Introduction

Volleyball is a highly technical sport. As a team game where players pass the ball to each other by letting it bounce off of their forearms, one of the most crucial skills for a player to have is ball control. With each team allowed to touch the ball three times before it must be passed over the net, the ability of a player to redirect the ball in just the correct direction and at a good height for their next teammate to execute a play is critical. As a volleyball player myself, I know all too well the frustration of failing to absorb the impact of a hard hit and allowing the ball to fly far out of bounds. One thing I had noticed in my years of playing was that my teammates always chose the balls they wanted to use in training by pressing on them to ensure that their air pressure was 'just right'. Of course, volleyballs tend to deflate with use and need to be reinflated from time to time, but players seemed to prefer the ones that were neither too deflated nor inflated. This led me to consider how the internal air pressure of volleyballs affects gameplay; how should players adjust their technique to accommodate a more or less inflated ball, especially if they are not able to change the air pressure itself, as in the case in many competitions?

Research Question

How does the internal air pressure of an indoor volleyball affect its coefficient of restitution?

Background Information

Mechanics of Bounce

As a ball falls, its gravitational potential energy is converted to kinetic energy, and then to elastic potential energy on impact with the ground as the ball deforms slightly. The ball, being made of an elastic material like synthetic leather, exerts a restoring force on the ground; by Newton's third law, then, the ground exerts an upward force on the ball, causing it to bounce upwards. As a ball hits the ground, it undergoes an inelastic collision with the ground and bounces to a height lower than its drop height as a result of its loss of some of its mechanical energy to thermal and sound energy at collision.

Coefficient of Restitution

The coefficient of restitution of the collision of two bodies is the ratio of their relative velocities just after and before the collision (Kim, 2013). For the ball bouncing against the stationary ground, this simplifies to:

$$C_R = \frac{|v_{final}|}{|v_{initial}|} \qquad \text{—----------(1)}$$

(Kim, 2013) where v_{final} and $v_{initial}$ refer to the ball's final and initial velocities respectively.

Since the ball only falls and bounces a short distance, the assumption that the energy loss to air resistance as the ball moves through the air is minimal is valid (Mohazzabi, 2011), allowing for the conservation of energy through the ball's flight. The ball's gravitational potential energy at its initial height is thus equal to its kinetic energy just before colliding with the ground. Also, its kinetic energy just after colliding with the ground is equal to its gravitational potential energy at its maximum rebound height. Hence,

$$\frac{1}{2}mv^2 = mgh$$

$$|v| = \sqrt{2gh}$$

where m is the ball's mass, v is its velocity, g is the acceleration due to gravity, and h is the ball's height, which refers to its drop height prior to the bounce, or its maximum rebound height after the bounce.

$$C_R = \sqrt{\frac{h_{final}}{h_{initial}}} \qquad \text{—----------(2)}$$

This definition for coefficient of restitution will be the one used for ease of measurement in this investigation.

Effect of Air Pressure

The internal air pressure of the ball describes the force exerted by the air molecules inside the ball on the walls of the ball per unit area of material. During the ball's collision with the ground, the ball gets deformed, reducing its volume. The decrease in volume results in a proportional increase in pressure, applied over the flattened area of the ball in contact with the ground, causing a net upward force exerted by the ground on the ball as a result of Newton's Third Law. As internal air pressure increases as adjusted by manually inflating the ball, the pressure applied by the ball over the ground increases, and thus the net upward force exerted by the ground on the ball correspondingly increases, thus increasing rebound height.

Theoretical Derivation and Hypothesis

To relate coefficient of restitution to the internal air pressure of the ball, a simplified version of the generalised model derived by Georgallas and Landry (2015) is formulated. Since the material of the ball being used remains constant as the investigation only studies one type of volleyball, unlike the study, the model has been adapted to ignore differences in restoring force resulting from material elasticity, instead focusing only on the restoring force arising from the internal air pressure of the ball. Beginning with the ideal gas law for an environment with constant temperature,

$$PV = P_0 V_0$$
$$P = \frac{4\pi r^3 P_0}{3V}$$

(Ott, 2001) where P and V represent instantaneous air pressure and volume of the ball at a given point in time, P_0 and V_0 represent initial air pressure and volume for the uncompressed ball, and $r$represents the uncompressed spherical ball's radius.

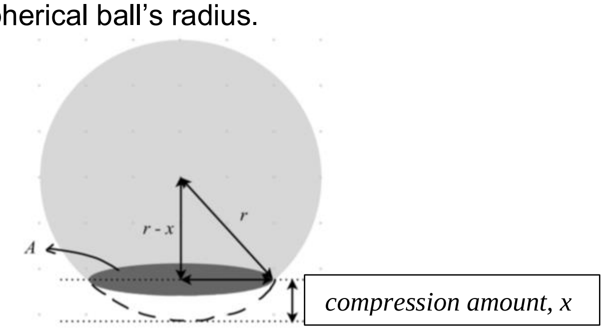


Figure 1: Diagram of a ball's cross-section upon compression, drawn by author.

With reference to Figure 1, as the ball undergoes a compression of length x,

$$\begin{aligned} V(x) &= \frac{4}{3}\pi r^3 - \frac{1}{3}\pi x^2(3r - x) \\ &= \frac{4}{3}\pi r^3 - \pi r x^2 + \frac{1}{3}\pi x^3 \end{aligned}$$

The restoring force is applied over the area of flattened ball in contact with the ground, which changes with the amount compressed as follows:

$$\begin{aligned} A(x) &= \pi\left[r^2 - (r - x)^2\right] \\ &= 2\pi r x - \pi x^2 \end{aligned}$$

The restoring force can then be given by the product of the contact area and pressure for a given compression x and initial pressure P_0:

$$F_{restore}(x) = \frac{A(x) \cdot \frac{4}{3}\pi r^3}{V(x)} P_0$$
$$= \frac{\frac{4}{3}\pi^2 r^3 x(2r - x)}{\frac{4}{3}\pi r^3 - \pi r x^2 + \frac{1}{3}\pi x^3} P_0$$
$$= \frac{8\pi r^4 x - 4\pi r^3 x^2}{4r^3 - 3rx^2 + x^3} P_0$$

Only the first term of the function's Maclaurin series is taken; not only is it a sufficiently accurate approximation from graphical inspection, but it also allows the rewriting of the function for restoring force in the form of Hooke's law:

$$F = kx$$

(Cantor, 2020) where k is a constant and x is the distance through which the compression occurs.

$$F_{restore}(x) = \frac{F'_{restore}(0)}{1!}(x - 0)^1$$
$$= \frac{4r^3 \cdot 8\pi r^4 \cdot P_0}{16r^6} x$$
$$= 2\pi r P_0 x$$

where the 'spring constant' is $2\pi r P_0$. The initial and final kinetic energies of the ball just before and just after the collision can then be found, accounting for the energy loss during the compression x and decompression x as a result of the dissipative forces F_D during the collision:

$$KE_{initial} = \frac{1}{2}\left(2\pi r P_0\right)x^2 + F_D x$$
$$= \pi r P_0 x^2 + F_D x$$
$$KE_{final} = \pi r P_0 x^2 - F_D x$$
$${C_R}^2 = \frac{v^2_{final}}{v^2_{initial}}$$

Hence undergoing algebraic manipulation to eliminate the x-term,

$$C_R^2 \cdot KE_{initial} = \pi r P_0 x^2 - F_D x$$
$$KE_{initial} + KE_{initial} \cdot C_R^2 = 2\pi r P_0 x^2$$
$$KE_{initial}\left(1 + C_R^2\right) = 2\pi r P_0 x^2$$

$$KE_{initial} - KE_{initial} \cdot C_R^2 = 2F_D x$$
$$KE_{initial}\left(1 - C_R^2\right) = 2F_D x$$

$$\frac{1 + C_R^2}{\left(1 - C_R^2\right)^2} = \frac{\pi r KE_{initial} P_0}{2F_D^2}$$
$$\frac{1 + C_R^2}{\left(1 - C_R^2\right)^2} = \alpha P_0 \qquad \text{-----------(3)}$$

where α is an experimentally determined parameter. The theoretical relationship found above (Georgallas and Landry, 2015) is not the technique commonly used to determine the relationship between coefficient of restitution and internal air pressure. Most of the literature relied on the empirical method for determining the equation governing the connection, finding instead a logarithmic-like trend that, when graphically plotted out, does appear similar to the relationship above for the specified domain, validating this model.

Hypothesis

From Equation 3, the relationship between coefficient of restitution and initial internal air pressure should resemble a logarithmic graph. Thus, it is hypothesised that the coefficient of restitution increases with internal air pressure, but this increase occurs at a decreasing rate and approaches a limit. Additionally, plotting $\frac{1+C_R^2}{(1-C_R^2)^2}$ against P_0 should give a linear graph with a gradient of α.

Methodology

Variables

Independent variable: Internal air pressure of the volleyball was varied from 1.0 to 5.0 psi, in intervals of 0.5 psi, as this was the range that the laboratory pressure gauge was able to measure that was also within the safe limit of inflation of the volleyball, avoiding damage to the ball and the risk of overinflation. However, pressure was converted into and recorded in kPa for ease of calculations in SI units. An air pump was used to inflate the ball and the pressure gauge was used to adjust the air pressure to each value.

Dependent variable: The maximum height achieved by the ball after the bounce was determined through video analysis. A mobile phone camera was used to record the ball bouncing at 60 frames per second, and Loggerpro's tracking software was used to mark the ball's position in each frame. Similar to the technique used by Kareem and Kim (2009), a quadratic best-fit curve of vertical displacement against time is then plotted for the bounce, and the y-value for the turning point of the curve is calculated to better approximate the maximum bounce height without needing to be limited by the camera's frame rate. In the case where maximum height might be achieved in between frames, plotting a best-fit curve improves the accuracy whereby an approximation closer to the actual maximum height can be taken, rather than just the maximum height that can be recorded in each frame.

Controlled Variables

Variable	Method of Control	Reason for Control
Material and properties (type, size) of the ball	The same ball was used for all the trials.	To prevent changes in ball elasticity from affecting rebound height.
Height above the ground from which the ball was dropped	A metre rule was fixed in place against a wall. Prior to dropping, the bottom of the ball was visually aligned with the top of the ruler.	To prevent rebound height and coefficient of restitution from being affected by impact velocity as a result of drop height.
Environmental conditions: temperature, humidity, presence of wind	All trials were conducted in the same location, an enclosed room, on the same day.	Changes in temperature and humidity could affect the degree to which air behaves as an ideal gas, and the presence of wind could cause the ball to be blown sideways.

Figure 2: Table of Controlled Variables

Apparatus List

Equipment	Uncertainty	Justification for Uncertainty
Metre rule	$\pm 0.001\ m$	The smallest interval of measurement of the metre rule is $0.001m$, so the uncertainty of each measurement is $\pm 0.0005m$. However, since two points of reference are used at the start and end of the metre rule, the uncertainty is doubled to $\pm 0.001\ m$.
Mobile phone camera	$\pm 0.01\ m$	Video analysis is used for

		determination of position, so the width of the motion blur is used to determine the uncertainty of the position marked in each frame.
Pressure gauge	± 0.3 psi ± 2 kPa	The smallest interval of measurement of the pressure gauge used is 0.5 psi, so the uncertainty of each measurement is ± 0.25 psi.
Air pump	-	-
Mikasa v200w indoor volleyball		
Tripod stand with mobile phone attachment		

Figure 3: Apparatus List

Procedure

Set-up

1. The apparatus was set up as follows in Figure 4.
    a. A metre rule was placed upright along the wall, perpendicular to the ground, fixed in place using blue tack.
    b. A tripod was set up, holding an attached smartphone camera 2m from the metre rule and 0.5m above the ground, halfway between the top of the metre rule and floor, to prevent excessive parallax error when the ball is either high or close to the ground.

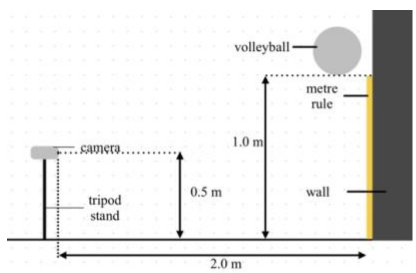


Figure 4: Diagram of experimental set-up

2. Using an air pump, the ball was inflated to 34.474 kPa (5.0 psi), as measured with the pressure gauge.
3. Video recording was started on the camera, and the ball was held up such that its bottom was in line with the top edge of the metre rule. The ball was then released, with care having been taken to ensure that no spin or additional force was applied to it.
4. After the ball had finished bouncing at least twice, video recording was stopped.
5. Repeat steps 3 to 4 four times for a total of five trials.
6. Using the deflation mechanism on the pressure gauge, the ball's air pressure was adjusted to 27.579 kPa (4.5 psi). Steps 3 to 5 were repeated for the new pressure level.
7. Step 6 was repeated for the following pressure readings of 24.132 kPa (4.0 psi), 20.684 kPa (3.5 psi), 17.237 kPa (3.0 psi), 13.790 kPa (2.5 psi), 10.342 kPa (2.0 psi), and 6.8948 kPa).

Data Collection Procedure

1. For a given trial, the video recording taken was imported to Loggerpro.
2. Using the video analysis tool, the scale was set with the metre rule for reference. The origin was set at the top of the ball at its drop position, as seen in Figure 5.

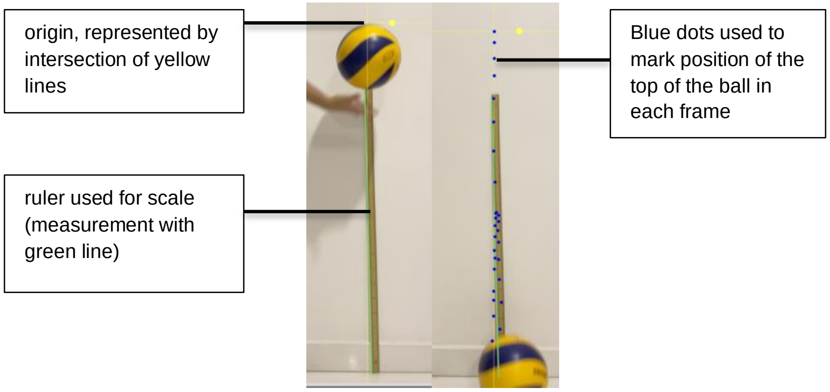


Figure 5: Screenshot from Loggerpro during video analysis

3. Through the fall and bounce, a point was marked on the top of the ball in each frame, generating a curve of Y (vertical displacement) against time in Figure 6.

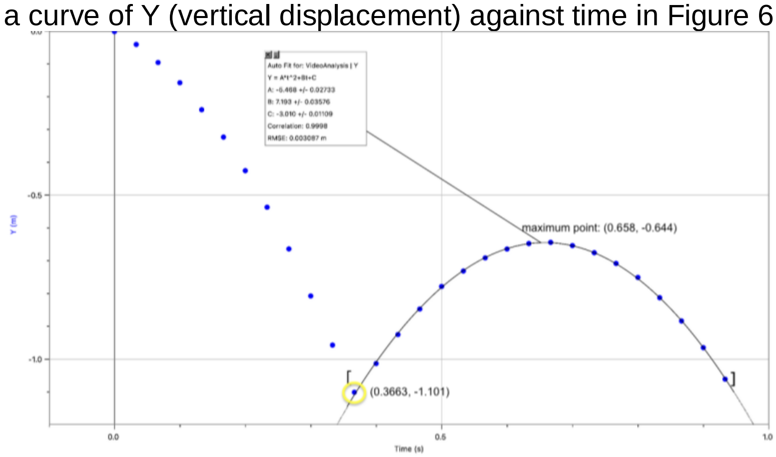


Figure 6: Graph of vertical displacement against time, plotted in Loggerpro. Author's own.

4. A quadratic curve of best-fit was automatically generated using Loggerpro. From the given equation, the maximum point was calculated. Bounce height was then found from the subtraction of the magnitude of the maximum vertical displacement from the magnitude of the lowest point recorded, circled in yellow in the above example.

$$bounce\ height = 1.101 - (0.644)$$
$$= 0.457$$

Environmental, ethical, and safety risks

This investigation does not pose any environmental or ethical risks as all apparatus used are not environmentally unfriendly or hazardous. However, to minimise risk of injury, care was taken to ensure pressure values remained within the recommended safe range of the volleyball to avoid overinflation and potential explosion.

Results

Qualitative Observations

At times, possibly due to unevenness of the tiled floor, the minute presence of wind, or accidental hand movements in releasing the ball, the ball ended up with significant horizontal displacement, sometimes grazing the wall. In such cases, the recording was not included and the trial was repeated until it was visually observed that the ball had minimal horizontal and rotational movement.

Table of Raw Data

Drop height: $1.00 \pm 0.0005\ m$

Pressure, P/Pa (± 2 kPa)	Rebound height, h/m ($\pm 0.01 m$)						
	h_1	h_2	h_3	h_4	h_5	h_{ave}	Δh_{ave}
7	0.590	0.587	0.584	0.593	0.589	0.589	0.004
10	0.614	0.612	0.605	0.608	0.613	0.611	0.004
14	0.629	0.625	0.624	0.636	0.628	0.628	0.006
17	0.656	0.654	0.657	0.651	0.655	0.655	0.003
21	0.663	0.670	0.657	0.670	0.664	0.665	0.007
24	0.673	0.688	0.684	0.669	0.689	0.681	0.009
28	0.696	0.696	0.681	0.694	0.694	0.692	0.007
31	0.700	0.70800	0.695	0.714	0.707	0.705	0.009
34	0.705	0.714	0.714	0.723	0.705	0.712	0.009

Figure 7: Table of Raw Data

Data Processing

Table of C_R

Pressure, P/Pa (± 2 kPa)	Coefficient of restitution, C_R	ΔC_R
7	0.767	0.003
10	0.781	0.003
14	0.793	0.004
17	0.809	0.002
21	0.815	0.004
24	0.825	0.006
28	0.832	0.004
31	0.840	0.006
34	0.844	0.005

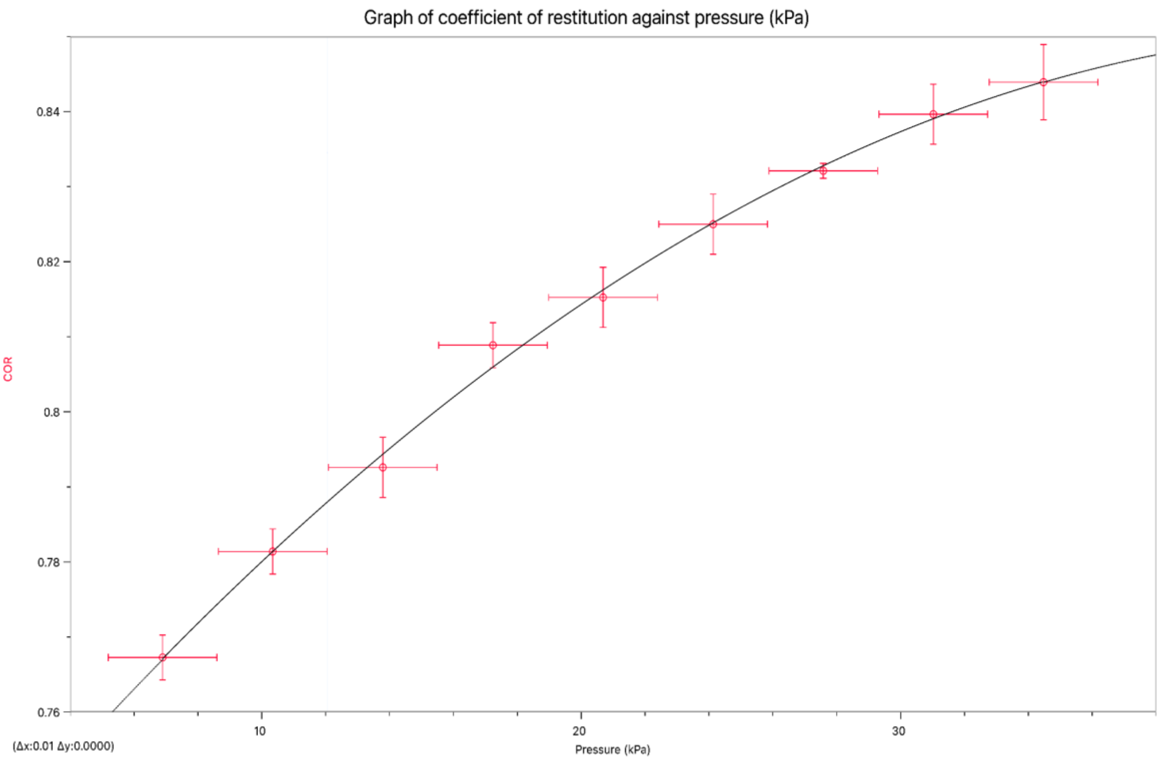
Figure 8: Table of Processed Data

Sample Calculations and Uncertainty Propagation (6.8948 kPa)

Sample Calculation	Uncertainty Propagation
$h_{ave} = \frac{h_1 + h_2 + h_3 + h_4 + h_5}{5} = \frac{0.58975 + 0.58722 + 0.58448 + 0.59298 + 0.58919}{5} = 0.58872\ (5sf) = 0.589 \pm 0.004\ (3dp)$	$\Delta h_{ave} = \frac{h_{max} - h_{min}}{2} = \frac{0.59298 - 0.58448}{2} = 0.004\ (1sf)$
$C_R = \sqrt{\frac{h_{final}}{h_{initial}}}$	$\frac{\Delta C_R}{C_R} = \left(\frac{1}{2}\right) \cdot \left(\frac{\Delta h_{ave}}{h_{ave}}\right)$

$= \sqrt{\frac{0.58872}{1.00}}$ $= 0.76728\ (5sf)$ $= 0.767\ (3dp)$	$\frac{\Delta C_R}{0.76728} = \left(\frac{1}{2}\right) \cdot \left(\frac{0.004}{0.58872}\right)$ $\Delta C_R = 0.0027695\ (5sf)$ $= 0.003\ (1sf)$

Figure 9: Sample Calculations



Graph 1: Graph of coefficient of restitution against pressure (kPa)

It is observed from Graph 1 that, as pressure increases for all of the bounces, the coefficient of restitution also increases, but at a decreasing rate. The coefficient of restitution tends toward, but can never reach or exceed 1, as the energy loss during the inelastic collision prevents the rebound height from reaching the initial drop height. This so far confirms the hypothesis, but a graph of $\frac{1+C_R^2}{(1-C_R^2)^2}$ against P_0 be plotted to confirm a linear relationship between the two.

Analysis

Table of Processed Data

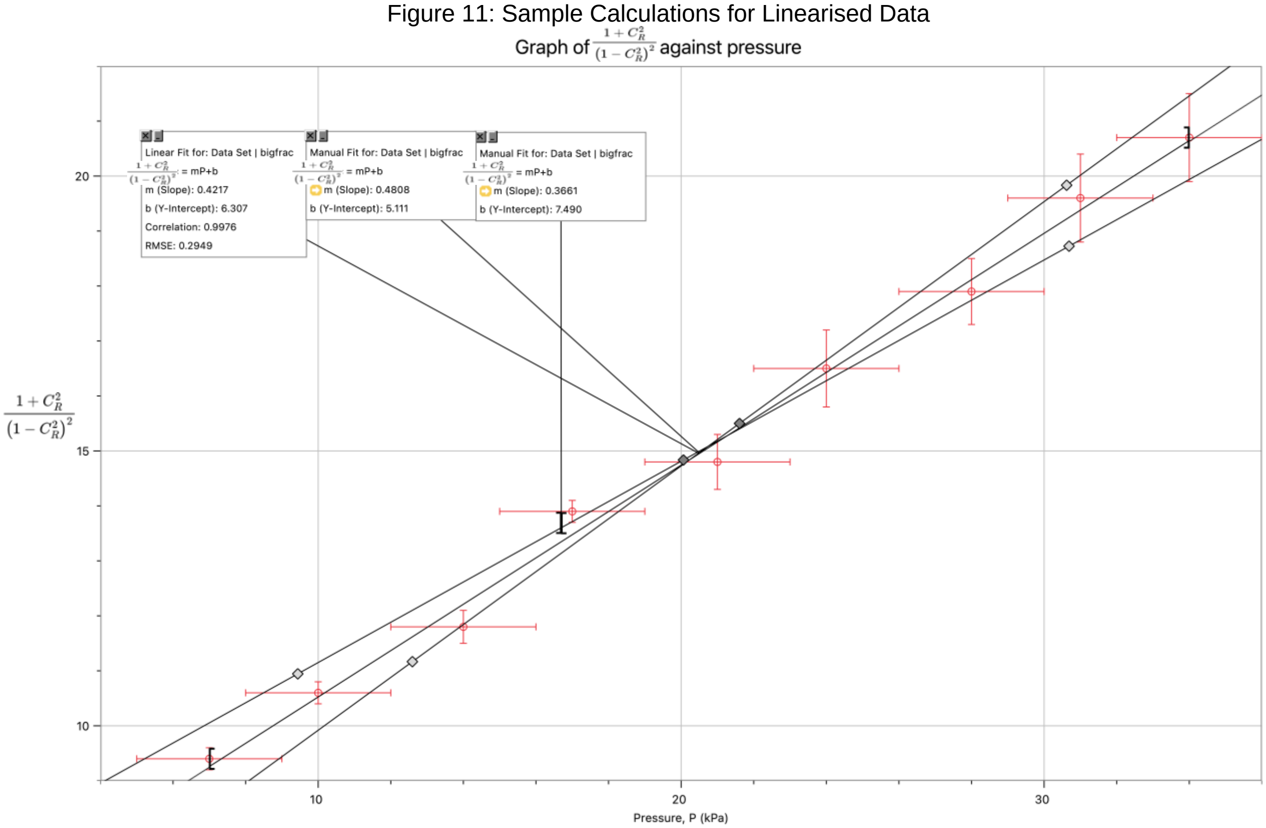
$P_0\ (\pm 2\ kPa^{-1})$	$\frac{1+C_R^2}{(1-C_R^2)^2}$	$\Delta\frac{1+C_R^2}{(1-C_R^2)^2}$
7	9.4	0.2
10	10.6	0.2
14	11.8	0.3
17	13.9	0.2
21	14.8	0.5
24	16.5	0.7
28	17.9	0.6
31	19.6	0.8
34	20.7	0.8

Figure 10: Table of Processed Linearised Data

Sample Calculations and Uncertainty Propagation (6.89476 kPa)

Sample Calculation	Uncertainty Propagation
$\frac{1+C_R^2}{\left(1-C_R^2\right)^2} = \frac{1+0.76728^2}{\left(1-0.76728^2\right)^2}$ $= 9.39223$ $= 9.4\ (1dp)$	$\frac{\Delta\frac{1+C^2{}_R}{(1-C^2{}_R)^2}}{\frac{1+C^2{}_R}{(1-C^2{}_R)^2}} = \frac{(2)(\Delta C_R)}{C_R} + \frac{(4)(\Delta C_R)}{C_R}$ $\Delta\frac{1+C^2{}_R}{(1-C^2{}_R)^2} = [\frac{(6)(0.003)}{(0.76728)}][\frac{1+0.76728^2}{(1-0.76728^2)^2}]$ $= 0.20341\ (5sf)$ $= 0.2\ (1sf)$

Figure 11: Sample Calculations for Linearised Data



Graph 2: Graph of $\frac{1+C_R^2}{(1-C_R^2)^2}$ against pressure

$gradient = 0.4217\ (4sf)$ $= 0.42\ (2dp)$	$gradient\ uncertainty = \frac{\max gradient - \min gradient}{2}$ $= \frac{0.4808 - 0.3661}{2}$ $= 0.06\ (1\ sf)$ $percentage\ uncertainty = \left(\frac{0.06}{0.42}\right)(100\%)$ $= 14\%\ (2sf)$

Figure 12: Gradient Uncertainty Calculations

Graphical Analysis and Discussion

From Graph 2, it is observed that $\frac{1+C_R^2}{(1-C_R^2)^2}$ is linearly related to internal air pressure P_0 according to the equation:

$$\frac{1+C_R^2}{(1-C_R^2)^2} = 0.42\ P_0 + 6.3$$

The high correlation of 0.9976 and relatively low percentage uncertainty of the gradient of 14% shows that the applied linear trendline is a good fit for the data. Upon inspection, however, it is obvious that the graph does not intersect the y-axis at the origin when extrapolated, but instead has a non-zero y-intercept of 6.3. This can be attributed to the fact that the simplified model accounts only for the upward restoring force that arises from the changes in internal air pressure of the ball, but not the force contributed by the elasticity of the deformed ball's material. As such, the actual coefficient of restitution is higher than theoretically predicted, and by extension, $\frac{1+C_R^2}{\left(1-C_R^2\right)^2}$ is higher than theoretically predicted, resulting in the positive y-intercept. An improvement to the model can then be made by adding the parameter β to account for this, as in Georgallas and Landry (2015):

$$\frac{1+C_R^2}{\left(1-C_R^2\right)^2} = \alpha P_0 + \beta$$

where β is an experimentally determined parameter describing the effect of the restoring force arising from the ball's elasticity. Although including the β term allows for a more accurate evaluation of the actual coefficient of restitution, the linear nature of the relationship between $\frac{1+C_R^2}{\left(1-C_R^2\right)^2}$ and pressure remains true. The fact that both α and β need to be experimentally determined means that the equation stated above can only be used to describe the relationship between the two variables, but it cannot be used to mathematically calculate the actual value of the coefficient of restitution from pressure, should conditions be changed. This is due to both parameters containing F_D, the average dissipative force exerted on the ball during compression and decompression, which is difficult to theoretically quantify. According to Georgallas and Landry (2015), from the full derivation, α is as follows:

$$\alpha = \frac{\pi r KE_{initial}}{2F_D^2} \qquad \text{---------(4)}$$

An increase in initial internal air pressure corresponds with an increased collision rate between air particles and the inner surface of the ball's material per unit area of inner ball material. After compression, the momentary increase in pressure (as a result of decreased volume) causes a downward force to be exerted on the floor by the ball, in turn causing the upward force that allows the ball to reach its second maximum height. A higher initial internal pressure therefore allows a greater force to be exerted on the floor by the ball upon deformation, consequently causing a greater upward force on the ball. However, as seen in Graph 1, at higher initial internal pressures such as from upwards of 28 kPa, the additional increase in pressure becomes less significant as the frequency of collisions between air particles and the inner walls of the ball is already high and the increase in the number of particles involved is quite small relative to the number of particles already in the ball. Hence, the increase in pressure gives rise to a less than proportional increase in coefficient of restitution at higher pressures, allowing for the coefficient of restitution to taper towards a maximum value of 1.

Since the internal air pressure of the ball has negligible effect on shape, as the ball remains fairly spherical for most realistic pressures, the magnitude of the dissipative forces acting on the ball during collision is independent of pressure. Instead, the changes in coefficient of restitution with pressure are a result of the variation in the length of compression (and decompression) of the ball's walls. The resultant compressive force exerted by the ground on the ball material is given by subtracting the downward force exerted by the inner air particles on the bottom of the ball from the upward normal reaction force exerted by the ground on the ball. As internal air pressure increases, the force exerted by the inner air particles on the ball material increases, causing the resultant compressive force exerted by the ground on the ball to decrease. This causes a reduction in the length through which the ball is compressed, allowing for less energy to be lost to dissipative forces, causing coefficient of restitution to increase with internal air pressure.

Although this causes a drop in the elastic restoring force exerted by the ball against the floor as a result of the ball's material, this drop is offset by the aforementioned increase in the force exerted by the air particles on the ball, and consequently by the ball on the floor, as a result of the increased pressure. This allows for the overall effect to be for the coefficient of restitution to increase with pressure.

From Graph 2, the vertical lengths of the error bars increase rather notably towards higher pressures, reaching the highest uncertainty in $\frac{1+C_R^2}{(1-C_R^2)^2}$ of 0.8 at the highest pressure of 34 kPa. This could have been a result of the ball's greater degree of random horizontal movement, as will be elaborated in a later evaluation section.

Conclusion

In conclusion, the results obtained support the hypothesis that coefficient of restitution is related to internal air pressure of the ball via the equation:

$$\frac{1+C_R^2}{(1-C_R^2)^2} = 0.42\ P_0 + 6.3 \qquad \text{——------(3)}$$

where α and β are experimentally determined. In other words, the coefficient of restitution increases with pressure, but at a decreasing rate, approaching but not reaching 1. This trend arises from the higher internal air pressure resulting in a greater force exerted by the air particles inside the ball on the bottom of the ball, and thus by the ball on the ground, causing a greater resultant force on the ball which allows it to reach a higher rebound height. However, the coefficient of restitution cannot reach 1 as the rebound height cannot reach its initial drop height as a result of the energy loss in the inelastic collision.

Although some variation is expected due to differences in environmental and ball conditions, the experimental parameters found of α = 0.42 and β = 6.3 are comparable with those found by Georgallas and Landry (2015), where a range of α was found to be from 0.1916 to 0.5140, and a range of β from 1.29 to 6.60 for a variety of different balls and drop heights. The correspondence with other literature values confirms the reliability of the results obtained. Additionally, the results obtained corroborates the trend observed by the literature, including those that describe the relationship as logarithmic (based on empirical observation rather than theoretical derivation). For comparison, the results of a few previous studies are included below. The study on the left of Figure 13, (Georgallas and Landry, 2015) used a similar derivation to this investigation, and the study on the right (Kareem and Kim, 2009) found a logarithmic relationship from an empirical observation of the trend. Both show a similar trend to that of the raw data collected here (Graph 1), with coefficient of restitution increasing with pressure at a decreasing rate, tending toward but not reaching 1. The similarity of the trends found across several studies supports the accuracy of the results found here, as well as the compatibility of the theoretical derivation with experimental results in a number of different conditions.

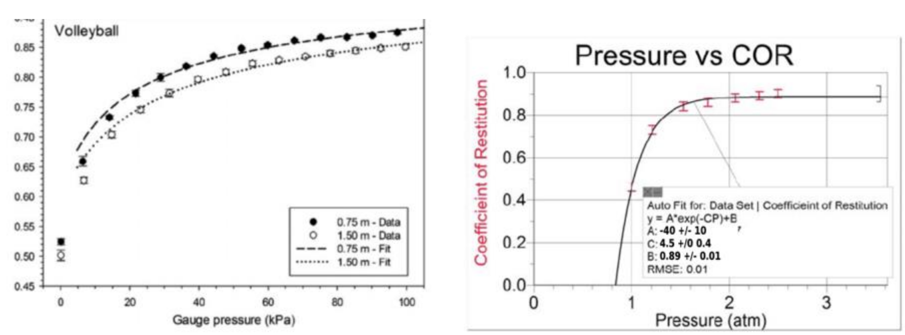


Figure 13: Graphs of coefficients of restitution against pressure, found by existing studies. Left: Georgallas and

Landry (2015). Right: Kareem and Kim (2009).

Evaluation

A strength of this investigation was that the calculation of maximum rebound height was performed based on the software-generated best-fit curve of the ball's vertical displacement, with its position marked in every frame. In this way, the maximum height is more accurate as it is no longer limited by the camera's frame rate since the ball might have achieved its maximum height between the frames captured by the camera. Additionally, the effect of the random human errors involved in pinpointing the location of the ball's position is minimised, since the best-fit curve makes an approximation based on the plotting of many points rather than a single point. Assuming that the random human errors are roughly evenly distributed between too high and too low, the best-fit curve presents a better estimate of the actual maximum height than simply using the highest point plotted.

One limitation of this investigation is the relatively small range of pressures that could be investigated. Due to the specifications of the volleyball used, it was necessary to use pressures that stay below the maximum capacity of inflation. Although the range of pressures used was effective in mimicking real-life conditions, it would have been advantageous to have had more data points with a wider range of pressures, allowing for a more general trend to be determined with applications in other contexts.

Another limitation is the large uncertainty, particularly with regards to the pressure values. The pressure gauge available only had a precision of ± 0.25 psi, which was rather close to the intervals between each pressure reading at 0.5 psi. As a result, there was a comparatively large uncertainty in the values of pressure, evidenced from the relatively large horizontal error bars in graph 2 on Page 9. The similarity in magnitude of the uncertainty of each interval and the actual pressure values results in a relatively high random error.

At higher pressures, it was observed that the ball tended to have more random horizontal movement as it rebounded. This could be attributed to similar reasons for the increase in coefficient of restitution with pressure; with a greater upward force exerted by the floor on the ball, any minute imperfections on the floor would cause the lateral bounce to be observed to a greater effect. As a result, although an effort was maintained to only use the video recordings in which minimal lateral movement was observed, a higher degree of random error occurred at higher pressures, where more horizontal motion was observed. A greater degree of horizontal motion indicates that the vertical component of the ball's velocity is smaller and the proportion of kinetic energy converted to gravitational potential energy as a result of vertical motion is smaller. Overall, this could have caused an underestimation of the average rebound height and therefore coefficient of restitution.

Extensions for Future Work

It could be worth investigating how temperature affects the coefficient of restitution of volleyballs. As mentioned above, temperature affects the degree to which air behaves as an ideal gas. For competitions played in different conditions around the world, temperature could therefore have a noticeable impact on coefficient of restitution and thus gameplay.

Furthermore, the relationship between different ball materials and coefficient of restitution for different pressures could be studied. In recent years, with the release of different volleyball models, several companies have been experimenting with using new synthetic materials. These materials might behave differently as pressure changes, and it would be interesting to investigate the material behaviour under different conditions.

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