 32.47382
## [120] 64.87136  4.70450 41.73026 83.38424 92.64068 23.21738 32.47382
## [127] 83.38424 88.01246 97.26890 27.84560 97.26890 60.24314 32.47382
## [134] 97.26890 60.24314 69.49958  4.70450 60.24314 83.38424 32.47382
## [141] 83.38424  4.70450 18.58916 27.84560 41.73026 69.49958 92.64068
## [148] 18.58916 64.87136 97.26890 88.01246 18.58916 69.49958 88.01246
## [155] 83.38424 92.64068  9.33272 69.49958  9.33272 13.96094 83.38424
## [162] 74.12780 41.73026 13.96094 88.01246 88.01246 18.58916 78.75602
## [169] 74.12780 97.26890 78.75602  4.70450 55.61492 74.12780 78.75602
## [176] 83.38424 78.75602  9.33272 64.87136 27.84560 88.01246 50.98670
## [183] 74.12780 92.64068 46.35848  9.33272 97.26890 78.75602 37.10204
## [190] 27.84560 88.01246  9.33272 64.87136  9.33272  4.70450 74.12780
## [197] 37.10204 78.75602 41.73026 78.75602 50.98670 32.47382 50.98670
## [204] 92.64068 78.75602 97.26890 83.38424 97.26890  4.70450 37.10204
## [211] 27.84560 27.84560 32.47382 55.61492 88.01246 88.01246 60.24314
## [218]  9.33272 64.87136 74.12780 37.10204 23.21738 64.87136 69.49958
## [225] 83.38424 92.64068 64.87136 78.75602 55.61492 69.49958 41.73026
## [232]  4.70450  4.70450 83.38424 46.35848 18.58916 50.98670 13.96094
```

```
## [239] 32.47382 37.10204 88.01246 88.01246 41.73026 78.75602 69.49958
## [246] 41.73026 50.98670 46.35848 18.58916 88.01246 74.12780 32.47382
## [253] 74.12780 64.87136 37.10204 97.26890 27.84560 64.87136 92.64068
## [260] 32.47382 55.61492  4.70450 46.35848 55.61492 50.98670 92.64068
## [267] 74.12780  4.70450 92.64068 41.73026 32.47382 64.87136 23.21738
## [274] 88.01246 41.73026 13.96094 46.35848  4.70450 74.12780 46.35848
## [281] 18.58916 83.38424 18.58916  9.33272 55.61492 92.64068 41.73026
## [288] 37.10204 74.12780 46.35848 55.61492 23.21738  4.70450 32.47382
## [295] 46.35848 92.64068  9.33272  9.33272 64.87136 92.64068  9.33272
## [302] 13.96094  9.33272 64.87136 83.38424  9.33272 97.26890 83.38424
## [309] 27.84560  4.70450 74.12780 78.75602 13.96094 88.01246 69.49958
## [316] 69.49958 92.64068 60.24314 60.24314 23.21738 64.87136 23.21738
## [323] 83.38424 74.12780 64.87136  4.70450 32.47382 55.61492 37.10204
## [330] 41.73026 55.61492 32.47382 23.21738 50.98670 32.47382  4.70450
## [337] 60.24314  9.33272  4.70450 50.98670 92.64068 55.61492 97.26890
## [344] 32.47382  4.70450 13.96094 60.24314 18.58916 55.61492 50.98670
## [351] 97.26890 97.26890 83.38424 13.96094 46.35848 69.49958 13.96094
## [358] 13.96094 92.64068 64.87136 60.24314 18.58916 83.38424 50.98670
## [365] 27.84560 78.75602 23.21738 18.58916 50.98670 92.64068 46.35848
## [372] 46.35848 64.87136  9.33272 78.75602 46.35848 23.21738 92.64068
## [379] 50.98670 88.01246 50.98670 27.84560 32.47382 55.61492  9.33272
## [386] 37.10204 13.96094 92.64068 46.35848 55.61492 27.84560 60.24314
## [393] 18.58916 60.24314 83.38424 18.58916 92.64068 46.35848 83.38424
## [400] 13.96094 18.58916 13.96094 37.10204 23.21738 27.84560 78.75602
## [407] 27.84560 23.21738 55.61492 27.84560 55.61492 23.21738 46.35848
## [414] 13.96094 78.75602  4.70450 97.26890 69.49958 23.21738 27.84560
## [421] 13.96094 92.64068 74.12780 64.87136 60.24314  4.70450 69.49958
## [428] 50.98670 60.24314 74.12780  4.70450 97.26890 74.12780  4.70450
## [435] 83.38424 97.26890 18.58916 55.61492 97.26890 46.35848 78.75602
## [442] 83.38424 69.49958 13.96094 92.64068 60.24314 83.38424 46.35848
## [449] 32.47382 83.38424 55.61492 27.84560 50.98670 60.24314 97.26890
## [456] 55.61492 60.24314 78.75602 46.35848 37.10204 69.49958 97.26890
## [463] 32.47382 92.64068 83.38424 46.35848 60.24314 69.49958 32.47382
## [470] 97.26890 83.38424 78.75602 37.10204 27.84560 60.24314 18.58916
## [477] 69.49958 92.64068 32.47382 97.26890 41.73026 78.75602 55.61492
## [484] 69.49958 41.73026 37.10204  9.33272 60.24314 55.61492 64.87136
## [491] 78.75602 69.49958 74.12780 13.96094 46.35848 69.49958  9.33272
## [498] 23.21738 78.75602 92.64068 41.73026 97.26890 13.96094 32.47382
## [505] 64.87136 41.73026 13.96094 88.01246 69.49958 13.96094 13.96094
## [512] 78.75602 97.26890 23.21738 74.12780  4.70450 27.84560 32.47382
## [519] 92.64068 55.61492 69.49958 37.10204  9.33272 92.64068 37.10204
## [526] 13.96094 78.75602 92.64068 46.35848 27.84560 60.24314  9.33272
## [533] 97.26890 78.75602 23.21738 88.01246  9.33272 50.98670 69.49958
## [540] 41.73026 92.64068 69.49958  9.33272 23.21738 18.58916 74.12780
## [547] 60.24314 60.24314 18.58916 83.38424 18.58916 55.61492 46.35848
## [554]  9.33272 78.75602 55.61492 46.35848 78.75602  9.33272 83.38424
## [561] 74.12780 55.61492 41.73026 46.35848  4.70450 13.96094  9.33272
## [568] 41.73026 18.58916 27.84560 23.21738 13.96094 13.96094 69.49958
## [575] 74.12780 50.98670 92.64068 32.47382 55.61492 27.84560 78.75602
## [582] 69.49958 18.58916 32.47382 37.10204 46.35848 27.84560 13.96094
## [589] 13.96094 27.84560 64.87136 60.24314 37.10204 41.73026 74.12780
## [596] 50.98670 55.61492  4.70450 23.21738 78.75602  4.70450 23.21738
## [603] 18.58916 74.12780 13.96094 13.96094  4.70450  4.70450 64.87136
## [610] 13.96094 92.64068  4.70450 50.98670 97.26890 41.73026 88.01246
```

```
## [617] 97.26890 32.47382 23.21738 92.64068 37.10204 92.64068  4.70450
## [624] 60.24314 83.38424 50.98670 64.87136 69.49958 41.73026  9.33272
## [631]  9.33272 92.64068 32.47382  4.70450 23.21738 83.38424 88.01246
## [638]  4.70450 46.35848 64.87136 92.64068 27.84560  4.70450 88.01246
## [645] 78.75602  9.33272 97.26890 27.84560 78.75602 88.01246 46.35848
## [652] 46.35848 64.87136 27.84560 92.64068 55.61492  9.33272 37.10204
## [659] 74.12780 13.96094 18.58916 78.75602 78.75602 69.49958 37.10204
## [666] 46.35848 83.38424 46.35848 37.10204 74.12780 23.21738  9.33272
## [673] 13.96094 88.01246 32.47382 46.35848 60.24314 92.64068 37.10204
## [680] 97.26890 78.75602 37.10204 83.38424 32.47382 64.87136  4.70450
## [687] 27.84560 18.58916 83.38424 32.47382 97.26890 60.24314 37.10204
## [694] 83.38424 55.61492 46.35848 50.98670 88.01246 41.73026 78.75602
## [701] 64.87136  4.70450 13.96094 64.87136 69.49958 92.64068 78.75602
## [708]  4.70450 18.58916 83.38424 92.64068 92.64068 37.10204 32.47382
## [715] 46.35848 88.01246 37.10204 92.64068 69.49958 23.21738 60.24314
## [722] 18.58916 69.49958 97.26890 74.12780 97.26890 27.84560 32.47382
## [729] 41.73026 37.10204 92.64068 23.21738 64.87136 83.38424 74.12780
## [736] 13.96094 46.35848 69.49958 23.21738 41.73026  4.70450 74.12780
## [743] 78.75602 13.96094 74.12780 88.01246 83.38424  4.70450 32.47382
## [750] 74.12780 55.61492 78.75602 32.47382 18.58916 32.47382 64.87136
## [757] 23.21738 23.21738 13.96094 32.47382  4.70450 32.47382  9.33272
## [764] 64.87136 50.98670 41.73026 55.61492 46.35848 13.96094 83.38424
## [771] 64.87136 13.96094 18.58916 18.58916 92.64068 88.01246 23.21738
## [778] 64.87136  4.70450 37.10204 50.98670 23.21738 27.84560 69.49958
## [785] 92.64068 97.26890  9.33272 13.96094 88.01246 97.26890 64.87136
## [792] 13.96094 37.10204  4.70450 41.73026  9.33272 74.12780 41.73026
## [799]  9.33272 64.87136 92.64068 88.01246 55.61492 37.10204 64.87136
## [806] 37.10204  9.33272 60.24314 37.10204  4.70450  4.70450 41.73026
## [813] 97.26890 69.49958 13.96094 64.87136 60.24314 37.10204 13.96094
## [820] 88.01246 78.75602 60.24314 97.26890 78.75602 83.38424 13.96094
## [827] 78.75602 27.84560  4.70450 32.47382 37.10204 32.47382 27.84560
## [834] 60.24314 27.84560  4.70450 23.21738 78.75602 37.10204 46.35848
## [841] 92.64068 23.21738 32.47382 92.64068 74.12780 13.96094 23.21738
## [848] 69.49958 50.98670 92.64068 97.26890  4.70450 83.38424 83.38424
## [855] 92.64068 97.26890 83.38424 27.84560 13.96094 46.35848  9.33272
## [862] 97.26890 32.47382 92.64068  4.70450 37.10204 88.01246 37.10204
## [869] 27.84560 46.35848  9.33272 27.84560 23.21738  9.33272 88.01246
## [876] 83.38424 69.49958 23.21738 64.87136 74.12780  9.33272 23.21738
## [883] 92.64068 74.12780 50.98670 83.38424 18.58916 13.96094 46.35848
## [890] 69.49958 60.24314 55.61492  9.33272 64.87136 97.26890 97.26890
## [897] 13.96094 50.98670 97.26890  9.33272 23.21738 27.84560 23.21738
## [904] 27.84560 55.61492 27.84560 41.73026 13.96094 83.38424 13.96094
## [911]  9.33272 88.01246  9.33272 83.38424 97.26890 55.61492  9.33272
## [918] 13.96094 46.35848 13.96094 55.61492 83.38424 88.01246 92.64068
## [925] 23.21738 23.21738 13.96094 41.73026 23.21738 64.87136 27.84560
## [932] 97.26890 83.38424 78.75602  4.70450 18.58916 13.96094 92.64068
## [939] 41.73026 97.26890 60.24314 83.38424 97.26890 50.98670 37.10204
## [946] 64.87136 55.61492 60.24314 83.38424 74.12780 13.96094 78.75602
## [953]  9.33272 27.84560 55.61492 18.58916 32.47382 37.10204 27.84560
## [960] 74.12780 46.35848 92.64068 23.21738 23.21738 97.26890 55.61492
## [967] 55.61492 27.84560  9.33272 50.98670 78.75602 46.35848 78.75602
## [974] 13.96094 13.96094 18.58916 13.96094 88.01246 32.47382  4.70450
## [981] 74.12780 74.12780 97.26890 23.21738 69.49958 37.10204 18.58916
## [988] 27.84560 60.24314 32.47382 32.47382 92.64068 64.87136 27.84560
```

```
## [995] 78.75602 41.73026 64.87136 32.47382 13.96094 97.26890
```

Exercise 11

```r
res <- m1$res
y_pop <- res + y_hat
y_pop
```

```
##            1            2            3            4            5 
##  59.51285495  65.91040140 -12.53867151  42.14108248  10.99998356 
##            6            7            8            9           10 
##  67.53862356  66.14107495  28.88463925  34.99997602  65.28218033 
##           11           12           13           14           15 
##  87.05149925  48.39751710  83.79505925  60.91039925  56.97420095 
##           16           17           18           19           20 
##  29.51286033   4.32022203  76.53861818  31.51285925  87.42327925 
##           21           22           23           24           25 
##  16.99997818  30.51286140  68.91039925  68.28217602  56.99996849 
##           26           27           28           29           30 
##  33.51286356   4.11531710  46.60243064  11.11531818   7.74353925 
##           31           32           33           34           35 
##  81.16683710  67.02573602  79.02573495  14.62820356  65.53861818 
##           36           37           38           39           40 
##  36.14108033  58.65395818  40.14107710 100.30794248  75.79505925 
##           41           42           43           44           45 
##   6.23065602  59.76929710  50.76929925 100.05150033   3.85887710 
##           46           47           48           49           50 
##  29.37175387  61.02573818  78.16683925  48.76930033  54.48709172 
##           51           52           53           54           55 
##  45.39752140  24.25642356  63.02573710   3.74354033  15.99998140 
##           56           57           58           59           60 
##  17.11531925  86.42327818  48.02574356  87.79505925  62.91040033 
##           61           62           63           64           65 
##  21.11531602  62.91040248  84.14107279  13.48709710  72.39751387 
##           66           67           68           69           70 
##  34.25641602  17.62820248  63.51285279  22.37175818  63.23064741 
##           71           72           73           74           75 
##  71.16684356  43.88463818  64.91040033 102.76929064  48.88463925 
##           76           77           78           79           80 
##  33.51286033  43.39752248  13.74353818  19.62820140  32.23064957 
##           81           82           83           84           85 
##  55.74353064  77.91039818  84.28217710  14.99998033  48.02574356 
##           86           87           88           89           90 
##   1.11532033  78.28217602  49.14107818  48.39752140  69.76930033 
##           91           92           93           94           95 
##  14.71777172  31.25641602  63.99997064  32.51286140  44.14108140 
##           96           97           98           99          100 
##  17.99998356  90.42327818  63.25641064  17.23065172  74.65395602 
##          101          102          103          104          105 
##  49.02574248  43.51285710  43.88463602  45.14107602  19.74353710 
##          106          107          108          109          110 
##  19.62820140  34.25641925  35.88463710  66.74352526  62.76929279 
##          111          112          113          114          115 
##   4.85887818  64.91040140  94.67972356  11.23065495  62.65395925 
```

```
##         116          117          118          119          120 
##  20.37176140  20.71776957   0.11531602  42.74353495  63.91040140 
##         121          122          123          124          125 
##  -9.02578505  34.14108033  73.28217602  87.79505818  17.99997925 
##         126          127          128          129          130 
##  19.85887172  90.42328140 106.02573279 114.53861495  28.25642033 
##         131          132          133          134          135 
##  91.42327818  70.28218140  25.62819818  94.79505710  55.65396033 
##         136          137          138          139          140 
##  44.39751602  17.08955172  41.76929710  68.02573387  35.51286140 
##         141          142          143          144          145 
##  93.91039710  13.99998356  13.60243279  38.62819925  50.76930140 
##         146          147          148          149          150 
##  65.53862140  91.42327925  14.23065387  62.99996957 111.56438356 
##         151          152          153          154          155 
## 110.62818526  24.80733773  74.91040033  94.05150140  62.28217602 
##         156          157          158          159          160 
##  94.53861602   9.37176140  69.91040033  -6.02578613  11.48709710 
##         161          162          163          164          165 
##  93.05150248  66.39751495  39.25641710  13.48709710  81.28217495 
##         166          167          168          169          170 
##  74.02573279  37.83310634  86.88462957  78.02573602  96.42327818 
##         171          172          173          174          175 
##  84.16684033  13.23065710  57.28218248  78.79506248  79.42328248 
##         176          177          178          179          180 
##  89.99996526  77.76929279   3.48709818  75.53862248  22.37175710 
##         181          182          183          184          185 
## 107.16683818  57.39752033  67.91039925  93.67972140  51.02574248 
##         186          187          188          189          190 
##  24.97421387  89.67972033  86.39751387  28.62819710  24.37175710 
##         191          192          193          194          195 
##  94.14106957   7.23065602  44.39751710  20.99998248  13.23065710 
##         196          197          198          199          200 
##  83.16684140  31.62819710  64.91039818  54.14108033  92.51285064 
##         201          202          203          204          205 
##  55.39752033  39.76930356  48.88463602 100.79505818  69.91039818 
##         206          207          208          209          210 
##  91.67972033  89.14107064  83.42327818  -0.39756398  34.14108140 
##         211          212          213          214          215 
##  32.74353602  22.25642033  42.14108248  71.11530849  91.42328033 
##         216          217          218          219          220 
##  90.42328033  63.28218140  17.85887710  70.65395925  74.16684140 
##         221          222          223          224          225 
##  29.99997602  29.74353710  66.51285387  77.53862140  81.53861818 
##         226          227          228          229          230 
##  98.28217387  68.91040140  72.91039818  59.62819279  61.51285279 
##         231          232          233          234          235 
##  38.62819602  10.11532033  -2.51290075  68.02573387  33.51285818 
##         236          237          238          239          240 
##  25.88464356  47.76929925  25.97421279  22.37175602  36.37175495 
##         241          242          243          244          245 
##  93.16683818  94.67972248  -7.14113043  89.02573495  70.51285279 
##         246          247          248          249          250 
##  37.25641710  58.39752033  27.25641602  14.71776957  86.05150140 
```

```
##         251         252         253         254         255 
##  63.02573602  30.62819818  65.51285172  56.51285387  32.88463925 
##         256         257         258         259         260 
## 105.30794140  29.37175710  62.76929602 101.67972140  29.11531387 
##         261         262         263         264         265 
##  71.99997172  -0.39756398  33.14107925  59.39751925  53.02574140 
##         266         267         268         269         270 
##  83.28217387  57.65395710  -4.51290075  97.16683710  36.88463818 
##         271         272         273         274         275 
##  24.25641925  53.51285387  25.88464248  85.05150140  62.34598526 
##         276         277         278         279         280 
##   6.48709710  54.37175279   2.11532033  65.28217818  42.11531064 
##         281         282         283         284         285 
##  17.37175925  81.42328140   3.11531710  14.62820356  53.39751925 
##         286         287         288         289         290 
##  84.53861602  43.76930140  35.99997602  72.28217818  45.37175279 
##         291         292         293         294         295 
##  53.62819279  -0.51290505   3.23065710  34.99997710  52.37175279 
##         296         297         298         299         300 
##  88.65395279  10.57666634  20.62820356  76.16684356  83.16683710 
##         301         302         303         304         305 
##   8.11531925  12.23065495   2.23065602  47.39751710  85.79506033 
##         306         307         308         309         310 
##   1.23065602 106.93616248  57.02573387  31.37175710 -11.02578505 
##         311         312         313         314         315 
##  95.76929387  80.14107172  26.46132849  81.16683818  77.14107387 
##         316         317         318         319         320 
##  60.28217925  96.30794248  48.37174957  57.39751818   2.85887387 
##         321         322         323         324         325 
##  51.14107495  31.97421064  83.16683925  64.91039925  73.51285387 
##         326         327         328         329         330 
##  -7.51290075  28.51286140  43.14107710  19.62819710  31.85886957 
##         331         332         333         334         335 
##  32.74352957  15.62819818  10.99997925  27.99997279  34.85887172 
##         336         337         338         339         340 
##  -9.14112182  58.39751818   7.83310849  -7.14112182  30.88463602 
##         341         342         343         344         345 
##  92.67972140  26.88463495  82.42327818  25.88464033   0.11532033 
##         346         347         348         349         350 
##   6.37176033  38.76929710   2.74353818  33.62819279  28.51285710 
##         351         352         353         354         355 
##  79.79505710  77.91039387  91.05150248   7.83310741  30.25641602 
##         356         357         358         359         360 
##  67.51285279   7.34599172  21.83310741  81.16683710  46.76929602 
##         361         362         363         364         365 
##  60.28218140  35.88464356  80.88462849  51.39752033  20.62819925 
##         366         367         368         369         370 
##  68.28217710  15.60243172   4.74353818  58.02574140 103.30794248 
##         371         372         373         374         375 
##  39.74353172  81.85886849  70.25641172   7.71777172  76.53861925 
##         376         377         378         379         380 
##  85.97420526  13.99997925  94.02573172  49.51285710 108.88462741 
##         381         382         383         384         385 
##  63.37175172  30.62819925   6.48709279  53.62819279   2.71777172
```

```
##         386         387         388         389         390 
##  44.48709172  13.99998140 103.79505818  55.65396356  45.25641387 
##         391         392         393         394         395 
##  30.25642033  45.51285495  16.37175925  60.28218140  82.02573387 
##         396         397         398         399         400 
##  25.62820140 116.76928957  25.37175279 111.99996526  42.08954957 
##         401         402         403         404         405 
##   5.48709602  17.48709710  35.51286033  18.71776849  34.62819925 
##         406         407         408         409         410 
##  76.16684033  32.25642033  14.11531602  58.39751925  34.88464140 
##         411         412         413         414         415 
##  62.76929818  38.48709495  41.62819495  10.37176033  70.53861925 
##         416         417         418         419         420 
##   0.08955172  91.05149925  86.28217925  27.62820033   5.11531495 
##         421         422         423         424         425 
##  22.37176033  93.79505818  86.79506248  67.25641172  67.14107602 
##         426         427         428         429         430 
##   0.23065710  76.39751602  30.51285710  52.02573925  83.16684140 
##         431         432         433         434         435 
##   5.37176248  82.16683602  86.42328356  -4.02578505  79.16683925 
##         436         437         438         439         440 
##  92.67972033  16.37175925  37.76929818  98.42327818  39.25641602 
##         441         442         443         444         445 
##  78.16684033  75.28217602  60.76929495  -4.51290290 105.02573172 
##         446         447         448         449         450 
##  42.14107602  85.79506033  81.74353172  62.34598741  80.53861818 
##         451         452         453         454         455 
##  48.02574033  30.25642033  36.88463602  59.65396033  98.67972033 
##         456         457         458         459         460 
##  43.14107710  55.39751818  65.91039818  63.85886849  43.62819710 
##         461         462         463         464         465 
##  57.28217925 146.28217279  44.23065064  84.42327925  78.76929172 
##         466         467         468         469         470 
##  40.14107925  58.02573925  52.76929495  31.25641925  99.16683602 
##         471         472         473         474         475 
##  77.28217602  84.53861925  30.51286033  16.85887279  51.39751818 
##         476         477         478         479         480 
##  13.11531710  64.99996849  83.16683710  17.62819818  87.05149925 
##         481         482         483         484         485 
##  31.88463818  79.42328248  63.91040356  62.14107387  45.39752248 
##         486         487         488         489         490 
##  55.85887064  27.11531925  61.91040248  52.25641387  52.39751710 
##         491         492         493         494         495 
##  67.91039818  57.76929495  79.16684140  22.99998140  52.02574248 
##         496         497         498         499         500 
##  66.37174741   5.11531925   1.48709495  87.79506140  86.91039495 
##         501         502         503         504         505 
##  41.37175387  83.79505710  20.11531818  33.25641925  60.39751710 
##         506         507         508         509         510 
##  46.76930140   4.85887602 115.79505925  92.65395818  15.60243387 
##         511         512         513         514         515 
##  10.85887602  96.62818741  90.16683602  22.74353710  78.79506248 
##         516         517         518         519         520 
##  -4.39756398  16.37175710  38.88464033  89.42327925  62.51285602 
```

```
##          521          522          523          524          525 
##  75.91040033  49.76930248   2.23065602  80.79505818  43.97420741 
##          526          527          528          529          530 
##   3.11531818  90.28217710 107.02573172  51.14107925  25.74353602 
##          531          532          533          534          535 
##  59.11530741  15.85887710 108.56438356  70.28217710  15.48709495 
##          536          537          538          539          540 
##  86.91039602   5.23065602  57.02574140  86.99996849  43.25641710 
##          541          542          543          544          545 
## 104.51284741  70.53862140   3.48709818  16.37175818  -3.14112505 
##          546          547          548          549          550 
##  59.65395710  70.74352849  61.76929710  -2.39756721  81.79506033 
##          551          552          553          554          555 
##  23.99998033  51.88463495  32.25641602   8.23065602  64.02573495 
##          556          557          558          559          560 
##  58.65396140  48.39752140 104.11530311   4.85887710  80.53861818 
##          561          562          563          564          565 
##  76.76929387  35.51285602  20.37175387  59.48708957  11.74354140 
##          566          567          568          569          570 
##  22.99998140  -0.51290182  48.48709064  13.74353818  28.08954634 
##          571          572          573          574          575 
##  49.97421064   7.23065495  15.11531818  46.51285279  76.91039925 
##          576          577          578          579          580 
##  41.14107818 102.16683710  28.62819818  69.37175064  26.23065172 
##          581          582          583          584          585 
##  79.02573495  68.91040033  17.97421172  14.37175602  24.62819710 
##          586          587          588          589          590 
##  55.62819495  18.37175710  15.74353925   8.48709710  21.23065172 
##          591          592          593          594          595 
##  66.28218033  56.02573925  29.99997602  46.99997495  79.79506248 
##          596          597          598          599          600 
##  51.51285710  44.25641387  16.99998356  18.11531602  73.65395602 
##          601          602          603          604          605 
##  33.46133064   8.74353710  20.11531710  50.76929387  29.57666526 
##          606          607          608          609          610 
##   2.85887602  -0.51290075  -3.65400613  66.14107495   8.48709710 
##          611          612          613          614          615 
##  87.42327925  -2.02578505  41.88463602 101.30794140  40.11531172 
##          616          617          618          619          620 
## 102.28217495  97.91039387  25.85887172  32.62820033  99.42327925 
##          621          622          623          624          625 
##  32.99997602  79.53861602  25.83310957  75.62819172  75.02573387 
##          626          627          628          629          630 
##  46.14107818  68.76929602  78.53862140  36.11531172  24.60243495 
##          631          632          633          634          635 
##  13.37176140 106.91039495  29.25641925   2.85887818  22.62820033 
##          636          637          638          639          640 
## 133.62818634  99.30794356 -10.28222721  60.39752140  63.39751710 
##          641          642          643          644          645 
##  78.16683710  27.99997818 -14.28222721  92.91039602  82.28217710 
##          646          647          648          649          650 
##  17.60243495 108.79505710  21.62819925  86.99996634  87.05150140 
##          651          652          653          654          655 
##  64.85886849  47.62819495  80.11530634  31.62819925  84.53861602 
```

```
##         656         657         658         659         660 
##  50.14107710  32.71777172  26.62819710  75.28217818  22.99998140 
##         661         662         663         664         665 
##  19.85887495  80.39751387 109.62818741  59.51285279  44.14108140 
##         666         667         668         669         670 
##  55.11531064  77.65395495  53.14107925  35.37175495  81.91039925 
##         671         672         673         674         675 
##  25.88464248   7.97421387   5.48709710  92.16683818 -14.39757043 
##         676         677         678         679         680 
##  44.62819495  68.76929710  87.91039495  16.99997602 100.91039387 
##         681         682         683         684         685 
##  73.88462957  40.88463925  83.91039710  30.88464033  58.65395925 
##         686         687         688         689         690 
##  17.74354140  32.51286248  19.62820140  96.39751279  33.14108248 
##         691         692         693         694         695 
##  98.67972033  66.99997064  33.14108140  91.53861818  61.39751925 
##         696         697         698         699         700 
##  39.14107925  64.99997279  94.30794356  24.99997495  78.76929279 
##         701         702         703         704         705 
##  71.91040140  35.46133064  21.71777064  47.02573818  65.91040033 
##         706         707         708         709         710 
##  83.79505818  57.28217710  20.08955172  31.37175925  80.79506033 
##         711         712         713         714         715 
##  92.53861602 122.88462634  36.74353387  -1.14112828  26.88463710 
##         716         717         718         719         720 
##  82.79505925  44.37175495  95.67972140  89.39751602  24.88464248 
##         721         722         723         724         725 
##  56.14107602  30.88464356  66.14107387  96.28217279  62.28217818 
##         726         727         728         729         730 
##  97.05149925  37.14108356  34.62819818  27.62819602  33.88463925 
##         731         732         733         734         735 
##  98.53861602  18.11531602  74.16684356  83.42328140  64.28217818 
##         736         737         738         739         740 
##  20.85887602  35.51285818  75.16684248   3.74353710  48.14108033 
##         741         742         743         744         745 
##   2.23065710  78.88463064  68.28217710  12.85887602  88.14107279 
##         746         747         748         749         750 
##  76.76929064  80.16683925  -9.76934290  22.62819818  81.42328356 
##         751         752         753         754         755 
##  61.65396140  82.88462957  58.83310311  14.23065387  38.62819818 
##         756         757         758         759         760 
##  55.65395925  28.71776849  44.34598957   5.34599172  43.14108248 
##         761         762         763         764         765 
##   3.11532033  22.62819818   9.99998248  79.25641172  57.11530957 
##         766         767         768         769         770 
##  42.88463818  62.28218248  49.14107925   0.85887602  88.67972356 
##         771         772         773         774         775 
##  70.25641172  25.25642356  20.99998033  20.99998033 109.93616356 
##         776         777         778         779         780 
##  88.67972248  16.99997925  56.28218033   2.74354140  44.39752356 
##         781         782         783         784         785 
##  48.14107818  18.11531602  24.85887279  72.91040033  91.02573172 
##         786         787         788         789         790 
##  94.16683602  13.46132957  25.97421279  89.16683818 123.28217279 
```

```
##         791         792         793         794         795
##  71.53862248  -5.39756613  43.76930248  15.99998356  45.51285925
##         796         797         798         799         800
##   0.48709818  83.91039925  50.88463818   9.74354033  82.28218033
##         801         802         803         804         805
##  77.65395279  97.16683818  40.76929818  43.88463925  72.02573818
##         806         807         808         809         810
##  49.76930248  40.97421387  50.14107602  36.51286033   4.23065710
##         811         812         813         814         815
##  15.23065710  30.48709064  88.42327818  61.91040033  33.48709710
##         816         817         818         819         820
##  74.16684356  71.39751818  25.37175495  11.48709710  89.67972248
##         821         822         823         824         825
##  82.39751387  63.14107602 110.93616248  80.16684033  77.16683925
##         826         827         828         829         830
##  21.25642356  52.02573495  29.62819925  15.48709925  20.99997710
##         831         832         833         834         835
##  51.14108140  38.14108248  31.88464140  67.53862356  33.51286248
##         836         837         838         839         840
##  12.37176248  15.11531602  81.02573495  25.48709172  36.37175279
##         841         842         843         844         845
##  88.91039495  38.99997925  17.48709279  88.39751064  79.16684140
##         846         847         848         849         850
##  20.62820248  41.48709495  68.65395818  53.99997279 118.02573172
##         851         852         853         854         855
## 103.93616248   8.60243602  82.14107064  89.65395495  97.79505818
##         856         857         858         859         860
## 103.56438356  95.28217602  33.51286248  12.37176033  44.76930033
##         861         862         863         864         865
##  18.62820356 111.16683602  50.14108248  91.67972140  -1.02578505
##         866         867         868         869         870
##  26.37175495  89.05150140  52.14108140  22.46132526  81.08954203
##         871         872         873         874         875
##  15.37176140  31.99997818  12.85887387   8.11531925  98.30794356
##         876         877         878         879         880
##  84.79506033  78.53862140  25.37175818  54.14107495  78.91039925
##         881         882         883         884         885
##  13.37176140  21.25642140  72.65395279  75.16684140  54.74353064
##         886         887         888         889         890
##  85.53861818  21.11531710 -15.02578721  47.37175279  70.99996849
##         891         892         893         894         895
##  46.51285495  62.65396140   5.20488741  71.28218033 103.93616248
##         896         897         898         899         900
##  96.05149925  41.60243387  68.14107818 106.91039387  37.62820356
##         901         902         903         904         905
##  22.37175818  16.85887279  23.85887387  30.99997818  57.65396140
##         906         907         908         909         910
##  50.88464140  61.76930140 -16.28222936  85.79506033   1.34599172
##         911         912         913         914         915
##  26.85887710  69.53861710  22.11531925  92.16683925 114.91039387
##         916         917         918         919         920
##  39.99997172  23.37176140   6.85887602  55.65396356  50.20488634
##         921         922         923         924         925
##  71.74352957 114.39751279  96.42328033  94.30794248  34.60243172
```

```
##          926          927          928          929          930
##  31.37175818 -10.39756613  33.23064849  26.11531602  52.14107495
##          931          932          933          934          935
##  27.99997818 106.30794140 100.91039710  74.28217710  46.94844634
##          936          937          938          939          940
##  59.08954849  29.25642356 104.39751064  39.76930140 117.16683602
##          941          942          943          944          945
##  56.65396033  68.65395495  94.79505710  69.48708849  22.37175495
##          946          947          948          949          950
##  54.39751710  63.65396140  53.11530741  62.28217602  72.02573602
##          951          952          953          954          955
##  18.11531818  82.62818741  15.48709818  39.51286248  73.91040356
##          956          957          958          959          960
##  31.99998033  22.37175602  40.74353387  30.99997818  92.02573602
##          961          962          963          964          965
##  40.51285818  92.91039495  24.62820033  36.88464248  88.76928849
##          966          967          968          969          970
##  72.28218248  53.88463495  33.88464140  -7.39756505  26.48708849
##          971          972          973          974          975
##  69.53861925  60.11531064  64.65395602   2.60243387  -5.39756613
##          976          977          978          979          980
##  36.62820140  37.74353925  81.79505925  24.88464033 -21.65400613
##          981          982          983          984          985
##  90.14107279  58.02573602  99.91039387  26.62820033  58.65395818
##          986          987          988          989          990
##  39.14108140   9.34599064  13.11531495  67.53862356  28.37175602
##          991          992          993          994          995
##  31.88464033 101.05150033  83.51285387  40.14108356  74.79506140
##          996          997          998          999         1000
##  40.51285925  62.76929602  38.62819818   0.11531818 118.99996203
```

```r
res_squared = (m1$res)^2
sum(res_squared)
```

```
## [1] 131999.6
```

```r
sum(res_squared)/998
```

```
## [1] 132.2641
```

$$\hat{\sigma}^2 = RSS/n - 2$$

=

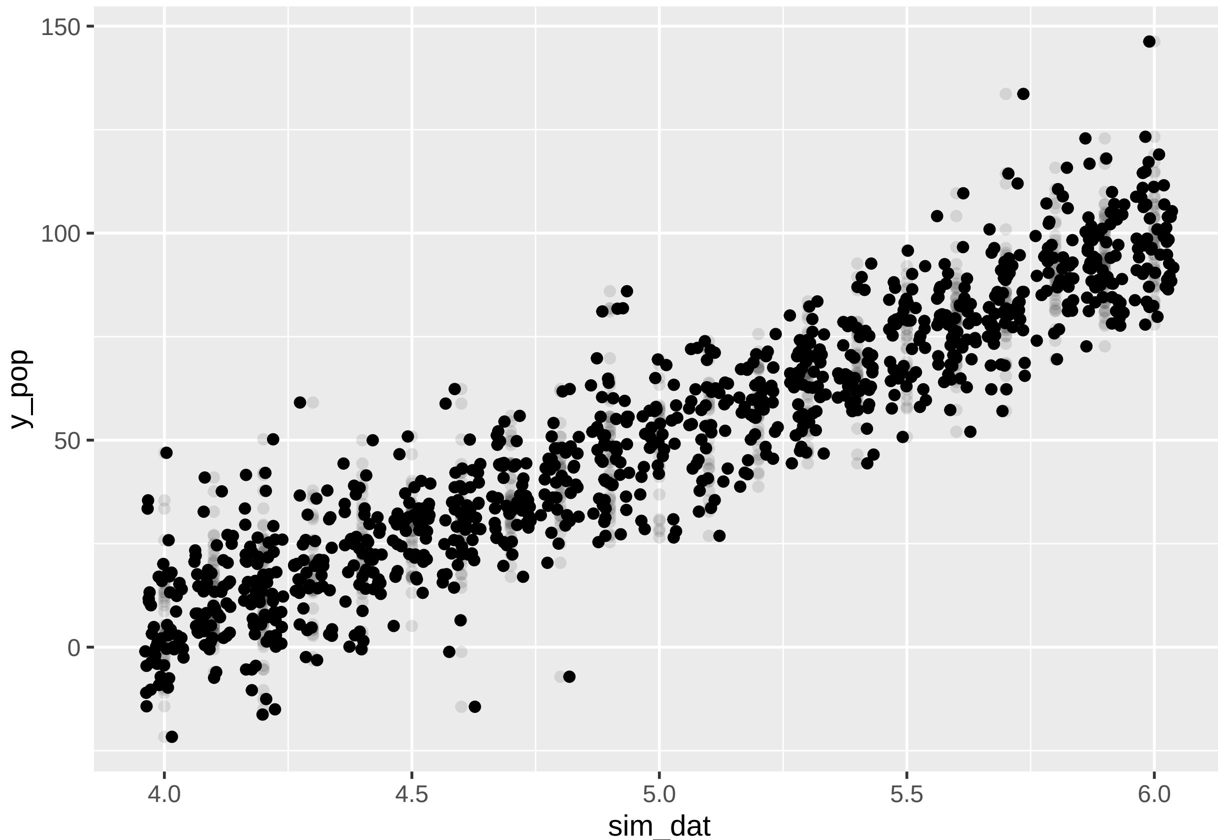
$$\hat{\sigma}^2 = 131999.6/998$$

= =

$$\hat{\sigma}^2 = 132.2641$$

Exercise 12

```r
ggplot(quakes, aes(sim_dat, y_pop)) + geom_point(alpha = 0.1) + geom_jitter()
```




```r
m2 = lm(y_pop ~ sim_dat, data = quakes)
summary(m2)
```

```
##
## Call:
## lm(formula = y_pop ~ sim_dat, data = quakes)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -48.820  -7.090  -0.380   6.807  50.051
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -181.7793     2.9176  -62.30   <2e-16 ***
## sim_dat       46.5538     0.5803   80.22   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.5 on 998 degrees of freedom
## Multiple R-squared:  0.8657, Adjusted R-squared:  0.8656
## F-statistic:  6435 on 1 and 998 DF,  p-value: < 2.2e-16
```

As with the original data, there is a strong, positive, and linear relationship between the magnitude of an earthquake and number of stations detecting it in the simulated data. However, the intercept is higher, and the slope is a little bit lower, suggesting that for every unit increase in the magnitude of an earthquake on the Richter scale, the number of stations that detect it increases by 45.6. To increase my model, I might find an intercept that lies around the mean of -180.4243 and -177.0066 and a slope that lies betwen mean of 45.6 (the slope of the simulated data) and the original slope of 46.2822. #### PROBLEM SET 2: #### Exercise 1

1)

$$H_0$$

: While holding all other predictors constant, the amount of spending on TV advertising has no effect on sales.

The model suggests that the relationship between the amount of spending on TV advertising and sales is statistically significant, allowing us to reject the null hypthesis ($p < 0.0001$). Spending an additional $1000 on TV advertising leads to an increase in sales by approzimately 46 units.

2)

$$H_0$$

: While holding all other predictors constant, the amount of spending on radio advertising has no effect on sales.

The model also suggests that relationship between the amount of spending on radio advertising and sales is statistically significant, allowing us to reject the null hypothesis ($p < 0.0001$). Spending an additional $1000 on radio advertising leads to an increases in sales by approximately 189 units.

3)

$$H_0$$

: While holding all other predictors constant, the amount of spending on newspaper advertising has no effect on sales.

While the model reports that spending an additional $1000 on newspaper advertising may lead to a decrease in sales by one unit, this relationship is not statistically significant ($p = 0.8599$).

When there is no spending on either TV, radio, or newspaper advertising the model makes the statistically significant prediction that the mean value of sales would be 2.939 units ($p < 0.0001$).

Exercise 4

a) Despite the fact that the true relationship is linear, I would expect the training RSS of the cubic regression to be lower because its greater flexibility gives it the capacity to fit greater variance in the training data.

b) In this case, the greater flexibility cubic regression model will overfit the training data whereas the greater bias of the linear regression model is mediated by the true linear relationship. Consequently I would expect the test RSS of the linear regression to be lower.

c) As in a), I would expect the training RSS of the cubic regression to be lower because its greater flexibility allows it to encompass the greater variance in training data regardless of the true relationship between X and Y.

d) There is not enough information to tell because we do not know the linearity of the true relationship between X and Y. If the true relationship is closer to linear, the linear regression model would be a better fit with a lower test RSS. However, if the relationship is extremely non-linear then it is possible that the greater flexibility of the cubic regression model wll be a better fit and yield a lower RSS. #### Exercise 5

1. If we are given the form of the ith fitted value as:

$$\hat{y} = x_i\hat{B}$$

where:

$$\hat{B} = (\sum_{i=1}^{n} x_i y_i)/(\sum_{i'=1}^{n} x_{i'}^2)$$

2. Then, after substitution:

$$\hat{y} = (x_i \sum_{i=1}^{n} x_{i'} y_{i'})/(\sum_{i"=1}^{n} x_{i"}^2)$$

3. We were also given that:

$$\hat{y} = \sum_{i'=1}^{n} a_{i'} y_{i'}$$

,

So:

$$\hat{y} = \sum_{i'=1}^{n} a_{i'} y_{i'} = (x_i \sum_{i=1}^{n} x_{i'} y_{i'})/(\sum_{i"=1}^{n} x_{i"}^2)$$

=

$$a_{i'} y_{i'} = x_i x_{i'} y_{i'} / \sum_{i"=1}^{n} x_{i"}^2$$

=

$$a_i = x_i x_{i'} / \sum_{i"=1}^{n} x_{i"}^2$$

ADDITIONAL EXERCISES

Exercise 1

1) Variance:

a) We know that k-nearest regression is defined as:

$$[\hat{f}(x) = \frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} y_i]$$

b) To find the variance:

$$Var(\hat{f}(x)) = Var \frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} y_i$$

=

$$Var(\hat{f}(x)) = \frac{1}{k^2} Var(\sum_{x_i \in \mathcal{N}(x)} y_i)$$

=

$$Var(\hat{f}(x)) = \frac{1}{k^2} \sum_{x_i \in \mathcal{N}(x)} Var(y_i)$$

=

$$Var(\hat{f}(x)) = \frac{1}{k^2} \sum_{x_i \in \mathcal{N}(x)} (\sigma^2)$$

=

$$Var(\hat{f}(x)) = \frac{1}{k^2} k(\sigma^2)$$

=

$$Var(\hat{f}(x)) = \frac{k}{k^2} (\sigma^2)$$

=

$$Var(\hat{f}(x)) = \frac{\sigma^2}{k}$$

2) Bias

a) We know that k-nearest regression is defined as:

$$[\hat{f}(x) = \frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} y_i]$$

b)

$$[Bias(\hat{f}(x))]^2 = [f(x) - E[\hat{f}(x)]]^2$$

=

$$[Bias(\hat{f}(x))]^2 = [f(x) - E[\frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} y_i]^2$$

=

$$[Bias(\hat{f}(x))]^2 = [f(x) - [\frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} E(y_i)]^2$$

=

$$[Bias(\hat{f}(x))]^2 = [f(x) - [\frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} f(x_i)]^2$$

2) Bias

a) We know that k-nearest regression is defined as:

$$[\hat{f}(x) = \frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} y_i]$$

b)

$$[Bias(\hat{f}(x))]^2 = [f(x) - E[\hat{f}(x)]]^2$$

=

$$[Bias(\hat{f}(x))]^2 = [f(x) - E[\frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} y_i]^2$$

=

$$[Bias(\hat{f}(x))]^2 = [f(x) - [\frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} E(y_i)]^2$$

=

$$[Bias(\hat{f}(x))]^2 = [f(x) - [\frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} f(x_i)]^2$$

c) Decomposition of MSE:

$$MSE = Var(f(x)) + Bias(\hat{f}(x))^2 + Var(\hat{f}x)$$

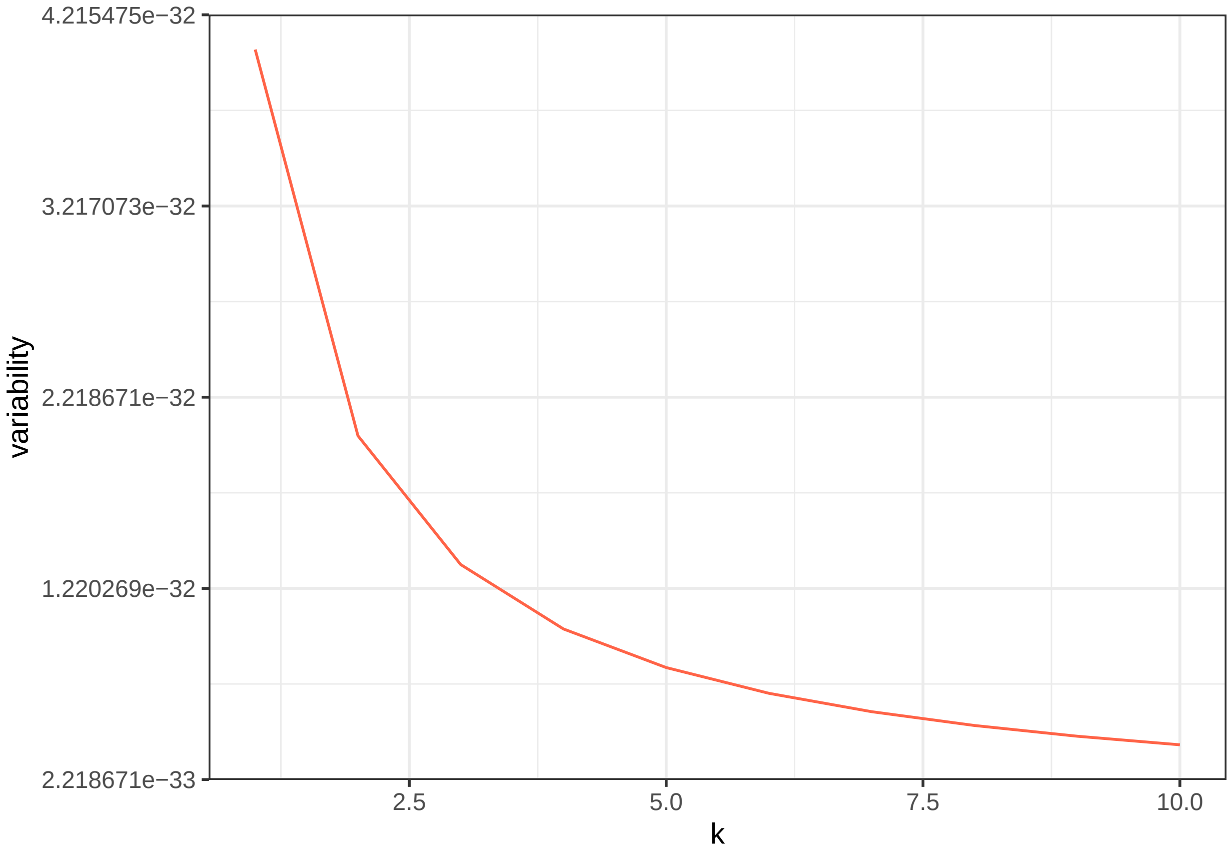
=

$$MSE = Var(f(x)) + [f(x) - [\frac{1}{k} \sum_{x_i \in \mathcal{N}(x)} f(x_i)]^2 + \frac{\sigma^2}{k}$$

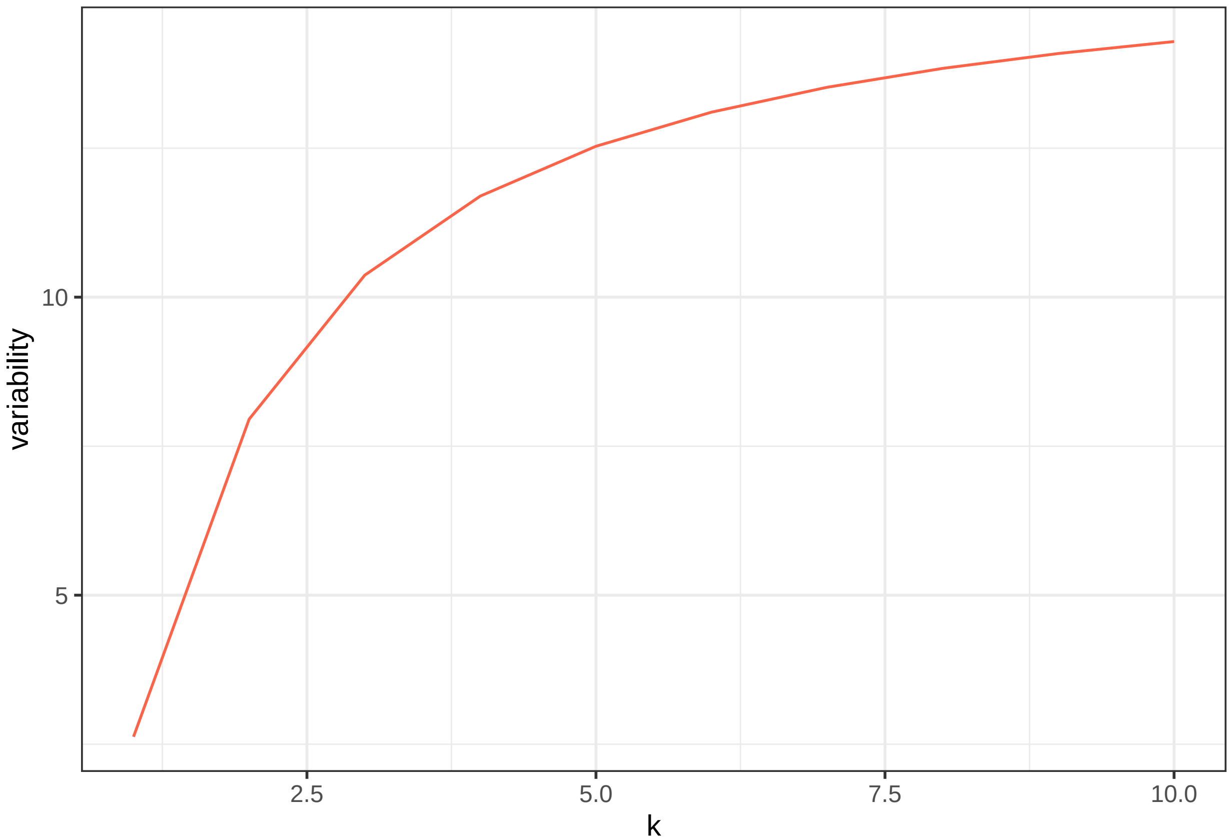
Exercise 2

```r
library(tidyverse)
x <- c(1:3, 5:12)
y <- c(-7.1, -7.1, .5, -3.6, -2, -1.7, -4, -.2, -1.2, -1.2, -3.5)
y_mean = mean(y)
```

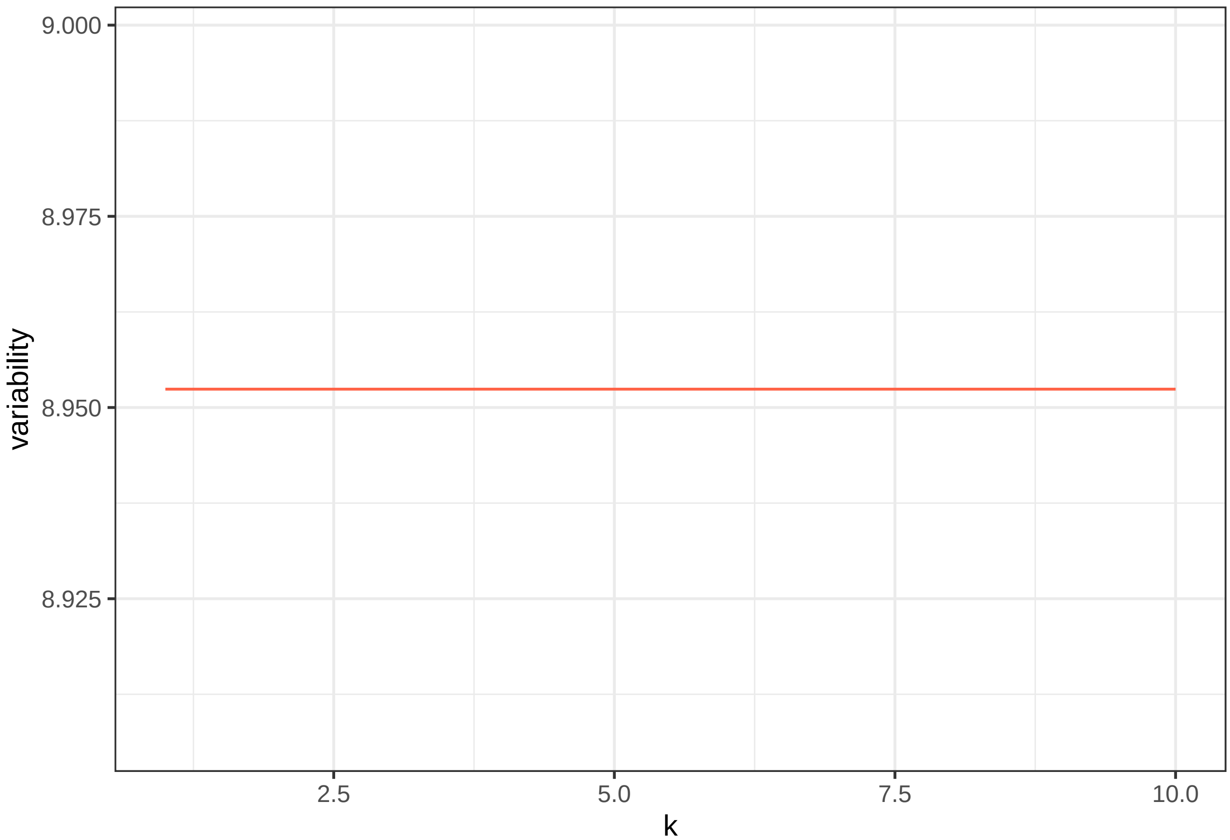
```r
std_dev <- function(y) {(sum(y-y_mean)^2)/11}
std_dev <- std_dev(y)



var_fun <- function(k, x, y) {
  v_k <- rep(NA, length(k))
  for (i in 1:length(k)) {
    v_k[i] <- std_dev/k[i]
  }
  v_k
}

f <- function(x) {
  f = -9.3 + 2.6 * x - 0.3 * x^2 + .01 * x^3
}
bias_fun <- function(k, x, y) {
  b_k <- rep(NA, length(k))
  for (i in 1:length(k)) {
    b_k[i] <- (-4.02 - (1/k[i]*-2.4))^2
  }
  b_k
}


MSE_fun <- function(k, x, y) {
  m_k <- rep(NA, length(k))
  for (i in 1:length(k)) {
    m_k[i] <- 7.9524 + 1 + std_dev/k[i]
  }
  m_k
}

k <- 1:10
b_k <- bias_fun(k, x, y)

k <- 1:10
m_k <- MSE_fun(k, x, y)



k <- 1:10
v_k <- var_fun(k, x, y)

df <- tibble(k = k, v_k = v_k)
ggplot(df, aes(x = k, y = v_k)) + geom_line (col = "tomato") +
  theme_bw() +
  ylab("variability")
```




```r
df <- tibble(k = k, b_k = b_k)
ggplot(df, aes(x = k, y = b_k)) + geom_line (col = "tomato") +
  theme_bw() +
  ylab("variability")
```




```
df <- tibble(k = k, m_k = m_k)
ggplot(df, aes(x = k, y = m_k)) + geom_line (col = "tomato") +
  theme_bw() +
  ylab("variability")
```



Exercise 7

Exercise 7

Exercise 7

Exercise 7

Exercise 7