**Explanation:** The electric field intensity due to a system of discrete charges in free space can be determined by the equation given below:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{N} \frac{q_k(\mathbf{r} - \mathbf{r}_k)}{|\mathbf{r} - \mathbf{r}_k|^3} \quad [\text{V/m}].$$

Here $\varepsilon_0$ is the permittivity (dielectric constant) of the medium, $N$ is the number of charges, $q_k$ and $\mathbf{r}_k$ denote the amount and location of the $k$th charge, respectively, and $\mathbf{r}$ denotes the location of the observation point. The permittivity of the free space is

$$\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9} \quad [\text{F/m}].$$

Note that the position vectors can be given as $\mathbf{r} = (x, y, z) = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$ and $\mathbf{r}_k = (x_k, y_k, z_k) = x_k\hat{\mathbf{x}} + y_k\hat{\mathbf{y}} + z_k\hat{\mathbf{z}}$, where $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ represent the unit vectors at the direction of $x$, $y$, and $z$, respectively.

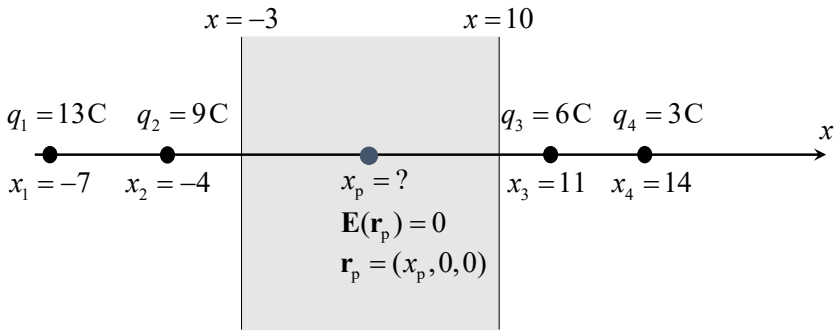


Figure 1: Illustration of the problem.

**Question:** Four charges are located to the $x$ axis as shown in Figure 1. The coordinates of the charge locations are $(-7, 0, 0)$, $(-4, 0, 0)$, $(11, 0, 0)$, and $(14, 0, 0)$, and the amount of charges are $q_1 = 13$ C, $q_2 = 9$ C, $q_3 = 6$ C, and $q_4 = 3$ C, respectively. It can be predicted that the $x$ component of the electric field intensity in the interval $x \in [-4, 11]$ will be in positive $\hat{\mathbf{x}}$ direction when $x$ is close to $x = -4$, it will be zero at a certain point ($x = x_{\text{p}}$), and then it will be in negative $\hat{\mathbf{x}}$ direction as $x$ gets closer to 11.

Determine the location where the electric field intensity is zero at the interval $x \in [-3, 10]$ with a tolerance of $tol = 10^{-10}$, using

(a) The Bisection Method,

(b) Newton's Method,

(c) The Secant Method.

**Notes:**

1. A report should be prepared as explained in **Homework and Project Report Preparation Guideline**.
2. The codes and the report should be student's own work.
3. Include the following data and formulation to your report:
   - Show the iteration results and errors in a table.
   - Plot the errors in a single graph using **loglog(·)** or **semilogy(·)** command.
   - (Optional) Find the exact point that makes the electric field intensity zero analytically.
4. Try to answer the following questions:
   - Compare and comment on the convergence rates and performances of the aforementioned methods.
   - If the tolerance is changed to $10^{-15}$, how is the result affected? Why?
   - If the tolerance is changed to $10^{-16}$, how is the result affected? Why?
5. Be careful on calculating the derivative of the function for the Newton's Method, since the function includes absolute value/norm operator at the denominator. Any point can be chosen as the initial approximation, specifically you can choose the midpoint as the initial approximation.
6. End points can be chosen as the initial approximations for the Secant Method.