2 2 0 6 0 2 8 9 3 2

ALDEN LUTHFI

1. tahun 1 : K_t K_t

tahun 2 : K_t K_t K_B K_B

tahun 3 : $6K$ $2K_s$ $2K_B$

tahun 4 : $14K_t$ $6K_B$ $4K_s$

tahun 5 : $34K_t$ $14K_B$ $10K_s$

K_t =	koki training
K_B =	koki tetap
K_S =	koki senior

Jumlah koki = $K_t(n) + K_B(n) + K_S(n)$

$K(0) = 2$
$K(1) = 4$
$K(2) = 10 \rightarrow 2.4+2$
$K(3) = 24 \rightarrow 2.10+4$
$K(4) = 2.24 + 10 = 58$
$K(n) = 2K(n-1) + K(n-2)$

maka $K(7) = 816$

2. a. $a_n \rightarrow \langle 4, 1, -2, -5 \ldots \rangle$

cek pakai telescoping :

$a_n = 2a_{n-1} - a_{n-2} \rightarrow a_n - 2a_{n-1} + a_{n-2} = 0$

$\downarrow$

$$\begin{aligned}
a_n - 2a_{n-1} + a_{n-2} &= 0 \\
a_{n-1} - 2a_{n-2} + a_{n-3} &= 0 \\
a_{n-2} - 2a_{n-3} + a_{n-4} &= 0 \\
&\vdots \\
a_2 - 2a_1 + a_0 &= 0 \quad + \\
\hline
a_n - a_{n-1} &= a_1 - a_0 \\
&= 1 - 4 = -2
\end{aligned}$$

$$\begin{aligned}
a_n - a_{n-1} &= -3 \\
a_{n-1} - a_{n-2} &= -3 \\
&\vdots \\
a_1 - a_0 &= -3 \quad + \\
\hline
a_n - 4 &= -3.n \\
a_n &= 4 - 3n
\end{aligned}$$

2206028932
ALDEN LUTHFI

Pakai FP

(i) kalikan dengan z^n

$$a_n z^n = 2a_{n-1} z^n - a_{n-2} z^n$$

(ii) jadikan notasi sigma

$$\sum_{n=2}^{\infty} a_n z^n = \sum_{n=2}^{\infty} 2a_{n-1} z^n - \sum_{n=2}^{\infty} a_{n-2} z^n$$

(iii) asumsi $G(z) = x_0 + x_1 z + x_2 z^2 \ldots$

$$G(z) - a_0 - a_1 z = 2z(G(z) - a_0) - (G(z) \cdot z^2)$$

(iv) ubah ke bentuk tertutup

$$G(z) - 4 - z = 2zG(z) - 8z - z^2 G(z)$$

$$G(z)(z^2 - 2z + 1) = 4 - 7z$$

$$G(z) = \frac{4-7z}{z^2-2z+1} = \frac{4-7z}{(z-1)^2}$$

$$= \frac{4}{(z-1)^2} - \frac{7z}{(z-1)^2}$$

$$= 4(n+1) - 7n = 4 - 3n$$

(b.) $a_n = 7a_{n-1} - 5^n$, $a_0 = 8$

(i)

$$a_n z^n = 7a_{n-1} z^n - 5^n z^n$$

(ii)

$$\sum_{n=1}^{\infty} a_n z^n = \sum_{n=1}^{\infty} 7a_{n-1} z^n - \sum_{n=1}^{\infty} 5^n z^n$$

(iii)

$$G(z) - a_0 = 7z\,G(z) - \left(\frac{1}{1-5z} - 1\right)$$

$$G(z) - 8 = 7z\,G(z) - \frac{1}{1-5z} + 1$$

2206028932
ALDEN LUTHFI

(iv)

$$G(z) = \frac{8 - \frac{1}{1-5z} + 1}{1-7z}$$

$$= \frac{8 - 45z}{(1-5z)(1-7z)} = \frac{A}{1-5z} + \frac{B}{1-7z}$$

$$= \frac{5}{2(1-5z)} + \frac{11}{2(1-7z)}$$

$$= \frac{5}{2}(5^n) + \frac{11}{2}(7^n)$$

3 (a) ya jika $a_0 = 0$ dan $a_1 = 0$ maka $a_n = 0$

(b) tidak → proof by Contradiction

asumsi $a_n = 1$ untuk $n \geq 0$

maka $a_0 = 1$ dan $a_1 = 1$ dan $a_2 = 1$

$$a_2 = 10a_1 - 25a_0$$

$$= 10 - 25 = -15$$

hal ini menyalahi asumsi bahwa $a_2 = 1$ maka $a_n = 1$ tidak merupakan solusi

(c) ya, jika kita selesaikan dengan FP

(i)

$$a_n = 10a_{n-1} - 25a_{n-2}, \quad n \geq 2$$

$$a_n z^n = 10a_{n-1} z^n - 25a_{n-2} z^n$$

(ii)

$$\sum_{n=2}^{\infty} a_n z^n = \sum_{n=2}^{\infty} 10a_{n-1} z^n - \sum_{n=2}^{\infty} 25a_{n-2} z^n$$

(iii)

$$G(z) - a_1 z - a_0 = 10z(G(z) - a_0) - 25z^2 G(z)$$

2206028932
ALDEN LUTHFI

(iv) $G(z)(25z^2-10z+1) = z(a_1-10a_0)+a_0$

$$G(z) = \frac{(a_1-10a_0)z + a_0}{(1-5z)^2}$$

Karena $\frac{1}{(1-u)^2} \to 1+2u+3u^2\cdots$ maka $\frac{1}{(1-5z)^2} \to 1+2.5z+3.(5z)^2\dots$

$x_n = 5^n(n+1)$

maka $a_n = (a_1-10a_0)5^n(n) + a_0 5^n(n+1)$

Solusi umum dari $a_n = 5^n(n(a_1-9a_0)+a_0)$
untuk sembarang a_1 dan a_0 bulat

maka dari itu (c) bisa didapat dari $a_1=1$, $a_0=0$

(d.) tidak karena tidak ada nilai a_0 dan a_1 yang dapat memenuhi persamaan

$$a_n = 5^n(n(a_1-9a_0)+a_0) \neq 5^n n^2$$

(4) (a) $\frac{z}{1-2z^2} \to z(1+2z^2+4z^4+8z^6)$

$= z + 2z^3 + 4z^5 \dots$

maka $3 + 3z + 3z^2 + 3z^3$
$z + 2z^3 + 4z^5$

suku ke-0 s/d ke-5 $\to \langle 3, 4, 3, 5, 0, 4 \rangle$

2206028932
ALDEN LUTHFI

(b.) $\frac{4z^2+z-1}{(1-z)^2} = \frac{4z^2}{(1-z)^2} + \frac{z}{(1-z)^2} - \frac{1}{(1-z)^2}$

$\to \frac{4z^2}{(1-z)^2} \Rightarrow 4z^2(1+2z+3z^2+\dots)$

$= 4z^2+8z^3+12z^4+16z^5$

$\to \frac{z}{(1-z)^2} \Rightarrow z(1+2z+3z^2+\dots)$

$= z+2z^2+3z^3+4z^4+5z^5$

$\to -\frac{1}{(1-z)^2} \Rightarrow -1(1+2z+3z^2+\dots)$

$= -1-2z-3z^2-4z^3-5z^4-6z^5$

maka suku ke-0 s/d suku ke-5 $= \langle -1, -1, 3, 7, 11, 15 \rangle$

(c.) $-2z^2-z+1 = (1-2z)(z+1)$

$$\frac{3z^2+7z-2}{(1-2z)(z+1)} = \frac{3z^2}{(1-2z)(z+1)} + \frac{7z}{(1-2z)(z+1)} - \frac{2}{(1-2z)(z+1)}$$

$$-\frac{2}{(1-2z)(z+1)} = -\frac{4}{3(1-2z)} - \frac{2}{3(z+1)}$$

$\to -\frac{4}{3}(1+2z+4z^2+8z^3+16z^4+32z^5+\dots)$

$-\frac{2}{3}(1-z+z^2-z^3+z^4-z^5+\dots)$

$= -2+(-2)z+(-6)z^2+(-10)z^3+(-22)z^4+(-42)z^5 + \dots$

22060289 32
ALDEN LUTHFI

$$\frac{7z}{(1-2z)(z+1)} = \frac{7}{3(z+1)} - \frac{7}{3(1-2z)}$$

$$\rightarrow \frac{7}{3}(1+2z+4z^2+8z^3+16z^4+32z^5+\dots)$$

$$\frac{7}{3}(1-z+z^2-z^3+z^4-z^5+\dots)$$

$$= 0 + 7z + 7z^2 + 21z^3 + \quad z^4 + \quad z^5 + \dots$$

$$\rightarrow z^2\left(\frac{3}{(1-2z)(z+1)}\right) = \frac{2}{1-2z} + \frac{1}{z+1}$$

$$\rightarrow 2(1+2z+4z^2+8z^3+16z^4+32z^5+\dots)$$
$$+ (1-z+z^2-z^3+z^4-z^5+\dots)$$

$$= z^2(3+3z+9z^2+15z^3+33z^4+63z^5+\dots$$
$$= 3z^2+3z^3+9z^4+15z^5+\dots$$

maka suku ke-0 s/d suku ke 5 = <-2, 5, 4, 14, 22, 50>

⑤ $L_n \rightarrow \langle 3, 4, 5, 6 \dots \rangle$

$$L(z) = \frac{\frac{1}{(1-z)^2} - (1+2z)}{z^2}$$

$$M(z) = z\left(\frac{1}{1-3z}\right)$$

$$K(z) = L(z) \cdot M(z) = \frac{\frac{1}{(1-z)^2} - (1+2z)}{z(1-3z)}$$

$$(1-3z)K(z) = \frac{1}{z^3-2z^2+z} - \frac{1+2z}{z} = \frac{3z-2z^2}{z^2-2z+1}$$

$$K(z) = \frac{3z-2z^2}{(z-1)^2(1-3z)}$$

⑥ ⓐ $\frac{2}{1+z} = G(z)$ ▨

↘ lookup table

ⓑ $\langle 0,2,5,9,14,20,27\dots\rangle = \langle 0,2,3,4,5\dots\rangle + \langle 0,0,2,5,9,14\dots\rangle$

$$G(z) = \frac{1}{(1-z)^2} - 1 + G(z).z$$

$$G(z)(1-z) = \frac{1-(1-z)^2}{(1-z)^2}$$

$$G(z) = \frac{(1+1-z)(1-1+z)}{(1-z)^3} = \frac{(2-z)(z)}{(1-z)^3}$$

$$\therefore G(z) = \frac{2z-z^2}{(1-z)^3}$$ ▨

ⓒ Pakai hint

$\langle 4, 5, 9, 27, 123, \dots\rangle = \langle 3, 3, 3, 3, 3, \dots\rangle + \langle 1, 2, 6, 24\dots\rangle$

$$= \frac{3}{1-z} + \sum_{n=0}^{\infty} n!\,z^n - 1$$

$$G(z) = \frac{3}{1-z} + \frac{e^{-\frac{1}{z}}E_1(-\frac{1}{z})}{z} - 1$$ ▨

⑦ ⓐ sifat $\sum a_n+b_n = \sum a_n + \sum b_n$

$$G(z) = \sum_{n=1}^{\infty} 2^n z^n + z^n = \sum_{n=1}^{\infty} 2^n z^n + \sum_{n=1}^{\infty} z^n$$

$$G(z) = \frac{1}{1-2z} + \frac{1}{1-z}$$ ▨

ⓑ $$\sum_{n=1}^{\infty} \frac{nz^n - z^n}{n} = \sum_{n=1}^{\infty} z^n - \sum_{n=1}^{\infty}\frac{z^n}{n} = G(z)$$

$$= \frac{1}{1-z} - 1 - \left(\sum_{n=1}^{\infty}\frac{z^n}{n}\right)$$

$$z + \frac{z^2}{2} + \frac{z^3}{3} + \dots \qquad 1 + z + z^2 + z^3 + \dots$$

22 06 0 28 9 3 2
ALDEN LUTHFI

$$\to \sum_{n=1}^{\infty} \frac{z^n}{n} = \int \sum_{n=1}^{\infty} z^{(n-1)} \, dz$$

$$= \int \frac{1}{1-z} \, dz$$

$$= -\ln|1-z|$$

maka $G(z) = \frac{1}{1-z} - 1 + \ln|1-z|$ $\blacksquare$

(8.) $$\frac{1+3z-z^2}{(1-z)(1-2z)(1+z)} = \frac{A}{1-z} + \frac{B}{1-2z} + \frac{C}{1+z}$$

$\to 1-z \to z=1$

$$\frac{1+3-1}{-2} = A \to A = -\frac{3}{2}$$

$\to 1-2z \to z = \frac{1}{2}$

$$\frac{1+\frac{3}{2}-\frac{1}{4}}{1-\frac{1}{4}} = B \to B = 3$$

$\to 1+z \to z=-1$

$$\frac{1-3-1}{2 \cdot 3} = C \to C = -\frac{1}{2}$$

$$\frac{1+3z-z^2}{(1-z)(1-2z)(1+z)} = \frac{3}{1-2z} - \frac{3}{2(1-z)} - \frac{1}{2(1+z)}$$

$$X_n = 3(2^n) - \frac{3}{2} - \frac{1}{2}(-1)^n \quad \blacksquare$$

22 06 0 28 9 3 2
ALDEN LUTHFI

(9) (a) Dek Depe = 200 - 25 - 35 = 140
Pak Esde = 200 - 20 - 35 = 145
Sofita = 200 - 20 - 25 = 155

(b) Dek Depe (z) = $(z^{20} + z^{21} + z^{22} + \dots + z^{140})$
Pak Esde (z) = $(z^{25} + z^{26} + z^{27} + \dots + z^{145})$
Sofita (z) = $(z^{36} + z^{36} + z^{37} + \dots + z^{155})$
minimal 80 eskrim

Cara $\to \sum_{i=80}^{200}$ koefisien $[z^i]$

pakai kombinatorik $a+b+c \leq 200 \to a+b+c+d = 200$
$a \geq 20,\ b \geq 25,\ c \geq 35$
Sisa $\to 200 - 20 - 25 - 35 = 120$
stars and bars (120 stars, 3 bars).

$$\frac{122!}{3!\,119!} = \frac{122 \cdot 121 \cdot 120}{2 \cdot 3} = 61 \cdot 40 \cdot 121 \text{ cara}$$

(c) Vanilla = $\left(\frac{z^0}{0!} + \frac{z^1}{1!} + \frac{z^2}{2!} + \dots + \frac{z^7}{7!}\right)$
Strawberry = $\left(\frac{z^0}{0!} + \frac{z^1}{1!} + \frac{z^2}{2!} + \dots + \frac{z^{10}}{10!}\right)$
matcha = $\left(\frac{z^0}{0!} + \frac{z^1}{1!} + \frac{z^2}{2!} + \dots + \frac{z^{18}}{18!}\right)$
coklat = $\left(\frac{z^0}{0!} + \frac{z^1}{1!} + \frac{z^2}{2!} + \dots + \frac{z^{15}}{15!}\right)$

Cara $\to$ koef $z^{50} \cdot 50! \to \frac{50!}{7!\,10!\,18!\,15!}$