Causal Heterogeneity

1. Heterogeneous Treatment Effects
   - Same treatment may affect different individuals differently
   - Conditional Average Treatment Effect (CATE)

$$\tau(\mathbf{x}) = \mathbb{E}(Y_i(1) - Y_i(0) \mid \mathbf{X}_i = \mathbf{x}) \quad \text{where} \quad \mathbf{x} \in \mathcal{X}$$

   - Individualized treatment rule $f : \mathcal{X} \longrightarrow \{0, 1\}$
   - We can never identify an individual causal effect $\tau_i = Y_i(1) - Y_i(0)$
   - Individualized treatment rule depends on the choice of $\mathbf{X}_i$

2. Causal interaction
   - Different combinations of treatments may have different effects
   - Interaction among treatment variables instead of interaction between a treatment and covariates
   - Factorial designs, e.g., conjoint analysis