PS03-05

January 19, 2018

Given: A language $L = \{a^n b^m c^l : n, m, l > 0 \text{ and } n^2 + m^2 = l^2\}$
Prove: L is not regular.

Proof. I will begin by playing a game with a demon.
Demon: $k \geq 0$
Me: $w = a^{3k} b^{4k} b^{5k}$
Demon: $w = xyz, \mid xy \mid \leq k, \mid y \mid > 0$
Me: $t = 23$

Since $\mid xy \mid \leq k$, y must be solely composed of a's. This means that if we apply y 22 more times, the ratio of n to m to l will be incorrect.

$\therefore xyz \notin L$.

□