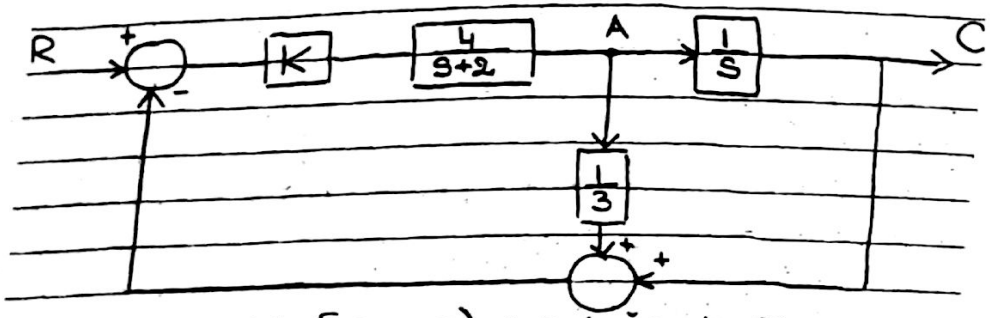
02.01.2010.

Automatsko upravljanje I

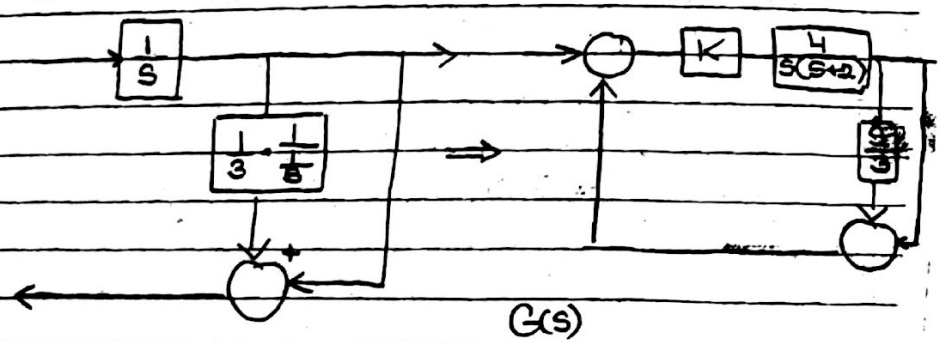
GMK ☺

* Konstruisati GMK sistema na slici:

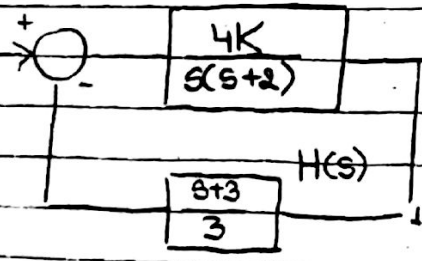


$K \in [0, +\infty)$ → pojačanje ☺

pomjeramo tačku grananja A iza bloka $\frac{1}{s}$ ☺



G(s)



1,2°

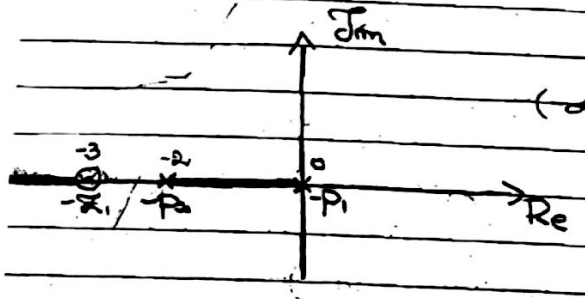
$$G(s) = \frac{4}{s(s+2)} \qquad H(s) = \frac{s+3}{3}$$

$$1 + KG(s)H(s) = 1 + \frac{4}{3}K\frac{s+3}{s(s+2)}$$

3°

$n_p = 2$, $n_z = 1$ $\quad -p_1 = 0$, $-p_2 = -2$ $\quad -z_1 = -3$

4°



$(-\infty, -3] \cup [-2, 0] \in GMK$

5°

$n_g = n_p = 2$ grane

6° GMK je simetričan u odnosu na realnu osu

7°.

$n_p = 2$, $n_z = 1$, $n_p - n_z = 1$

$$\Phi_A = \frac{2q+1}{n_p - n_z} 180° \qquad q = 0, 1 \ldots n_p - n_z - 1$$

$q = 0$

$$q = 0 \Rightarrow \Phi_A = \frac{1}{1} 180° = 180°$$

8° $1+\frac{4}{3}K\frac{s+3}{s(s+2)}=\frac{3s(s+2)+4K(s+3)}{3s(s+2)}$

$=\frac{3s^2+6s+4Ks+12K}{3s(s+2)}$

$3s^2+(6+4K)s+12K=0$

$s^2+(2+\frac{4}{3}K)s+4K=0$

s^2	1	$4K$
s^1	$2+\frac{4}{3}K$	0
s^0	$4K$	

9° tačka odvajanja (spajanja) na realnom osu

$$\sum_{i=1}^{m}\frac{1}{\delta+p_i}=\sum_{j=1}^{n}\frac{1}{\delta+z_j}$$

$\frac{1}{\delta}+\frac{1}{\delta+2}=\frac{1}{\delta+3}\ /\cdot\delta(\delta+2)(\delta+3)$

$(\delta+2)(\delta+3)+\delta(\delta+3)=\delta(\delta+2)$

$\delta^2+2\delta+3\delta+\delta^2+3\delta=\delta^2+2\delta$

$\delta^2+6\delta+6=0$

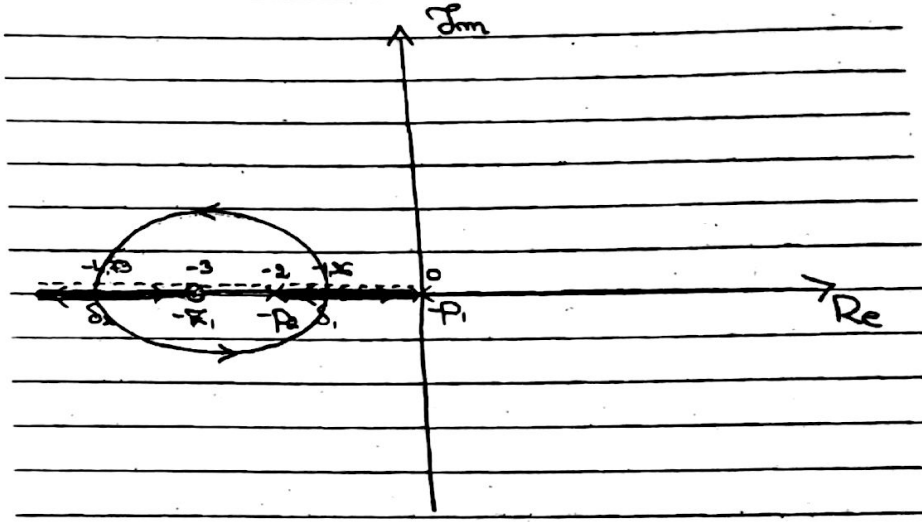
$\delta_{1,2}=\frac{-6\pm\sqrt{36-24}}{2}=\frac{-6\pm2\sqrt{3}}{2}=-3\pm\sqrt{3}$

$\delta_1=-1{,}26$ → tačka napuštanja $\Rightarrow \in$ GMK

$\delta_2=-4{,}73$ → tačka spajanja

10° nema kompleksnih polova i nula pa ne računamo

11° crtanje GMK



* Konstruisati GMK sistema čija je prenosna funkcija $G(s) = K\frac{s+1}{s(s-1)(s^2+4s+16)}$

1,2°

$$1+G(s) = 1+K\frac{s+1}{s(s-1)(s^2+4s+16)} = 0$$

3°

$n_p = 4$ $\quad -p_1 = 0$ $\quad p_{3/4} = \frac{-4\pm\sqrt{16-64}}{2}$

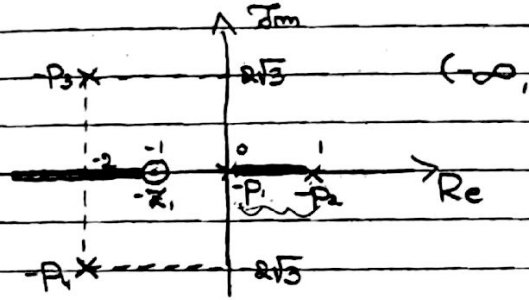
$n_z = 1$ $\quad -p_2 = 1$

$-p_3 = -2+j2\sqrt{3}$ $\quad p_{3/4} = \frac{-4\pm\sqrt{-48}}{2}$

$-z_1 = -1$ $\quad -p_4 = -2-j2\sqrt{3}$

$p_{3/4} = -2\pm j2\sqrt{3}$

4°



$(-\infty,-1]\cup[0,1] \in GMK$

5° $n_g = n_p = 4$ grane

6° GMK je simetričan u odnosu na realnu osu.

7°

$n_p = 4$

$n_z = 1$

$$\phi_A = \frac{2q+1}{n_p-n_z}180°$$

$q = 0,1,\dots n_p-n_z-1$

$q=0 \Rightarrow \phi_A = \frac{1}{3}\cdot 180° = 60°$

$q=1 \Rightarrow \phi_A = \frac{3}{3}\cdot 180° = 180°$

$q=2 \Rightarrow \phi_A = \frac{5}{3}\cdot 180° = 300°$

$$\sigma_A = \frac{\sum(-p_i)-\sum(-z_i)}{n_p-n_z} = \frac{(0+1-2-2)+1}{3} = \frac{-2}{3}$$

↳ tačka spajanja asimptotica ☺

8°

$$1+G(s) = 1+K\frac{s+1}{s(s-1)(s^2+4s+16)}$$

$$\frac{s(s-1)(s^2+4s+16)+K(s+1)}{s(s-1)(s^2+4s+16)}$$

$$s(s-1)(s^2+4s+16)+K(s+1)=0$$

$$(s^2-s)(s^2+4s+16)+Ks+K=0$$

$$s^4-s^3+4s^3-4s^2+16s^2-16s+Ks+K=0$$

$$s^4+3s^3+12s^2+(K-16)s+K=0$$

s^4	1	12	K
s^3	3	K-16	0
s^2	A_1	A_2	
s^1	B_1	B_2	
s^0	C_1		

$$A_1 = \frac{3\cdot 12-K+16}{3} = \frac{52-K}{3}$$

$$A_2 = \frac{3K-0\cdot 1}{3} = K$$

$$B_1 = \frac{\frac{52-K}{3}(K-16)-3\cdot K}{\frac{52-K}{3}}$$

$$B_2 = \frac{A_1\cdot 0-0\cdot 3}{A_1} = 0$$

$$B_1 = \frac{52K-K^2-832+16K-9K}{52-K}$$

$$C_1 = \frac{B_1\cdot A_2-B_2\cdot A_1}{B_1} = K$$

s^4	1	K
s^3	3	$K\cdot 16$
s^2	$\frac{52-K}{3}$	K
s^1	$\frac{-K^2+59K-832}{52-K}$	0
s^0	K	

→ ako nepojavi multi red 0

$52-K \neq 0$

$K \neq 52$

$-K^2+59K-832=0$

$K_{1/2} = \frac{-59 \pm \sqrt{3481-3328}}{-2}$

$K_{1/2} = \frac{-59 \pm 12,37}{-2}$

$K_1 = 35,7$ $\quad$ $K_2 = 23,3$

$\frac{52-K}{3}s^2 + K = 0$

$(52-K)s^2 + 3K = 0$

$52s^2 - Ks^2 + 3K = 0$

$s_{1/2} = \pm\sqrt{\frac{-3K}{52-K}}$

$K_1 = 35,7 \Rightarrow s_{1/2} = \pm\sqrt{\frac{-3\cdot 35,7}{52-35,7}} = \pm j2,56$

$K_2 = 23,3 \Rightarrow s_{3/4} = \pm j1,56$

9° tačka spajanja (odvajanja)

$1 + K\frac{s+1}{s(s-1)(s^2+4s+16)} = 0$

$K = -\frac{s(s-1)(s^2+4s+16)}{s+1}$

p.p. $s \in [0,1]$ $\quad s_{i+1} = s_i - \frac{f(s_i)}{f'(s_i)}$

s	0	0,2	0,4	0,6	0,8	1
K	0	2,245	3,045	2,814	1,764	0

s	0,2	0,25	0,3	0,35	0,4	0,45	0,5	0,55	0,6
K	2,245	2,669	2,793	2,953	3,045	3,063	3,042	2,9	2,814

→ max. najedmost

$s_1 = 0,45$

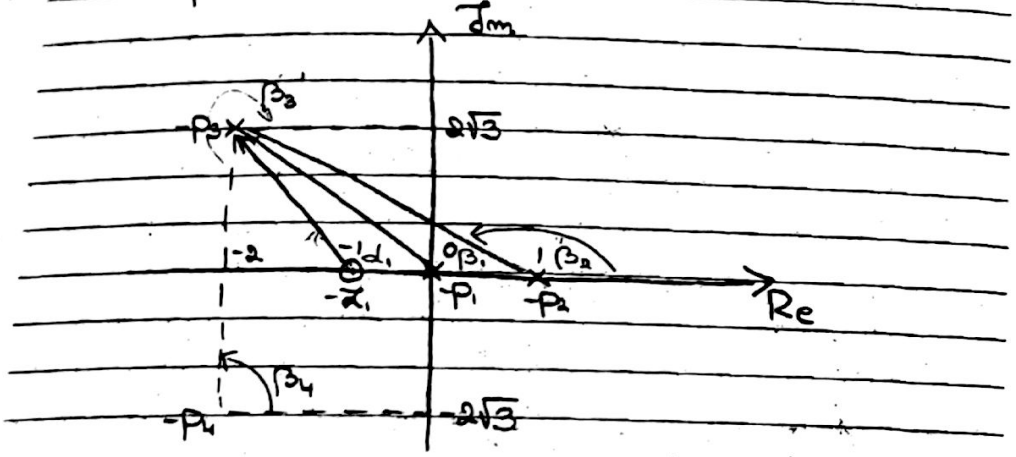
p.p. $s \in [-3,-1]$

s	-3	-2,6	-2,2	-1,8	-1,4	-1
K	78	72,306	70,625	75,185	100,38	∞

→ min. najedmost

$s_2 = -2,2$

10° uglasi napuštanja konjugovano - kompleksnih polova



$$\alpha_1 - (\beta_1 + \beta_2 + \beta_4 + \beta_3') - 180° + 360° = 0$$

$$\beta_3' = \alpha_1 - (\beta_1 + \beta_2 + \beta_4) - 180°$$

$$\beta_1 = 180° - \operatorname{arctg}\frac{2\sqrt{3}}{2} = 120°$$

$$\beta_2 = 180° - \operatorname{arctg}\frac{2\sqrt{3}}{3} = 130{,}89°$$

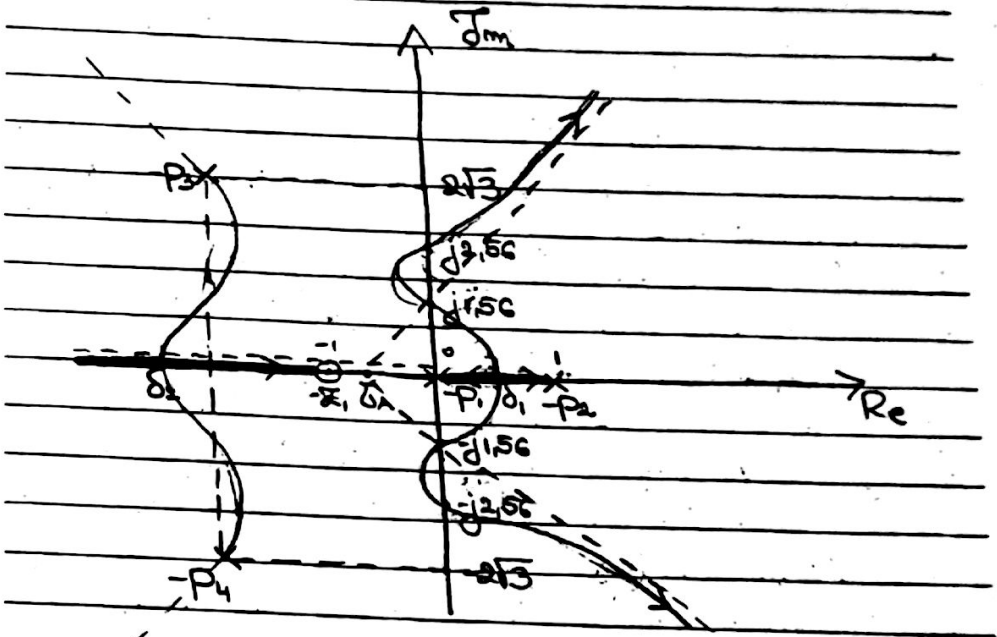
$$\alpha_1 = 180° - \operatorname{arctg}\frac{2\sqrt{3}}{1} = 106{,}1°$$

$$\beta_4 = 90°$$

$$\beta_3' = 106{,}1° - (120 + 130{,}89° + 90°) - 180° = -54{,}79°$$

$$\beta_4' = -\beta_3' = 54{,}79°$$

11° crtanje GMK



* Konstruisati GMK sljedećeg sistema:

$$G(s) = K\frac{(s+2)}{(s+1)^2}$$

1,2°

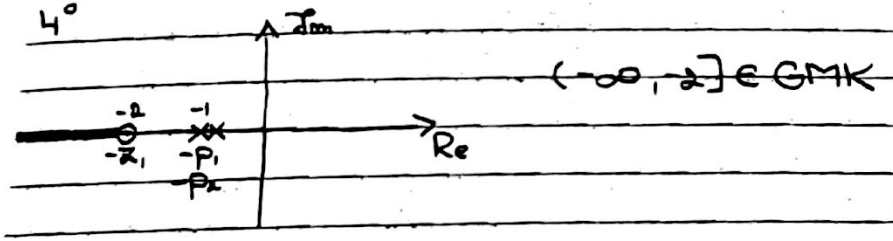
$$1 + K\frac{(s+2)}{(s+1)^2} = 0$$

3°

$n_p = 2$ $\quad -p_1 = -1$ $\quad -z_1 = -2$

$n_z = 1$ $\quad -p_2 = -1$

4°



$(-\infty, -2] \in GMK$

5° $n_g = n_p = 2$ grane

6° GMK je simetričan u odnosu na realnu osu

7°

$$\phi_A = \frac{2q+1}{n_p - n_z} \cdot 180° \qquad q = 0, 1 \ldots n_p - n_z - 1$$

$$q = 0 \Rightarrow \phi_A = \frac{1}{1} \cdot 180° = 180°$$

8°

$$1 + K\frac{(s+2)}{(s+1)(s+1)} = \frac{s^2+2s+1+Ks+2K}{(s+1)(s+1)}$$

$$s^2 + 2s + 1 + Ks + 2K = 0$$

$$s^2 + (2+K)s + 2K + 1 = 0$$

s^2	1	$2K+1$
s^1	$2+K$	0
s^0	$2K+1$	

→ GMK ne siječe imaginarnu osu

9° tačka odvajanja (spajanja)

$$\sum \frac{1}{\delta + p_i} = \sum \frac{1}{\delta + z_i}$$

$$\frac{1}{\delta+1} + \frac{1}{\delta+1} = \frac{1}{\delta+2} \quad / (\delta+1)^2(\delta+2)$$

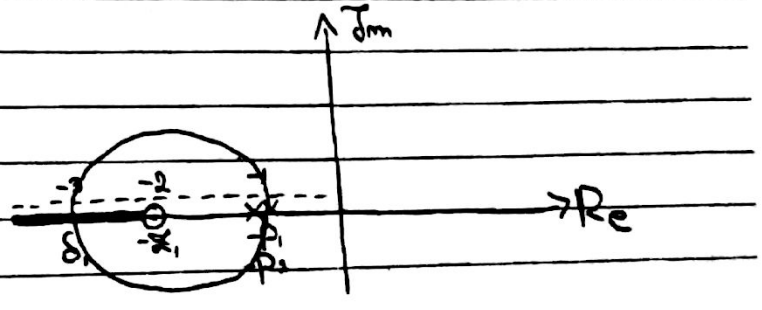
$$(\delta+1)(\delta+2) + (\delta+1)(\delta+2) = (\delta+1)^2$$

$$\delta^2 + \delta + 2\delta + 2 + \delta^2 + \delta + 2\delta + 2 = \delta^2 + 2\delta + 1$$

$$\delta^2 + 4\delta + 3 = 0$$

$$\delta_{1/2} = \frac{-4 \pm \sqrt{16 - 12}}{2} = \frac{-4 \pm 2}{2}$$

$\delta_1 = -3$ $\quad \delta_2 = -1$



* Konstruisati GMK sljedećeg sistema:

$$G(s) = \frac{24}{s(s+4)^2}$$

1,2°

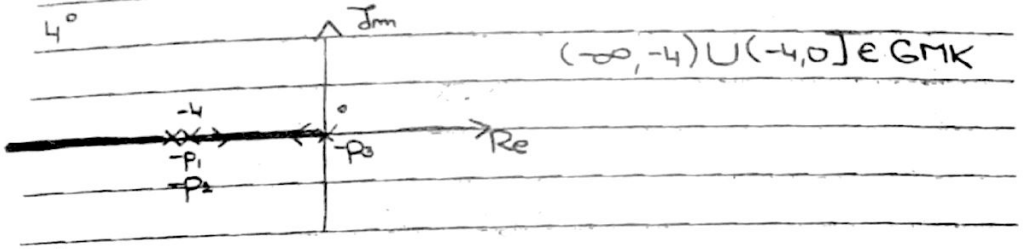
$$1 + \frac{24}{s(s+4)^2} = 0$$

3°

$n_p = 3$ $\quad -p_1 = -p_2 = -4$

$n_z = 0$ $\quad -p_3 = 0$

4°



$(-\infty, -4) \cup (-4, 0]$ ∈ GMK

5°

$n_g = n_p = 3$ grane

6° GMK je simetričan u odnosu na realnu osu.

7°

$$\phi_A = \frac{2q+1}{n_p - n_z} \cdot 180° \qquad q = 0, 1 \dots n_p - n_z - 1$$

$$q = 0, 1, 2$$

$q = 0 \Rightarrow \phi_A = \frac{1}{3} \cdot 180° = 60°$

$q = 1 \Rightarrow \phi_A = \frac{3}{3} \cdot 180° = 180°$

$q = 2 \Rightarrow \phi_A = \frac{5}{3} \cdot 180° = 300°$

$$\sigma_A = \frac{\sum(-p_i) - \sum(-z_i)}{n_p - n_z} = \frac{(-4-4-0)+0}{3} = \frac{-8}{3} = -2{,}67$$

8°

$$1 + \frac{24}{s(s+4)^2} = \frac{s(s^2+8s+16)+24}{s(s+4)^2}$$

$$s(s^2+8s+16)+24 = 0$$

$$s^3 + 8s^2 + 16s + 24 = 0$$

s^3	1	16	0
s^2	8	24	
s^1	13	0	
s^0	24		

GMK ne sječe imaginarnu osu

9° tačka spajanja (odvajanja)

$$\frac{1}{\delta} + \frac{1}{\delta+4} + \frac{1}{\delta+4} = 0 \;/\; \delta(\delta+4)^2$$

$$(\delta+4)^2 + \delta(\delta+4) + \delta(\delta+4) = 0$$

$$\delta^2 + 8\delta + 16 + \delta^2 + 4\delta + \delta^2 + 4\delta = 0$$

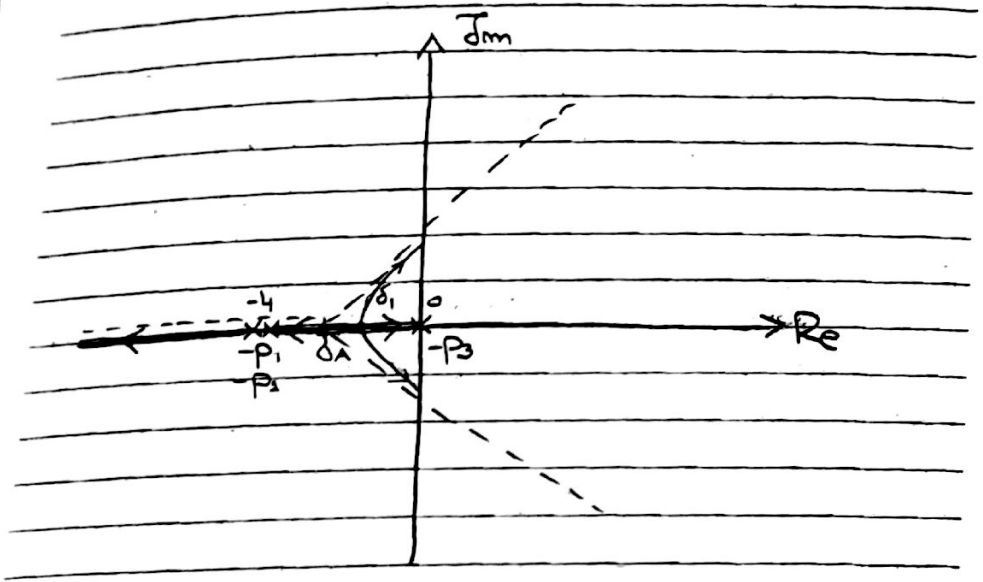
$$3\delta^2 + 16\delta + 16 = 0$$

$$\delta_{1/2} = \frac{-16 \pm \sqrt{256 - 192}}{6} = \frac{-16 \pm 8}{6}$$

$\delta_1 = -4 \qquad \delta_2 = -1{,}33$

10° nema konjugovano - kompleksnih polova pa me

11° ostanje GMK



* Konstruisati GMK sistema sa prenosnom funkcijom $G(s) = \frac{K}{(s+0,5)(s-1,5)}$

1,2°

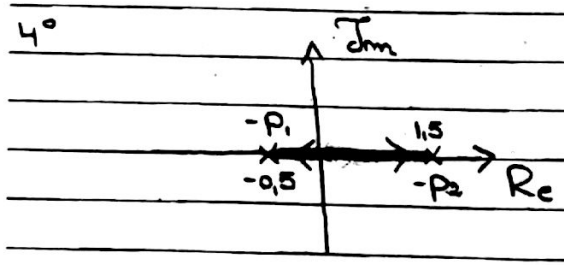
$$1 + K\frac{1}{(s+0,5)(s-1,5)} = 0$$

3°

$m_p = 2$ $\quad -p_1 = -0,5$

$m_z = 0$ $\quad -p_2 = 1,5$

4°



$[-0,5 \ , \ 1,5] \in GMK$

5°

$m_g = m_p = 2$ grane

6° GMK je simetričan u odnosu na realnu osu.

7°

$$\varphi_A = \frac{2q+1}{m_p - m_z} \cdot 180° \qquad q = 0,1 \ldots m_p - m_z + 1$$

$q = 0,1$

$q = 0 \rightarrow \varphi_A = \frac{1}{2} \cdot 180° = 90°$

$q = 1 \Rightarrow \varphi_A = \frac{3}{2} \cdot 180° = 270°$

$$\sigma_A = \frac{\sum(-p_i) - \sum(-z_i)}{n_p - n_z} = \frac{-0,5 + 1,5}{2} = 0,5$$

8°

$$1 + K\frac{1}{(s+0,5)(s-1,5)} = \frac{(s+0,5)(s-1,5)+K}{(s+0,5)(s-1,5)}$$

$$(s+0,5)(s-1,5) + K = 0$$
$$s^2 + 0,5s - 1,5s - 0,75 + K = 0$$
$$s^2 - s + K - 0,75 = 0$$

s^2	1	$K-0,75$
s^1	-1	0
s^0	$K-0,75$	

=> GMK ne sječe imaginarnu osu ☺

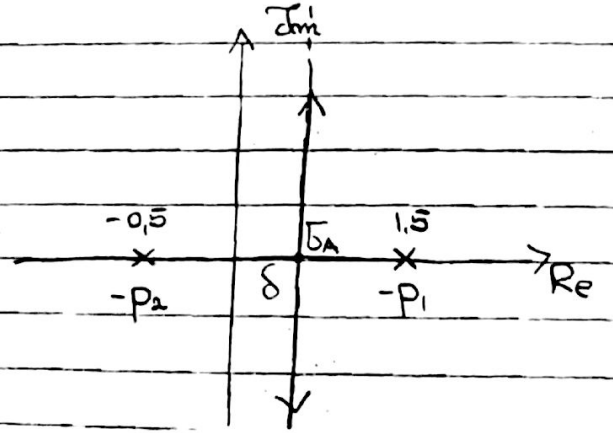
9° tačka spajanja (odvajanja)

$$\frac{1}{\delta+0,5} + \frac{1}{\delta-1,5} = 0 \;/\,(\delta+0,5)(\delta-1,5)$$

$$\delta - 1,5 + \delta + 0,5 = 0$$

$$2\delta = 1$$

$$\boxed{\delta = 0,5}$$



* Konstruisati GMK za sistem na prenosnom funkcijom $G(s) = \frac{K(s-0,5)}{(s-1)^2}$

1,2°

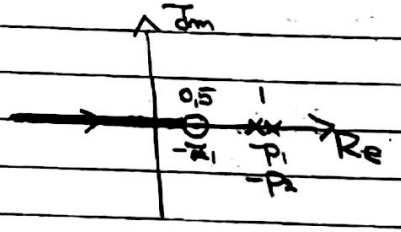
$$1 + K\frac{(s-0,5)}{(s-1)^2} = 0$$

3°

$n_p = 2$ $\quad -p_1 = 1$ $\quad -p_2 = 1$ $\quad z_1 = 0,5$

$n_z = 1$

4°



$(-\infty, 0,5] \in GMK$

5° $n_g = n_p = 2$ grane

6° GMK je nimetričan u odnosu na realnu osu.

7°

$$\phi_A = \frac{2g+1}{n_p - n_z}\,180° \qquad g = 0, 1, \ldots n_p - n_z - 1$$

$$g = 0$$

$$g = 0 \Rightarrow \phi_A = \frac{1}{1}\cdot 180° = 180°$$

8°

$$1+K\frac{(s-0,5)}{(s-1)^2}=\frac{(s-1)^2+K(s-0,5)}{(s-1)^2}$$

$$(s-1)^2+K(s-0,5)=0$$
$$s^2-2s+1+Ks-0,5K=0$$
$$s^2+(K-2)s+1-0,5K=0$$

s^2	1	$1-0,5K$
s^1	$K-2$	0
s^0	$1-0,5K$	

$\Rightarrow$ GMK ne sječe imaginarnu osu ☺

9° tačka spajanja (odvajanja)

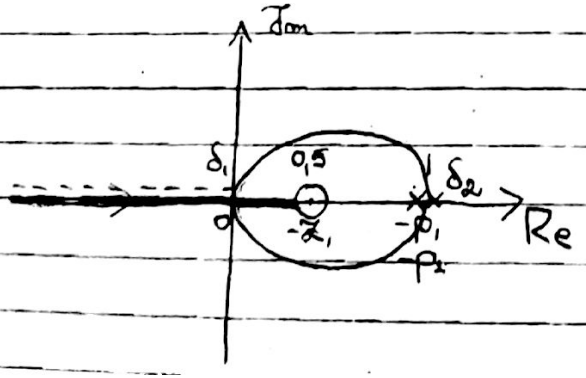
$$\frac{1}{s-1}+\frac{1}{s-1}=\frac{1}{s-0,5} \;/\cdot(s-1)^2(s-0,5)$$

$$(s-1)(s-0,5)+(s-1)(s-0,5)=(s-1)^2 \;/:(s-1)$$

$$(s-0,5)(s^2-2s+1)=s^2-2s+1$$
$$s^3-0,5s^2-2s^2+s+s-0,5=s^2-2s+1$$
$$s^3-3,5s^2+4s-2,5=0$$

$$2s-1=s-1$$

$\boxed{s=0}$ $\boxed{s=1}$



* Konstruisati GMK sistema sa prenosnom funkcijom $G(s)=\frac{K(s+2)}{(s+1+j\sqrt{3})(s+1-j\sqrt{3})}$

12°

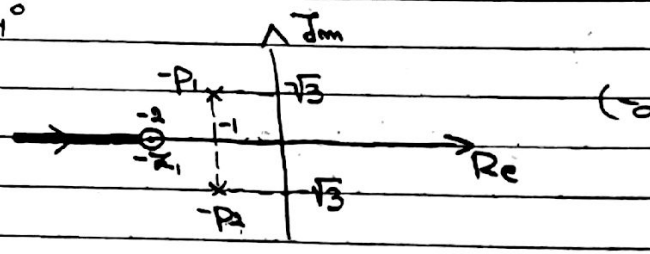
$$1+K\frac{(s+2)}{(s+1+j\sqrt{3})(s+1-j\sqrt{3})}=0$$

3°

$n_p=2$ $\quad -p_1=-1-j\sqrt{3}$ $\quad -z_1=-2$

$n_z=1$ $\quad -p_2=-1+j\sqrt{3}$

4°



$(-\infty,-2]\in GMK$

5° $n_g=n_p=2$ grane

6° GMK je simetričan u odnosu na realnu osu

7°

$$\Phi_A=\frac{2q+1}{n_p-n_z}180° \qquad q=0,1\ldots n_p-n_z+1$$

$q=0$

$$q=0\rightarrow\Phi_A=\frac{1}{1}\cdot 180°=180°$$

8°

$$1+K\frac{(s+2)}{(s+1+j\sqrt{3})(s+1-j\sqrt{3})}=\frac{(s+1+j\sqrt{3})(s+1-j\sqrt{3})+K(s+2)}{(s+1+j\sqrt{3})(s+1-j\sqrt{3})}$$

$$(s+1+j\sqrt{3})(s+1-j\sqrt{3})+K(s+2)=0$$

$$s^2+s+j\sqrt{3}s+s+1+j\sqrt{3}-j\sqrt{3}s-j\sqrt{3}+3+Ks+2K=0$$

$$s^2+2s+4+Ks+2K=0$$

$$s^2+(2+K)s+2K+4=0$$

s^2	1	$2K+4$
s^1	$K+2$	0
s^0	$2K+4$	

GMK ne ryješe imaginarnu ose ☺

s nula šta imaginarno

9° tačka spajanja (odvajanja)

$$\frac{1}{s+1+j\sqrt{3}}+\frac{1}{s+1-j\sqrt{3}}=\frac{1}{s+2}\ /(s^2+2s+4)(s+2)$$

$$(s+1-j\sqrt{3})(s+2)+(s+1+j\sqrt{3})(s+2)=s^2+2s+4$$

$$(s+2)(2s+2)=s^2+2s+4$$

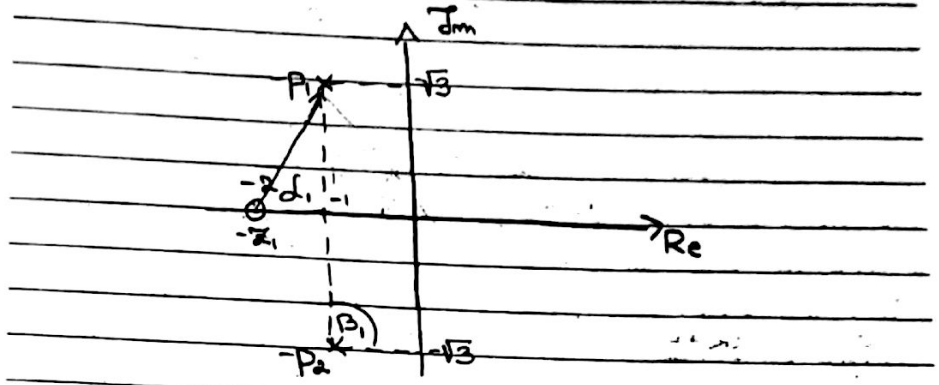
$$2s^2+4s+2s+4=s^2+2s+4$$

$$s^2+4s=0$$

$$s(s+4)=0$$

$$\boxed{s=0}\quad\boxed{s=-4}$$

10. ugao napuštanja konjugovano - kompleksnih polova



$\beta_1 = 90°$

$\delta_1 = \operatorname{arctg}\frac{\sqrt{3}}{1} = 60°$

$\beta_2' = \delta_1 - \beta_1 - 180° + k360°$

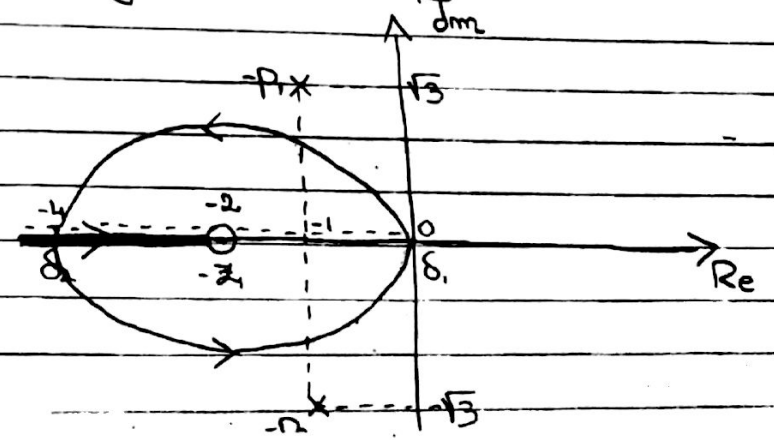
$\beta_2' = 60° - 90° - 180° + k360°$

$\boxed{\beta_2' = 150°}$

gdje je prvi ugao ovog tipa

$\beta_1' = -\beta_2' = -150°$

11. crtanje GMK



* Konstruisati GMK sistema sa prenosnom funkcijom $G(s) = K\frac{(s-2)(s-1)}{(s+1)(s+2)(s+3)}$

1,2°

$$1 + K\frac{(s-2)(s-1)}{(s+1)(s+2)(s+3)} = 0$$

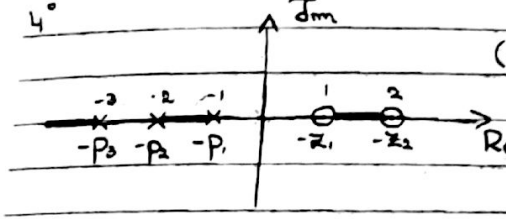
3°

$n_p = 3$ $\quad -p_1 = -1$ $\quad -z_1 = 1$

$n_z = 2$ $\quad -p_2 = -2$ $\quad -z_2 = 2$

$-p_3 = -3$

4°



$(-\infty, -3) \cup [-2, -1] \cup [1, 2]$

5°

$n_g = n_p = 3$ grane

6° GMK je simetričan u odnosu na realnu osu

7°

$$\phi_A = \frac{2q+1}{n_p - n_z} \cdot 180° \qquad q = 0, 1 \dots n_p - n_z - 1$$

$q = 0$

$q = 0 \Rightarrow \phi_A = \frac{1}{1} \cdot 180° = 180°$

8°

$$1 + K\frac{(s-2)(s-1)}{(s+1)(s+2)(s+3)} = \frac{(s+1)(s+2)(s+3) + K(s-2)(s-1)}{(s+1)(s+2)(s+3)}$$

$$(s+1)(s+2)(s+3) + K(s-2)(s-1) = 0$$

$$(s^2+s+2s+2)(s+3) + K(s^2-2s-s+2) = 0$$

$$s^3 + 3s^2 + 2s + 3s^2 + 9s + 6 + Ks^2 - 3Ks + 2K = 0$$

$$s^3 + (6+K)s^2 + (11-3K)s + 6 + 2K = 0$$

s^3	1	$11-3K$
s^2	$6+K$	$6+2K$
s^1	A_1	A_2
s^0	B_1	

$$A_1 = \frac{(6+K)(11-3K) - 6 - 2K}{6+K} = \frac{66 + 11K - 18K - 3K^2 - 6 - 2K}{6+K}$$

$$\boxed{A_1 = \frac{-3K^2 - 9K + 60}{6+K}}$$

$$A_2 = \frac{(6+K)\cdot 0 - 0 \cdot 1}{6+K} = 0$$

$$B_1 = \frac{A_1(6+2K) - 0(6+K)}{A_1} = 6 + 2K$$

$$\frac{-3K^2 - 9K + 60}{6+K} = 0$$

$$6 + K \neq 0$$

$$-3K^2 - 9K + 60 = 0$$

$K_1 = -6,22$ | $K_2 = 3,22$

$(6+K)S^2 + 6 + 2K = 0$ → ponoćni polinom

$(6+3,22)S^2 + 6 + 2 \cdot 3,22 = 0$

$9,22S^2 = -12,44$

$S = \pm\sqrt{\frac{-12,44}{9,22}} = \pm j1,16$ → tačke presjeka sa imaginarnom osom

9° tačke spajanja (odvajanja)

$$1 + K\frac{(S-2)(S-1)}{(S+1)(S+2)(S+3)} = 0$$

$$K = -\frac{(S+1)(S+2)(S+3)}{(S-2)(S-1)}$$

$$K = -\frac{(S+1)(S^2+2S+3S+6)}{S^2-2S-S+2} = \frac{-S^3-6S^2-11S-6}{S^2-3S+2}$$

$$\frac{dK}{dS} = \frac{(-S^3-6S^2-11S-6)'(S^2-3S+2) + (S^3+6S^2+11S+6)(S^2-3S+2)'}{(S^2-3S+2)^2}$$

$$\frac{dK}{dS} = \frac{(-3S^2-12S-11)(S^2-3S+2) + (S^3+6S^2+11S+6)(2S-3)}{(S^2-3S+2)^2}$$

$$\frac{dK}{dS} = \frac{-3S^4-12S^3-11S^2+9S^3+36S^2+33S-6S^2-24S-22}{+2S^4+12S^3+22S^2+12S-3S^3-18S^2-33S-18}$$
$$(S^2-3S+2)^2$$

$$\frac{dK}{dS} = \frac{-S^4+6S^3+23S^2-12S-40}{(S^2-3S+2)^2} = 0$$ → ne znam ovaj način :(

$\delta_1 = 1,3882 \in GMK$

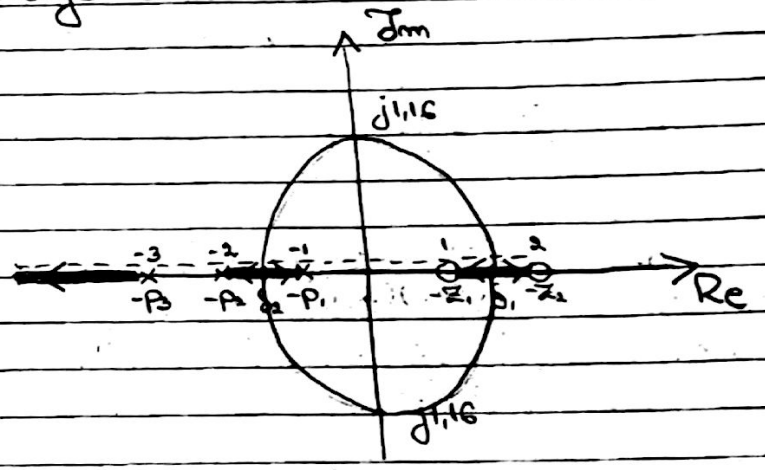
$\delta_2 = -1,3489 \in GMK$ → tačke spajanja (odvajanja) na realnom osom

$\delta_3 = -2,5191 \notin GMK$

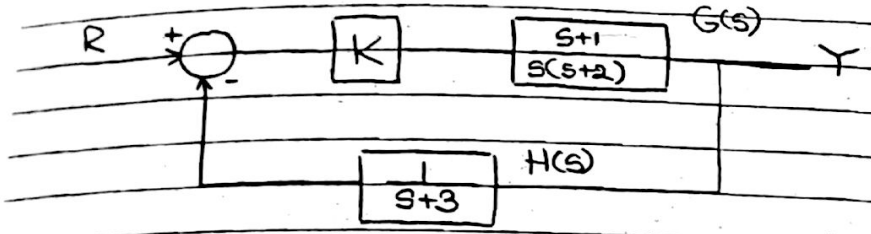
$\delta_4 = 8,4798 \notin GMK$

10° nemamo konjugovano kompleksnih polova pa ne računamo uglove mapiranja

11° crtanje GMK



* Konstruisati GMK sistema sa slike:



1,2°

$$1+KG(s)H(s)=0$$

$$1+K\frac{s+1}{s(s+2)(s+3)}=0$$

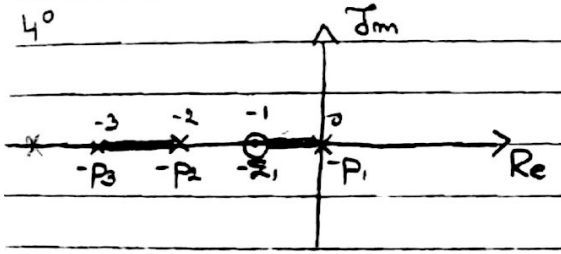
3°

$n_p=3$ $\quad -p_1=0$

$n_z=1$ $\quad -p_2=-2$ $\quad -z_1=-1$

$-p_3=-3$

4°



$[-3,-2]\cup[-1,0]\in GMK$

5°

$n_g=n_p=3$ grane

6° GMK je simetričan u odnosu na realnu osu.

7°

$$\Phi_A=\frac{2q+1}{n_p-n_z}180° \qquad q=0,1\ldots n_p-n_z+1$$

$$q=0,1$$

$$q=0\Rightarrow\Phi_A=\frac{1}{2}180°=90°$$

$$q=1\Rightarrow\Phi_A=\frac{3}{2}\cdot180°=270°$$

$$\sigma_A=\frac{\sum(-p_i)-\sum(-z_i)}{n_p-n_z}=\frac{(0-2-3)+1}{2}=-2$$

8°

$$1+K\frac{(s+1)}{s(s+2)(s+3)}=\frac{s(s+2)(s+3)+K(s+1)}{s(s+2)(s+3)}$$

$$s(s+2)(s+3)+K(s+1)=0$$

$$(s^2+2s)(s+3)+Ks+K=0$$

$$s^3+2s^2+3s^2+6s+Ks+K=0$$

$$s^3+5s^2+(6+K)s+K=0$$

s^3	1	$6+K$	
s^2	5	K	$A_1=\frac{5(6+K)-1K}{5}=\frac{30+4K}{5}$
s^1	A_1	A_2	
s^0	B_1		$A_2=\frac{5\cdot0-0\cdot1}{5}=0$

$$B_1=\frac{A_1\cdot K-0\cdot5}{A_1}=K$$

$$\frac{30+4K}{5}=0$$

$$30+4K=0$$

$$K=-15$$

9° tačke spajanja (odvajanja)

$$1+K\frac{(s+1)}{s(s+2)(s+3)}=0$$

$$K=-\frac{s(s+2)(s+3)}{s+1}$$

$$K=-\frac{(s^2+2s)(s+3)}{s+1}=\frac{s^3+5s^2+6s}{s+1}$$

$$\frac{dK}{ds}=0$$

$-(s^3+\dots+\dots)=0$

$s=0 \quad K=0$

$s^3+5s^2+s=0$

$$\frac{dK}{ds}=\frac{(3s^2+10s+6)(s+1)-(s^3+5s^2+6s)}{(s+1)^2}$$

$$\frac{dK}{ds}=\frac{3s^3+10s^2+6s+3s^2+10s+6-s^3-5s^2-6s}{(s+1)^2}$$

$$\frac{dK}{ds}=\frac{2s^3+8s^2+10s+6}{(s+1)^2}$$

$$2s^3+8s^2+10s+6=0 \;/:2$$

$$s^3+4s^2+5s+3=0$$

$s_0=-2{,}5 \rightarrow$ tačka između polova

$$s_1=s_0-\frac{f(s_0)}{f'(s_0)} \rightarrow \text{Newton-Raphsonov metod}$$

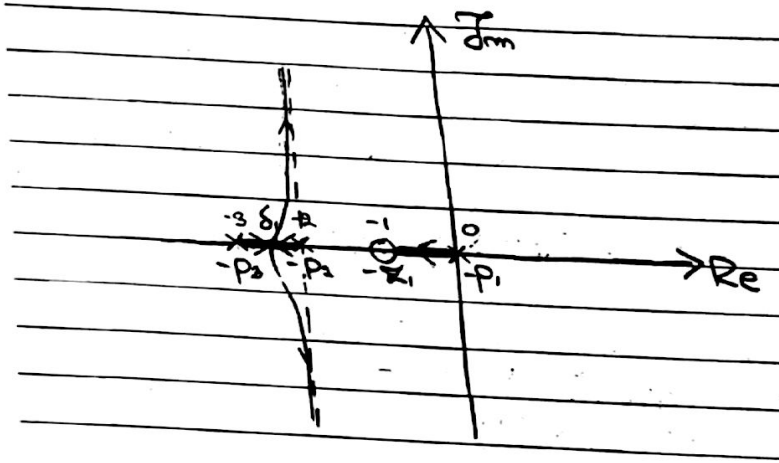
$$f(s)=s^3+4s^2+5s+3$$

$$f'(s)=3s^2+8s+5$$

$$s_1=-2{,}5-\frac{-0{,}125}{-3{,}75}=-2{,}47$$ tačka odvajanja

10° nema konjugovano-kompleksnih polova pa ne računamo uglove napuštanja

1° crtanje GMK



* Konstruisati GMK za sistem sa prenosnom funkcijom $G(s) = \frac{K}{s(s+4)(s^2+8s+32)}$

1,2°

$$1 + K\frac{1}{s(s+4)(s^2+8s+32)}$$

3°

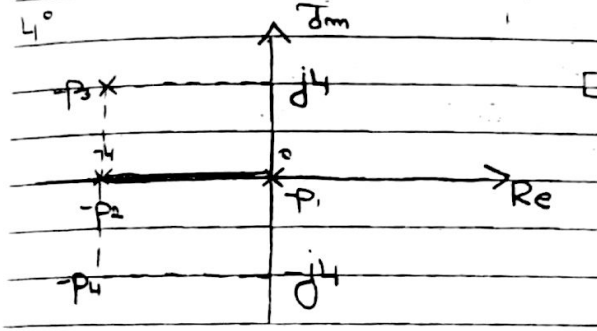
$n_p = 4$ $\quad -p_1 = 0$ $\quad s^2+8s+32=0$

$n_z = 0$ $\quad -p_2 = -4$ $\quad s_{1/2} = \frac{-8 \pm \sqrt{64-128}}{2}$

$-p_3 = -4+j4$

$-p_4 = -4-j4$ $\quad s_{1/2} = \frac{-8 \pm j8}{2} = -4 \pm j4$

4°



$[-4,0] \in GMK$

5°

$n_g = n_p = 4$ grane

6° GMK je simetričan u odnosu na realnu osu

7°

$$\phi_A = \frac{2g+1}{n_p - n_z} \cdot 180° \qquad g = 0,1,2,3$$

$g=0 \Rightarrow \phi_A = \frac{1}{4} \cdot 180° = 45°$

$g=1 \Rightarrow \phi_A = \frac{3}{4} \cdot 180° = 135°$

$g=2 \Rightarrow \phi_A = \frac{5}{4} \cdot 180° = 225°$

$g=3 \Rightarrow \phi_A = \frac{7}{4} \cdot 180° = 315°$

$$\sigma_A = \frac{\sum(-p_i) - \sum(-z_i)}{n_p - n_z} = \frac{-12}{4} = -3$$

8°

$$1 + \frac{K}{s(s+4)(s^2+8s+32)} = \frac{s(s+4)(s^2+8s+32)+K}{s(s+4)(s^2+8s+32)}$$

$$s(s+4)(s^2+8s+32)+K=0$$

$$(s^2+4s)(s^2+8s+32)+K=0$$

$$s^4+4s^3+8s^3+32s^2+32s^2+128s+K=0$$

$$s^4+12s^3+64s^2+128s+K=0$$

s^4	1	64	K
s^3	12	128	0
s^2	53,3	K	
s^1	$\frac{6822,4-12K}{53,3}$	0	
s^0	K		

$$\frac{6822,4-12K}{53,3} = 0$$

$$\boxed{K = 568,6}$$

$53,3s^2 + K = 0 \rightarrow$ pomoćni polinom

$53,3s^2 + 568,6 = 0$

$$s_{1/2} = \pm\sqrt{\frac{-568,6}{53,3}} = \pm j3,26$$

9° tačka spajanja (odvajanja)

$$1+K\frac{1}{(s^2+4s)(s^2+8s+32)}=0$$

$$K=-(s^4+4s^3+8s^3+32s^2+32s^2+128s)$$

$$K=-(s^4+12s^3+64s^2+128s)$$

I način $\quad \frac{dK}{ds}=0$

$$\frac{dK}{ds}=-(4s^3+36s^2+128s+128)=0$$

$$4s^3+36s^2+128s+128=0 \;/:4$$

$$s^3+9s^2+32s+32=0$$

$$f(s)=s^3+9s^2+32s+32$$
$$f'(s)=3s^2+18s+32$$

$$s_0=-2$$

$$s_1=s_0-\frac{f(s_0)}{f'(s_0)}=-2-\frac{-4}{8}=-2+0{,}5=-1{,}5$$

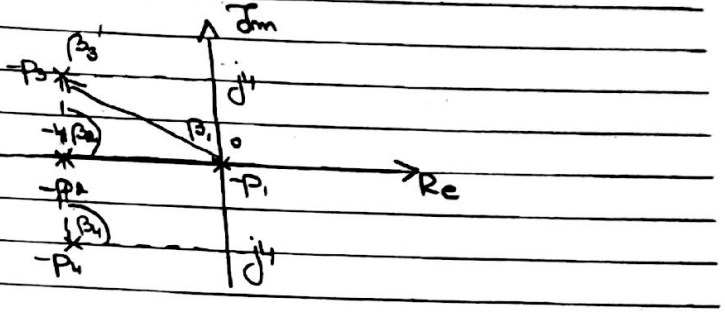
$$s_2=s_1-\frac{f(s_1)}{f'(s_1)}=-1{,}5-\frac{0{,}875}{11{,}75}=-1{,}574$$

II način

p.p. $s\in[-4,0]$

s	-4	-3	-2,5	-2	-1,5	-1	0
K	0	51	68,5	80	85	75	0

10° uglovi napuštanja konjugovano-kompleksnih polova



$$\beta_1=180^\circ-\operatorname{arctg}\frac{4}{4}=180^\circ-45^\circ=135^\circ$$

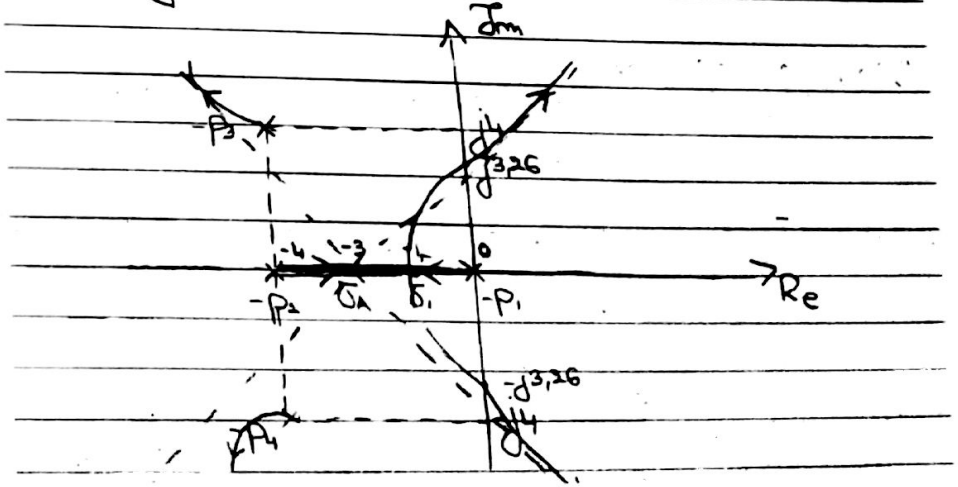
$$\beta_2=\beta_4=90^\circ$$

$$\beta_3'=-\beta_1-\beta_2-\beta_4-180^\circ\pm k360^\circ$$

$$\beta_3'=-135^\circ-90^\circ-90^\circ-180^\circ\pm k360^\circ$$

$$\boxed{\beta_3'=-135^\circ} \qquad \boxed{\beta_4'=135^\circ}$$

11° crtanje GMK



* Konstruisati GMK sistema sa prenosnom funkcijom $G(s) = K\frac{(s+2)}{(s+1)(s+3+j)(s+3-j)}$

1,2°

$$1 + K\frac{s+2}{(s+1)(s+3+j)(s+3-j)}$$

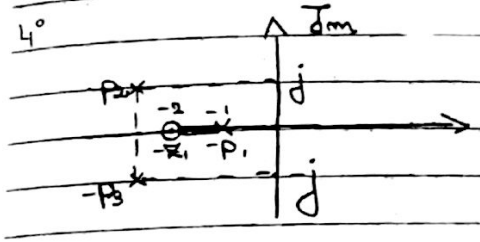
3°

$n_p = 3$ $\quad -p_1 = -1$

$n_z = 1$ $\quad -p_2 = -3-j$ $\quad -z_1 = -2$

$-p_3 = -3+j$

4°



$[-2,-1] \in GMK$

5°

$n_g = n_p = 3$ grane

6° GMK je simetričan u odnosu na realnu osu.

7°

$$\phi_A = \frac{2q+1}{n_p - n_z} \cdot 180° \qquad q = 0,1$$

$$q = 0 \Rightarrow \phi_A = \frac{1}{2} \cdot 180° = 90°$$

$$q = 1 \Rightarrow \phi_A = \frac{3}{2} \cdot 180° = 270°$$

$$\gamma_A = \frac{\sum(-p_i) - \sum(-z_i)}{n_p - n_z} = \frac{-5}{2} = -2.5$$

8°

$$1 + K\frac{s+2}{(s+1)(s+3+j)(s+3-j)} = \frac{(s+1)(s+3+j)(s+3-j) + K(s+2)}{(s+1)(s+3+j)(s+3-j)}$$

$$(s+1)(s+3+j)(s+3-j) + K(s+2) = 0$$

$$(s+1)((s+3)^2 - j^2) + Ks + 2K = 0$$

$$(s+1)(s^2+6s+9+1) + Ks + 2K = 0$$

$$s^3 + s^2 + 6s^2 + 6s + 10s + 10 + Ks + 2K = 0$$

$$s^3 + 7s^2 + (16+K)s + 10 + 2K = 0$$

s^3	1	$16+K$
s^2	7	$10+2K$
s^1	$\frac{5K+102}{7}$	0
s^0	$10+2K$	

$\Rightarrow$ nema presjeka sa imaginarnom osom

9°

$$1 + K\frac{s+2}{(s+1)(s^2+6s+10)} = 0$$

$$K = -\frac{(s+1)(s^2+6s+10)}{s+2}$$

$$K = -\frac{(s^3+6s^2+10s+s^2+6s+10)}{s+2} = \frac{s^3+7s^2+16s+10}{s+2}$$

$$\frac{dK}{ds} = 0$$

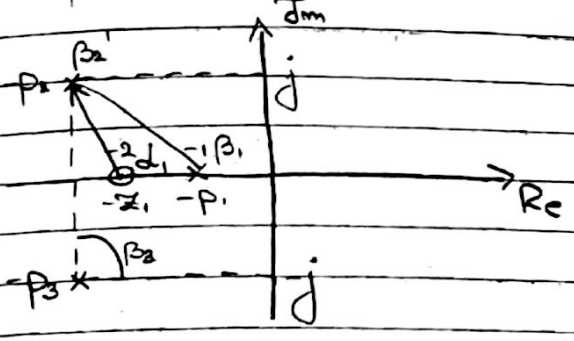
$$\frac{dK}{ds} = \frac{(3s^2+14s+16)(s+2) - (s^3+7s^2+16s+10)}{(s+2)^2}$$

$$\frac{dK}{ds} = \frac{3s^3+14s^2+16s+6s^2+28s+32-s^3-7s^2-16s-10}{(s+2)^2}$$

$$\frac{dK}{ds} = \frac{2s^3+13s^2+28s+22}{(s+2)^2}$$

$2s^3+13s^2+28s+22=0$ → nema presjeka na realnom dijelu

10° uglovi mapiranja kompleksno-konjugovanih polova



$\beta_3 = 90°$

$\beta_1 = 180° - \operatorname{arctg}\frac{1}{2} = 180° - 26{,}56° = 153{,}43°$

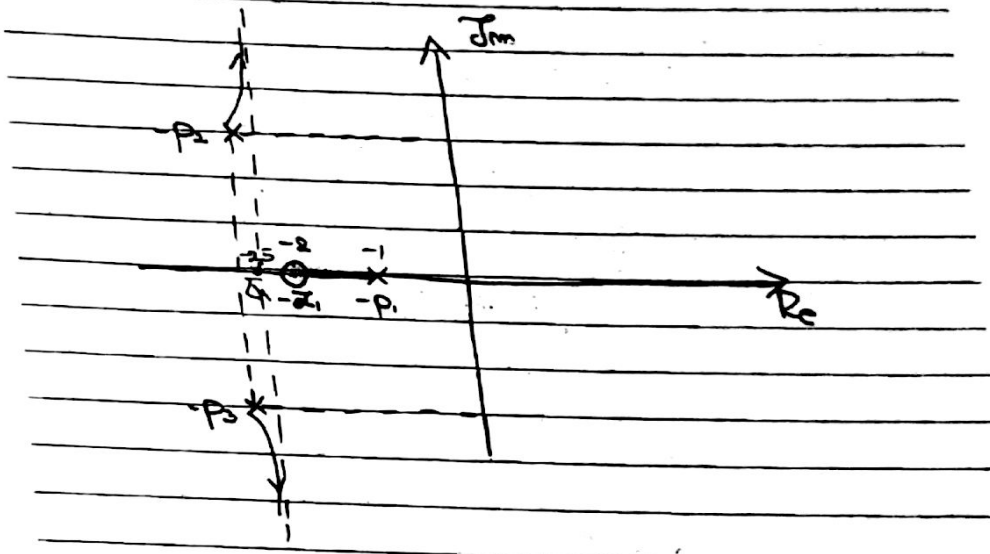
$\alpha_1 = 180° - \operatorname{arctg}\frac{1}{1} = 180° - 45° = 135°$

$\beta_2' = \alpha_1 - \beta_1 - \beta_3 - 180° \pm k360°$

$\beta_2' = 135° - 153{,}43° - 90° - 180° \pm k360°$

$\beta_2' = \mp 1{,}57°$ $\quad \beta_3' = -\mp 1{,}57°$

11° ortanje GMK



$$B_1 = \frac{(94-K)(18+K) - 49(8+3K)}{94-K}$$

$$B_1 = \frac{1692 - 18K + 94K - K^2 - 392 - 147K}{94-K}$$

$$B_1 = \frac{-K^2 - 71K + 1300}{94-K} \qquad B_2 = 0$$

$$C_1 = \frac{B_1 \cdot A_2 - B_2 \cdot A_1}{B_1} = 0$$

$$K^2 + 71K + 1300 = 0$$

$$1K^2 + 71K - 1300 = 0$$

$$K_{1/2} = \frac{-71 \pm \sqrt{5041 + 5200}}{2}$$

$$K_{1/2} = \frac{-71 \pm 101,2}{2}$$

$$K_1 = -86,1 \qquad \boxed{K_2 = 15,1}$$

$$\frac{94-K}{7} s^2 + 8 + 3K = 0$$

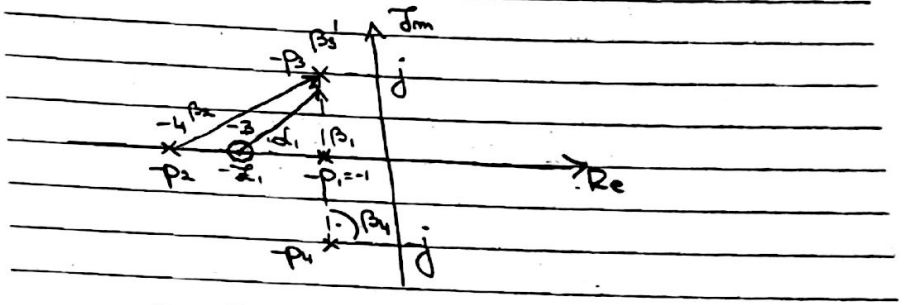
$$\frac{94-15,1}{7} s^2 + 8 + 3 \cdot 15,1 = 0$$

$$11,8 s^2 + 53,3 = 0$$

$$s_{1/2} = \pm \sqrt{\frac{-53,3}{11,3}} = \pm j\,2,17 \rightarrow \text{tačke preseka sa imaginarnom osom}$$

9° nema tačaka odvajanja (spajanja) sa realnom osom

10° uglovi napuštanja konjugovano-kompleksnih polova



$\beta_1 = \beta_4 = 90°$

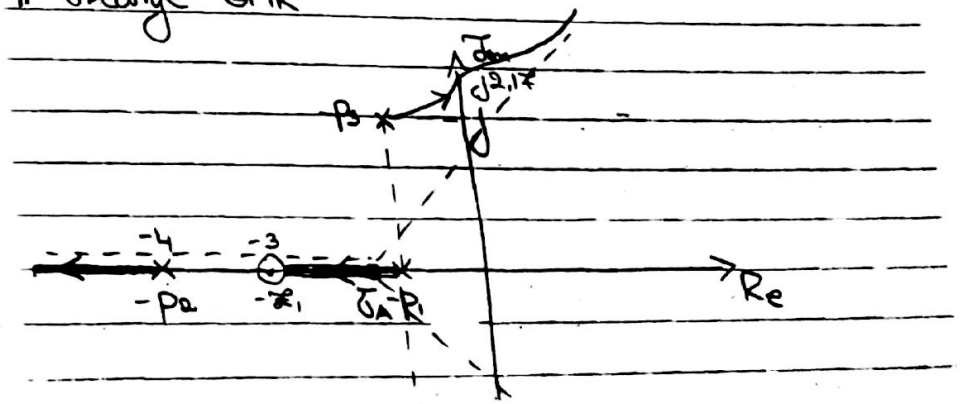
$\beta_2 = \text{arctg}\,\frac{1}{3} = 18,43°$

$\alpha_1 = \text{arctg}\,\frac{1}{2} = 26,565°$

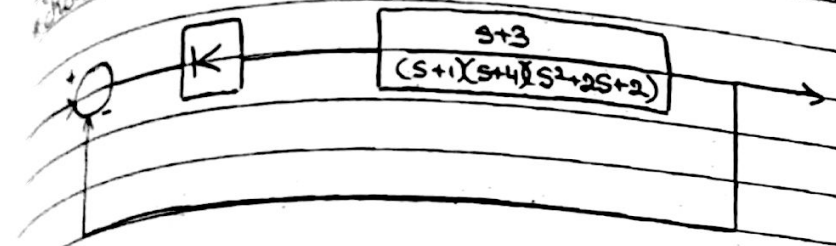
$\beta_3' = \alpha_1 - \beta_1 - \beta_2 - \beta_4 - 180° + k360° = 8,135$

$\beta_4' = 8,135°$

11° crtanje GMK



Konstruisati GMK za sistem dat na slici:



1.2°

$$1+K\frac{s+3}{(s+1)(s+4)(s^2+2s+2)}=0$$

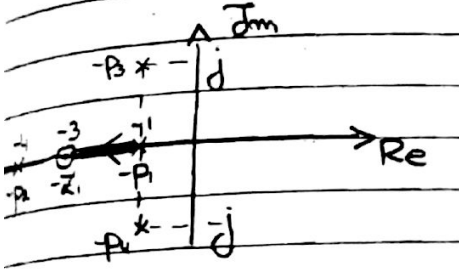
3°

$n_p=4$ $\quad -p_1=-1$ $\quad s^2+2s+2=0$

$n_z=1$ $\quad -p_2=-4$ $\quad s_{1/2}=\frac{-2\pm\sqrt{4-8}}{2}$

$-p_3=-1+j$

$-z_1=-3$ $\quad -p_4=-1-j$ $\quad s_{1/2}=\frac{-2\pm j2}{2}=-1\pm j$



$(-\infty,-4]\cup[-3,-1]\in GMK$

$n_g=n_p=4$ grane

GMK je numerican u odnosu na realnu osu

$$\phi_A=\frac{2q+1}{n_p-n_z}\cdot 180^\circ \qquad q=0,1,2$$

$q=0 \Rightarrow \phi_A=\frac{1}{3}\cdot 180^\circ=60^\circ$

$q=1 \Rightarrow \phi_A=\frac{3}{3}\cdot 180^\circ=180^\circ$

$q=2 \Rightarrow \phi_A=\frac{5}{3}\cdot 180^\circ=300^\circ$

$$\sigma_A=\frac{\sum(-p_i)-\sum(-z_i)}{n_p-n_z}=\frac{(-1-4-1-1)+3}{3}=\frac{-4}{3}$$

8°

$$1+K\frac{s+3}{(s+1)(s+4)(s^2+2s+2)}$$

$$=\frac{(s+1)(s+4)(s^2+2s+2)+K(s+3)}{(s+1)(s+4)(s^2+2s+2)}$$

$$(s+1)(s+4)(s^2+2s+2)+K(s+3)=0$$

$$(s^2+s+4s+4)(s^2+2s+2)+Ks+3K=0$$

$$s^4+5s^3+4s^2+2s^3+10s^2+8s+2s^2+10s+8+Ks+3K=0$$

$$s^4+7s^3+16s^2+(18+K)s+8+3K=0$$

s^4	1	16	$8+3K$
s^3	7	$18+K$	0
s^2	A_1	A_2	
s^1	B_1	B_2	
s^0	C_1		

$$A_1=\frac{7\cdot 16-18-K}{7}=\frac{94-K}{7}$$

$$A_2=8+3K$$

$$B_1=\frac{A_1\cdot(18+K)-A_2\cdot 7}{A_1}$$

$$B_1=\frac{\frac{94-K}{7}(18+K)-(8+3K)7}{\frac{94-K}{7}}$$

*Konstruisati GMK sistema sa prenosnom funkcijom $G(s) = \frac{K}{s(s+1)(s+3)(s+4)}$

1,2°

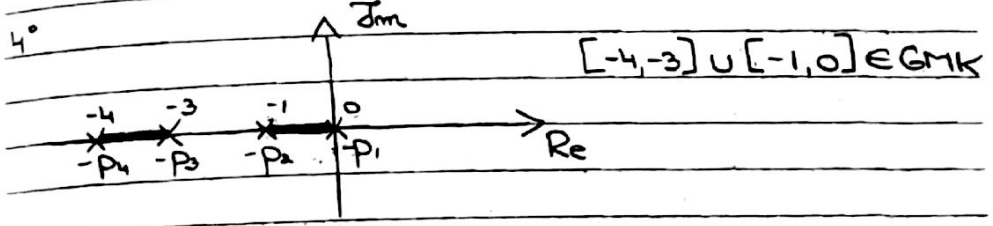
$$1 + \frac{K}{s(s+1)(s+3)(s+4)} = 0$$

3°

$n_p = 4$ $\quad -p_1 = 0$ $\quad -p_3 = -3$

$n_z = 0$ $\quad -p_2 = -1$ $\quad -p_4 = -4$

4°



$[-4,-3] \cup [-1,0] \in GMK$

5° $n_g = n_p = 4$ grane

6° GMK je simetričan u odnosu na realnu osu.

7°

$$\phi_A = \frac{2q+1}{n_p - n_z} \cdot 180° \qquad q = 0,1,2,3$$

$q = 0 \Rightarrow \phi_A = \frac{1}{4} \cdot 180° = 45°$

$q = 1 \Rightarrow \phi_A = \frac{3}{4} \cdot 180° = 135°$

$q = 2 \Rightarrow \phi_A = \frac{5}{4} \cdot 180° = 225°$

$q = 3 \Rightarrow \phi_A = \frac{7}{4} \cdot 180° = 315°$

$$\sigma_A = \frac{\sum(-p_i) - \sum(-z_i)}{n_p - n_z} = \frac{(0-1-3-4)+0}{4} = -2$$

8°

$$1 + \frac{K}{s(s+1)(s+3)(s+4)} \cdot \frac{s(s+1)(s+3)(s+4)+K}{s(s+1)(s+3)(s+4)}$$

$$s(s+1)(s+3)(s+4) + K = 0$$

$$(s^2+s)(s^2+3s+4s+12) + K = 0$$

$$s^4 + s^3 + 3s^3 + 8s^2 + 4s^3 + 4s^2 + 12s^2 + 12s + K = 0$$

$$s^4 + 8s^3 + 19s^2 + 12s + K = 0$$

s^4	1	19	K
s^3	8	12	0
s^2	17,5	K	
s^1	$\frac{210-8K}{17,5}$	0	
s^0	K		

$$\frac{210-8K}{17,5} = 0$$

$$210 - 8K = 0$$

$$\boxed{K = 26,25}$$

$17,5\,s^2 + K = 0 \rightarrow$ pomoćni polinom

$17,5\,s^2 + 26,25 = 0$

$$s_{1/2} = \pm\sqrt{\frac{26,25}{17,5}} = \pm j1,225$$

9°

$$1 + \frac{K}{s(s+1)(s+3)(s+4)} = 0$$

$$K = -s(s+1)(s+3)(s+4)$$

pp. $s \in [-1, 0]$

s	-1	-0,8	-0,6	-0,4	-0,2	0
K	0	1,1264	1,9584	2,2464	1,7024	0

↳ maximalna vrijednost

$\delta_1 = -0,4$

↳ tačka odvajanja

p.p. $s \in [-4, -3]$

s	-4	-3,8	-3,6	-3,4	-3,2	-3
K	0	1,7024	2,2464	1,9584	1,1264	0

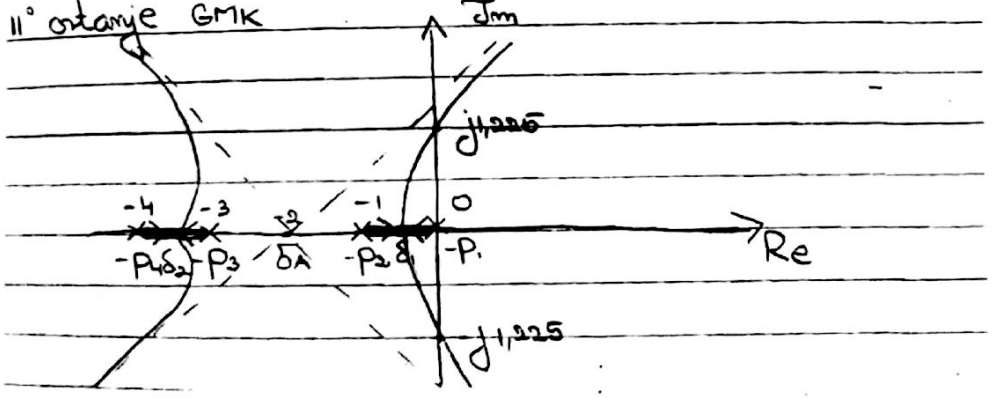
↳ maximalna vrijednost

$\delta_2 = -3,6$

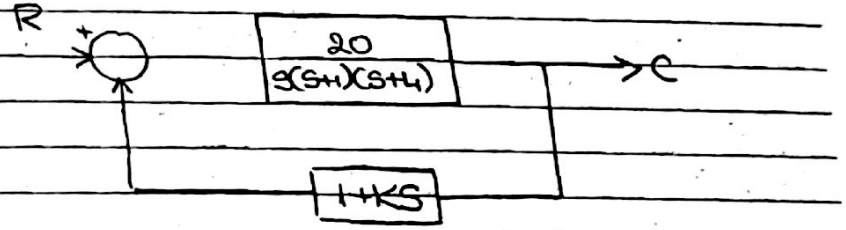
↳ tačka odvajanja

10° nema konjugovano - kompleksnih polova pa ne računamo uglove napuštanja.

11° crtanje GMK



* Na slici je prikazan sistem sa međujedinom povratnom spregom. Nacrtati GMK datog sistema pri promjeni pojačanja K linijske povratne sprege.



1,2°

$$1 + \frac{20(1+KS)}{s(s+1)(s+4)} = 0$$

$$s(s+1)(s+4) + 20 + 20KS = 0$$

$$s^3 + 5s^2 + 4s + 20 + 20KS = 0$$

$$(s^3 + 5s^2 + 4s + 20)\left(1 + \frac{20KS}{s^3 + 5s^2 + 4s + 20}\right)$$

$$20K = K_1$$

$$1 + \frac{K_1 \cdot S}{s^3 + 5s^2 + 4s + 20} = 0$$

$$(s^3 + 5s^2 + 4s + 20) : (s+5) = s^2 + 4$$

$s^3 + 5s^2$

$4s + 20$

$4s + 20$

$= \quad =$

$s^2 + 4 = 0$

$s_{1,2} = \pm j2$

$$1 + \frac{K_1 \cdot S}{(S+5)(S+j2)(S-j2)} = 0$$

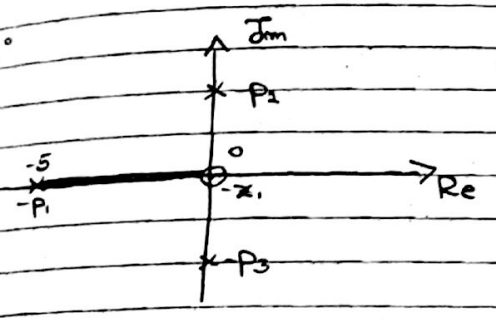
3°

$n_p = 3$ $\quad -p_1 = -5$

$n_z = 1$ $\quad -p_2 = j2$, $\quad -z_1 = 0$

$-p_3 = j2$

4°



$[-5, 0]$ ∈ GMK

5° $n_g = n_p = 3$ grane

6° GMK je simetričan u odnosu na realnu osu

7°

$$\phi_A = \frac{2q+1}{n_p - n_z} \cdot 180° \qquad q = 0, 1$$

$q = 0 \Rightarrow \phi_A = \frac{1}{2} \cdot 180° = 90°$

$q = 1 \Rightarrow \phi_A = \frac{3}{2} \cdot 180° = 270°$

$$\sigma_A = \frac{\Sigma(-p_i) - \Sigma(-z_i)}{n_p - n_z} = \frac{-5}{2} = -2{,}5$$

8°

$$1 + \frac{K_1 \cdot S}{(S+5)(S+j2)(S-j2)} = \frac{(S+5)(S+j2)(S-j2) + K_1 S}{(S+5)(S+j2)(S-j2)}$$

$$(S+5)(S+j2)(S-j2) + K_1 S = 0$$

$$S^3 + 5S^2 + 4S + 20 + K_1 S = 0$$

$$S^3 + 5S^2 + (4 + K_1)S + 20 = 0$$

S^3	1	$4 + K_1$	
S^2	5	20	
S^1	K_1	0	⇒ GMK ne ulazi imaginarnu osu
S^0	20		

9° tačke spajanja (odvajanja)

$$K_1 = \frac{S^3 + 5S^2 + 4S + 20}{S}$$

$$\frac{dK_1}{dS} = \frac{(3S^2 + 10S + 4)S - (S^3 + 5S^2 + 4S + 20)}{S^2}$$

$$\frac{dK_1}{dS} = \frac{3S^3 + 10S^2 + 4S - S^3 - 5S^2 - 4S - 20}{S^2}$$

$$\frac{dK_1}{dS} = \frac{2S^3 + 5S^2 - 20}{S^2} = 0$$

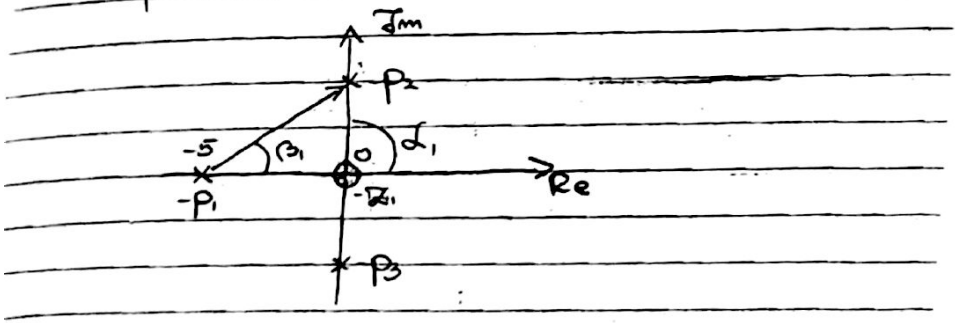
$$F(S) = 2S^3 + 5S^2 - 20$$

$F(-5) = -250 + 125 - 20 = -145$

$F(0) = -20$

funkcija ne mijenja znak, GMK nema tačaka odvajanja (spajanja)

10° ugao napuštanja konjugovano - kompleksnih polova



$\alpha_1 = 90°$

$\beta_3 = 90°$

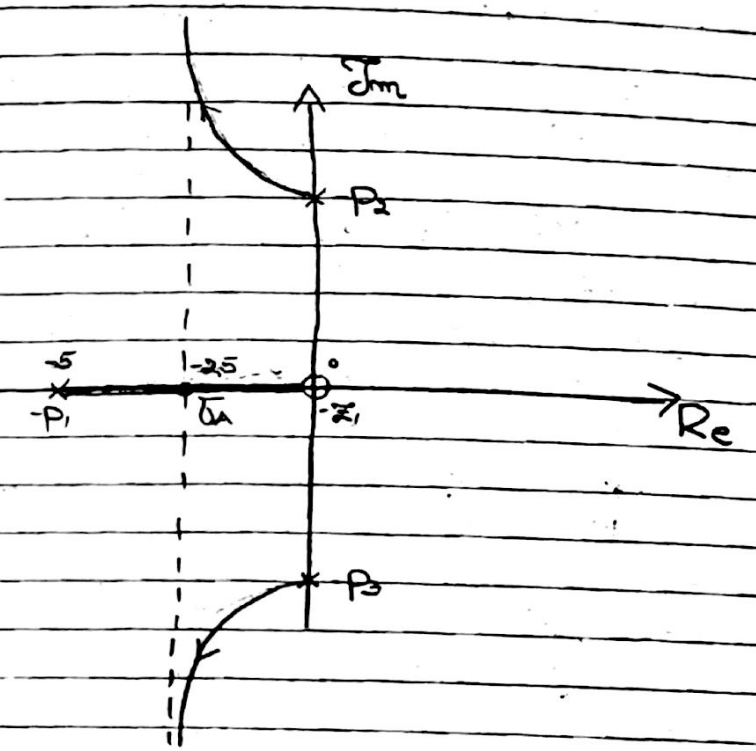
$\beta_1 = \text{arctg}\frac{2}{5} = 21,8°$

$\beta_2' = \alpha_1 - \beta_1 - \beta_3 - 180° \pm k360°$

~~$\beta_2' = 90° - 90° - 21,8° - 180° \pm k360° = -158,2°$~~

$\beta_3' = -158,2°$

11° ostanje GMK



*Nacrtati GMK sistema čija je prenosna funkcija na otvorenom povratnom spregom data kao: $G(s) = \frac{1}{s+1} e^{-\tau s}$, $\tau = 1$

$$e^{-\tau s} \approx \frac{1}{\frac{\tau}{2}s + 1}$$

$$e^{-\tau s} = \frac{e^{-\frac{\tau}{2}s}}{e^{\frac{\tau}{2}s}} = \frac{1 - \frac{\tau}{2}s + \frac{(\tau s)^2}{8}s^2 - \dots}{1 + \frac{\tau}{2}s + \frac{(\tau s)^2}{8} + \dots}$$

$$e^{-s} = \frac{2 + \sum \frac{(-s)^i}{i!}}{2 + \sum \frac{s^i}{i!}} = \frac{2 - s + \frac{1}{2}s^2}{2 + s + \frac{1}{2}s^2} = \frac{s^2 - 2s + 4}{s^2 + 2s + 4}$$

$s^2 - 2s + 4 = 0$

$s_{n1} = \frac{2 \pm \sqrt{4-16}}{2}$

$s_{n1} = \frac{2 \pm j2\sqrt{3}}{2}$

$\boxed{s_{n1} = 1 \pm j\sqrt{3}}$

$s^2 + 2s + 4 = 0$

$s_{n2} = \frac{-2 \pm \sqrt{4-16}}{2}$

$s_{n2} = \frac{-2 \pm j2\sqrt{3}}{2}$

$\boxed{s_{n2} = -1 \pm j\sqrt{3}}$

$$G(s) = \frac{(s-1-j\sqrt{3})(s-1+j\sqrt{3})}{(s+1-j\sqrt{3})(s+1+j\sqrt{3})(s+1)}$$

2°

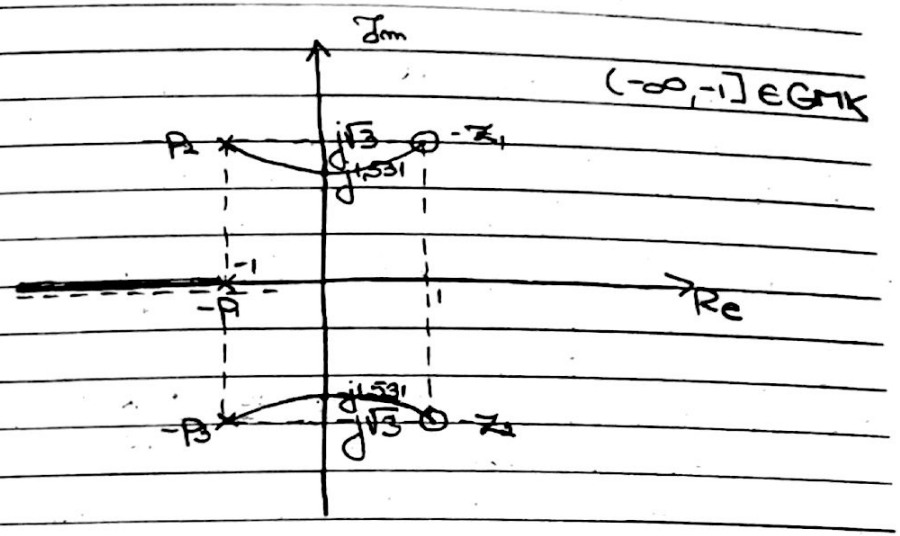
$$1 + K\frac{(s-1-j\sqrt{3})(s-1+j\sqrt{3})}{(s+1-j\sqrt{3})(s+1+j\sqrt{3})(s+1)} = 0$$

3°

$m_p = 3$ $\quad -p_1 = -1$ $\quad -z_1 = 1 + j\sqrt{3}$

$m_z = 2$ $\quad -p_2 = -1 + j\sqrt{3}$ $\quad -z_2 = 1 - j\sqrt{3}$

4°



$(-\infty, -1] \in GMK$

5°

$m_g = m_p = 3$ grane

6° GMK je simetričan u odnosu na realnu osu

7°

$$\phi_A = \frac{2q+1}{m_p - m_z} 180° \qquad q = 0$$

$q = 0 \Rightarrow \phi_A = \frac{1}{1} \cdot 180° = 180°$

8°

$$1 + K\frac{(s-1-j\sqrt{3})(s-1+j\sqrt{3})}{(s+1-j\sqrt{3})(s+1+j\sqrt{3})(s+1)} =$$

$$\frac{(s+1-j\sqrt{3})(s+1+j\sqrt{3})(s+1) + K(s-1-j\sqrt{3})(s-1+j\sqrt{3})}{(s+1-j\sqrt{3})(s+1+j\sqrt{3})(s+1)}$$

$$(s+1)(s^2+2s+4)+K(s^2-2s+4)=0$$

$$s^3+s^2+2s^2+2s+4s+4+Ks^2-2Ks+4K=0$$

$$s^3+(K+3)s^2+(6-2K)s+4+4K=0$$

s^3	1	$6-2K$
s^2	$K+3$	$4K+4$
s^1	A_1	A_2
s^0	B_1	

$$A_1=\frac{(K+3)(6-2K)-4K-4}{K+3}=\frac{6K+18-2K^2-6K-4K-4}{K+3}$$

$$\boxed{A_1=\frac{-2K^2-4K+14}{K+3}}$$

$$A_2=\frac{(K+3)\cdot 0-0\cdot 1}{K+3}=0$$

$$B_1=\frac{A_1(4K+4)-0\cdot(K+3)}{A_1}=4K+4$$

$$\frac{-2K^2-4K+14}{K+3}=0$$

$$2K^2+4K-14=0$$

$$K_{1/2}=\frac{-4\pm\sqrt{16+112}}{4}=\frac{-4\pm 11{,}31}{4}$$

$$K_1=3{,}828 \qquad \boxed{K_2=1{,}828}$$

$(-K+3)s^2+4K+4=0 \rightarrow$ pomožni polinom

$$(-1{,}828+3)s^2+4(-1{,}828+1)=0$$

$$4{,}8286s^2+11{,}312=0$$

$$s_{1/2}=\pm\sqrt{\frac{-11{,}312}{4{,}828}}=\pm j1{,}531$$

↳ točke preseka na imaginarnem osi

s^0

$$1+K\frac{(s^2-2s+4)}{(s+1)(s^2+2s+4)}=0$$

$$K=\frac{(s+1)(s^2+2s+4)}{s^2-2s+4}$$

$$K=\frac{s^3+s^2+2s^2+2s+4s+4}{s^2-2s+4}=\frac{s^3+3s^2+6s+4}{s^2-2s+4}$$

$$\frac{dK}{ds}=\frac{s^4-4s^3+16s+32}{(s^2-2s+4)^2}$$

$p_{1/2}=3{,}2101\pm j1{,}6702$

$p_{3/4}=-1{,}21\pm j0{,}9897$

$\in \mathbb{C} \rightarrow$ nema točaka odvajanja

10° uglovi napuštanja konjugovano-kompleksnih polova i nula

$$\beta_2' = \alpha_1 + \alpha_2 - \beta_1 - \beta_3 - 180° \pm k360°$$

$$\beta_1 = \beta_3 = 90°$$

$$\alpha_1 = 180°$$

$$\alpha_2 = 180° - \operatorname{arctg}\frac{2\sqrt{3}}{2} = 180° - 60° = 120°$$

$$\beta_2' = 180° + 120° - 90° - 90° - 180° = -60°$$

$$\beta_3' = \beta_2' = 60°$$

$$\alpha_1' = -\alpha_2 + \beta_1 + \beta_2 + \beta_3 = -90° + 40,89° + 60° + 0 = 190,89°$$

$$\beta_1 = \operatorname{arctg}\frac{\sqrt{3}}{2} = 40,89°$$

$$\beta_2 = 0°$$

$$\beta_3 = \operatorname{arctg}\frac{2\sqrt{3}}{2} = 60°$$

$$\alpha_2 = 90°$$

$$\alpha_2' = -\alpha_1' = -190,89°$$

STABILNOST

* Ispitati stabilnost sistema:

$$G(s) = \frac{1}{s^4 + 4s^3 + 10s^2 + 8s + 2}$$

$$P(s) = s^4 + 4s^3 + 10s^2 + 8s + 2$$

s^4	1	10	2
s^3	4	8	0
s^2	8	2	
s^1	7	0	
s^0	2		

$$A_1 = \frac{4\cdot 10 - 8\cdot 1}{4} = 8$$

$$A_2 = \frac{4\cdot 2 - 0\cdot 1}{4} = 2$$

$$B_1 = \frac{8\cdot 8 - 4\cdot 2}{8} = 7$$

$$B_2 = \frac{8\cdot 0 - 0\cdot 4}{8} = 0$$

$$C_1 = \frac{7\cdot 2 - 8\cdot 0}{7} = 2$$

polinom je AS stabilan jer nisu na elementi prve kolone istog znaka

* Ispitati stabilnost sistema:

$$G(s) = \frac{s+5}{s^6 + 3s^5 + 7s^4 + 3s^3 + 4s^2 + 6s + 2}$$

s^6	1	7	4	2
s^5	3	3	6	0
s^4	6	2	2	
s^3	2	5	0	
s^2	-13	2		

sistem nije AS stabilan

imamo 2 promjene znaka što implicira

* Ispitati stabilnost sistema u funkciji neperzinatog parametra k:

$$G(s) = \frac{s+10}{s^4+6s^3+13s^2+12s+k}$$

$$P(s) = s^4+6s^3+13s^2+12s+k$$

s^4	1	13	k
s^3	6	12	0
s^2	11	k	
s^1	$(132-6k)/11$	0	
s^0	k		

$$A_1 = \frac{6\cdot 13 - 12\cdot 1}{6} = 11$$

$$A_2 = \frac{6\cdot k - 0\cdot 1}{6} = k$$

$$B_1 = \frac{11\cdot 12 - 6k}{11} = \frac{132-6k}{11}$$

$$B_2 = \frac{11\cdot 0 - 0\cdot 6}{11} = 0$$

$$\frac{132-6k}{11} > 0$$

$$k > 0$$

$$\Rightarrow \quad 0 < k < 22$$

* Ispitati stabilnost sistema:

$$G(s) = \frac{6s+1}{s^5+2s^4+3s^3+4s^2+5s+6}$$

s^5	1	3	5
s^4	2	4	6
s^3	1	2	0
s^2	ε	6	
s^1	$\frac{2\varepsilon-6}{\varepsilon}$	0	

sistem nije AS stabilan ☹

* Ispitati stabilnost sistema:

$$P(s) = s^5+2s^4+2s^3+4s^2+11s+10$$

s^5	1	2	11
s^4	2	4	10
s^3	$\varepsilon\to 0$	6	0
s^2	$-\infty$	10	
s^1	6		
s^0	10		

$$A_1 = \frac{2\cdot 2 - 4\cdot 1}{2} = 0$$

$$A_2 = \frac{2\cdot 11 - 10\cdot 1}{2} = 6$$

$$B_1 = \frac{4\cdot\varepsilon - 6\cdot 2}{\varepsilon} = \frac{4\varepsilon-12}{\varepsilon} \qquad \lim_{\varepsilon\to 0} B_1 = \lim_{\varepsilon\to 0}\frac{4\varepsilon-12}{\varepsilon} = \infty$$

$$B_2 = \frac{10\cdot\varepsilon - 0\cdot 2}{\varepsilon} = 10$$

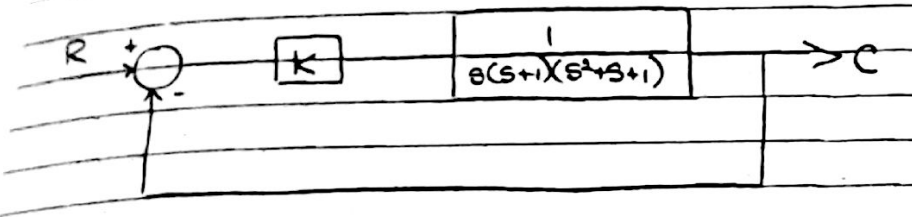
$$C_1 = \frac{\frac{4\varepsilon-12}{\varepsilon}\cdot 6 - 10\varepsilon}{\frac{4\varepsilon-12}{\varepsilon}} = \frac{24\varepsilon-72-10\varepsilon^2}{4\varepsilon-12}$$

$$\lim_{\varepsilon\to 0} C_1 = \lim_{\varepsilon\to 0}\frac{-10\varepsilon^2+24\varepsilon-72}{4\varepsilon-12} = 6$$

$$C_2 = 10$$

sistem je nestabilan, ima dva nestabilna pola budući da u prvoj koloni imamo 2 promjene znaka ☹

* Ispitati stabilnost sistema prikazanog na slici:



$$G(s) = \frac{\frac{K}{s(s+1)(s^2+s+1)}}{1+\frac{K}{s(s+1)(s^2+s+1)}} = \frac{K}{s(s+1)(s^2+s+1)+K}$$

$$P(s) = s(s+1)(s^2+s+1)+K = (s^2+s)(s^2+s+1)+K$$

$$P(s) = s^4+s^3+s^3+s^2+s^2+s+K = s^4+2s^3+2s^2+s+K$$

s^4	1	2	K
s^3	2	1	0
s^2	$\frac{3}{2}$	K	
s^1	$\frac{3-4K}{3}$	0	
s^0	K		

$$A_1 = \frac{2\cdot 2 - 1\cdot 1}{2} = \frac{3}{2}$$

$$A_2 = \frac{2\cdot K - 0\cdot 1}{2} = K$$

$$B_1 = \frac{\frac{3}{2}\cdot 1 - 2\cdot K}{\frac{3}{2}} = \frac{3-4K}{3}$$

$$B_2 = 0 \qquad C_1 = K$$

$K>0$ i $\frac{3-4K}{3}>0$

$3-4K>0$

$3>4K$

$K<\frac{3}{4}$

$0<K<\frac{3}{4}$

↳ sistem je asimptotski stabilan ☺

b) $K=\frac{3}{4} \Rightarrow \frac{3-4K}{3}=0$

imamo multi red pa formiramo pomoćni polinom ☺

$$U(s) = \frac{3}{2}s^2+\frac{3}{4}$$

$$\frac{dU(s)}{ds} = \frac{3}{2}\cdot 2\cdot s = 3s$$

s^4	1	2	$\frac{3}{4}$
s^3	2	1	0
s^2	$\frac{3}{2}$	$\frac{3}{4}$	
s^1	3	0	
s^0	$\frac{3}{4}$		

sistem je marginalno stabilan ☺

* Ispitati stabilnost sistema Hurwitzovim kriterijem ako je karakteristična jednačina:

a) $s^3+s^2+2s+24$

$$\Delta_3 = \begin{vmatrix} 1 & 24 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 24 \end{vmatrix}$$

$a_n = 1 > 0$

$$\Delta_2 = \begin{vmatrix} 1 & 24 \\ 1 & 2 \end{vmatrix} = 2-24 = -22 < 0$$

$\Delta_1 = a_{n-1} = 1 > 0$

$\Delta_3 = 24\cdot\Delta_2 = -528 < 0$