Relation: Part 1 - Definition

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Reference: Rosen, *Discrete Mathematics and Its Applications*, 8ed, 2019, Sec. 9.1 - 9.2

What is relation?

Intuition: relation $\rightsquigarrow$ association between several objects.

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  `divisibleBy(12,4)`, `divisibleBy(0,7)`.

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- divisibleBy between an integer and an(other) integer that divides it;
  `divisibleBy(12,4)`, `divisibleBy(0,7)`.

- Arrival between a flight, airline, origin airport, landing date, landing time, and debarking terminal;
  `Arrival(QZ 0691, Air Asia, SUB, 2020-03-20, 13:00, 2E)`,
  `Arrival(ID 6519, Batik Air, DPS, 2020-03-20, 09:30, 2E)`.

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For every $n \in \mathbb{N}$, an **n-tuple** is a sequence (or ordered list) containing n elements $a_1, a_2, \ldots, a_n$ written using the notation: $(a_1, a_2, \ldots, a_n)$

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- 3-tuples or **triples**, e.g., $(3, 3, a)$, $(0, 1, 2)$, (adila, UI, depok)

Cartesian product

Definition

The **Cartesian product** of n sets $A_1, A_2, \ldots, A_n$ is the following set

$$A_1 \times A_2 \times \ldots \times A_n = \{(c_1, c_2, \ldots, c_n) \mid c_i \in A_i \text{ for each } i = 1, 2, \ldots, n\}$$

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- $A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$
- $B \times B \times A = \{(a, a, 1), (a, a, 2), (a, a, 3), (a, b, 1), (a, b, 2), (a, b, 3), (b, a, 1), (b, a, 2), (b, a, 3), (b, b, 1), (b, b, 2), (b, b, 3)\}$

Suppose $A = \{1, 2, 3\}$, $B = \{a, b\}$, $C = \{0, 1\}$, and $D = \emptyset$. Then:

- $B \times A =$
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- $B \times C \times A =$
- $A \times B \times C \times D =$

Relation

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Let $A_1, A_2, \ldots, A_n$ be sets.

- An **n-ary relation** R over $A_1, \ldots, A_n$ is a subset of the Cartesian product $A_1 \times \ldots \times A_n$
- We write the above n-ary relation $R \subseteq A_1 \times \ldots \times A_n$.
- The sets $A_1, \ldots, A_n$ are the **domain** of R dan n is the **arity** of R.
- A tuple $(a_1, \ldots, a_n)$ is an **instance** of an n-ary relation R iff $(a_1, \ldots, a_n) \in R$.
    - For the above, we usually write $R(a_1, \ldots, a_n)$.
    - If R is binary ($n = 2$), an instance $(a, b) \in R$ is also written $a\ R\ b$.

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  Yes: If any of domain set is infinite, a subset of the Cartesian product may be infinite.

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- $n = 2$: **binary** relations, i.e., sets of pairs $\rightsquigarrow$ **focus of this chapter**.
  - A binary relation $R \subseteq A \times B$ is also called a **binary relation from A to B**.
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- General n-ary relations are used in relational databases studied in a later course.

Let A be the set of cities/town in Java headed by a major and B the set of Indonesian provinces in Java. Define the relation R such that $(a, b) \in R$ if city/town a is located in the province b.

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- (Bandung, Jawa Tengah), (Madiun, Jawa Barat), etc.

Exercise

1. Let $A = \{0, 1, 2\}$ and $B = \{a, b\}$. Give two examples of binary relation R from A ke B. How many such relations are possible?
2. Check if **all** of $(1,1), (1,2), (2,1), (1,-1), (2,2)$ are in the relations below.
   - $R_1 = \{(a, b) \in \mathbb{Z}^2 \mid a \leqslant b\}$
   - $R_2 = \{(a, b) \in \mathbb{Z}^2 \mid a > b\}$
   - $R_3 = \{(a, b) \in \mathbb{Z}^2 \mid a = b \text{ or } a = -b\}$
   - $R_4 = \{(a, b) \in \mathbb{Z}^2 \mid a = b + 1\}$
   - $R_5 = \{(a, b) \in \mathbb{Z}^2 \mid a + b \leqslant 3\}$
3. For each of the following ternary relations, write its degree and domains, and give two triples that belong to it and two that do not belong to it.
   - $R_1 = \{(a, b, c) \in \mathbb{N}^3 \mid a < b < c\}$
   - $R_2 = \{(a, b, c) \in \mathbb{Z}^3 \mid b = a + k \text{ and } c = a + 2k \text{ for some } k \in \mathbb{Z}\}$
   - $R_3 = \{(a, b, c) \in \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}^+ \mid a \equiv b \pmod{c}\}$

Let $A = \{0, 1, 2\}$ and $B = \{a, b\}$. Give two examples of binary relation R from A to B. How many such relations are possible?

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$R_5 = \{(a,b) \in \mathbb{Z}^2 \mid a + b \leqslant 3\}$ ☐ Yes ☐ No

For each of the following ternary relations, write its domains, and give two triples that belong to it and two that do not belong to it.

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