Circuit Analysis: DC

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Introduction

Welcome to *Circuit Analysis: DC*! Today you will be learning the basics of DC circuits. You may be asking, "What does DC mean?" Wonderful question, dear student. DC stands for **Direct Current**, and refers to circuits in which *current only flows in one direction*. You see these types of current sources every day: batteries, solar cells, and USB chargers, to name a few. We need to have a good understanding of DC signals because [1] as noted before, you will encounter them on a daily basis, and [2] they are a building block toward understanding more complicated signals.

In this course, you will be analyzing, building, measuring, and understanding several DC circuits, and using three new tools as well: the digital multimeter, the DC power supply, and several types of cables. These tools are staples in the electrical engineering laboratory.

What concepts are we going to cover?

- Kirchhoff's Voltage Law
- Voltage Dividers
- Kirchhoff's Current Law
- Capacitance

In what context are we going to be covering them?

- Using digital multimeters
- Using DC power supplies
- Potentiometers
- RC circuits

What should I know before taking this course?

You should have already taken Basic Electronics (or an equivalent), and should be able to read simple schematics, understand breadboards, and solder. If you do not feel comfortable building a simple resistance circuit based on a schematic, inform your instructor.

Where can I go from here?

TechShop's Electronics and Electrical Engineering (EEE) curriculum teaches the fundamentals of modern low-power electronics. You are currently taking the first **electrical engineering** course, in which you learn about DC signals. The next course in the series is **Circuit Analysis: AC**, in which you will learn about AC signals.

There is also a concurrent **Microcontrollers** path, in which you learn how to control tiny computers. These courses can be taken before, after, or in tandem with the electrical engineering courses.

Let's get started!

Review: Ohm’s Law et al.

A quick review is in order, such that we’re all starting with the same fundamentals. If you’ve taken *Basic Electronics*, this will look very familiar!

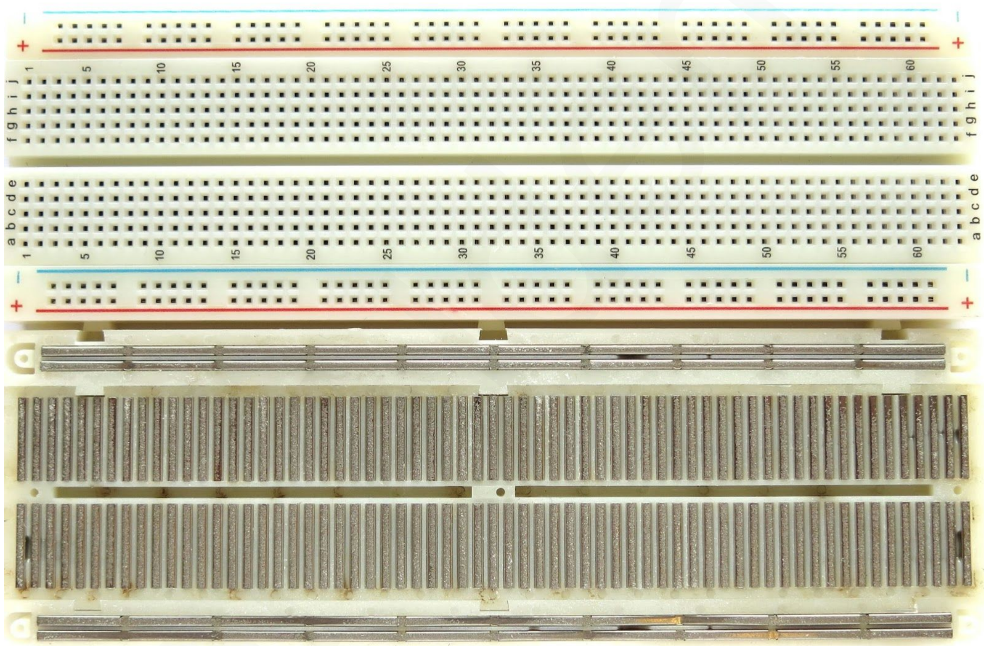
Ohm’s Law

This law relates the three basic quantities in electrical engineering. V is the voltage (measured in Volts), I is the current (in Amps), and R is the resistance (in Ohms):

$$V = IR$$

Breadboards

Your best friend, your compatriate, your electrical soulmate; breadboards allow you to rapidly prototype and test a circuit without the need for soldering. Typically, they have a pair of rows on both the top and the bottom, which are used for power distribution; they also have two sets of vertical columns (which are electrically isolated from each other), in which you connect components.



Series and parallel systems

Components can be wired such that current will flow through from one into the next; those components are said to be in **series**. The current can also split apart and take a forked path through two or more components simultaneously, and then reconnect on the other side; those components are said to be in **parallel**.

An important prepositional lesson

One more note to add to this section: we’ll be concerning ourselves with measuring the three fundamental quantities of electricity: voltage, current, and resistance. They each represent different manifestations of electricity, and are therefore measured differently. Associating a preposition to each will help you remember how to measure each quantity:

- Voltage is measured ***across***; therefore, the DMM probes are placed on either side of the component(s).

- Current is measured ***through*** a section of the circuit; therefore, the DMM must be inserted *into* the circuit!
- Resistance is an inherent property ***of*** a component or a series of components; therefore, it must be measured in isolation from any other sections of the circuit.

These will be explained in more detail in the following activities.

Activity 1: Meet your new tools

Please give a warm welcome to the following three tools: the **digital multimeter**, the **DC power supply**, and **electronics workbench cabling**. Each play an important role in the electrical engineering laboratory.

Digital multimeter

The digital multimeter is *the* staple tool in any electrical engineer's toolkit. It comes in both handheld and benchtop forms, and your laboratory setup may have one or both of these available. Typically, a **DMM** will be able to measure the following quantities:

Resistance	Continuity	Voltage (DC)	Current (DC)

While some DMMs will be able to handle other types of quantities (such as capacitance, forward voltage, et al.), we'll only be concerning ourselves today with measuring these four.

DC power supply unit

In order for your circuits to function, you'll need a source of power! Just as the 9V battery powered your *Basic Electronics* circuits, a DC power supply will supply the necessary unidirectional voltage and current.

Benchtop DC power supplies come in varied shapes, sizes, and ratings, but all typically have the following controls and connectors:

- Voltage control, in which you set the voltage, and the supply varies the current based on the resistance.
- Constant current mode, in which the supply varies the voltage to provide the same current to the system.
- Output engage, which turns the output lines on or off.
- Lugs with which to connect the power supply to your system.

Cables

In your quest through the world of electronics, you will encounter many different types of cables. Most will be able to perform two basic tasks: **signal measurement**, and **signal transportation**. (The one common exception to this rule is an oscilloscope probe, which is only used for measurement, and will be covered in *Circuit Analysis: AC*.) Your cables choices are dependant primarily on convenience; that is to say, pick whatever is easiest! There are several different types of **connectors** that cables may come with:

Probe	Banana	Alligator	BNC

Activity 2: Calculating and measuring resistance

Resistance is the opposition to electron flow. Every component has some inherent resistance; even wires themselves! But when analyzing systems, electrical engineers idealize components so that the math is more straightforward; in our circuits, we will only be concerned with the resistances of the resistors themselves.

Calculating resistance: series and parallel

Set up the following circuits (in isolation) on your breadboard:

Network 1	Network 2	Network 3

We need to find the **total resistance** (R_T) of these networks. But why should we even care about this quantity? For many circuits, the total resistance will ultimately determine the total current draw, and the current draw is very important - too little or too much, and the circuit may not behave as intended.

When resistors are wired together, the way they are wired together (in series or in parallel) will determine how their resistances are combined. **Series** resistance is very straightforward: their resistances should simply add together linearly. Therefore, the total resistance is higher than either of the individual resistors. **Parallel** resistance is a little less obvious, but still pretty easy: if you have one path for current to flow through, and you connect another path for current to go through, *even if the resistance of that path is more*, there will be more *total* current. If there is more total current at the same voltage, by Ohm's Law, *the total resistance must be lower than either of the individual resistors*!

Expressing these two concepts mathematically:

Series resistance	**Parallel resistance**
$R_T = R_1 + R_2 + \ldots$	$R_T = \left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots\right)^{-1}$

Calculate the total resistance of each network:

Network 1	Network 2	Network 3
$R_T = \qquad \Omega\ (calculated)$	$R_T = \qquad \Omega\ (calculated)$	$R_T = \qquad \Omega\ (calculated)$

Measuring resistance: series and parallel

Now let's use the DMM as an **ohmmeter** to measure the actual resistance of these networks. Attach two **banana to probe** cables to the DMM: a red one to the jack which is labelled with the Ω symbol; and a black one in the "COM" jack. ("COM" in this case stands for "common", which is a reference point. When you're using a DMM, you'll almost always have a black lead connected here.)

As with every measurement you make with the DMM, you'll have two considerations:

- You must set the DMM to the correct **quantity** to be measured. On handheld DMMs, this is typically done by rotating the large dial to the correct place; on benchtops, it's typically a button. In this case, we'll be measuring resistance; set your DMM to that now.
- Unless your DMM automatically does it, you'll also need to set the **range** that it is operating in: that is, the scale at which the values you're measuring will be in. The scale you choose should be as close to the measured value as possible, without the measured value going over!

You're ready to take your first measurements! For each network you've created earlier, touch the red probe to point A, and the black probe to point B; then observe the measured total resistance and write them down below.

Network 1	Network 2	Network 3
$R_T =$ Ω $(measured)$	$R_T =$ Ω $(measured)$	$R_T =$ Ω $(measured)$

With any luck, your measured values should be very close to the values you calculated earlier! If not, your system is likely wired incorrectly; ask your instructor for help. Note that these networks you created earlier are all in isolation; this is because resistance is an inherent property ***of*** a system, and so needs to be measured while disconnected from the rest of the system!

Two considerations:

1. If you switched the position of the test leads (Black to A, Red to B), would you expect there to be a change in readings? Try it.
2. If you were indeed to wire these systems together, would their total resistances change?

Activity 3: Kirchhoff's Voltage Law

Voltage - if you remember from *Basic Electronics* - is **energy per charge**. When you create a complete circuit which includes a voltage source, that voltage will then "drop" across the various components; that is to say, because the source *increases* the energy per charge, the components need to *decrease* the energy per charge on the way back to the source, to return them to their original value. (For the record: this is essentially an extension of the concept of conservation of energy, directly applied to electromagnetism.)

Why do we care about this? Sections of your circuit will present resistance to the voltage source; these are typically called **loads**, and loads by their very nature will draw power. In many applications, it is important to know the power draw of a load so that you don't overload the power source, overheat the circuit, etc. Calculating voltage drops allows you to (by extension) calculate power draw and protect your systems.

There is an easy way of mathematically representing the concept of voltage drop - it was discovered by Gustav Kirchhoff, an influential physicist from the 1800s. Called **Kirchhoff's Voltage Law** (abbreviated **KVL**), it says that in a closed network, **the sum of all the voltages must equal zero**; or equivalently, **the source voltage must equal the voltage drops**. Mathematically, it looks like this[1]:

$$V_1 + V_2 + \ldots + V_n = 0 \qquad \text{or equivalently} \qquad V_{source} = V_{drops}$$

These equations say exactly the same thing. In the first equation, n is the number of voltages in the system (both source and drops), and voltage sources will be positive, while voltage drops will be negative.

Calculating voltages using KVL

Set up the following circuit on your breadboard:

Calculating the voltage drops across two resistors in a series circuit is a four-step process:

1. **Label all the voltages** (that is, the voltage sources as well as the voltage drops):

2. **Write out the KVL equation for your particular circuit**. The source voltage (V_S) must equal the voltage drops across the two resistors - which we will call (V_{R_1}) and (V_{R_2}); therefore:

$$V_{source} - V_{R_1} - V_{R_2} = 0 \qquad \text{or equivalently} \qquad V_{source} = V_{R_1} + V_{R_2}$$

3. **Do the substitution dance to solve for current**. Remembering that $V_S = IR_T$, substitute in known values:

$$V_{source} = IR_T \quad \Rightarrow \quad 9 = I(1k\Omega + 100\Omega) \quad \Rightarrow \quad I = 8.2mA$$

4. Knowing the the current is the same through both resistors, and using Ohm's law once again, we can **substitute and solve for each voltage drop**:

[1] Technically, this is in expanded form. You will typically see KVL in Sigma notation; see Appendix A.

$$V_{R_1} = IR_1 = 8.2mA(1k\Omega) = 8.2V \quad \text{and} \quad V_{R_2} = IR_2 = 8.2mA(100\Omega) = 0.8V$$

Now that you know how to use KVL, set up and analyze the following two circuits:

Circuit 1	Circuit 2
$V_{R_1} =$ $\qquad$ $V(calculated)$ $V_{R_2} =$ $\qquad$ $V(calculated)$	$V_{R_1} =$ $\qquad$ $V(calculated)$ $V_{R_2} =$ $\qquad$ $V(calculated)$

Measuring voltages using the DMM

Now that you've got the nominal voltage drops across the resistors, dust off that trusty DMM once again and turn it into a **voltmeter**. Make sure that:

- your probes are connected to the "V" and "COM" lugs;
- your DMM is in DC Volts mode;
- you're in the right scale.

Now recall that wonderful preposition and place the probes **across** the individual resistors to measure their voltage drops.

Circuit 1	Circuit 2
$V_{R_1} =$ $\qquad$ $V(measured)$ $V_{R_2} =$ $\qquad$ $V(measured)$	$V_{R_1} =$ $\qquad$ $V(measured)$ $V_{R_2} =$ $\qquad$ $V(measured)$

With any luck, your measured values should be very close to the values you calculated earlier! If not, your system is likely wired incorrectly; ask your instructor for help.

Consideration: if you switched positions of the test leads, would you expect a change in readings? Try it.

Activity 4: Voltage Divider

KVL is an incredibly useful tool. In some cases, however, there is an even quicker way to calculate the voltage drop across a resistor. This only occurs *when resistors are in series*; the technique is called the **voltage divider**. It says:

$$V_{R_x} = \left(\frac{R_x}{R_T}\right) V_S$$

If you do a little bit of mathematical rearranging, you'll see that both of these approaches are essentially the same. But remember the hugely important caveat: **the voltage divider only works in series networks**.

Calculating voltages with the Voltage Divider

Use the Voltage Divider to calculate the voltage drops across the resistors in the following two circuits, and then set them up and measure them using your DMM.

Circuit 1	Circuit 2
$V_{R_1} =$ $V(calculated)$ $V_{R_1} =$ $V(measured)$ $V_{R_2} =$ $V(calculated)$ $V_{R_2} =$ $V(measured)$	$V_{R_1} =$ $V(calculated)$ $V_{R_1} =$ $V(measured)$ $V_{R_2} =$ $V(calculated)$ $V_{R_2} =$ $V(measured)$

Pretty cool, right?

Activity 5: Kirchhoff’s Current Law

Sometimes, you will want to know the current through a section of your circuit. When circuits present only one path for current to take, finding the current is pretty darn easy - just use Ohm’s Law! But when another path is present for current to take, such as this one:

then another approach must be employed.

Kirchhoff luckily had another trick up his sleeve. Like his Voltage Law, he derived an incredibly useful law for currents. It’s (unsurprisingly) called **Kirchhoff’s Current Law**, and it states that for any given point (or **node**) in a circuit, **the sum of the currents entering and leaving that node must equal zero**; or even more simply, that **the sum of the currents entering a node must equal the sum of the currents leaving that node**. Mathematically, it looks like this[2]:

$$I_1 + I_2 + \ ... \ + I_n = 0$$

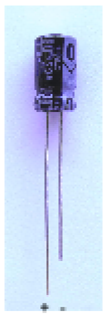
where *n* is the number of paths connected to the node. This should make sense: if you have some amount of electricity flowing into a splitting point, then you should have an equivalent amount flowing out. Otherwise, you’re breaking the fundamental laws of physics!

Calculating currents using KCL

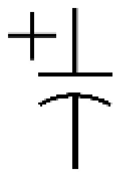
Measuring currents using the DMM

[2] KCL, like KVL, is typically represented in Sigma notation. See Appendix A.

Activity 6: Capacitors



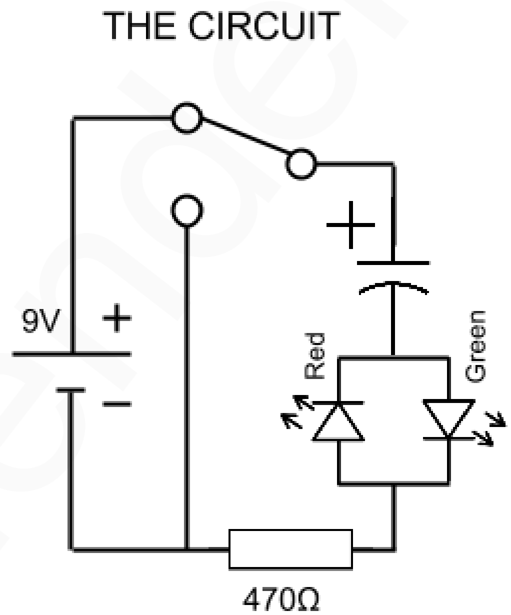
electrolytic capacitor



schematic symbol

Capacitors store energy! They do this by accumulating charges on their two plates. Capacitors come in many flavors; some are polarized (such as the **electrolytic** type you'll be using today), and some are not (such as **ceramic** capacitors, which you will see in later courses). They are used for many things; today, we'll be using one to demonstrate that capacitors store charge!

When the switch connects the + terminal to the capacitor, current begins to flow clockwise through the circuit, from positive terminal to negative terminal, and the *green* LED lights up. The capacitor will also begin to charge. As it charges, the current slowly drops (and the LED begins to dim), until it is done charging, at which point no current flows (and the LED turns off). When you flip the switch, the capacitor is connected in reverse, and acts as a voltage source, supplying current *counterclockwise* through the *red* LED!



The RC circuit

Tau: τ

$$\tau = RC$$

Summary: What have I learned?

Ahead: Where can I go from here?

Appendix A: Sigma notation

When you need to *add* a group of values - those that appear in a series, or those that are of the same type, for example - you'd typically just fully write it out. For example, this circuit:

would produce this equation:

The disadvantage here is that this equation describes this system and this system only. What if you want to abstract the equation, to create one that could describe adding *any* resistor network? Sigma to the rescue!

Sigma: Σ

In mathematics, a capital **Sigma** is used to represent *summations* - sums of values. The symbology here requires a little explanation, but once you've grasped what each term means, Sigma notation becomes a powerful tool to describe additive systems.

To describe the same system as above (and in fact *any* additive system) in Sigma notation, all that is needed is the following:

$$V_T = \sum_{i=m}^{n} V_k$$

- *m* is the number of the first voltage (otherwise known as the *lower bound*) - aka where to start counting - which in this case will always be 1
- *n* is the number of voltages in the network (otherwise known as the *upper bound*) - aka where to stop counting
- *i* is the current index of the voltages; that is, which voltage you are currently talking about

But why should you care? It abstracts the mathematics in a way such that the concept can be (and in fact is) used in many different contexts, across many disciplines. Mathematicians, engineers, and physicists all use Sigma notation in their work. It also saves quite a bit of handwriting!

Application to KVL and KCL

Both KVL and KCL are typically written in Sigma notation. For a closed loop system:

$$\sum_{k=1}^{n} V_k = 0$$

And for any node in a circuit:

$$\sum_{k=1}^{n} I_k = 0$$

Appendix B: Reading data sheets

Data sheets store a wealth of information about components. Let's explore the datasheet of the LEDs you'll be working with today: the Lite-On 565nm (green) LED.

There's quite a bit of information in here, but what you're looking for is the **forward voltage**. Recalling from *Basic Electronics*, you will remember that LEDs need a certain voltage dropped across them before they open up and allow current to flow through them. Dig through the data sheet to find this value:

$$V_F = \qquad\qquad V\,(from\ datasheet)$$

To measure the forward voltage of an LED, connect two banana to alligator cables to your benchtop DMM; red to the lug which has the schematic symbol of an LED over it, and black to the COM lug. Now switch your DMM into forward voltage measurement mode by pressing the schematic symbol for an LED button. Connect the red cable to the anode, and the black cable to the cathode. The LED should light, and the DMM should be displaying a forward voltage:

$$V_F = \qquad\qquad V\,(measured)$$