GEBZE TECHNICAL UNIVERSITY

ELECTRONIC ENGINEERING

MATH214 – NUMERICAL ANALYSIS

FINAL PROJECT

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Problem definition and formulas;

The voltage-current characteristic of a diode at forwarded biased region (VD > 0) is measured at a junction temperature of 25 C and the measured values can be shown in Figure 2 with the approximated value. In the data file given the first column is the voltage in [V] and the second column is the measured current in [A]. This diode is used in the RL circuit shown in Figure 1, where the switch is closed at t = t0. Formulate and write a code to calculate the current i(t), voltages v1(t), v2(t), and vD(t) for VS = 2 V, L = 0.98 H, R = 14.2 Ω, t0 = 0 s, and i0 = 0 A for the time interval t ∈ [0, 600] ms using the step sizes Δt = 25 ms and Δt = 2.5 ms. Assuming that vD(0) = 0 V for t = 0 s.

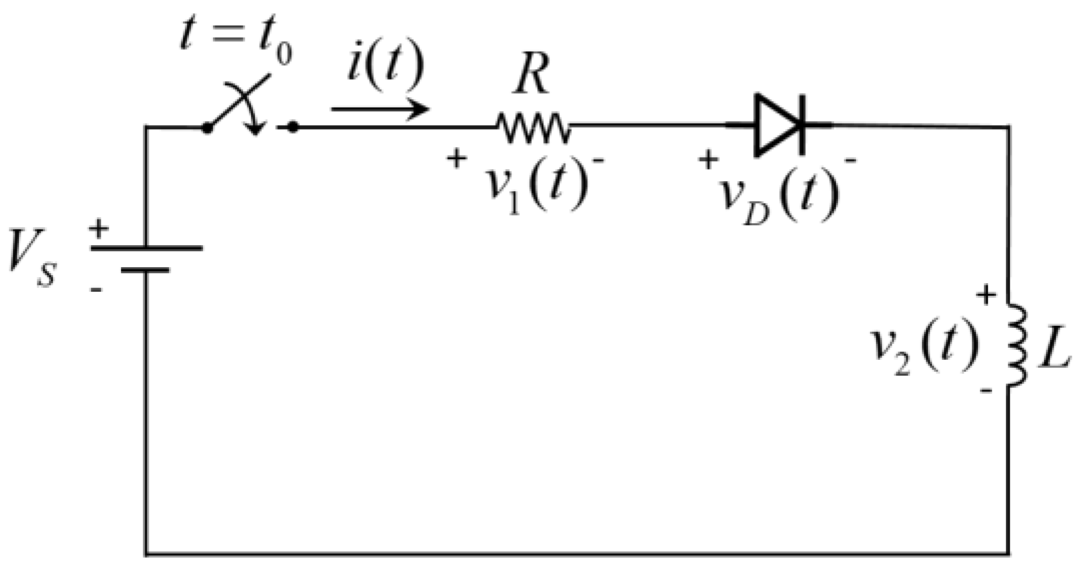

Figure 1 The circuit to analyze for the final project.

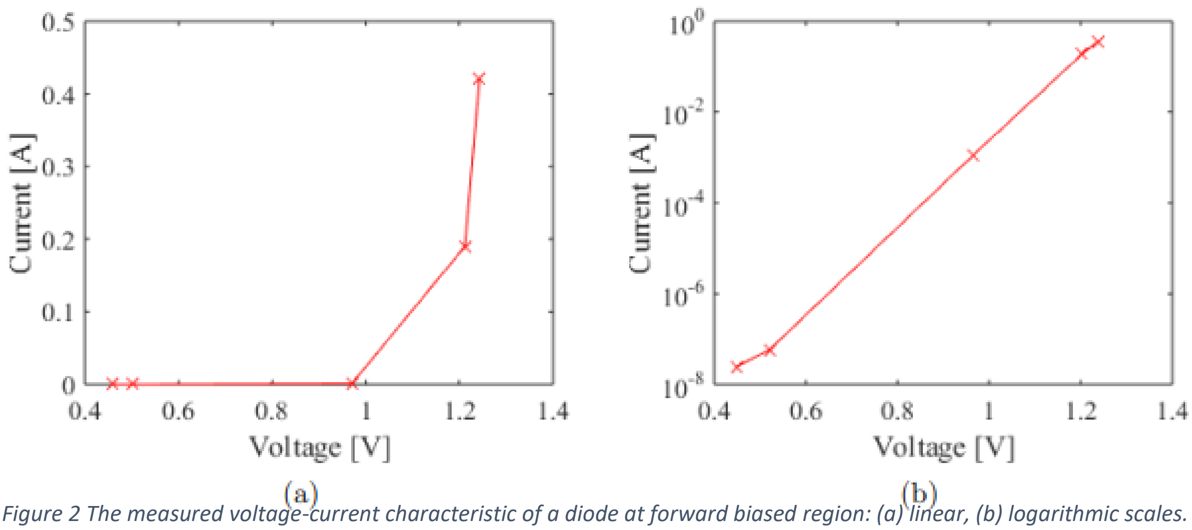

Figure 2 The measured voltage-current characteristic of a diode at forward biased region: (a) linear, (b) logarithmic scales.

Least Square Approximation for Exponential Functions;

The Figure 2 was given in the Project guideline to show us Current-Voltage characteristic of a diode. The Exponential Least Square Approximation can be used to specify the characteristics of a diode since the model seen in Figure 2 is an exponential model. The Exponential Least Square Approximation equations can be seen below;

$$a = \frac{\left(n\ \sum_{i=0}^{n} x_i lny_i\right) - \left(\sum_{i=0}^{n} x_i \sum_{i=0}^{n} lny_i\right)}{\left(n\ \sum_{i=0}^{n} x_i^2\right) - \left(n\ \sum_{i=0}^{n} x_i\right)^2} \tag{1.1}$$

$$lnb = \frac{\left(\sum_{i=0}^{n} lny_i \sum_{i=0}^{n} x_i^2\right) - \left(\sum_{i=0}^{n} x_i \sum_{i=0}^{n} x_i lny_i\right)}{\left(n\ \sum_{i=0}^{n} x_i^2\right) - \left(n\ \sum_{i=0}^{n} x_i\right)^2} \tag{1.2}$$

$$y = a\ e^{bx} \tag{1.3}$$

Where “x” is the voltage values, “y” is the current values and “n” is the number of data we have in the data file. So when we edit the equation of The Exponential Least Square Approximation, it becomes;

$$i_D = ae^{bV_D} \tag{1.4}$$

Also it is required to find voltage values by using the Exponential Least Square Approximation. The equation (1.4) can be converted into the one below to find voltage values;

$$V_D = \frac{\ln\left(i_D\right) - lnb}{a} \tag{1.5}$$

Kirchoff’s Voltage Law;

Kirchoff’s Voltage Law states that all the voltage around the elements in the circuit should be equal to the voltage value of the source therefore to calculate the total voltage, following equation can be used for the circuit in Figure 1.

$$V_S = V_R + V_D + V_L \tag{2.1}$$

Where “S” stands for source, “R” stands for resistor, “D” is for diode, “L” is for inductance. The following equation actually represents how we can calculate the voltage values in the circuit shown in Figure 1.

$$Vs = i(t)R + V_D + L\frac{di(t)}{dt} \tag{2.2}$$

Some of the values in the equation above is already been given in the project guideline and fprdata.dat file has 5 rows in it and first row represents voltage values and second column represents current values for the circuit. Placing given values in the equation above gives (First row in the fprdata.dat file is considered in the equation below.);

$$2 = 0 \, x \, 14.2 + 0 + 0.98 \frac{di(t)}{dt} \tag{2.3}$$

The result can be gathered is “2.04 A” when the equation above solved.

Euler’s Method;

Euler’s Method, is a method to analyze a Differential equation, which uses the idea of local linearity or linear approximation, where we use small tangent lines over a short distance to approximate the solution to an initial-value problem.

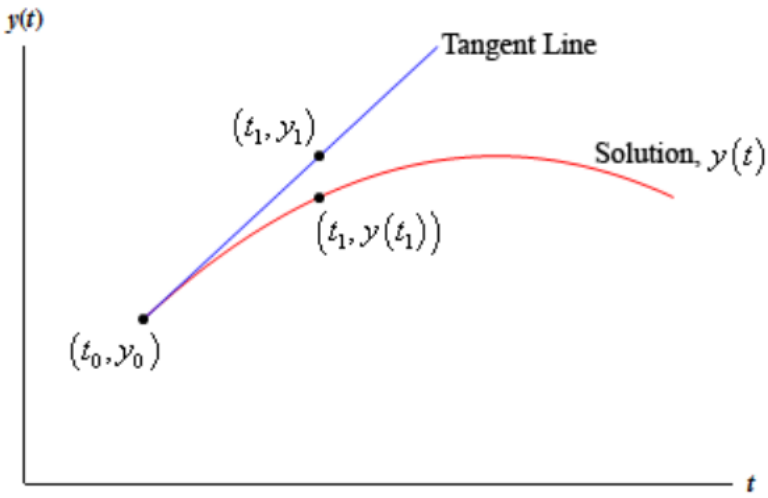

Figure 3 Tangent line and solution line's graph.

It is better how much ever t_1 is close to t_0 since the actual value of the solution at t_1 or y(t1). “y_1“ can be calculated easily. Only thing needed to be done is to put t_1 in the equation of the tangent line. As a result of this, equation becomes;

$$W_{i+1} = W_i + f(t_i, y_i)(t_{i+1} - t_i) \tag{3.1}$$

The only thing needed here is W_i value of the equation. The other values can be found by using first value of the W.

To find the current values in the circuit in Figure 1, the equation in 3.1 becomes;

$$i_{t+1} = i(t_i) + h \frac{di(t_i)}{dt}$$

(3.2)

All the equations that is used in this report explained above. Now, the code for the project can be written since all the necessary equations and topics were explained in the previous section.

Codes, Inputs and Discussion;

At the very first part of the code, the fprdata.dat file imported and split into 2 variables according to columns. First column of the fprdata represents voltage values and second column represents current values. These values can be seen in Table 1 below;

Table 1: Inside of fprdata.dat file

Iterations \ Type	Voltage values	Current values
1	0.46	2.3994e-08
2	0.50	5.4703e-08
3	0.97	0.0011599
4	1.212	0.19031
5	1.243	0.42157

Some constant values in the code are already given in the project guideline such as;

```
VS = 2; L = 0.98; R = 14.2;
DELTA_T1 = 0.025; DELTA_T2 = 0.0025;
```

Also it is needed to find how many times the loop that are going to be done in Euler's method is going to repeat itself. The following variables is defined to find exact number of iteration of the for loop in Euler's method.

```
STEP_COUNT = 0:DELTA_T1:0.6;
STEP_COUNT2 = 0:DELTA_T2:0.6;
```

The DELTA_T1 and T2 were already specified in the constants' section and from here it can be seen that the for loop in the Euler's method is going to repeat 24 times for Euler's method when Δt = 25 ms and 240 times when Δt = 2.5 ms. (Actually 24-1 and 240-1 times since the loop starts from 2 not 1.) Smaller steps seem to be more successful.

After that to make the graph in Figure 2-b it is needed to convert all values into logarithmic scale with a for loop (lines 35-37).

Then the Equations 1.1, 1.2 and 1.3 is interpreted in the code (lines 40-59).

Since all the necessary inputs are defined in the first part of the code and found via for loops in the code, let us move onto the next part which is to calculate Euler's Method. Here, the Euler's method shall be done for 2 times by using different step sizes. In the first for loop 25 ms is used and in the second one 2.5 ms is used. (lines 62-79)

Now that all values gathered via the methods interpreted in MATLAB. The plots can be drawn. (lines 84-121).

Results of what is made in the code;

It can be seen that the approximated svalues are really precise according to Figure 3.

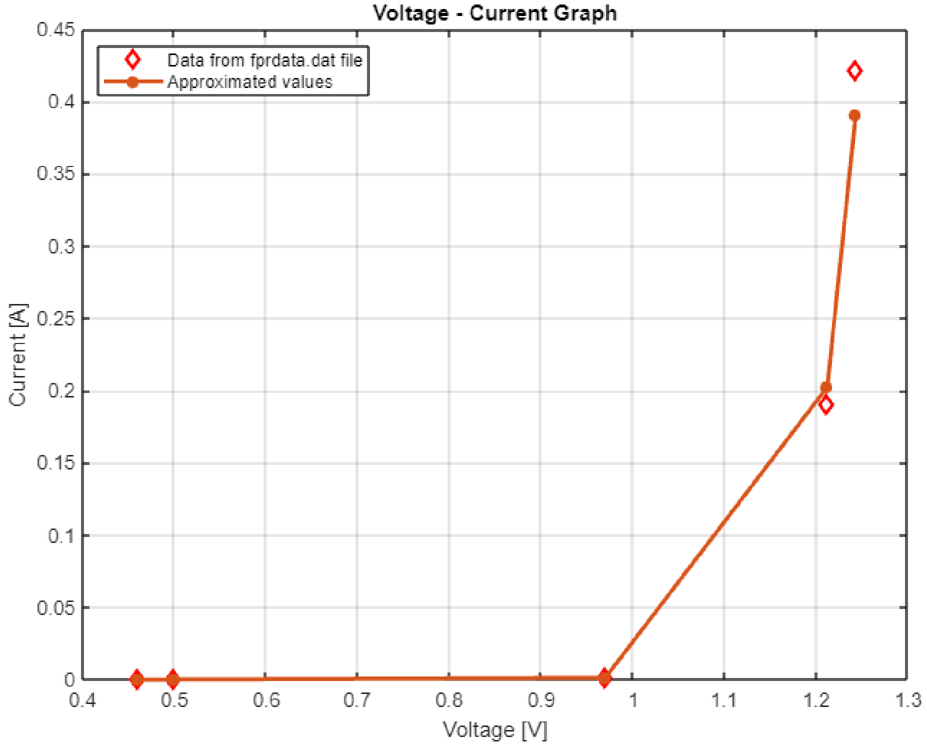

Figure 4 Approximated current values - Voltage values graph.

In the lines 35-37 the current values were converted into logarithmic scale and when it is plotted the result can be seen that all the values completely fits in Figure 5.

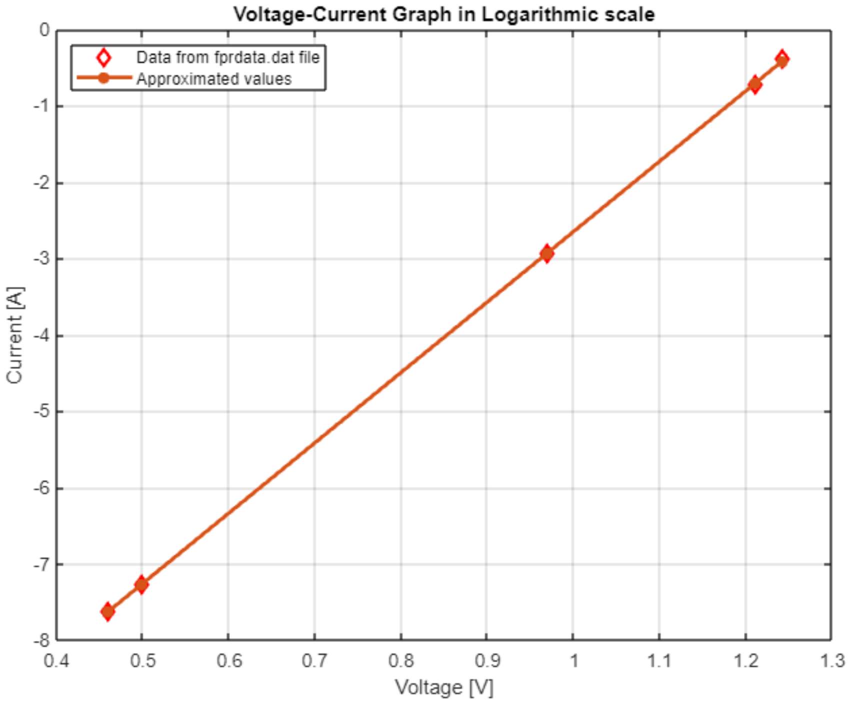

Figure 5 Voltage - Current Graph in Logarithmic scale.

All of the values that are found via Euler's method can be seen in Figure 6 below. The value of current in the circuit approximates to 0.06 A. Also the Voltage around the diode approximates to 1.2 V and Voltage around resistor approximates a bit 0.845 V. It can be seen in the figures that when the voltages around resistor and diode increases the voltage around inductance decreases up to 0 to keep Vsource value constant as specified in Kirchoff's Voltage Law section with equation (2.1).

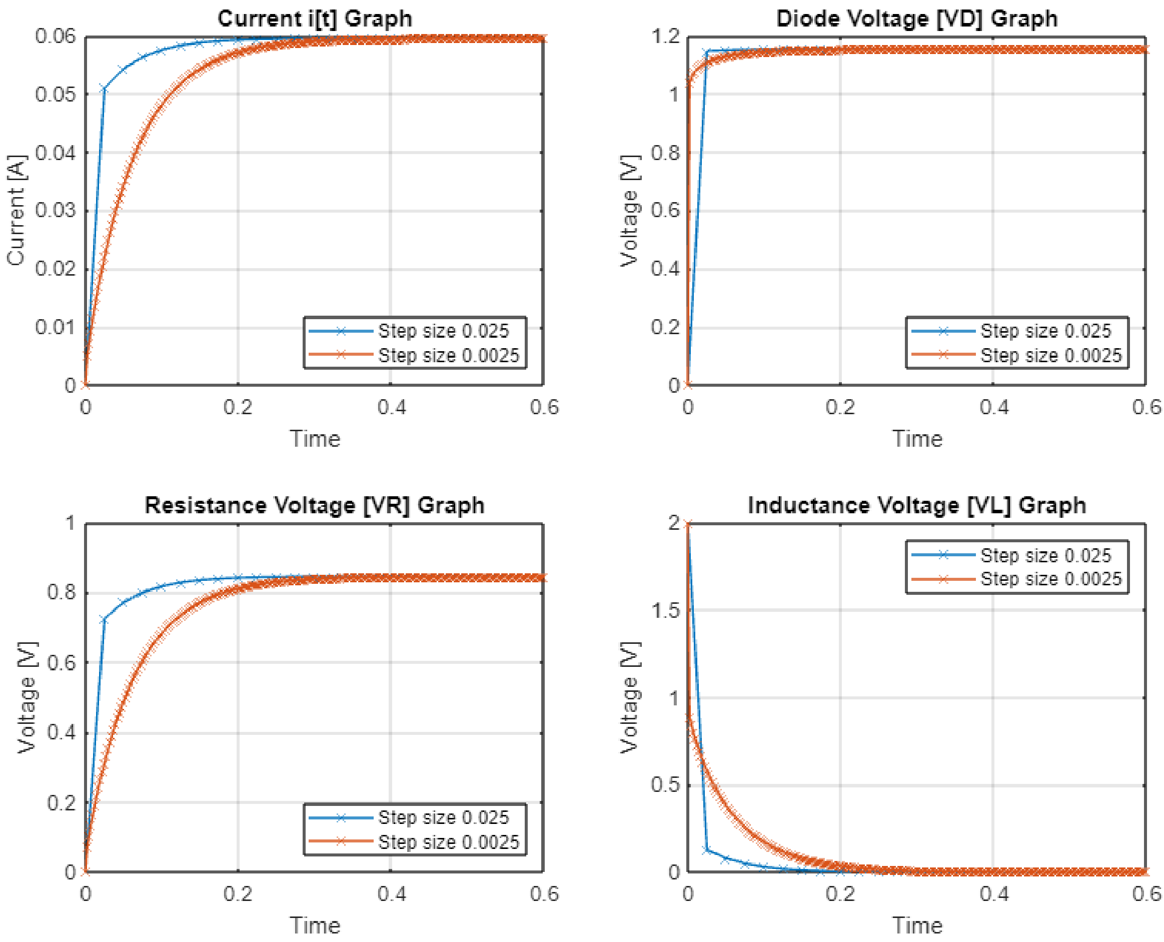

Figure 6 Shows all of the essential values in the code

Conclusion

As conclusion, It can be seen that all of the values found via approximations were very precise. Sum of voltage values are always same with the source voltage value which is 2 V. There are too many voltage values on the plot and workspace since we started loop from 2 to 240. All of the results can be seen in the MATLAB's Workspace section when the code is run. Moreover, It is crystal clear that when we use smaller step sizes to calculate Euler's Method always gives more precise results as expected. Small step for an Euler approximation, giant leap for the result.

All codes that can be seen in APPENDIX part are written by me.

APPENDIX

```
%% Alican Bayındır 200102002087
% MATH 214 - Final Project
% 22.01.2021
clc; clear; close all;
format long;


% Load the data
load fprdata.dat


% Edit the data to be used easily in future operations
voltage_values = fprdata(:, 1)';
current_values = fprdata(:, 2)';


% Constant values given in the project guideline
VS = 2; L = 0.98; R = 14.2;
DELTA_T1 = 0.025; DELTA_T2 = 0.0025;


% It is required to define the part below since we use it in Euler's Method
% to make for loop run according to STEP_COUNT numbers.
STEP_COUNT = 0:DELTA_T1:0.6;
STEP_COUNT2 = 0:DELTA_T2:0.6;
elsize = length(fprdata);


% The part below iss defined for Euler's Method.
derivative_of_current1(1) = VS/L; derivative_of_current2(1) = VS/L;
current_data1(1) = 0; current_data2(1) = 0;
step_one(1) = 0; step_two(1) = 0;


VD_one(1) = 0; VD_two(1) = 0;
VR_one(1) = 0; VR_two(1) = 0;
VL_one(1) = VS; VL_two(1) = VS;


% The part below takes the log of the current values that are given
% in the fprdata.dat file.
for i = 1:elsize
log_of_current(i) = log10(current_values(i));
end


% The following values is needed to find a, b and lnb values.
ln_current = 0; sqrt_current = 0; ln_currents = 0; sum_of_voltages = 0;
for i = 1:elsize
sum_of_voltages = sum_of_voltages + voltage_values(i);
ln_current = ln_current + log(current_values(i));
sqrt_current = sqrt_current + voltage_values(i).^2;
ln_currents = ln_currents + voltage_values(i) * log(current_values(i));
end


% These values will be used to calculate The Exponential form of Least
% Square Approximation.
a = (5 * ln_currents - sum_of_voltages * ln_current) / (5 * sqrt_current - sum_of_voltages.^2);
lnb = (sqrt_current * ln_current - ln_currents * sum_of_voltages) / (5 * sqrt_current -
sum_of_voltages.^2);
b = exp(lnb);
```

```
% The part below is the definition of the equation 1.3 in the project
% report in MATLAB platform.
for i = 1:elsize
current(i) = b * exp(voltage_values(i) * a);
function_log(i) = log10(current(i));
end


% Euler's method's definition when the DELTA_T1 equals 0.025 ms
for i = 2:length(STEP_COUNT)
    current_data1(i) = current_data1(i-1) + DELTA_T1 * derivative_of_current1(i-1);
    VD_one(i) = (log(current_data1(i)) - log(b)) / a;
    derivative_of_current1(i) = (VS - current_data1(i) * R - VD_one(i)) / L;
    step_one(i) = step_one(i-1) + DELTA_T1;
    VR_one(i) =  current_data1(i) * R;
    VL_one(i) = derivative_of_current1(i) * L;
end


% Euler's method's definition when the DELTA_T2 equals 0.0025 ms
for i = 2:length(STEP_COUNT2)
    current_data2(i) = current_data2(i-1) + DELTA_T2 * derivative_of_current2(i-1);
    VD_two(i) = (log(current_data2(i)) - log(b) ) / a;
    derivative_of_current2(i) = (VS - current_data2(i)*R - VD_two(i)) / L;
    step_two(i) = step_two(i-1) + DELTA_T2;
    VR_two(i) = current_data2(i) * R;
    VL_two(i) = derivative_of_current2(i) * L;
end


% Plotting the Current-Voltage graph's values that are given in the fprdata.dat file
% and the current values that is calculated by using equation (1.3) in the
% report.
figure(1);
plot(voltage_values, current_values, "rd", voltage_values, current, "-*", 'LineWidth', 2);
title('Voltage - Current Graph'); xlabel('Voltage [V]'); ylabel('Current [A]');
legend('Data from fprdata.dat file','Approximated values', 'Location', 'northwest');
grid on;


% Plotting the Log of the current and log of the function with respect to voltage values.
figure(2);
plot(voltage_values, log_of_current, "rd", voltage_values, function_log, "-*", 'LineWidth', 2);
title('Voltage-Current Graph in Logarithmic scale'); xlabel('Voltage [V]'); ylabel('Current [A]');
legend('Data from fprdata.dat file','Approximated values', 'Location', 'northwest');
grid on;


% The subplots below is the plotting part of the Euler's method's values.
figure(3);
subplot(2,2,1);
plot(step_one, current_data1, "-x", step_two, current_data2, "-x");
title('Current i[t] Graph'); xlabel('Time'); ylabel('Current [A]');
legend('Step size 0.025','Step size 0.0025', 'Location', 'southeast');
grid on;


subplot(2,2,2);
plot(step_one, VD_one, "-x", step_two, VD_two, "-x");
title('Diode Voltage [VD] Graph'); xlabel('Time'); ylabel('Voltage [V]');
legend('Step size 0.025','Step size 0.0025', 'Location', 'southeast');
grid on;


subplot(2,2,3);
plot(step_one, VR_one, "-x", step_two, VR_two, "-x");
```

```
title('Resistance Voltage [VR] Graph'); xlabel('Time'); ylabel('Voltage [V]');
legend('Step size 0.025','Step size 0.0025', 'Location', 'southeast');
grid on;


subplot(2,2,4);
plot(step_one, VL_one, "-x", step_two, VL_two, "-x");
title('Inductance Voltage [VL] Graph'); xlabel('Time'); ylabel('Voltage [V]');
legend('Step size 0.025','Step size 0.0025');
grid on;
```