Hypothesis Testing

Fakultas Ilmu Komputer

Universitas Indonesia

2021

References

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Sub topics

- There is one normal population
  - Test Concerning the **Mean** of a Normal Population
    - Case of Known Variance
    - Case of Unknown Variance
- There are two normal populations
  - Testing the **Equality of Means** of Two Normal Population
    - Independent Samples
      - Case of Known Variances
      - Case of Unknown Variances, but The relation is known
        - $\boldsymbol{\sigma^1 = \sigma^2}$
        - $\boldsymbol{\sigma^1 \neq \sigma^2}$
      - Case of Unknown Variances, but The relation is unknown ($\boldsymbol{\sigma^1 \ ? \ \sigma^2}$)
    - Dependent Samples (**paired t-test**)

Sebuah jurnal mengklaim bahwa rata-rata tinggi badan dari semua mahasiswa UI adalah 165 CM.

Kemudian, Anda diminta untuk memverifikasi klaim tersebut !

Bagaimana caranya ?

Rektorat UI menyatakan bahwa rata-rata nilai SBMPTN seluruh mahasiswa UI adalah 930. Seseorang kemudian mengambil beberapa mahasiswa untuk menjadi sampel dan menemukan bahwa rata-rata nilai SBMPTN pada sampel mahasiswa tersebut adalah 960.

Bisakah kita simpulkan bahwa pernyataan yang dikeluarkan oleh pihak Rektorat UI adalah salah ?

Seorang pakar mengatakan bahwa rata-rata IPK mahasiswa FASILKOM lebih tinggi dari mahasiswa FH.

Bagaimana caranya menguji pernyataan tersebut ?

Diketahui ada 2 buah metode pengajaran, yaitu **metode Tradisional** dan **metode Baru**.

Seorang guru ingin mengetahui apakah metode pengajaran baru lebih baik dari metode pengajaran tradisional.

Bagaimana caranya ?

Perusahaan ingin mengetahui apakah **pelatihan motivasi** dapat meningkatkan kinerja karyawan atau tidak berpengaruh sama sekali.

Bagaimana caranya ?

Pertanyaan pada 5 kotak sebelumnya dapat dijawab menggunakan **hypothesis testing**.

Kita ingin melakukan verifikasi sebuah klaim atau hipotesis !

Hypothesis Testing = Pengadilan

Test-Statistic & prosedur uji hipotesis

Hakim & Decision-making procedure

Bukti & Saksi

Sample

Klaim tentang sesuatu

Hipotesis

Contoh untuk sebuah proses uji hipotesis

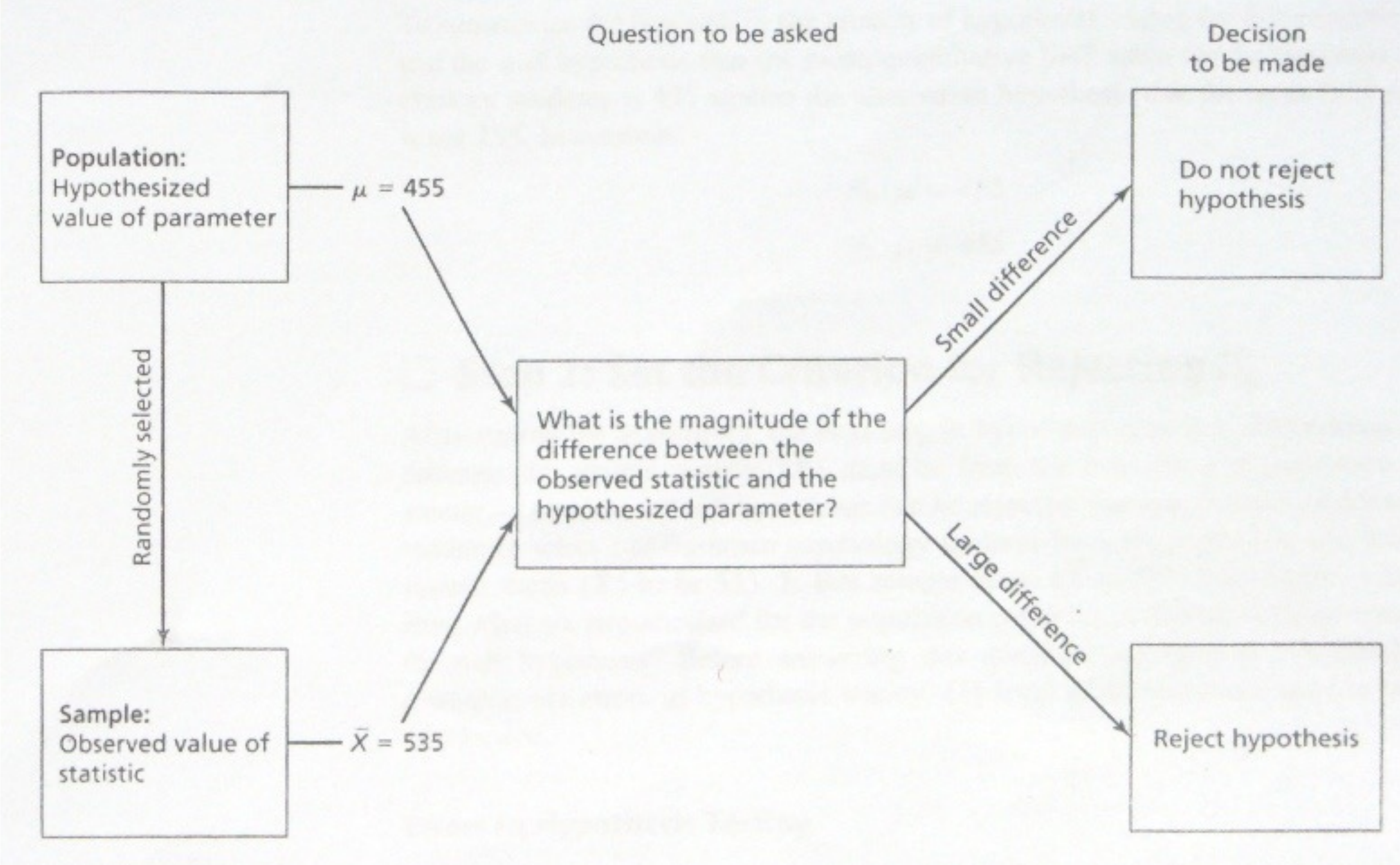


FIGURE 8.1
Logic of hypothesis testing

Hypothesis testing in **inferential statistics** involves **making inferences about the nature of the population** (parameter) on the basis of **observations of a sample** drawn from the population.

The steps in testing a Hypothesis

- **Step 1:** State the hypothesis
- **Step 2:** Set the criterion for rejecting Null Hypothesis ($\boldsymbol{H_0}$)
- **Step 3:** Compute the Test-Statistic
- **Step 4:** Decision about Null Hypothesis ($\boldsymbol{H_0}$)
  - **Accept** or **Reject** $\boldsymbol{H_0}$
  - **Write the conclusion !**

STEP 1 : State the Hypothesis

A **hypothesis** is a conjecture about one or more **population parameter**. This conjecture **may** or **may not be true**.

There are two types of hypothesis for each situation:

Null Hypothesis ($\boldsymbol{H_0}$) : is a statistical hypothesis that states that **there is no difference** between a parameter and a specific value, or that **there is no difference** between two parameters.

Alternative Hypothesis ($\boldsymbol{H_1}$) : is a statistical hypothesis that states **the existence of a difference** between a parameter and a specific value, or states that **there is a difference** between two parameters.

- Can be supported only by rejecting the null hypothesis

We need them both !

STEP 1 : State the Hypothesis

Example for **One-Sample Case**, from one population

Seorang pakar psikologi merasa bahwa memutar musik saat ujian dapat mengubah nilai ujian siswa. Dari pengalaman yang telah lalu, rata-rata nilai ujian siswa adalah 68.

Nyatakan hipotesisnya (Null & Alternatif) !

$$H_0 : \mu = 68 \qquad \textbf{Versus} \qquad H_1 : \mu \neq 68$$

We call this as **Two-Tailed Test** !

To state hypotheses correctly, You must translate the *conjecture* or *claim* from words into mathematical symbols !

STEP 1 : State the Hypothesis

Example for **One-Sample Case**, from one population

Sebuah jurnal mengklaim bahwa rata-rata tinggi badan dari semua mahasiswa UI kurang dari 190 CM.

Nyatakan hipotesisnya (Null & Alternatif) !

$$H_0 : \mu = 190 \quad \textbf{Versus} \quad H_1 : \mu < 190$$

We call this as **One-Tailed Test (left-tailed)** !

STEP 1 : State the Hypothesis

Example for **One-Sample Case**, from one population

Seorang dokter menduga bahwa jika seorang ibu hamil mengkonsumsi sebuah **pil vitamin**, maka berat bayi yang lahir akan meningkat.

Sejauh ini diketahui bahwa rata-rata bayi yang baru lahir adalah **8.6 pounds**.

Nyatakan hipotesisnya (Null & Alternatif) !

$$H_0 : \mu = 8.6 \quad \textbf{Versus} \quad H_1 : \mu > 8.6$$

We call this as **One-Tailed Test (right-tailed)** !

STEP 1 : State the Hypothesis

Example for **Two-Sample Case**, from two population

Diketahui ada 2 buah metode pengajaran, yaitu **metode Tradisional** dan **metode Baru**.

Seorang pakar pendidikan mengklaim bahwa siswa yang diajar menggunakan metode baru memberikan nilai ujian yang **lebih baik** dibandingkan siswa yang diajar menggunakan metode tradisional.

Nyatakan hipotesisnya (Null & Alternatif) !

$$H_0 : \mu_1 = \mu_2 \qquad \textbf{Versus} \qquad H_1 : \mu_1 > \mu_2$$

We call this as **One-Tailed Test (right-tailed)** !

STEP 1 : State the Hypothesis

Table 8–1 Hypothesis-Testing Common Phrases

>	<
Is greater than	Is less than
Is above	Is below
Is higher than	Is lower than
Is longer than	Is shorter than
Is bigger than	Is smaller than
Is increased	Is decreased or reduced from
=	**≠**
Is equal to	Is not equal to
Is the same as	Is different from
Has not changed from	Has changed from
Is the same as	Is not the same as

	One-Sample Case			Two-Sample Case		
Two-tailed test:	$H_0 : \mu = k$	**VS**	$H_1 : \mu \neq k$	$H_0 : \mu_1 = \mu_2$	**VS**	$H_1 : \mu_1 \neq \mu_2$
One-tailed test:	$H_0 : \mu = k$	**VS**	$H_1 : \mu > k$	$H_0 : \mu_1 = \mu_2$	**VS**	$H_1 : \mu_1 > \mu_2$
	$H_0 : \mu = k$	**VS**	$H_1 : \mu < k$	$H_0 : \mu_1 = \mu_2$	**VS**	$H_1 : \mu_1 < \mu_2$

k adalah sebuah nilai konstan

STEP 1 : State the Hypothesis

In each of the following situations, state whether it is a **correctly stated hypothesis** testing problem and why.

$$H_0 : \mu = 190 \quad \textbf{Versus} \quad H_1 : \mu < 190$$

$$H_0 : \sigma = 3.4 \quad \textbf{Versus} \quad H_1 : \sigma \neq 3.4$$

$$H_0 : \bar{x} = 4 \quad \textbf{Versus} \quad H_1 : \bar{x} > 4$$

$$H_0 : \sigma_1 = \sigma_2 \quad \textbf{Versus} \quad H_1 : \sigma_1 > \sigma_2$$

$$H_0 : \mu > 190 \quad \textbf{Versus} \quad H_1 : \mu = 190$$

Case I
There is one normal population
(one sample case)

Test Concerning the **Mean** of a Normal Population
Known Variance

STEP 2 : Set the criterion for rejecting H_0

Suppose, we want to test the hypothesis that the mean **SAT score** for students is 455.

$$H_0 : \mu = 455 \quad \textbf{Versus} \quad H_1 : \mu \neq 455$$

We then randomly selected **144 students** as our sample, and found that the sample mean ($\bar{X}$) is **535**.

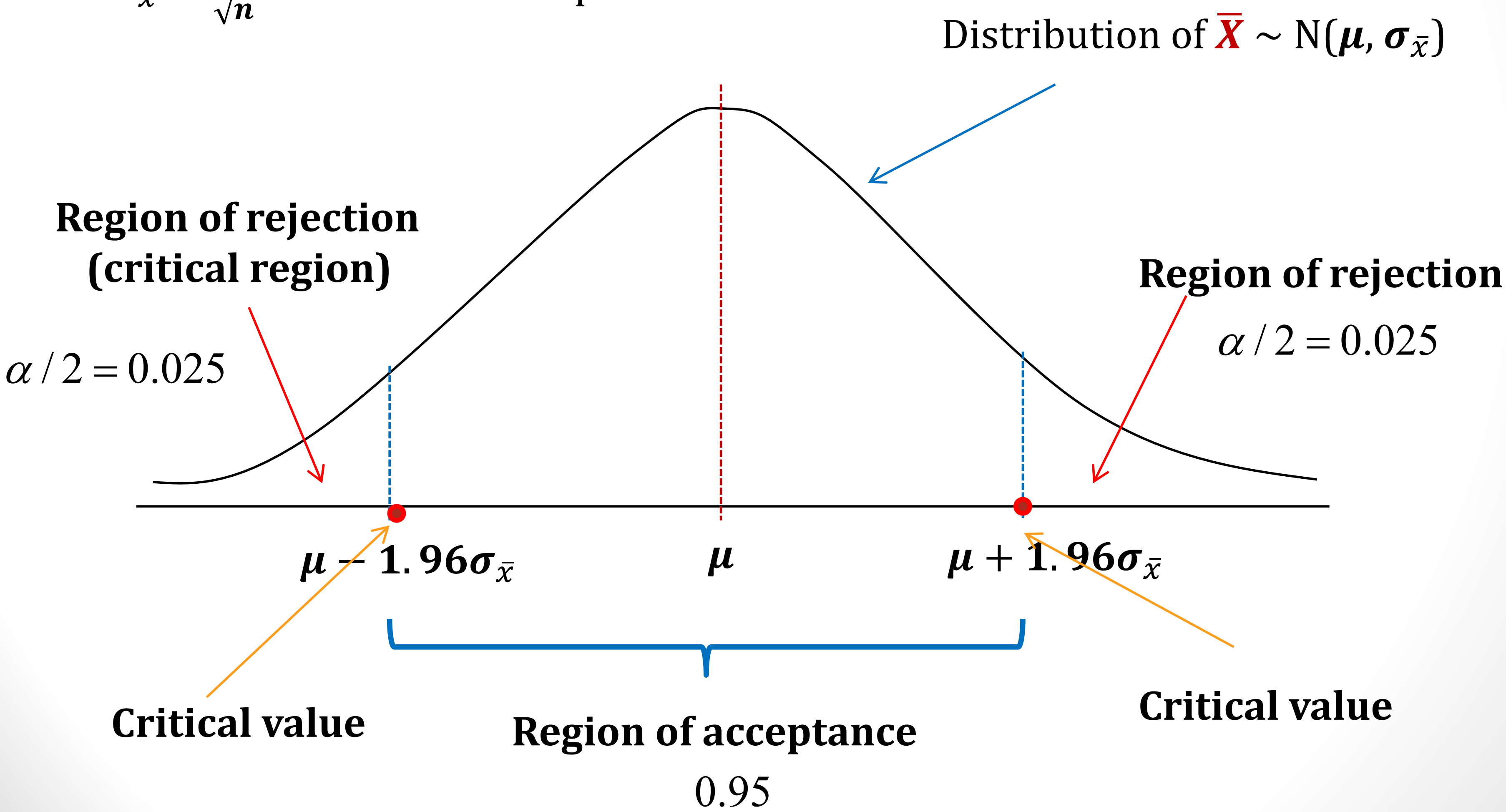
Idea:

A value of sample mean ($\bar{X}$) that falls **close** to the hypothesized value of $\mu = 455$ is **evidence** that the true mean is really 455 (supports the H_0).

STEP 2 : Set the criterion for rejecting H_0

Misal, jika $\mu - 1.96\sigma_{\bar{x}} \leq \bar{X} \leq \mu + 1.96\sigma_{\bar{x}}$, kita **tidak akan tolak** H_0.

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ is std. dev. of sample mean

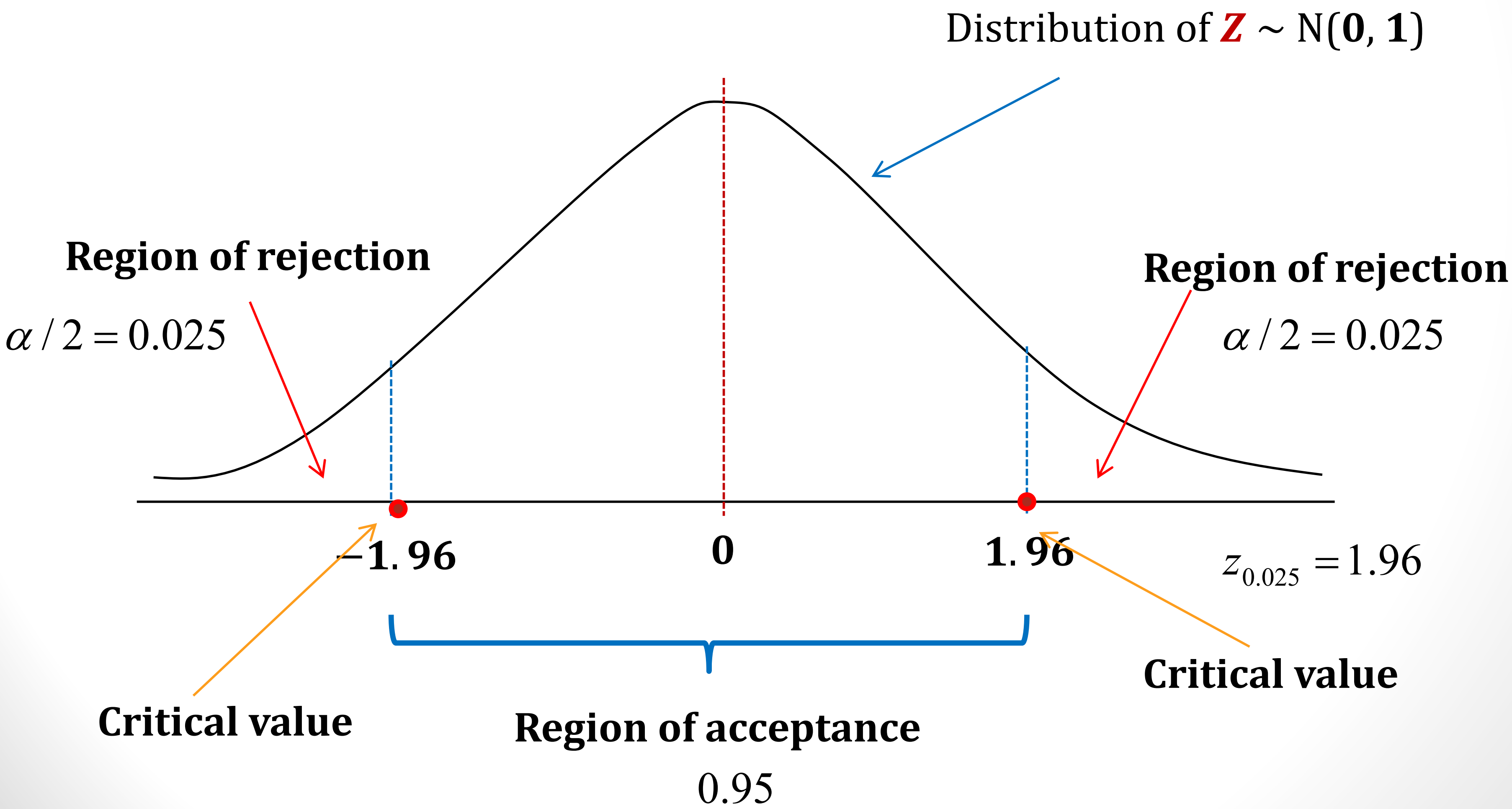


STEP 2 : Set the criterion for rejecting H_0

Now, if we use **standard-score z** to determine how different $\bar{X}$ is from $\boldsymbol{\mu}$.

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

artinya, jika $-\mathbf{1.96} \leq Z \leq \mathbf{1.96}$, kita **tidak akan tolak** H_0 !

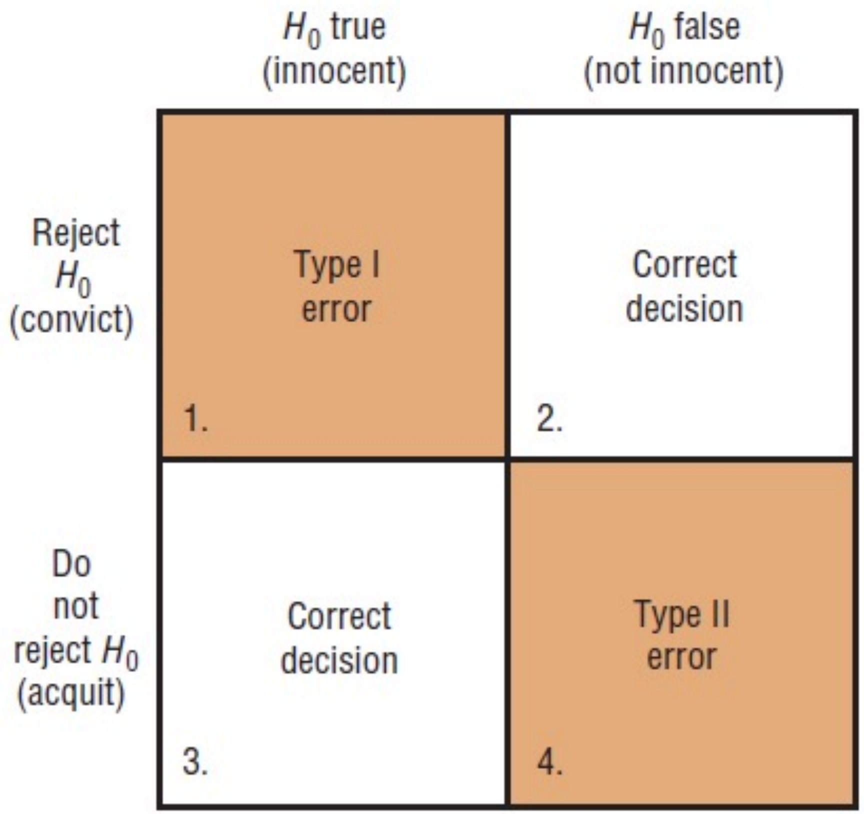


STEP 2 : Set the criterion for rejecting H_0

The decision procedure can lead to either of **two wrong conclusions**.

Type I error : Rejecting the null hypothesis H_0 when it is true.

Type II error : Failing to reject the null hypothesis H_0 when it is false.



STEP 2 : Set the criterion for rejecting H_0

Level of Significance (α)

Level of Significance is:

- **α**
- ***Luas area region of rejection***
- **Probability of making Type I Error**
- **Probability of rejecting H_0 when we know that H_0 is true**

Generally, the analyst **controls the type I error probability (i.e level of significance α)** when he or she **selects the critical values.**

Researchers usually **established α before** collecting any data.

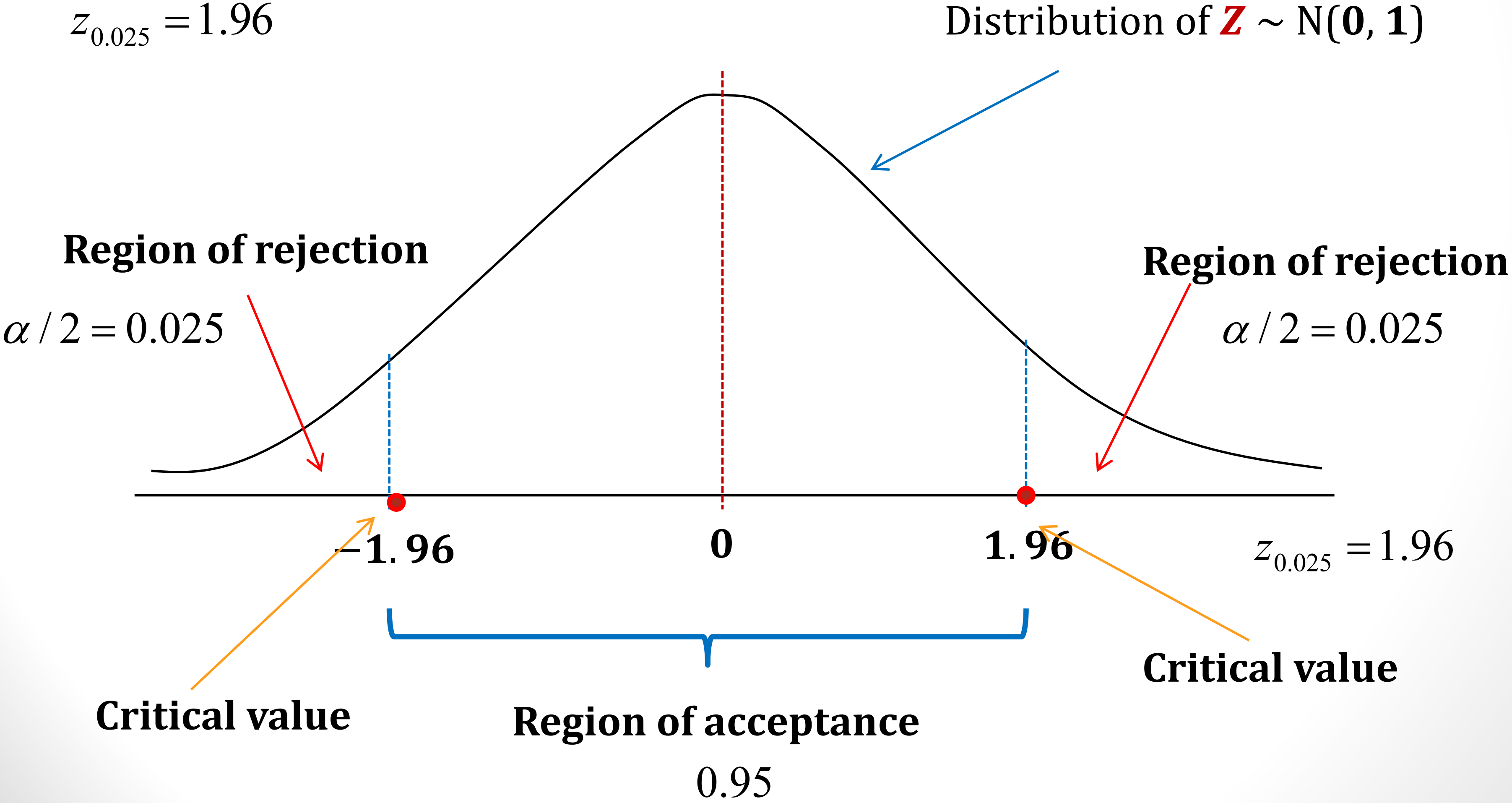
The most frequently used: 0.05 and 0.01 → the researcher knows that the decision to reject H_0 may be incorrect 5% or 1% of the time, respectively.

STEP 2 : Set the criterion for rejecting H_0

In the previous example (SAT Score), can you mention the **significance level α** being used ?

Answer: **α = 0.05 or 5%**

$z_{0.025} = 1.96$



STEP 3 : Compute the Test-Statistic (TS)

Test-Statistic : sample statistic used to decide whether to reject the **null hypothesis**.

A **statistical test** uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected.

In the previous example (SAT score), we use

$$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$$

Sebuah contoh test-statistic

as our **Test-Statistic**.

- if $-\mathbf{1.96} \leq \boldsymbol{Z} \leq \mathbf{1.96}$, we **accept** $\boldsymbol{H_0}$!
- otherwise

STEP 3 : Compute the Test-Statistic (TS)

Suppose, the previous example (SAT score) has the following additional information regarding the population & sample.

$$\mu = 455$$

$$n = 144$$

$$\bar{X} = 535$$

$$\sigma = 100$$

Now, we compute the Test-Statistic:

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{535 - 455}{100 / \sqrt{144}} = 9.60$$

9.60 is a **test value**, i.e. The numerical value obtained from a statistical test.

- if $\mathbf{-1.96 \le Z \le 1.96}$, we **accept** $\boldsymbol{H_0}$!
- Otherwise, we **reject**

STEP 4 : Decision about the Null Hypothesis

There are only two decisions:

- **Accept** the Null Hypothesis (H_0)
- **Reject** the Null Hypothesis (Take the alternative one)

In the previous example (SAT score), the observed value of the test statistic (**+9.60**) exceeds the critical value (**± 1.96**).

We also used **level of significance α = 0.05**.

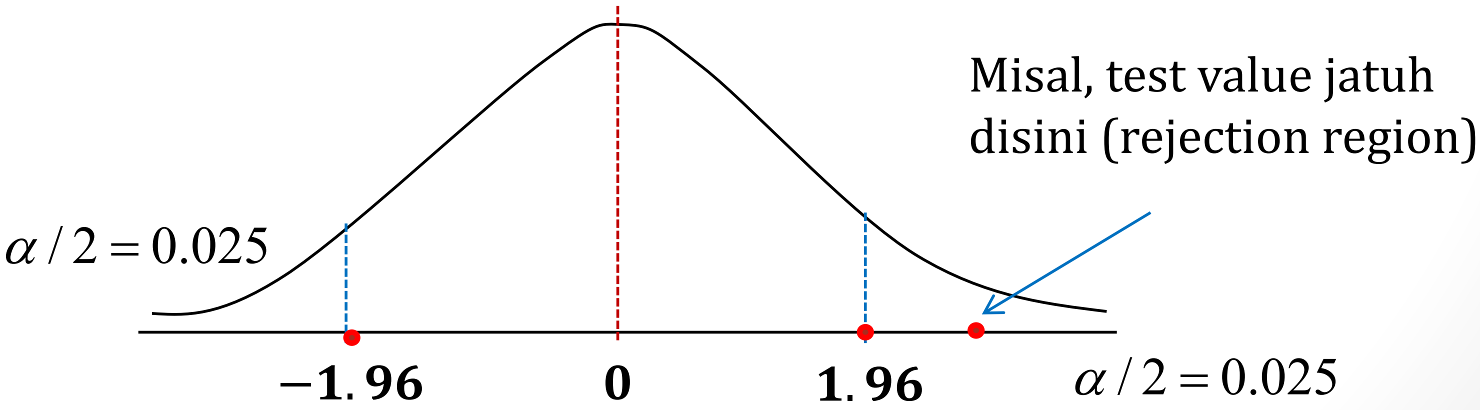
- if $-1.96 \le Z \le 1.96$, we **accept** H_0 !
- Otherwise, we **reject**

So, we reject H_0 !   But, what does it mean ??

STEP 4 : Decision about the Null Hypothesis

If the observed value of test statistic falls in the **rejection region** with, **for instance, α=0.05**, we can say for observed sample mean:

- The **sample mean** is considered as **significantly different from the H_0 at the 0.05 level of significance**.
- The probability is less than 0.05 that the observed sample mean will have occurred by chance if the null hypothesis is true → $p < 0.05$.

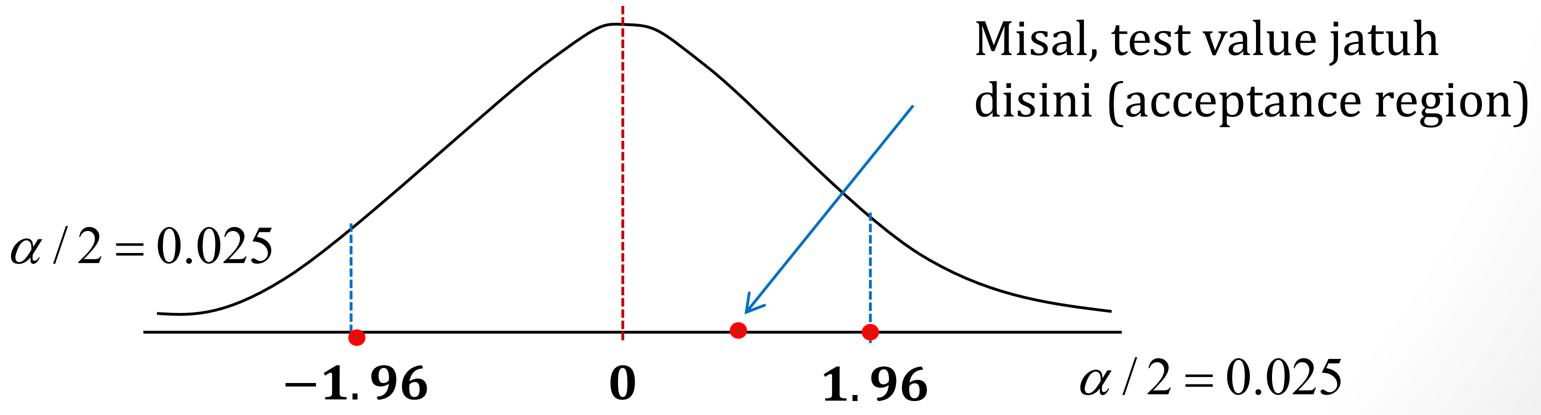


STEP 4 : Decision about the Null Hypothesis

If the observed value of test statistic does **not** fall in the **rejection region** with, **for instance, α=0.05**, we can say for observed sample mean:

- The **sample mean** is not **sufficiently different from the H_0 at the 0.05 level of significance**. (**non-significant difference**)

- The probability is greater than 0.05 that the observed sample mean will have occurred by chance if the null hypothesis is true → $p > 0.05$.



STEP 4 : Decision about the Null Hypothesis

- Choosing level of significance
  - Tendency to guard against making type I error → set too conservative α (0.05)
  - It results in retaining the null hypothesis even when p is relatively small (e.g. $p < 0.07$ but $p > 0.05$ → retain H_0)

- Statistical precision
  - Larger sample → smaller standard error → more precision → tend to reject any H_0
  - Smaller sample → bigger standard error → tend to retain any H_0 even though the difference between the hypothesized value and the observed value is seemingly large.

Case of Known Variance

summary

Our **Test-Statistics (TS)** is $$TS = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

[Two-tailed test]

Given the following hypothesis statements and significance level **α** :

$$H_0 : \mu = \mu_0 \quad \textbf{VS} \quad H_1 : \mu \neq \mu_0$$

- we **reject** $\boldsymbol{H_0}$ if $|TS| > z_{\alpha/2}$
- we **accept** $\boldsymbol{H_0}$ if $|TS| \leq z_{\alpha/2}$

[One-tailed #1]

Given the following hypothesis statements and significance level **α** :

$$H_0 : \mu = \mu_0 \quad \textbf{VS} \quad H_1 : \mu > \mu_0$$

- we **reject** $\boldsymbol{H_0}$ if $TS > z_{\alpha}$
- we **accept** $\boldsymbol{H_0}$ if $TS \leq z_{\alpha}$

[One-tailed #2]

Given the following hypothesis statements and significance level **α** :

$$H_0 : \mu = \mu_0 \quad \textbf{VS} \quad H_1 : \mu < \mu_0$$

- we **reject** $\boldsymbol{H_0}$ if $TS < -z_{\alpha}$
- we **accept** $\boldsymbol{H_0}$ if $TS \geq -z_{\alpha}$

Case II
There is one normal population
(one sample case)

Test Concerning the **Mean** of a Normal Population
Unknown Variance

Case of Unknown Variance (t-test)

Previously, we assume that the population variance **σ** is known.
However, the most common situation is when **σ** is **unknown** !

Now when **σ** is **no longer known**, we use the following proposition:

$$T_{n-1} = \frac{\overline{X} - \mu_0}{S / \sqrt{n}} \sim t_{n-1}$$

Where,

$$S^2 = \frac{\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2}{n-1}$$

n : sample size

For **two-tailed test**, $\boldsymbol{H_0}$ is rejected when $\left| \frac{\overline{X} - \mu_0}{S / \sqrt{n}} \right|$ is **large** !

Case of Unknown Variance (t-test)

summary

Our **Test-Statistics (TS)** is $$TS = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$

[Two-tailed test]

Given the following hypothesis statements and significance level **α** :

$$H_0 : \mu = \mu_0 \quad \textbf{VS} \quad H_1 : \mu \neq \mu_0$$

- we **reject** ***H*₀** if $|TS| > t_{\alpha/2, n-1}$
- we **accept** ***H*₀** if $|TS| \leq t_{\alpha/2, n-1}$

[One-tailed #1]

Given the following hypothesis statements and significance level **α** :

$$H_0 : \mu = \mu_0 \quad \textbf{VS} \quad H_1 : \mu > \mu_0$$

- we **reject** $\boldsymbol{H_0}$ if $TS > t_{\alpha, n-1}$
- we **accept** $\boldsymbol{H_0}$ if $TS \leq t_{\alpha, n-1}$

[One-tailed #2]

Given the following hypothesis statements and significance level **α** :

$$H_0 : \mu = \mu_0 \quad \textbf{VS} \quad H_1 : \mu < \mu_0$$

- we **reject** $\boldsymbol{H_0}$ if $TS < -t_{\alpha, n-1}$
- we **accept** $\boldsymbol{H_0}$ if $TS \geq -t_{\alpha, n-1}$

LATIHAN CASE I DAN CASE II

Seorang peneliti mengklaim bahwa rataan harga dari sepatu olahraga pria adalah kurang dari \$80. Dia kemudian memilih sampel secara random yang berisi 36 pasang sepatu dari katalog dan menemukan bahwa rataan biaya dari sepatu-sepatu pada sampel adalah \$75.

Asumsikan populasi harga sepatu mengikuti distribusi normal dengan variansi 368.84

Apakah ada cukup bukti untuk mendukung klaim dari peneliti tersebut dengan level of significance 0.1 ?

Solusi:

Step 1: State the hypothesis and identify the claim

$$H_0 : \mu = 80 \quad \textbf{VS} \quad H_1 : \mu < 80 \; (claim)$$

Step 2: Set the rejection criteria

Since **α = 0.1** and the test is one-tailed test (left) (**σ is known**),

we **reject H_0** if $TS < -z_{\alpha}$

The critical value is $-z_{\alpha} = -z_{0.1} = -1.28$

Step 3: Compute the Test-Statistics

$$TS = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{75 - 80}{19.2 / \sqrt{36}} = -1.56$$

Step 4: Make the decision and conclusion

since $TS = -1.56 < -z_{\alpha} = -1.28$, we **reject H_0** !

Conclusion: **There is enough evidence to support the claim** that the average cost of men's athletic shoes is less than $80.

Sebuah pusat rehabilitasi medis melaporkan bahwa rata-rata biaya untuk rehabilitasi penderita stroke adalah $24,672.

Untuk melihat apakah laporan ini berbeda di sebuah rumah sakit tertentu, seorang peneliti mengambil sampel secara acak yang berisi 35 penderita stroke di sebuah rumah sakit dan menemukan bahwa rataan biaya rehabilitasi mereka adalah $26.343.

Standar deviasi dari populasi adalah $3251. Pada level of significance 0.01, bisakah kita simpulkan bahwa rataan biaya rehabilitasi stroke pada rumah sakit tersebut berbeda dari $24.672 ?

Solusi:

Step 1: State the hypothesis and identify the claim

$$H_0 : \mu = 24.672 \quad \textbf{VS} \quad H_1 : \mu \neq 24.672 \ (claim)$$

Step 2: Set the rejection criteria

Since **α = 0.01** and the test is two-tailed test (**σ is known**),

we **reject** $\boldsymbol{H_0}$ if $|TS| > z_{\alpha/2}$ $\quad z_{\alpha/2} = z_{0.005} = 2.58$

The critical value is $z_{0.005} = 2.58 \quad and \quad -z_{0.005} = -2.58$

Step 3: Compute the Test-Statistics

$$TS = \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{26.343 - 24.672}{3251 / \sqrt{35}} = 3.04$$

Step 4: Make the decision and conclusion

since $|TS| = 3.04 > z_{\alpha/2} = 2.58$, we **reject** $\boldsymbol{H_0}$!

<u>**Conclusion:**</u> **There is enough evidence to support the claim** that the average cost of rehabilitation at the particular hospital is different from $24,672.

Sebuah investigasi medis mengeluarkan klaim bahwa rata-rata banyaknya infeksi per minggu pada sebuah rumah sakit adalah 16.3

Sebuah sampel acak dari 10 minggu mempunyai rataan banyaknya infeksi 17.2. Standar deviasi dari sampel adalah 1.8

Apakah cukup bukti untuk menyangkal klaim dari si investigator pada level of significance 0.05 ?

Solusi:

Step 1: State the hypothesis and identify the claim

$$H_0 : \mu = 16.3\ (claim) \quad \textbf{VS} \quad H_1 : \mu \neq 16.3$$

Step 2: Set the rejection criteria

Since **α = 0.05** and the test is two-tailed test (**σ is unknown**),

we **reject** ***H*₀** if $|TS| > t_{\alpha/2, n-1}$ $\quad t_{\alpha/2,\, n-1} = t_{0.025,\, 9} = 2.262$

The critical value is $t_{0.025,9} = 2.262 \quad and \quad -t_{0.025,9} = -2.262$

Step 3: Compute the Test-Statistics (t-test)

$$TS = \frac{\overline{X} - \mu_0}{S/\sqrt{n}} = \frac{17.2 - 16.3}{1.8/\sqrt{10}} = 1.58$$

Step 4: Make the decision and conclusion

since $|TS| = 1.58 < t_{\alpha/2,n-1} = 2.262$, we **accept** $\boldsymbol{H_0}$!

Conclusion: There is **not** enough evidence to **reject** the claim that the average number of infections is 16.3.

Berdasarkan eksperimen di lapangan, sebuah varietas baru dari gandum diharapkan dapat menghasilkan 12 Kwintal per hektar.

10 ladang yang menanam gandum baru tersebut dipilih secara acak sebagai sampel. Masing-masing ladang pada sampel dihitung dan hasilnya (Kwintal/Hektar) adalah sebagai berikut :

14.3	12.6	13.7	10.9	13.7
12.0	11.4	12.0	12.6	13.1

Asumsikan hasil "kwintal/hektar" dari gandum tersebut mengikuti distribusi **Normal**.

Apakah hasil yang diperoleh dari sampel sesuai dengan harapan ? (ujilah dengan level of significance **5%**)

There are two normal populations
(Two Sample Case)

Testing the **Equality of Means** of Two Normal Population
(for independent samples)

Let $X_1, \dots, X_n$ be a sample of size n from a normal population having mean μ_1 and variance ${\sigma_1}^2$

Let $Y_1, \dots, Y_m$ be a sample of size m from a different normal population having mean μ_1 and variance ${\sigma_2}^2$

Suppose that the two samples are **independent of each other**.

the parameters μ_1, μ_2 are **unknown** !

Now, we want to test the hypothesis:

$$H_0 : \mu_1 = \mu_2 \quad \textbf{vs} \quad H_1 : \mu_1 \neq \mu_2$$

Case of Known Variances

The null hypothesis can be written as $H_0 : \mu_1 - \mu_2 = 0$

It seems reasonable to reject $\boldsymbol{H_0}$ when $\boldsymbol{\bar{X} - \bar{Y}}$ is **far from zero**. That is,

Reject $\boldsymbol{H_0}$ if $\left|\bar{X} - \bar{Y}\right| > c$ for some value **c**

Accept $\boldsymbol{H_0}$ otherwise

We need to determine the distribution of $\boldsymbol{\bar{X} - \bar{Y}}$:

$$\bar{X} - \bar{Y} \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}\right)$$

when $\boldsymbol{H_0}$ is true: $\mu_1 - \mu_2 = 0$

$$\frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}} \sim N(0,1) \quad \Rightarrow \quad P_{H_0}\left(-z_{\alpha/2} \le \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}} \le z_{\alpha/2}\right) = 1 - \alpha$$

Case of Known Variances

So, we can conclude that for two-sided test

Reject $\boldsymbol{H_0}$ if $$\left| \frac{\bar{X}-\bar{Y}}{\sqrt{\sigma_1^2/n + \sigma_2^2/m}} \right| > z_{\alpha/2}$$

One – Tailed Test

For example, for testing : $H_0 : \mu_1 = \mu_2 \ (or\ \mu_1 \le \mu_2)$ **vs** $H_1 : \mu_1 > \mu_2$

Reasonable to reject $\boldsymbol{H_0}$ if $\bar{X}-\bar{Y} > c$ for some value **c**

Using the distribution of $\bar{X}-\bar{Y}$ and the same way as before, we conclude:

Reject $\boldsymbol{H_0}$ if $$\frac{\bar{X}-\bar{Y}}{\sqrt{\sigma_1^2/n + \sigma_2^2/m}} > z_{\alpha}$$

Case of Known Variances

summary

Our **Test-Statistics (TS)** is

$$TS = \frac{\bar{X} - \bar{Y}}{\sqrt{\sigma_1^2/n + \sigma_2^2/m}}$$

[Two-tailed test]

Given the following hypothesis statements and significance level **α** :

$$H_0 : \mu_1 = \mu_2 \quad \textbf{VS} \quad H_1 : \mu_1 \neq \mu_2$$

- we **reject** $\boldsymbol{H_0}$ if $|TS| > z_{\alpha/2}$
- we **accept** $\boldsymbol{H_0}$ if $|TS| \leq z_{\alpha/2}$

[One-tailed #1]

Given the following hypothesis statements and significance level **α** :

$$H_0 : \mu_1 = \mu_2 \quad \textbf{VS} \quad H_1 : \mu_1 > \mu_2$$

- we **reject** $\boldsymbol{H_0}$ if $TS > z_{\alpha}$
- we **accept** $\boldsymbol{H_0}$ if $TS \leq z_{\alpha}$

[One-tailed #2]

Given the following hypothesis statements and significance level **α** :

$$H_0 : \mu_1 = \mu_2 \quad \textbf{VS} \quad H_1 : \mu_1 < \mu_2$$

- we **reject** $\boldsymbol{H_0}$ if $TS < -z_{\alpha}$
- we **accept** $\boldsymbol{H_0}$ if $TS \geq -z_{\alpha}$

Data 2 buah sampel menyatakan bahwa rataan sewa hotel per malam di Depok adalah $88.42 dan rataan sewa hotel di Bandung adalah $80.61. Data tersebut terdiri dari 50 hotel untuk masing-masing kota.

Standar deviasi dari kedua populasi (dalam hal harga sewa) adalah $5.62 dan $4.83 untuk Depok dan Bandung, secara berurutan.

Pada level of significance 0.05, bisakah kita simpulkan bahwa ada perbedaan signifikan dalam hal harga sewa hotel antara di kota Depok dan Bandung ?

Asumsikan populasi harga sewa hotel mengikuti distribusi normal.

Solusi:

Step 1: State the hypothesis and identify the claim

$$H_0 : \mu_1 = \mu_2 \qquad \textbf{VS} \qquad H_1 : \mu_1 \neq \mu_2 \ (claim)$$

Step 2: Set the rejection criteria

Since **α = 0.05** and the test is two-tailed test (**σ is known**),

we **reject** ***H*₀** if $|TS| > z_{\alpha/2}$ $\qquad z_{\alpha/2} = z_{0.025} = 1.96$

The critical value is $z_{0.025} = 1.96 \quad and \quad -z_{0.025} = -1.96$

Step 3: Compute the Test-Statistics (t-test)

$$TS = \frac{\overline{X} - \overline{Y}}{\sqrt{\sigma_1^2/n + \sigma_2^2/m}} = \frac{88.42 - 80.61}{\sqrt{5.62^2/50 + 4.83^2/50}} = 7.45$$

Step 4: Make the decision and conclusion

since $|TS| = 7.45 > z_{\alpha/2} = 1.96$, we **reject** ***H*₀** !

Conclusion: There is enough evidence to support the claim that the means are not equal. Hence, there is a significant difference in the rates.