PS04-02

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a. *Given:* A language A homomorphically defined as $A := \{01^n2^n3 : n \geq 0\}$.
Prove: A is a non-regular language.

Proof. I will begin by playing a game with a demon.
Demon: $k \geq 0$
Me: $w = 01^k2^k3$
Demon: $w = xyz, \mid xy \mid \leq k, \mid y \mid > 0$
Me: t = 17
Since $\mid xy \mid < k$, y must conist of all 1s. Therefore $xy^tz \notin A$ because the number of 1s and 2s must be equal. Therefore, by the contrapositive of the pumping lemma, A is a non-regular language. □

b. CFG for A:

$$S \to 0A3$$
$$A \to 1A2 \mid \varepsilon$$