PAKET 5

PELATIHAN ONLINE

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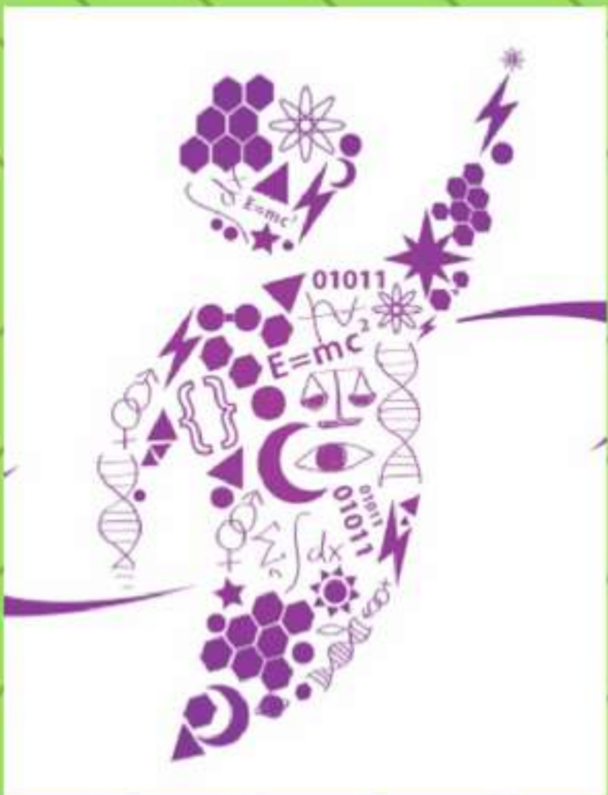

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PEMBAHASAN PAKET 5

1. Gunakan aturan rantai

$$a = -kv^2$$
$$\frac{dv}{dt} = -kv^2$$
$$\int \frac{dv}{v^2} = -\int k\,dt$$
$$\frac{1}{v_0} - \frac{1}{v} = -kt$$
$$v = \frac{v_0}{v_0kt + 1}$$

Percepatan merupakan turunan pertama dari kecepatan

$$a = \frac{dv}{dt} = -\frac{v_0^2 k}{(v_0kt + 1)^2}$$

(a)

2. Sesuai dengan pembahasan sebelumnya

$$v = \frac{v_0}{v_0kt + 1}$$

(b)

3. Posisi merupakan integral dari kecepatan terhadap waktu

$$s = \int v\,dt = \int \frac{v_0}{v_0kt + 1}\,dt = \frac{1}{k}\ln(v_0kt + 1)$$

(d)

4. Gunakan aturan rantai dalam menurunkan persamaan $y(x) = sin^3 4x\; tan^2 3x$

$$u = sin^3 4x$$
$$v = tan^2 3x$$
$$\frac{dy}{dx} = \frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx} = 6sin^2 4x \tan 3x(\sin 4x\, sec^2 3x + 2\tan 3x \cos 4x)$$

(c)

5. Gunakan aturan rantai dalam menurunkan persamaan $y(x) = e^{\sec x} sin^2 x$

$$u = e^{\sec x}$$
$$v = sin^2 x$$
$$\frac{dy}{dx} = \frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$$

Turunan untuk $e^{\sec x}$. Gunakan manipulasi

$$\ln u = \sec x$$

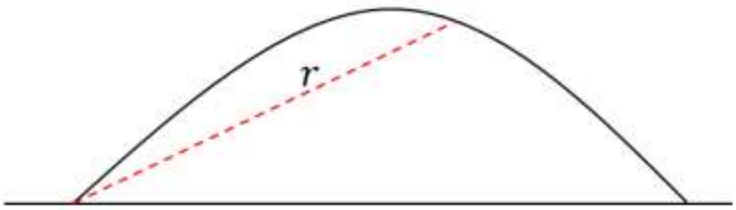
$$\frac{d}{dx}\frac{d}{du}(\ln u) = \frac{d}{dx}\frac{d}{du}(\sec x)$$
$$\frac{du}{dx}\frac{1}{u} = \sec x \tan x$$
$$\frac{du}{dx} = e^{\sec x}\sec x \tan x$$

Turunan untuk $sin^2 x$

$$\frac{dv}{dx} = 2\sin x \cos x = \sin 2x$$
$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx} = e^{\sec x}(\sin 2x + \sin x\, tan^2 x)$$

(a)

6. Perhatikan gambar dibawah ini



Besar r tidak boleh berkurang tiap waktu, harus semakin besar.

$$x = v_0 \cos\theta\, t$$
$$y = v_0 \sin\theta\, t - \frac{1}{2}gt^2$$
$$r^2 = x^2 + y^2 = (v_0\cos\theta\, t)^2 + \left(v_0 \sin\theta\, t - \frac{1}{2}gt^2\right)^2 = v_0^2 t^2 - v_0\sin\theta\, gt^3 + \frac{1}{4}g^2t^4$$

$\frac{dr}{dt} > 0$ agar objek semakin jauh (tidak mendekat).

$$\frac{d}{dt}r^2 = 2r\frac{dr}{dt} > 0$$
$$\frac{d}{dt}r^2 = 2v_0^2 t - 3v_0\sin\theta\, gt^2 + g^2t^3 > 0$$
$$2v_0^2 - 3v_0 \sin\theta\, gt + g^2t^2 > 0$$

Agar $\frac{dr}{dt} > 0$ dapat terwujud, maka persamaan diatas harus mempunyai akar yang real.

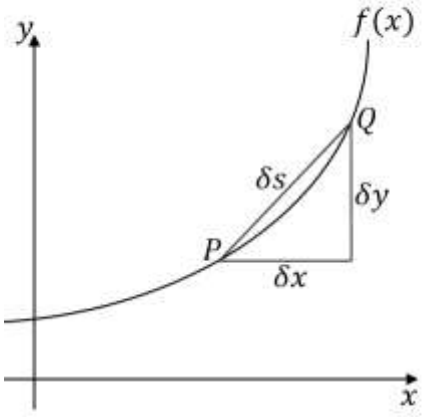
$$D = b^2 - 4ac > 0$$
$$(3v_0 \sin\theta\, gt)^2 - 4(g^2t^2)(2v_0^2) > 0$$
$$sin^2\theta < \frac{8}{9}$$
$$\theta < \arcsin\sqrt{\frac{8}{9}}$$

(a)

7. Berikut merupakan persamaan matematika panjang kurva

$$(ds)^2 = (dx)^2 + (dy)^2$$

$$s = \int \sqrt{(dx)^2 + (dy)^2}$$



$$s = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Persamaan gerak parabola

$$x = v_0 \cos\theta\, t$$

$$y = v_0 \sin\theta\, t - \frac{1}{2} g t^2$$

$$\frac{dx}{dt} = v_0 \cos\theta$$

$$\frac{dy}{dt} = v_0 \sin\theta - gt$$

$$ds = \sqrt{(v_0 \cos\theta)^2 + (v_0 \sin\theta - gt)^2} dt = v_0 \cos\theta \sqrt{1 + \left(\tan\theta - \frac{gt}{v_0 \cos\theta}\right)^2}\, dt$$

Asumsikan subtitusi integralnya

$$\tan\theta - \frac{gt}{v_0 \cos\theta} \equiv \tan\varphi$$

$$\frac{d}{d\varphi}\left(\tan\theta - \frac{gt}{v_0 \cos\theta}\right) = sec^2\varphi$$

$$\frac{dt}{d\varphi} = -\frac{v_0 sec^2\varphi \cos\theta}{g}$$

$$ds = -\frac{v_0^2 cos^2\theta sec^2\varphi}{g} \sqrt{1 + tan^2\varphi}\, d\varphi = -\frac{v_0^2 cos^2\theta}{g} sec^3\varphi\, d\varphi$$

$$s = -\frac{v_0^2 cos^2\theta}{g} \int sec^3\varphi\, d\varphi$$

Menghitung integral $sec^3\varphi$

$$\int sec^3\varphi\, d\varphi = \int sec^2\varphi \sec\varphi\ \ d\varphi$$

Gunakan metode parsial

$$dv = sec^2\varphi\, d\varphi$$

$$u = \sec\varphi$$
$$\int sec^3\varphi\, d\varphi = \int sec^2\varphi \sec\varphi\ \ d\varphi = \sec\varphi\tan\varphi - \int tan^2\varphi \sec\varphi\, d\varphi$$
$$tan^2\varphi \sec\varphi = sin^2\varphi sec^3\varphi = (1 - cos^2\varphi)sec^3\varphi = sec^3\varphi - \sec\varphi$$

$$\int sec^3\varphi\, d\varphi = \int sec^2\varphi \sec\varphi\ \ d\varphi = \sec\varphi\tan\varphi - \int (sec^3\varphi - \sec\varphi)\, d\varphi$$
$$2\int sec^3\varphi\, d\varphi = \sec\varphi\tan\varphi + \int \sec\varphi\, d\varphi = \sec\varphi\tan\varphi + \ln(\sec\varphi + \tan\varphi)$$
$$\int sec^3\varphi\, d\varphi = \frac{1}{2}\sec\varphi\tan\varphi + \frac{1}{2}\ln(\sec\varphi + \tan\varphi)$$

Kita mengetahui bahwa

$$\tan\theta - \frac{gt}{v_0\cos\theta} \equiv \tan\varphi$$
$$\sec\varphi = \frac{\sqrt{v_0^2 - 2v_0\sin\theta\, gt + g^2t^2}}{v_0\cos\theta}$$
$$\sec\varphi\tan\varphi = \frac{(v_0\sin\theta - gt)\sqrt{v_0^2 - 2v_0\sin\theta\, gt + g^2t^2}}{v_0^2 cos^2\theta}$$
$$\ln(\sec\varphi + \tan\varphi) = \ln\left(\frac{\sqrt{v_0^2 - 2v_0\sin\theta\, gt + g^2t^2}}{v_0\cos\theta} + \tan\theta - \frac{gt}{v_0\cos\theta}\right)$$

Maka, panjang kurva lintasan

$$s = -\frac{v_0^2 cos^2\theta}{g}(\sec\varphi\tan\varphi + \ln(\sec\varphi + \tan\varphi))\Big|_{t=0}^{t=\frac{v_0\sin\theta}{g}}$$
$$\sec\varphi\tan\varphi\Big|_{t=0}^{t=\frac{v_0\sin\theta}{g}} = -sec^2\theta$$
$$\ln(\sec\varphi + \tan\varphi)\Big|_{t=0}^{t=\frac{v_0\sin\theta}{g}} = -\ln\left(\frac{1+\sin\theta}{\cos\theta}\right)$$
$$s = -\frac{v_0^2 cos^2\theta}{g}(\sec\varphi\tan\varphi + \ln(\sec\varphi + \tan\varphi))\Big|_{t=0}^{t=\frac{v_0\sin\theta}{g}}$$
$$= \frac{v_0^2}{g}\left(\sin\theta + cos^2\theta\ln\left(\frac{1+\sin\theta}{\cos\theta}\right)\right)$$

Untuk mencari besar θ agar s maksimal adalah harus memenuhi $\frac{ds}{d\theta} = 0$

$$\frac{ds}{d\theta} = 0 = \frac{d}{d\theta}\left(\sin\theta + cos^2\theta\ln\left(\frac{1+\sin\theta}{\cos\theta}\right)\right)$$

Differensial

$$\ln\left(\frac{1+\sin\theta}{\cos\theta}\right) = \ln(1+\sin\theta) - \ln(\cos\theta)$$
$$\frac{d}{d\theta}\left(\ln\left(\frac{1+\sin\theta}{\cos\theta}\right)\right) = \frac{d}{d\theta}\ln(1+\sin\theta)\frac{d(1+\sin\theta)}{d(1+\sin\theta)} - \frac{d}{d\theta}\ln(\cos\theta)\frac{d(\cos\theta)}{d(\cos\theta)}$$

$$\frac{d}{d\theta}\left(\ln\left(\frac{1+\sin\theta}{\cos\theta}\right)\right)=\frac{\cos\theta}{1+\sin\theta}+\tan\theta$$

$$\frac{ds}{d\theta}=\cos\theta-2\cos\theta\sin\theta\ln\left(\frac{1+\sin\theta}{\cos\theta}\right)+cos^2\theta\left(\frac{\cos\theta}{1+\sin\theta}+\tan\theta\right)=0$$

$$1=\sin\theta\ln\left(\frac{1+\sin\theta}{\cos\theta}\right)$$

(a)

8. Sesuai di pembahasan sebelumnya

$$L=\frac{v_0^2}{g}\left(\sin\theta+cos^2\theta\ln\left(\frac{1+\sin\theta}{\cos\theta}\right)\right)$$

(e)

9. Untuk memudahkan pengerjaan, akan dilakukan manipulasi persamaan

$$\frac{x^2-14}{x^2+16}=1-\frac{30}{x^2+16}$$

$$\int\frac{x^2-14}{x^2+16}\,dx=\int dx-\int\frac{30}{x^2+16}dx$$

$$x=4\tan\varphi$$

$$dx=4sec^2\varphi\,d\varphi$$

$$\int\frac{x^2-14}{x^2+16}\,dx=x-\frac{30}{4}\int d\varphi=x-\frac{15}{2}\arctan\left(\frac{x}{4}\right)+c$$

(d)

10. Tinjau gerak parabola

$$x=\frac{v^2\sin 2\theta}{g}$$

$$t=\frac{2v\sin\theta}{g}$$

Tinjau gerak miring

$$v=g\sin\varphi\,t$$

$$x=\frac{v^2}{2a}=\frac{v^2}{2g\tan\varphi}$$

Objek 1 dan objek 2 masing-masing menempuh x yang sama

$$\frac{v^2\sin 2\theta}{g}=\frac{v^2}{2g\tan\varphi}$$

$$\tan\varphi=\frac{1}{2\sin 2\theta}$$

Objek 1 dan objek 2 masing-masing menempuh t yang sama

$$\frac{2v\sin\theta}{g} = \frac{v}{g\sin\varphi}$$
$$2\sin\theta = \frac{1}{\sin\varphi}$$

Dengan 2 persamaan diatas dapat diselesaikan untuk masing-masing sudut

$$\sin\theta = \sqrt{\frac{3+\sqrt{13}}{8}}$$
$$\theta = \arcsin\sqrt{\frac{3+\sqrt{13}}{8}}$$

(a)

11. Sesuai persamaan sebelumnya

$$\sin\varphi = \sqrt{\frac{2}{3+\sqrt{13}}}$$
$$\varphi = \arcsin\sqrt{\frac{2}{3+\sqrt{13}}}$$

(b)

12. Gunakan sifat logaritma

$$\log_7 3x = \frac{\ln 3x}{\ln 7}$$
$$\int \log_7 3x\ \ dx = \frac{1}{\ln 7}\int \ln 3x\, dx$$

Lakukan metode parsil

$$u = \ln 3x$$
$$\frac{du}{dx} = \frac{1}{x}$$
$$dv = dx$$
$$\int \log_7 3x\ \ dx = \frac{1}{\ln 7}\left(x\ln 3x - \int dx\right) = \frac{1}{\ln 7}(x\ln 3x - x) + c$$

(a)

13. Persamaan geraknya

$$y = v_0\sin\theta\, t - \frac{1}{2}gt^2$$
$$x = v_0\cos\theta\, t$$

Subtitusikan persamaan x pada persamaan y

$$0 = \frac{1}{2}g\frac{x^2}{v_0^2}tan^2\theta - x\tan\theta + \frac{1}{2}g\frac{x^2}{v_0^2} + y$$

Namun, kita memerlukan satu nilai θ yang real, maka $D = 0$ untuk $x = d$ dan $y = d$

$$b^2 - 4ac = 0$$
$$(x\tan\theta)^2 - 4\left(\frac{1}{2}g\frac{x^2}{v_0^2}tan^2\theta\right)\left(\frac{1}{2}g\frac{x^2}{v_0^2}+y\right) = 0$$
$$v_0^4 - 2gdv_0^2 - g^2d^2 = 0$$
$$v_0 = \sqrt{gd\left(1+\sqrt{2}\right)}$$

(c)

14. Untuk mendapatkan nilai θ, subtitusikan kecepatan awal pada persamaan sebelumnya untuk $x = d$ dan $y = d$

$$0 = \frac{1}{2}g\frac{x^2}{v_0^2}tan^2\theta - x\tan\theta + \frac{1}{2}g\frac{x^2}{v_0^2} + y$$
$$0 = \frac{1}{2}g\frac{d^2}{gd\left(1+\sqrt{2}\right)}tan^2\theta - d\tan\theta + \frac{1}{2}g\frac{d^2}{gd\left(1+\sqrt{2}\right)} + d$$
$$\tan\theta = 1+\sqrt{2}$$
$$\theta = \arctan\left(1+\sqrt{2}\right)$$

(c)

15. Ketinggian maksimum dicapai saat $v_y = 0$

$$v_y = v_0\sin\theta - gt$$
$$t = \frac{v_0\sin\theta}{g}$$
$$y = v_0\sin\theta\left(\frac{v_0\sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{v_0\sin\theta}{g}\right)^2 = \frac{v_0^2\,sin^2\theta}{2g}$$
$$y = \frac{gd\left(1+\sqrt{2}\right)}{2g}\left(\frac{1+\sqrt{2}}{\sqrt{4+2\sqrt{2}}}\right)^2 \approx 1{,}03d$$

(c)