Lab 2: Taking the Temperature of a Cold Brown Dwarf

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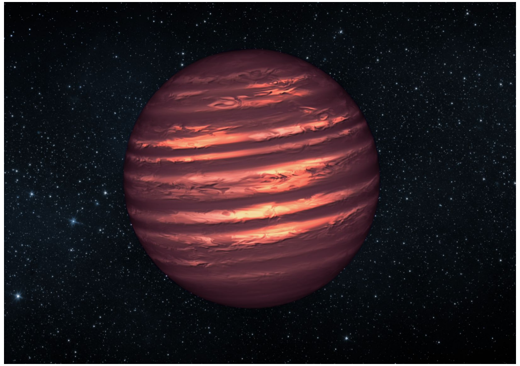
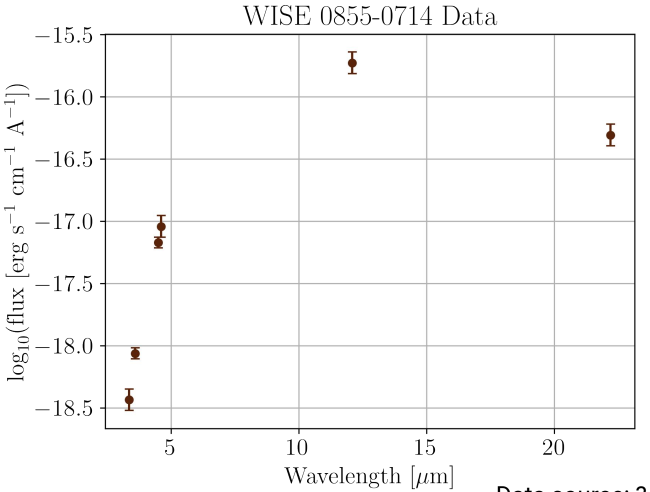
The WISE space telescope has measured the flux and wavelengths of light from the nearby brown dwarf WISE 0855-0714 (Luhman et al. 2014)





Data source: ?

This data appears to follow Planck's law given by the equation:

$$f(\lambda, T) = \alpha \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1}$$

The shape parameters of such a fit would be a multiplicative factor, α, and the BD's temperature T. **Can we fit the data to the Planck function and obtain a posterior distribution for the temperature of WISE 0855-0714?**

We wanted to fit data to a nonlinear model so we used a MCMC algorithm following the Metropolis rule.

The (log) probability was calculated assuming a wide, uninformative uniform prior on log α and three different temperature priors

The initial guess bounds were [-50, -50] for log α and [0K, 1024K] for temperature

The jump scales (for Gaussian proposals) were tuned to 0.05 and 3K for log α and temperature respectively.

Model

$$f(\lambda, \alpha, T) = \alpha \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1}$$

Likelihood

$$\mathcal{L} = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma_i} \mathrm{e}^{-\frac{r_i^2}{2\sigma_i^2}}$$

Priors

Uniform Prior: $\mathcal{P}(T) \propto 1$

Log-Uniform Prior: $\mathcal{P}(T) \propto T^{-1}$

T^4-Uniform Prior: $\mathcal{P}(T) \propto T^{-3}$

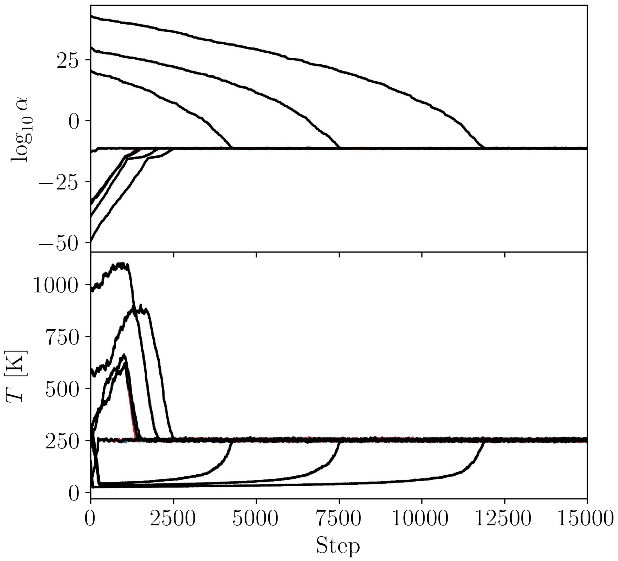
Metropolis Rule

if $\mathscr{P}_{\text{trial}} > \mathscr{P}_i$,
accept the jump, so
$\theta_{i+1} = \theta_{\text{trial}}$

if $\mathscr{P}_{\text{trial}} < \mathscr{P}_i$,
accept the jump with probability $\mathscr{P}_{\text{trial}}/\mathscr{P}_i$

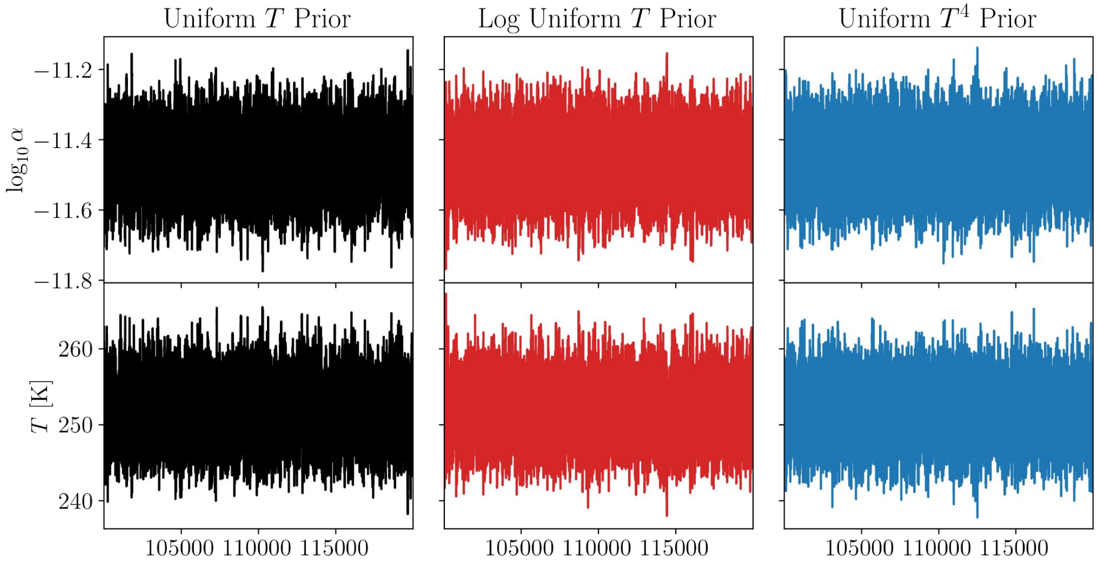
For each of the three T priors, we ran walkers started on eight different random seeds (same seeds between priors). We obtained 1,048,576 samples from each walker, totaling 8,388,608 per prior.

Burn-In Check



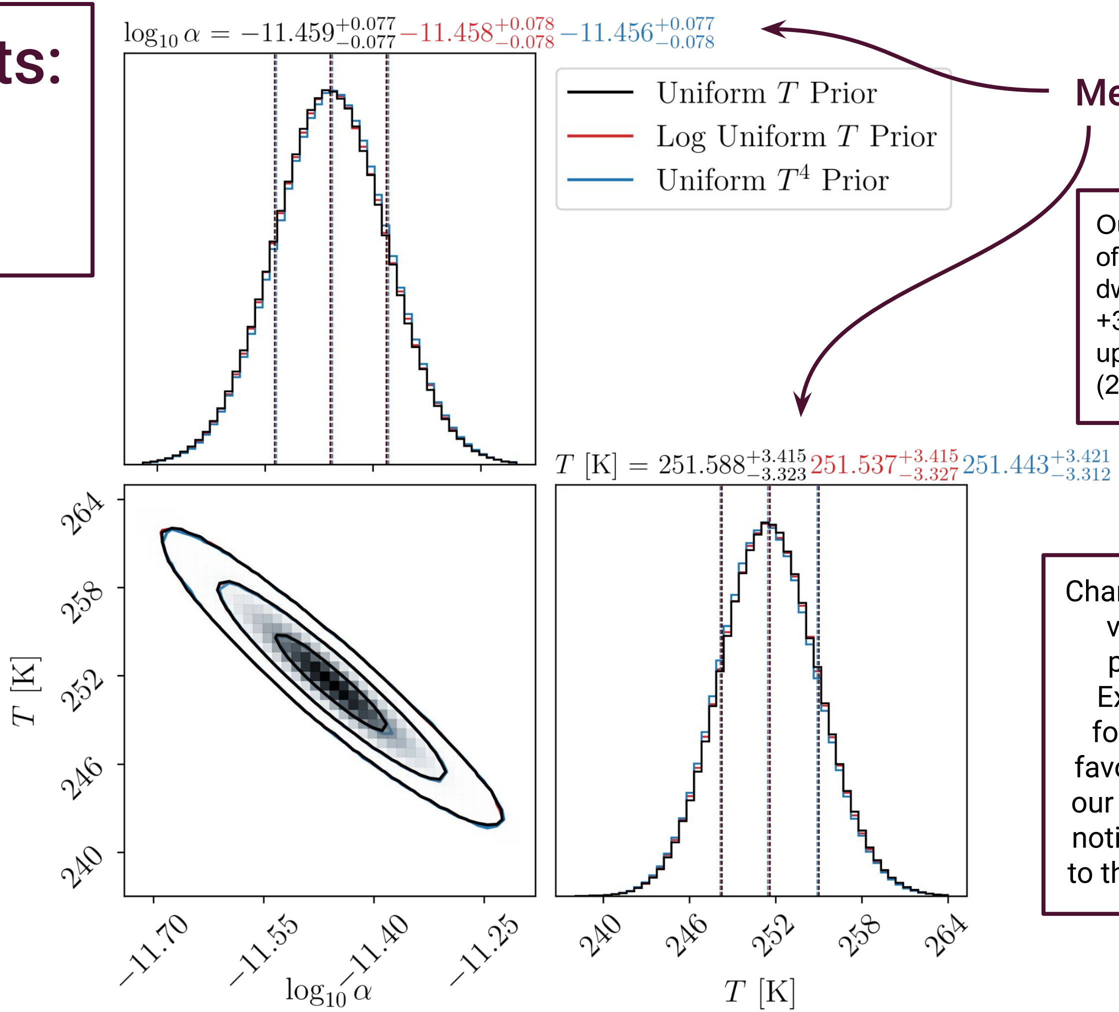
The first 15,000 steps are plotted here to check the burn-in time. Burn-in was complete by 12,500 steps at most. All three priors are plotted here, but the lines overlap (same seeds).

Mixing Check



20,000 steps from #100,000 to #120,000 of a walker is displayed here to check for mixing. The sampling appears to be sufficiently mixed for each prior.

MCMC Results: Posterior Distributions



Median ± 1 σ

Our aggregated results were α of -11.46 ± 0.08 and a brown dwarf temperature of 251.5K +3.4K/-3.3K, close to the upper limit of WISE's results (225K-260K).

log α and T are highly correlated. Our fit parameters are not independent. Makes sense referring to Planck equation (for small changes, increasing alpha and decreasing T have similar effect on flux).

Changing the prior on T only very slightly alters the posterior distribution. Expect log-uniform and fourth-power-uniform to favor lower values of T, but our choice of prior is barely noticeable when compared to the width of the posterior