On Meteorite Paths Through the Earth's Atmosphere

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1 Introduction

Over 1.8 billion years ago, a meteorite struck the surface of the Earth in Northern Ontario creating the Sudbury basin and depositing many of the materials that drive the city's mining-based economy today. The author's exposure to this city prompted a strong interest in meteorite impacts on Earth. Through this investigation we attempt to analyze how different properties of the meteorite that collided with what is now Sudbury influenced its kinetic energy at impact. We hope to elucidate some of the reasons for the size of the basin and the distribution of materials throughout it through

Modelling the movement of a meteorite through a fluid and its eventual impact requires consideration of a wide variety of areas of physics. These areas, which include thermal physics and fluid mechanics, have different degrees of effect on the movement of a meteorite. Each of these fields in turn can be modelled in varying degrees of complexity and accuracy. Gravitation, for example, is a key concept that must be considered in order to understand the movement of a meteorite as it nears a planet. A simple implementation may consider gravitational acceleration to be constant throughout a meteorite's trajectory whereas a more complicated one would internally represent acceleration due to gravity as a function of its distance from the Earth. By better approximating each component, simulations can be built out from broad abstractions to very accurate models.

In this investigation, a simulation is created to model the kinetic energy of the meteorite that created the Sudbury basin as it enters the Earth's atmosphere and eventually collides with its surface. The first step is creating a base simulation that incorporates only the most important variables including gravity and drag in a very general manner. The final simulation will then refine these crude approximations to produce a more accurate result. The completed simulation will be used to model the meteorite impact in Sudbury using public data. The final section of this investigation will explore what the results reveal about the nature of meteorite impacts and their effects on the Earth.

2 Background

2.1 Classification of Meteorites

A piece of space debris ranging in size from a speck of dust to a large asteroid is defined broadly as a meteoroid. This classification develops two further distinctions after a meteoroid has entered the atmosphere of a planet: meteorites and meteors. Meteors burn up in a planet's atmosphere before reaching the surface as a result of the intense thermal energy they are subjected to during their flight. A meteorite, on the other hand, survives its flight through the atmosphere and makes contact with the surface of a planet while still intact.

Further classification of meteorites is largely based on their mineralogical and petrographic characteristics. The Weisberg et al. (2006) scheme [1] groups meteorites into three distinct groups: chondrites, primitive achondrites, and achondrites. For the purposes of this study, we classify the meteorite solely based on qualitative data about is composition. Classifying the meteorite, even broadly, allows us to estimate its mass which is a necessary component of our calculations. Based on the materials the meteorite discharged around the Sudbury area, we believe the meteorite to be ferrous; that is mostly made of iron or an iron nickel alloy. We classify it as an achrondite because its lack of silicon makes it unfit for the other two categories.

2.2 Ablation and the Knudsen-Langmuir Equation

Ablation describes the process by which a meteoroid loses mass during its flight through an atmosphere. This effect is proportional to the kinetic energy transferred to the air molecules colliding with the meteorite. Ablation can have a substantial effect on the mass of a meteoroid if the atmosphere is sufficiently dense (which it is on Earth). Aeteoroids smaller than 25 m in diameter will almost completely burn up before reaching the surface of the Earth [2].

Ablation due to evaporation can be calculated with the Knudsen-Langmuir equation [3] which is analyzed here.

$$\frac{dm}{dt} = -4\pi r_o^2 P_{vap}(T_m)\sqrt{\frac{\mu}{2\pi k_B T_m}} \tag{1}$$

Equation (1) expresses the change in mass with respect to time using factors that take into account properties of the atmosphere and the meteorite. r_o refers to the outer radius of the isothermal shell (the portion of the meteorite that is being heated). During its flight through a dense atmosphere a meteorite's surface will often evaporate before its core temperature has risen by a measurable quantity. Therefore the value of r_o is dependent on temperature which influences the rate of evaporation. As a simplification, the value of r_o will be set as a constant 0.001 m (THIS NEEDS TO BE CHECKED) for this investigation.

$P_{vap}(T)$ is a factor that represents the control of the mass loss by vapour pressure. Vapour pressure refers simply to the pressure exerted on the meteorite by the gaseous particles in the atmosphere. It is more meaningful than melting and vaporization points in the rarefied upper atmosphere where the meteorite begins its flight. It can be calculated with the Clausius-Clapeyron equation [4].

$$log_{10} P_{vap}(T_m) = A - \frac{B}{T_m} \tag{2}$$

In equation (2) A and B are material dependent constants. Substituting values for an iron meteorite taken from Podolak et al [5] gives:

$$log_{10}P_{vap}(T_m) = 12.509 - \frac{20014}{T_m}$$

T_m is simply the temperature of the meteorite, and μ and k_B are both constants. They are the average mass of ablated particles and the Boltzmann constant respectively. The constant k_B was taken from the IB data booklet, and μ is $1.4 * 10^-8$ kg [3].

3 Methods

3.1 Creation of the Base Simulation

The base simulation was designed as a crude approximation of the flight of a meteorite throught an atmosphere. Namely, it assumed constant gravitational acceleration, it did not account for ablation, and it did not factor in the large quantities of thermal energy heating the meteorite. It only described the vertical position of the meteorite (z) and it's velocity along the same axis (v) accounting for atmospheric drag.

Theorem 3.1. Letting F_d be the force due to atmospheric drag on the meteorite and F_g be the force of gravity we can derive the equation for the velocity (v) of the meteorite.

$$F_d = \frac{C_d \rho v^2 A}{2} \ and \ F_g = mg$$

Let

$$k = \frac{C_d \rho A}{2}$$

We have

$$F_{net} = F_g - F_d$$

$$ma = mg - kv^2$$

Substituting $a = \frac{\Delta v}{\Delta t}$ *we have*:

$$m(\frac{\Delta v}{\Delta t}) = mg - kv^2$$
$$\frac{\Delta v}{\Delta t} = g - \frac{kv^2}{m}$$
$$\Delta v = (g - \frac{kv^2}{m})\Delta t$$

Substituting the original values for k we have

$$v = v_0 + (g - \frac{C_d \rho A}{2m} v_0^2)\Delta t \tag{3}$$

Theorem 3.1. We can now derive the equation for the vertical position of the meteorite (z).

Assuming constant acceleration we have

$$\Delta z = v\Delta t + \frac{a\Delta t^2}{2}$$

Subtituting $g - \frac{kv^2}{m}$ for a gives

$$\Delta z = v\Delta t + (\frac{g - kv^2}{2m})\Delta t^2$$

Therefore we can express z as

$$z = z_0 + v\Delta t + (\frac{g - C_d \rho A v^2}{4m})\Delta t^2 \tag{4}$$

Equations (1) and (2) were used to recalculate velocity and vertical displacement after an arbitrarily small time step (epsilon) until the meteorite made contact with the ground (i.e. its vertical displacement (z) is equal to the thickness of the atmosphere). This is shown in figure 1.

```
while z > 0:

    time += epsilon

    # update position
    z -= v*epsilon + 1/2 *(acceleration - k*v**2) * epsilon**2
    # update velocity
    v += (acceleration - k*v**2)*epsilon
```

Figure 1: Initial code for the main loop

3.2 Improving Calculations of Gravitational Acceleration

We improved the accuracy of the gravitational acceleration from the base simulation using Newton's Law of Gravitation. Instead of using a constant, approximated parameter for this value, we will calculate here it as a function of the meteorite's distance from the Earth.

$$g = \frac{GM}{r^2} \tag{5}$$

Newton's law of gravitation (equation (3)) calculates the gravitational acceleration (g) using radius (r) and two constant parameters.

Let that acceleration due to gravity at distance x be $f(x)$. We know the mass of the earth (M) is $5.972 * 10^2 4$ kg and that the radius of the Earth is $6.371 * 10^6$ meters. The distance from the meteorite to the center of the Earth is the sum of the radius of the Earth and the meteorite's distance from its surface:

$$r = 6371000 + x$$

Substituting r in equation (3) we have

$$f(x) = \frac{G(5.972 * 10^24}{(6371000 + x)^2} \quad (6)$$

Equation (6) was coded as a function as shown in Figure 2.

```
def calc_g(position: int):
    """g = GM/r^2"""
    # mass of the earth
    m_planet = 5.972 * 10**24
    # radius of the earth plus the distance
    r = (6371 * 1000) + position

    return 6.67 * 10**-11 * m_planet / r**2
```

Figure 2: Function for calculating acceleration due to gravity

3.3 Calculating Mass Loss due to Ablation

The mass of the meteorite was also recalculated at each time step to reflect the mass lost due to evaporation as a result of intense thermal reactions between the meteorite and the atmosphere. This was done using the Knudsen-Langmuir equation (equation (1)).

```
def calc_mass(temp: int, t: int):
    """Using the Knudsen-Langmuir equation to calculate mass loss due to ablation"""
    p_vap = 10**(12.509-20014/temp)
    c = math.sqrt(1.4*10**-8/2*math.pi*1.38 * 10**-23 * temp) * t

    return -4 * math.pi * 0.001**2*p_vap * c
```

Figure 3: Function to calculate mass loss due to ablation

The code shown in Figure 3 shows a functional implementation of equation (1). The temperature value used in the equation (T_m) was calculated using energy transformations.

We have

$$\Delta E = F_d z$$

where F_d *is the force of drag and* z *is the distance the meteorite has fallen. We will assume that all of this energy is being used to heat the meteorite.*

Substituting the definition for thermal energy we have

$$F_d x = mc\Delta T$$

Replacing c *with the specific heat capacity of iron and isolating* ΔT *gives*

$$\Delta T = \frac{F_d x}{0.45m}$$

$$T_m = T_0 + \frac{F_d x}{0.45m} \tag{7}$$

T_m is incremented until it reaches the melting point of iron (1538 deg C) at which point all further energy is assumed to go into melting the meteorite (This is, of course, a simplification). This increment is performed at each time step.

```
if (temp < 1538):
    temp += DRAG * DENSITY * A/2 * v**2
mass += calc_mass(temp, time)
```

Figure 4: Condition to decrease the mass by the quantity ablated

4 Results

Display tables and graphs that display your findings. You may need multiple sections.

5 Discussion

Analyze your results and address your hypothesis/ engineering goals. Also, discuss them within the context of the problem and previous research.

6 Future Work

Explain what you plan to do in the future with your project.

7 Conclusion

Provide a results-oriented summary of what your project has accomplished.

8 Acknowledgments

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References

[1] M. K. Weisberg, T. J. McCoy, A. N. Krot, et al. Systematics and evaluation of meteorite classification. *Meteorites and the early solar system II*, 19, 2006.

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[4] V. A. Bronshten. *Physics of meteoric phenomena*, volume 22. Springer Science & Business Media, 2012.

[5] M. Podolak, J. B. Pollack, and R. T. Reynolds. Interactions of planetesimals with protoplanetary atmospheres. *Icarus*, 73(1):163–179, 1988.

A Appendix Title

This is a good place to put code, raw data, extra figures, etc.