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STATPROB 3

STATPROB E - 12

Dengan ini saya menyatakan bahwa PR ini adalah hasil pekerjaan saya sendiri

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① Corengan (C) = 5
Kudapan (K) = 2
Makanan berat (M) = 3

Sample space : $C_5^{10} - C_5^7 = 231$

② Tabel $P_{XY}(x, y)$

x \ y	1	2	3	Σ_y
0	0	0	$\frac{1}{231}$	$\frac{1}{231}$
1	0	$\frac{5}{77}$	$\frac{10}{231}$	$\frac{25}{231}$
2	$\frac{10}{77}$	$\frac{20}{77}$	$\frac{10}{231}$	$\frac{100}{231}$
3	$\frac{20}{77}$	$\frac{10}{77}$	0	$\frac{30}{77}$
4	$\frac{5}{77}$	0	0	$\frac{5}{77}$
5	0	0	0	0
Σ_x	$\frac{5}{11}$	$\frac{5}{11}$	$\frac{1}{11}$	

Cara mendapatkan $P(x, y)$:

$$P_{XY}(x, y) = \begin{cases} 0 & x+y > 5 \text{ atau } x+y < 5-2 \\ \frac{5Cx\ 3Cy\ 2C(5-x-y)}{231} & \text{otherwise} \end{cases}$$

i) 0 karena jumlah yang dipilih harus 5

ii) rumus kombinasi $x = G$, $y = M$

Sisanya K

③ $\Sigma_x = F_x(x)$, $\Sigma_y = F_y(y)$

$F(2,3) = f_x(0) + F_x(1) + f_x(2)$ karena $y \leq 3$

$= \frac{6}{11}$

④ $P_{X|Y}(1,3) = \frac{P_{XY}(3,1)}{P_Y(3)} = \frac{20}{30} = \frac{2}{3}$

(a.) $Cov(X,Y) = E[XY] - E[X]E[Y]$

$E[X] = 1 \cdot P_X(1) + 2P_X(2) + 3P_X(3) + 4P_X(4) + 5P_X(5) + 0P_X(0)$

$= \frac{25}{231} + \frac{200}{231} + \frac{90}{77} + \frac{20}{77} + 0 = \frac{185}{77}$

$E[Y] = 1P_Y(1) + 2P_Y(2) + 3P_Y(3) = \frac{5}{11} + \frac{10}{11} + \frac{3}{11} = \frac{18}{11}$

$E[XY] = \sum_{i=0}^{5} \sum_{j=1}^{3} ij\, P(i,j) = \frac{40}{11}$

$cov(X,Y) = \frac{40}{11} - \frac{185}{77} \cdot \frac{18}{11} = \frac{1}{11}\left(40 - \frac{185 \cdot 18}{77}\right)$

$= \frac{1}{11}\left(40 - \frac{3330}{77}\right) = \frac{1}{11}\left(40 - 43\frac{19}{77}\right)$

$= \frac{1}{11}\left(-3\frac{19}{77}\right) = -\frac{250}{11 \cdot 77} \approx -0,295\ldots$

(2.) Independen
(a.) Jumlah tidak diketahui } Bernoulli

X = Peluang mendapatkan undian, $X \sim Ber(0,12)$

$P = 0,12$

$E[X] = P = 0,12$

Varians $= E[X^2] - E[X]^2 = p(1-p) = 0,12 \cdot 0,88 = 0,1056$

(b) Binomial $n = 3$

X = Kesuksesan setiap membeli album semua fans, $X \sim Bin(3, 0,12)$

$P(X \geq 1) = 1 - F(0) = 1 - 3C0\,(0,12)^0 (0,88)^3$

$= 0,319$

(c) $n = 50$ $X \sim Bin(50, 0,12)$

$E[X] = n \cdot p = 50 \cdot 0,12 = 6$

(d) Geometric, $X \sim Geo(0,12)$

X = Berapa kali beli album sebelum sukses

$P(X > 16 \mid X > 12) = P(X > 4)$ (memoryless Property) $= 1 - f(4) = (1 - 0,12)^4 \approx 0,5996$

(e) X = Berapa kali beli album sebelum berhasil
rata-rata → $E[X] = 1/0,12 = 8,333...$

(3) (a.) $\lambda = 576$
$t = 10 \text{ menit} = \frac{1}{144} \text{ hari}$

$\alpha = \lambda t = 4$

$X \sim Poi(4)$
$P(X \geq 2) = 1 - P(0) - P(1)$
$= 1 - \frac{e^{-4} 4^0}{0!} - \frac{e^{-4} 4}{1!} = 1 - 0,092 = 0,908$

(b.) X = penumpang check in dalam 1 minggu Nab Air
Y = penumpang check in dalam 1 minggu eastair
t = 7 hari
$Var(X) = Var(Y) = \alpha = 576.7 = 4032$

$X \sim Poi(4032)$
$Y \sim Poi(4032)$
Varians X dan Y = $Var(X) + Var(Y) = 8064$

(c.) $t = \frac{1}{144}$ hari, $\alpha = 4$

X = check in 3 maskapai dalam 10 menit
$X \sim Poi(4.3) = Poi(12)$

$P(X \geq 2) = 1 - P(0) - P(1) = 1 - e^{-12} \cdot \frac{1}{1} - e^{-12} \cdot \frac{12}{1} = 0,959$

(d.) $t = 15 \text{ menit} = \frac{1}{96}$ hari → $\alpha = 576 \cdot \frac{1}{96} = 6$

X = Jumlah penumpang check in dalam 15 menit
$X \sim Poi(6)$
$P(X = 15) = e^{-6} \frac{6^{15}}{15!} = 0,00089 \simeq 0,001$

(a) $\lambda = \frac{2}{5}$ orang/menit $\Rightarrow$ 2,5 menit/orang

maka penumpang selanjutnya check in 12.36.30

(c) X = rentang waktu check in, $\lambda = \frac{2}{5}$ orang/menit

$X \sim Exp(0,4)$

$P(8 < X < 14) = F(14) - F(8)$

$= e^{-\frac{2}{5} \cdot 8} - e^{-\frac{2}{5} \cdot 14}$

$\approx 0,037$

(g) $P(X > 23 \mid X > 18) = P(X > 5)$ Memoryless Property

$= 1 - f(5) = e^{-\frac{2}{5} \cdot 5} \rightarrow e^{-2} \approx 0,135$

(4) Uniform distribution

(a) $Uni[0, 10]$

$E[X] = \frac{10+0}{2} = 5$ km

(b) $F(1,5) = \frac{1,5}{10} = 0,15$

(c) kemungkinan bangun hotel di jalan tidak rusak

$1 - P(8 < X < 9,5) = 1 - \frac{1,5}{10} = 0,85$