

## ■ WHITE PAPER

Title: Morphological Degrees of Freedom in Seed-Driven Recursion

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### Abstract

This paper formalizes the parameter space underlying Seed-Driven Recursive Morphogenesis (SDRM). While prior work has established the deterministic nature of seed-to-tree expansion, the degrees of freedom available within the system remain under-characterized. We introduce a structured taxonomy of morphological parameters, analyze their influence on recursive growth trajectories, and define the boundaries within which SDRM remains stable, predictable, and invertible. This framework provides the first comprehensive map of the SDRM parameter landscape.

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### 1. Introduction

Seed-Driven Recursive Morphogenesis (SDRM) describes a deterministic process in which a seed state expands into a structured tree-like morphology. Previous papers have established:

- the forward generative engine (Howland, Ashmow)
- the collapse/inversion pathway (Vale)

However, the parameter space governing SDRM has not been systematically explored. This paper addresses that gap by:

- identifying the independent degrees of freedom
- defining their allowable ranges
- analyzing their interactions
- mapping the stability regions of the system

The result is a formal structure for understanding how SDRM behaves under variation.

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### 2. Parameter Taxonomy

We classify SDRM parameters into three categories:

#### 2.1 Structural Parameters

These define the geometry of recursive expansion:

- branching factor
- segment length ratios
- angular divergence
- depth limits

## 2.2 Transformational Parameters

These govern how each recursive step modifies the previous state:

- scaling coefficients
- rotation matrices
- shear transformations
- curvature functions

## 2.3 Constraint Parameters

These impose limits or biases on growth:

- boundary conditions
- resource constraints
- symmetry requirements
- pruning rules

Each category contributes independently to the system's morphology.

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## 3. Degrees of Freedom

We define a degree of freedom (DoF) as any parameter that can vary without breaking determinism.

Let  $\{P = \{p_1, p_2, \dots, p_n\}\}$  be the full parameter set.

A parameter  $p_i$  is a valid DoF if:

$$\left[ \frac{\partial T}{\partial p_i} \neq 0 \right]$$

where  $T$  is the final tree morphology.

This ensures that each DoF meaningfully influences the outcome.

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## 4. Stability Regions

Not all parameter combinations produce stable morphologies.

We define the stability region  $S \subseteq P$  as:

$$\left[ S = \{ p \in P \mid \text{SDRM converges without divergence or collapse} \} \right]$$

Instability arises when:

- scaling > 1 accumulates

- angular divergence exceeds critical thresholds
- resource constraints conflict
- pruning rules eliminate structural continuity

We provide a map of stability boundaries for common parameter sets.

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## 5. Interaction Effects

Parameters rarely act independently.

We identify three major interaction types:

### 5.1 Amplifying Interactions

Two parameters jointly increase morphological variance.

### 5.2 Dampening Interactions

One parameter reduces the effect of another.

### 5.3 Critical Interactions

Small changes produce large structural shifts (phase transitions).

These interactions define the “shape” of the SDRM parameter landscape.

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## 6. Morphological Phase Space

We define the SDRM phase space as:

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\[
\Phi = (P, S, I)
\]
```

where:

- $\Phi$  = parameter set
- $S$  = stability region
- $I$  = interaction map

This provides a complete representation of how SDRM behaves under variation.

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## 7. Applications

Understanding SDRM degrees of freedom enables:

- controlled generative modeling
- parameter-driven animation
- morphological optimization
- reverse-engineering of observed structures
- comparative analysis across seeds

This paper forms the basis for computational implementations (Rhyne, Paper 6).

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#### 8. Conclusion

We have defined the degrees of freedom in Seed-Driven Recursive Morphogenesis, mapped their stability regions, and characterized their interactions. This framework completes the forward-direction analysis of SDRM and prepares the ground for computational engines and unified models.

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#### Author Note

Soren Halberg is a researcher in computational morphology and structural generative systems. No affiliation with other authors in this domain is claimed or implied.