

## TLDR

- Texture Synthesis: Goal is to generate a realistic, yet different, version of a reference texture.
- We propose a modified Sliced Wasserstein Loss to capture long-range constraints in neural texture synthesis.
- An additional height-dimension loss term/multi-scale approach to improve structure without manual masks or alternative regularization terms.

## Texture Synthesis and SW Loss

- Let layer  $\ell$  of an  $L$  layer convolutional neural network have  $N_\ell$  channels and  $M_\ell$  pixels in each channel.
- $p^\ell, \hat{p}^\ell$ : probability density functions for vectors  $\{F_m^\ell\}$  and  $\{\hat{F}_m^\ell\}$  associated to images  $I_1$  and  $I_2$ .
- We assume that the probability density functions take the form

$$p^\ell(x) = \frac{1}{M_\ell} \sum_{m=1}^{M_\ell} \delta_{F_m^\ell}(x). \quad (1)$$

The Sliced Wasserstein Loss between two images,  $\{w_\ell\}$  are weight terms

$$\mathcal{L}_{\text{SW}}(I_1, I_2) = \sum_{\ell=1}^L w_\ell \mathcal{L}_{\text{SW},\ell}(p^\ell, \hat{p}^\ell), \quad (2)$$

where the Sliced Wasserstein Distance between two feature distributions is given by

$$\mathcal{L}_{\text{SW},\ell}(p^\ell, \hat{p}^\ell) = \mathbb{E}_V[\mathcal{L}_{\text{SW1D}}(p_V^\ell, \hat{p}_V^\ell)]. \quad (3)$$

Let  $V$  be a random direction on the unit sphere of dimension  $N_\ell$ . Here, we define (with corresponding definitions for  $\hat{p}^\ell$ )  $p_V^\ell := \{\langle F_m^\ell, V \rangle\}$  as a set consisting of batched projections of the feature maps  $F_m^\ell$  onto  $V$ ; define vector  $P_V^\ell$  consisting of the elements of  $p_V^\ell$  and

$$\mathcal{L}_{\text{SW1D}}(P_V^\ell, \hat{P}_V^\ell) = \frac{1}{\text{len}(P_V^\ell)} \left\| \text{sort}(P_V^\ell) - \text{sort}(\hat{P}_V^\ell) \right\|_2^2. \quad (4)$$

- Convolution operators are local, so (1) ignores correlations between distant pixels (e.g. long-range structure).
- Consider a set of feature maps  $F^\ell \in \mathbb{R}^{H_\ell \times W_\ell \times N_\ell}$  and a feature vector of shape  $W_\ell \times N_\ell$ , which we denote by  $F_{H,n}^\ell \in \mathbb{R}^{H_\ell}$ , where  $n \in \{1, \dots, W_\ell \times N_\ell\}$ .
- We have another set of probability distributions to match, which incorporate locality:

$$p_H^\ell(x) = \frac{1}{W_\ell \times N_\ell} \sum_{n=1}^{W_\ell \times N_\ell} \delta_{F_{H,n}^\ell}(x). \quad (5)$$

Consider distributions  $p_H^\ell$  and  $\hat{p}_H^\ell$  associated with  $I_1$  and  $I_2$ . The corresponding additional loss term is

$$\mathcal{L}_{\text{SW},H}(I_1, I_2) = \sum_{\ell=1}^L w_\ell \mathcal{L}_{\text{SW},\ell}(p_H^\ell, \hat{p}_H^\ell), \quad (6)$$

## Texture Synthesis and SW Loss Continued

We minimize

$$\mathcal{L}_{\text{Slicing}}(I_1, I_2) = \mathcal{L}_{\text{SW}}(I_1, I_2) + \mathcal{L}_{\text{SW},H}(I_1, I_2). \quad (7)$$

### Algorithm 1: Synthesis Algorithm

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Denote the feature map extraction of image  $I$  from VGG19 as  $\text{Extract}(I)$ ;  
 $I_{\text{WN}} \leftarrow$  white noise to be updated by optimizer via backpropagation;  
 $I_{\text{Ref}} \leftarrow$  reference texture;  
**for**  $k \leftarrow 1$  **to**  $M$  ; // Epochs  
  **do**  
    Calculate  $\text{Extract}(I_{\text{WN}})$ ;  
    Calculate  $\text{Extract}(I_{\text{Ref}})$ ;  
    Calculate  $\mathcal{L}_{\text{Slicing}}(I_{\text{WN}}, I_{\text{Ref}})$ ;  
    Backpropagate and update  $I_{\text{WN}}$ ;  
**return**  $I_{\text{WN}}$  as synthesized texture;

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## Example Results

For more periodic textures (first two rows), the performance varies between algorithms, but proposed algorithm has more consistent performance on nonstationary textures (last two rows).

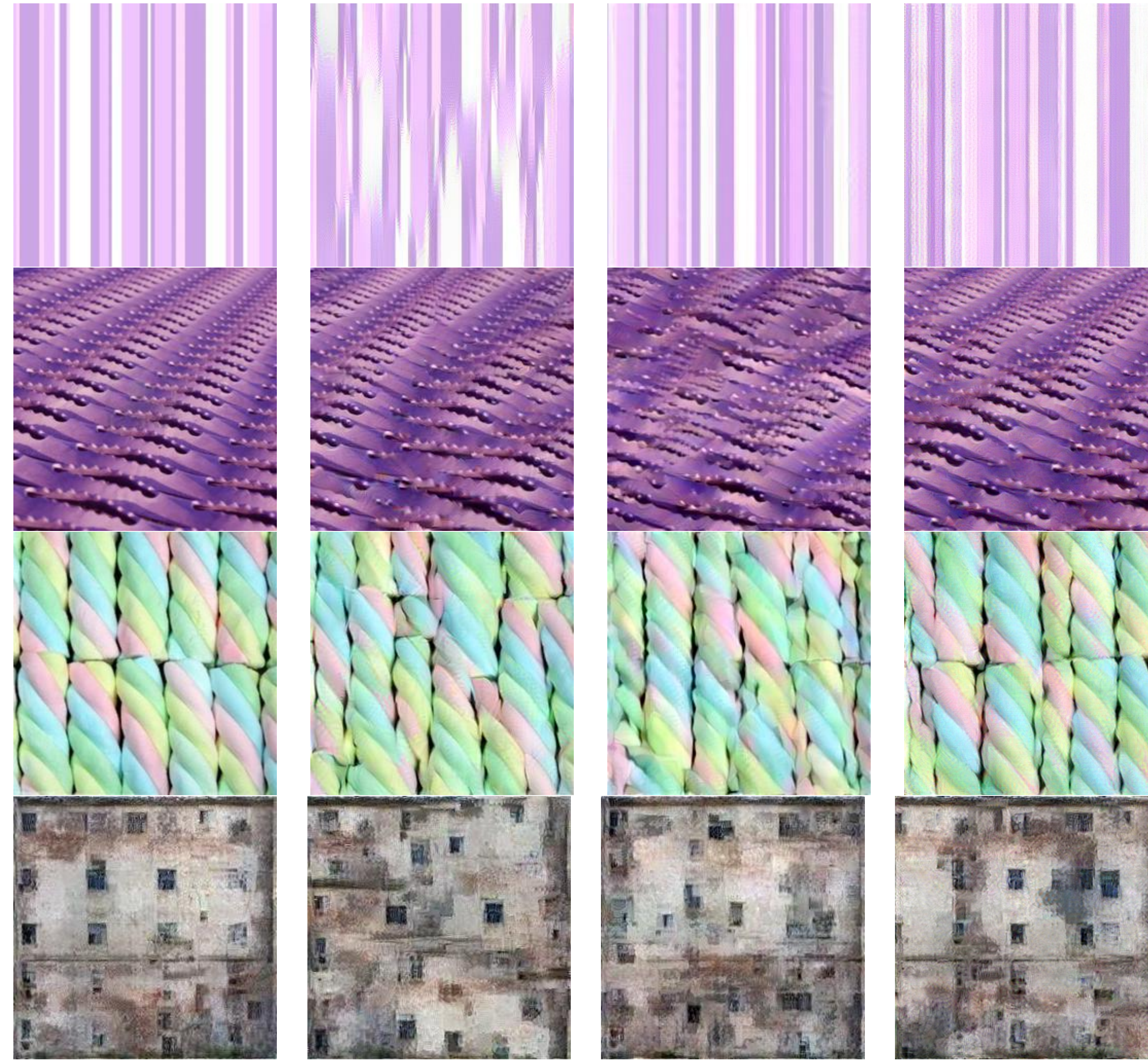


Fig. 1: Comparison of results for textures. **Left:** Reference. **Mid Left:** SW Loss. **Mid Right:** Spectrum. **Right:** Using new loss (Ours).

## Multi-scale Algorithm

To improve the results of our algorithm, we incorporate a multi-scale approach previously seen in [1].

### Algorithm 2: Multi-scale Algorithm

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Initialize  $I_{\text{Synthesis}}$  as white noise, equal to the reference texture downsampled by  $2^K$ ;  
Let  $I_{\text{ref},i}$  be the reference texture downsampled by  $2^i$ ;  
Let  $\text{SWSynthesis}$  be synthesis using Algorithm 1;  
**for**  $i \leftarrow 0$  **to**  $K$  **do**  
   $I_{\text{Synthesis}} \leftarrow \text{SWSynthesis}(I_{\text{Synthesis}}, I_{\text{ref},K-i})$ ;  
   $I_{\text{Synthesis}} \leftarrow 2 \times \text{Upsample}(I_{\text{Synthesis}})$ ;  
**return**  $I_{\text{Synthesis}}$  as the synthesized texture;

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More scales, means better synthesis, but possible repetition.



Fig. 2: Multi-scale procedure at different scales. **Left:** Reference. **Mid Left:**  $K = 0$ . **Mid Right:**  $K = 1$ . **Right:**  $K = 2$ .

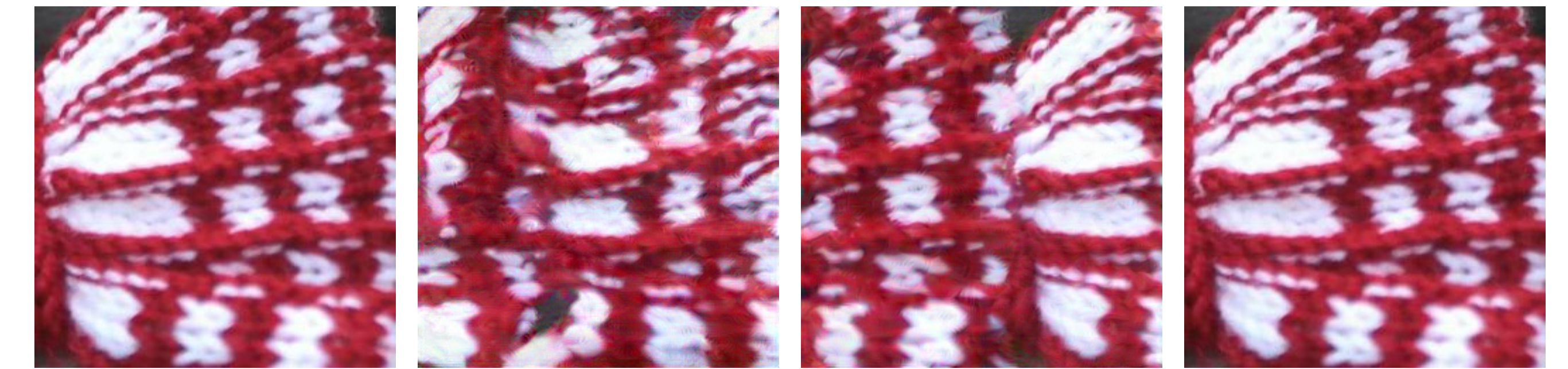


Fig. 3: Progression of synthesis that lead to repetitions. **Left:** Reference Texture. **Middle Left:**  $K = 0$ . **Middle Right:**  $K = 1$ . **Right:**  $K = 2$ .

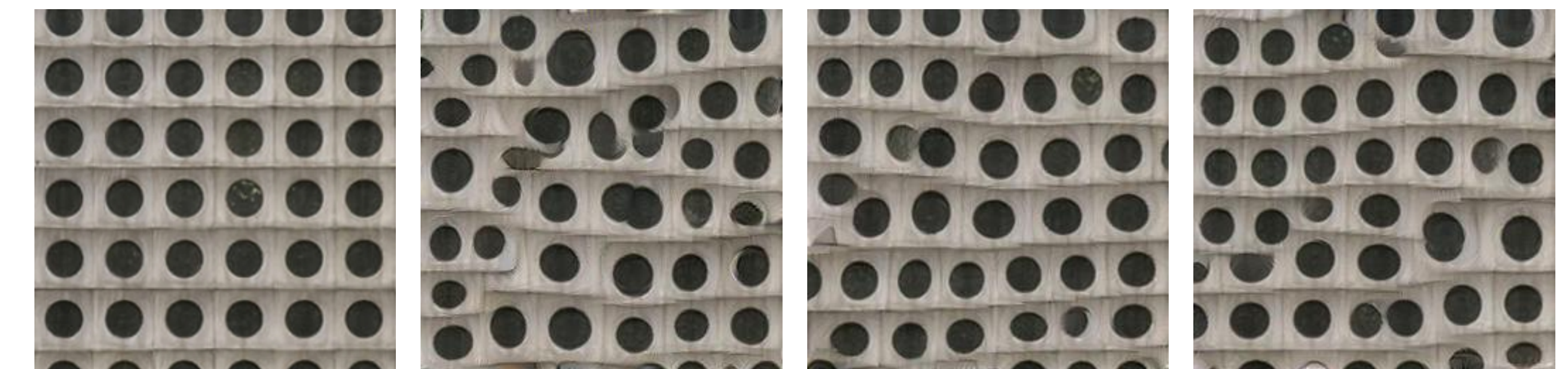


Fig. 4: Ablation study: results with only (2) **Left:** Reference. **Mid Left:**  $K = 0$ . **Mid Right:**  $K = 1$ . **Right:**  $K = 2$ .

- Quant comparison with other methods using set of 34 textures.
- $K = 1$  shows the best results for  $256 \times 256$  images.

Method	LPIPS	FID	c-FID	KID	c-KID
$K = 0$	.44	107.2	$72.3 \pm 0.34$	-.014	$.073 \pm 0.0008$
SW	.45	101.8	$78.7 \pm 0.48$	-.016	$.084 \pm 0.001$
Spec.	.45	99.6	$78.3 \pm 0.57$	-.016	$.085 \pm 0.001$
Gonthier	.42	77.6	$68.3 \pm 0.35$	-.018	$.069 \pm 0.0009$
$K = 1$	.38	67.1	$53.5 \pm 0.32$	-.018	$.043 \pm 0.0006$
$K = 2$	.25	38.3	$40.5 \pm 0.6$	-.022	$.028 \pm 0.0009$

Tab. 1: Perceptual Metric Comparison, (C-FID/C-KID scores [2] have SE for runs)

## References

- [1] Nicolas Gonthier, Yann Gousseau, and Saïd Ladjal. "High resolution neural texture synthesis with long range constraints," in *Journal of Mathematical Imaging and Vision* 64:478–492, 2022.
- [2] Guilin Liu, Rohan Taori, Ting-Chun Wang, Zhiding Yu, Shiqiu Liu, Fitsum A. Reda, Karan Sapra, Andrew Tao, and Bryan Catanzaro. "Transposer: Universal Texture Synthesis Using Feature Maps as Transposed Convolution Filter," in *arXiv:2007.07243 [cs.CV]*, July 2020.