

① ^(a) $n^2 + 4n + 17 = O(n^3)$

$$\exists c \forall n > n_0 (0 \leq n^2 + 4n + 17 \leq cn^3)$$

misal $c = 1, n_0 = 10$

Proof by Induction

$$P(n) : 0 \leq n^2 + 4n + 17 \leq n^3$$

Base case

$$P(10) : 0 \leq 157 \leq 1000 = T$$

Inductive Case

$$P(k) : 0 \leq k^2 + 4k + 17 \leq k^3 \text{ diasumsikan } T$$

$$P(k+1) : 0 \leq (k+1)^2 + 4(k+1) + 17 \leq (k+1)^3$$

$$\equiv 0 \leq k^2 + 6k + 22 \leq k^3 + 3k^2 + 3k + 1$$

$$\equiv 0 \leq (k^2 + 4k + 17) + 2k + 5 \leq k^3 + 3k^2 + 3k + 1$$

$$\equiv 0 \leq k^3 + 2k + 5 \leq k^3 + 3k^2 + 3k + 1$$

memiliki penyelesaian $k \geq 1$

sehingga bernilai T Q.E.D

② $n^3 = O(n^2 + 4n + 17)$

$$\exists c \forall n > n_0 (0 \leq n^3 \leq c(n^2 + 4n + 17))$$

untuk setiap c dan n_0 bisa diambil

$n = \max(|2c|, n_0 + 1)$ sehingga

$$0 \leq n^3 \not\leq c(n^2 + 4n + 17)$$

Proof by Contradiction, Q.E.D