

## PS05-03

February 7, 2018

What languages can be recognized by Turing Machines with the option to keep the head still. Can any more be recognized...? any less?

No more or less can be recognized.

*Proof.* Given Turing machine  $S$ , which can whose movement options are  $\{L, R, S\}$  and Turing machine  $M$ , which is a standard Turing machine with identical states as  $S$ .

$$A = L(S) \text{ and } B = L(M)$$

Proof Idea: If  $M$  can mimic  $S$ 's behaviour with a finite number of additional states, then  $M$  is just as powerful as  $S$ , which means that  $S$  has no additional function and cannot recognize any more languages than  $M$ .

Implementation: In any place  $S$  would stay, add a state to  $M$ , where  $M$ 's head moves one direction once, and returns it to where it was. Regardless of whether or not the respective transition in  $S$  requires a (different) write and a stay, the return from the new state would not alter nothing on the tape. Now  $M$  can read and write while effectively staying in the same tape position. Since it can exactly emulate this function,  $S$  has no more function than  $M$  and thus cannot recognize any more languages.  $\square$