PS09-01

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Make an NTM M. On input $\langle G, H \rangle$:

1. Verify that $|V| = |V'|$ and $|E| = |E'|$.
2. If that is not the case, then there is no way G and H are *isomorphic*. Reject.
3. We know that in order for the graphs to be isomorphic, there needs to exist a bejective function that maps V to V'. Essentially, we will "guess" by mapping permutations of V to V', then check by seeing if all of the $f(u)$s and $f(v)$s behave in E' like they do in E. If so, accept. If not, then make another guess. If there are no more guesses, reject.

There are three components of this algorithm. The verifying, the guessing, and the checking.
The verification process involves summing and comparing the lengths of the sets, making it a linear time process.
Guessing involves translating value-by-value with a bit of overhead per value, which is also a linear time process.
The checking process is also linear because it involves direct, one-to-one comparison.
In the worst case scenario, the machine will have to make $\binom{|V|}{2}$ guesses.

Lemma: n choose 2 is $O(n^2)$

$$\begin{aligned}\binom{n}{2} &= \frac{n!}{(n-2)!2!} \\ &= \frac{n(n-1)}{2!} \\ &= n^2 \qquad \text{disregard constants}\end{aligned}$$

little box.

Which is a $O(n^2)$ operation. Therefore, this machine runs in polynomial time, which means the GraphIso in in NP.
Box.