Pravila za crtanje GMK

Kapić 100/

① GMK ima n-grana; n → broj polova prenosne fun. otvorenog sistema $G_0(s)$. Svaka grana čini zaseban GMK za svoje vrijednosti $k \in [0, +\infty)$

② grane GMK počinju ($k=0$) u polovima P_j, a završavaju ($k \to \infty$) u nulama prenosne fun. $G_0(s)$; kako je $m < n$, preostalih $n-m$ nula koje nedostaju nadomještase tačkama u beskonačnosti;

③ GMK mora biti simetričan u odnosu na Re osu

④ GMK leži na Re osi ako se desno od tih tačaka Re ose nalazi neparan br. nula i polova prenosne fun. $G_0(s)$.

⑤ grana GMK za $k \to \infty$ asimptotski se približavaju pravcima čiji su uglovi nagiba:

$$\varphi_k = \frac{(2k+1)\pi}{n-m} \; ; \; k = 0, 1, \ldots, n-m-1$$

⑥ Pravci (asimptote) sijeku realnu osu u tački

$$\sigma_a = \frac{\sum_{1} pol - \sum nule}{n-m}$$

⑦ Tačka odvajanja σ_a, gdje se GMK odvaja od Re ose se dobija kao rj. jedn. po σ_a:

$$\sum_{j=1}^{n} \frac{1}{\sigma_a - P_j} - \sum_{i=1}^{m} \frac{1}{\sigma_a - N_i} = 0$$

⑧ Kada GMK napušta kompleksni pol ugao odlaska iznosi $\beta = 180° + \arg\{G_0'(s)\}$, gdje je $\{G_0'(s)\}$ → fazni pomak $G_0(s)$ bez doprinosa posmatranog pola.

(1.)

$$G_0(S) = \frac{K}{S(S+1)(S+2)}$$

1° $S=0$
$S=-1$ } polovi
$S=-2$ } $n=3$ (3 grane GMK)

2° Sve grane će se nadomjestati u tačkama $+\infty$

3° crtamo samo gornji dio GMK zbog simetrije

4° GMK leži na Re osi jer se desno od polova i nula nalazi neparan br. (3 pola)

5° asimp.

$$\sigma_a = \frac{\Sigma pol - \Sigma nule}{n-m} = \frac{0-1-2-0}{3} = -1$$

6° uglovi asimp

$k = n-m = 3$

$k = 0, 1, 2$

$$\phi_0 = \frac{(2\cdot 0+1)\pi}{3} = \frac{\pi}{3} = 60°$$

$$\phi_1 = \frac{3\pi}{3} = \pi = 90°$$

$$\phi_2 = \frac{5\pi}{3} = 300° = -60°$$

7° $$\sum_{j=1}^{n} \frac{1}{\sigma_e - P_j} - \sum_{i=1}^{m} \frac{1}{\sigma_e - N_i} = 0$$ tačka odvajanja i spajanja

$$\Rightarrow \frac{1}{\sigma_e - 0} + \frac{1}{\sigma_e(-1)} + \frac{1}{\sigma_e + 2} = 0$$

$$\frac{\sigma_e^2 + 3\sigma_e + 2 + \sigma_e^2 + 2\sigma_e + \sigma_e^2 + \sigma_e}{\sigma_e(\sigma_e+1)(\sigma_e+2)} = 0$$

$$3\sigma_e^2 + 6\sigma_e + 2 = 0$$

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$$\sigma_{c_{1,2}} = \frac{-6 \pm \sqrt{36-24}}{6}$$

$$\sigma_{c_1} = \frac{-3+\sqrt{3}}{3} \approx -0{,}4226497 \Rightarrow u\ opsegu\ (0,-1)$$

$$\sigma_{c_2} = \frac{-3-\sqrt{3}}{3} \approx -1{,}57735\ (-1,-2)\ \times$$

8° stabilnost zatvorenog sis.

$$G_z = \frac{G_o}{1+G_o}$$

$$1+G_o=0$$

$$1+\frac{K}{s(s+1)(s+2)}=0$$

$$\frac{s(s+1)(s+2)+K}{s(s+1)(s+2)}=0$$

$$s(s^2+3s+2)+K=0$$

$$s^3+3s^2+2s+K=0$$

s^3	1	2
s^2	3	K
s^1	$\frac{6-K}{3}$	
s^0	K	

$a_n=1 \quad s^3$

$a_{n-1}=3 \quad s^2$

$a_{n-2}=2 \quad s^1$

$a_{n-3}=K \quad s^0$

$$b_1 = \frac{3\cdot 2-K\cdot 1}{3} = \frac{6-K}{3}$$

$$\frac{6-K}{3}>0 \quad \wedge \quad K>0$$

$$6-K>0 \qquad K>0$$

$$-K>-6$$

$$K<6$$

$K \in (0,6)$ zatvoreni sis. je stabilan

$K=6 \Rightarrow$ kritično

Još presjek sa imag. osom:

$s=j\omega \quad \sigma=0$

$$s^3+3s^2+2s+K=0$$

$$(j\omega)^3+3(j\omega)^2+2j\omega+K=0$$

$$-j\omega^3-3\omega^2+2j\omega+K=0$$

$$K-3\omega^2+j(2\omega-\omega^3)=0$$

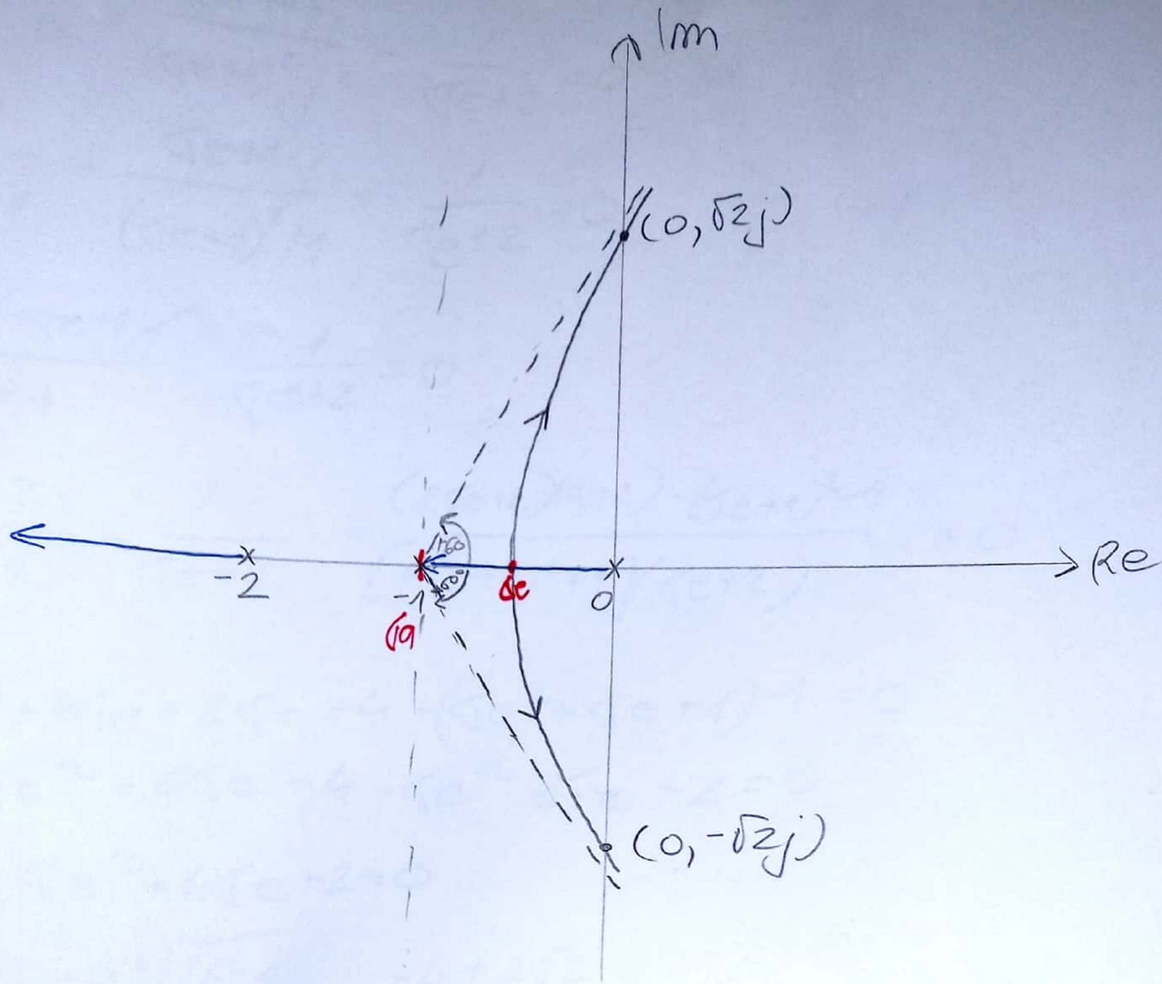
$$K-3\omega^2=0$$

$$2\omega-\omega^3=0$$

$$\omega(2-\omega^2)=0 \qquad \omega_1=0$$

$$2-\omega^2=0$$

$$2=\omega^2 \Rightarrow \omega_{1,2}=\pm\sqrt{2}$$


Im
Re
$(0, \sqrt{2}j)$
$(0, -\sqrt{2}j)$
-2
-1
0
σ_a
σ_c
60°
60°

② $G_o = \frac{K(s+2)}{s^2+2s+2}$

1° nule $\Rightarrow m=1$

$s=-2$

polovi $\Rightarrow n=2$

$s^2+2s+2=0$

$s_{1,2} = \frac{-2\pm\sqrt{4-8}}{2}$

$s_{1,2} = \frac{-2\pm 2j}{2}$

$s_{1,2} = -1\pm j$

$s_1 = -1-j$

$s_2 = -1+j$

2° 1 grana će se nadomjestiti u ∞

3° crtamo gornji dio zbog simetrije

4° GMK se nalazi na Re osi ako se desno od tih tačaka nalazi neparan br. nula i polova

5° asimptote

$$\sigma_a = \frac{\sum_{i=1} Pol - \sum nule}{n-m} = \frac{-1-j-1+j+2}{1} = 0$$

6° uglovi

$k=m-n$

$k=1$

$k=0$

$$\phi_0 = \frac{(2\cdot 0+1)\pi}{1} = \pi$$

7° $\sigma_e \Rightarrow$ tačka odvajanja

$$\sum_{j=1}^{n} \frac{1}{\sigma_e - P_j} - \sum_{i=1}^{m} \frac{1}{\sigma_e - N_i} = 0$$

$$\frac{1}{\sigma_e+1+j} + \frac{1}{\sigma_e+1-j} - \frac{1}{\sigma_e+2} = 0$$

$$\frac{1}{\sigma_e+1+j}\cdot\frac{\sigma_e+1-j}{\sigma_e+1-j} + \frac{1}{\sigma_e+1-j}\cdot\frac{\sigma_e+1+j}{\sigma_e+1+j} + \frac{1}{\sigma_e+2} = 0$$

$$\frac{\sigma_e+1-j}{(\sigma_e+1)^2-j^2}+\frac{\sigma_e+1+j}{(\sigma_e+1)^2-j^2}-\frac{1}{\sigma_e+2}=0$$

$$\frac{\sigma_e+1-j}{(\sigma_e+1)^2+1}+\frac{\sigma_e+1+j}{(\sigma_e+1)^2+1}-\frac{1}{\sigma_e+2}=0$$

$$\frac{\sigma_e+1\cancel{+j}+\sigma_e+1\cancel{-j}}{(\sigma_e+1)^2+1}-\frac{1}{\sigma_e+2}=0$$

s_1

-2

$$\frac{2\sigma_e+2}{(\sigma_e+1)^2+1}-\frac{1}{\sigma_e+2}=\frac{(2\sigma_e+2)(\sigma_e+2)-(\sigma_e+1)^2-1}{[(\sigma_e+1)^2+1](\sigma_e+2)}=0$$

$$2\sigma_e^2+4\sigma_e+2\sigma_e+4-(\sigma_e^2+\sigma_e+1)-1=0$$

$$2\sigma_e^2+6\sigma_e+4-\sigma_e^2\ \cancel{2}\sigma_e-2=0$$

$$\sigma_e^2+4\sigma_e+2=0$$

$$\sigma_{e_{1,2}}=\frac{-4\pm\sqrt{16-8}}{2}=\frac{-4\pm2\sqrt{2}}{2}$$

$$\sigma_{e_{1,2}}=-2\pm\sqrt{2}$$

$$\sigma_{e_1}=-2-\sqrt{2}=-3,4142 \quad (-3,-4)$$

$$\sigma_{e_2}=-2+\sqrt{2}=-0,5857 \quad (0,-1)$$

90°

δ° ugao odlaska iz kompl polaje:

$$\measuredangle^{\circ}=180^\circ+\arg\{G_0'(s)\}$$

$$G_0(s)=\frac{k(s+2)}{(s+1+j)(s+1-j)}$$

$(s-a)$

$(s-(-1-j))$

$(s+1+j)$ ✓

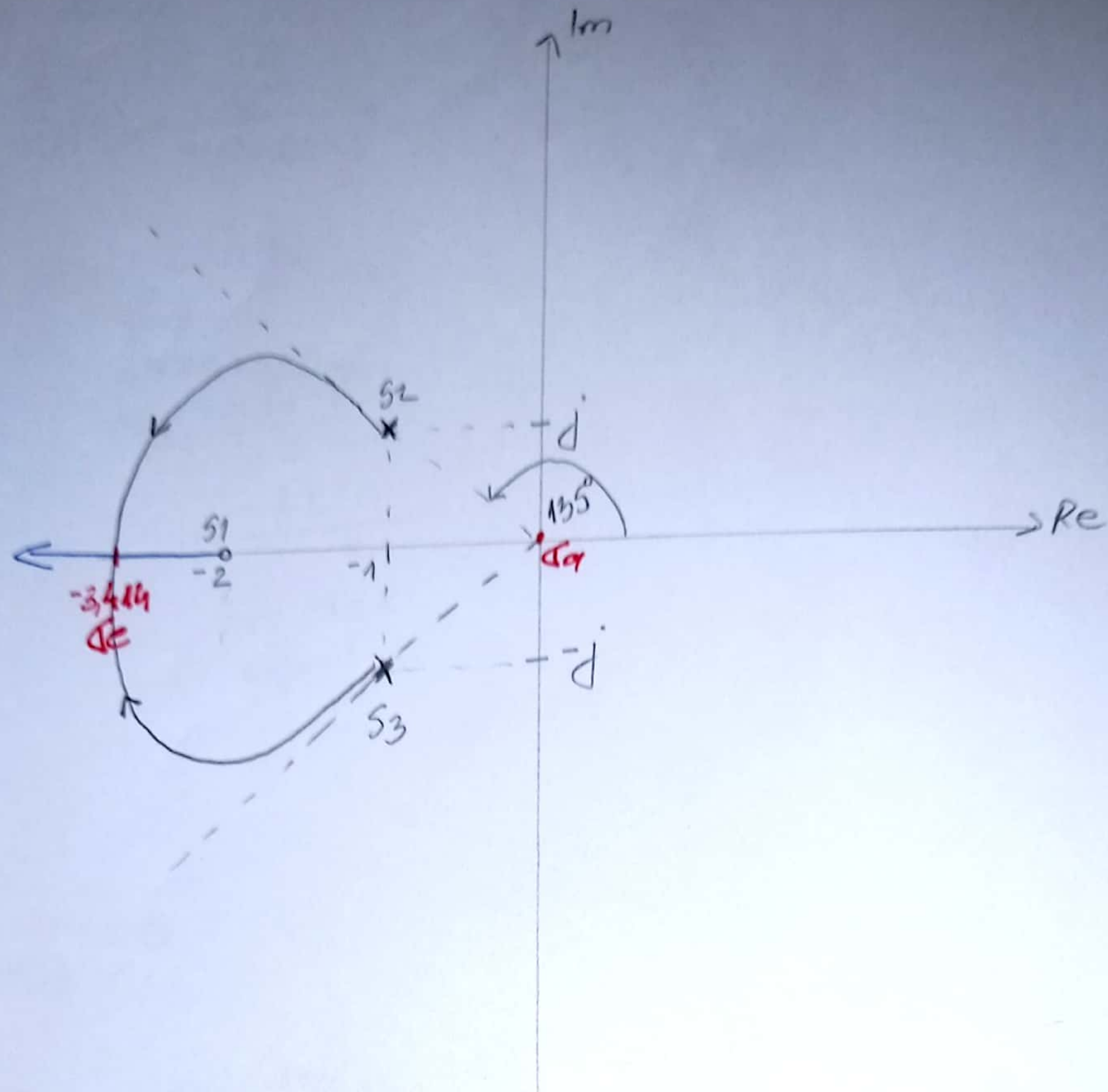
$$G_0'(s)=\frac{k(s+2)}{(s+1+j)}$$

$s=s_1=-1+j$

$$\arg\{G_0'(s)\}=\arg\{s+2\}-\arg\{s+1+j\}$$

$$=\arg\{-1+j+2\}-\arg\{-1+j+1+j\}=\arg\{1+j\}-\arg\{2j\}$$

$$=\frac{\pi}{4}-\frac{\pi}{2}=-\frac{\pi}{4}\quad(-45^\circ)$$



$$\varphi = 180° + (-45°) = 135°$$

GMK SA ROKA

$$W_p(s) = \frac{s+4}{s(s^2+2s+5)}$$

nule (1)

$s=-4$

polovi (3)

$s=0$

$s^2+2s+5=0$

$$s_{1,2}=\frac{-2\pm\sqrt{4-20}}{2}$$

$$s_{1,2}=\frac{-2\pm 4i}{2}$$

$s_1=-1-2i$

$s_2=-1+2i$

$$1+\frac{s+4}{s(s^2+2s+5)}=0$$

Asimp.

$$\sigma_a=\frac{\Sigma p-\Sigma n}{p-n}=\frac{0-1-\cancel{2i}-1+\cancel{2i}+4}{2}=1$$

Uglovi $k=0,1$

$$\varphi_0=\frac{\pi}{2}=90^\circ \quad \varphi_1=\frac{3\pi}{2}=270$$

Stabilnost

$$G(s)=\frac{\frac{s+4}{s(s^2+2s+5)}}{1+\frac{s+4}{s(s^2+2s+5)}}=\frac{s+4}{s(s^2+2s+5)+s+4}=\frac{s+4}{s^3+2s^2+5s+s+4}=\frac{s+4}{s^3+2s^2+6s+4}$$

$a_3\,(a_n)=1$

$a_2\,(a_{n-1})=2$

$a_1\,(a_{n-2})=6$

$a_0\,(a_{n-3})=4$

s^3	1	6
s^2	2	4
s^1	4	
s^0	4	

sistem je stabilan