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GEBZE
TEKNİK ÜNİVERSİTESİ

MAT214 - Numerical Methods
Homework - 4

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Note: Show all your operations in detail. The solutions that do not have enough details will be graded with zero points.

1. (P.180 Q.6c) Use the most accurate three point formula to determine derivative of $f(x)$ at each point in the following table.

x	$f(x)$
1.1	1.52918
1.2	1.64024
1.3	1.70470
1.4	1.71277

2. (P.200 Q.2a-6a-10a) Approximate the following integral

$$\int_{-0.25}^{0.25} \cos^2(x)dx$$

using

 (a) Trapezoidal rule,
 (b) Simpson's rule,
 (c) Midpoint rule.

3. (P.208 Q.2a-4a-6a) Approximate the following integral

$$\int_{-0.5}^{0.5} \cos^2(x)dx$$

using

 (a) Composite trapezoidal rule with $n = 4$,
 (b) Composite Simpson's rule with $n = 4$,
 (c) Composite midpoint rule with $n + 2 = 4 + 2 = 6$ subintervals.

1. (P.180 Q.6c) Use the most accurate three point formula to determine derivative of $f(x)$ at each point in the following table.

x	$f(x)$
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For the first and the last point we should use three point endpoint formula since $h=0,1$ for the other two cases we should use the central three point formula.

Three point end point formula

$$\frac{1}{2h}\left[-3f(x_0)+4f(x_0+h)-f(x_0+2h)\right]$$

$$f'(1,1)=\frac{1}{0,2}\left[-3(1,52918)+4(1,64024)-1,70470\right]$$

$$f'(1,1)=\frac{0.26872}{0,2}=1,3436.$$

Three point midpoint formula

$$\frac{1}{2}\left[f(x_0+h)-f(x_0-h)\right]$$

$$f'(1,2)=\frac{1}{0,2}(1,70470-1,52918)$$

$$f'(1,2)=0.8776$$

$$f'(1.3) = \frac{1}{0.2}\left[1.71277 - 1.64024\right]$$

$$f'(1.3) = 0.36265$$

three point midpoint formula

Here, we are going to use three point endpoint

$h = -0.1$

$$-\frac{1}{0.2}\left[-1.64024 + 4(1.70470) - 3(1.71277)\right]$$

$$f'(1.4) = -0.20125$$

2. (P.200 Q.2a-6a-10a) Approximate the following integral

$$\int_{-0.25}^{0.25} \cos^2(x)dx$$

using

(a) Trapezoidal rule,

(b) Simpson's rule,

(c) Midpoint rule.

$$\boxed{\begin{array}{l}\text{a) Trapezoidal Rule (2.1);} \\ (b-a)\cdot\dfrac{f(a)+f(b)}{2}\end{array}}$$

$\cos^2(-0.25) = 0.938791280$

$\cos^2(0.25) = 0.93879128094$

$$0.5\cdot\frac{\cos^2(-0.25)+\cos^2(0.25)}{2} = 0.46939564047$$

b) Simpson's rule (2,2);

$$\frac{f(a)+4f\left(\frac{a+b}{2}\right)+f(b)}{6}(b-a)$$

$$0.5\left(\frac{0{,}93879128094+4+0{,}93879128094}{6}\right)$$

$$=0{,}489779854682$$

c) Midpoint Rule (2,3);

$$f\left(\frac{a+b}{2}\right)(b-a)$$

$$\int_{-0{,}25}^{0{,}25}\cos^2(x)dx = 0{,}5$$

3. (P.208 Q.2a-4a-6a) Approximate the following integral

$$\int_{-0.5}^{0.5} \cos^2(x)dx$$

using

(a) Composite trapezoidal rule with $n = 4$,

(b) Composite Simpson's rule with $n = 4$,

(c) Composite midpoint rule with $n + 2 = 4 + 2 = 6$ subintervals.

Recall the formulas specified in (2,1), (2,2) and (2,3).

$$\Delta x = \frac{(0{,}5) - (-0{,}5)}{4} = \frac{1}{4} = 0{,}25$$

$$\underbrace{\int_{-0{,}5}^{-0{,}25} \cos^2(x)\, dx}_{a} + \underbrace{\int_{-0{,}25}^{0} \cos^2(x)\, dx}_{b} + \underbrace{\int_{0}^{0{,}25} \cos^2(x)\, dx}_{c} + \underbrace{\int_{0{,}25}^{0{,}5} \cos^2(x)\, dx}_{d}$$

$$a = \int_{-0{,}5}^{-0{,}25} \cos^2(x)\, dx = \frac{\left((0{,}25)\left(f(-0{,}25), f(-0{,}5)\right)\right)}{2}$$

$$= \frac{0{,}93879128094 + 0{,}77015115293}{8}$$

$$= 0{,}21361780423$$

$$b = \int_{-0,25}^{0} \cos^2(x)\,dx = \frac{(0{,}25)\left(f(0) + f(-0{,}25)\right)}{2}$$

$$\frac{1 + 0{,}93879128094}{8} = 0{,}24234891011$$

$$c)\ \int_{0}^{0,25} \cos^2(x)\,dx = (0{,}25)\left(\frac{f(0.25) + f(0)}{2}\right)$$

$$\frac{1 + 0{,}93879128094}{8} = 0.24234891011$$

$$d = \int_{0,25}^{0,5} (0{,}25)\left(\frac{f(0{,}5) + f(0{,}25)}{2}\right)$$

$$\frac{0{,}93879128094 + 0{,}77015115293}{8} = 0.21361780423$$

$$2(0.24234891011) + 2(0{,}21361780423)$$

$$\boxed{= 0{,}91193428}$$

d) Simpson's rule when $n=4$; $\Delta x = 0.25$

$$\underbrace{\int_{-0,5}^{-0,25} \cos^2(x)\,dx}_{a} + \underbrace{\int_{-0,25}^{0} \cos^2(x)\,dx}_{b} + \underbrace{\int_{0}^{0,25} \cos^2(x)\,dx}_{c} + \underbrace{\int_{0,25}^{0,5} \cos^2(x)\,dx}_{d}$$

a) $$\int_{-0,5}^{-0,25} \cos^2(x)\,dx = (0,25)\left(\frac{f(-0,25)+f(-0,5)+f(0,375)\cdot 4}{6}\right)$$

$$\frac{0,9387912809 4 + 0,7701511529 3 + 3,4633777377 5}{24}$$

$$= 0,2155133404 8$$

b) $$\int_{-0,25}^{0} \cos^2(x)\,dx = (0,25)\left(\frac{f(0)+f(-0,25)+4f(-0,125)}{6}\right)$$

$$\frac{1 + 0,9387912809 4 + 3,9378248634 2}{24}$$

$$= 0,2448590051 8$$

$$c = \int_0^{0{,}25} \cos^2(x)\,dx = (0{,}25)\left(\frac{f(0{,}25) + f(0) + 4f(0{,}125)}{8}\right)$$

$$= 0{,}244859005 18$$

$$d = \int_{0{,}25}^{0{,}5} \cos^2(x)\,dx = (0{,}25)\left(\frac{f(0{,}5) + f(0{,}25) + 4f(0{,}375)}{8}\right)$$

$$= 0{,}2155133 4048$$

Sum of all partial integrations' results:

$$= 2(0{,}21551334048) + 2(0{,}24485900518)$$

$$= 0.92074469132$$

c) $n+2 = n+4 = 6$ $\qquad h = \frac{b-a}{n+2}$

$x_j = a + (j+1) \cdot h$

$$\int_a^b f(x) \cdot dx = 2 \cdot h \sum_{j=0}^{n/2} f(x_{2j}) + \frac{b-a}{6} h^2 f''(\mu)$$

$$\int_{-0{,}5}^{0{,}5} \cos^2 x \, dx = 2h \left[f(x_0) + f(x_2) + f(x_4) \right] \quad h = \frac{1}{6}$$

$x_0 = a + h = \frac{-2}{6}$ $\qquad x_2 = 0$

$$= \frac{2}{6} \left[\cos^2\left(\frac{-2}{6}\right) + \cos^2(0) + \cos^2\left(\frac{2}{6}\right) \right]$$

$= 0{,}928629086$