Advanced Counting: Recurrence Relation

Adila A. Krisnadhi

Fakultas Ilmu Komputer, Universitas Indonesia

Version date: 2023-02-28 10:32:22+07:00

Reference: Rosen, Ed.8, Ch.8

Recap: Sum rule and product rule

How many possible license plates can the DKI Jakarta's police department issue?

Recap: Inclusion-exclusion principle

How many bit strings of length 8 begins with a 1 or ends with 00?

Recap: Pigeonhole principle

How many students must register to the Discrete Math 2 class at least so that at least two students can get the same final grade?

Recap: Permutation (without repetition)

How many ways to give out one gold, one silver, and one bronze medal to 100 marathoners?

Recap: Combination (without repetition)

How many volleyball teams (6 volleayballer per team) can be formed of 10 volleyballers who are ready to compete?

Motivation

An example of rather complex problems: "How many bit strings of length n do not contain two consecutive 0s?"

Motivation

An example of rather complex problems: "How many bit strings of length n do not contain two consecutive 0s?"

- Suppose a_n is the number of bit strings of length n that do not contain two consecutive 0s.

Motivation

An example of rather complex problems: "How many bit strings of length n do not contain two consecutive 0s?"

- Suppose a_n is the number of bit strings of length n that do not contain two consecutive 0s.
- $a_1 =$

Motivation

An example of rather complex problems: "How many bit strings of length n do not contain two consecutive 0s?"

- Suppose a_n is the number of bit strings of length n that do not contain two consecutive 0s.
- $a_1 = 2$
- $a_2 =$

Motivation

An example of rather complex problems: "How many bit strings of length n do not contain two consecutive 0s?"

- Suppose a_n is the number of bit strings of length n that do not contain two consecutive 0s.
- $a_1 = 2$
- $a_2 = 3$
- $a_3 =$

Motivation

An example of rather complex problems: "How many bit strings of length n do not contain two consecutive 0s?"

- Suppose a_n is the number of bit strings of length n that do not contain two consecutive 0s.
- $a_1 = 2$
- $a_2 = 3$
- $a_3 = 5$
- $a_n =$

Motivation

An example of rather complex problems: "How many bit strings of length n do not contain two consecutive 0s?"

- Suppose a_n is the number of bit strings of length n that do not contain two consecutive 0s.
- $a_1 = 2$
- $a_2 = 3$
- $a_3 = 5$
- $a_n = a_{n-1} + a_{n-2}$ untuk $n > 2$.

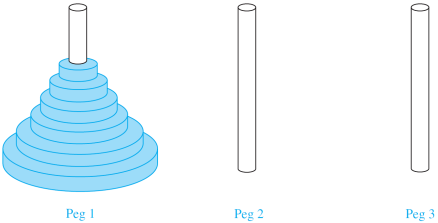
The last equation is called a **recurrence relation** representing the sequence 2, 3, 5, 8, 13, ... Each term in this particular recurrence relation is defined in terms of two preceding terms. Thus, the first two terms of the recurrence relation need to be defined as initial conditions.

Recurrence relation can be solved by finding an **explicit** formula for a_n as a function of n.

Example 1

A pair of very young rabbits (one of each sex) is left on an island. Every pair of rabbits does not breed until they reach the age of 2 months old. After reaching 2 months old, each pair produces another pair (of opposite sex) each month. How many pairs of rabbits are there in that island after n months if the rabbits live forever and every rabbit is paired exactly to another rabbit?

Example 2



How many steps are needed to move all disks from peg 1 to peg 3 such that a disk is never stacked on top of a smaller disk throughout the process of moving?

Example 3

A special PIN is made of a string containing n decimal digits. The PIN is valid if it contains an even number of 0s. How many valid PINs of length n are there?

Example 4

How many bit strings of length n that do not have two consecutive 0s?

Example 5

Find the number of ways to parenthesize the product of $n+1$ numbers, $x_0, x_1, \ldots, x_n$ to specify the order of multiplication. For example, there are five ways to parenthesize the product $x_0 \cdot x_1 \cdot x_2 \cdot x_3$, namely $((x_0 \cdot x_1) \cdot x_2) \cdot x_3$, $(x_0 \cdot (x_1 \cdot x_2)) \cdot x_3$, $(x_0 \cdot x_1) \cdot (x_2 \cdot x_3)$, $x_0 \cdot ((x_1 \cdot x_2) \cdot x_3)$, and $x_0 \cdot (x_1 \cdot (x_2 \cdot x_3))$

Solving recurrence relation

Techniques to solve a recurrence relation:

- Telescoping
- Iteration
- Characteristic equation [Rosen, Section 8.2]
- Generating function (discussed here)