PAKET 1

PELATIHAN ONLINE

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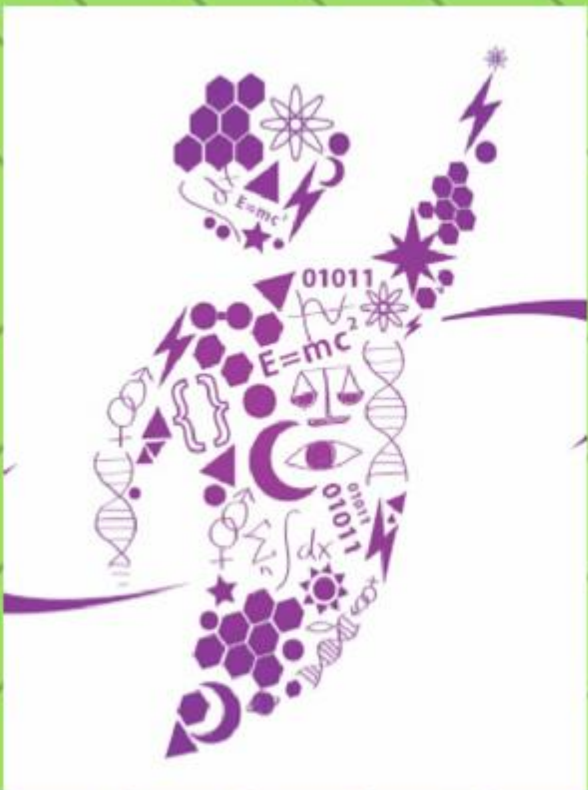

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PEMBAHASAN PAKET 1

1. Gunakan ekspansi untuk trigonometri

$$\sqrt{\frac{x}{\sin x}} = \sqrt{\frac{x}{x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\frac{x^9}{9!}-\cdots}} = \left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\frac{x^8}{9!}-\cdots\right)^{-\frac{1}{2}} \approx 1+\frac{x^2}{6}$$

Maka,

$$\sqrt{\frac{\theta}{\sin\theta}}-1=\frac{\theta^2}{6}\text{ (B)}$$

2. Ekspansikan masing-masing persamaan.

$$\ln[1+x] = x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots = x\left(1-\frac{x}{2}+\frac{x^2}{3}-\cdots\right)$$
$$x(1+ax)^b = x\left(1+abx+\frac{b(b-1)}{2}a^2x^2+\cdots\right)$$

Samakan untuk masing-masing variabel

$$ab=-\frac{1}{2}$$
$$\frac{b(b-1)}{2}a^2=\frac{1}{3}$$

Maka, jika persamaan diatas diselesaikan, akan didapatkan

$$a=\frac{5}{6};b=-\frac{3}{5}$$
$$a+b=\frac{7}{30}\text{ (C)}$$

3. Gunakan Binomial Newton. Asumsikan $\alpha \equiv dx^2+2\beta x$

$$\frac{1}{\sqrt{1+\alpha}}=(1+\alpha)^{-\frac{1}{2}}=1-\frac{\alpha}{2}+\frac{3}{8}\alpha^2+\cdots$$
$$=1-\frac{1}{2}(dx^2+2\beta x)+\frac{3}{8}(d^2x^4+4\beta dx^3+4\beta^2x^2)$$
$$\frac{1}{\sqrt{1+\alpha}}\approx 1-\beta x+\frac{1}{2}(3d-\beta)x^2\text{ (E)}$$

4. Gunakan L'Hopital

$$\lim_{x\to 0}\left(\frac{\sin x+\sin 5x}{6x}\right)=\lim_{x\to 0}\left(\frac{\cos x+5\cos x}{6}\right)=1\text{ (B)}$$

5. Gunakan L'Hopital

$$\lim_{x\to 5}\frac{\sqrt{x+4}-\sqrt{14-x}}{x^2-2x-15}=\lim_{x\to 5}\frac{\frac{1}{2}\frac{1}{\sqrt{x+4}}+\frac{1}{2}\frac{1}{\sqrt{14-x}}}{2x-2}=\frac{1}{24}\text{ (A)}$$

6. Tentukan variabel a dari persamaan garis.

$$0 = 1^2 + 1 + a$$
$$a = -2$$
$$0 = b + 1$$
$$b = -1$$

Gunakan L'Hopital

$$\lim_{x \to 1} \frac{x^2+x+a}{bx+1} = \lim_{x \to 1} \frac{x^2+x-2}{-x+1} = \lim_{x \to 1} \frac{2x+1}{-1} = -3 \text{ (E)}$$

7. Gunakan L'Hopital

$$\lim_{x \to \infty} \frac{\ln x}{x} = \lim_{x \to \infty} \frac{1}{x} = 0 \text{ (C)}$$

8. Gunakan L'Hopital

$$\lim_{x \to 1} \frac{\cos\left(\frac{1}{2}\pi x^k\right)}{\ln x} = -\lim_{x \to 1} \left(\frac{1}{2}\pi k x^k \sin\left(\frac{1}{2}\pi x^k\right)\right) = -\frac{1}{2}\pi k \text{ (B)}$$

9. Turunan pertama $y(x)$

$$\frac{dy}{dx} = 5x^4 - 12x^3 + 26x$$

Turunan kedua $y(x)$

$$\frac{d^2y}{dx^2} = 20x^3 - 36x^2 + 26 \text{ (C)}$$

10. Turunan pertama $y(x)$

$$\frac{dy}{dx} = 3(\sin x + \sec x)^2(\cos x + \sec x \tan x) \text{ (D)}$$

11. Turunan pertama $y(x) = 2^{x^2+8}$

Gunakan aturan rantai dalam menyelesaikannya

$$\ln y = (x^2 + 8)\ln 2$$
$$\frac{d}{dx}\frac{d}{dy}(\ln y) = (2x + 8)\ln 2 \frac{d}{dy}$$
$$\frac{dy}{dx}\frac{1}{y} = (2x + 8)\ln 2$$
$$\frac{dy}{dx} = x2^{x^2+9}\ln 2 \text{ (D)}$$

12. Gunakan aturan differensial implisit

$$\frac{d}{dx}(y^3 + 4x^2y) = \frac{d}{dx}\left(\frac{2x}{y} + y^2x\right)$$

$$\frac{d}{dx}y^3\frac{dy}{dy}+8xy+4x^2\frac{dy}{dx}=\frac{2}{y}+2x\frac{d}{dx}\frac{1}{y}\frac{dy}{dy}+y^2+x\frac{d}{dx}y^2\frac{dy}{dy}$$
$$3y^2\frac{dy}{dx}+8xy+4x^2\frac{dy}{dx}=\frac{2}{y}-\frac{2x}{y^2}\frac{dy}{dx}+y^2+2yx\frac{dy}{dx}$$
$$\frac{dy}{dx}\left(3y^2+4x^2+\frac{2x}{y^2}-2yx\right)=\frac{2}{y}-8xy+y^2$$
$$\frac{dy}{dx}=\frac{2y-8xy^3+y^4}{3y^4+4x^2y^2+2x-2xy^3}\text{ (A)}$$

13. Gunakan aturan rantai dalam menyelesaikan soal ini dan juga sifat-sifat logaritma
Sifat Logaritma
$$\frac{\log_a b}{\log_a c}=\log_c b$$
Maka, $\log_4 f(x)=\frac{\log_e f(x)}{\log_e 4}=\frac{\ln f(x)}{\ln 4}$
$$y(x)=\log_4 f(x)=\frac{\ln f(x)}{\ln 4}$$
$$\frac{dy}{dx}=\frac{1}{\ln 4}\frac{d}{dx}\ln f(x)\frac{df(x)}{df(x)}=\frac{1}{\ln 4}f'(x)\times\frac{1}{f(x)}\text{ (A)}$$

14. Karena *range* sudutnya $-\frac{\pi}{8}<\delta<\frac{\pi}{8}$, maka nilai $\tan\delta$ mempunyai *range* $-1<\tan\delta<1$.
Dapat dikatakan sebagai deret geometri dimana
$$S=1-tan^2 2\delta+tan^4 2\delta-tan^6 2\delta+\cdots$$

Rasionya adalah $r=-tan^2 2\delta$ dan $a=1$
Untuk menghitung geometri tak hingga untuk rasio tersebut adalah
$$S=\frac{a}{1-r}=\frac{1}{1+tan^2 2\delta}=cos^2 2\delta$$
Hitung integralnya
$$\int\sqrt{1-tan^2 2\delta+tan^4 2\delta-tan^6 2\delta+\cdots}\,d\delta=\int\sqrt{S}\,d\delta=\int\cos 2\delta\,d\delta=\frac{1}{2}\sin\delta+c\text{ (D)}$$

15. Gunakan manipulasi pada persamaannya
$$4^x=e^{\ln 4^x}=e^{x\ln 4}$$
$$\int 4^x\,dx=\int e^{x\ln 4}\,dx=\frac{e^{x\ln 4}}{\ln 4}+c=\frac{4^x}{\ln 4}+c\text{ (C)}$$

16. Gunakan metode integral parsial untuk $\int x^2\ln x\;dx$ dimana $dv=x^2dx$ dan $u=\ln x$
$$\int x^2\ln x\;dx=\int u\,dv=uv-\int v\,du=\frac{1}{3}x^3\ln x-\frac{1}{3}\int x^2\,dx=\frac{1}{3}x^3\left(\ln x-\frac{1}{3}\right)+c\text{ (C)}$$

17. Gunakan subtitusi trigonometri untuk menyelesaikannya. Kita akan selesaikan dengan subtitusi variabel.
$$cos^7\varphi=\cos\varphi\,(1-sin^2\varphi)(1-sin^2\varphi)(1-sin^2\varphi)$$

$$cos^7\varphi = \cos\varphi - 3sin^2\varphi\cos\varphi + 3sin^4\varphi\cos\varphi - sin^6\varphi\cos\varphi$$
$$\int cos^7\varphi\ d\varphi = \int(\cos\varphi - 3sin^2\varphi\cos\varphi + 3sin^4\varphi\cos\varphi - sin^6\varphi\cos\varphi)\,d\varphi$$

Karena sinus dan cosinus saling mempunyai hubungan pada differensial, maka jadikan $u = \sin\varphi$.

$$\int cos^7\varphi\ d\varphi = \int \cos\varphi\,d\varphi - 3\int sin^2\varphi\cos\varphi\,d\varphi + 3\int sin^4\varphi\cos\varphi\,d\varphi - \int sin^6\varphi\cos\varphi\,d\varphi$$
$$\frac{du}{d\varphi} = \cos\varphi$$
$$\int cos^7\varphi\ d\varphi = \sin\varphi - sin^3\varphi + \tfrac{3}{5}sin^5\varphi - \tfrac{1}{7}sin^7\varphi + c \text{ (E)}$$

18. Lakukan manipulasi persamaan

$$\operatorname{cosec}x \times \frac{\operatorname{cosec}x + \cot x}{\operatorname{cosec}x + \cot x} = \frac{cosec^2x + \cot x\operatorname{cosec}x}{\operatorname{cosec}x + \cot x}$$
$$\int \operatorname{cosec}x\ dx = \int \frac{cosec^2x + \cot x\operatorname{cosec}x}{\operatorname{cosec}x + \cot x}dx$$

Jadikan $u = \operatorname{cosec}x + \cot x$

$$\frac{du}{dx} = -(\operatorname{cosec}x\cot x + cosec^2x)$$
$$\int\frac{cosec^2x+\cot x\operatorname{cosec}x}{\operatorname{cosec}x+\cot x}dx = -\int\frac{du}{u} = -\ln u + c = -\ln(\operatorname{cosec}x + \cot x) + c \text{ (B)}$$

19. Gunakan metode parsial untuk menyelesaikannya dimana $u = \arcsin x$ dan $dv = dx$

$$\int \arcsin x\ dx = \int u\,dv = uv - \int v\,du$$

Untuk menyelesaikan persamaan differensial $\frac{du}{dx}$, lakukan invers terlebih dahulu

$$\sin u = x$$
$$\frac{d}{dx}\frac{d}{du}\sin u = \frac{d}{dx}\frac{d}{du}x$$
$$\frac{du}{dx} = \frac{1}{\cos x} = \frac{1}{\sqrt{1-x^2}}$$

$$\int \arcsin x\ dx = x\arcsin x + \sqrt{1-x^2} + c \text{ (C)}$$

20. Gunakan sedikit manipulasi dalam mengerjakanya, yaitu pecahan rasional dikarenakan tidak bisa di subtitusikan secara langsung.

$$\frac{9x+8}{4x^2+11x+6}=\frac{9x+8}{(4x+3)(x+2)}=\frac{A}{4x+3}+\frac{B}{x+2}=\frac{x(A+4B)+3B+2A}{(4x+3)(x+2)}$$

Lakukan eliminasi untuk mendapatkan variabel A dan B.

$$A=1$$
$$B=2$$
$$\frac{9x+8}{4x^2+11x+6}=\frac{1}{4x+3}+\frac{2}{x+2}$$

$$\int\frac{9x+8}{4x^2+11x+6}dx=\int\left(\frac{1}{4x+3}+\frac{2}{x+2}\right)dx=\int\frac{dx}{4x+3}+2\int\frac{dx}{x+2}$$
$$=\frac{1}{4}\ln(4x+3)+2\ln(x+2)+c \text{ (B)}$$