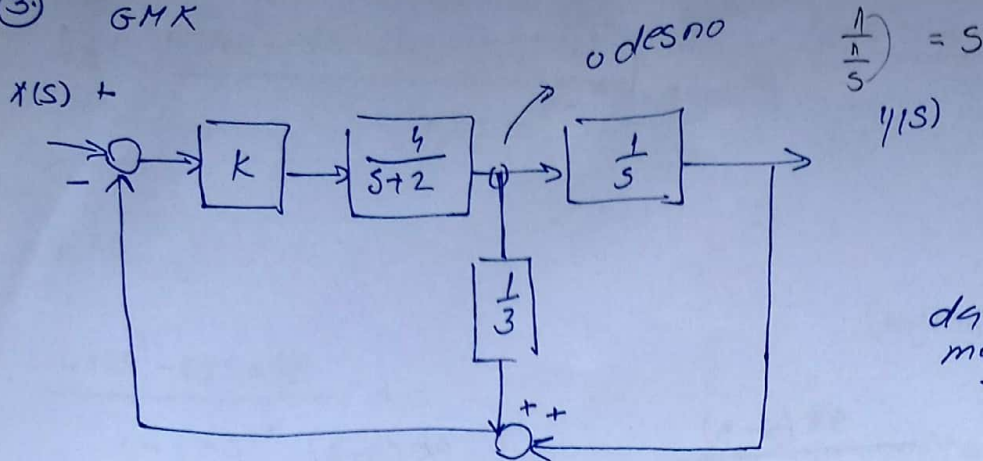
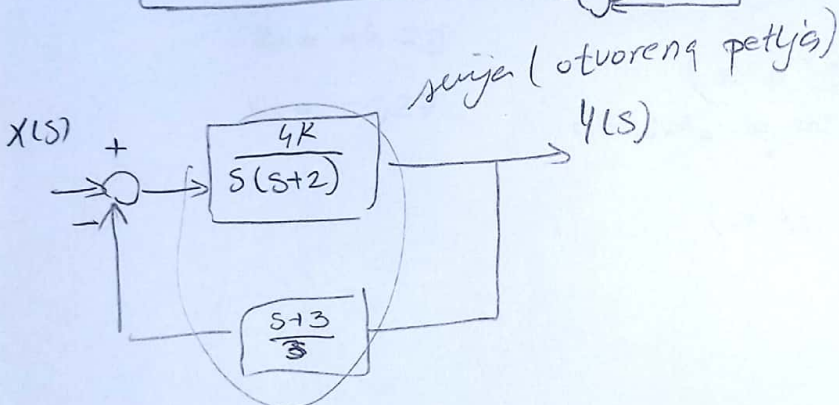
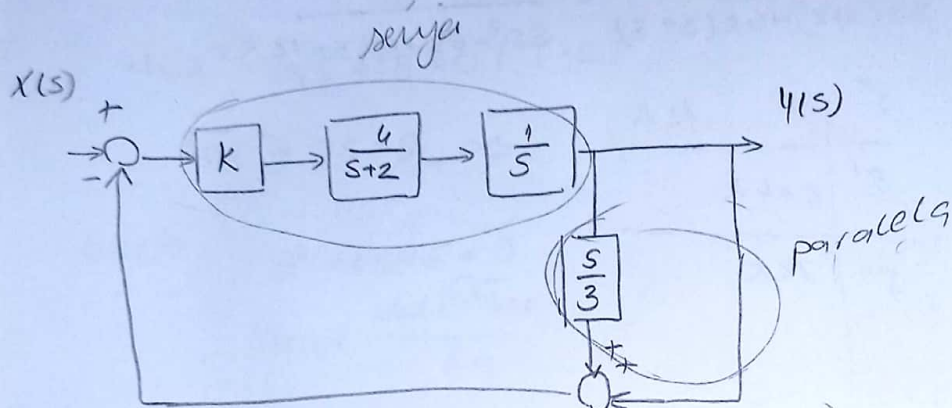


3) GPK



$$\frac{1}{\frac{1}{s}} = s$$

da bi radili GPK moramo napraviti sis da ga napravimo sa gov. delom pa smo premjestili jednu tacnu u desno



$$1 + \frac{4K}{s(s+2)} \frac{s+3}{3} = 0$$

$$1 + \frac{4K(s+3)}{3s(s+2)} = 0$$

nule

$$s = -3$$

polovi

$$s = 0$$

$$s = -2$$

asimpt

$$\sigma_a = \frac{\sum n - \sum m}{n - m} = \frac{-2 - 0 + 3}{1} = 1$$

$$\varphi = \frac{(2k+1)\pi}{n-m}$$

$$k = 1$$

$$k = 0$$

$$\varphi = \frac{\pi}{1} = \pi$$

STABILNOST (zatu. petlja)

$$G(s) = \frac{\frac{4K(s+3)}{3s(s+2)}}{1 + \frac{4K(s+3)}{3s(s+2)}} = \frac{4K(s+3)}{3s(s+2) + 4K(s+3)} = \frac{4K(s+3)}{3s^2 + 6s + 4Ks + 12K}$$

$$a_2(a_n) = 3$$

$$a_1(a_{n-1}) = 6 + 4K$$

$$a_0(a_{n-2}) = 12K$$

s^2	3	12K
s^1	6 + 4K	
s^0	12K	

$$12K > 0$$

$$K > 0$$

$$6 + 4K > 0$$

$$4K > -6$$

$$K > -\frac{3}{2}$$

$$K \in (0, \infty)$$

sys je stabilan

TAČKA GRANANJA (IZVOD OD K)

$$\frac{dk}{ds} = 0$$

$$1 + \frac{4K(s+3)}{3s(s+2)} = 0$$

$$4K(s+3) = -3s(s+2)$$

$$K = \frac{-3s(s+2)}{4(s+3)}$$

$$K' = \frac{[-3s(s+2)]' \cdot (4s+3) - [-3s(s+2)] \cdot (4s+3)'}{(4s+3)^2}$$

$$K' = \frac{(-3s^2 - 6s)' \cdot (4s+3) - (-3s^2 - 6s) \cdot (4s+3)'}{(4s+3)^2}$$

$$K' = \frac{(-6s - 6) \cdot (4s+3) - (-3s^2 - 6s) \cdot 4}{(4s+3)^2}$$

$$k' = \frac{-24s^2 - 18s - 24s + 18 - (-12s^2 - 24s)}{(4s+3)^2} = \frac{-24s^2 - 42s + 18 + 12s^2 + 24s}{(4s+3)^2}$$

$$k' = 0$$

$$\frac{-12s^2 - 18s + 18}{(4s+3)^2} = 0$$

\Rightarrow nije nešto ok

$$-12s^2 - 18s + 18 = 0 \quad | \cdot (-2)$$

$$6s^2 + 9s - 9 = 0$$

bude

$$s^2 + 6s + 6 = 0$$

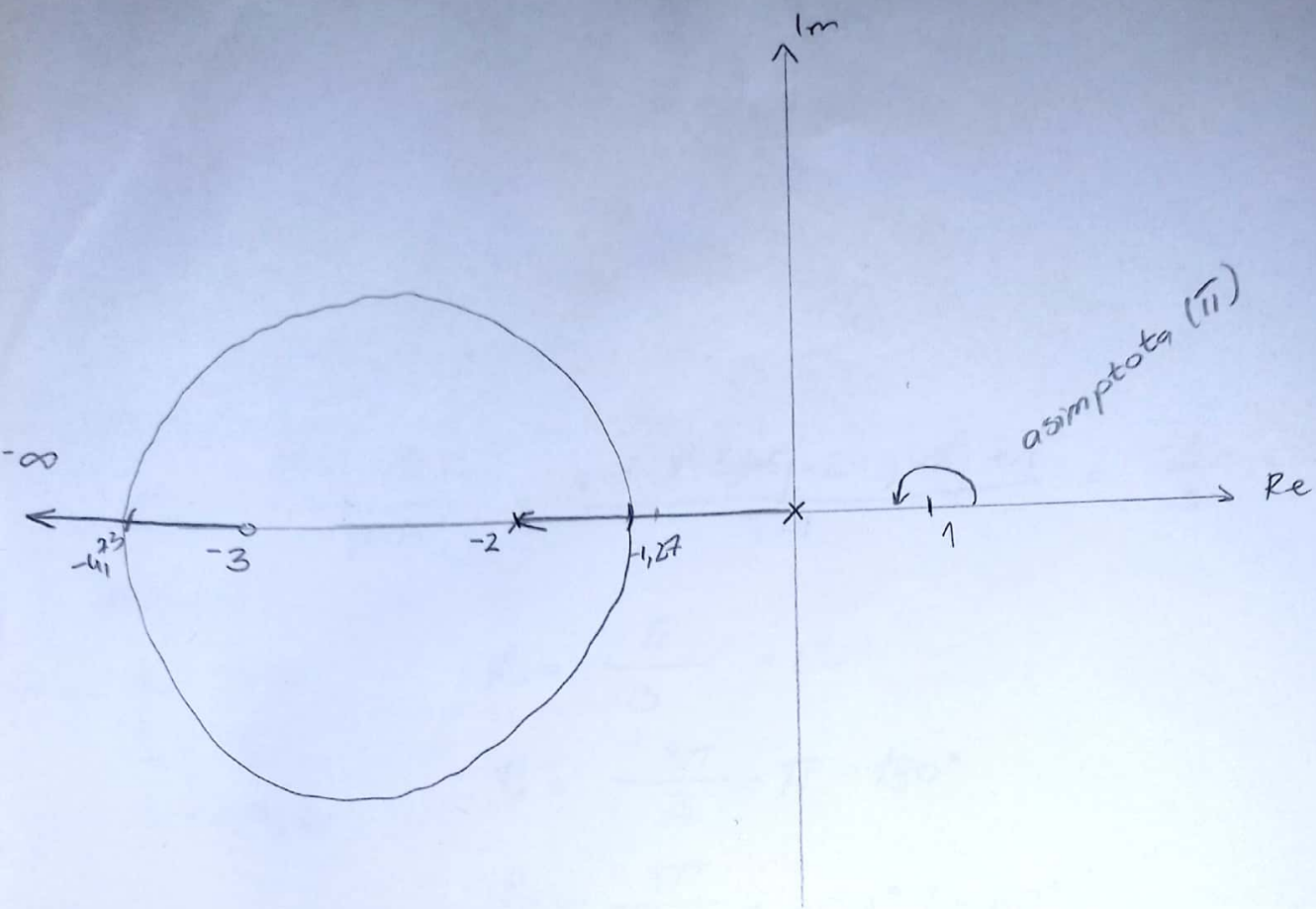
$$s_{1,2} = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1}$$

$$s_1 = -4,73$$

$$s_2 = -1,27$$

nule

polovi



$$s+4=0$$

$$9. \quad G(s) = K \frac{s+1}{s(s-1)(s^2+4s+16)}$$

$(s+1)^2$
 $s = -1$
 tu su 2 pola
 u -1 !!

$$1 + \frac{K(s+1)}{s(s-1)(s^2+4s+16)} = 0$$

isto je i sa
 nule

nule

$$s = -1$$

polovi

$$s = 0$$

$$s = 1$$

$$s^2 + 4s + 16 = 0$$

$$s_{1,2} = \frac{-4 \pm \sqrt{16 - 64}}{2}$$

$$s_{1,2} = -2 \pm 3,46j$$

asimp.

$$\sigma_a = \frac{\sum p - \sum n}{p - n} = \frac{0 + 1 - 2 - 3,46j - 2 + 3,46j + 1}{4 - 1} = \frac{-2}{3} = -0,666$$

*glori

$$k = n - p$$

$$\phi_0 = \frac{\pi}{3} = 60^\circ$$

$$n = 3$$

$$\phi_1 = \frac{3\pi}{3} = \pi = 180^\circ$$

$$k = 0, 1, 2$$

$$\phi_2 = \frac{5\pi}{3} = 300^\circ (-60^\circ)$$

* STABILNOST (odgovor petlja)

$$G_0(s) = \frac{\frac{K(s+1)}{(s-1)s(s^2+4s+16)}}{1 + \frac{K(s+1)}{s(s-1)(s^2+4s+16)}} = \frac{K(s+1)}{s(s-1)(s^2+4s+16) + K(s+1)} = \frac{K(s+1)}{s(s^3+4s^2+16s-s^2-4s-16) + Ks + K}$$

$$= \frac{K(s+1)}{s^4 + 4s^3 + 16s^2 - s^3 - 4s^2 - 16s + Ks + K} = \frac{K(s+1)}{s^4 + 3s^3 + 12s^2 + s(K-16) + K}$$

$$a_4(a_n) = 1$$

$$a_3(a_{n-1}) = +3$$

$$a_2(a_{n-2}) = 12$$

$$a_1(a_{n-3}) = K - 16$$

$$a_0(a_{n-4}) = K$$

s^4	1	12	K
s^3	+3	K-16	0
s^2	$\frac{52-K}{3}$	K	
s^1	$\frac{K^2+59K-832}{52-K}$		
s^0	K		

$$b_1 = \frac{+36 - K + 16}{-3} = \frac{52-K}{3}$$

$$c_1 = \frac{\frac{52-K}{3} \cdot (K-16) - 3K}{\frac{52-K}{3}} = \frac{52K - 832 - K^2 + 16K - 9K}{\frac{52-K}{3}} = \frac{-K^2 + 59K - 832}{52-K}$$

$$52-K > 0$$

$$-K > -52$$

$$K < 52$$

$$3^0$$

$$K > 0$$

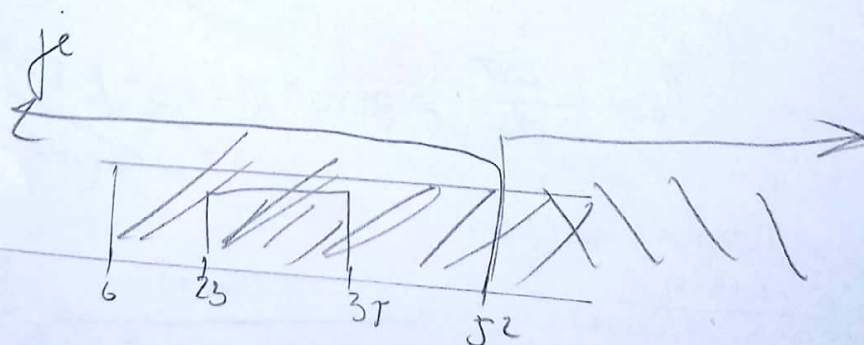
$$-K^2 + 59K - 832 = 0$$

$$K_{1,2} = \frac{-5 \pm \sqrt{b^2 - 4ac}}{2a}$$

$$K_1 =$$

$$K \in (23, 3; 35, 7) \cup (52, +\infty)$$

proszę mi



$$K \in (23, 3; 35, 7)$$

2. $11/s = \frac{s+4}{s}$
 \Rightarrow PRESJEK IM OSE

$K \in (23,3; 35,7)$

29 23,3 K de sjeći osu, a tako i 29 35,7.

to odredujemo iz:

$$b_1 s^2 + b_2 = 0$$

$$\frac{52-K}{3} s^2 + K = 0$$

1. da $K = 23,3$

$$\frac{52-23,3}{3} s^2 + 23,3 = 0$$

$$9,556 s^2 + 23,33 = 0$$

$$9,556 s^2 = -23,33$$

$$s^2 = -2,44$$

$$s_{1,2} = \pm 1,56j$$

2. da $K = 35,7$

$$\frac{52-35,7}{3} s^2 + 35,7 = 0$$

$$5,433 s^2 + 35,7 = 0$$

$$s_{1,2} = \pm 2,56j$$

\Rightarrow PRESJEK RE OSE

$$1 + \frac{K(s+1)}{s(s-1)(s^2+4s+16)} = 0$$

$$K = \frac{s(s-1)(s^2+4s+16)}{(s+1)}$$

greška $s+1$!

$$(s^2-s)(s^2+4s+16)$$

$$s^4 + 4s^3 + 16s^2 - s^3 - 4s^2 - 16s$$

$$s^4 + 3s^3 + 12s^2 - 16s$$

$$\frac{dK}{dt} = \frac{(s^4 + 3s^3 + 12s^2 - 16s)' \cdot (s+1) - (s^4 + 3s^3 + 12s^2 - 16s) \cdot (s+1)'}{(s+1)^2}$$

$$K' = \frac{(4s^3 + 9s^2 + 24s - 16)(s+1) - (s^4 + 3s^3 + 12s^2 - 16s)(s+1)'}{(s+1)^2}$$

$$K' = \frac{(4s^4 + 9s^3 + 24s^2 - 16s - 4s^4 - 3s^3 - 12s^2 - 16s)(s+1)'}{(s+1)^2}$$

$$K' = \frac{3s^4 + 2s^3 + 3s^2 - 24s + 16}{(s+1)^2}$$

nijedno nije ok
 rade

$$K' = 3s^4 + 10s^3 + 21s^2 + 24s - 16$$

$$3s^4 + 12s^3 + 24s^2 + 24s - 16 = 0$$

ali radit se kao on na yt

$$3s^4 + 10s^3 + 21s^2 + 24s - 16 = 0 \quad | \Rightarrow f(s)$$

$$s_1 = s_0 - \frac{f(s_0)}{f'(s_0)}$$

$$f'(s) = 12s^3 + 30s^2 + 42s + 24$$

$$s_0 = 0,5 \text{ (pretpostavka)}$$

$$s_1 = 0,5 - \frac{3(0,5)^4 + 10(0,5)^3 + 21(0,5)^2 + 24(0,5) - 16}{12(0,5)^3 + 30(0,5)^2 + 42(0,5) + 24}$$

$$s_1 = 0,45$$

s_1 i s_0 su razlikujući za 0,05 i jedno od ta dva je
gđ. (bilo koje)
npr. 0,5 ← dobro je gđ. za prvi dio BMK
od 0 do 1

isto radimo iz drugog dioa
od -1 do $-\infty$... ← prvo gđ.!

$$(3s^4 + 10s^3 + 21s^2 + 24s - 16) : (s - 0,5)$$

$$3s^2 + 11,5s^2 + 25,75s + 36,87 = 0$$

$$s_0 = -2$$

s_0 i $s_1 \Rightarrow$ veliko odstupanje

$$s_1 = s_0 - \frac{f(s_0)}{f'(s_0)}$$

$$s_1 = -1,86$$

s_1 i $s_2 \Rightarrow$ veliko odstupanje

$$s_2 = s_1 - \frac{f(s_1)}{f'(s_1)}$$

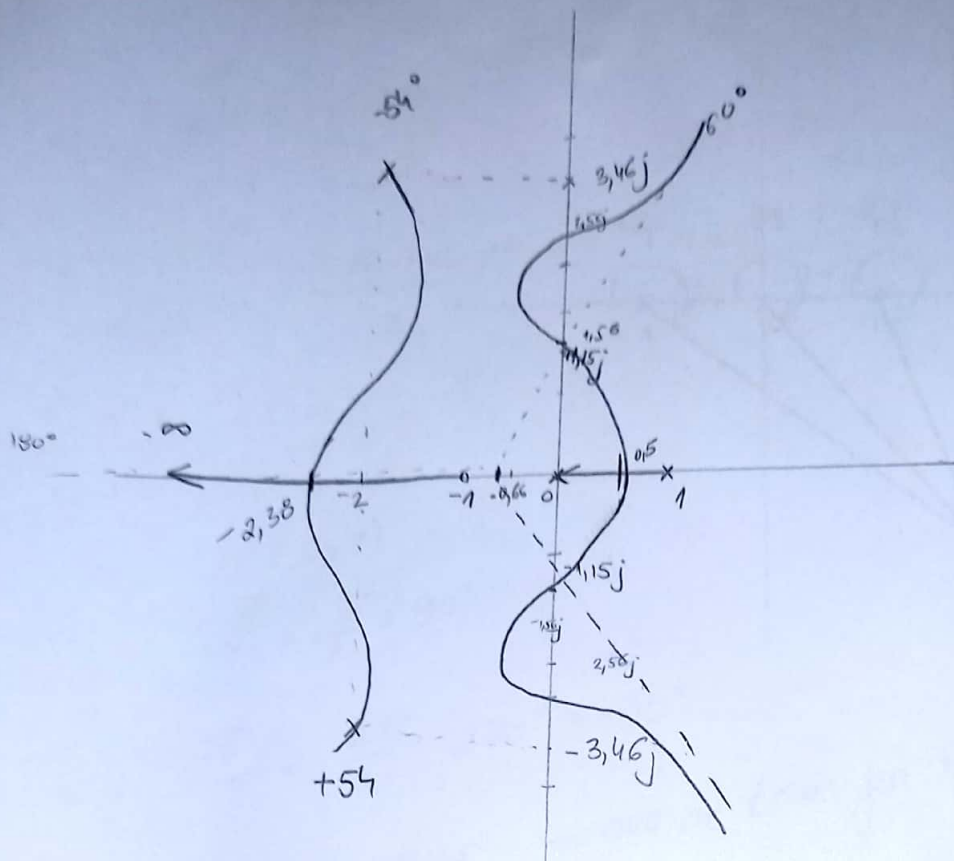
$$s_2 = -2,45$$

s_2 i $s_3 \Rightarrow$ malo odstupanje
jedno od ta dva
je gđ.

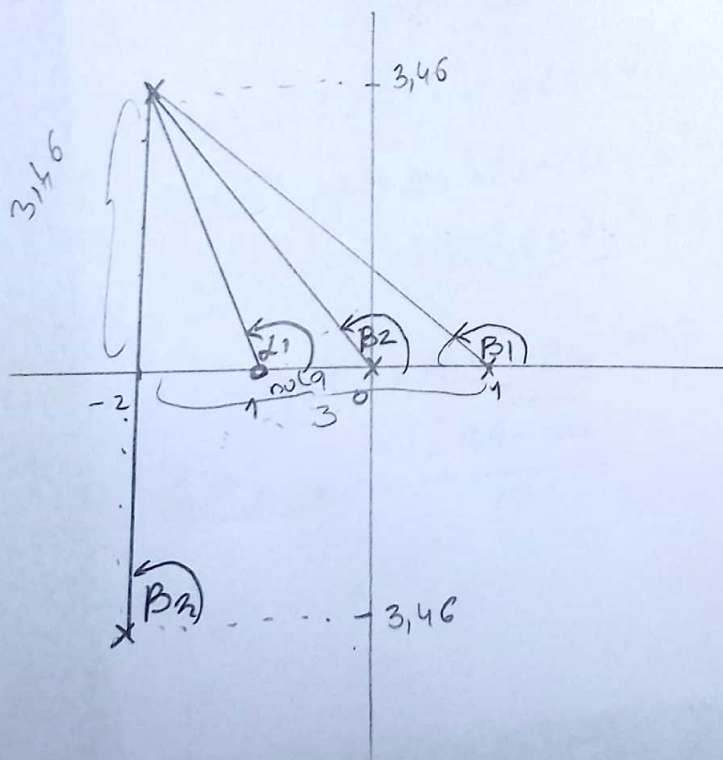
$$s_3 = s_2 - \frac{f(s_2)}{f'(s_2)}$$

$$s_3 = -2,38$$

bilo koje
ili $\frac{s_2 + s_3}{2}$



$$\tan 60^\circ = \frac{\text{suprotn.} =}{\text{na lew.}}$$



$$\varphi = \sum \beta_i - \sum \alpha_i$$

$$P = \sum \beta_i + (-90^\circ)$$

$$\beta_1 = 180^\circ - \arctan \frac{3.46}{3} = 130^\circ$$

$$\beta_2 = 180^\circ - \arctan \frac{3.46}{2} = 120^\circ$$

$$\alpha_1 = 180^\circ - \arctan \frac{3.46}{1} = 106.1^\circ$$

$$\beta_3 = 90^\circ$$

$$\varphi_1 = 180^\circ - \beta_1 - \beta_2 + \alpha_1 - \beta_3$$

$$\varphi_1 = 180^\circ - 130^\circ - 120^\circ + 106.1^\circ - 90^\circ$$

$$\varphi_1 = -54^\circ (306^\circ)$$

(2.)

$$W(s) = \frac{s+4}{s(s^2+2s+5)}$$

nule \rightarrow brojnik \circ ($1=n$)

polovi \rightarrow nazivnik \times ($p=3$)

$$s+4=0$$

$$s=-4$$

$$z_1 = -4$$

BR. GRANA

$$n=3$$

ASIMPTOTE

$$\sigma_a = \frac{\sum p - \sum z}{n-m} = \frac{-1-2-1+2+4}{2}$$

$$\sigma_a = 1$$

UGLOVI

$$k=n-m=2 \quad (0,1)$$

$$\varphi_0 = \frac{(2 \cdot 0 + 1)\pi}{2} = \frac{\pi}{2} = 90^\circ$$

$$\varphi_1 = \frac{(2 \cdot 1 + 1)\pi}{2} = \frac{3\pi}{2} = 270^\circ = -90^\circ$$

\rightarrow SPAJANJE I GRANANJE \rightarrow ovo ne treba jer imamo na Re osi 1 granu

$$\sum \frac{1}{s-p} = \sum \frac{1}{s-z}$$

$$\frac{1}{s} + \frac{1}{s^2+2s+5} = \frac{1}{s+4} \quad | \cdot s(s^2+2s+5)(s+4)$$

$$(s^2+2s+5)(s+4) + s(s+4) = s(s^2+2s+5)$$

$$s^3+4s^2+2s^2+8s+5s+20 + s^2+4s = s^3+2s^2+5s$$

$$s^3+4s^2+2s^2+13s+20 - s^3-2s^2-5s = 0$$

$$5s^2+12s+20=0$$

$$s_{1,2} = \frac{-12 \pm \sqrt{144-80}}{10} = \frac{-12 \pm 8}{10} \Rightarrow s_1 = -2$$

$$s_2 = 0,4$$

POJACANJE K

$$F(s) = 1 + K \cdot W(s)$$

$$F(s) = 1 + K \cdot \frac{s+4}{s(s^2+2s+5)} \Rightarrow F(s) = \frac{s(s^2+2s+5) + K(s+4)}{s(s^2+2s+5)}$$

$$\psi(s) = s^3 + 2s^2 + 5s + Ks + 4K$$

$$\psi(s) = s^3 + 2s^2 + s(K+5) + 4K$$

$$a_n(s^3) = 1$$

$$a_{n-1}(s^2) = 2$$

$$a_{n-2}(s^1) = K+5$$

$$a_{n-3}(s^0) = 4K$$

$$b_1 = \frac{2K+10-4K}{2} = \frac{10-2K}{2}$$

$$b_1 = 5-K$$

$$K+5 > 0$$

$$K > -5$$

$$5-K > 0$$

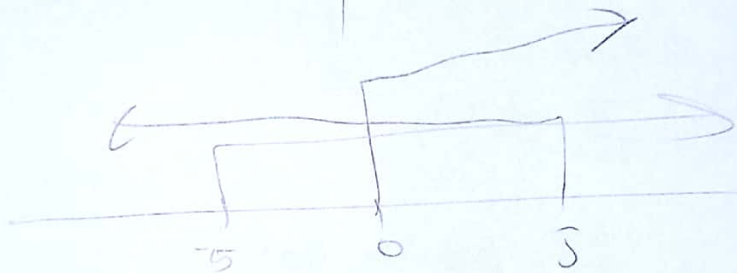
$$-K > -5$$

$$K < 5$$

$$4K > 0$$

$$K > 0$$

$$K \in (0, 5)$$

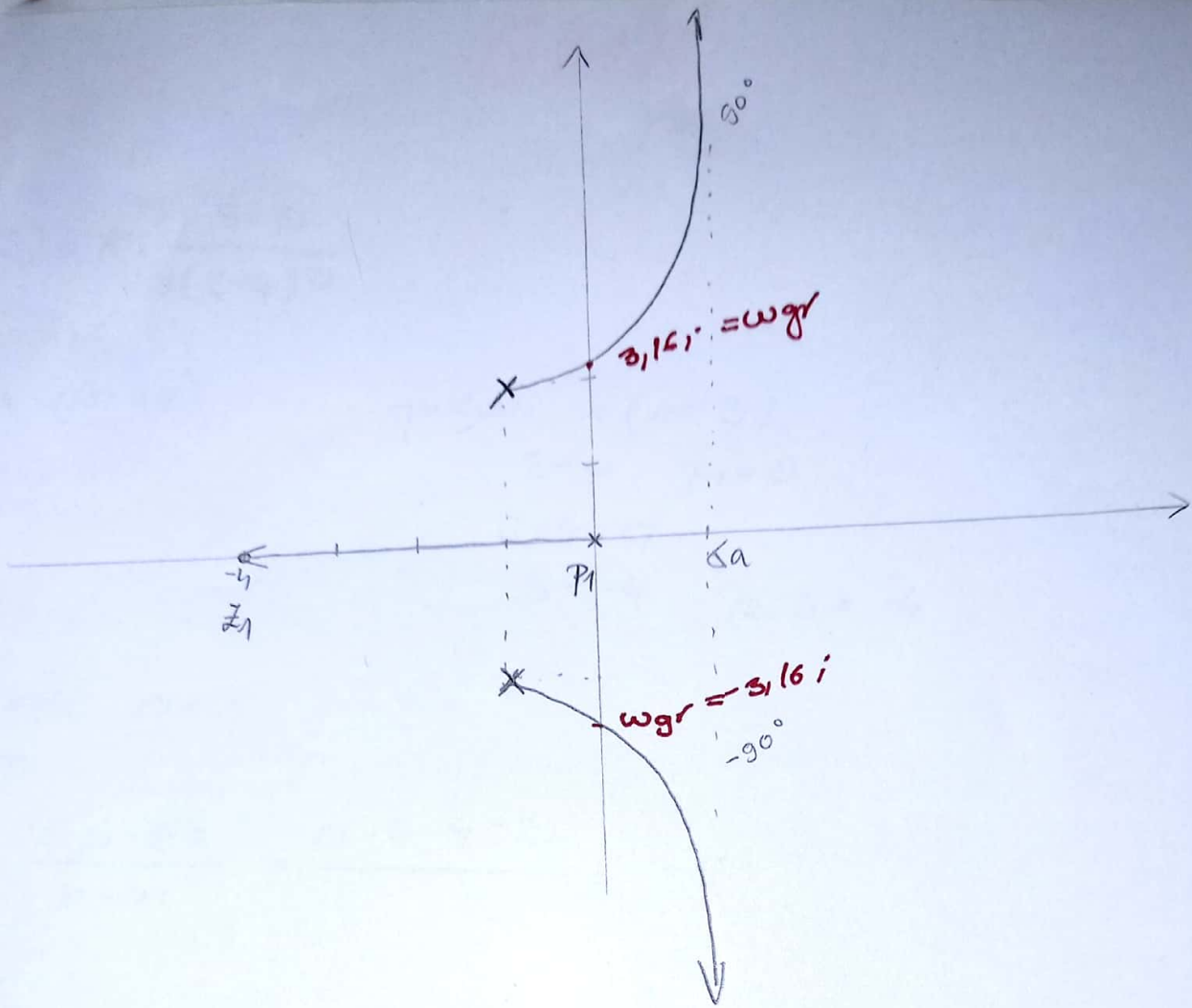


$$\omega_{gr} = \sqrt{\frac{a_0}{a_2}} = \sqrt{\frac{7}{2}} = 1,87 \quad \leftarrow \text{me treba}$$

$$2s^2 = 4K$$

$$s = \sqrt{\frac{4 \cdot 5}{2}} = \sqrt{10}$$

$$\omega_{gr} = 3,16$$



(3)

$$G(s) = K \cdot \frac{s-2}{s(s+4)^2}$$

nule o ($m=1$)

$$s-2=0$$

$$s=2$$

$$z_1=2$$

polovi x ($n=3$)

$$s=0 \quad p_1=0$$

$$s+4=0$$

$$s=-4 \quad p_{2,3}=-4$$

GRANE $n=3$

ASIMPT.

$$\sigma_a = \frac{\sum p - \sum z}{n-m} = \frac{0-4-4-2}{3} = -5$$

UGAO

$$k=n-m=2(0,1)$$

$$\varphi_0 = \frac{(20+1)\pi}{2} = 90^\circ$$

$$\varphi_1 = 270^\circ = -90^\circ$$

$$\angle \in (0,2) \cup (-4)$$

SPAJANJE I GRANANJE

$$\sum \frac{1}{s-p} = \sum \frac{1}{s-z}$$

$$\frac{1}{s} + \frac{1}{s+4} + \frac{1}{s+4} = \frac{1}{s-2} \quad | \cdot s(s+4)(s+4)(s-2)$$

$$(s+4)(s+4)(s-2) + s(s+4)(s-2) + s(s+4)(s-2) = s(s+4)(s+4)$$

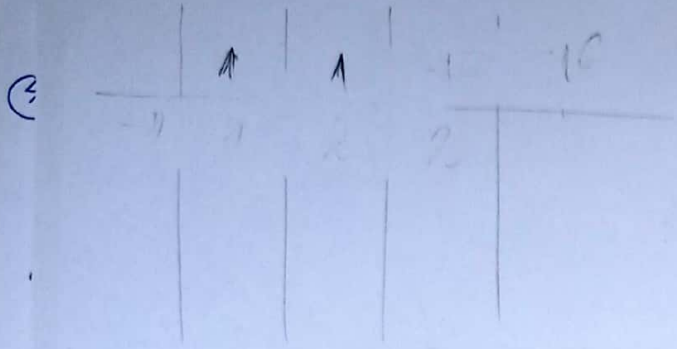
$$(s^2+8s+16)(s-2) + (s^2+4s)(s-2) + (s^2+4s)(s-2) = s(s^2+8s+16)$$

$$s^3-2s^2+8s^2-16s+16s-32 + s^3-2s^2+4s^2-8s + s^3-2s^2+4s^2-8s = s^3+8s^2+16s$$

$$3s^3+10s^2-16s-32 - s^3-8s^2-16s = 0$$

$$2s^3+2s^2-32s-32 = 0 \quad | : 2$$

$$s^3+s^2-16s-16 = 0$$



STABILITY

$$F(s) = 1 + G(s)$$

$$F(s) = 1 + \frac{K(s-2)}{s(s+4)^2}$$

$$\Rightarrow F(s) = \frac{s(s+4)^2 + K(s-2)}{s(s+4)^2}$$

$$f(s) = s(s^2 + 8s + 16) + Ks - 2K$$

$$f(s) = s^3 + 8s^2 + 16s + Ks - 2K$$

$$f(s) = s^3 + 8s^2 + s(K+16) - 2K$$

$$a_n(s^3) = 1$$

$$a_{n-1}(s^2) = 8$$

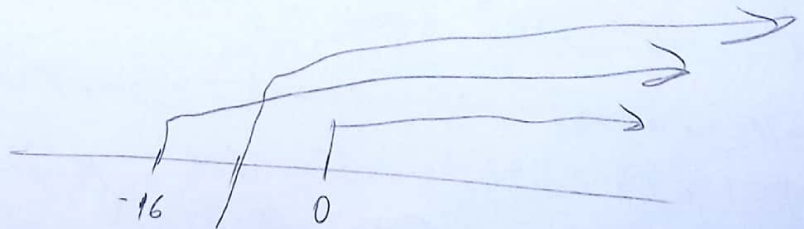
$$a_{n-2}(s) = K+16$$

$$a_{n-3}(s^0) = -2K$$

$$b_1 = \frac{8K + 128 + 2K}{8}$$

$$b_1 = \frac{128 + 10K}{8}$$

s^3	1	$K+16$	0
s^2	8	$-2K$	0
s^1	$\frac{128+10K}{8}$		
s^0	$-2K$		



$$-2K > 0$$

$$K > 0$$

$$K+16 > 0$$

$$K > -16$$

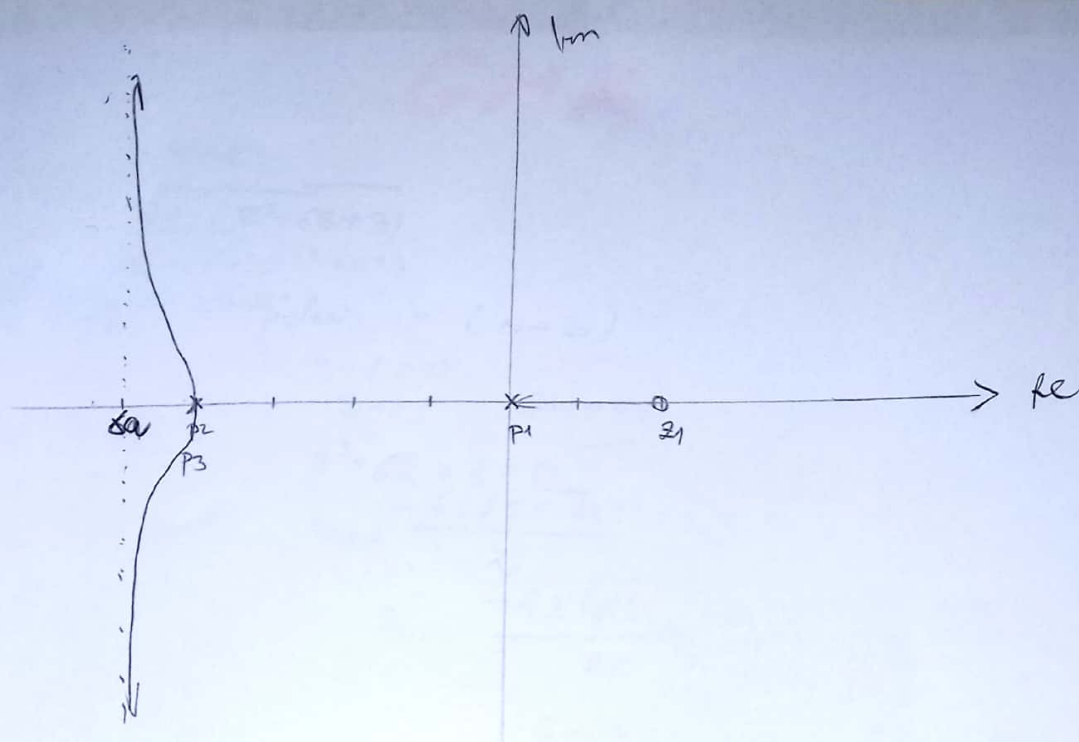
$$128 + 10K > 0$$

$$10K > -128$$

$$K > -12.8$$

$$K \in (0, +\infty)$$

5.



4.

$$W(s) = K \frac{s+2}{(s+1)(s^2+6s+3)}$$

rule 0 (m=1) $s^3 + 6s^2 + 3s + 3$ polovi x (n=3)

$$s+2=0$$

$$s=-2$$

$$z_1 = -2$$

$$s+1=0$$

$$s=-1$$

$$p_1 = -1$$

$$s^2+6s+3=0$$

$$s_{1,2} = \frac{-6 \pm \sqrt{36-12}}{2}$$

$$s_{1,2} = \frac{-6 \pm \sqrt{24}}{2}$$

$$s_1 = \frac{-6 - 2\sqrt{6}}{2} \Rightarrow p_2 = -3 - \sqrt{6} \Rightarrow p_2 = -5,45$$

$$p_3 = -3 + \sqrt{6} \Rightarrow p_3 = -0,5$$

GRANE

$$n=3$$

ASIMPTOTE

$$\sigma_a = \frac{\sum p - \sum z}{n-m} = \frac{-1 - 5,45 - 0,5 + 2}{2} = \frac{-5}{2} = -2,5$$

UGLOVI

$$k = n - m = 2 (0,1)$$

$$\varphi_0 = \frac{(2 \cdot 0 + 1)\pi}{2} = 90^\circ$$

$$\varphi_1 = \frac{(2 \cdot 1 + 1)\pi}{2} = \frac{3\pi}{2} = 270^\circ = -90^\circ$$

SPAJANJE I GRANANJE

$$\sum \frac{1}{s-p} = \sum \frac{1}{s-z}$$

$$\frac{1}{s+1} + \frac{1}{s^2+6s+3} = \frac{1}{s+2} \quad | \cdot (s+1)(s+2)(s^2+6s+3)$$

$$(s+2)(s^2+6s+3) + (s+1)(s+2) = (s+1)(s^2+6s+3)$$

$$s^3 + 6s^2 + 3s + 2s^2 + 12s + 6 + s^2 + 3s + 2 = s^3 + 6s^2 + 3s + s^2 + 6s + 3$$

$$3s^2 + 18s + 8 - 3s - s^2 - 6s - 3 = 0$$

$$2s^2 + 9s + 5 = 0$$

$$s_{1,2} = \frac{-9 \pm \sqrt{81-40}}{4} = \frac{-9 \pm 6,4}{4}$$

$$s_1 = -3,85 \notin \text{GMK}$$

$$s_2 = -0,65$$

$$\Delta v = -0,65$$

STABILITY

$$F(s) = 1 + W(s)$$

$$F(s) = 1 + \frac{k(s+2)}{(s+1)(s^2+6s+3)}$$

$$F(s) = \frac{(s+1)(s^2+6s+3) + k(s+2)}{(s+1)(s^2+6s+3)}$$

$$f(s) = (s+1)(s^2+6s+3) + ks + 2k$$

$$f(s) = s^3 + 6s^2 + 3s + s^2 + 6s + 3 + ks + 2k$$

$$f(s) = s^3 + 7s^2 + (3+k)s + 3+2k$$

$$a_n(s^3) = 1$$

$$a_{n-1}(s^2) = 7$$

$$a_{n-2}(s^1) = 3+k$$

$$a_{n-3}(s^0) = 3+2k$$

$$b_1 = \frac{63+7k-3-2k}{7}$$

$$b_1 = \frac{60+5k}{7}$$

s^3	1	$3+k$	0
s^2	7	$3+2k$	0
s^1	$\frac{60+5k}{7}$		
s^0	$3+2k$		

$$3+k > 0$$

$$k > -3$$

$$3+2k > 0$$

$$2k > -3$$

$$k > -\frac{3}{2}$$

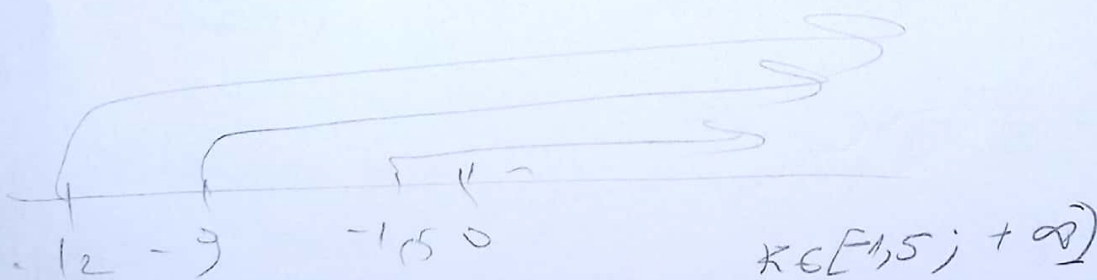
$$k > -1.5$$

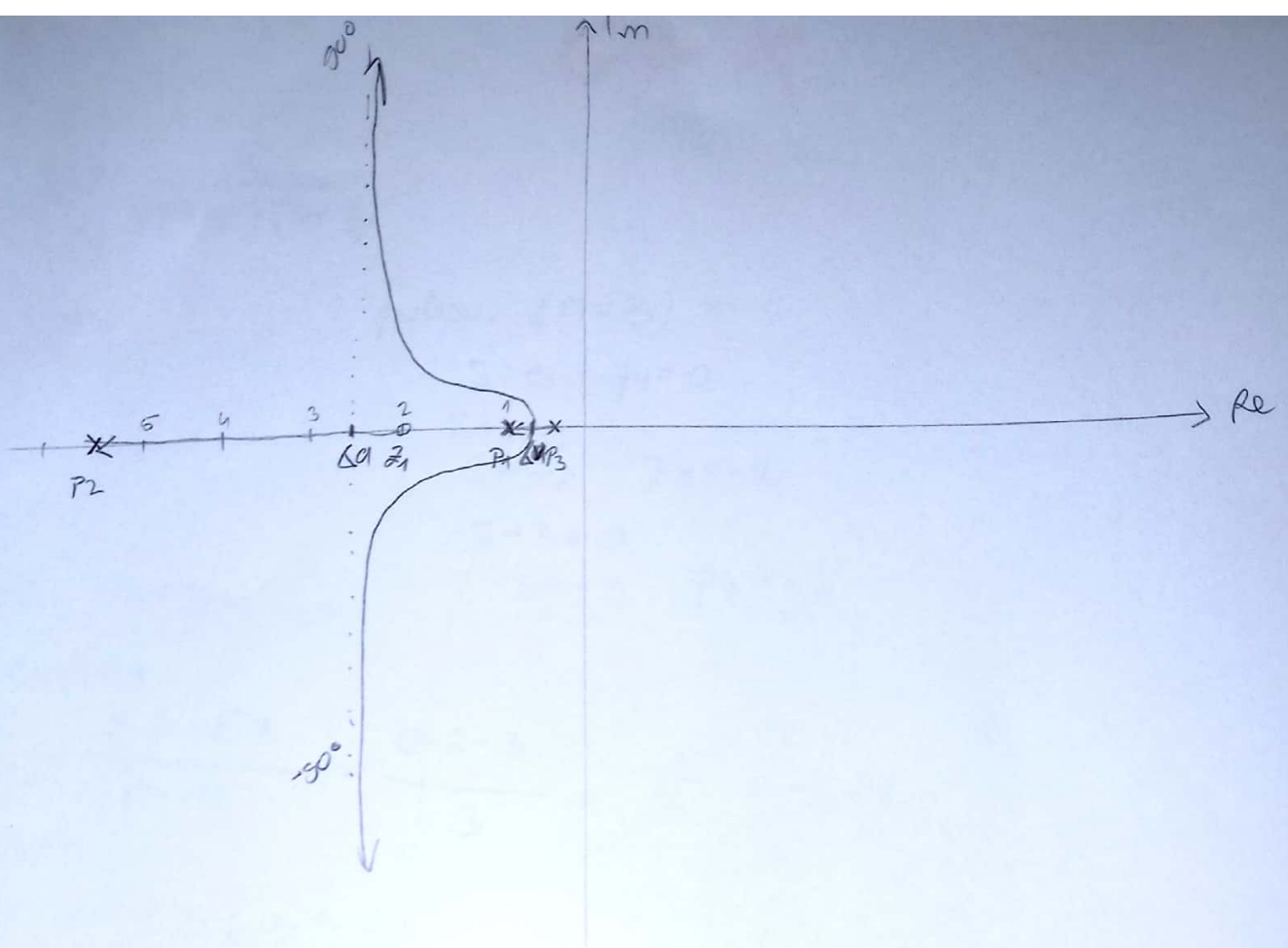
$$60+5k > 0$$

$$5k > -60$$

$$k > \frac{-60}{5}$$

$$k > -12$$





$$\frac{s+2}{(s+1)(s^2+6s+3)}$$

(5)

$$W(s) = \frac{K}{s(s+2)(s+3)}$$

nule ($m=0$)

NEHA 1H

polovi ($n=3$) x

$$s=0 \quad p_1=0$$

$$s+2=0$$

$$s=-2 \quad p_2=-2$$

$$s+3=0$$

$$s=-3 \quad p_3=-3$$

GRANJE $n=3$

ASIMPTOTE

$$\sigma_c = \frac{\sum p - \sum z}{n-m} = \frac{0-2-3}{3} = \frac{-5}{3} = -1,66$$

UGLOVI

$$k = n-m = 3 (0, 1, 2)$$

$$\varphi_0 = \frac{(2 \cdot 0 + 1)\pi}{3} = 60^\circ$$

$$\varphi_1 = \frac{(2 \cdot 1 + 1)\pi}{3} = \frac{3\pi}{3} = 180^\circ$$

$$\varphi_2 = \frac{(2 \cdot 2 + 1)\pi}{3} = \frac{5\pi}{3} = 300^\circ = -60^\circ$$

SPAJANJE I GRANANJE

$$\sum \frac{1}{s-p} = \sum \frac{1}{s-z}$$

$$\frac{1}{s} + \frac{1}{s+2} + \frac{1}{s+3} = 0 \quad | \cdot s(s+2)(s+3)$$

$$(s+2)(s+3) + s(s+3) + s(s+2) = 0$$

$$s^2 + 5s + 6 + s^2 + 3s + s^2 + 2s = 0$$

$$3s^2 + 10s + 6 = 0$$

$$s_{1,2} = \frac{-10 \pm \sqrt{100 - 72}}{6} = \frac{-10 \pm 5,3}{6}$$

$$s_1 = -2,55 \quad \text{GMK}$$

$$s_2 = -0,78 \quad \checkmark$$

$$\sigma_v = -0,78$$

STABILITY SIS.

$$F(s) = 1 + W(s)$$

$$F(s) = 1 + \frac{k}{s(s+2)(s+3)}$$

$$F(s) = \frac{s(s+2)(s+3) + k}{s(s+2)(s+3)}$$

$$f(s) = s(s^2 + 5s + 6) + k$$

$$f(s) = s^3 + 5s^2 + 6s + k$$

$$a_n(s^3) = 1$$

$$a_{n-1}(s^2) = 5$$

$$a_{n-2}(s) = 6$$

$$a_{n-3}(s^0) = k$$

$$s_1 = \frac{30-k}{5}$$

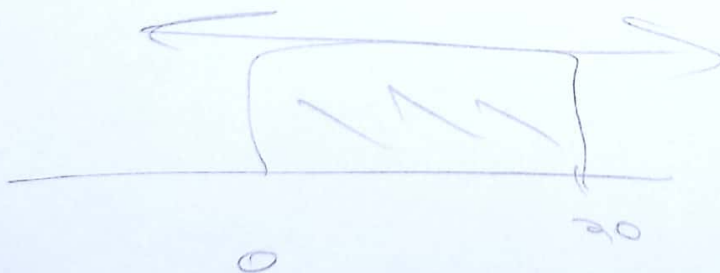
$$k > 0$$

$$30 - k > 0$$

$$-k > -30$$

$$k < 30$$

$$k \in (0, 30)$$



$$k = 30 \Rightarrow a_0 = 30$$

$$\omega_{gr} = \sqrt{\frac{a_0}{a_{n-1}}} = \sqrt{\frac{30}{5}} = \sqrt{6} = 2,45$$

