PAKET 11

PELATIHAN ONLINE

2019

SMA

FISIKA

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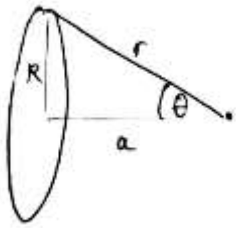
PEMBAHASAN PAKET 11

1. Perhatikan gambar dibawah ini



$$g_x = G\int \frac{\lambda\, dl}{r^2}\cos\theta = G\lambda \int \frac{dl}{a^2+R^2}\left(\frac{a}{\sqrt{a^2+R^2}}\right)$$

$$g_x = \frac{G\lambda a}{(a^2+R^2)^{\frac{3}{2}}}\int_{l=0}^{2\pi R} dl = \frac{2\pi R G\lambda a}{(a^2+R^2)^{3/2}}$$

(a)

2. Gunakan aproksimasi dalam menyederhanakan persamaan dimana $a = x \ll R$

$$g_x = \frac{2\pi R G\lambda x}{(x^2+R^2)^{3/2}} = \frac{2\pi R G\lambda x}{R^3\left(1+\frac{x^2}{R^2}\right)^{3/2}} = \frac{2\pi G\lambda x}{R^2}$$

Jika massa disimpang sejauh x, akan berosilasi di sekitar pusat massa cincin

$$F_{pemulih} = m\ddot{x}$$

$$-mg_x = -m\frac{2\pi G\lambda x}{R^2} = m\ddot{x}$$

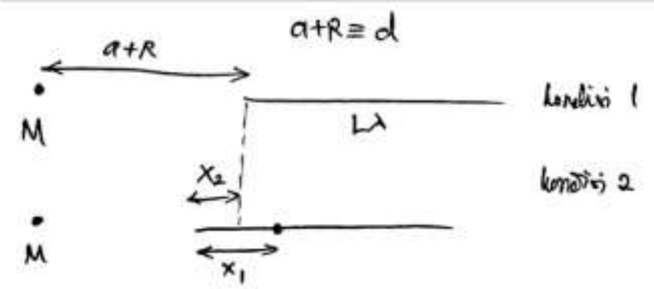
$$-\frac{2\pi G\lambda x}{R^2} = \ddot{x}$$

Maka, periode osilasi

$$T = 2\pi R\sqrt{\frac{1}{2\pi G\lambda}}$$

(b)

3. Kita akan beri acuan untuk integralnya. Perhatikan gambar dibawah ini (kita asumsikan bahwa planet merupakan massa titik).



Yang perlu diperhatikan adalah, dalam melakukan integrasi persamaan (pada gravitasi), jangan gunakan kondisi mula-mula (awal)/jangan di awal atau di akhir, melainkan di suatu waktu t. Agar persamaan universal dan fisis.

Percepatan di x_1

$$g = \frac{GM}{(d - x_2 + x_1)^2}$$

Gaya yang dirasakan oleh batang akibat planet di suatu jarak x_2

$$F = \int g\, dm = GM \int_{x_1=0}^{L} \frac{\lambda\, dx_1}{(d - x_2 + x_1)^2} = GM\lambda \left(\frac{1}{d - x_2} - \frac{1}{d - x_2 + L}\right)$$

Persamaan dinamika

$$F = m\ddot{x}_2 = L\lambda\ddot{x}_2 = GM\lambda \left(\frac{1}{d - x_2} - \frac{1}{d - x_2 + L}\right)$$

Gunakan aturan rantai

$$\int_{\dot{x}_2=0}^{\dot{x}_2} \dot{x}_2\, d\dot{x}_2 = GM\lambda \int_{x_2=0}^{a} \left(\frac{1}{d - x_2} - \frac{1}{d - x_2 + L}\right) dx_2$$

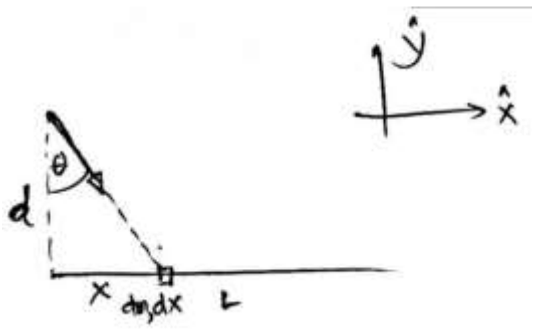
$$\dot{x}_2^2 = \frac{2GM}{L}\left(\ln\left(\frac{d}{d-a}\right) + \ln\left(\frac{d-a+L}{d+L}\right)\right)$$

Kita mengetahui bahwa $d \equiv a + R$

$$\dot{x}_2 = v = \frac{2GM}{L}\ln\left(\frac{(a+R)(R+L)}{R(a+R+L)}\right)$$

(a)

4. Tentukan acuan integral



$$g_x = G\int \frac{\lambda\, dx}{d^2 + x^2}\sin\theta = G\int \frac{\lambda\, dx}{d^2 + x^2}\frac{x}{\sqrt{d^2 + x^2}} = G\lambda \int_{x=0}^{L} \frac{x}{(d^2 + x^2)^{\frac{3}{2}}}\, dx$$

Lakukan subtitusi variabel, dimana

$$u = d^2 + x^2$$

$$\frac{du}{dx} = 2x$$

$$g_x = \frac{G\lambda}{2}\int_{x=0}^{L}\frac{1}{u^{\frac{3}{2}}}du = G\lambda\left(\frac{1}{d} - \frac{1}{\sqrt{d^2+L^2}}\right)$$

(c)

5. Mencari percepatan vertikal

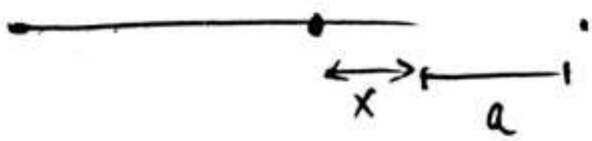
$$g_y = G\int\frac{\lambda\,dx}{d^2+x^2}\cos\theta = G\lambda\int\frac{dx}{d^2+x^2}\frac{d}{\sqrt{d^2+x^2}} = G\lambda d\int_{x=0}^{L}\frac{1}{(d^2+x^2)^{\frac{3}{2}}}dx$$

Lakukan subtitusi trigonometri dimana

$$x = d\tan\theta$$
$$\frac{dx}{d\theta} = d\,sec^2\theta$$
$$g_y = G\lambda d\int\frac{d\,sec^2\theta\,d\theta}{d^3sec^3\theta} = \frac{G\lambda}{d}\int\cos\theta\,d\theta = \frac{G\lambda}{d}\sin\theta| = \frac{G\lambda}{d}\left(\frac{x}{d^2+x^2}\right)\Bigg|_{x=0}^{L}$$
$$g_y = G\lambda\frac{L}{d}\frac{1}{\sqrt{d^2+L^2}}$$

(b)

6. Tentukan acuan integral. Perhatikan gambar dibawah ini



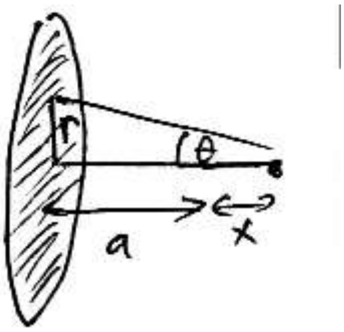
$$g_x = G\int\frac{\lambda\,dx}{(a+x)^2} = G\lambda\int\frac{dx}{(a+x)^2} = G\lambda\int_{x=0}^{L}\frac{dx}{(a+x)^2}$$

Lakukan subtitusi variabel, dimana

$$u = a + x$$
$$\frac{du}{dx} = 1$$
$$g_x = G\lambda\int\frac{du}{u^2} = G\lambda\left(\frac{1}{a+x}\right)\Bigg|_{x=L}^{0} = G\lambda\left(\frac{1}{a} - \frac{1}{a+L}\right)$$

(c)

7. Tentukan acuan integral. Perhatikan gambar dibawah ini



Partisi massa dari plat adalah $dm = \sigma r\, dr\, d\varphi$

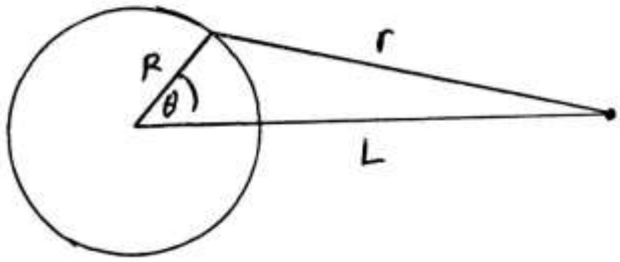
$$g_x = G\int \frac{\sigma\, dA}{r^2}\cos\theta = G\sigma \int \frac{rdr\, d\varphi}{a^2+r^2}\frac{a}{\sqrt{a^2+r^2}} = 2\pi G\sigma a \int \frac{rdr}{(a^2+r^2)^{\frac{3}{2}}}$$

Subtitusi variabel, dimana

$$u = a^2 + r^2$$
$$\frac{du}{dr} = 2r$$
$$g_x = \pi G\sigma a \int \frac{du}{u^{\frac{3}{2}}} = 2\pi G\sigma a \left(\frac{1}{\sqrt{a^2+r^2}}\right)\Bigg|_R^0 = 2\pi G\sigma a\left(\frac{1}{a} - \frac{1}{\sqrt{a^2+R^2}}\right)$$

(a)

8. Perhatikan gambar dibawah ini



Kita mengetahui bahwa potensial newton adalah

$$\Phi = -G\int \frac{dm}{r}$$

$$\Phi = -G\int \frac{\lambda R}{\sqrt{L^2+R^2-2LR\cos\theta}}d\theta = -\frac{G\lambda R}{L}\int \left(1+\frac{R^2}{L^2} - 2\frac{R}{L}\cos\theta\right)^{-\frac{1}{2}} d\theta$$

Aproksimasi

$$\left(1+\frac{R^2}{L^2} - 2\frac{R}{L}\cos\theta\right)^{-\frac{1}{2}} \approx 1 + \frac{R}{L}\cos\theta + \frac{1}{2}\left(\frac{R}{L}\right)^2 (3cos^2\theta - 1) + \cdots$$

Namun, kita akan mengabaikan orde 2 dan diatasnya, $\left(\frac{R}{L}\right)^3 \approx 0$

$$\left(1+\frac{R^2}{L^2} - 2\frac{R}{L}\cos\theta\right)^{-\frac{1}{2}} \approx 1 + \frac{R}{L}\cos\theta + \frac{1}{2}\left(\frac{R}{L}\right)^2 (3cos^2\theta - 1)$$

$$\Phi = -\frac{G\lambda R}{L}\int_{\theta=0}^{2\pi}\left(1 + \frac{R}{L}\cos\theta + \frac{1}{2}\left(\frac{R}{L}\right)^2 (3cos^2\theta - 1)\right)d\theta$$

Menghitung nilai integralnya

$$\int_{\theta=0}^{2\pi}\left(1+\frac{R}{L}\cos\theta+\frac{1}{2}\left(\frac{R}{L}\right)^2(3cos^2\theta-1)\right)d\theta$$

$$=\left(2\pi+0+\frac{1}{2}\left(\frac{R}{L}\right)^2\left(-2\pi+3\int_{\theta=0}^{2\pi}cos^2\theta\,d\theta\right)\right)$$

$$=2\pi\left(1+\left(\frac{R}{L}\right)^2\right)$$

Maka, potensial gravitasi adalah

$$\Phi=-\frac{G\lambda R}{L}2\pi\left(1+\left(\frac{R}{L}\right)^2\right)$$

(a)

9. Gunakan teorema gauss untuk gravitasi

$$\oint g\,dA=4\pi GM_{enc}$$

$$g4\pi r^2=4\pi G\frac{4}{3}\pi\left(\rho_1a^3+\rho_2(b^3-a^3)\right)$$

$$g=\frac{4\pi G}{3r^2}\left(\rho_1a^3+\rho_2(b^3-a^3)\right)$$

(c)

10. Gunakan teorema gauss untuk gravitasi

$$\oint g\,dA=4\pi GM_{enc}$$

$$g4\pi r^2=4\pi G\frac{4}{3}\pi\left(\rho_1a^3+\rho_2(r^3-a^3)\right)$$

$$g=\frac{4\pi G}{3r^2}\left(\rho_1a^3+\rho_2(r^3-a^3)\right)$$

(b)

11. Gunakan teorema gauss untuk gravitasi

$$\oint g\,dA=4\pi GM_{enc}$$

$$g4\pi r^2=4\pi G\frac{4}{3}\pi\rho_1r^3$$

$$g=\frac{4\pi\rho_1Gr}{3}$$

(a)