

Introduction to Algorithm

DAA Term 2 2023/2024





- Any well-defined computational procedure
- A tool for solving a well-specified computational problem
- Takes some value as input and produce some value as output
- Describes a specific computational procedure for achieving that input/output relationship

 Algorithms exist before computers, and now with computers there are even more algorithms. Algorithms lie at the heart of computing.



What are Algorithms?

- Input. An algorithm has input values from a specified set.
- *Output*. From each set of input values, an algorithm **produces output values** from a specified set. The output values are the solution to the problem.
- Definiteness. The steps of an algorithm must be defined precisely.
- *Correctness*. An algorithm should **produce the correct output values** for each set of input values.
- Finiteness. An algorithm should produce the desired output after a finite (but perhaps large)
 number of steps for any input in the set.
- Effectiveness. It must be possible to perform each step of an algorithm exactly and in a finite
 amount of time.
- *Generality*. The procedure should be applicable for all problems of the desired form, **not just** for a particular set of input values.

(Kenneth H. Rosen, 2002)





- Example: Sorting Problem
 - Input: A sequence of number $< a_1, a_2, a_3, ..., a_n >$ (the instance of the sorting problem)

Sorting Algorithm

• Output: A permutation of the input sequence: $< a'_1, a'_2, a'_3, ..., a'_n >$ such that $a'_1 \le a'_2 \le a'_3 \le \cdots$, $\le a'_n$ (the solution to the problem)

What are Algorithms?



- An algorithm can be specified:
 - in a natural language
 - in a pseudocode
 - as a computer program
 - as a hardware design
- What kind of problems are solved by algorithms?

Algorithmic Paradigm

- Divide and Conquer (Recurrence)
- Greedy approach
- Dynamic Programming
- Incremental Improvement: Network Flow
- Randomization





Correctness of an Algorithm

- An algorithm is said to be correct if for every input instance, it halts with the correct output.
 - A correct algorithm solves the given computational problem.

- An **incorrect algorithm** might **not halt at all** on some input instances, or it might halt with an **undesirable answer**.
 - Incorrect algorithms can sometime be useful when the error rate can be controlled, for example to find the large prime numbers.



Hard Problems

- Most of this course is about efficient algorithms. The usual measure of
 efficiency is speed, i.e., how long an algorithm takes to produce its result.
 There are some problems for which no efficient algorithm is known.
 - NP-Complete problem
- Although no efficient algorithm for NP-Complete has ever been found, nobody has ever proven that an efficient algorithm for one cannot exist.
 - If an efficient algorithm exists for any one of them, then efficient algorithms exist for all of them.

• Example: Traveling Salesman Problem





Why do we need to study Algorithms?

- Even if we are given superfast computer with free memory:
 - We still need to show that **our programs are** terminated and produce the right output as desired (**correct**).
 - We need to maintain/control the time and space complexity, and the efficient algorithm will help us.
- Computers may be fast, but they are not infinitely fast; and memory may be cheap, but it is not free.
 - Computing time is therefore a bounded resource, and so is space in memory. These
 resources should be used wisely, and algorithms that are efficient in terms of time or
 space will help you do so.





 Algorithms devised to solve the same problem often differ dramatically in their efficiency. These differences can be much more significant than differences due to hardware and software.

Computer A:

Executes 109 instructions/second

Insertion Sort \rightarrow 2 n^2 (worst case)

Computer B:

Executes 10⁷ instructions/second

Merge Sort \rightarrow 50 n lg n

To sort one million numbers (10^6) :

Computer A:

2(10⁶)² instructions 10⁹ instructions/second

2000 second

Computer B:

50(10⁶)lg(10⁶) instructions 10⁷ instructions/second

100 second



Algorithms and other Technologies

- Total system performance depends on choosing efficient algorithms as much as choosing fast hardware.
- Algorithms are at the core of most technologies used in contemporary computers
- With modern technology, we can accomplish some task without knowing much about algorithms.
- But with a good background of algorithms, we can do much more.
 - In case we cannot find an algorithm that suitable for our problem, by learning the techniques of algorithm design and analysis, we can develop algorithms on our own, show that they give the correct answer, and understand their efficiency.





- Lecturer Slides by Bapak L. Yohanes Stefanus
- Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein. 2009. Introduction to Algorithms, Third Edition (3rd. ed.). The MIT Press.
- Kenneth H. Rosen. 2002. Discrete Mathematics and Its Applications (5th. ed.). McGraw-Hill Higher Education.