GEBZE TECHNICAL UNIVERSITY

ELECTRONIC ENGINEERING

MATH214 NUMERICAL ANALYSIS

2020 – 2021 FALL TERM

Project - 2

Student's

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Abstract:

Following report includes the numerical analysis project two, in this project we are given an RL circuit to find impressed voltage E(t) by using different types of methods. As methods, we used forward and backward difference methods, three point midpoint method to discover derivative of the curve of the currents. These current values are given to us in advance within the current1-2-3-4 files as .dat format. We have loaded all the datas to MATLAB and tried to process all operations due to project guideline. Obtained data such as derivative of the current files and voltage values that are ascertained from the methods are plotted on the screen to make comments on them freely.

Introduction:

In the first place the circuit given in this project can be seen in the Figure 1 below;

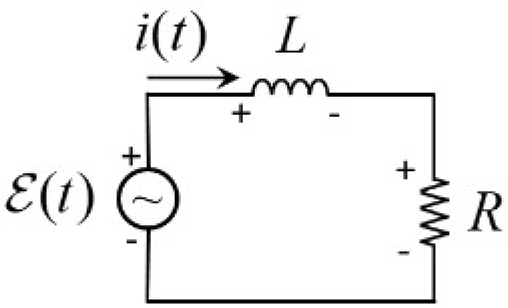

Figure 1: An RL circuit.

The methods forward difference, backward difference and three point midpoint formula have been used to solve the problem. All these formulas can be used to converge. Forward differences are useful in solving ordinary differential equations by single-step predictor-corrector methods. For instance, the forward difference we used in this project predicts the voltage values from the derivative of the current values inside the files and from the amount of change in time. Backward differences are useful for approximating the derivatives if data in the future are not yet available. Moreover, data in the future may depend on the derivatives approximated from the data in the past. If the data values are equally spaced with the step size delta t, the truncation error of the backward difference approximation has the order of O(t) (as bad as the forward difference approximation). The general problem in this project was to find voltage E values in an RL circuit. In order to find it, the following equation has been used.

$$\mathcal{E}(t) = V_L + V_R$$

$$\mathcal{E}(t) = (L * d(i) / dt) + R * i(t)$$

Backward and forward difference methods and three point midpoint formulas has following equations applied on the datasets. First, an array which contains all datasets has been created. All the datas loaded in the MATLAB platform with the help of a for loop. Then, forward difference - backward difference methods and three point midpoint formulas to find all voltage and derivatives' values. All values plotted at the end of the code after all operations are done. All the converge rates are almost same with each other. There is slight difference among all methods this is probably because of their error rate.

The following equation shows us how we can apply the forward difference method to find function f'(K) both on the paper and on the MATLAB.

$$f'(K) = i_{K+1} - i_K \; / \; \Delta t$$

The following equation shows us how we can apply the backward difference method to find function f'(K) both on the paper and on the MATLAB.

$$f'(K) = i_K - i_{K-1} \; / \; \Delta t$$

Finally, the three point midpoint formula can be seen below;

$$f'(K) = i_{K+1} - i_{K-1} \; / \; 2*\Delta t$$

Therefore, while doing the project there was some errors within my results. For instance, I was finding my voltage values with the huge percentage of mistake while applying three point mid point formulas for all current files. I tried to find it's source by asking questions in order and finally I found out that there was a matrix indexing problem in my "for" loop that I created to find three point mid point formula. The problem can be seen below in Figure 2;

```
% Forward difference formula for current1.dat file.
for i = 1:8
    derivat_cur1_forward(i) = (current1(i+1, 2) - current1(i, 2)) / delta_t1;
    voltage_e_cur1_forward(i) = (derivat_cur1_forward(i) * L) + (R * current1(i, 2));
end
```

Figure 2: The reason of the error in my code can be seen in red rectangle.

I changed the part in the red rectangle as below [Figure 3] and problem solved.

```
% Forward difference formula for current1.dat file.
for i = 1:8
    derivat_cur1_forward(i) = (current1(i+1, 2) - current1(i, 2)) / delta_t1;
    voltage_e_cur1_forward(i) = (derivat_cur1_forward(i) * L) + (R * current1(i, 2));
end
```

Figure 3: Changed part of the code.

Other thing I struggled while doing the project was that I tried doing plots as 6 subplots and put 4 of them in an order and doing legend table horizontal to others as common. However, I was not able to do it due to some errors on MATLAB. Since it can be done with other ways I tried to find new ways and finally I decided not to put a common legend so I put legens on every subplots.

The other problem that I faced during this project was finding converge rate errors. I confused while making the project I took a break to be more relax. I understood the problem and fixed it when I came back.

<u>Conclusion:</u>

As a result, the error rates are so accurate as we see in the Table 1 below:

Table 1: Includes converge error rates of methods while finding voltage values.

Data files ↓	**Methods →**	**Forward difference method**	**Backward difference method**	**Three point mid point formula**
current1.dat		0.137195	0.091433	0.024596
current2.dat		0.087307	0.066486	0.013135
current3.dat		0.041159	0.035900	0.003916
current4.dat		0.015792	0.014951	0.000693

As we see on the table [Table 1], the error rates of forward difference and backward difference formulas are really close to each other and three point midpoint formula has less error rate than others. Additionally, when we look at the column of the table's, it shows that more data bring much more success to our methods.

The plots of voltage values we have gathered with MATLAB code can be seen in the Figure 4 below;

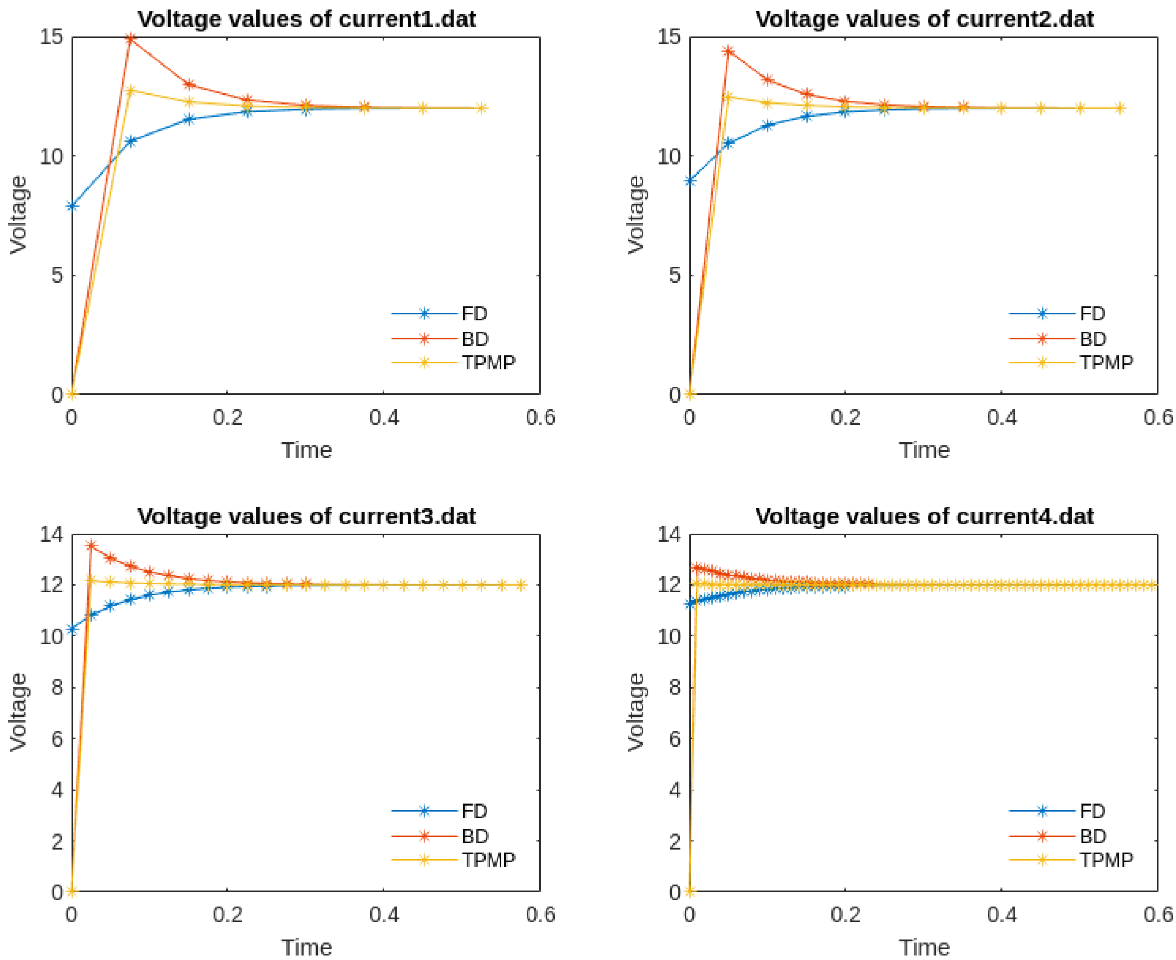

Figure 4: The plots of all voltage values for all current.dat files.

I found out that all results we gathered from methods starts from zero except forward difference method. I could not find what is underneath the hood but probably it is because of I started my loop from two while doing backward difference and three point mid point formulas to prevent matrix overflow. Therefore, the first element of the results became zero. We can ignore the first part of the plots (at the time zero) since they are wrong.

The plots of derivative values of the function we have gathered with MATLAB code can be seen in the Figure 5 below;

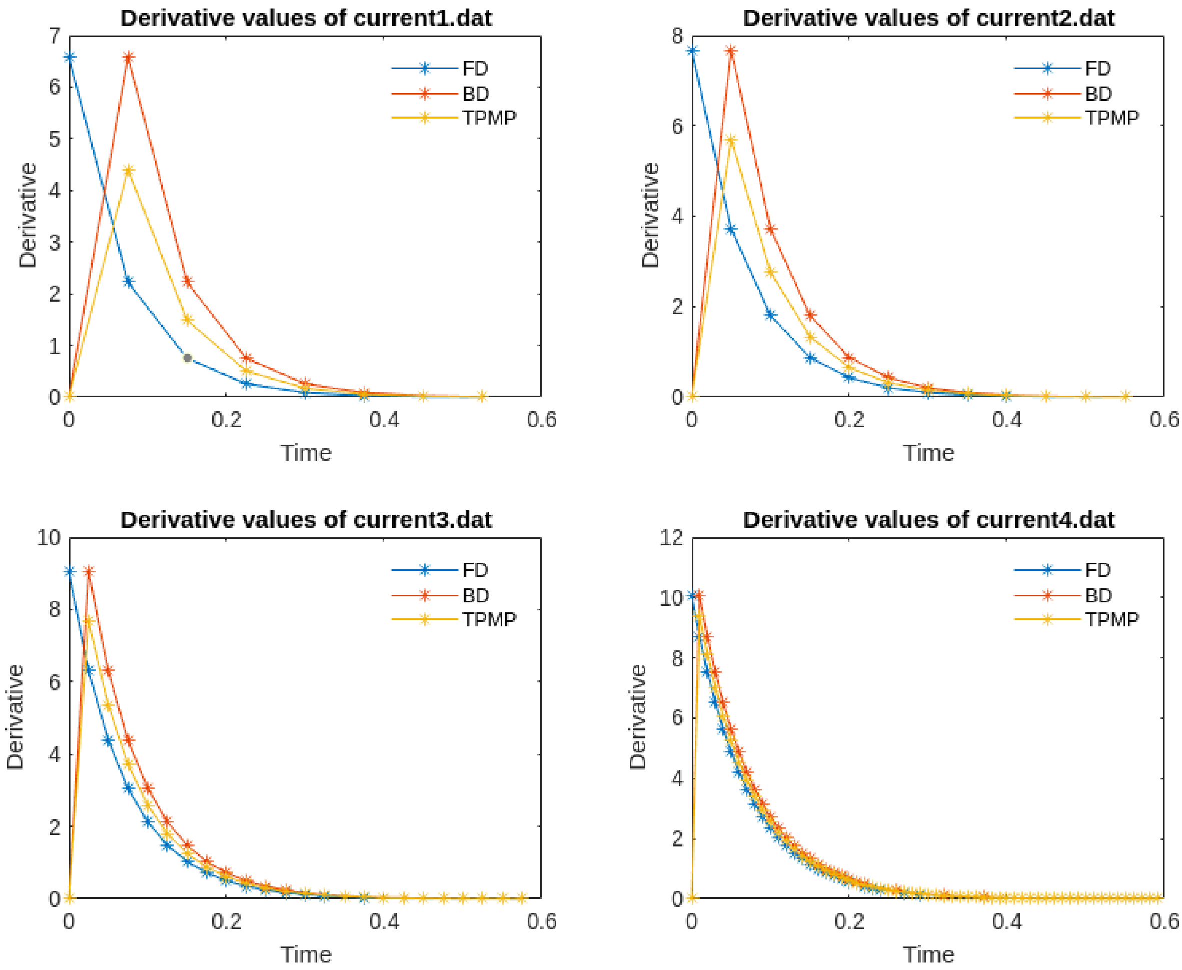

Figure 5: Plot of all derivatives for all current.dat files.

I would wonder why there is such a difference among the derivatives of the values in the current.dat files. Remember what was inside the current.dat files, there were values of the circuit given in Figure 1. All those values are measured on the same circuit with different time intervals. We can know understand that why the fourth graph is more optimal than the first one. We have more data in the current4.dat file and it provides us to find all the derivatives more often than others. This gives us more optimized data. The plot of current1.dat file is more cornered that causes a bit more difference in the derivatives for every time value. The current files' plot can be seen in Figure 6 below;

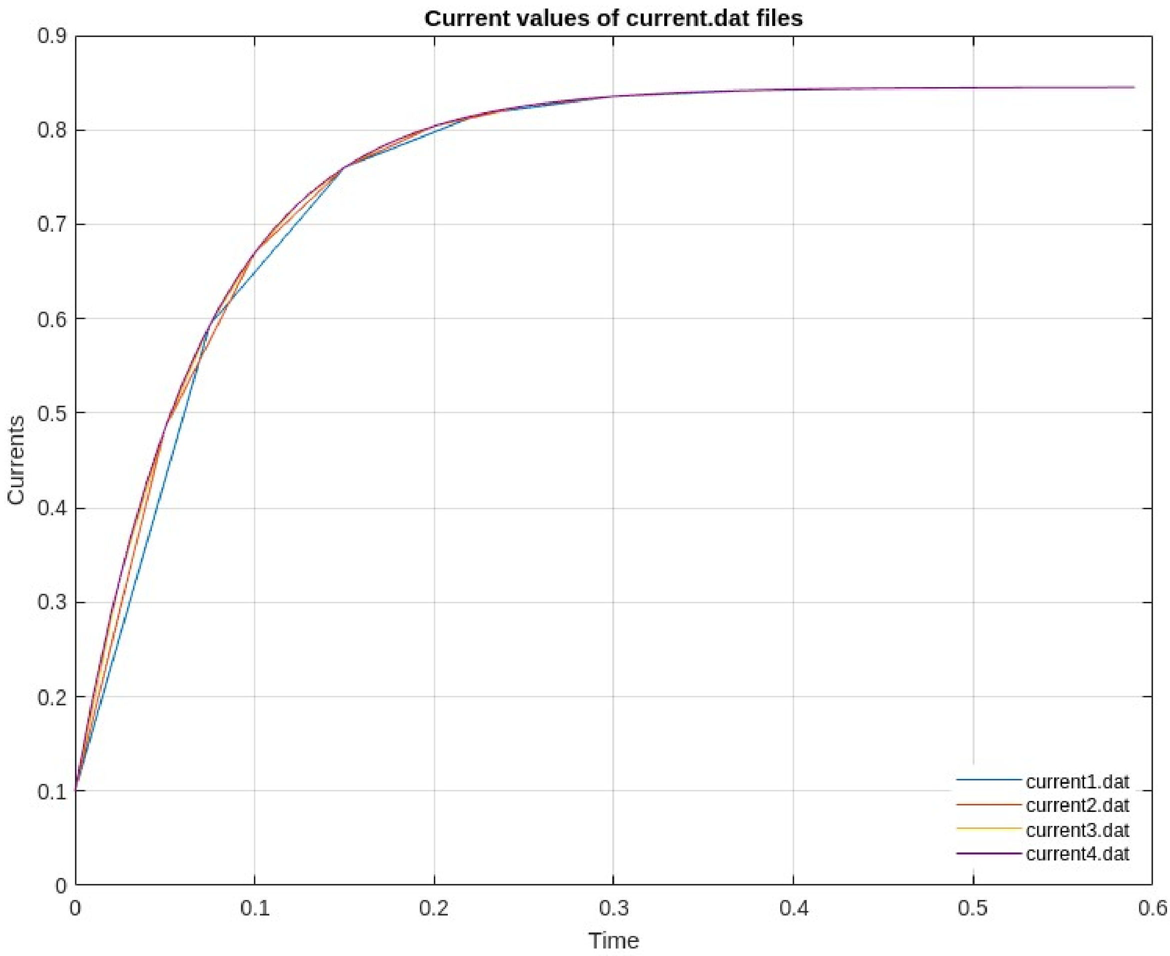

Figure 6: Current - Time graph of current.dat files all.

In this report, all the methods are compared with their error rates and values on the same circuit with changing the amount of the data we have. The methods were pretty simple and we tested them all. Three point midpoint methos is more accurate than others it is because of we use more data points we already have. All the code I wrote for this project belongs to me and can be seen in the APPENDIX part.

APPENDIX

/ › MATLAB Drive › Project2

CURRENT FOLDER

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```matlab
clear all;
close all;
clc;

% to decrease truncation error I used format long.
format long

L= 0.98; % henry
R= 14.2; % ohm

% Delta values it shows how often we measured the currents.
delta_t1 = 0.075;
delta_t2 = 0.050;
delta_t3 = 0.025;
delta_t4 = 0.010;

% Reading the datas
datafiles = {'current1.dat' 'current2.dat' 'current3.dat' 'current4.dat'};

for i = 1:numel(datafiles)
    load( datafiles{i} )
end

% to prevent matrix overflow we need to decrease number of total elements
% in the matrix minus one.
% the format of the variables are type_file_method so if you see
% derivat_cur2_forward it means it is derivative of the current 2 function
% to apply forward difference method.
% Then, calculated the voltage in the circuit that can be seen in Figure 1
% in the project report.

% Forward difference formula for current1.dat file.
% The operations that I wrote in comments above are applied on the codes
% below.
for i = 1:8
    derivat_cur1_forward(i, 1) = (current1(i+1, 2) - current1(i, 2)) / delta_t1;
    voltage_e_cur1_forward(i, 1) = (derivat_cur1_forward(i) * L) + (R * current1(i, 2));
```

COMMAND WINDOW

```matlab
    voltage_e_cur1_forward(i, 1) = (derivat_cur1_forward(i) * L) + (R * current1(i, 2));
end

% Backward difference formula for current1.dat file.
for i = 2:8
    derivat_cur1_backward(i, 1) = (current1(i, 2) - current1(i-1, 2)) / delta_t1;
    voltage_e_cur1_backward(i, 1) = (derivat_cur1_backward(i) * L) + (R * current1(i, 2));
end

% Three point midpoint formula for current1.dat file.
for i = 2:8
    derivat_cur1_tpmp(i, 1)=((current1(i+1, 2) - current1(i-1, 2)) / (2*delta_t1));
    voltage_e_cur1_tpmp(i, 1)=(derivat_cur1_tpmp(i) * L) + (R*current1(i,2));
end
```

CURRENT FOLDER / > MATLAB Drive > Project2

m_200102002087_bayindir_PR2_MATH214.m

```
% Forward difference formula for current2.dat file.
for i = 1:12
    derivat_cur2_forward(i, 1) = (current2(i+1, 2) - current2(i, 2)) / delta_t2;
    voltage_e_cur2_forward(i, 1) = (derivat_cur2_forward(i) * L) + (R * current2(i, 2));
end

% Backward difference formula for current2.dat file.
for i = 2:12
    derivat_cur2_backward(i, 1) = (current2(i, 2) - current2(i-1, 2)) / delta_t2;
    voltage_e_cur2_backward(i, 1) = (derivat_cur2_backward(i) * L) + (R * current2(i, 2));
end

% Three point midpoint formula for current2.dat file.
for i = 2:12
    derivat_cur2_tpmp(i, 1)=(current2(i+1, 2) - current2(i-1, 2)) / (2*delta_t2);
    voltage_e_cur2_tpmp(i, 1)=(derivat_cur2_tpmp(i) * L) + (R*current2(i,2));
end

% Forward difference formula for current3.dat file.
for i = 1:24
    derivat_cur3_forward(i, 1) = (current3(i+1, 2) - current3(i, 2)) / delta_t3;
    voltage_e_cur3_forward(i, 1) = (derivat_cur3_forward(i) * L) + (R * current3(i, 2));
end

% Backward difference formula for current3.dat file.
for i = 2:24
    derivat_cur3_backward(i, 1) = (current3(i, 2) - current3(i-1, 2)) / delta_t3;
    voltage_e_cur3_backward(i, 1) = (derivat_cur3_backward(i) * L) + (R * current3(i, 2));
end

% Three point midpoint formula for current3.dat file.
for i = 2:24
    derivat_cur3_tpmp(i, 1)=(current3(i+1, 2) - current3(i-1, 2)) / (2*delta_t3);
    voltage_e_cur3_tpmp(i, 1)=(derivat_cur3_tpmp(i) * L) + (R * current3(i,2));
end

% Forward difference formula for current4.dat file.
```

COMMAND WINDOW

```
% Forward difference formula for current4.dat file.
for i = 1:60
    derivat_cur4_forward(i, 1) = (current4(i+1, 2) - current4(i, 2)) / delta_t4;
    voltage_e_cur4_forward(i, 1) = (derivat_cur4_forward(i) * L) + (R * current4(i, 2));
end

% Backward difference formula for current4.dat file.
for i = 2:60
    derivat_cur4_backward(i, 1) = (current4(i, 2) - current4(i-1, 2)) / delta_t4;
    voltage_e_cur4_backward(i, 1) = (derivat_cur4_backward(i) * L) + (R * current4(i, 2));
end

% Three point midpoint formula for current4.dat file.
for i = 2:60
    derivat_cur4_tpmp(i, 1)=(current4(i+1, 2) - current4(i-1, 2)) / (2*delta_t4);
    voltage_e_cur4_tpmp(i, 1)=(derivat_cur4_tpmp(i) * L) + (R * current4(i,2));
end
```

/ > MATLAB Drive > Project2

m_200102002087_bayindir_PR2_MATH214.m

CURRENT FOLDER

```matlab
% In the datafiles, we have 9 rows in time section. To plot the data, I
% needed to delete last row of the time column which means I deleted the
% 0.6 ms and my plots does not show any value at the time 0.6 ms.
current1(end,:) = [];
current2(end,:) = [];
current3(end,:) = [];
current4(end,:) = [];

% I created new figures and plotted subplots with respect to appropriate
% data file and decorate it to make more understandable in the section
% below.
figure(1);
subplot(2,2,1)
plot(current1(:, 1), voltage_e_cur1_forward, '-*', current1(:, 1), voltage_e_cur1_backward, '-*', current1(:, 1), voltage_e_cur1_tpmp, '-*');
title('Voltage values of current1.dat');
xlabel('Time');
ylabel('Voltage');
legend('FD', 'BD', 'TPMP', 'Location', 'southeast');
legend('boxoff');

subplot(2,2,2)
plot(current2(:, 1), voltage_e_cur2_forward, '-*', current2(:, 1), voltage_e_cur2_backward, '-*', current2(:, 1), voltage_e_cur2_tpmp, '-*');
title('Voltage values of current2.dat');
xlabel('Time');
ylabel('Voltage');
legend('FD', 'BD', 'TPMP','Location','southeast');
legend('boxoff');

subplot(2,2,3)
plot(current3(:, 1), voltage_e_cur3_forward, '-*', current3(:, 1), voltage_e_cur3_backward, '-*', current3(:, 1), voltage_e_cur3_tpmp, '-*');
title('Voltage values of current3.dat');
xlabel('Time');
ylabel('Voltage');
legend('FD', 'BD', 'TPMP','Location','southeast');
legend('boxoff');

subplot(2,2,4)
```

COMMAND WINDOW

/ > MATLAB Drive > Project2

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CURRENT FOLDER

```matlab
subplot(2,2,4)
plot(current4(:, 1), voltage_e_cur4_forward, '-*', current4(:, 1), voltage_e_cur4_backward, '-*', current4(:, 1), voltage_e_cur4_tpmp, '-*');
title('Voltage values of current4.dat');
xlabel('Time');
ylabel('Voltage');
legend('FD', 'BD', 'TPMP','Location','southeast');
legend('boxoff');

figure(2);
subplot(2,2,1)
plot(current1(:, 1), derivat_cur1_forward, '-*', current1(:, 1), derivat_cur1_backward, '-*', current1(:, 1), derivat_cur1_tpmp, '-*');
title('Derivative values of current1.dat');
xlabel('Time');
ylabel('Derivative');
legend('FD', 'BD', 'TPMP', 'Location', 'northeast');
legend('boxoff');

subplot(2,2,2)
plot(current2(:, 1), derivat_cur2_forward, '-*', current2(:, 1), derivat_cur2_backward, '-*', current2(:, 1), derivat_cur2_tpmp, '-*');
title('Derivative values of current2.dat');
xlabel('Time');
ylabel('Derivative');
legend('FD', 'BD', 'TPMP', 'Location', 'northeast');
legend('boxoff');
```

/ > MATLAB Drive > Project2

m_200102002087_bayindir_PR2_MATH214.m

```
subplot(2,2,3)
plot(current3(:, 1), derivat_cur3_forward, '-*', current3(:, 1), derivat_cur3_backward, '-*', current3(:, 1), derivat_cur3_tpmp, '-*');
title('Derivative values of current3.dat');
xlabel('Time');
ylabel('Derivative');
legend('FD', 'BD', 'TPMP', 'Location', 'northeast');
legend('boxoff');

subplot(2,2,4)
plot(current4(:, 1), derivat_cur4_forward, '-*', current4(:, 1), derivat_cur4_backward, '-*', current4(:, 1), derivat_cur4_tpmp, '-*');
title('Derivative values of current4.dat');
xlabel('Time');
ylabel('Derivative');
legend('FD', 'BD', 'TPMP', 'Location', 'northeast');
legend('boxoff');

% Plot of currents with respect to time in the datafiles.
figure(3);
plot(current1(:,1), current1(:,2), current2(:,1), current2(:,2), current3(:,1), current3(:,2), current4(:,1), current4(:,2))
grid on;
title('Current values of current.dat files');
xlabel('Time');
ylabel('Currents');
legend('current1.dat', 'current2.dat', 'current3.dat', 'current4.dat', 'Location', 'southeast');
legend('boxoff');

% ERROR ANALYSIS
disp('***************ERRORS********************');
disp('The error for the current1.dat file;');
fprintf('When applied forward difference method = %f\n', (norm(voltage_e_cur1_forward - 12) / norm(voltage_e_cur1_forward)));
fprintf('When applied backward difference method = %f\n', (norm(voltage_e_cur1_backward - 12) / norm(voltage_e_cur1_backward)));
fprintf('When applied three point midpoint formula = %f\n', (norm(voltage_e_cur1_tpmp - 12) / norm(voltage_e_cur1_tpmp)));

fprintf('\nThe error for the current2.dat file;\n');
fprintf('When applied forward difference method = %f\n', (norm(voltage_e_cur2_forward - 12) / norm(voltage_e_cur2_forward)));
fprintf('When applied backward difference method = %f\n', (norm(voltage_e_cur2_backward - 12) / norm(voltage_e_cur2_backward)));
fprintf('When applied three point midpoint formula = %f\n', (norm(voltage_e_cur2_tpmp - 12) / norm(voltage_e_cur2_tpmp)));
```

CURRENT FOLDER

COMMAND WINDOW


```
% ERROR ANALYSIS
disp('***************ERRORS********************');
disp('The error for the current1.dat file;');
fprintf('When applied forward difference method = %f\n', (norm(voltage_e_cur1_forward - 12) / norm(voltage_e_cur1_forward)));
fprintf('When applied backward difference method = %f\n', (norm(voltage_e_cur1_backward - 12) / norm(voltage_e_cur1_backward)));
fprintf('When applied three point midpoint formula = %f\n', (norm(voltage_e_cur1_tpmp - 12) / norm(voltage_e_cur1_tpmp)));

fprintf('\nThe error for the current2.dat file;\n');
fprintf('When applied forward difference method = %f\n', (norm(voltage_e_cur2_forward - 12) / norm(voltage_e_cur2_forward)));
fprintf('When applied backward difference method = %f\n', (norm(voltage_e_cur2_backward - 12) / norm(voltage_e_cur2_backward)));
fprintf('When applied three point midpoint formula = %f\n', (norm(voltage_e_cur2_tpmp - 12) / norm(voltage_e_cur2_tpmp)));
```



```

fprintf('\nThe error for the current3.dat file;\n');
fprintf('When applied forward difference method = %f\n', (norm(voltage_e_cur3_forward - 12) / norm(voltage_e_cur3_forward)));
fprintf('When applied backward difference method = %f\n', (norm(voltage_e_cur3_backward - 12) / norm(voltage_e_cur3_backward)));
fprintf('When applied three point midpoint formula = %f\n', (norm(voltage_e_cur3_tpmp - 12) / norm(voltage_e_cur3_tpmp)));

fprintf('\nThe error for the current4.dat file;\n');
fprintf('When applied forward difference method = %f\n', (norm(voltage_e_cur4_forward - 12) / norm(voltage_e_cur4_forward)));
fprintf('When applied backward difference method = %f\n', (norm(voltage_e_cur4_backward - 12) / norm(voltage_e_cur4_backward)));
fprintf('When applied three point midpoint formula = %f\n', (norm(voltage_e_cur4_tpmp - 12) / norm(voltage_e_cur4_tpmp)));
```

COMMAND WINDOW