2 2 0 6 0 2 8 9 3 2

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1. tahun 1 : K_t K_t
tahun 2 : K_B K_B
tahun 3 : K_t K_t K_S K_S
tahun 4 : K_t K_t K_t K_t K_B K_B K_S K_S
tahun 5 : K_t K_t K_t K_t K_t K_t K_S K_S K_S K_S K_B K_B K_B K_B

K_t = koki training
K_B = koki tetap
K_S = koki senior

Jumlah koki $= k_t(n) + k_B(n) + k_S(n)$

$k(1) = 2$
$k(2) = 2$
$k(3) = 2 + 2 = 4$
$k(\ ) = 2 + 2 + 4 = 8$
$k(5) = 2 + 4 + 8 = 14$
$k(n) = k(n-1) + k(n-2) + k(n-3)$

maka $k(7) = 48$

2. a. $a_n \rightarrow \langle 4, 1, -2, -5 \ldots \rangle$
cek pakai telescoping :
$a_n = 2a_{n-1} - a_{n-2} \rightarrow a_n - 2a_{n-1} + a_{n-2} = 0$

$$\begin{aligned}
a_n - 2a_{n-1} + a_{n-2} &= 0 \\
a_{n-1} - 2a_{n-2} + a_{n-3} &= 0 \\
a_{n-2} - 2a_{n-3} + a_{n-4} &= 0 \\
&\vdots \\
a_2 - 2a_1 + a_0 &= 0 \quad + \\
\hline
a_n - a_{n-1} &= a_1 - a_0 \\
&= 1 - 4 = -2
\end{aligned}$$

$$\begin{aligned}
a_n - a_{n-1} &= -3 \\
a_{n-1} - a_{n-2} &= -3 \\
&\vdots \\
a_1 - a_0 &= -3 \quad + \\
\hline
a_n - 4 &= -3 . n \\
a_n &= 4 - 3n
\end{aligned}$$

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Pakai FP

(i) kalikan dengan z^n

$a_n z^n = 2a_{n-1} z^n - a_{n-2} z^n$

(ii) jadikan notasi sigma

$$\sum_{n=2}^{\infty} a_n z^n = \sum_{n=2}^{\infty} 2a_{n-1} z^n - \sum_{n=2}^{\infty} a_{n-2} z^n$$

(iii) asumsi $G(z) = x_0 + x_1 z + x_2 z^2 \ldots$

$G(z) - a_0 - a_1 z = 2z(G(z) - a_0) - (G(z) \cdot z^2)$

(iv) ubah ke bentuk tertutup

$G(z) - 4 - z = 2z\,G(z) - 8z - z^2 G(z)$

$G(z)(z^2 - 2z + 1) = 4 - 7z$

$$G(z) = \frac{4-7z}{z^2-2z+1} = \frac{4-7z}{(z-1)^2}$$

$$= \frac{4}{(z-1)^2} - \frac{7z}{(z-1)^2}$$

$= 4(n+1) - 7n = 4 - 3n$ $\blacksquare$

(b.) $a_n = 7a_{n-1} - 5^n$, $a_0 = 8$

(i)

$a_n z^n = 7a_{n-1} z^n - 5^n z^n$

(ii) $$\sum_{n=1}^{\infty} a_n z^n = \sum_{n=1}^{\infty} 7a_{n-1} z^n - \sum_{n=1}^{\infty} 5^n z^n$$

(iii) $$G(z) - a_0 = 7z\,G(z) - \left(\frac{1}{1-5z} - 1\right)$$

$$G(z) - 8 = 7z\,G(z) - \frac{1}{1-5z} + 1$$

(iv) $$G(z) = \frac{8 - \frac{1}{1-5z} + 1}{1-7z}$$

$$= \frac{8-45z}{(1-5z)(1-7z)} = \frac{A}{1-5z} + \frac{B}{1-7z}$$

$$= \frac{5}{2(1-5z)} + \frac{11}{2(1-7z)}$$

$$= \frac{5}{2}(5^n) + \frac{11}{2}(7^n)$$

(3) (a) ya jika $a_0 = 0$ dan $a_1 = 0$ maka $a_n = 0$

(b) tidak $\rightarrow$ proof by contradiction

asumsi $a_n = 1$ untuk $n \geq 0$

maka $a_0 = 1$ dan $a_1 = 1$ dan $a_2 = 1$

$a_2 = 10a_1 - 25a_0$

$= 10 - 25 = -15$

hal ini menyalahi asumsi bahwa $a_2 = 1$ maka $a_n = 1$ tidak merupakan solusi

(c) ya, jika kita selesaikan dengan FP

(i)

$a_n = 10a_{n-1} - 25a_{n-2}$, $n \geq 2$

$a_n z^n = 10a_{n-1} z^n - 25a_{n-2} z^n$

(ii) $$\sum_{n=2}^{\infty} a_n z^n = \sum_{n=2}^{\infty} 10a_{n-1} z^n - \sum_{n=2}^{\infty} 25a_{n-2} z^n$$

(iii) $G(z) - a_1 z - a_0 = 10z(G(z) - a_0) - 25z^2 G(z)$

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(iv) $G(z)(25z^2 - 10z + 1) = z(a_1 - 10a_0) + a_0$

$G(z) = \dfrac{(a_1 - 10a_0)z + a_0}{(1-5z)^2}$

karena $\dfrac{1}{(1-u)^2} \rightarrow 1 + 2u + 3u^2 \cdots$ maka $\dfrac{1}{(1-5z)^2} \rightarrow 1 + 2.5z + 3.(5z)^2 \ldots$

$x_n = 5^n(n+1)$

maka $a_n = (a_1 - 10a_0)5^n(n) + a_0 5^n(n+1)$

solusi umum dari $a_n = 5^n(n(a_1 - 9a_0) + a_0)$
untuk sembarang a_1 dan a_0 bulat

maka dari itu (c) bisa didapat dari $a_1 = 1$, $a_0 = 0$

(d) tidak karena tidak ada nilai a_0 dan a_1 yang dapat memenuhi persamaan

$a_n = 5^n(n(a_1 - 9a_0) + a_0) \neq 5^n n^2$

(4) (a) $\dfrac{z}{1-2z^2} \rightarrow z(1 + 2z^2 + 4z^4 + 8z^6)$

$= z + 2z^3 + 4z^5 \ldots$

maka $3 + 3z + 3z^2 + 3z^3$
$z \quad + 2z^3 + 4z^5$

suku ke-0 s/d ke-5 $\rightarrow \langle 3, 4, 3, 5, 0, 4 \rangle$

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(b) $\dfrac{4z^2 + z - 1}{(1-z)^2} = \dfrac{4z^2}{(1-z)^2} + \dfrac{z}{(1-z)^2} - \dfrac{1}{(1-z)^2}$

$\rightarrow \dfrac{4z^2}{(1-z)^2} \Rightarrow 4z^2(1 + 2z + 3z^2 + \ldots)$

$= 4z^2 + 8z^3 + 12z^4 + 16z^5$

$\rightarrow \dfrac{z}{(1-z)^2} \Rightarrow z(1 + 2z + 3z^2 + \ldots)$

$= z + 2z^2 + 3z^3 + 4z^4 + 5z^5$

$\rightarrow -\dfrac{1}{(1-z)^2} \Rightarrow -1(1 + 2z + 3z^2 + \ldots)$

$= -1 - 2z - 3z^2 - 4z^3 - 5z^4 - 6z^5$

maka suku ke-0 s/d suku ke-5 $= \langle -1, -1, 3, 7, 11, 15 \rangle$

(c) $-2z^2 - z + 1 = (1-2z)(z+1)$

$\dfrac{3z^2 + 7z - 2}{(1-2z)(z+1)} = \dfrac{3z^2}{(1-2z)(z+1)} + \dfrac{7z}{(1-2z)(z+1)} - \dfrac{2}{(1-2z)(z+1)}$

$\dfrac{2}{(1-2z)(z+1)} = \dfrac{-4}{3(1-2z)} - \dfrac{2}{3(z+1)}$

$\rightarrow -\frac{4}{3}(1 + 2z + 4z^2 + 8z^3 + 16z^4 + 32z^5 + \ldots)$

$-\frac{2}{3}(1 - z + z^2 - z^3 + z^4 - z^5 + \ldots)$

$= -2 + (-2)z + (-6)z^2 + (-10)z^3 + (-22)z^4 + (-42)z^5 + \ldots$

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$$\frac{7z}{(1-2z)(z+1)} = -\frac{7}{3(z+1)} + \frac{7}{3(1-2z)}$$

$$\rightarrow \frac{7}{3}(1 + 2z + 4z^2 + 8z^3 + 16z^4 + 32z^5 + \dots)$$

$$\rightarrow -\frac{7}{3}(1 - z + z^2 - z^3 + z^4 - z^5 + \dots)$$

$$= 0 + 7z + 7z^2 + 21z^3 + 35z^4 + 77z^5 + \dots$$

$$\rightarrow z\left(\frac{3z}{(1-2z)(z+1)}\right) = \frac{1}{1-2z} - \frac{1}{z+1}$$

$$\rightarrow (1 + 2z + 4z^2 + 8z^3 + 16z^4 + 32z^5 + \dots)$$

$$\rightarrow (1 - z + z^2 - z^3 + z^4 - z^5 + \dots)$$

$$= z(2 + z + 5z^2 + 7z^3 + 17z^4 + 31z^5 + \dots$$

$$= 2z + z^2 + 5z^3 + 7z^4 + 17z^5 + \dots$$

maka suku ke-0 s/d suku ke5 = $\langle -2, 7, 2, 16, 20, 52 \rangle$

5. $L_n \rightarrow \langle 3, 4, 5, 6 \dots \rangle$

$$L(z) = \frac{\frac{1}{(1-z)^2} - (1+2z)}{z^2}$$

$$M(z) = z\left(\frac{1}{1-3z}\right)$$

$$K(z) = L(z)\cdot M(z) = \frac{\frac{1}{(1-z)^2} - (1+2z)}{z(1-3z)}$$

$$(1-3z)K(z) = \frac{1}{z^3-2z^2+z} - \frac{1+2z}{z} = \frac{3z-2z^2}{z^2-2z+1}$$

$$K(z) = \frac{3z-2z^2}{(z-1)^2(1-3z)}$$

6. (a) $\frac{2}{1+z} = G(z)$ $\blacksquare$

lookup table

(b) $\langle 0, 2, 5, 9, 14, 20, 27 \dots \rangle = \langle 0, 2, 3, 4, 5 \dots \rangle + \langle 0, 0, 2, 5, 9, 14 \dots \rangle$

$$G(z) = \frac{1}{(1-z)^2} - 1 + G(z).z$$

$$G(z)(1-z) = \frac{1-(1-z)^2}{(1-z)^2}$$

$$G(z) = \frac{(1+1-z)(1-1+z)}{(1-z)^3} = \frac{(2-z)(z)}{(1-z)^3}$$

$$\therefore G(z) = \frac{2z-z^2}{(1-z)^3} \quad \blacksquare$$

(c) Pakai hint

$\langle 4, 5, 9, 27, 123, \dots \rangle = \langle 3, 3, 3, 3, 3, \dots \rangle + \langle 1, 2, 6, 24 \dots \rangle$

$$= \frac{3}{1-z} + \sum_{n=0}^{\infty} n!\,z^n - 1$$

$$G(z) = \frac{3}{1-z} + \frac{e^{-\frac{1}{z}}E_1(-\frac{1}{z})}{z} - 1 \quad \blacksquare$$

7. (a) sifat $\Sigma\, a_n + b_n = \Sigma a_n + \Sigma b_n$

$$G(z) = \sum_{n=1}^{\infty} 2^n z^n + z^n = \sum_{n=1}^{\infty} 2^n z^n + \sum_{n=1}^{\infty} z^n$$

$$G(z) = \frac{1}{1-2z} + \frac{1}{1-z} \quad \blacksquare$$

(b) $$\sum_{n=1}^{\infty} \frac{nz^n - z^n}{n} = \sum_{n=1}^{\infty} z^n - \sum_{n=1}^{\infty} \frac{z^n}{n} = G(z)$$

$$= \frac{1}{1-z} - 1 - \left(\sum_{n=1}^{\infty} \frac{z^n}{n}\right)$$

$z + \frac{z^2}{2} + \frac{z^3}{3} + \dots$ $\quad 1 + z + z^2 + z^3 + \dots$

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$$\rightarrow \sum_{n=1}^{\infty} \frac{z^n}{n} = \int \sum_{n=1}^{\infty} z^{(n-1)} \, dz$$

$$= \int \frac{1}{1-z} \, dz$$

$$= -\ln|1-z|$$

maka $G(z) = \frac{1}{1-z} - 1 + \ln|1-z|$ $\blacksquare$

(8) $\frac{1+3z-z^2}{(1-z)(1-2z)(1+z)} = \frac{A}{1-z} + \frac{B}{1-2z} + \frac{C}{1+z}$

$\rightarrow 1-z \rightarrow z=1$

$\frac{1+3-1}{-2} = A \rightarrow A = -\frac{3}{2}$

$\rightarrow 1-2z \rightarrow z=\frac{1}{2}$

$\frac{1+\frac{3}{2}-\frac{1}{4}}{1-\frac{1}{4}} = B \rightarrow B = 3$

$\rightarrow 1+z \rightarrow z=-1$

$\frac{1-3-1}{2 \cdot 3} = C \rightarrow C = -\frac{1}{2}$

$$\frac{1+3z-z^2}{(1-z)(1-2z)(1+z)} = \frac{3}{1-2z} - \frac{3}{2(1-z)} - \frac{1}{2(1+z)}$$

$$X_n = 3(2^n) - \frac{3}{2} - \frac{1}{2}(-1)^n \quad \blacksquare$$

22 06 02 8 9 3 2

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(9) (a) Dek Depe $= 200 - 25 - 35 = 140$

Pak Esde $= 200 - 20 - 35 = 145$

Sofita $= 200 - 20 - 25 = 155$

(b) Dek Depe $(z) = (z^{20} + z^{21} + z^{22} + \dots + z^{140})$

Pak Esde $(z) = (z^{25} + z^{26} + z^{27} + \dots + z^{145})$

Sofita $(z) = (z^{36} + z^{36} + z^{37} + \dots + z^{155})$

Cara $\rightarrow$ koef z^{200}

pakai kombinatorik $a+b+c = 200$

$a \geq 20$, $b \geq 25$, $c \geq 35$

Sisa $\rightarrow 200 - 20 - 25 - 35 = 120$

stars and bars (120 stars, 2 bars).

$\frac{122!}{2!\,120!} = \frac{122 \cdot 121}{2} = 61 \cdot 121 = 7381$ cara

(c) Vanilla $= \left(\frac{z^0}{0!} + \frac{z^1}{1!} + \frac{z^2}{2!} + \dots + \frac{z^7}{7!}\right)$

Strawberry $= \left(\frac{z^0}{0!} + \frac{z^1}{1!} + \frac{z^2}{2!} + \dots + \frac{z^{10}}{10!}\right)$

matcha $= \left(\frac{z^0}{0!} + \frac{z^1}{1!} + \frac{z^2}{2!} + \dots + \frac{z^{18}}{18!}\right)$

coklat $= \left(\frac{z^0}{0!} + \frac{z^1}{1!} + \frac{z^2}{2!} + \dots + \frac{z^{15}}{15!}\right)$

Cara $\rightarrow$ koef $z^{50} \cdot 50! \rightarrow \frac{50!}{7!\,10!\,18!\,15!}$