PS07-04

February 19, 2018

Proof. by contradiction Suppose for the sake of contradiction that f is computable and the machine F takes $[D, M]$ and returns D' such that $L(D') = L(D)/L(M)$
Let D be an all-accept DFA.
If $L(M) = \varnothing$, then $L(D') = \varnothing$
If $L(M) \neq \varnothing$, then $L(D') = \Sigma^*$
I will try to solve E_{TM} using a new machine N.
On input $[M]$:

1. Run F on $[D, M]$
2. Run resulting DFA through M_{DFA}†
3. accept if M_{DFA} accepts, reject if M_{DFA} rejects

† This machine was proven recursive in class.

$$\begin{aligned}[M] \in \mathrm{E}_{\mathrm{TM}} &\Leftrightarrow M \in L(N) \\ &\Leftrightarrow L(M) = \varnothing\end{aligned}$$

$$\begin{aligned}[M] \notin \mathrm{E}_{\mathrm{TM}} &\Leftrightarrow M \notin L(N) \\ &\Leftrightarrow L(M) \neq \varnothing\end{aligned}$$

This decides E_{TM}, which is a provably non-recursive language, thus a Contradiction! $\square$