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GEBZE
TEKNİK ÜNİVERSİTESİ

MAT214 - Numerical Methods
Homework - 5

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Elektronik Mühendisliği

Note: Show all your operations in detail. The solutions that do not have enough details will be graded with zero points.

1. (P.396 Q.8) Consider the four 3×3 linear systems having the same coefficient matrix:

$$\begin{aligned} 2x_1 - 3x_2 + x_3 &= 2 \\ x_1 + x_2 - x_3 &= -1 \\ -x_1 + x_2 - 3x_3 &= 0 \end{aligned} \qquad \begin{aligned} 2x_1 - 3x_2 + x_3 &= 6 \\ x_1 + x_2 - x_3 &= 4 \\ -x_1 + x_2 - 3x_3 &= 5 \end{aligned}$$

$$\begin{aligned} 2x_1 - 3x_2 + x_3 &= 0 \\ x_1 + x_2 - x_3 &= 1 \\ -x_1 + x_2 - 3x_3 &= -3 \end{aligned} \qquad \begin{aligned} 2x_1 - 3x_2 + x_3 &= -1 \\ x_1 + x_2 - x_3 &= 0 \\ -x_1 + x_2 - 3x_3 &= 0 \end{aligned}$$

(a) Solve the linear systems by applying Gaussian elimination to the augmented matrix

$$\left[\begin{array}{ccc:cccc} 2 & -3 & 1 & 2 & 6 & 0 & -1 \\ 1 & 1 & -1 & -1 & 4 & 1 & 0 \\ -1 & 1 & -3 & 0 & 5 & -3 & 0 \end{array}\right].$$

(b) Solve the linear systems by finding and multiplying by the inverse of

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & -3 \end{bmatrix}.$$

(c) Which method requires more operations?

1. (P.396 Q.8) Consider the four 3×3 linear systems having the same coefficient matrix:

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$$\begin{aligned} 2x_1 - 3x_2 + x_3 &= 0 \\ x_1 + x_2 - x_3 &= 1 \\ -x_1 + x_2 - 3x_3 &= -3 \end{aligned} \qquad \begin{aligned} 2x_1 - 3x_2 + x_3 &= -1 \\ x_1 + x_2 - x_3 &= 0 \\ -x_1 + x_2 - 3x_3 &= 0 \end{aligned}$$

(a) Solve the linear systems by applying Gaussian elimination to the augmented matrix

$$\left[\begin{array}{ccc:cccc} 2 & -3 & 1 & 2 & 6 & 0 & -1 \\ 1 & 1 & -1 & -1 & 4 & 1 & 0 \\ -1 & 1 & -3 & 0 & 5 & -3 & 0 \end{array}\right] \xrightarrow{R_2 - \left(\frac{1}{2}\right) R_1 \to R_2} \left[\begin{array}{ccc:cccc} 2 & -3 & 1 & 2 & 6 & 0 & -1 \\ 0 & 5/2 & -3/2 & -2 & 1 & 1 & 1/2 \\ -1 & 1 & -3 & 0 & 5 & -3 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 - \left(\frac{-1}{2}\right) R_1 \to R_3} \left[\begin{array}{ccc:cccc} 2 & -3 & 1 & 2 & 6 & 0 & -1 \\ 0 & 5/2 & -3/2 & -2 & 1 & 1 & 1/2 \\ 0 & -1/2 & -5/2 & 1 & 8 & -3 & -1/2 \end{array}\right]$$

$$\xrightarrow{R_3 \leftarrow R_3 - \left(\frac{-1}{5}\right) R_2} \left[\begin{array}{ccc:cccc} 2 & -3 & 1 & 2 & 6 & 0 & -1 \\ 0 & 5/2 & -3/2 & -2 & 1 & 1 & 1/2 \\ 0 & 0 & -14/5 & \frac{3}{5} & \frac{41}{5} & \frac{-14}{5} & \frac{-4}{5} \end{array}\right]$$

1. Denklem

$$2x_1 - 3x_2 + x_3 = 2$$

$$0x_1 + \frac{5x_2}{2} - \frac{3x_3}{2} = -2$$

$$0x_1 + 0x_2 - \frac{14x_3}{5} = 3/5$$

$x_1 = \frac{-2}{7}$ $\quad x_2 = \frac{-13}{14}$ $\quad x_3 = \frac{-3}{14}$

2. Denklem

$$2x_1 - 3x_2 + x_3 = 6$$

$$0x_1 + \frac{5x_2}{2} - \frac{3x_3}{2} = 1$$

$$0x_1 + 0x_2 - \frac{14x_3}{5} = \frac{41}{5}$$

$x_1 = \frac{17}{5}$ $\quad x_2 = \frac{-19}{14}$ $\quad x_3 = \frac{-41}{14}$

3. Denklem

$$2x_1 - 3x_2 + x_3 = 0$$

$$0x_1 + \frac{5x_2}{2} - \frac{3x_3}{2} = 1$$

$$0x_1 + 0x_2 - \frac{14x_3}{5} = \frac{-14}{15}$$

$$x_{1,2,3} = 1$$

4. Denklem

$$2x_1 - 3x_2 + x_3 = -1$$

$$0x_1 + \frac{5x_2}{2} - \frac{3x_3}{2} = \frac{1}{2}$$

$$0x_1 + 0x_2 - \frac{14x_3}{5} = \frac{-4}{10}$$

$$x_1 = \frac{-1}{7} \quad x_2 = \frac{2}{7} \quad x_3 = \frac{1}{7}$$

(b) Solve the linear systems by finding and multiplying by the inverse of

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & -3 \end{bmatrix}. \qquad A^{(-1)} = ?$$

$$= \left[\begin{array}{ccc|ccc} 2 & -3 & 1 & 1 & 0 & 0 \\ 1 & 1 & -1 & 0 & 1 & 0 \\ -1 & 1 & -3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1/2 \to R_1} \left[\begin{array}{ccc|ccc} 1 & -3/2 & 1/2 & 1/2 & 0 & 0 \\ 1 & 1 & -1 & 0 & 1 & 0 \\ -1 & 1 & -3 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 - R_1 \to R_2} \left[\begin{array}{ccc|ccc} 1 & -3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 5/2 & -3/2 & -1/2 & 1 & 0 \\ -1 & 1 & -3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 + R_1 \to R_3} \left[\begin{array}{ccc|ccc} 1 & -3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 5/2 & -3/2 & -1/2 & 1 & 0 \\ 0 & -1/2 & -5/2 & 1/2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2/\left(\frac{5}{2}\right) \to R_2} \left[\begin{array}{ccc|ccc} 1 & -3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & -3/5 & -1/5 & 2/5 & 0 \\ 0 & -1/2 & -5/2 & 1/2 & 0 & 1 \end{array}\right] \xrightarrow{R_3 + \left(\frac{1}{2} R_2\right) \to R_3} \left[\begin{array}{ccc|ccc} 1 & -3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & -3/5 & -1/5 & 2/5 & 0 \\ 0 & 0 & -14/5 & 2/5 & 1/5 & 1 \end{array}\right]$$

$$\xrightarrow{R_3/\frac{-14}{5} \to R_3} \left[\begin{array}{ccc|ccc} 1 & -3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & -3/5 & -1/5 & 2/5 & 0 \\ 0 & 0 & 1 & -1/7 & -1/14 & -5/14 \end{array}\right] \xrightarrow{R_2 + \left(\frac{3}{5}\right) R_3 \to R_2} \left[\begin{array}{ccc|ccc} 1 & -3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & -2/7 & 5/14 & -3/14 \\ 0 & 0 & 1 & -1/7 & -1/14 & -5/14 \end{array}\right]$$

$$\xrightarrow{R_1 - \frac{R_3}{2} \to R_1} \left[\begin{array}{ccc|ccc} 1 & -3/2 & 0 & 4/7 & 1/28 & 5/28 \\ 0 & 1 & 0 & -2/7 & 5/14 & -3/14 \\ 0 & 0 & 1 & -1/7 & -1/14 & -5/14 \end{array}\right] \xrightarrow{R_1 + \left(\frac{3}{2}R_2\right) \to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/7 & 4/7 & -1/7 \\ 0 & 1 & 0 & -2/7 & 5/14 & -3/14 \\ 0 & 0 & 1 & -1/7 & -1/14 & -5/14 \end{array}\right]$$

$$A^{(-1)} = \begin{bmatrix} \frac{1}{7} & \frac{4}{7} & \frac{-1}{7} \\ \frac{-2}{7} & \frac{5}{7} & \frac{-3}{7} \\ \frac{-1}{7} & \frac{-1}{14} & \frac{-5}{14} \end{bmatrix}$$

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1. Denklem;

$$A^{(-1)} \times \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1/7 & 4/7 & -1/7 \\ -2/7 & 5/7 & -3/14 \\ -1/7 & -1/14 & -5/14 \end{bmatrix} \times \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2/7 \\ -13/14 \\ -3/14 \end{bmatrix}$$

$x_1 = -2/7$

$x_2 = \frac{-13}{14}$

$x_3 = \frac{-3}{14}$

2. Denklem;

$$A^{(-1)} \times \begin{bmatrix} 6 \\ 4 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1/7 & 4/7 & -1/7 \\ -2/7 & 5/7 & -3/14 \\ -1/7 & -1/14 & -5/14 \end{bmatrix} \times \begin{bmatrix} 6 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 17/7 \\ -19/14 \\ -41/14 \end{bmatrix}$$

$x_1 = 17/7$

$x_2 = -19/14$

$x_3 = -41/14$

3. Denklem;

$$A^{(-1)} \times \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1/7 & 4/7 & -1/7 \\ -2/7 & 5/7 & -3/14 \\ -1/7 & -1/14 & -5/14 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$x_1 = 1$

$x_2 = 1$

$x_3 = 1$

4. Denklem;

$$A^{(-1)} \times \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1/7 & 4/7 & -1/7 \\ -2/7 & 5/7 & -3/14 \\ -1/7 & -1/14 & -5/14 \end{bmatrix} \times \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1/7 \\ 2/7 \\ 1/7 \end{bmatrix}$$

$x_1 = -1/7$

$x_2 = 2/7$

$x_3 = 1/7$

(c) Which method requires more operations?

The second way (b) requires more operations than first way does because taking the inverse of a matrix takes a bit time to calculate.