Relation: Part 2 - Representation

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Reference: Rosen, *Discrete Mathematics and Its Applications*, 8ed, 2019, Sec. 9.3

Representation of relation

Different ways relations are represented:

- The standard mathematical notation (already discussed earlier).
- Table/tabular form, i.e., list of tuples.
- Matrix (only for binary relations)
- Graph (only for binary relations)

Tabular representation

- Table = list of tuples in the relation.
- Number of columns = arity of relation.
- Columns may have names (as commonly seen in relational databases).
- Infinite relation $\rightsquigarrow$ infinite table. Practical table is normally finite.

$R_1 = \{(a,b) \in \mathbb{N}^2 \mid a + b = 5\}$

R_1

a	b
0	5
1	4
2	3
3	2
4	1
5	0

$R_2 = \{(a,b,c) \in (\mathbb{Z}^+)^3 \mid a^2 + b^2 = c^2\}$

R_2

a	b	c
3	4	5
5	12	13
6	8	10
7	24	25
8	15	17
$\vdots$	$\vdots$	$\vdots$

List, using a table, all 4-tuples in the relation $\{(a, b, c, d) \in (\mathbb{Z}^+)^4 \mid abcd = 6\}$.

Matrix representation

Let $A = \{a_1, a_2, \ldots, a_m\}$ and $B = \{b_1, b_2, \ldots, b_n\}$ be two finite sets.

- We pick a particular ordering of the elements of A and B, and consistently stick to it, say $a_1, \ldots, a_m$ for elements of A and $b_1, \ldots, b_n$ for elements of B.
- Let $R \subseteq A \times B$ be a binary relation over A, B.
- R can be represented by a **zero-one matrix** $\mathbf{M}_R$ where
  - Element $m_{i,j}$ at the position (i, j) satisfies $m_{i,j} = 1$ iff $(a_i, b_j) \in R$.
  - Also, $m_{i,j} = 0$ iff $(a_i, b_j) \notin R$.
- If $A = B$, then by convention, we use the same ordering of elements for A and B.

Let $A = \{4, 5, 6\}$, $B = \{4, 5\}$, and $R = \{(a, b) \in A \times B \mid a > b\}$.
Give a matrix representation for R where the integers are listed in increasing order.

$$\begin{array}{cc}
\begin{array}{cc} b_1 = 4 & b_2 = 5 \end{array} & \\
\mathbf{M}_R = \begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix} & \begin{array}{l} a_1 = 4 \\ a_2 = 5 \\ a_3 = 6 \end{array}
\end{array}$$

Let $A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2, b_3, b_4, b_5\}$ with the matrix $\mathbf{M}_R$ below representing $R \subseteq A \times B$. What is R?

$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

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$$R = \{(a_1, b_2), (a_2, b_1), (a_2, b_3), (a_2, b_4),$$

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Give a matrix representation of the relation R on $\{1, 2, 3, 4\}$ where $R = \{(1, 2), (2, 1), (2, 3), (3, 3), (4, 1), (4, 3)\}$

Give the relation R on $\{1, 2, 3, 4\}$ whose matrix representation is below where the rows and columns correspond to the integers listed in increasing order.

$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Directed graph (digraph) representation

- Digraph consists of **nodes/vertices** and **edges/arrows**, which connects one node to another (hence can only represent binary relation).
    - An edge may connect one node to itself: it is called a **loop**.
- Given binary relation $R \subseteq A \times B$, the graph representation is constructed as follows:
    - The nodes are all unique elements of $A \cup B$.
    - Construct an edge from node a to node b iff $(a, b) \in R$.

Let $A = \{4, 5, 6\}$, $B = \{4, 5\}$, and $R = \{(a, b) \in A \times B \mid a > b\}$. Give its graph representation.

Let $A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2, b_3, b_4, b_5\}$ with the matrix $\mathbf{M}_R$ below representing $R \subseteq A \times B$. Give the graph representation for R.

$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Give a graph representation of the relation R on $\{1, 2, 3, 4\}$ where $R = \{(1, 2), (2, 1), (2, 3), (3, 3), (4, 1), (4, 3)\}$

Give a graph representation of the relation R on $\{1, 2, 3, 4\}$ whose matrix representation is below where the rows and columns correspond to the integers listed in increasing order.

$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$