Determining the launch parameters that allow for a safe landing for Billy Morgan's 2015 quadruple cork 1800 using calculus.

Introduction

Snowboarding has always been a passion of mine. I was particularly fascinated with big air, which was just introduced as an Olympic event[1] in 2018, in which competitors ride down a large ramp and perform a single aerial trick, scored according to its difficulty, execution, amplitude, and landing.[2] As the culmination of snowboarders' ever-growing interest in landing increasingly challenging tricks, the quadruple cork 1800, consisting of five rotations and four flips, has emerged as one of the most physically demanding feats performed yet, with only a few recorded successful attempts after the first performed by Billy Morgan in 2015.

Figure 1: A composite photo[3] illustrating a double cork, in which the snowboarder is flipped upside-down twice.

During my own attempts at performing tricks off ramps, like many other amateurs, I tend to slow myself down in fear of gaining too much speed; this inevitably results in me not going fast enough to complete a rotation, nor far enough to land safely on the downhill section of the jump. This investigation focuses on launch velocity and rate of spin as the two determinants of an attempt's success, with the prior affecting the safety of the snowboarder and the latter governing the ability of the snowboarder to land accurately after executing five complete spins.

Research Aim

Figure 2 illustrates the key regions of a ramp. Region 1 is the takeoff, from which the rider is launched. With a predetermined launch angle, the trajectory of flight is therefore only dependent on the snowboarder's launch velocity, which the snowboarder controls by making side-to-side turns prior to reaching the takeoff, slowing themself down. Region 2 is the flat 'knuckle' of the landing area. Riders with insufficient initial launch velocity land in this area and risk injury due to the lack of a gradient. Region 3 is the optimal landing zone, where the gradient of the landing ramp is sufficiently steep so as to minimise the rider's equivalent fall height. A rider with too much initial velocity lands in region 4, at which point the slope tapers off and becomes flatter.

Figure 2: A side-view profile of a jump on Mammoth Mountain's terrain park.[4] Annotated by author.

Objectives

In this investigation, I will derive the range of launch velocities that allow for a safe landing, and then Morgan's 2015 jump will be analysed to check whether his initial velocity lies within this range. Next, I will reference Morgan's rate of spin to describe how snowboarders should change their body's effective radius through the jump to successfully complete five spins.

Mathematical Background

Key Concepts

Equivalent Fall Height

Equivalent fall height is a concept employed by terrain park designers to minimise injury risk to snowboarders, in which the effective fall experienced by a rider achieving a very high maximum height is much less than it actually is,[5,6] as a result of the gradient of the landing area. This works by reducing the normal reaction force exerted on the rider by the ground at their landing, as illustrated by Figure 3 as $F\cos\theta$, where θ represents the local angle of slope incline.

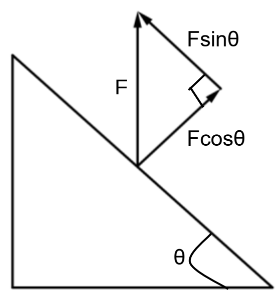

Figure 3: Vector decomposition of force exerted by ground on rider

According to the American College of Surgeons' Committee on Trauma,[7] the critical threshold is about 8 metres; a fall from a height greater than this likely results in critical injuries. The equivalent fall height of the snowboarder on the landing zone must therefore be equal to or less than 8 metres, that is, the component of the force exerted by the ramp on the snowboarder that is normal to the ramp must be equal to or less than the force exerted by flat ground on the snowboarder after a snowboarder has fallen 8 metres.

Approach

First, we determine the range of launch velocities at which a snowboarder can take off while maintaining a safe equivalent fall height. To do this, a function of normal reaction force on impact against horizontal displacement along the ramp must be attained. However, due to the incline of the ramp, the height through which a snowboarder falls increases with horizontal displacement. Thus, it is necessary to model the ramp to determine the actual fall height for each point along the landing area. Once we have obtained a function relating actual fall height to horizontal displacement, we can then relate normal reaction force to horizontal displacement, solving to find the safe launch velocity range. After the determination of the launch velocity range, Billy Morgan's 2015 jump will be sampled for analysis, to be compared against the theoretical range.

The other determinant of a quadruple cork 1800's success or failure is the snowboarder's ability to control their rate of spin. After takeoff, a snowboarder cannot change their angular momentum; instead, to vary their rate of spin, they must change their effective radius and therefore their moment of inertia by extending or contracting their arms, as elaborated in a later section. As the first complete attempt, Morgan's jump will be analysed as a guide for snowboarders on the technical aspect of how effective radius should change through the jump for a successful execution.

Derivations

For the purposes of this inquiry, the effects of air resistance will be ignored; horizontal velocity is therefore assumed to be constant through the motion. The first objective of this exploration is to investigate the launch velocity for which equivalent fall height is less than or equal to 8 metres. Since the gradient of the landing ramp changes with respect to distance, the domain of horizontal displacement for which equivalent fall height is safe, that is, with the rider experiencing a normal reaction force less than that experienced from an 8-metre fall on flat ground, must be calculated. A table of definitions, Figure A1 in the appendix, describes the abbreviated parameters referenced in this section. Calculations are made to five significant figures and rounded to three significant figures at the final step.

Fall on Flat Ground

The general form of Newton's second law states that a force is equal to its change in momentum per change in time. Mathematically, for an object with constant mass, as is the case for a snowboarder,

$$F = \frac{m \cdot \Delta v}{\Delta t}$$

One of the kinematic equations of motion for an object undergoing constant acceleration, for final velocity v, initial velocity u, acceleration a, and displacement s is:

$$v^2 = u^2 + 2 \cdot a \cdot s$$

The magnitude of the final velocity at which an object falling 8 metres is thus given by substituting $u = 0$, $a = g$, and $s = 8$:

$$|v_{final}| = \sqrt{16g}$$

Since impact with the floor brings the object to rest, the change in velocity is equal to the magnitude of the final impact velocity. The normal reaction force exerted by the ground on an object falling 8 metres is represented as:

$$F_{max} = \frac{m\left(\sqrt{16g}\right)}{\Delta t}$$

Fall on Slope

As a result of the angle of incline of the slope on which a snowboarder lands, the normal reaction force is given by the component of the force exerted by the slope on the snowboarder that is perpendicular to the slope:

$$F_{slope} = \frac{m \cdot \Delta v}{\Delta t} \cdot \cos\theta$$

The magnitude of the final velocity at which the snowboarder lands is:

$$|v_{final}| = \sqrt{2 \cdot g \cdot h_{fall}}$$

in which h_{fall} describes the actual fall height, as defined in Figure 4.

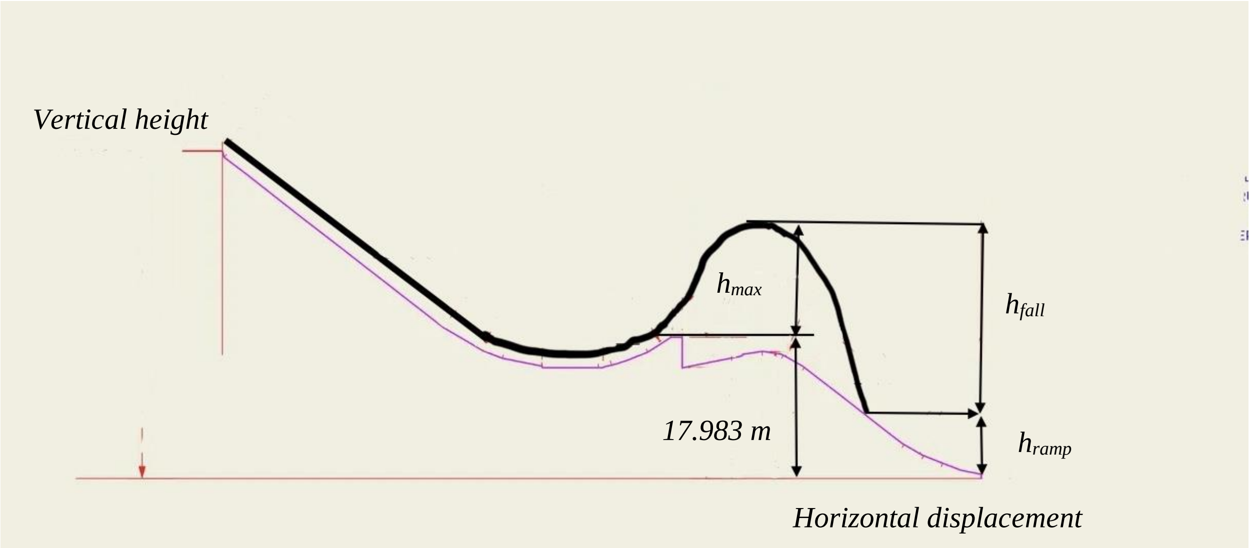

Figure 4: A blueprint of the Olympic big air ramp.[1] The black line indicates example snowboarder trajectory, with which h_{fall}, h_{max}, and h_{ramp} are defined.

Figure 4 defines the parameters used to calculate actual fall height, h_{fall}.

$$\mathrm{h}_{fall} = h_{max} + 17.983 - h_{ramp}$$

h_{max} can be determined using the kinematic equation shown above, substituting $s = h_{max}$, $a = g$, and $v = 0$, as the instantaneous vertical velocity at the maximum height achieved is zero.

$$h_{max} = \frac{u_y^2}{2g}$$

Attaining a function of h_{fall} with reference to horizontal displacement is necessary to determine the region of the ramp where landing is safe. From timing the videos of 25 big air jumps at the Beijing 2022

Olympics, I found that the average airtime is about 3 seconds (Figure A2, Appendix). Hence, to relate u_y to s_x:

$$u_x = \frac{s_x}{3}$$
$$u_y = u_x \tan 0.66131$$
$$= \frac{s_x \tan 0.66131}{3}$$

where 0.66131 is the angle of incline of the takeoff ramp in radians. Hence,

$$h_{max} = \frac{\left(\frac{s_x \tan 0.66131}{3}\right)^2}{2g}$$

For the region between the knuckle and the end of the sloped landing zone, Loggerpro is used to find a function of h_{ramp} in s_x. With reference to the blueprint, points of position are marked along the curve of the landing ramp, and a line of best fit is automatically plotted by Loggerpro's software (Figure 5) for the region between the knuckle and the end of the ramp. The ramp dimensions from the blueprint are used for scale.

Modelling the Ramp Between Knuckle and the End

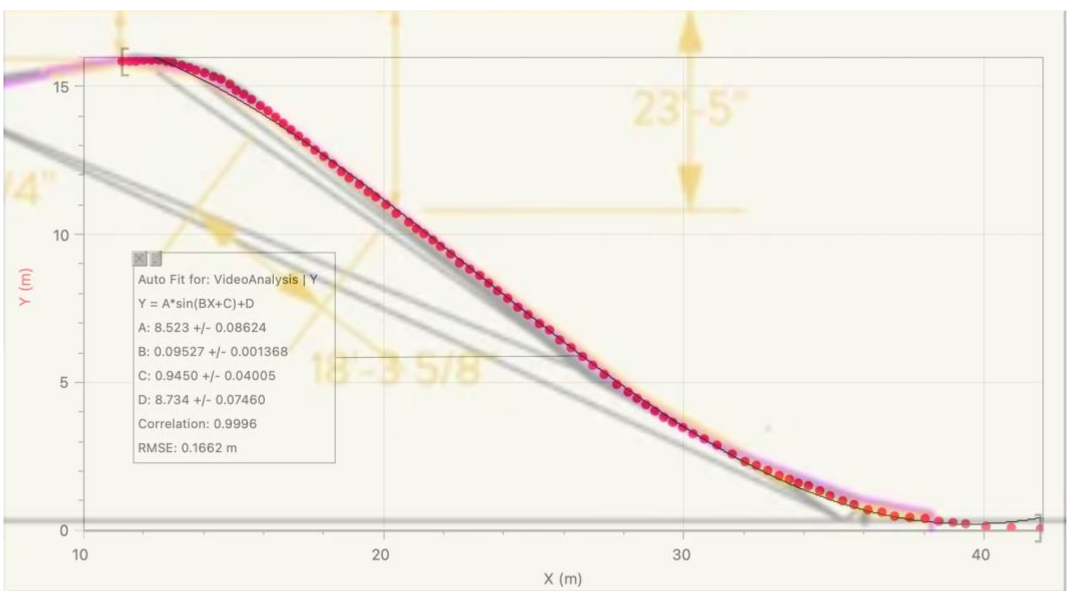

Figure 5: A comparison of the best-fit graph with the landing ramp illustrated on the blueprint.

x displacement (m)	y displacement (m)	x displacement, continued (m)	y displacement, continued (m)

11.27900944	15.82291362	23.81099475	8.111470326
11.53698553	15.84437211	24.13888999	7.837678578
11.75542644	15.85793696	24.46366688	7.551881156
11.99819054	15.88397524	24.81962733	7.272632283
12.26746067	15.90362089	25.17714697	6.99556626
12.51817657	15.87041039	25.54760778	6.778216775
12.7743496	15.82223177	25.87363201	6.424127333
12.99996274	15.78559108	26.2638944	6.175126524
13.27889977	15.71293336	26.62936584	5.86110797
13.5297716	15.61610837	26.98158427	5.560810187
13.76349246	15.5403323	27.35126549	5.270958901
14.04710703	15.46720682	27.75805517	4.93854204
14.32027511	15.34902109	28.14597879	4.679406571
14.60576069	15.22849659	28.44799166	4.456288125
14.87581041	15.07600893	28.7487572	4.232390089
15.09659009	14.87097696	29.03860849	4.038272364
15.35323087	14.73704352	29.34062137	3.811255971
15.61657613	14.57036734	29.6413869	3.655338119
15.89005604	14.34366278	29.97957272	3.462467737
16.15667557	14.16108298	30.29624188	3.275210397
16.39772456	13.95573917	30.71082745	3.096528538
16.64469844	13.73838968	31.12057956	2.876684367

16.89291966	13.51386798	31.63947417	2.582779216
17.16218979	13.33534204	32.03987122	2.323175993
17.42849748	13.11222359	32.42561198	2.171156087
17.70868186	12.84731916	32.81337968	2.013990893
18.01163025	12.6109477	33.1690283	1.842481256
18.32393371	12.38471089	33.50752596	1.706676807
18.61378499	12.12245707	33.82107676	1.593324528
18.86902252	11.88624152	34.14335896	1.503671764
19.18974554	11.67247815	34.53175033	1.328731934
19.48786047	11.45419315	34.89535076	1.181389564
19.75245307	11.25726891	35.29450046	1.006449734
20.06428877	11.00343464	35.67182166	0.8556771711
20.42929246	10.70703481	36.14175807	0.6902483303
20.85182984	10.39488727	36.61075896	0.5860952052
21.10051881	10.20591483	37.04795262	0.476796791
21.36557916	10.00540447	37.53987344	0.4191071859
21.63032767	9.791797016	38.05299909	0.375762023
21.91737244	9.572108763	38.5290163	0.2900072045
22.21907348	9.338387903	38.96605403	0.2516514129
22.54930749	9.054305576	39.41213501	0.202069536
22.87907375	8.843660559	40.07119977	0.1395464774
23.21507672	8.601520135	40.91190883	0.09277112187

23.49947088	8.342072829	41.91274552	0.06174346934

Figure 6: Table of coordinates from the modelling of the ramp from the knuckle to the end

The Loggerpro graphing software found the trend of best fit to follow a sine relationship, as seen in Figure 5, with a correlation of 0.9996 and therefore an R-squared value of 0.9992. The proximity of both values to 1 indicate that this model is able to describe the ramp well. After a downward translation of 0.21100 to better fit the graph to the real-life geometry of the ramp, the equation obtained from Loggerpro is as follows:

$$h_{ramp} = 8.5230 \sin\left(0.09527 x_{land} + 0.94500\right) + 8.5230$$

where x_{land} represents the distance of landing from the first point plotted. Translating rightward by 12.499 metres, the distance between the origin (at which the snowboarder leaves the ramp) and the knuckle of the landing to obtain h_{ramp} in s_x,

$$\begin{aligned} h_{ramp} &= 8.5230 \sin\left(0.09527(s_x - 12.499) + 0.94500\right) + 8.5230 \\ &= 8.5230 \sin\left(0.09527 s_x - 0.24580\right) + 8.5230 \end{aligned}$$

To account for the fact that, due to errors in human judgement when plotting position points, the first point was not exactly at the knuckle, but instead 6.5688 metres behind, the above equation describing the landing between the knuckle and the end of the slope is valid for the domain of:

$$19.068 \leq s_x \leq 52.044$$

The equation for h_{fall} can now be rewritten as:

$$\begin{aligned} \mathrm{h}_{fall} &= h_{max} + 17.983 - h_{ramp} \\ &= \frac{\left(\frac{s_x \cdot \tan 0.66131}{3}\right)^2}{2 \cdot g} + 17.983 - 8.5230 \sin\left(0.09527 s_x - 0.24580\right) + 8.5230 \\ &= \frac{s_x^2 \cdot \tan^2 0.66131}{18 \cdot g} - 8.5230 \sin\left(0.09527 s_x - 0.24580\right) + 9.4602 \end{aligned}$$

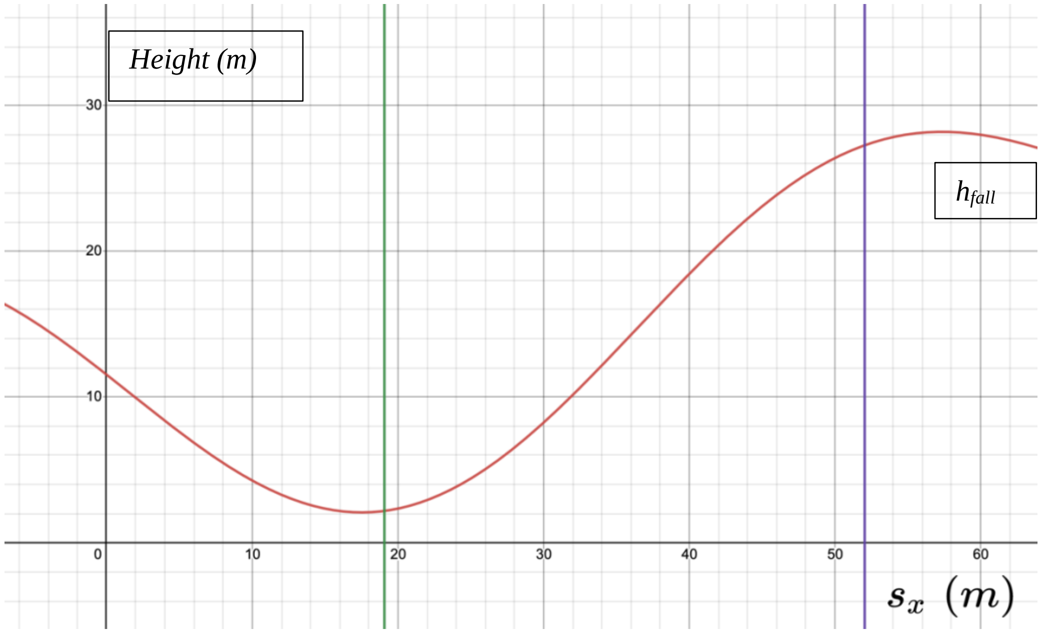

Figure 7: The graph of h_{fall} plotted in Desmos.

Figure 7 shows the aforementioned functions plotted in the graphing software Desmos, for the domain of $19.068 \leq s_x \leq 52.044$, enclosed by the green and purple vertical lines.

At this point, F_{normal} can be represented in terms of s_x and θ. To attain an equation in terms of only s_x, a function relating s_x and θ must be found. $\tan\theta$ is given by the local gradient of the ramp and is thus found by differentiating the function of h_{ramp} in s_x.

$$\tan\theta = \frac{\mathrm{d}}{\mathrm{d}s_x}(8.5230\sin(0.09527s_x - 0.24580) + 8.5230)$$
$$= 0.81199\cos(0.09527s_x - 0.24580)$$
$$\theta = \arctan(|0.81199\cos(0.09527s_x - 0.24580)|)$$

Finally, F_{slope} can now be rewritten in terms of s_x alone:

$$F_{slope} = \frac{m \cdot \Delta v}{\Delta t} \cdot \cos\theta$$
$$= \frac{m}{\Delta t} \cdot \sqrt{2 \cdot g \cdot h_{fall}} \cdot \cos\theta$$

Where:

$$h_{fall} = \frac{s_x^2 \cdot \tan^2 0.66131}{18 \cdot g} - 8.5230\sin(0.09527s_x - 0.24580) + 9.4602$$
$$\theta = \arctan(|0.81199\cos(0.09527s_x - 0.24580)|)$$

Thus, for $F_{slope} \leq F_{normal}$,

$$v_{final} \leq \sqrt{16g}$$

$$\sqrt{2 \cdot g \cdot h_{fall}} \cdot \cos\theta \leq \sqrt{16g}$$

For the region between the knuckle and the slope, as the domain defined above, solving graphically using Desmos (Figure 8) gives:

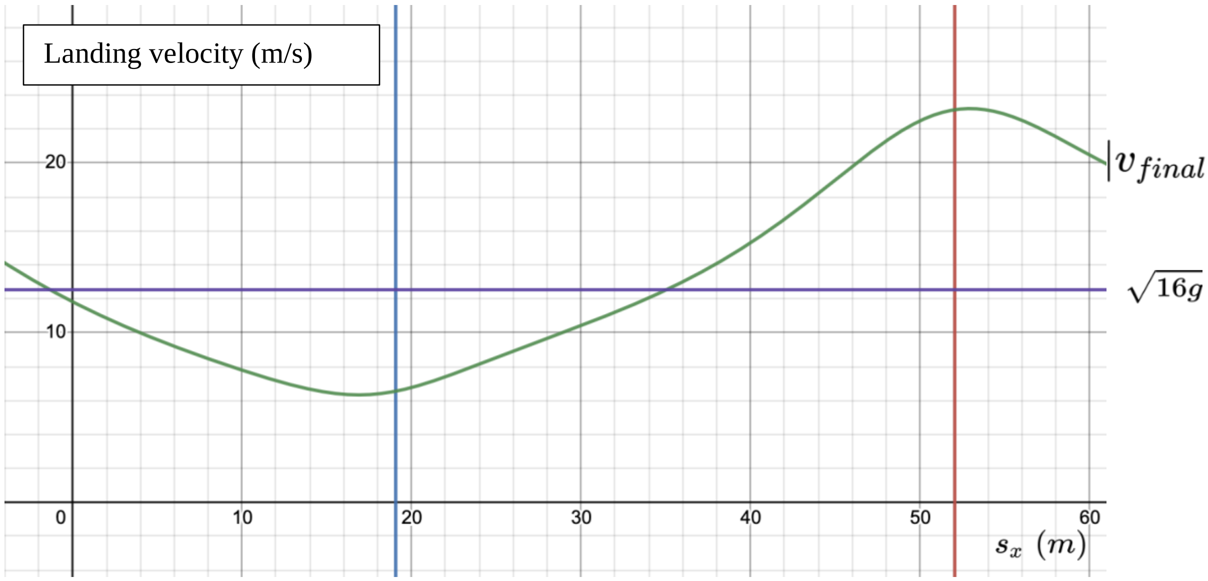

Figure 8: Graphical solution for the domain of s_x, plotted using Desmos.

Figure 8 shows the plot of $|v_{final}|$ against s_x, represented by the green curve. $y = \sqrt{16g}$ is represented by the horizontal purple line. Within the enclosed domain, the intersection of the two graphs occurs at $(35.049\ ,\ 12.528)$. Hence,

$$19.068 \leq s_x \leq 35.049$$
$$19.1 \leq s_x \leq 35.0 \quad (3\,sf)$$

Interestingly, this indicates that falling on the flat part of the knuckle itself results in an equivalent fall height that is still within the safe limit, due to the low speed at which the rider hits the ground. Overshooting is thus significantly more dangerous than undershooting. From the graphical solution, the maximum horizontal velocity as is safe is 12.528 metres per second. Referring to Figure 3 to calculate initial launch velocity,

$$
\begin{aligned}
v &= \frac{12.528}{\cos 0.66131} \\
&= 15.875 \quad (5sf) \\
&= 15.9 \quad (3sf)
\end{aligned}
$$

Hence, the maximum safe velocity for launch is 15.9 metres per second.

Data Collection

Trajectory Validation

I will now use video analysis to model Billy Morgan's attempt and compare it with the theoretical trajectory calculated in the previous section. I chose to plot points of position using frame-by-frame video analysis rather than the composite photograph, as more points could be taken for improved accuracy, as well as the availability of the time domain allowing for the analysis of object velocity. Since numerous rotations are being executed, points are plotted at an estimated centre of Morgan's body instead of the snowboard itself, which would be rotating three-dimensionally around the snowboarder's actual centre of mass. Using the software Loggerpro, a graph of best fit is automatically plotted. Figure 8 shows the best-fit graph plotted by Loggerpro overlaid with a composite image of the jump, provided by Red Bull Media.

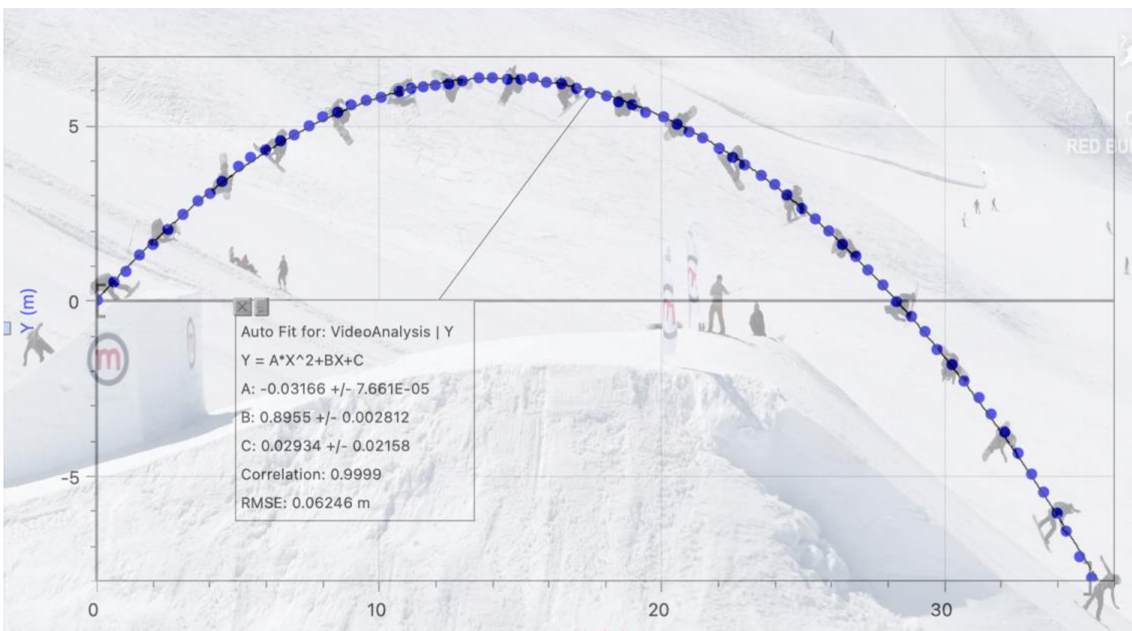

Figure 9: A comparison of the best-fit graph with the path traced by Morgan. Composite photo provided by Red Bull Media.[8]

The table of data points collected as coordinates are included in the appendix (Figure A3).

From Loggerpro's automatic graph fit tool, with the ramp height used for scale and the origin set at the point where Morgan just leaves the takeoff ramp, the function of vertical displacement against horizontal displacement is given by:

$$s_y = -0.031660s_x^2 + 0.89550s_x + 0.029340$$

Additionally, the functions of horizontal and vertical displacement against time are as follows:

$$s_x(t) = 12.160t - 0.078590$$
$$s_y(t) = -4.7360t^2 + 10.950t - 0.039300$$

The non-zero value attained when t = 0 is substituted could indicate that the origin was not plotted exactly at the point of launch, but the discrepancy is small enough to be ignored. Morgan attained an airtime of 2.850 seconds, which is lower than the average estimated for Olympic big air jumps. Substituting this value for *t*, solving for horizontal displacement gives:

$$\begin{aligned} s_x(2.8500) &= 12.160(2.8500) - 0.078590 \\ &= 34.577 \end{aligned}$$

Differentiating the function of horizontal displacement in time to obtain horizontal velocity gives:

$$\begin{aligned} v_x(t) &= \frac{d}{dt}(12.160t - 0.078590) \\ &= 12.160 \end{aligned}$$

Morgan travels a horizontal distance of 34.6 metres and at a horizontal velocity of 12.2 m/s, which lies within the safe range as calculated in the previous section. Referring to Figure 3, the Morgan's initial launch speed is:

$$\begin{aligned} v &= \frac{12.16}{\cos 0.66131} \\ &= 15.408 \ \ (5\,sf) \\ &= 15.4 \ \ (3\,sf) \end{aligned}$$

Modelling the Spins

The quadruple cork 1800 consists of its four eponymous four off axis flips and five spins, denoted by the 1800. In snowboarding, a flip is defined as a full inversion whereby the snowboarder goes upside-down and back upright again. A spin, on the other hand, is a rotation around an axis passing vertically through the snowboarder's body. These terms are differentiated in Figure 10, which has been annotated in red to indicate each point where Morgan has completed a full flip, and in black to indicate each point where he has completed a full spin. This investigation focuses on studying the spinning motion, as getting a full five spins yet being able to land in the correct orientation is regarded as the more challenging technical aspect of the trick. Spinning too slowly results in not being able to complete all five spins. However, landing with too much angular speed results in the snowboarding not landing in the correct orientation and falling.

Figure 10: A composite photograph of Billy Morgan's quadruple cork[8] 1800, with spins labelled in black and flips labelled in red.

A graph of rotation angle against time can be found using video analysis by determining the time taken for a rotation of $\frac{\pi}{2}$. This is achieved through marking a point on Loggerpro every time Morgan's snowboard is perpendicular from its previous position, starting off as facing forward. This process is illustrated in Figure 11:

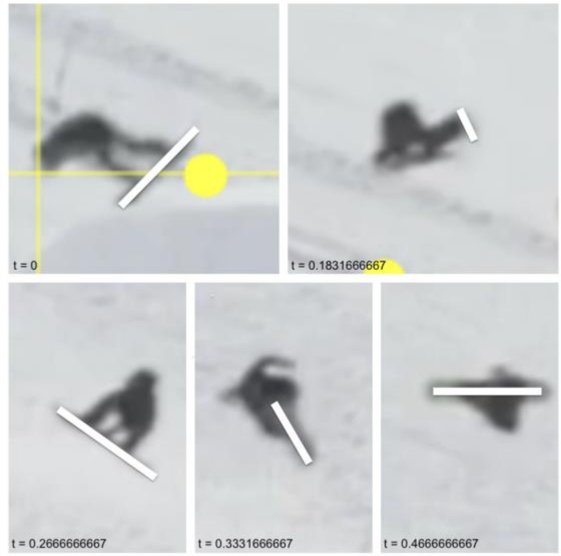

Figure 11: Annotation of plotting spin angle, with the snowboard used for reference highlighted as a white rectangle for clarity.

As shown, from t = 0 to t = 0.1831666667 (timestamps taken from Loggerpro), the Morgan has rotated a quarter spin with his snowboard now perpendicular to the original position. Similarly, Morgan rotates a quarter spin between each of the next frames in Figure 11, and these timestamps are then plotted against spin angle in intervals of $\frac{\pi}{2}$.

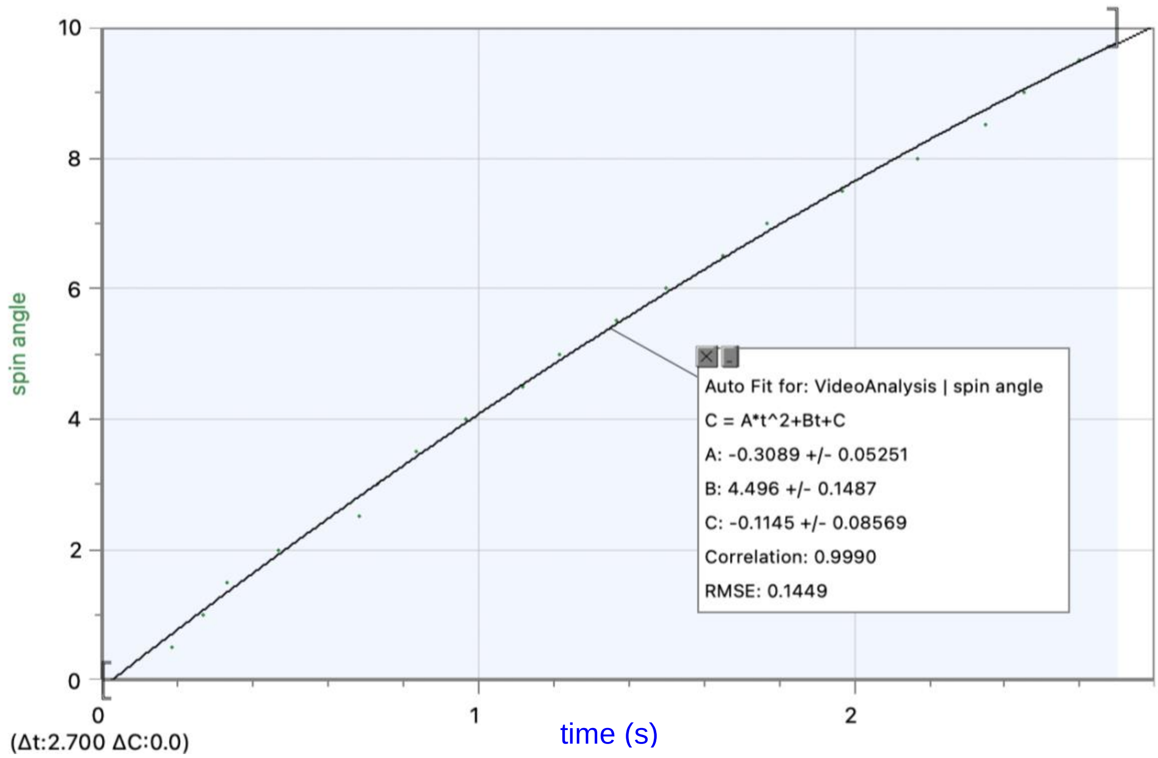

Figure 12: Graph of spin angle in terms of π radians against time.

$$\phi(t) = -0.30890\pi t^2 + 4.4960\pi t - 0.11450\pi$$

The table of data points collected is included in the appendix (Figure A4).

The model shows correlation of 0.9990 and hence an R-squared value of 0.9980 to the data points, indicating a very good fit.

Relating Spin Rate to Horizontal Displacement

Recalling the function for horizontal displacement against time, the substitution can be made such that:

$$\phi(s_x) = -0.30890\pi\left(\frac{s_x + 0.07859}{12.16}\right)^2 + 4.4960\pi\left(\frac{s_x + 0.07859}{12.16}\right) - 0.11450\pi$$
$$= -0.0020890\pi s_x^2 + 0.36941\pi s_x - 0.085455\pi$$

This function provides a spatial description relating Morgan's rotation motion through the jump to his flight path.

Investigating Change of Effective Radius through the Jump

The shape of the graph plotted in Figure 12 describes the spin angle increasing at a decreasing rate. In other words, the rate at which the snowboarder spins slows down through the jump. Angular frequency ω, or rate of spin, is the instantaneous change of angular displacement per unit time.

This can be found by differentiating the function of spin angle with respect to time:

$$\omega(t) = -0.61780\pi t + 4.4960\pi$$

The negative linear relationship found exemplifies that Morgan's rate of spin slows with time, which is very reasonable, and snowboarders must be careful not to land with too much angular velocity in order to avoid 'catching an edge', where they land on the wrong part of the snowboard and end up falling. While in flight, a snowboarder is not able to 'push off' of anything, as he is only subject to the influence of his weight and air resistance. As a result, the only way for Morgan to change his rate of spin is by varying his effective radius by changing the shape of his body as he rotates through the air.

A snowboarder spinning around an axis of their body can be approximated as cylindrical, with a mass m and effective radius k, which is varied by extending or contracting their arms. The moment of inertia of such a body is:

$$I = mk^2$$

From Team Great Britain's athlete statistics websites,[9] in Morgan's case, $m = 72$. Angular momentum, L, is given as the product of the instantaneous angular frequency and the object's moment of inertia.

$$L = 72 \cdot \omega \cdot k^2$$

From the law of conservation of angular momentum, the angular momentum possessed by the body remains constant throughout the flight. Mathematically,

$$\frac{dL}{dt} = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(72 \cdot \omega \cdot k^2\right) = 0$$

$$\frac{\mathrm{d}(72\omega)}{\mathrm{d}t} \cdot k^2 + 72\omega \cdot \frac{\mathrm{d}k^2}{\mathrm{d}t} = 0$$

$$\frac{\mathrm{d}(72)(-0.61780\pi t + 4.4960\pi)}{\mathrm{d}t} \cdot k^2 + 72(-0.61780\pi t + 4.4960\pi) \cdot \frac{\mathrm{d}k^2}{\mathrm{d}t} = 0$$

$$-44.482\pi k^2 + (-44.482\pi t + 323.71\pi) \cdot \frac{\mathrm{d}k^2}{\mathrm{d}t} = 0$$

$$\frac{\mathrm{d}k^2}{\mathrm{d}t} = \frac{44.482\pi k^2}{-44.482\pi t + 323.71\pi}$$

$$\frac{1}{44.482\pi k^2}\,dk^2 = \frac{1}{-44.482\pi t + 323.71\pi}\,dt$$

Taking the indefinite integral to get a function of k^2, and subsequently k, in terms of t,

$$\int \frac{1}{44.482\pi k^2}\,dk^2 = \int \frac{1}{-44.482\pi t + 323.71\pi}\,dt$$

$$\frac{\ln\left(k^2\right)}{44.482\pi} + C_1 = \frac{\ln\left(-44.482\pi t + 323.71\pi\right)}{-44.482\pi} + C_2$$

$$\ln k = -\frac{1}{2}\ln\left(-44.482\pi t + 323.71\pi\right) + C_3$$

$$k = e^{-\frac{1}{2}\ln(-44.482\pi t + 323.71\pi) + C_3}$$

$$= C_4 \cdot (-44.482\pi t + 323.71\pi)^{-\frac{1}{2}}$$

At the start of the jump, Morgan begins with an initial effective radius *k = 0.3180* metres as measured in Loggerpro. To determine the constant of integration,

$$0.3180 = C_4 \cdot \left[-44.4816\pi(0)^2 + 323.712\pi\right]^{-\frac{1}{2}}$$

$$C_4 = \frac{0.3180}{(323.712\pi)^{-\frac{1}{2}}}$$

$$= 10.141 \quad (5sf)$$

Therefore,

$$k = 10.141(-44.482\pi t + 323.71\pi)^{-\frac{1}{2}}$$

$$= 10.1(-44.5\pi t + 324\pi)^{-\frac{1}{2}}$$

for the domain of $0 \leq t \leq 2.850$, the duration of time for which Morgan is airborne. This relationship reveals that Morgan's effective radius gradually increases with time.

Discussion and Evaluation

The quadruple cork 1800 has only been performed by a few athletes worldwide. This leads to one key weakness of this study: analysing a single athlete's performance may not be enough to make a sufficiently useful guide for snowboarders hoping to accomplish this feat. As more snowboarders achieve the quadruple cork, it may be useful for future investigations to study the different approaches other riders may use successfully execute the jump. For example, snowboarders of different statures and shorter arm lengths than Morgan may need to compensate for being unable to change their effective radius as significantly by taking off with lower initial rotational momentum. A larger database of information collected over time as a diverse range of snowboarders add the quadruple cork 1800 to their repertoire would be beneficial for a more comprehensive understanding of the mechanics necessary to achieve this feat.

Another limitation of this investigation was in the video analysis of Morgan's motion. While the coordinates of the points plotted were generated by the Loggerpro software, human judgement was still involved in pinpointing an estimate of the snowboarder's centre of mass. This is particularly significant in the spin analysis, as it is even more challenging for a human to approximate the rotated angle in intervals any more precise than a quarter of a spin. Additionally, the frame rate of the camera was noticeably rather low, which also limited the precision of the timestamps that could be used. This could have been due to the availability of only one video of the jump from a single perspective; a more accurate model could have been constructed with a more precise method of measuring spin angle or position, such as by attaching lightweight sensors to the snowboard in motion, or by using a drone-mounted slow-motion camera to record the jump from a closer distance, allowing for a more accurate process of pinpointing translational and rotational motion.

Conclusion

From our theoretical derivations, a snowboarder can make a safe landing of equivalent fall height below the critical threshold for horizontal displacements of less than 35 metres, which, for the average airtime of about 3 seconds, translates to a maximum launch velocity of 15.9 metres per second. Our video analysis of Morgan's successful attempt found his horizontal displacement to be 34.6 metres, with an initial launch velocity of 15.4 metres per second. This falls within our calculated domain, validating our model. As a professionally trained athlete, it is understandable and impressive that Morgan's launch and landing parameters lie so close to the limit, with his body likely being able to withstand the high impact from the large normal reaction force. Remarkably, Morgan's horizontal velocity was found to be fairly constant, proving our approximation of negligible air resistance to be quite accurate, potentially due to a snowboarder's relatively slow speed and small cross-sectional area.

From our analysis of Morgan's rotational kinematics, it was found that he very gradually increased his effective radius from the initial $k = 0.318$ metres at the beginning of the jump to $k = 0.408$ metres at landing, likely to prevent an over-rotation at landing that might cause him to fall. However, one limitation of our investigation is that our simplified model of the human body is unable to account for the complexities of the twisting motions a snowboarder undergoes. As the body moves through the three-dimensional space, the moment of inertia in real life usually cannot be accurately calculated with the simple formula used in the previous section and can only be approximated at best.

This investigation of the limits and mechanics of the quadruple cork 1800 has showcased the extreme technicality and strength required for such a feat, further highlighting the skill and athleticism of big air snowboarders like Billy Morgan. Even slight miscalculations, especially in terms of underestimations of launch velocity, brings a high risk of injury to the inexperienced rider. Demonstrating precise control over one's body orientation is also a very important aspect that leads to the successful execution of big air snowboarding tricks. For many amateur snowboarders such as myself, although massive tricks like the quadruple cork 1800 may be out of the question, the incredible amount of skill exhibited by professionals is always an inspiration to marvel at.

Bibliography

(1)

Gonzalez, R. *The Big Engineering Behind Olympic Snowboarding's Big Air Event*. Wired. https://www.wired.com/story/big-engineering-big-air-olympics/.

(2)

Snowboarding 101: Scoring | NBC Olympics. www.nbcolympics.com. https://www.nbcolympics.com/news/snowboarding-101-scoring.

(3)

Red Bull Media. Watch Anna Gasser land a world-first Cab Double Cork 1260 by a female rider. Red Bull. https://www.redbull.com/sg-en/anna-gasser-cab-double-cork-1260 (accessed 2023-11-07).

(4)

Behn, E. *North America's Top 10 Terrain Parks*. SnowBrains. https://snowbrains.com/north-americas-top-10-terrain-parks/ (accessed 2023-10-15).

(5)

Petrone, N.; Cognolato, M.; McNeil, J. A.; Hubbard, M. Designing, Building, Measuring, and Testing a Constant Equivalent Fall Height Terrain Park Jump. *Sports Engineering* **2017**, *20* (4), 283–292. https://doi.org/10.1007/s12283-017-0253-y.

(6)

Hubbard, M. *Safer Ski Jump Landing Surface Design Limits Normal Impact Velocity*. Semantic Scholar. https://doi.org/10.1520/STP47480S.

(7)

American College of Surgeons, Committee on Trauma. *Resources for Optimal Care of the Injured Patient*. https://www.facs.org/quality-programs/trauma/quality/verification-review-and-consultation-program/standards/.

(8)

Horton, J. *Billy Morgan lands first-ever snowboard Quad Cork*. Red Bull. https://www.redbull.com/za-en/billy-morgan-lands-world-first-backside-quadruple-cork (accessed 2023-10-15).

(9)

Team GB. *Billy Morgan*. www.teamgb.com. https://www.teamgb.com/athlete/billy-morgan/2Od8VSasbLmlvwdUHya9Rn (accessed 2023-10-15).

Appendix

Abbreviation	Parameter
F	Force exerted by ground on snowboarder (Newtons)
m	Mass of snowboarder (kilograms)
θ	Angle of incline of landing ramp (radians)
v_x/v_y	Instantaneous horizontal/vertical velocity respectively (metres per second)
u_x/u_y	Initial horizontal/vertical velocity respectively (metres per second)
s_x/s_y	Instantaneous horizontal/vertical displacement respectively (metres)
t	Time elapsed from beginning of takeoff (seconds)
g	Gravitational acceleration, taken to equal 9.81 metres per second squared
ϕ	Spin angle (radians)
ω	Angular frequency (radians per second)
I	Moment of inertia (kilogram metre squared)
k	Effective radius (metres)
L	Angular momentum (kilogram metre squared per second)

Figure A1: Table of Definitions

Jump Number	Duration of Jump (s)
1	3.142
2	3.312
3	3.010
4	3.163
5	3.063
6	3.018
7	3.037
8	2.636
9	2.909
10	2.968
11	3.183
12	3.256
13	3.006
14	2.739
15	3.401
16	2.564
17	2.911
18	3.165
19	2.262
20	3.291
21	3.188
22	2.635
23	2.873
24	3.233
25	3.035

Figure A2: Average durations of Beijing 2022 big air jumps

Time (s)	Horizontal displacement (m)	Vertical displacement (m)
0.0445	0.006470929702	0.01495007897
0.06116666667	0.6140689151	0.5203520022
0.0945	1.048513747	0.8423365388
0.1445	1.50080942	1.288607553
0.1776666667	1.983228386	1.613715986
0.2111666667	2.498671407	2.003756852
0.2611666667	3.027279423	2.421243386
0.2945	3.522417113	2.802805103
0.3445	3.936110343	3.033750353
0.3776666667	4.402686688	3.376486491
0.4111666667	4.96922774	3.787502095
0.4611666667	5.367301484	4.030050391
0.4945	5.87359595	4.263896402
0.5445	6.43076531	4.494841652
0.5776666667	6.87078853	4.695663608
0.6111666667	7.416354845	4.94736046
0.6611666667	7.863518401	5.196825957
0.6945	8.386771164	5.301030239
0.7445	8.874768518	5.520595578
0.7776666667	9.39913696	5.66451798
0.8111666667	9.944033867	5.729896683
0.8611666667	10.54181389	5.901041617
0.8945	11.00191931	5.972668115
0.9445	11.37477874	6.031575889
0.9776666667	11.79828993	6.075087312
1.011166667	12.27423797	6.12038382
1.061166667	12.74059118	6.195803622

1.0945	13.31382629	6.276578675
1.1445	13.82659169	6.272785372
1.177666667	14.31793607	6.256050209
1.211166667	14.82132978	6.255157666
1.261166667	15.24461784	6.26787639
1.2945	15.71610316	6.154969824
1.3445	16.22440585	6.114582297
1.377666667	16.76260869	5.999890647
1.411166667	17.2267002	5.850718192
1.461166667	17.76740715	5.763533196
1.4945	18.22412855	5.621157697
1.5445	18.68772877	5.503737904
1.5945	19.14221056	5.328248104
1.611166667	19.75362534	5.178033954
1.661166667	20.26153793	4.986226963
1.6945	20.67810624	4.740189303
1.7445	21.15674388	4.575257686
1.777666667	21.70209003	4.301224845
1.811166667	22.19464526	4.031991177
1.861166667	22.62769074	3.828506195
1.8945	23.17463662	3.524878447
1.9445	23.64399584	3.270522219
1.977666667	24.06376361	3.000168744
2.011166667	24.61422888	2.609036054
2.061166667	25.08742744	2.307327975
2.0945	25.56510524	1.980984136
2.1445	26.01222829	1.588571666
2.177666667	26.48446703	1.237592067
2.211166667	26.88807757	0.8613368164

2.261166667	27.40606843	0.4340503483
2.2945	27.92117978	-0.02171121951
2.3445	28.43437147	-0.4464381281
2.377666667	28.88485394	-0.8751643484
2.411166667	29.31022074	-1.344843521
2.461166667	29.82501215	-1.801404951
2.4945	30.23118225	-2.229491282
2.5445	30.77524862	-2.732764673
2.577666667	31.21997209	-3.183407122
2.611166667	31.66005636	-3.706197155
2.661166667	32.14349316	-4.255862563
2.6945	32.61893134	-4.847120813
2.7445	33.02446156	-5.387507818
2.777666667	33.48198282	-5.966768133
2.811166667	33.80656696	-6.465402322
2.861166667	34.30744077	-7.182238962
2.8945	34.6913747	-7.790294321

Figure A3: Table of Coordinates of Billy Morgan's Jump Trajectory, with the origin set as the point where a snowboarder leaves the ramp.

Time (s)	Spin angle (π rad)
0	0
0.1831666667	0.5
0.2666666667	1
0.3331666667	1.5
0.4666666667	2
0.6831666667	2.5
0.7331666667	3
0.8331666667	3.5
0.9666666667	4
1.116666667	4.5
1.216666667	5
1.366666667	5.5
1.5	6
1.65	6.5
1.766666667	7
1.966666667	7.5
2.166666667	8
2.35	8.5
2.45	9
2.6	9.5
2.7	10

Figure A4: Table of data points for Morgan's spin.