Homework 01

Josep Fortiana

Facultat de Matemàtiques i Informàtica, UB

2020-03-16

Instructions: Solutions should be submitted by groups of about three students (meaning: two is OK, four is not advisable but possible upon request. Individual homeworks will also be accepted but collaborative work is preferable).

Only one copy of each group's work must be uploaded (by any member), clearly stating the names of all contributors.

Format: As a Jupyter or R Markdown notebook. Exercise 1 can be submitted as a `.pdf` document. When more than one file needs to be entered, prepare a single compressed (e.g., a `.zip`) file.

Completed assignments are due on Monday, March 30. They are to be uploaded to the Virtual Campus.

Exercise 1. *Consider the following experiment:*

1. *We toss n fair coins. We know the number X_1 of heads is a binomial r.v.* $\sim \mathrm{B}\left(n, \frac{1}{2}\right)$.
2. *We toss again the remaining $R_1 = n - X_1$ coins and denote by X_2 the number of heads obtained this second time.*
3. *We toss again the remaining $R_2 = n - X_1 - X_2$ coins and denote by X_3 the number of heads obtained this second time.*
4. *The process continues in the same manner, stopping when all n coins show heads.*

Questions:

1. *Find:*
   (a) *The pmf of X_2 conditional to a given value of X_1.*
   (b) *The joint pmf of (X_1, X_2) and that of (X_1, X_2, R_2)*
   (c) *Thge marginal pmf of X_2 and that of R_2.*
2. *Try to generalize the previous, $k = 2$, case 1., for $k > 2$.*

   Remark: this is an open question, with more than one possible or correct answer. Some extension is feasible, others give unwieldy expressions as far as I know. Start with $k = 3$, observe what you get and follow any practicable path. Do not worry if none appears.
3. *Define the r.v.*

   $$Y = \text{“Total number of tosses. ”}$$

   Obtain the cdf of Y. Hint: Focus on the separate behaviour of each coin.

Exercise 2. *Write code to generate random numbers following a Rayleigh distribution by the acceptance-rejection method from an exponential candidate distribution, following Exercise 5 in the Simulation 02 Notebook. Evaluate the efficiency (acceptance rate) of the method. Use the statement from the new* `Simul.02.Ex.05.pub.ipynb` *notebook, as it features a (slight but important) correction to the original*

statement. Especially relevant is the change of parameters $y = x/\sigma$ and $\alpha = \lambda\sigma$ which simplifies the necessary computations.

Exercise 3. *Complete* Exercise 02 - Further computations *from the* Placenta Previa *Case Study. See the statements in the* `Binomial.model.02.Placenta.previa.ipynb` *notebook.*
