Statement ①

Q4) ⓐ $i: 1, 1+2(1), 1+2(2), \dots 1+2k \leq n-1$

$1+2k = n-1 \Rightarrow \boxed{k = \frac{n-2}{2}}$

$$\sum_{k=0}^{n-2/2} 1 = \boxed{\frac{n-2}{2} + 1}$$

Statement ② $i: 1, 2, 2^2, 2^3 \dots 2^m \leq n$

$2^k = n \Rightarrow k = \log_2 n$

$$\sum_{m=0}^{\log_2 n} \sum_{k=1}^{i} 1 = \sum_{m=0}^{\log n} i = \sum_{m=0}^{\log n} 2^m = \frac{2^{\log_2 n+1} - 1}{2-1}$$

$$= \boxed{2n-1}$$

Statement ③ : $i: 1, 2, 2^2 \dots 2^k$

$2^r = n^2 \longrightarrow \log r = 2\log_2 n$

$$\sum_{k=0}^{2\log_2 n} 1 = \boxed{2\log_2 n + 1}$$

Statement ④ $i = 2^{2(1)}, 2^{2(2)}, 2^{2(3)} \dots 2^{2(k)}$

$2^{2r} = n \Rightarrow r = \frac{1}{2}\log_2 n$

$$\sum_{i=1}^{n-1} \sum_{k=0}^{\frac{1}{2}\log_2 n} 1 = \boxed{\sum_{i=1}^{n-1} \frac{1}{2}\log_2 n + \sum_{i=1}^{n-1} 1}$$

$$= \frac{1}{2}\log_2 n\,(n-1) + (n-1)$$

Q4) (b)

Best case is when Stmnt 2 is never executed ie, $x \geq 5$

to find best case we calculate $\Omega = (\min(Stm1, Stm3, Stm4))$

$2\log n \rightarrow \Omega(\log n)$

(c) worst case when Stmnt 2 is always triggired ie, $x < 5$ $O(n \log n)$

(2) $f_1(n) = n^2 + 100n$ $\quad O(f_1) = n^2$

$f_2(n) = n^2 + 100n + 1000$ $\quad O(f_2) = n^2 > g(n)$

(3) (a) if $f(n)$ is $\Theta(g(n))$ $\therefore$ $O(g(n)) = \Omega(g(n))$

but, $2^{f(n)} = O(2^{f(n)+i})$ $\quad i > 0$

and $2^{fn} = \Omega(2^{f(n)+i})$ $\quad i < 0$

$\therefore$ $O(gn) \neq \Omega(g(n))$ Contradiction

So the Statement is false

(B) $f(n) + g(n)$ is $\Theta(\min(f(n), g(n)))$

false by the definition of Big Θ

(c) 2^{na} is $O(2^n)$ where a is Const

true using fact 4 of Big O

2^{n+a}