PS04-04

January 29, 2018

a. $\{a^n b^{2n} c^k : n, k \geq 1\}$

$$S \to NC$$
$$C \to Cc \mid c$$
$$N \to abb \mid aNbb$$

b. $\{a^n b^k a^n : n, k \geq 1\}$

$$S \to aSA \mid aBA$$
$$A \to a$$
$$B \to b \mid bB$$

c. $\{a^k b^m c^n : k, m, n \geq 1, 2k \geq n\}$ My CFG for this(original, not Chomsky-ified):

$$S \to AScc \mid ABcc$$
$$A \to AA \mid a$$
$$B \to BB \mid b$$

Now, in Chomsky Normal Form:

$$S \to AD \mid AE$$
$$T \to AD \mid AE$$
$$A \to AA \mid a$$
$$B \to BB \mid b$$
$$C \to FF$$
$$D \to SC$$
$$E \to BC$$
$$F \to c$$

d. $\{a, b\}^* - \{palindromes\}$

$$
\begin{aligned}
S &\to XSX \mid N \\
N &\to aRb \mid bRa \\
R &\to XRX \mid X \mid \varepsilon \\
X &\to a \mid b
\end{aligned}
$$

e. Using union, I will create a double-barrel CFG. One side will match $x\#y$ of mismatching size, and the other will reliably match $x\#y$ of the same length mismatching in the ith character. I say reliably, because sometimes when the strings are different lengths, it doesn't let them through. However, since I will union the two mini-CFGs, the different length CFG *will* let them through.
CFG for language A: $\{x\#y : |x| \neq |y|\}$

$$
\begin{aligned}
S_A &\to XSX \mid A\# \mid \#A \\
A &\to AX \mid X \\
X &\to 0 \mid 1
\end{aligned}
$$

CFG for language B: $\{x\#y : x_i \neq y_i\}$

$$
\begin{aligned}
S_B &\to R^1T^1 \mid R^0T^0 \\
R^1 &\to XR^1X \mid 1Y\# \\
T^1 &\to 0Y \\
R^0 &\to XR^0X \mid 0Y\# \\
T^0 &\to 1Y \\
X &\to 0 \mid 1 \\
Y &\to YX \mid \varepsilon
\end{aligned}
$$

Where R^1 and T^1 detect $i1...\#i0...$ and R^0 and T^0 detect $i0...\#i1...$
If we take the union of these CFGs(by problem 5):

$$S \to S_A \mid S_B$$

then the result will recognize either $\{x\#y : |x| \neq |y|\}$ or $\{x\#y : x_i \neq y_i\}$, which means that the strings that the CFG recognizes will be in the language $\{x\#y : x \neq y\}$.