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Hansen, Asger Bendix; Carstensen, Stefan

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EXPERIMENTAL INVESTIGATION OF THE PERFORMANCE OF TILT CURRENT METERS IN WAVE-DOMINATED FLOWS

ASGER BENDIX HANSEN(1) & STEFAN CARSTENSEN(2)

(1) Fluid Mechanics, Coastal and Maritime Engineering, Department of Mechanical Engineering, Technical University of Denmark,

e-mail asbha@mek.dtu.dk

(2) Fluid Mechanics, Coastal and Maritime Engineering, Department of Mechanical Engineering, Technical University of Denmark,

e-mail scar@mek.dtu.dk

ABSTRACT

In recent years, tilt current meters (TCMs) have received renewed attention as they provide an inexpensive method for measuring currents in the coastal zone. However, previous studies have focused mainly on current dominated flows or the current component of the flow. This study investigates the performance of tilt current meters in wave-dominated flows and capturing the wave orbital velocities. A series of laboratory experiments have been performed in which tilt current meters were used to measure flow velocities in pure current, pure wave and combined wave-current flows. Both spherical and cylindrical TCMs have been investigated in order to assess the effect of TCM shape on its performance. The measured TCM tilt is compared with the flow velocity measured by conventional methods. Furthermore, the ability of a TCM to measure wave orbital and wave-averaged velocities is discussed.

Keywords: tilt current meter, observational techniques, surf zone, coastal hydrodynamics

1 INTRODUCTION

The tilt current meter (TCM) is an old measurement technique, which uses the tilt of a tethered object as an indicator for measuring current speed in water. Although the technique had been replaced by more advanced and accurate methods such as acoustic Doppler velocimetry, it is now again receiving attention as it provides an inexpensive alternative to the costly acoustic techniques. Due to the availability of low cost accelerometers, magnetometers and microcontrollers, it is now possible to construct a TCM for a cost well below 100 USD. Since TCMs are also easy to deploy they allow researchers to obtain velocity measurements with a larger spatial resolution than what would have been possible using acoustic instruments.

With this as the motivation, several studies have recently studied the performance of TCMs in different coastal flows (Figurski et. al. 2011; Lowell et. al. 2015; Marchant et. al. 2014, Radermacher et. al. 2015). The flows considered have mainly been tidal flows with current speeds smaller than 1 m/s. One exception is Figurski et al. (2011) who used a TCM to estimate wave conditions by looking at the standard deviation of the TCM tilt. Results from previous studies show that TCMs can deliver quite good accuracies (0.05 m/s, Marchant et. al (2014) or 2% Lowell et. al. (2015)). These results encourage the further development of the use of TCMs so that they might be applied to measure under a wider range of flow conditions.

We are interested in extending the use of TCMs so that they may also be used to measure velocities in and around the surf zone. The surf zone presents a very challenging environment with very large flow velocities, unsteady flow due to wave orbital motion and significant turbulence. In order to be able to measure such flows it is necessary to develop a TCM with a measurement range much larger than the previous ones and which performs well even in unsteady flow.

This paper presents an experimental study of the behavior of spherical and cylindrical TCMs. The objective of this study is two-fold. First, we examine how varying the proportions of a cylindrical TCM affects the relationship between tilt angle and flow velocity in order to optimize the measurement range. Secondly, we wish to examine how well a given TCM performs in unsteady flows with either pure wave motion or combined waves and currents.

2 MEASUREMENT PRINCIPLE

A TCM estimates the flow speed (U) from the measured tilt angle (θ). This requires that the relationship between tilt angle and flow speed (here referred to as the response curve) is known. A theoretical response curve can be determined by considering the force balance in the angular direction (see Figure 1) between the in-line force (F_x) and the net buoyancy force (F_B) acting on the TCM.

$$\tan\theta = \frac{F_{\chi}}{F_{R}} \tag{1}$$

The net buoyancy force can be expressed as

$$F_B = g(\rho V - m) \tag{2}$$

where ρ denotes the density of the water, g the acceleration of gravity, V the volume and m the mass of the floatation body.

In steady current, the in-line force is solely due to drag. The drag force on a spherical TCM exposed to steady current is from a geometrical perspective insensitive to the tilt angle and can be expressed as

$$F_{x} = F_{D} = \frac{1}{2}\rho C_{D}AU^{2}$$
 [3]

Here, C_D is the drag coefficient and A is the frontal projection area of the floatation body. Inserting the expressions for the forces ([2] and [3]) into equation [1] and solving for U gives

$$U = k\sqrt{\tan\theta} \tag{4}$$

where k is a function of the TCM properties (essentially mass and diameter) as well as the drag coefficient

$$k = \sqrt{\frac{2g(\rho V - m)}{\rho C_D A}}$$
 [5]

Hence, the shape of the response curve is the same for all spherical TCMs but the steepness will increase with increasing k. Although the theoretical response curve goes to infinity when the tilt angle approaches 90°, in practice a TCM will only give useful results up to a certain angle that then defines the measurement range of the TCM. Equations [4] and [5] thus show that the larger and lighter a TCM is the larger the measurement range.

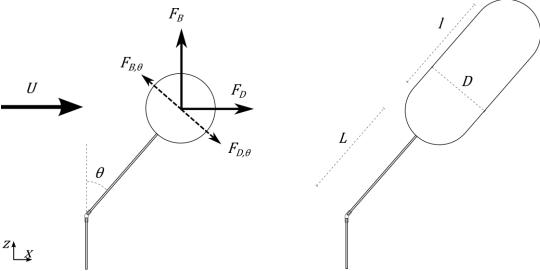


Figure 1. Definition sketch. Left: Spherical TCM with indication of current speed, tilt angle and forces on the floatation body. Subscript θ indicates projection onto the angular direction. Right: Cylindrical TCM with indication of tether length, cylinder length and diameter.

The drag coefficient will generally vary considerably with Reynolds number (Re = DU/v) but in the range $1 \cdot 10^3 < Re < 2 \cdot 10^5$ the drag coefficient for a smooth sphere assumes an approximately constant value between 0.4 and 0.5 (Schlichting and Gersten 2017). A gradual variation in the drag coefficient would affect the tilt response but this can be accounted for and hence does not pose a problem for the flow measurement. However, an abrupt change in the drag coefficient as experienced for a stationary sphere at the drag crisis would result in a vertical step in the response curve. This discontinuity will cause a range of velocities to give the same tilt angle and hence be undistinguishable by the TCM. When designing a TCM it is therefore important to ensure that the drag crisis occurs outside of the desired measurement range.

In an unsteady flow the relationship between flow speed and tilt angle becomes considerably more complicated since inertia forces can potentially be of importance, the TCM itself is moving and the natural frequency of the TCM may coincide with frequencies in the flow causing resonance. The combined drag and inertia forces are described by the Morison equation (Sumer and Fredsøe, 2006) and depend on both the flow velocity and acceleration relative to the TCM. The relative importance of the drag force and inertia forces can be described by the Keulegan-Carpenter number

$$KC = \frac{U_m T}{D}$$
 [6]

where U_m is the maximum wave orbital velocity, T is the wave period and D is the diameter of the body. The drag force becomes increasingly important and the inertia forces less so as KC is increased. For example, a fixed cylinder may be considered drag-dominated for KC numbers larger than 20-30 (Sumer and Fredsøe, 2006). This suggests that inertia may be neglected as long as the TCM is sufficiently small.

The natural frequency of the TCM may be calculated by considering the TCM as an oscillator damped by viscous friction and inertia forces (Sumer and Fredsøe, 2006). Hence, the natural frequency is given as

$$f_n = \frac{1}{2\pi} \sqrt{\frac{g(\rho V - m)}{L(m + m')}}$$
 [7]

where L is the tether length, $m' = C_m \rho V$ is the hydrodynamic mass of the TCM and C_m (= ½ for a sphere, hence ½ $\leq C_m < 1$ for $l \geq 0$ in an ideal fluid) is the hydrodynamic mass coefficient. If the TCM is exposed to wave motion with frequencies close to its resonant frequency it may resonate which could result in a frequency dependent response. To avoid this, the TCM should be chosen to have a resonant frequency that is significantly larger than the frequencies of the waves.

When choosing a TCM for measuring currents and wave orbital motion in the surf zone one is faced with a number of challenges: (1) The measurement range should be relatively high, reaching several meters per second; (2) The *KC* numbers should be large; (3) The drag coefficient should not experience abrupt change; and (4) The natural frequency should be considerably higher than the wave frequency.

One way of increasing the achievable measurement range of a TCM is to make it cylindrical. In this case, the angle of attack of the flow on the floatation body changes with the tilt angle. An analytical expression for the tilt response of a cylindrical can be obtained by applying the cross flow principle (Sumer and Fredsøe, 2006) according to which the drag force in the angular direction should be calculated by only considering the component of the flow which is orthogonal to the cylinder. This gives the expression

$$F_{D,\theta} = \frac{1}{2} \rho C_D(\cos \theta \, U)^2 A \tag{8}$$

For the cylindrical TCM, the theoretical expression for the response curve then becomes

$$U = k \sqrt{\frac{\tan \theta}{\cos \theta}} \tag{9}$$

where k is given by the same expression as for the sphere but with the volume and frontal projection area relevant to the cylinder. It is important to note that A is the frontal projection of the floatation body when the TCM is in the vertical position. Due to the factor $\cos \theta$ in the denominator of equation

[9], the response curve for a cylinder becomes considerably steeper than for a sphere with the same value of k thus giving a larger measuring range. The drag coefficient, however, will be different for the cylinder. For Reynolds numbers in the range $1 \cdot 10^3$ to $3 \cdot 10^5$ an infinitely long smooth cylinder has a drag coefficient between 1.0 and 1.3 (Schlichting and Gersten, 2017). This suggest that the drag coefficient increases and causes a reduced impact of the change in geometry.

3 EXPERIMENTAL SETUP

The experiments were carried out in the hydraulic laboratory at the Technical University of Denmark (DTU) in a 3 m wide, 1 m deep and 35 m long current flume. The flume is equipped with a carriage mounted on rails, which run along the length of the flume. The TCMs were mounted on a streamlined frame that was attached to the carriage. For low velocities (less than $1.4 \, \text{m/s}$), constant currents were simulated by towing the TCMs through still water. Higher velocities were simulated by generating a current in the flume and then towing the TCMs against the current. By utilizing both the movement of the carriage as well as the flow in the flume, it was possible to test the TCMs in current speeds up to approximately 2 m/s. The duration of each of the steady current tests was 20 seconds or more depending on towing speed.

The TCMs consisted of a hollow plastic floatation body attached to the end of a tether made from a carbon fiber tube (2 mm outer diameter). The tether was attached to an instrument box through a joint made from silicone tube. Tilt angle and direction were measured with a 3-axis accelerometer placed inside the floatation body and logged at the highest frequency possible (approximately 20 Hz). The flow velocity relative to the TCM was measured using a combination of an Armfield H33 propeller velocimeter mounted on the carriage and a laser distance meter, which recorded the position of the carriage at a frequency of 25 Hz.

A total of 1 spherical and 12 "cylindrical" TCMs were tested in the experiments (see Table 1). The floatation bodies of the cylindrical TCMs were circular cylinders with semispherical endings (see Figure 1). The properties of the cylindrical TCMs were varied by varying the tether length and the cylinder length. Three tether lengths (L) and four cylinder lengths (l) were tested. The spherical TCM was included mainly as a reference since it can be viewed as a cylinder with l=0.

Tilt angles were determined by comparing the instantaneous acceleration vector to the acceleration vector measured when the TCM was at rest in the vertical position. Comparing the vertical acceleration vectors between consecutive tests showed changes in direction of a few degrees in the direction of the flow in the preceding test. This is interpreted as a symptom of some stiffness in the joint.

Table 1. Properties of the tested TCMs. The optimal cylindrical TCM has been highlighted. $m^* = m/\rho V$ denotes the dimensionless mass of the floatation body. The small difference in diameter, and hence KC numbers, between the sphere and cylindrical TCMs has no practical significance for the measurement range.

Type	<i>L</i> [mm]	l [mm]	<i>D</i> [mm]	<i>V</i> [cm ³]	A [cm ³]	m [g]	k	m*	f_n [s ⁻¹]
	[111111]	[111111]	[111111]	[cm·]	[CIII*]	LěJ			I ₂ I
Sphere	15	0	30	14	7	5	0.44	0.35	1.11
Cylinder	50	25	32	37	16	22	0.38	0.60	1.30
Cylinder	100	25	32	37	16	22	0.37	0.60	0.94
Cylinder	150	25	32	37	16	22	0.36	0.60	0.77
Cylinder	50	50	32	57	24	28	0.43	0.49	1.42
Cylinder	100	50	32	57	24	28	0.42	0.49	1.06
Cylinder	150	50	32	57	24	28	0.41	0.49	0.88
Cylinder	50	100	32	98	40	41	0.46	0.42	1.38
Cylinder	100	100	32	98	40	41	0.45	0.42	1.10
Cylinder	150	100	32	98	40	41	0.44	0.42	0.93
Cylinder	50	150	32	138	56	53	0.47	0.39	1.28
Cylinder	100	150	32	138	56	53	0.46	0.39	1.05
Cylinder	150	150	32	138	56	53	0.46	0.39	0.92

4 RESULTS

4.1 Steady current

The spherical TCM was first tested in steady current to document the presented measurement principle and serve as a reference when comparing to the different cylindrical TCMs. The measured tilt response is shown in Figure 2 (left panel). The right panel shows the apparent drag coefficient calculated using equation [4]. The apparent drag coefficient of approximately 1 that was observed for tilt angles between 10 and 60 degrees is larger than the value of approximately 0.5 normally found for a fixed smooth sphere. This agrees well with observations made by Williamson and Govardhan (1997) who found a 100% increase in drag coefficient due to vortex-induced oscillations of the sphere. In the present case, the drop in apparent drag coefficient observed at very small tilt angles was found to coincide with a reduction of the amplitude of the vortex induced oscillations. Despite this, there is a good agreement between the theoretical expression ([4]) with $C_D = 1$ and the measured tilt response for tilt angles up to 60° .

For larger velocities, the tilt angle continued to increase approximately linearly. Quite remarkably, this steady increase continued up to tilt angles larger than 100° corresponding to the TCM pointing slightly downwards. This behavior cannot be explained by the analytical framework presented above. However, the present results suggest that the measurement range of spherical TCMs effectively extends up to tilt angles of 100° .

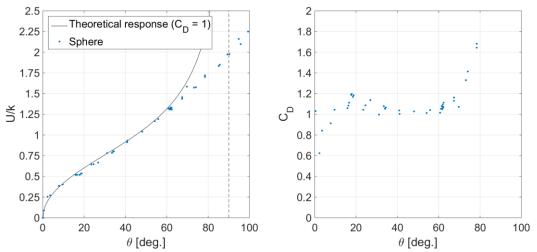


Figure 2. Results from the experiments with the spherical TCM in steady current. Left: Tilt response shown together with theoretical response curve ([4]) for $C_D = 1$. Right: Apparent drag coefficients.

Figure 3 shows the measured tilt responses for the tested cylindrical TCMs. Each panel shows TCMs with the same cylinder length but with different tether lengths. Included in each panel is also the theoretical response curve (

[9]) corresponding to C_D = 1.5. This facilitates easy comparison between panels. The results show that the tether length has little to no effect on the tilt response. Only panel a (l = 2.5 cm) shows some effect of the tether length namely that increasing the tether length results in a flattening of the response curve (increased drag coefficient).

Comparing the four panels (a to d) in Figure 3 shows the effect of extending the length of the cylinder. The shortest cylinder with $l=2.5\,\mathrm{cm}$ (panel a) has a tilt response similar to that of the spherical TCM in the sense that it is relatively flat for tilt angles larger than 60° . As the cylinder length increases, the general tendency is for the response curve to become steeper for tilt angles larger than 60° and flatter for tilt angles smaller than 60° . The behavior for tilt angles less than 60° could be expected since the drag coefficient for a cylinder is larger than for a sphere.

At very large tilt angles, some of the cylinders show a "drop-off" where the tilt response suddenly increases dramatically. This suggests a sudden increase in the apparent drag coefficient similar to what was seen for the spherical TCM at tilt angles larger than 60°. Although this is not necessarily a problem for the velocity measurement, it does suggest that one should be cautious if using the TCM to measure in this velocity range.

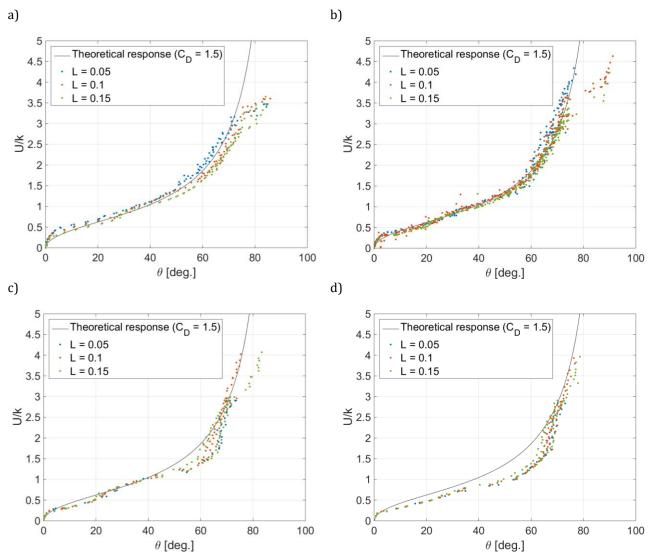


Figure 3. Results of tests with cylindrical TCMs exposed to steady current. Left: Tilt response including analytical response curve ([9]) for $C_D = 1.5$. a) l = 25 mm, b) l = 50 mm, c) l = 100 mm, d) l = 150 mm.

The shape of the response curve of the long cylinders is undesirable as it gives a very high accuracy for a small range of velocities but a very poor accuracy in the rest of the measurement range. In fact, Lowell et al. (2015) who used a very long cylindrical TCM reported that for tilt angles larger than 70° they were unable to detect changes in the current speed. Thus, there is a clear trade-off when selecting the cylinder length; spheres or very short cylinders give rather small measurement ranges while long cylinders give larger uncertainties. Of the cylinders tested in this study, the one with L = 10 cm and l = 5 cm is considered to be the optimal as it provides the larges measurement range while still giving a nicely shaped tilt response. The theoretical response curve is not a perfect match to the measured response curve. However, improved accuracy of the TCM is obtained by fitting a curve to the measured data and using this to estimate flow velocities based on measured tilt angle. This is utilized in the following.

Figure 4 shows the tilt response of the optimal cylindrical TCM together with that of the spherical TCM. The figure shows that choosing this cylindrical TCM instead of a sphere gives a general steepening of the response curve but the main benefit occurs above 60° . It should be noted that whereas the spherical TCM was very lightweight in its construction the cylindrical TCM was heavier and could potentially be optimized. The potentially achievable measurement range of a 32 mm cylindrical TCM with l=50 mm is also shown in Figure 4.

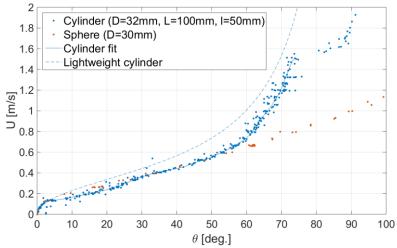


Figure 4. Comparison of observed tilt responses for the spherical and the optimal cylindrical TCM. The dashed line indicates the expected response curve of the optimal cylinder if it were constructed as lightweight as the sphere.

4.2 Pure waves

The optimal cylindrical TCM was tested in a range of different wave conditions with periods ranging from 1.7 to 30 seconds. KC numbers ranged from around 30 to 300. The TCM has a natural frequency of approximately $1.06~\rm s^{-1}$, which is larger than the largest wave frequency ($0.6~\rm s^{-1}$). Therefore, resonance is not expected to be an issue.

Figure 5 shows corresponding values of tilt angle and maximum orbital velocity (crosses). The figure shows that there is good agreement between the tilt response found in pure waves and the one found in pure current.

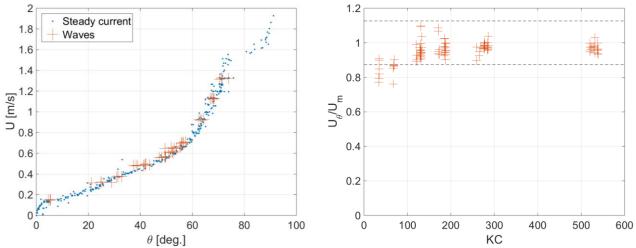


Figure 5. Results from the tests with the optimal cylindrical TCM (L = 100 mm and l = 50 mm) in pure wave motion. Left: Tilt response plotted together with tilt response found for current tests. Right: Ratio of velocity predicted by the TCM and the actual velocity plotted against KC number. Dashed lines indicate the standard deviation in pure current ($KC = \infty$).

A closer examination of the TCM behavior in waves was done by considering the ratio $U_{m,c}/U_m$ where $U_{m,c}$ is the amplitude of the orbital velocity estimated by applying the steady current response curve to the measured tilt angles. It should be noted that this ratio is directly related to the ratio of the drag coefficient in steady current $(C_{D,c})$ and in waves $(C_{D,w})$. Combining equations [5] and

[9] gives the relationship as

$$\frac{C_{D,w}}{C_{D,c}} = \left(\frac{U_{m,\theta}}{U_m}\right)^2$$
 [10]

The comparison in Figure 5 (right panel) shows that the predicted amplitude of the orbital velocity agrees very well with the actual one for *KC* numbers larger than 100. For *KC* number smaller than 100 the TCM tends to underestimate the velocities slightly. With increasing *KC* number the unsteady flow

becomes increasingly drag-dominated approaching the steady current case, hence the result is as expected. However, it is important to note that this experimental setup does not reproduce the horizontal pressure gradients present in real wave motion that give rise to the Froude-Krylow force (Sumer and Fredsøe, 2006).

4.3 Combined waves and current

The TCM performance in combined waves and currents was tested by running a steady current in the flume while moving the carriage in a sinusoidal motion. Two current speeds of 15 cm/s and 46 cm/s were tested. Each of these were combined with a test series of wave conditions which approximately corresponded to those considered for the pure wave case.

In the same way as was done for the pure waves tests, the performance of the TCM was evaluated by considering the velocities close to the peaks of the wave orbital motion. However, in the case of combined waves and current it was found necessary to consider the forestroke of the wave (current and wave flow in the same direction) separately from the backstroke (current and wave flow in opposite directions) since TCM performance was different for the two. For the forestroke it was found that the steady current response curve gives a good estimate of the peak velocities (Figure 6). During the backstroke, however, the TCM tends to underestimate the magnitude of the peak orbital velocity.

The observed behavior has yet to be explained. Possible reasons are that this is the result of the flow regime; stiffness in the joint as noted in the section on the experimental setup or the relative turbulence level affecting the apparent drag. In the surf zone, current and waves are often at an angle to each other. Furthermore, the turbulence levels in the surf zone may affect the apparent drag coefficient. Hence, it is concluded that the logical next step is to carry out a field validation rather than exploring the discrepancy in the laboratory. The discrepancy may not present an issue in the field as this situation may not be present. The TCM for a field validation in the surf zone should be chosen to have a measurement range that covers the expected orbital flow velocities. Then both the orbital velocities and the wave-averaged velocities can be measured provided the above-mentioned issue is absent or has been resolved. In the surf zone, one may experience large cross-shore orbital flow velocities and small longshore currents. In this case, it may be important to choose the smallest possible measurement range in order to ensure adequate measurement accuracy for the longshore current. This is due to the shape of the response curve at small tilt angles and hence small flow velocities.

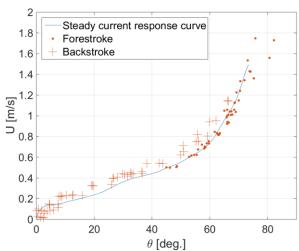


Figure 6. Tilt response observed in the wave-current tests with the optimal cylindrical TCM.

5 CONCLUSIONS

The behavior of spherical and cylindrical TCMs has been investigated with the objective of finding a TCM suitable for use in the surf zone. The main requirements for such a TCM is a measurement range of several meters per second while also being able to capture the unsteady flow of the wave orbital motion.

A large measurement range could in principle be achieved by increasing the size of the TCM. However, the size of the floatation body should be kept small to give large *KC* numbers ensuring a drag-dominated unsteady flow. Additionally, it is desirable to have a relatively constant apparent drag coefficient and necessary to have relatively high natural frequency.

The benefit of a cylindrical compared to a spherical floatation body was demonstrated. The results showed that a short cylinder gave a considerable increase in the measuring range without increasing diameter. For long cylinders, the response curve became very flat for tilt angles smaller than 60° while for larger angles the response curve became so steep that it would give very large uncertainties in the velocity estimate. The effect of tether length was investigated, but it was found to be negligible for the present cases. Out of the 12 TCMs considered in the present study, the one with tether length 10 cm (L/D = 3.1) and cylinder length 5 cm (l/D = 1.6) was found to be preferable.

This cylindrical TCM had a response curve in waves similar to the one in steady current. The noticeable discrepancies were found, as expected, for small KC values (KC < 100) The response curve in combined waves and current was similar to the steady current response curve, when the current and wave was in the same direction; however this response curve tends to underestimate the flow speed when the current and waves are in opposite directions.

Overall, the present findings show that it is possible to choose a TCM so that it is able to capture both mean current and orbital flow velocities as experience in the surf zone.

The present study, however, has only considered unidirectional flow whereas the current is expected to often be almost orthogonal to the wave motion in the surf zone. The turbulence levels in the in the surf zone could be an issue as well as the apparent drag coefficient may change. Such conditions are hard to establish in the laboratory. Therefore, the next step would logically be to carry out a field validation of the described TCM.

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