PS07-03

February 16, 2018

a. $GT_{23} = \{[\text{M}] : \text{M is a TM and L(M) containts at least 23 elements}\}$.
Proving GT_{23} is r.e.:

Proof. Make a machine N. Using the interweaved method mention in class, N runs all inputs on M ($L(M) = GT_{23}$). Accept when M has accepted 23 inputs. □

Proving GT_{23} is NOT recursive.

Proof. by reduction of HP.
let ($[M$,$x]$) = (M')
M' on input y:
ignore y
run M on x
Accept if M halts on x.

$$\begin{aligned}[M, x] \in HP &\Rightarrow M \text{ halts on} \\ &\Rightarrow M' \text{ accepts all inputs} \\ &\Rightarrow |L(M') \geq 23 \\ &\Rightarrow M' \in GT_{23} \\ &\Rightarrow [M, x] \in GT_{23}\end{aligned}$$

A contradiction! □

b. $LT_{23} = \{[\text{M}] : \text{M is a TM and L(M) containts at most 23 elements}\}$.
Proving LT_{23} is NOT r.e.:

Proof. by reduction of $\overline{\text{HP}}$
Let ($[M$,$x]$) = (M')
M' on input y:
ignore y

run M on x
Accept if M halts on x.

$$\begin{aligned}
[M, x] \in \overline{\text{HP}} &\Rightarrow M \text{ does not halt on } x \\
&\Rightarrow M' \text{ does not accept any input} \\
&\Rightarrow |L(M') \leq 23 \\
&\Rightarrow M' \in LT_{23} \\
&\Rightarrow [M, x] \in LT_{23}
\end{aligned}$$

A contradiction! □