

1. Consider the finite difference approximation

$$f'(x) \approx \frac{1}{2h}[-3f(x) + 4f(x+h) - f(x+2h)]$$

for  $f(x) = e^x$  at the point  $x = 1$ . Using the matlab function `Num_der_err.m` to fill in the Table 1. Then compute the convergence order based on the formula

$$\text{order} = \log_2 \left( \frac{Err_h}{Err_{h/2}} \right)$$

$h$	$Err_h$	$Err_{h/2}$	order
0.1	0.098	0.0024	2.0548
0.01	$9.1292e-05$	$2.2737e-05$	2.0054
0.001	$9.0678e-07$	$2.2661e-07$	2.005
0.0001	$9.0697e-09$	$2.2729e-09$	1.9965

2. Consider the polynomial interpolation on the interval  $[-1, 1]$  with two types of  $f(x)$ :

$$f_1(x) = \sin(x), \quad f_2(x) = \frac{1}{1 + 25x^2}.$$

The matlab script for polynomial interpolation of  $f_1(x)$  is given by `Err_f1.m`. Try to revise the matlab script to  $f_2(x)$  and fill in the Table 2.

*Hint:* Using the element-wise division `./` and the element-wise power `.^`

$n$	$f_1(x)$		$f_2(x)$	
	Vandermonde Err	Lagrange Err	Vandermonde Err	Lagrange Err
5	$6.9967e-05$	$6.9967e-05$	112.0662	12.0662
10	$8.7900e-10$	$8.7895e-10$	8.2077	8.2077
20	$4.7063e-09$	$5.4384e-13$	168.5423	168.5423
40	2.0079	$1.0558e-07$	11.9349e+05	1.9346e+05