PS07-06

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Proof. By contradiction
Suppose for the sake of contradiction that there exists a TM that can compute $f(k)$ and call it F.
F on input 1^n writes $f(n)$ 1s on the tape.
Construct another TM M that always halts on a blank input. This machine will:

1. write n 1s on the tape
2. double the number of 1s
3. run F on input 1^{2n}

Lemma: f is a strictly increasing function.

Proof. Let M be a TM with q states. At its most efficient, M will generate n 1s. Let M' be a TM with $q+1$ states. If M' first q states are identical to M(with the exception of the accept state), then it will generate n 1s. With the addition of the extra state, M' can generate 1 more 1, resulting in $n+1$ 1s. This means that, without any restructuring a machine with $q+1$ states can provably generate more 1s than a machine with q states. □

With this in mind, we tally up the states in M. We now know that it will take at most n states to print out n 1s. For step two, we can use an algorithm, which uses a constant number of states, as does running F. The number of states in M is $n+c$. This means that $f(n+c) \geq f(2n)$ because in running M, we doubled the number of 1s and should hold true $\forall n$. However, when n is greater than c, then $f(n+c) < f(2n)$(lemma: f is strictly increasing). Thus: A contradiction! □