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GEBZE
TEKNİK ÜNİVERSİTESİ

MAT214 - Numerical Methods
Homework - 1

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Elektronik Mühendisliği

1. Find the representation of the given numbers in the requested base. Show details of the calculations.

   (a) $(6023)_{10} = (?)_8$

   (b) $(367.35)_{10} = (?)_2$

   (c) $(103607)_8 = (?)_{10}$

a) $(6023)_{10} =$

$$6023 : 8 = 752 \text{ remainder } 7,\quad 752 : 8 = 94 \text{ remainder } 0,\quad 94 : 8 = 11 \text{ remainder } 6,\quad 11 : 8 = 1 \text{ remainder } 3,\quad 1$$

$(6023)_{10} = (13607)_8$

b) $(367,35)_{10} =$

integer part	remainder
367 : 2 = 133	1
133 : 2 = 66	1
66 : 2 = 33	0
33 : 2 = 16	1
16 : 2 = 8	0
8 : 2 = 4	0
4 : 2 = 2	0
2 : 2 = 1	0
1 : 2 = 0	1

Binary form of 367

after comma	integer part
0,35 × 2 = 0,7	0
0,7 × 2 = 1,4	1
0,4 × 2 = 0,8	0
0,8 × 2 = 1,6	1
0,6 × 2 = 1,2	1
0,2 × 2 = 0,4	0
0,4 × 2 = 0,8	0
0,8 × 2 = 1,6	1
0,6 × 2 = 1,2	1

$$(110100001,01\overline{0110})_2 = (367,35)_{10}$$

c) $(103607)_8 = 103607$

$8^0 \times 7 = 7$

$8^1 \times 0 = 0$

$8^2 \times 6 = 384$

$8^3 \times 3 = 1536$

$8^4 \times 0 = 0$

$+\ 8^5 \times 1 = 32768$

$(34695)_{10} = (103607)_8$

2. (P.25 Q.2) Compute the absolute error and relative error in approximations of p by p^*.

(a) $p = e^{10}$, $p^* = 22000$

(b) $p = 10^{\pi}$, $p^* = 1400$

(c) $p = 8!$, $p^* = 39900$

(d) $p = 9!$, $p^* = \sqrt{18\pi}(9/e)^9$

Absolute error $= |p - p^*|$

Relative error $= \dfrac{|p - p^*|}{|p|}$

a) $p = e^{10}$, $p^* = 22000$

$$p = e^{10} = 1 + 10 + \frac{10^2}{2!} + \frac{10^3}{3!} + \frac{10^4}{4!} \dots + \frac{10^{10}}{10!} = 22026.4658$$

$$\text{Absolute error} = |\underbrace{22026.4658}_{p} - \underbrace{22000}_{p^*}| = \underbrace{26,4658}_{\Delta p}$$

$$\text{Relative error} = \frac{\Delta p}{|p|} = \frac{26,4658}{|22026,4658|} = 1,2015 \cdot 10^{-3}$$

b) $p = 10^{\pi}$, $p^* = 1400$

$p = 1385,45573137$

$$A.E = |1385,45573137 - 1400| = 14,544269$$

$$R.E. = \frac{14,544269}{|1385,45573137|} = 0,01049782\,2$$

c) $p = 8!$, $p^* = 39900$ $\quad p = 8.7.6.5.4.3.2.1 = 40320$

$$A.E = |40320 - 39900| = 420$$

$$R.E = \frac{420}{|40320|} = 0,010416666$$

d) $P = 9!$, $P^* = \sqrt{18\pi}(9/e)^9 = 359536{,}872842$

$P = 362880$

$A.E = |362880 - 359536.872842| = 3343.1272$

$$R.E = \frac{|3343.1272|}{|362880|} = 9.21276 \times 10^{-3}$$

3. (P.27 Q.21) Suppose two points (x_0, y_0) and (x_1, y_1) are on a straight line with $y_1 \neq y_0$. Two formulas are available to find the x-intercept of the line:

$$x = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0} \quad \text{and} \quad x = x_0 - \frac{(x_1 - x_0) y_0}{y_1 - y_0}$$

(a) Show that both formulas are algebraically correct.

(b) Use the data $(x_0, y_0) = (1.31, 3.24)$ and $(x_1, y_1) = (1.93, 4.76)$ and three-digit rounding arithmetic to compute the x-intercept both ways. Which method is better, and why?

3-a) The equation that passes through (x_0, y_0) and (x_1, y_1) is

$$\frac{y - y_0}{x - x_0} = \frac{y_1 - y_0}{x_1 - x_0} = (y - y_0)(x_1 - x_0) = (y_1 - y_0)(x - x_0)$$

$$(y - y_0)(x_1 - x_0) + x_0(y_1 - y_0) = x(y_1 - y_0)$$

$$x = x_0 + \frac{(y - y_0)(x_1 - x_0)}{(y_1 - y_0)}$$

when $y = 0$ the x-intercept;

$$x = x_0 + \frac{(0 - y_0)(x_1 - x_0)}{(y_1 - y_0)} = x_0 - \frac{y_0(x_1 - x_0)}{(y_1 - y_0)}$$

$$\boxed{= x_0 - \frac{y_0 x_1 - y_0 x_0}{(y_1 - y_0)}}$$

If we take other equation;

$$x = x_0 - \frac{x_1 y_0 - x_0 y_0}{(y_1 - y_0)}$$

$$x = \frac{x_0 y_1 - \cancel{x_0 y_0} - x_1 y_0 + \cancel{x_0 y_0}}{(y_1 - y_0)} = \boxed{\frac{x_0 y_1 - x_1 y_0}{(y_1 - y_0)}}$$

3b)

$$x = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0} = \frac{1.31(4.76) - 1.93(3.24)}{4.76 - 3.24} = \frac{6.24 - 6.25}{1.52}$$

$$= \frac{-0.01}{1.52} = -0.00658$$

$$x = x_0 - \frac{(x_1 - x_0)\, y_0}{(y_1 - y_0)} = 1.31 - \frac{(1.93 - 1.31),\ 3.24}{4.76 - 3.24}$$

$$= 1.31 - \frac{2.01}{1.52} = 1.31 - 1.32 = -0.01$$

The real value of x is $-0.011578\ldots \cong -0.01158$ so that 2nd method is better than first one since it is more accurate

4. (P.35 Q.2) The number e is defined by $e = \sum_{n=0}^{\infty}(1/n!)$, where $n! = n(n-1)\cdots 2\cdot 1$ for $n \neq 0$ and $0! = 1$. Use four-digit chopping arithmetic to compute the following approximations to e and determine the absolute and relative errors.

(a) $e \approx \sum_{n=0}^{5} \frac{1}{n!}$

(b) $e \approx \sum_{j=0}^{5} \frac{1}{(5-j)!}$

(c) $e \approx \sum_{n=0}^{10} \frac{1}{n!}$

(d) $e \approx \sum_{j=0}^{10} \frac{1}{(10-j)!}$

4a) $\sum_{n=0}^{5} \frac{1}{n!} = \underbrace{1}_{\frac{1}{0!}} + \underbrace{1}_{\frac{1}{1!}} + \sum_{n=2}^{5} \frac{1}{n!} = 2 + \sum_{n=2}^{5} \frac{1}{n!}$ ①

$n=2 \Rightarrow 2.5 + \sum_{n=3}^{5} \frac{1}{n!}$ ②

$n=3 \Rightarrow 2{,}666 + \sum_{n=4}^{5} \frac{1}{n!}$ ③

$n=4 \Rightarrow 2{,}707 + \sum_{n=5}^{5} \frac{1}{n!}$ ④

$n=5 \Rightarrow 2{,}707 + \frac{1}{120} = \underbrace{2{,}715}_{p^*}$ ⑤

b

$p = e \qquad p^* = 2{,}715$

$e = 2{,}718281828$

$A.E = |p - p^*| = 3{,}2818 \times 10^{-3}$

$R.E = \frac{|p-p^*|}{|p|} = 1{,}2073 \times 10^{-3}$

4b) $\sum_{j=0}^{5} \frac{1}{(5-j)!} = \underbrace{\frac{1}{120}}_{\substack{(5-0)! \\ 8{,}333\times 10^{-3}}} + \underbrace{\frac{1}{24}}_{\substack{(5-1)! \\ 0.041}} + \sum_{=2} \frac{1}{(5-j)!} = 0{,}049 + \frac{1}{6} + \sum_{=3}^{5} \frac{1}{(5-j)!}$

$= 0{,}2166 + 0{,}5 + 2$

$= \underbrace{2{,}716}_{p^*}$

$A.E = |p - p^*| = 2{,}2818 \times 10^{-3}$

$R.E = \frac{|p-p^*|}{|p|} = 8{,}3943 \times 10^{-4}$

4c) $\sum_{n=0}^{10} \frac{1}{n!} = 1 + \underbrace{1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120}}_{} + \text{---} + \frac{1}{10!}$

It is enough to just compute these values since the rest is not affect the four-digit chopped value.

$p^* = 2{,}716$

$A.E = |p - p^*| = 2{,}281828 \times 10^{-3}$

$R.E = \frac{|p - p^*|}{|p|} = 8{,}3942 \times 10^{-4}$

4d) $\sum_{j=0}^{10} \frac{1}{(10-j)!} = 2.755 \times 10^{-7} + \frac{1}{9!} + \sum_{j=2}^{10} \frac{1}{(10-j)!}$

$= 3.031 \times 10^{-6} + \frac{1}{8!} + \sum_{j=3}^{10} \frac{1}{(10-j)!}$

$= 2.783 \times 10^{-5} + \frac{1}{7!} + \sum_{j=4}^{10} \frac{1}{(10-j)!} = 2{,}262 \times 10^{-4} + \frac{1}{6!} + \sum_{j=5}^{10} \frac{1}{(10-j)!}$

$= 1.615 \times 10^{-3} + \frac{1}{5!} + \sum_{j=6}^{10} \frac{1}{(10-j)!} = 9{,}948 \times 10^{-3} + \frac{1}{24} + \sum_{j=7}^{10} \frac{1}{(10-j)!}$

$= 0{,}05161 + \frac{1}{6} + \sum_{j=8}^{10} \frac{1}{(10-j)!} = 0{,}2182 + \underbrace{\frac{1}{2} + 1 + 1}_{2{,}5}$

$= \underbrace{2{,}718}_{p^*}$

$AE = |p - p^*| = 2.8182 \times 10^{-4}$

$R.E = \frac{|p - p^*|}{|p|} = 1.0367 \times 10^{-4}$

5. (P.37 Q.13) Describe the output of the following algorithm.

$$\boxed{\text{INPUT} \quad n, x_1, x_2, \ldots, x_n.}$$

OUTPUT $\quad SUM.$

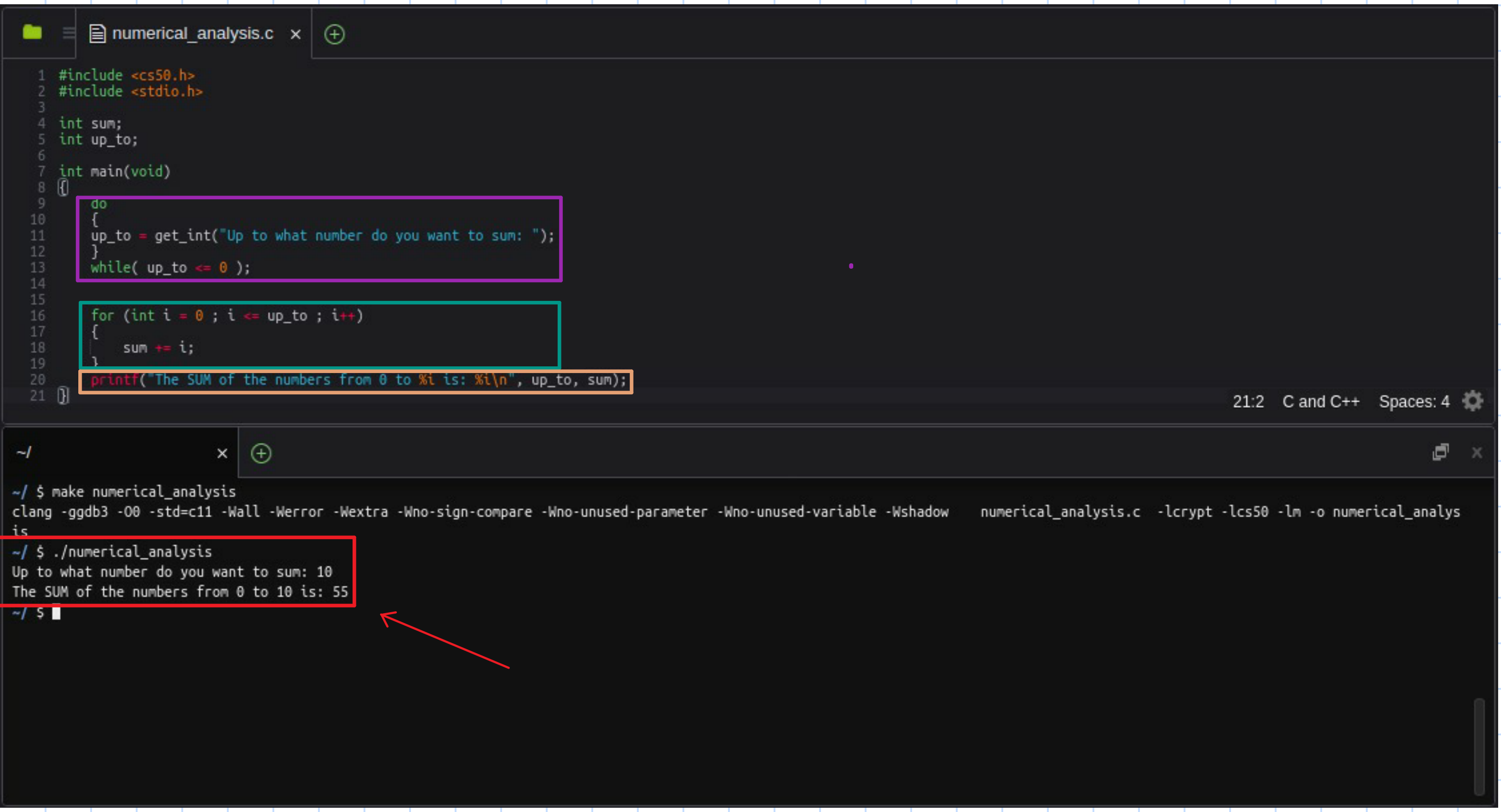
Step 1 $\quad$ Set $SUM = x_1$.

Step 2 $\quad$ For $i = 2, 3, \ldots, n$ do Step 3.

Step 3 $\quad SUM = SUM + x_i$.

Step 4 $\quad$ OUTPUT SUM;

STOP



numerical_analysis.c

```c
#include <cs50.h>
#include <stdio.h>

int sum;
int up_to;

int main(void)
{
    do
    {
    up_to = get_int("Up to what number do you want to sum: ");
    }
    while( up_to <= 0 );


    for (int i = 0 ; i <= up_to ; i++)
    {
        sum += i;
    }
    printf("The SUM of the numbers from 0 to %i is: %i\n", up_to, sum);
}
```

21:2 C and C++ Spaces: 4

```
~/ $ make numerical_analysis
clang -ggdb3 -O0 -std=c11 -Wall -Werror -Wextra -Wno-sign-compare -Wno-unused-parameter -Wno-unused-variable -Wshadow    numerical_analysis.c  -lcrypt -lcs50 -lm -o numerical_analys
is
~/ $ ./numerical_analysis
Up to what number do you want to sum: 10
The SUM of the numbers from 0 to 10 is: 55
~/ $
```

The algorithm above gets an up to integer number to sum all integers from 0 to number n. Then, gives it as output SUM.