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GEBZE
TEKNİK ÜNİVERSİTESİ

MAT214 - Numerical Methods
Homework-3

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Note: Show all your operations in detail. The solutions that do not have enough details will be graded with zero points.

1. (P.112 Q.6a) Use appropriate Lagrange interpolating polynomials of degrees one, two, and three to approximate $f(0.43)$, if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, and $f(0.75) = 4.48169$.

2. (P.113 Q.14b) Construct the Lagrange interpolating polynomials for $f(x) = \log_{10}(x)$, using the samples of $f(x)$ at $x_0 = 3.0$, $x_1 = 3.2$, $x_2 = 3.5$, and $n = 2$, and find a bound for the absolute error on the interval $[x_0, x_2]$.

3. (P.121 Q.2b) Use Neville's method to obtain the approximations for Lagrange interpolating polynomials of degrees one, two, and three to approximate $f(0)$, if $f(-0.5) = 1.93750$, $f(-0.25) = 1.33203$, $f(0.25) = 0.800781$, and $f(0.5) = 0.687500$.

4. (P.121 Q.6) Neville's method is used to approximate $f(0.5)$, giving the following table:

$x_0 = 0$	$P_0 = 0$		
$x_1 = 0.4$	$P_1 = 2.8$	$P_{0,1} = 3.5$	
$x_2 = 0.7$	P_2	$P_{1,2}$	$P_{0,1,2} = \frac{27}{7}$

Determine $P_2 = f(0.7)$.

1. (P.112 Q.6a) Use appropriate Lagrange interpolating polynomials of degrees one, two, and three to approximate $f(0.43)$, if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, and $f(0.75) = 4.48169$.

x	f(x)
0	1
0.25	1.64872
0,5	2.71828
0,75	4.48169

One;

$$\frac{(x-x_1)}{(x_0-x_1)} \cdot f(x_0) = \frac{(x-0{,}5)}{0{,}25-0{,}5} \cdot 1.64872$$

$$\frac{(x-x_0)}{(x_1-x_0)} \cdot f(x_1) = \frac{(x-0{,}25)}{0{,}5-0{,}25} \cdot 2.71828$$

$$P_1(0{,}43) = \underbrace{\frac{(0{,}43-0{,}5)}{(0{,}25-0{,}5)} \cdot 1.64872}_{} + \frac{(0.43-0{,}25)}{0{,}5-0{,}25} \cdot 2.71828$$

$$0.46164 16 + 1{,}9571616 = 2{,}4188032$$

Two;

$$P_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

$x_0 = 0 \quad x_1 = 0{,}25 \quad x_2 = 0{,}5$

$x_0 = 0 \quad x_1 = 0{,}25 \quad x_2 = 0{,}5 \quad x_3 = 0{,}75$

$$P_2(0{,}43) = \frac{(0{,}43-0{,}25)(0{,}43-0{,}5)}{(0-0{,}25)(0-0{,}5)} 1 + \frac{(0{,}43-0)(0{,}43-0{,}5)}{(0{,}25-0)(0{,}25-0{,}5)} 1.64872$$

$$+ \frac{(0{,}43-0)(0{,}43-0{,}25)}{(0{,}5-0)(0{,}5-0{,}25)} 2{,}71828$$

$$= -0{,}1008 + 0{,}794023552 + 1{,}683158976$$

$$= 2{,}376382528$$

Three:

$x_0 = 0 \qquad x_1 = 0{,}25 \qquad x_2 = 0{,}5 \qquad x_3 = 0{,}75$

$$P_3(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1)$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

$$P_3(0{,}43) = \frac{(0{,}43-0{,}25)(0{,}43-0{,}5)(0{,}43-0{,}75)}{(0-0{,}25)(0-0{,}5)(0-0{,}75)} 1$$

$$+ \frac{(0{,}43-0)(0{,}43-0{,}5)(0{,}43-0{,}75)}{(0{,}25-0)(0{,}25-0{,}5)(0{,}25-0{,}75)} 1{,}64872$$

$$+\frac{(0{,}43-0)(0{,}43-0{,}25)(0{,}43-0{,}75)}{(0{,}5-0)(0{,}5-0{,}25)(0{,}5-0{,}75)}\cdot 2{,}71828$$

$$+\frac{(0{,}43-0)(0{,}43-0{,}25)(0{,}43-0{,}5)}{(0{,}75-0)(0{,}75-0{,}25)(0{,}75-0{,}5)}\cdot 4{,}48164$$

$$= 2{,}3606047 3408$$

2. (P.113 Q.14b) Construct the Lagrange interpolating polynomials for $f(x) = \log_{10}(x)$, using the samples of $f(x)$ at $x_0 = 3.0$, $x_1 = 3.2$, $x_2 = 3.5$, and $n = 2$, and find a bound for the absolute error on the interval $[x_0, x_2]$.

x	f(x)
3,0	0,47712125 4
3,2	0,50514 9978
3,5	0,544068044

To calculate error;

$$f'(x) = \frac{1}{x\ln(10)}$$

$$f''(x) = \frac{-1}{x^2\ln(10)}$$

$$f'''(x) = \frac{2}{x^3\cdot\ln(10)}$$

$$P(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\cdot f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_0)} f(x_2)$$

$$P(x) = \frac{(x-3{,}2)(x-3{,}5)}{(3-3{,}2)(3-3{,}5)}\cdot f(3) + \frac{(x-3)(x-3{,}5)}{(3{,}2-3)(3{,}5-3)} f(3{,}2)$$

$$+\frac{(x-3)(x-3{,}2)}{(3{,}5-3)(3{,}5-3{,}2)} f(3{,}5)$$

$$P(x) = \frac{(x-3{,}2)(x-3{,}5)}{0{,}1}\, 0{,}477121254 + \frac{(x-3{,}0)(x-3{,}5)}{0{,}06}\, 0{,}505149978$$

$$+ \frac{(x-3{,}0)(x-3{,}2)}{0{,}15}\, 0{,}544068044$$

$$P(x) = \Bigg(\overbrace{\left(x^2-(6{,}7)x + 11{,}2\right)/10}^{L_0}\Bigg)\, \overbrace{0{,}477121254}^{f(x_0)} + \overbrace{\frac{x^2-(6{,}5)x+10{,}5}{-6\times10^{-2}}}^{L_1} \cdot$$

$$\ldots \underbrace{0{,}505149978}_{f(x_1)} + \underbrace{\frac{(x^2-(6{,}2)x+9{,}6)}{0{,}15}}_{L_2} \cdot \underbrace{0544068044}_{f(x_2)}$$

$$\boxed{\begin{aligned} P(x) &= (10x^2-67x+112)\cdot 0{,}477121254 + \frac{x^2-(6{,}5)x+10{,}5}{-6\cdot10^{-2}}\cdot 0{,}505149978 \\ &+ \frac{x^2-6{,}2x+9{,}6}{0{,}15}\cdot 0544068044 \end{aligned}}$$

Error calculation interval $[3, 3{,}5]$

$$\frac{f^{n+1}(K(x))}{n+1!}\,(x-x_0)(x-x_1)\text{------}(x-x_n)$$

To make $f'''(x)$ maximum $x = 3,$

$$\frac{f'''(K(x))}{3!}\,(x-3)(x-3{,}2)(x-3{,}5) \qquad (K(x))^{-3} = 3^{-3}$$

$$\frac{2}{27\ln 10 \cdot 3!}\,(x-3)(x-3{,}2)(x-3{,}5)$$

$x^3 - 9.7x^2 + 31.3x - 33.6 = g(x) \to$ Take its derivative to fin maximum

$3x^2 - 19.4x + 31.3 \longrightarrow$ We have critical points;

$x_1 = \dfrac{97+\sqrt{19}}{30}$ $\quad x_2 = \dfrac{97-\sqrt{19}}{30}$ $\quad x_1 \cong 3{,}37$ $\quad x_3 \cong 3{,}08$

$g(3.37) = |-0{,}082| = 0{,}0082$

$g(3{,}08) = |0{,}0040| = 0{,}0040$

Maximum value $g(3{,}37)$

So, maximum absolute error;

$$\frac{|f'''(\xi(x))|}{3!}\,|(x-3)(x-3{,}2)(x-3{,}5)| \le \frac{2}{27\ln 10}\cdot 0{,}0082 \cong 4{,}40\times 10^{-5}$$

3. (P.121 Q.2b) Use Neville's method to obtain the approximations for Lagrange interpolating polynomials of degrees one, two, and three to approximate $f(0)$, if $f(-0.5) = 1.93750$, $f(-0.25) = 1.33203$, $f(0.25) = 0.800781$, and $f(0.5) = 0.687500$.

x	f(x)
-0,5	1,93750
-0,25	1,33203
0	?
0,25	0.800781
0,5	0,687500

$Q_{(0,0)} = 1{,}93750$

$Q_{(1,0)} = 1{,}33203$

$Q_{(2,0)} = 800781$

$Q_{(3,0)} = 0{,}687500$

$$Q_{(1,1)} = \frac{(x-x_0)\cdot Q_{(1,0)} - (x-x_1)\,Q_{(0,0)}}{x_1 - x_0}$$

$$\frac{(x+0{,}5)\cdot 1.33203 - (x+0{,}25)\,1{,}95750}{0{,}25} = \boxed{0{,}72656}$$

$$Q_{(2,1)} = \frac{(x-x_1)\,Q_{(2,0)} - (x-x_2)\cdot Q_{(1,0)}}{x_2-x_1} \Rightarrow 1{,}0664055$$

$$Q_{(3,1)} = \frac{(x-x_2)\,Q_{(3,0)} - (x-x_3)\,Q_{(2,0)}}{x_3-x_2} = 0{,}914062$$

$$Q_{(2,2)} = \frac{(x-x_0)\,Q_{(2,1)} - (x-x_2)\cdot Q_{(1,1)}}{x_2-x_0} = \boxed{0{,}953123666}$$

$$Q_{(3,2)} = \frac{(x-x_1)\,Q_{(3,1)} - (x-x_3)\,Q_{(2,1)}}{x_3-x_1} = 1{,}01564333$$

$$Q_{(3,3)} = \frac{(x-x_0)\cdot Q_{(3,2)} - (x-x_3)\cdot Q_{(2,2)}}{x_3-x_0} = \boxed{0{,}984373666}$$

4. (P.121 Q.6) Neville's method is used to approximate $f(0.5)$, giving the following table:

$x_0 = 0$	$P_0 = 0$		
$x_1 = 0.4$	$P_1 = 2.8$	$P_{0,1} = 3.5$	
$x_2 = 0.7$	P_2	$P_{1,2}$	$P_{0,1,2} = \frac{27}{7}$

Determine $P_2 = f(0.5)$.

$$P_{0,1,2} = \frac{1}{x_2 - x_0}\left[(x - x_0) P_{(1,2)} - (x - x_2) P_{(0,1)}\right]$$

$$P_{0,1,2} = \frac{1}{0{,}7}\left(0{,}5 \cdot P_{(1,2)} + 0.2 \cdot 3{,}5\right) = \frac{27}{7}$$

$$P_{(1,2)} = 4$$

$$P_{1,2} = \frac{1}{x_2 - x_1}\left[(x - x_1) P_2 - (x - x_2) P_1\right]$$

$$P_{1,2} = \frac{1}{0{,}3}\left[(0{,}5 - 0{,}4) P_2 + 0{,}2 \cdot 2{,}8\right] = 4$$

$$P_2 = 6{,}4$$

$$(0{,}1) P_2 + 0{,}56 = 1{,}2$$