Supplementary Online Materials for

"A Multi-Area Architecture for Real-Time Feedback-Based Optimization of Distribution Grids"

Manuscript

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1 Introduction

This document serves as a supplementary material for the paper titled "A Multi-Area Architecture for Real-Time Feedback-Based Optimization of Distribution Grids" [ref] . In this document, we cover details that are not mentioned in the paper due to the limited space, or to help make the paper flow smooth.

The mathematical derivations and simulation setup shown here are based on the control structure described in the paper and MATDSS Application [ref] . MATDSS Application is a `MATLAB` -based application that integrates with OpenDSS©. OpenDSS©is a standalone application specialized in distribution network simulations, and contains an engine that can be interfaced with `MATLAB` through a `COM` interface [ref] . For more details about the capabilities and ways to integrate OpenDSS©with other IDEs and programming languages, refer to [ref] .

MATDSS Application is a *GUI* application that mainly focuses on modeling distribution networks/feeders in `MATLAB` using OpenDSS©engine as a solver. The dynamics and simulation time-series are handled by MATDSS Application, where it passes circuit components updates to OpenDSS©when they occur. For full details of MATDSS Application capabilities, please refer to [ref] . This supplementary material is prepared using MATDSS Application V0.92.

The rest of this document is divided as:

- Section 2: DER device parameters and model used inside MATDSS
- Section 3: Distribution network model and DERs' sensitivity matrices (**A**, **B**, **M**, **H**)
- Section 4: Simulations - Setup and Configuration Parameters
  - 5 Bus Feeder
  - IEEE-123 Feeder
  - IEEE-8500 Feeder

2 DER Device Parameters & Model within MATDSS

In this section, we briefly summarize the first-order dynamical model used in the simulations in Section 4, and explain the different parameters used to control the response of DERs.

2.1 DER Parameters

2.1.1 Active and reactive powers - x_i & x_i^{set}

Each DER device has an active and reactive power values ($x_i = \begin{bmatrix} p_i & q_i \end{bmatrix}^\mathsf{T}$) and set-points ($x_i^{\mathsf{set}} = \begin{bmatrix} p_i^{\mathsf{set}} & q_i^{\mathsf{set}} \end{bmatrix}^\mathsf{T}$). The controllers within MATDSS will assign different x_i^{set} values following the outcome of the optimization problem, and the DERs would respond accordingly. The units for active and reactive powers are W and Var, respectively.

2.1.2 Active and reactive power constraints set - $\mathcal{X}_i$

The set $\mathcal{X}_i$ represents the set of possible x_i that can be set for the ith DER. In the implementations we show, we have considered a box constraint $\mathcal{X}_i = [\underline{x}_i, \overline{x}_i]$, and we project the updated x_i^{set} on $\mathcal{X}_i$.

2.1.3 DER Cost function - $f_i(x_i)$

In the simulations, quadratic DERs cost functions were considered. The general form of a quadratic cost function is shown below.

$$f_i(x_i) = x_i^\mathsf{T} C_i'' x_i + x_i^\mathsf{T} C_i', \tag{1}$$

where $C_i'' \succ 0$ is a diagonal 2×2 matrix and $C_i' \in \mathbb{R}^2$.

2.2 DER 1st order model

The DERs are modeled within MATDSS as a 1st order differential equation; where the DERs are assumed to have decoupled active and reactive power controls. The general transfer function of the ith DER is given by

$$T_i(s) = \begin{bmatrix} \frac{1}{\tau_{p,i}s+1} & 0 \\ 0 & \frac{1}{\tau_{q,i}s+1} \end{bmatrix} \tag{2}$$

where $\tau_{p,i}$ and $\tau_{q,i}$ are the time-constants that represent how fast the DER responds to input signals (x_i^{set}). In the simulations, we consider the same time constant for active and reactive responses.

For more advanced and realistic models of DERs, one can change the dynamic simulation code (`MATDSS_DERUpdate` function) in MATDSS. For more details, refer to **[ref]** .

3 Distribution Network Model and DERs' sensitivity matrices (A, B, M, H)

In this section, we introduce the distribution network model used and then build the sensitivity matrices for the DERs following the defined model of DERs in Section 2. The distribution model discussed here is following the control structure described in the paper **[ref]** .

3.1 Distribution Network Model

We adopt the multi-phase distribution model presented in [1,2]. Within each control area, we consider the same model. For the ith CA, we let $\overline{\mathcal{N}}_i = \{0\} \cup \mathcal{N}_i$ with $\mathcal{N}_i := \{1, 2, \ldots, N_i\}$ denote the set of buses, where the interface bus of the CA is given the node "0". We consider a general setup where each bus is potentially multi-phase, with up to three phases. In the annotations below, we show the derivations and relationships for three phases, where for any configurations with less than three phases, the corresponding components can be set to zero. Also, we drop the control area indexing 'i' with the understanding that all parameters and variables belong to the ith CA.

In addition, we recall that $\mathcal{M}_v$ denotes the set of monitored and controlled bus voltages and $\mathcal{M}_i$ denotes the set of monitored and controlled branch currents. For more details about those sets and structure details, refer to **[ref]** .

We define $\mathbf{Y}$ as the three-phase admittance matrix for the distribution network as

$$\mathbf{Y} := \begin{bmatrix} \mathbf{Y}_{00} & \mathbf{Y}_{0L} \\ \mathbf{Y}_{L0} & \mathbf{Y}_{LL} \end{bmatrix} \in \mathbb{C}^{3(N+1)\times 3(N+1)} \tag{3}$$

where $\mathbf{Y}_{00} \in \mathbb{C}^{3\times 3}$, $\mathbf{Y}_{L0} \in \mathbb{C}^{3N\times 3}$, $\mathbf{Y}_{0L} \in \mathbb{C}^{3\times 3N}$ and $\mathbf{Y}_{LL} \in \mathbb{C}^{3N\times 3N}$ when considering an all 3-phase buses. In addition, we define $\mathbf{G}$ as $3N \times 3N$ block diagonal matrix that is considered as phase-to-ground to phase-to-phase (3-phase) voltage conversion matrix.

$$\mathbf{G} := \begin{bmatrix} \mathbf{\Gamma} & & \\ & \ddots & \\ & & \mathbf{\Gamma} \end{bmatrix}, \qquad \mathbf{\Gamma} := \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{4}$$

In [1, 2], the linear models for voltages, currents and interface bus powers that capture active sources or DERs sensitivities are developed for wye and delta connected devices separately. We show next the main results from [1, 2], where Table 1 lists the main parameters used in their linear model.

Table 1: Distribution network model parameters

Parameter	Description	Parameter	Description
$\mathbf{x}^Y$	Active and reactive powers for wye-connected sources and devices	$\hat{\mathbf{v}}$	Voltage profile of the network (for linearization purposes)
$\mathbf{x}^\Delta$	Active and reactive powers for delta-connected sources and devices	$\mathbf{w}$	Zero-load voltage profile
$\overline{c}$	denotes conjugate of c		

3.1.1 Linear models for $\mathbf{v}, \mathbf{i}$ and $\mathbf{s}_0$

Voltage Phasor : The voltage profile phasor is linearly modeled as a function of wye and delta powers of devices as

$$\tilde{\mathbf{v}} = \mathbf{M}^Y \mathbf{x}^Y + \mathbf{M}^\Delta \mathbf{x}^\Delta + \mathbf{a} \tag{5a}$$

where

$$\mathbf{M}^Y = \left[\mathbf{Y}_{LL}^{-1}\mathsf{diag}(\overline{\widehat{\mathbf{v}}})^{-1}, -\jmath\mathbf{Y}_{LL}^{-1}\mathsf{diag}(\overline{\widehat{\mathbf{v}}})^{-1}\right], \tag{5b}$$

$$\mathbf{M}^\Delta = \left[\mathbf{Y}_{LL}^{-1}\mathbf{G}^\top\mathsf{diag}\left(\mathbf{G}\overline{\widehat{\mathbf{v}}}\right)^{-1}, -\jmath\mathbf{Y}_{LL}^{-1}\mathbf{G}^\top\mathsf{diag}\left(\mathbf{G}\overline{\widehat{\mathbf{v}}}\right)^{-1}\right], \tag{5c}$$

$$\mathbf{a} = \mathbf{w}. \tag{5d}$$

Voltage magnitude : The voltage profile magnitude is obtained from the phasor profile described in (5) as

$$|\tilde{\mathbf{v}}| = \mathbf{K}^Y \mathbf{x}^Y + \mathbf{K}^\Delta \mathbf{x}^\Delta + \mathbf{b} \tag{6a}$$

where

$$\mathbf{W} = \mathsf{diag}(\mathbf{w}), \tag{6b}$$

$$\mathbf{K}^Y = |\mathbf{W}|\ \mathsf{Re}\{\mathbf{W}^{-1}\mathbf{M}^Y\}, \tag{6c}$$

$$\mathbf{K}^\Delta = |\mathbf{W}|\ \mathsf{Re}\{\mathbf{W}^{-1}\mathbf{M}^\Delta\}, \tag{6d}$$

$$\mathbf{b} = |\mathbf{w}|. \tag{6e}$$

Power flow at the interface bus : The complex power flow model at the interface bus is given by

$$\tilde{\mathbf{s}}_0 = \mathbf{G}^Y \mathbf{x}^Y + \mathbf{G}^\Delta \mathbf{x}^\Delta + \mathbf{c} \tag{7a}$$

where

$$\mathbf{G}^Y = \mathsf{diag}(\mathbf{v}_0)\overline{\mathbf{Y}}_{0L}\overline{\mathbf{M}}^Y, \tag{7b}$$

$$\mathbf{G}^\Delta = \mathsf{diag}(\mathbf{v}_0)\overline{\mathbf{Y}}_{0L}\overline{\mathbf{M}}^\Delta, \tag{7c}$$

$$\mathbf{c} = \mathsf{diag}(\mathbf{v}_0)\left(\overline{\mathbf{Y}}_{00}\overline{\mathbf{v}}_0 + \overline{\mathbf{Y}}_{0L}\overline{\mathbf{a}}\right). \tag{7d}$$

Branch currents : The current flowing in the line connecting buses i & j is given by

$$\tilde{\mathbf{i}}_{ij} = \mathbf{J}_{ij}^Y \mathbf{x}^Y + \mathbf{J}_{ij}^\Delta \mathbf{x}^\Delta + \mathbf{c}_{ij} \tag{8a}$$

where

$$\mathbf{Z}_{ij} \in \mathbb{C}^{3\times3} \text{ phase impedance matrix of line } (i,j), \tag{8b}$$

$$\mathbf{Y}_{ij}^{(s)} \in \mathbb{C}^{3\times3} \text{ shunt admittance matrix of line } (i,j), \tag{8c}$$

$$\mathbf{E}_i = \left[\mathbf{0}_{3\times3(i-1)}, \mathbf{I}_3, \mathbf{0}_{3\times3(N-i)}\right], \tag{8d}$$

$$\mathbf{J}_{ij}^Y = \left[\left(\mathbf{Y}_{ij}^{(s)} + \mathbf{Z}_{ij}^{-1}\right)\mathbf{E}_i - \mathbf{Z}_{ij}^{-1}\mathbf{E}_j\right]\mathbf{M}^Y, \tag{8e}$$

$$\mathbf{J}_{ij}^\Delta = \left[\left(\mathbf{Y}_{ij}^{(s)} + \mathbf{Z}_{ij}^{-1}\right)\mathbf{E}_i - \mathbf{Z}_{ij}^{-1}\mathbf{E}_j\right]\mathbf{M}^\Delta, \tag{8f}$$

$$\mathbf{c}_{ij} = \left[\left(\mathbf{Y}_{ij}^{(s)} + \mathbf{Z}_{ij}^{-1}\right)\mathbf{E}_i - \mathbf{Z}_{ij}^{-1}\mathbf{E}_j\right]\mathbf{w}. \tag{8g}$$

Following the main results, we note the following:

- The linear models shown above considers the wye and delta devices separately. However, in our control architecture, we aim to control all DERs according to their cost functions and utilize them to track power requests and maintain circuit constraints without explicitly separating them to: wye/delta or *1-/2-/3-* phases connected devices.
- Therefore, we aim to capture their connection type and number of phases effects within a unified sensitivity matrices that implicitly capture these information.
- In the following section, we regroup the DERs and develop a unified sensitivity matrices that map the DERs accordingly.

3.2 Modified Equations of distribution network model

Following the main results of [1, 2], we now derive a linearized model for the distribution network that groups wye and delta DERs to obtain a linear model of the form

$$\begin{aligned}
&\text{(voltage magnitude)} \quad \tilde{\mathbf{v}}_{\mathcal{M}_v}(\mathbf{x}_i) = \sum_{j\in\mathcal{D}_i} A_j x_j + a_j && = \mathbf{A}_i\mathbf{x}_i + \mathbf{a}_i && \text{(9a)}\\
&\text{(current magnitude)} \quad \tilde{\mathbf{i}}_{L,\mathcal{M}_i}(\mathbf{x}_i) = \sum_{j\in\mathcal{D}_i} B_j x_j + b_j && = \mathbf{B}_i\mathbf{x}_i + \mathbf{b}_i && \text{(9b)}\\
&\tilde{\mathbf{p}}_0(\mathbf{x}_i) = \sum_{j\in\mathcal{D}_i} M_j x_j + m_j && = \mathbf{M}_i\mathbf{x}_i + \mathbf{m}_i && \text{(9c)}\\
&\tilde{\mathbf{q}}_0(\mathbf{x}_i) = \sum_{j\in\mathcal{D}_i} H_j x_j + h_j && = \mathbf{H}_i\mathbf{x}_i + \mathbf{h}_i && \text{(9d)}
\end{aligned}$$

where $\mathbf{x}_i = \mathsf{col}(x_1, \dots, x_{D_i})$ is the vector of all DER/DER-aggregator set-points where $D_i = |\mathcal{D}_i|$ denotes the number of DERs within the control area and "$\sim$" indicates that the obtained variables are the linear modeled values. We recall that $x_1 = \mathsf{col}(p_1, q_1)$.

Mapping $\mathbf{A}, \mathbf{B}, \mathbf{M}, \mathbf{H}, \mathbf{a}, \mathbf{b}, \mathbf{m}$ & $\mathbf{h}$ to multi-phase distribution network model matrices (Eqs. 6a, 7a & 8a)

In the work below, we drop the control area indexing "i" with the understanding that these calculations are done per control area. To simplify the deviations, we consider that all buses consists of three phases. However, the same logic applies for buses with 1 or 2 phases, where the missing phases are simply omitted from the vectors and matrices.

3.2.1 A matrix

When building the matrix $\mathbf{A}$, we construct it by considering one DER at a time. Then, we combine all individual DERs A matrices together to obtain a unified sensitivity matrix for all DERs that can address all DERs at once. To begin with, we have from Eq. 6a that

$$\begin{aligned}
|\tilde{\mathbf{v}}| &= \mathbf{K}^Y \mathbf{x}^Y + \mathbf{K}^\Delta \mathbf{x}^\Delta + \mathbf{b} \\
&= \begin{bmatrix} \mathbf{K}^Y & \mathbf{K}^\Delta \end{bmatrix} \begin{bmatrix} \mathbf{x}^Y \\ \mathbf{x}^\Delta \end{bmatrix} + \mathbf{b} \\
&= \underbrace{\begin{bmatrix} \mathbf{K}^Y & \mathbf{K}^\Delta \end{bmatrix}}_{3N \times 12N} \underbrace{\begin{bmatrix} \mathbf{p}^Y \\ \mathbf{q}^Y \\ \mathbf{p}^\Delta \\ \mathbf{q}^\Delta \end{bmatrix}}_{12N \times 1} + \mathbf{b}
\end{aligned} \tag{10}$$

To extract only the measured values $|\mathbf{v}_{\mathcal{M}_\mathbf{v}}|$ out of $|\tilde{\mathbf{v}}|$, and to express the right-hand side as a function only of the DER point setpoints $\mathbf{x}$, we do the following manipulations. First, we split the powers $\mathbf{p}$ and $\mathbf{q}$ to DER and none-DER components, and then, we extract the required components. To do this, let $\mathbf{Q}_L^v \in \mathbb{R}^{3r_v \times 3N}$ select the desired measured voltages from the vector of all voltages, where the ij component is given by

$$(Q_L^v)_{ij} = \begin{cases} 1 & \text{if the } i\text{th voltage measurement is from bus } j \\ 0 & \text{otherwise} \end{cases}$$

and then set $\mathbf{Q}_L^v = Q_L^v \otimes I_3$. With this notation, we have that

$$|\tilde{\mathbf{v}}_{\mathcal{M}_v}| = \mathbf{Q}_L^v |\tilde{\mathbf{v}}|.$$

Next, we wish to express $|\tilde{\mathbf{v}}|$ in terms of $\mathbf{x}$.We define a matrix $\mathbf{Q}_R$ such that

$$\begin{bmatrix} \mathbf{p}^Y \\ \mathbf{q}^Y \\ \mathbf{p}^\Delta \\ \mathbf{q}^\Delta \end{bmatrix} = \mathbf{Q}_R \mathbf{x} + \mathbf{c}$$

Here, $\mathbf{Q}_R$ places DER powers at the correct phases and locations within the vector of wye and delta powers, accounting for their method of interconnection:

- For the jth DER connected at bus i (here $e_j \in \mathbb{R}^{2D}$ is a zero vector with 1 at the jth component)
  - ◇ Y-connected balanced DER (3-phases)

$$\begin{bmatrix} \mathbf{Q}_{R,\mathsf{rows}(i:i+2)} \\ \mathbf{Q}_{R,\mathsf{rows}(3N+i:3N+i+2)} \end{bmatrix} = \begin{bmatrix} \frac{1}{3}\mathbb{1}_3 e_{2j-1}{}^\top \\ \frac{1}{3}\mathbb{1}_3 e_{2j}{}^\top \end{bmatrix} \tag{11a}$$

  - ◇ Single-phase connected DER

$$\begin{bmatrix} \mathbf{Q}_{R,\mathsf{rows}(i:i+2)} \\ \mathbf{Q}_{R,\mathsf{rows}(3N+i:3N+i+2)} \end{bmatrix} = \begin{cases} \begin{bmatrix} e_{2j-1} & \mathbb{0} & \mathbb{0} & e_{2j} & \mathbb{0} & \mathbb{0} \end{bmatrix}^\top & \text{, if connected at phase } a \\ \begin{bmatrix} \mathbb{0} & e_{2j-1} & \mathbb{0} & \mathbb{0} & e_{2j} & \mathbb{0} \end{bmatrix}^\top & \text{, if connected at phase } b \\ \begin{bmatrix} \mathbb{0} & \mathbb{0} & e_{2j-1} & \mathbb{0} & \mathbb{0} & e_{2j} \end{bmatrix}^\top & \text{, if connected at phase } c \end{cases} \tag{11b}$$

◇ Δ-connected balanced DER (3-phases)

$$\begin{bmatrix} \mathbf{Q}_{R,\text{rows}(6N+i:6N+i+2)} \\ \mathbf{Q}_{R,\text{rows}(9N+i:9N+i+2)} \end{bmatrix} = \begin{bmatrix} \frac{1}{3}\mathbb{1}_3 e_{2j-1}{}^\top \\ \frac{1}{3}\mathbb{1}_3 e_{2j}{}^\top \end{bmatrix} \tag{11c}$$

◇ phase-to-phase connected DER

$$\begin{bmatrix} \mathbf{Q}_{R,\text{rows}(6N+i:6N+i+2)} \\ \mathbf{Q}_{R,\text{rows}(9N+i:9N+i+2)} \end{bmatrix} = \begin{cases} \begin{bmatrix} e_{2j-1} & \mathbb{0} & \mathbb{0} & e_{2j} & \mathbb{0} & \mathbb{0} \end{bmatrix}^\top & \text{, if } ab\text{-phase connected} \\ \begin{bmatrix} \mathbb{0} & e_{2j-1} & \mathbb{0} & \mathbb{0} & e_{2j} & \mathbb{0} \end{bmatrix}^\top & \text{, if } bc\text{-phase connected} \\ \begin{bmatrix} \mathbb{0} & \mathbb{0} & e_{2j-1} & \mathbb{0} & \mathbb{0} & e_{2j} \end{bmatrix}^\top & \text{, if } ca\text{-phase connected} \end{cases} \tag{11d}$$

◇ Other connections - treat similarly to what presented here. One can have two phase-to-phase, or phase-to-ground connected DERs ("ab" and "bc", or "a" and "b" for example). In both cases, we consider their corresponding e_j vectors, and we scale them by 1/2 (for balanced response).

With the defined $\mathbf{Q}_R$ rows above, we have

$$\underbrace{|\mathbf{v}_{\mathcal{M}_v}|}_{3r_v\times 1} = \overbrace{\underbrace{\mathbf{Q}_L^v}_{3r_v\times 3N} \underbrace{\begin{bmatrix}\mathbf{K}^Y & \mathbf{K}^\Delta\end{bmatrix}}_{3N\times 12N} \underbrace{\mathbf{Q}_R}_{12N\times 2D}}^{\mathbf{A}} \underbrace{\overbrace{\begin{bmatrix} p_1 \\ q_1 \\ \vdots \\ p_D \\ q_D \end{bmatrix}}^{\mathbf{x}}}_{2D\times 1} + \overbrace{\mathbf{Q}_L^v \left(\begin{bmatrix}\mathbf{K}^Y & \mathbf{K}^\Delta\end{bmatrix} \begin{bmatrix} \mathbf{p}^Y_{\text{none}-\text{DER}} \\ \mathbf{q}^Y_{\text{none}-\text{DER}} \\ \mathbf{p}^\Delta_{\text{none}-\text{DER}} \\ \mathbf{q}^\Delta_{\text{none}-\text{DER}} \end{bmatrix} + \mathbf{b} \right)}^{\mathbf{a}} \tag{12}$$

One can define the matrix $\mathbf{A}$ by defining the sensitivity matrices $\mathbf{A}_1$, $\mathbf{A}_2$,..., $\mathbf{A}_D$ as shown above then combine them to get

$$\mathbf{A} = \begin{bmatrix}\mathbf{A}_1 & \cdots & \mathbf{A}_D\end{bmatrix} \tag{13}$$

where $\mathbf{A}_i \in \mathbb{R}^{3r_v\times 2}, \forall i \in \{1,\ldots,D\}$

Note that both, $\mathbf{Q}_L^v$ and $\mathbf{Q}_R$ are "unitless". Therefore, the unit of $\mathbf{A}$ is the same as $\mathbf{K}^Y$ and $\mathbf{K}^\Delta = \frac{V}{W}$.

3.2.2 B matrix

Following a similar approach, we start with Eq. 8a.

$$\tilde{\mathbf{i}}_{ij} = \mathbf{J}^Y_{ij}\mathbf{x}^Y + \mathbf{J}^\Delta_{ij}\mathbf{x}^\Delta + \mathbf{c}_{ij} \tag{14}$$

Let the line ij map to the branch $k \in \mathcal{M}_i$. Then, we combine the J matrices, and collect all constants in new $\mathbf{b}$ vector, to get the updated equation of $\tilde{\mathbf{i}}_k$

$$\tilde{\mathbf{i}}_k = \tilde{\mathbf{i}}_{ij} = \begin{bmatrix} \mathbf{J}^Y_{ij} & \mathbf{J}^\Delta_{ij} \end{bmatrix} \begin{bmatrix} \mathbf{x}^Y \\ \mathbf{x}^\Delta \end{bmatrix} + \mathbf{c}_{ij} \tag{15a}$$

$$\tilde{\mathbf{i}}_k = \overbrace{\begin{bmatrix} \mathbf{J}^Y_{ij} & \mathbf{J}^\Delta_{ij} \end{bmatrix} \mathbf{Q}_R}^{\overline{\mathbf{B}}_k} \overbrace{\begin{bmatrix} p_1 \\ q_1 \\ \vdots \\ p_D \\ q_D \end{bmatrix}}^{\mathbf{x}} + \overbrace{\begin{bmatrix} \mathbf{J}^Y_{ij} & \mathbf{J}^\Delta_{ij} \end{bmatrix} \begin{bmatrix} \mathbf{p}^Y_{\mathsf{none-DER}} \\ \mathbf{q}^Y_{\mathsf{none-DER}} \\ \mathbf{p}^\Delta_{\mathsf{none-DER}} \\ \mathbf{q}^\Delta_{\mathsf{none-DER}} \end{bmatrix} + \mathbf{c}_{ij}}^{\overline{\mathbf{b}}_k} \tag{15b}$$

where $\overline{\mathbf{B}}_k \in \mathbb{R}^{3\times 2D}$. To calculate the current in all branches (similar to how we calculate the voltage), we can stack $\overline{\mathbf{B}}_k$ matrices next to each other to form a new $\mathbf{B} = \begin{bmatrix} \overline{\mathbf{B}}_1^\top & \cdots & \overline{\mathbf{B}}_{r_i}^\top \end{bmatrix}^\top$. We then can re-partition $\mathbf{B}$ per DER device (rather than per branch) as follows: $\mathbf{B} = \begin{bmatrix} \mathbf{B}_1 & \cdots & \mathbf{B}_{|\mathcal{D}|} \end{bmatrix}$, where $\mathbf{B}_i \in \mathbb{R}^{3r_i \times 2}$.

The unit of $\mathbf{B}$ is the same as $\mathbf{J}^Y$ and $\mathbf{J}^\Delta = \frac{A}{W}$.

3.2.3 M & H matrices

We start with Eq. 7a.

$$\tilde{\mathbf{s}}_0 = \mathbf{G}^Y \mathbf{x}^Y + \mathbf{G}^\Delta \mathbf{x}^\Delta + \mathbf{c} \tag{16}$$

We then combine G matrices and collect all constants in $\mathbf{g}$ as follows:

$$\tilde{\mathbf{s}}_0 = \begin{bmatrix} \mathbf{G}^Y & \mathbf{G}^\Delta \end{bmatrix} \begin{bmatrix} \mathbf{x}^Y \\ \mathbf{x}^\Delta \end{bmatrix} + \mathbf{c} \tag{17a}$$

$$\tilde{\mathbf{s}}_0 = \overbrace{\begin{bmatrix} \mathbf{G}^Y & \mathbf{G}^\Delta \end{bmatrix} \mathbf{Q}_R}^{\overline{\mathbf{G}}} \overbrace{\begin{bmatrix} p_1 \\ q_1 \\ \vdots \\ p_D \\ q_D \end{bmatrix}}^{\mathbf{x}} + \overbrace{\begin{bmatrix} \mathbf{G}^Y & \mathbf{G}^\Delta \end{bmatrix} \begin{bmatrix} \mathbf{p}^Y_{\mathsf{none-DER}} \\ \mathbf{q}^Y_{\mathsf{none-DER}} \\ \mathbf{p}^\Delta_{\mathsf{none-DER}} \\ \mathbf{q}^\Delta_{\mathsf{none-DER}} \end{bmatrix} + \mathbf{c}}^{\mathbf{g}} \tag{17b}$$

$$\tag{17c}$$

We then define $\mathbf{M}, \mathbf{H}, \mathbf{m}$ & $\mathbf{h}$ based on $\overline{\mathbf{G}}$ and $\mathbf{g}$ as follows:

$$\mathbf{M} = \mathsf{Re}(\overline{\mathbf{G}}) \tag{17d}$$

$$\mathbf{H} = \mathsf{Im}(\overline{\mathbf{G}}) \tag{17e}$$

$$\mathbf{m} = \mathsf{Re}(\mathbf{g}) \tag{17f}$$

$$\mathbf{h} = \mathsf{Im}(\mathbf{g}) \tag{17g}$$

The units of $\mathbf{M}$ & $\mathbf{H}$ are the same of $\overline{\mathbf{G}}$. $\overline{\mathbf{G}}$ has the same unit of $\mathbf{G}^Y$ and $\mathbf{G}^\Delta$, which are "unitless".

With this, we have obtained the new sensitivity matrices that embed the DERs wye and delta configuration within.

4 Simulations

In this section, we describe the configurations used for the simulations presented in [ref] . In addition, we discuss the changes made to the original feeder files in OpenDSS for IEEE-123 and IEEE-8500 feeders.

The Tables 2-4 summarize the configurations used for simulation time, DERs and VDER for all simulations.

Table 2: MATDSS simulation time settings for all test cases and circuits.

Parameter	Value
Duration*	[15s, 60s]
Simulation Time Step**	100ms
Controller Time Step	100ms
Measurement Time Step	100ms
Measurement Delay	0ms
Stabilization Time	0s

* Duration of 15s were used for step-change simulations while 60s were used for ramp-change simulations.

** For smaller circuits 10ms were used to have smoother curves (DERs' dynamics).

Table 3: DER configurations in MATDSS for all test cases and circuits.

Parameter	Value
$\boldsymbol{\tau}$	0.2s
DER Type	`PV`
Mode	`DSS_load`
$\underline{x}_j$	$(-10^6\text{W}, -10^6\text{Var})$
$\overline{x}_j$	$(10^6\text{W}, 10^6\text{Var})$
(ax, cx)	$(1, 1)$

Table 4: VDER configurations in MATDSS for all test cases and circuits.

Parameter	Value
$\boldsymbol{\tau}$***	10^{-6}
DER Mode***	`DSS_load`
$(P(x), Q(x))$	follow equation (5) in [ref]

*** This is not used in dynamic simulation, but is used to define the variable in `MATLAB` struct.

4.1 Low-pass filter and PID Controller setup

As discussed in the paper [ref] , we found that the performance is improved by integrating a low-pass filter and proportional-derivative controller with the integral controller. The low-pass filter is applied to the VDERs set-points where it mainly works on smoothing the set-point curve (minimizing oscillations). The proportional controller is used to boost the initial response of DERs when a disturbance occur or a switching in set-point is requested at the interface bus. The derivative controller is used to additionally boost the response initially for the VDERs to get additional response from lower-level DERs that are behind the VDER bus. This process resulted in speeding up the participation of lower-level DERs and achieve more stable response from the feeder when tracking the interface bus set-points. The effective gain for each parameters is the multiplication of the parameter coefficient (κ) with the main controller gain (K). Table 5 summarizes LPF-PID parameters used for all feeders.

Table 5: Low-pass filter and PID Control Configurations

		Feeder		
		5-bus	IEEE-123	IEEE-8500
LPF	T_c	0.9	1	1
	Applied to	VDERs Only	VDERs Only	VDERs Only
P - Controller Configuration	K_p	850	1500	1000
	$\kappa_{p,\lambda}$	0.001	0.001	0.002
	$\kappa_{p,\mu}$	0.001	0.001	0.002
	$\kappa_{p,\eta}$	0.0005	0.001	5e-6
	$\kappa_{p,\psi}$	0.0005	0.001	5e-6
	$\kappa_{p,\gamma}$	1e-12	1e-12	1e-12
	$\kappa_{p,\nu}$	1e-12	1e-12	1e-12
	$\kappa_{p,\zeta}$	1e-7	1e-7	1e-7
	Applied to	DERs & VDERs	DERs & VDERs	DERs & VDERs
D - Controller Configuration	K_d	330	1500	5
	$\kappa_{d,\lambda}$	0.001	0.001	0.001
	$\kappa_{d,\mu}$	0.001	0.001	0.001
	$\kappa_{d,\eta}$	0.001	0.001	1e-6
	$\kappa_{d,\psi}$	0.001	0.001	1e-6
	$\kappa_{d,\gamma}$	1e-12	1e-12	1e-12
	$\kappa_{d,\nu}$	1e-12	1e-12	1e-12
	$\kappa_{d,\zeta}$	1e-7	1e-7	1e-7
	D-Signal	Measurements (Y)	Measurements (Y)	Measurements (Y)
	Applied to	VDERs Only	VDERs Only	VDERs Only

4.2 5-Bus Feeder

The 5 bus feeder circuit used is based on the '`4Bus-DY-Bal`' IEEE test case in OpenDSS. The circuit has been modified by changing the loads powers and adding line 3 (`L3`) and load 2. The final OpenDSS code used to run 5-bus feeder is shown below (Code 1).

```
clear
! IEEE 5-bus test case D-Y Stepdown Balanced based on IEEE 4-bus test case in OpenDSS (D
   -Y Stepdown Balanced)
new circuit.5busDYBal basekV=12.47 phases=3
~ mvasc3=200000 200000

! **** DEFINE WIRE DATA
new wiredata.conductor Runits=mi Rac=0.306 GMRunits=ft GMRac=0.0244 Radunits=in Diam
   =0.721
new wiredata.neutral Runits=mi Rac=0.592 GMRunits=ft GMRac=0.00814 Radunits=in Diam
   =0.563
! **** DEFINE LINE GEOMETRY; REDUCE OUT THE NEUTRAL
new linegeometry.4wire nconds=4 nphases=3 reduce=yes
~ cond=1 wire=conductor units=ft x=-4 h=28
~ cond=2 wire=conductor units=ft x=-1.5 h=28
~ cond=3 wire=conductor units=ft x=3 h=28
~ cond=4 wire=neutral units=ft x=0  h=24
! **** 12.47 KV LINE
new line.L1 geometry=4wire length=2000 units=ft bus1=sourcebus bus2=n2
! **** 3-PHASE STEP-DOWN TRANSFORMER 12.47/4.16 KV Delta-Ygrd
new transformer.t1 xhl=6
~ wdg=1 bus=n2 conn=delta kV=12.47 kVA=6000 %r=0.5
~ wdg=2 bus=n3 conn=wye kV=4.16 kVA=6000 %r=0.5
! **** 4.16 KV LINE
new line.L2 bus1=n3 bus2=n4 geometry=4wire length=2500 units=ft !NormAmps=990
new line.L3 bus1=n4 bus2=n5 geometry=4wire length=2500 units=ft NormAmps=123

! **** WYE-CONNECTED LOADS
new load.load1 phases=3 bus1=n4 conn=wye kV=4.16 kW=400 pf=0.9 model=1
new load.load2 phases=3 bus1=n5 conn=wye kV=4.16 kW=770 pf=0.9 model=1
~ vminpu=0.75  ! model will remain const p,q down to 0.75 pu voltage

set voltagebases=[12.47, 4.16]
calcvoltagebases   ! **** let DSS compute voltage bases
```

Code 1: 5-bus feeder OpenDSS code.

Controllers configurations

Table 6: 5-bus feeder Controllers configurations

	1CA	2CAs		$\text{2CAs}_{\text{LPF-PID}}$	
# Control Area	1	1	2	1	2
Controller Type*	`l2c`	`l2c`	`l1c`	`l2c`	`l1c`
alpha	0.002	0.002	0.002	0.003	0.003
r_p	1e-4	1e-4	1e-4	1e-4	1e-4
$\overline{r}_d$	1e-3	1e-3	1e-3	1e-3	1e-3
E	1e2	1e2	1e2	1e2	1e2
v_{ul}	1.05	1.05	1.05	1.05	1.05
v_{ll}	0.95	0.95	0.95	0.95	0.95
i_{ul}	`Specified in DSS File`				
a_ρ^{**}	1	1	1	1	1
a_σ^{**}	1	1	1	1	1
a_λ	5e3	1e3	5e3	1e3	5e3
a_μ	5e3	1e3	5e3	1e3	5e3
a_η	1e3	1e3	1e3	1e3	1e3
a_ψ	1e3	1e3	1e3	1e3	1e3
a_γ	1e12	1e12	1e12	1e12	1e12
a_ν	1e12	1e12	1e12	1e12	1e12
a_ζ	1e7	1e7	1e7	1e7	1e7
c_ρ^{**}	1	1	1	1	1
c_σ^{**}	1	1	1	1	1
c_λ	1e-3	1e-3	1e-3	1e-3	1e-3
c_μ	1e-3	1e-3	1e-3	1e-3	1e-3
c_η	1e-3	1e-3	1e-3	1e-3	1e-3
c_ψ	1e-3	1e-3	1e-3	1e-3	1e-3
c_γ	1e-12	1e-12	1e-12	1e-12	1e-12
c_ν	1e-12	1e-12	1e-12	1e-12	1e-12
c_ζ	1e-7	1e-7	1e-7	1e-7	1e-7

* `l2c` indicates that the controller is tracking P_0^{set} and `l1c` indicates the controller is tracking P_0^{set} and Q_0^{set}.
** Those parameters are not used in the simulation, but were introduced at early stages of development. They are not considered in the current implementation and their values are not used.

4.3 IEEE-123 Feeder

For IEEE-123 feeder, the OpenDSS circuit named `IEEE123Master.dss`' is considered where we modified the following:

- Renamed bus '`150`' to '`sourcebus`' due to how MATDSS is developed.
- Updated all defined '`Lines`' to have '`NormAmps=900`' to avoid any current violation when analyzing power tracking performance.
- The same current limit is added to the 'switches' since they are defined as lines in OpenDSS (to maintain compatibility with MATDSS).

In addition, within MATDSS, we partition the feeder to 6 CAs and introduce 24 DERs (4 per CA) and place them randomly within each area. Figure 1 shows the partitioned feeder and the location of the DERs.

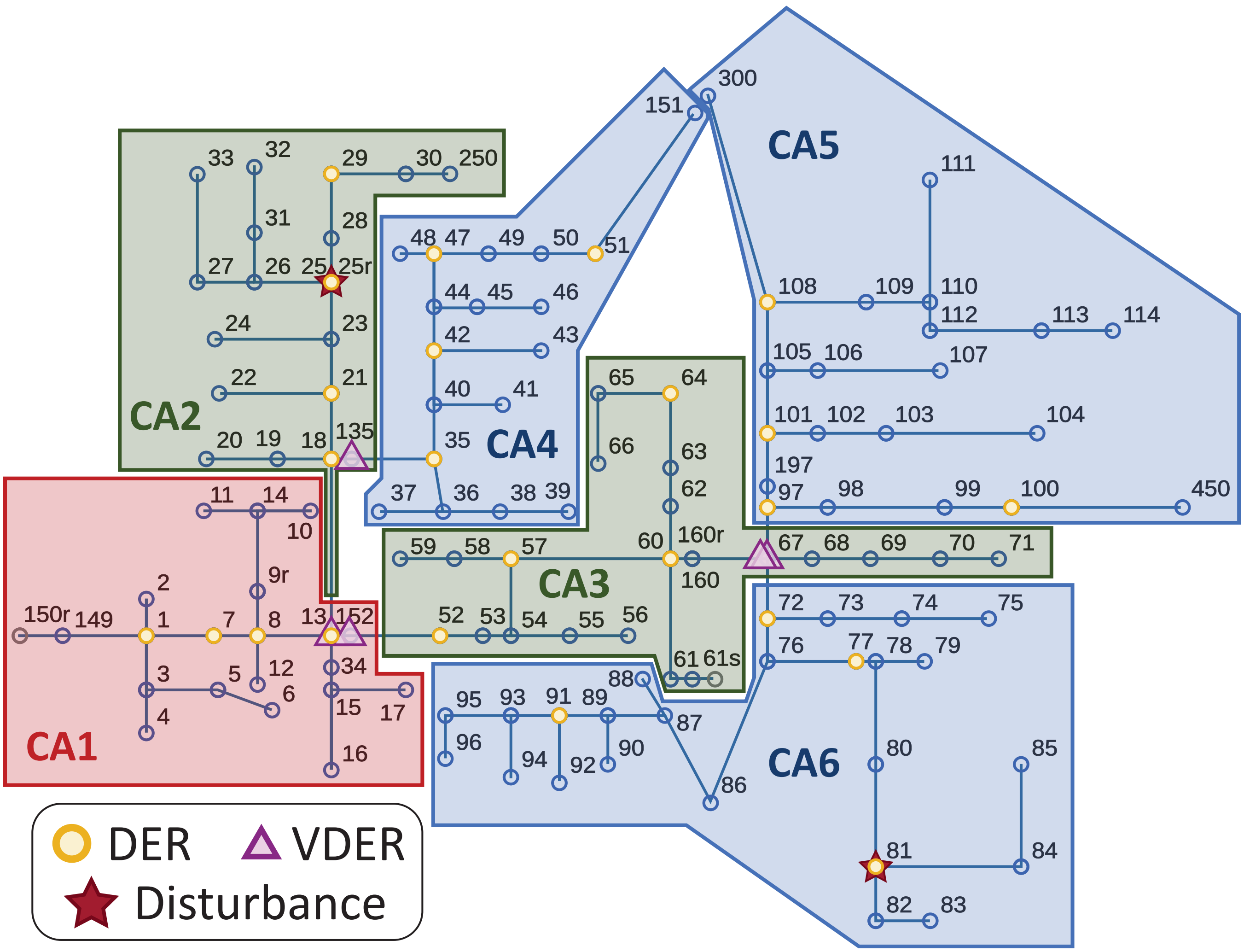

Figure 1: IEEE-123 bus feeder with six control areas.

The controllers configurations for the tests shown in the paper are summarized in Table 8.

Controllers configurations

Table 7: IEEE-123 feeder Controllers configurations

	Step Change			Stepped-Ramp Change		
	1CA	6CAs		1CA	6CAs	
# CA	1	1	{2,3,4,5,6}	1	1	{2,3,4,5,6}
Controller Type*	`l2c`	`l2c`	`llc`	`l2c`	`l2c`	`llc`
α	0.001	9.6e-4	4.32e-4	0.001	9.6e-4	4.32e-4
r_p	1e-4	1e-4	1e-4	1e-4	1e-4	1e-4
$\overline{r}_d$	1e-3	1e-3	1e-3	1e-3	1e-3	1e-3
E	1e2	1e2	1e2	1e2	1e2	1e2
v_{ul}	1.05	1.05	1.05	1.05	1.05	1.05
v_{ll}	0.95	0.95	0.95	0.95	0.95	0.95
i_{ul}	`Specified in DSS File`					
a_ρ^{**}	1	1	1	1	1	1
a_σ^{**}	1	1	1	1	1	1
a_λ	2e3	5e3	1e3	2e3	5e3	1e3
a_μ	2e3	5e3	1e3	2e3	5e3	1e3
a_η	1e3	1e3	1e3	1e3	1e3	1e3
a_ψ	1e3	1e3	1e3	1e3	1e3	1e3
a_γ	1e12	1e12	1e12	1e12	1e12	1e12
a_ν	1e12	1e12	1e12	1e12	1e12	1e12
a_ζ	1e8	1e8	1e8	1e8	1e8	1e8
c_ρ^{**}	1	1	1	1	1	1
c_σ^{**}	1	1	1	1	1	1
c_λ	1e-3	1e-3	1e-3	1e-3	1e-3	1e-3
c_μ	1e-3	1e-3	1e-3	1e-3	1e-3	1e-3
c_η	1e-3	1e-3	1e-3	1e-3	1e-3	1e-3
c_ψ	1e-3	1e-3	1e-3	1e-3	1e-3	1e-3
c_γ	1e-12	1e-12	1e-12	1e-12	1e-12	1e-12
c_ν	1e-12	1e-12	1e-12	1e-12	1e-12	1e-12
c_ζ	1e-8	1e-8	1e-8	1e-8	1e-8	1e-8

* `l2c` indicates that the controller is tracking P_0^{set} and `llc` indicates the controller is tracking P_0^{set} and Q_0^{set}.
** Those parameters are not used in the simulation, but were introduced at early stages of development. They are not considered in the current implementation and their values are not used.

4.4 IEEE-8500 Feeder

In this simulation, we consider the IEEE-8500 Feeder defined in OpenDSS '`Master.dss`' in the folder '`8500-Node`'. In our implementation, we have disabled the following internal switches:

- '`Line.WD701_48332_sw`'
- '`Line.V7995_48332_sw`'
- '`Line.WG127_48332_sw`'
- '`Line.WF856_48332_sw`'
- '`Line.WF586_48332_sw`'

In addition, we have partitioned the feeder into 49 control areas. Figures 2-3 below show the partitioned feeder and a tree graph illustrating the control areas defined. To partition the feeder into these areas, we have considered all main and load buses ('`m`' and '`l`'). The control areas were defined to obtain a nested control areas that are interfaced through these buses. Figure 4 shows the control areas tree structure.

Figure 2: IEEE-8500 bus feeder with 49 control areas.

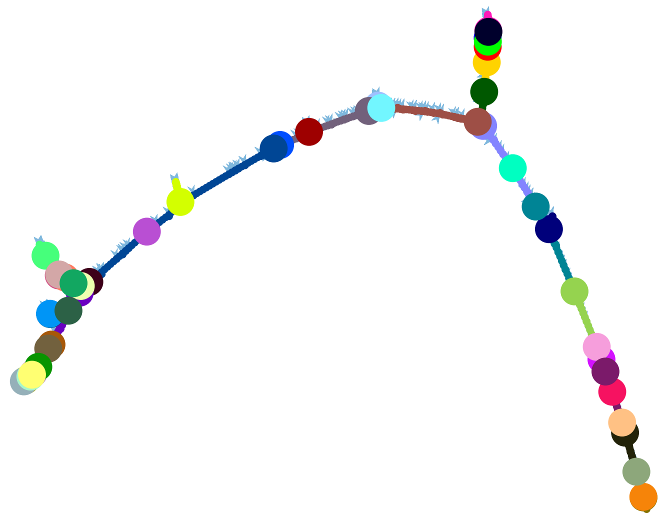

Figure 3: IEEE-8500 feeder tree graph with 49 control areas.

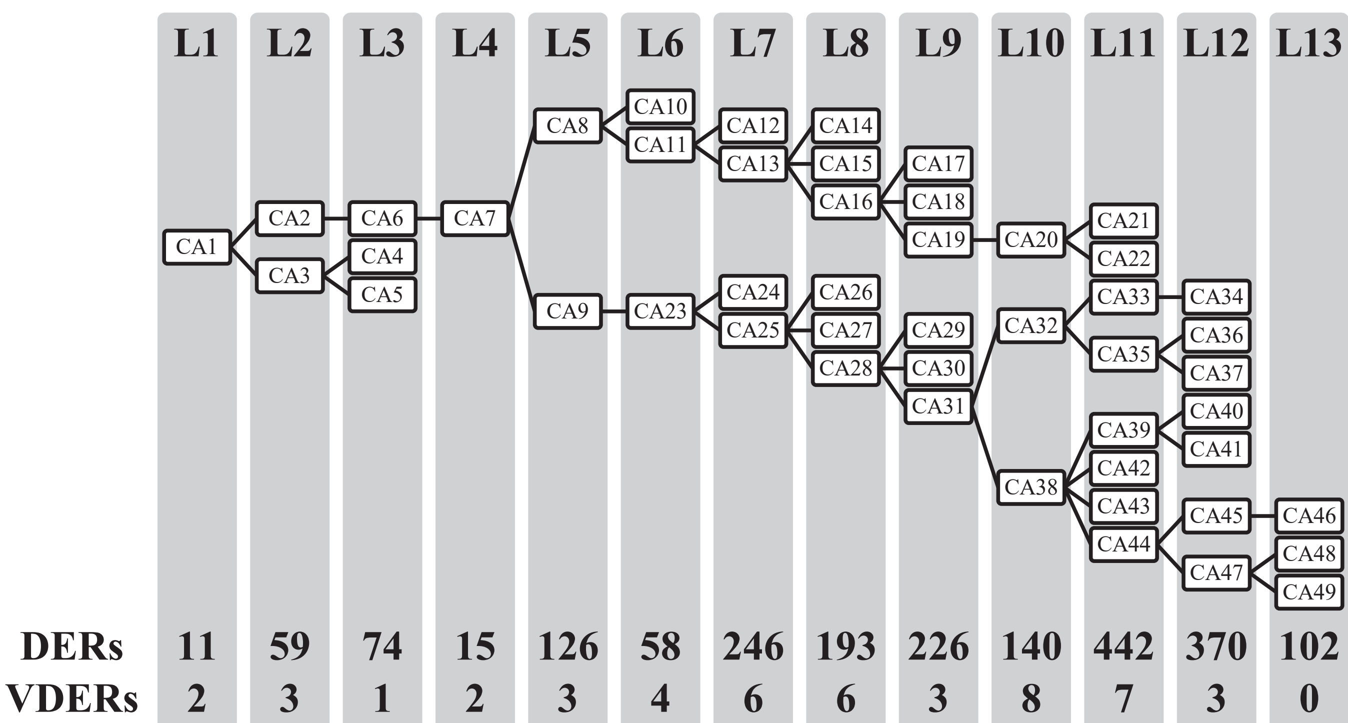

Figure 4: IEEE-8500 feeder control areas' tree with 13 nested levels of control areas.

Table 8: IEEE-8500 feeder Controllers configurations

	Stepped-Ramp Change		
	1CA	49CAs	
# CA	1	1	{2:49}
Controller Type*	`l2c`	`l2c`	`l1c`
α	0.0015	4e-4	2e-4
r_p	1e-4	1e-4	1e-4
$\overline{r}_d$	1e-3	1e-3	1e-3
E	1e2	1e2	1e2
v_{ul}	1.05	1.05	1.05
v_{ll}	0.95	0.95	0.95
i_{ul}	`Specified in DSS File`		
a_ρ^{**}	1	1	1
a_σ^{**}	1	1	1
a_λ	20	5e3	1e3
a_μ	20	5e3	1e3
a_η	20	5e3	1e3
a_ψ	20	5e3	1e3
a_γ^{***}	0	0	0
a_ν^{***}	0	0	0
a_ζ^{***}	0	0	0
c_ρ^{**}	1	1	1
c_σ^{**}	1	1	1
c_λ	1e-3	1e-3	1e-3
c_μ	1e-3	1e-3	1e-3
c_η	1e-3	1e-3	1e-3
c_ψ	1e-3	1e-3	1e-3
c_γ	1e-12	1e-12	1e-12
c_ν	1e-12	1e-12	1e-12
c_ζ	1e-8	1e-8	1e-8

* `l2c` indicates that the controller is tracking P_0^{set} and `l1c` indicates the controller is tracking P_0^{set} and Q_0^{set}.
** Those parameters are not used in the simulation, but were introduced at early stages of development. They are not considered in the current implementation and their values are not used.
*** To focus on power tracking performance, voltage and current control is disabled.

References

[1] A. Bernstein, C. Wang, E. Dall'Anese, J.-Y. Le Boudec, and C. Zhao, "Load flow in multiphase distribution networks: Existence, uniqueness, non-singularity and linear models," IEEE Transactions on Power Systems, vol. 33, no. 6, pp. 5832–5843, 2018.

[2] A. Bernstein and E. Dall'Anese, "Linear power-flow models in multiphase distribution networks," in 2017 IEEE PES Innovative Smart Grid Technologies Conference Europe (ISGT-Europe), 2017, pp. 1–6.