PS03-02

January 22, 2018

Given:
Prove:

Proof. Let N be an NFA such that $L(N) = A$.
$N = \langle Q_N, \Sigma, \delta_N, S_N, F_N \rangle$
where $\hat{\delta}(X, w) = Y$ and $X, Y \in Q_N$
(and N accepts w iff $Y \cap F_N \neq \varnothing$

Let N_R be an NFA that recognizes reverse(A).
$N_R = \langle Q_{N_R}, \Sigma \delta_{N_R}, S_{N_R}, F_{N_R} \rangle$
where $Q_{n_R} = Q_N \cup \{q_s\}$,
$\delta_{N_R}(q_s, \varepsilon) = F_N$,
$\delta_{N_R}(Y, w) = X$,
$S_{N_R} = \{q_s\}$,
$F_{N_R} = S_N$, and
N_R accepts w iff $Y \cap F_{N_R} \neq \varnothing$

Begin by induction on $\mid w \mid$.

i. Base Case: $\mid w \mid = 0$. By definition, $w = \varepsilon$ The reverse of empty string equals empty string.

ii. Inductive Case: $\mid w \mid = n$.
Let $w = xa$
N accepts xa iff $\hat{\delta}_N(\hat{\delta}_N(S_N, x), a) \cap F_N \neq \varnothing$
Therefore, N_R accepts reverse(w)$\big($= reverse(xa) = a $\circ$ reverse(x)$\big)$ iff $\hat{\delta}_{N_R}(\hat{\delta}_{N_R}(S_{N_R}, a), x) \cap F_{N_R} \neq \varnothing$, where $S_{N_R} = \{q_s\}$.
By the definitions of δ_N and δ_{N_R}, since a represents the first step in the reverse path of w, since by IH, we can assume that reverse holds for $\mid w \mid < n$, and since $\mid x \mid = n - 1$, $\exists$ a machine N_R that can express reverse(A). Therefore, if A is a regular language, reverse(A) is also a regular language.

$\square$