NYQUISTOV KRITERIJ

$$G(s) = k\,\frac{8s-3}{s^2-6s+11}$$

Routhov kriterij:

→ zatvorena petlja

→ pov. v.

$$G(s) = \frac{\dfrac{K(8s-3)}{s^2-6s+11}}{1+\dfrac{K(8s-3)}{s^2-6s+11}} = \frac{\dfrac{K(8s-3)}{s^2-6s+11}}{\dfrac{s^2-6s+11+8Ks-3K}{s^2-6s+11}} = \frac{K(8s-3)}{s^2-6s+11+8Ks-3K}$$

$s^2 = 1$

$s = (-6+8K)$

$s^0 = 11-3K$

$s^2 \to 1$ (a_n)

$s^1 \to 8K-6$ (a_{n-1})

$s^0 \to 11-3K$ (a_{n-2})

a_n	a_{n-2}	a_{n-4}
a_{n-1}	a_{n-3}	a_{n-5}

s^2	1	$11-3k$
s^1	$8k-6$	0
s^0	$11-3k$	

$8k-6>0$

$8k>6$

$k>\frac{3}{4}$

$11-3k>0$

$-3k>11$

$3k<11$

$k<\frac{11}{3}$

$$k \in \left[\frac{3}{4}, \frac{11}{3}\right]$$

m fajl

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s = tf('s');
G1 = (8s-3)/(s^2+6s+11);
nyquist(G1)
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DOBIJEM GRAF OVAKO

za $K=1$

$$G(s) = \frac{8s-3}{s^2-6s+11}$$

$s \rightarrow j\omega$

$$G(s) = \frac{8j\omega - 3}{(j\omega)^2 - 6j\omega + 11} = \frac{8j\omega - 3}{-\omega^2 - 6j\omega + 11} = \frac{8j\omega - 3}{(11-\omega^2) - 6j\omega} \cdot \frac{(11-\omega^2) + 6j\omega}{(11-\omega^2) + 6j\omega}$$

$$= \frac{(8j\omega - 3)\left[11 - \omega^2 + 6j\omega\right]}{(11-\omega^2)^2 + 36\omega^2} = \frac{88j\omega - 8j\omega^3 + 48(j\omega)^2 - 33 + 3\omega^2 - 18j\omega}{11^2 - 2\cdot 11\cdot\omega^2 + \omega^4 + 36\omega^2}$$

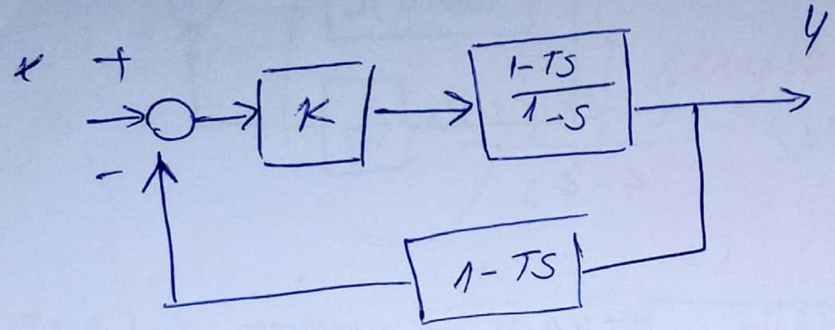
$$= \frac{88j\omega - 8j\omega^3 - 48\omega^2 - 33 + 3\omega^2 - 18j\omega}{121 - 22\omega^2 + \omega^4 + 36\omega^2} = \frac{70j\omega - 8j\omega^3 - 45\omega^2 - 33}{\omega^4 + 14\omega^2 + 121}$$

$$= \underbrace{\frac{-45\omega^2 - 33}{\omega^4 + 14\omega^2 + 121}}_{Re} + \underbrace{\frac{-8\omega^3 + 70\omega}{\omega^4 + 14\omega^2 + 121}}_{Im} \cdot j$$

ω	0	1	2
Im	0	0,45	0,39
Re	-0,27	-0,57	-1,104

NYQUISTOV K.

$T = 2$



Routhov kriterij

→ serijska pa pov veza

$\frac{K(1-2s)}{1-s}$ sa $1-2s$ $\Rightarrow G(s) = \dfrac{\frac{K(1-2s)}{1-s}}{1 + \frac{K(1-2s)}{1-s}(1-2s)} =$

$$\frac{\frac{K(1-2s)}{\cancel{1-s}}}{\frac{1-s+K(1-2s)(1-2s)}{\cancel{1-s}}} = \frac{K(1-2s)}{1-s+K(1-2s-2s+4s^2)}$$

$$= \frac{K(1-2s)}{1-s+K-4Ks+4Ks^2}$$

$-1-4K$

$s^2 \to 4K$ (a_n)

$s^1 \to -1-4K$ (a_{n-1})

$s^0 \to 1+K$ (a_{n-2})

s^2	$4K$	$1+K$
s^1	$-1-4K$	0
s^0	$1+K$	

a_n a_{n-2} a_{n-4}

a_{n-1} a_{n-3}

$4K > 0$ $\quad -1-4K > 0$ $\quad 1+K > 0$

$K > 0$ $\quad -4K > 1$ $\quad K > -1$

$4K < 1$

$K < \frac{1}{4}$ $\quad K \in [0, \frac{1}{4}]$

S obzirom da Nik

2a $K = 0{,}25 \ (\frac{1}{4})$

Nyquistov k.

$\Rightarrow$ otvorena petlja

sve u seriji!

$$G(s) = \frac{1}{4} \cdot \frac{1-2s}{1-s} \cdot (1-2s) = \frac{(1-2s)(1-2s)}{4-4s}$$

$s \Rightarrow j\omega$

$$G(j\omega) = \frac{(1-2j\omega)(1-2j\omega)}{4-4j\omega} \cdot \frac{4+4j\omega}{4+4j\omega}$$

$$G(j\omega) = \frac{(1-2j\omega-2j\omega+4(j\omega)^2)\cdot(4+4j\omega)}{4^2-(4j\omega)^2}$$

$$G(j\omega) = \frac{(1-4j\omega-4\omega^2)(4+4j\omega)}{16+16\omega} = \frac{4+4j\omega-16j\omega-16(j\omega)^2-16\omega^2-16j\omega^3}{16+16\omega}$$

$$G(j\omega) = \frac{4+4j\omega-16j\omega+\cancel{16\omega^2}-\cancel{16\omega^2}-16j\omega^3}{16+16\omega} = \frac{4-12j\omega-16j\omega^3}{16+16\omega}$$

$$G(j\omega) = \frac{4}{16+16\omega} + \frac{-12\omega-16\omega^3}{16+16\omega} \cdot j$$

$$\boxed{G(j\omega) = \frac{1}{4+4\omega} + \frac{-3\omega-4\omega^3}{4+4\omega} \cdot j}$$

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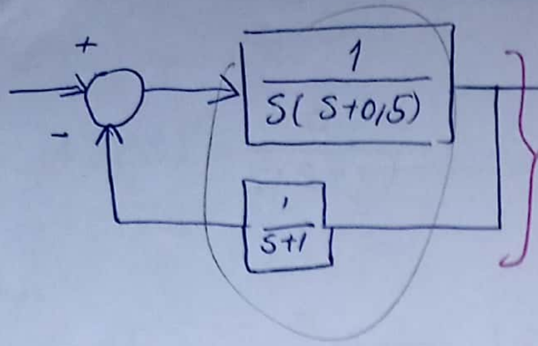
② Zadan je sistem na slici



serijska veza

S obzirom da Nikvist radi u otvorenoj petlji ✓

$$G(S) = \frac{1}{s(s+0,5)} \cdot \frac{1}{s+1} = \frac{1}{s(s+0,5)(s+1)}$$

$s \rightarrow j\omega$

$$G(S) = \frac{1}{j\omega(j\omega+0,5)(j\omega+1)} \cdot \frac{-j\omega(j\omega-0,5)(j\omega-1)}{-j\omega(j\omega-0,5)(j\omega-1)}$$

$$G(S) = \frac{-j\omega(j\omega-0,5)(j\omega-1)}{(-j\omega)^2\left[(j\omega)^2-0,5^2\right]\left[(j\omega)^2-1^2\right]} =$$

$$G(S) = \frac{-j\omega(j\omega-0,5)(j\omega-1)}{-\omega^2(-\omega^2-0,25)(-\omega^2-1)} = \frac{+j\omega(j\omega-0,5)(j\omega-1)}{+\omega^2(-1)\cdot(-1)(\omega^2+0,25)(\omega^2+1)}$$

$$G(S) = \frac{j\omega(j\omega-0,5)(j\omega-1)}{\omega^2(\omega^2+0,25)(\omega^2+1)} = \frac{j(j\omega-0,5)(j\omega-1)}{\omega(\omega^2+0,25)(\omega^2+1)}$$

$$= \frac{(j^2\omega-j0,5)(j\omega-1)}{\omega(\omega^4\omega^2+0,25\omega^2+0,25)} = \frac{(-\omega-j0,5)(j\omega-1)}{\omega(\omega^2+0,25)(\omega^2+1)} = \frac{-j\omega^2+\omega-0,5\omega j^2+j0,5}{\omega(\omega^2+0,25)(\omega^2+1)}$$

$$= \frac{\omega+0,5\omega+j0,5-j\omega^2}{\omega(\omega^2+0,25)(\omega^2+1)}$$

$$G(S)_{Re} = \frac{\cancel{\omega}(1+0,5)}{\cancel{\omega}(\omega^2+0,25)(\omega^2+1)} \Rightarrow \boxed{G(S)_{Re} = \frac{1,5}{(\omega^2+0,25)(\omega^2+1)}}$$

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$$G(S)Im = \frac{j(0{,}5-\omega^2)}{\omega(\omega^2+0{,}25)(\omega^2+1)}$$

$$G(S)Im = \frac{0{,}5-\omega^2}{\omega(\omega^2+0{,}25)(\omega^2+1)} j$$

ω	0	0,4	$\sqrt{0{,}5}$	1	∞
Re	-6	-3,15	-1,36	-0,6	0
Im	$-\infty$	-1,78	-0,013	0,2	0

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Nyquistov kriterij

1.

$$W_o(s) = \frac{5}{s(s+5)}$$

$s \to j\omega$

$$W_o(j\omega) = \frac{5}{j\omega(j\omega+5)}$$

$$W_o(j\omega) = \frac{5}{(j\omega)^2 + 5j\omega} \cdot \frac{(j\omega)^2 - 5j\omega}{(j\omega)^2 - 5j\omega}$$

$$W_o(j\omega) = \frac{5\cdot(j\omega)^2 - 25j\omega}{(j\omega)^4 - (5j\omega)^2}$$

$$W_o(j\omega) = \frac{5(-1)\omega^2 - 25j\omega}{\omega^4 + 25\omega^2}$$

$$W_o(j\omega) = \frac{-5\omega^2 - 25j\omega}{\omega^4 + 25\omega^2}$$

$$W_o(j\omega)_{Re} = \frac{-5\omega^2}{\omega(\omega^3+25\omega)}$$

$$W_o(j\omega)_{Re} = \frac{-5\omega}{\omega^3+25\omega} \Rightarrow \boxed{W_o(j\omega)_{Re} = \frac{-5}{\omega^2+25}}\ Re$$

$$W_o(j\omega)_{Im} = \frac{-25j\omega}{\omega(\omega^3+25\omega)} \Rightarrow W_o(j\omega)_{Im} = \frac{-25}{\omega^3+25\omega} j$$

$$\boxed{W_o(j\omega)_{Im} = \frac{-25}{\omega(\omega^2+25)} j}\ Im$$

$\cancel{j\omega^2}$

$j = j$

$j^2 = -1$

$j^3 = -j$

$j^4 = 1$

ω	0	1	5	∞
Re	-0,2	-0,19	-0,1	0
Im	-∞	-0,96	-0,1	0

— | 0,55 k_r | — | —

NYQUISTOV K.

$$W(s) = k \cdot \frac{8s-3}{s^2-6s+11}$$

1x ✓

→ Routhov k.

→ zatvorena petlja

→ jed. pov. k.

$$G(s) = \frac{\frac{k(8s-3)}{s^2-6s+11}}{1+\frac{k(8s-3)}{s^2-6s+11}} = \frac{\frac{k(8s-3)}{s^2-6s+11}}{\frac{s^2-6s+11+8ks-3k}{s^2-6s+11}} = \frac{k(8s-3)}{s^2-6s+8ks+11-3k}$$

$s^2 \Rightarrow 1$ (a_n)

$s^1 \Rightarrow 8k-6$ (a_{n-1})

$s^0 \Rightarrow 11-3k$ (a_{n-2})

a_n a_{n-2} a_{n-4}

a_{n-1} a_{n-3} a_{n-5}

s^2	1	11-K	0
s^1	8k-6	0	
s^0	11-3k		

$8k-6>0$

$8k>6$

$k>\frac{3}{4}$

$11-3k>0$

$-3k>11$

$3k<11$

$k<\frac{11}{3}$

$k \in [\frac{3}{4}, \frac{11}{3}]$

za $k=1$

nyquistov k.

$$\frac{8s-3}{s^2-6s+11} = G(s) \qquad s \to j\omega$$

$$G(j\omega) = \frac{8j\omega-3}{(j\omega)^2-6j\omega+11} = \frac{8j\omega-3}{-\omega^2-6j\omega+11} = \frac{8j\omega-3}{(11-\omega^2)-6j\omega} \cdot \frac{(11-\omega^2)+6j\omega)}{(11-\omega^2)+6j\omega}$$

ne znam dalje

NYQUISTOV K.

$$G(s) = \frac{K_R}{s^3+3s^2+2s}$$

Routhov k.
⇒ zatvorena petlja
⇒ jed. pov. v.

$$F(s) = \frac{\frac{K_R}{s^3+3s^2+2s}}{1+\frac{K_R}{s^3+3s^2+2s}} = \frac{\frac{K_R}{\cancel{s^3+3s^2+2s}}}{\frac{s^3+3s^2+2s+K_R}{\cancel{s^3+3s^2+2s}}} = \frac{K_R}{s^3+3s^2+2s+K_R}$$

$s^3 \Rightarrow 1$ (a_n)
$s^2 \Rightarrow 3$ (a_{n-1})
$s^1 \Rightarrow 2$ (a_{n-2})
$s^0 \Rightarrow K_R$ (a_{n-3})

a_n a_{n-2} a_{n-4}
a_{n-1} a_{n-3} a_{n-5}

s^3	1	2
s^2	3	K_R
s^1		
s^0	K_R	

$$b = \frac{3\cdot 2 - K_R}{3} = \frac{6-K_R}{3}$$

$$6-K_R > 0$$
$$-K_R > -6$$
$$K_R < 6$$

$$K_R \in [0, 6]$$

za $K_R = 1$
Nyquistov k.
⇒ otvorena petlja
$s \Rightarrow j\omega$

$$G(j\omega) = \frac{1}{(j\omega)^3+3(j\omega)^2+2j\omega} = \frac{1}{-j\omega^3-3\omega^2+2j\omega} = \frac{1}{(2j\omega-j\omega^3)-3\omega^2}$$

$$= \frac{1}{(2j\omega-j\omega^3)-3\omega^2} \cdot \frac{(2j\omega-j\omega^3)+3\omega^2}{(2j\omega-j\omega^3)+3\omega^2} =$$

$$= \frac{2j\omega - j\omega^3 + 3\omega^2}{(2j\omega - j\omega^3)^2 - (3\omega^2)^2} = \frac{2j\omega - j\omega^3 + 3\omega^2}{\left[-4\omega^2 - 2\cdot 2j\omega \cdot j\omega^3 + (j\omega^3)^2\right] - 9\omega^4}$$

$$= \frac{2j\omega - j\omega^3 + 3\omega^2}{-4\omega^2 + 4\omega^4 - \omega^5 - 9\omega^4} = \frac{2j\omega - j\omega^3 + 3\omega^2}{-4\omega^2 - \omega^5 - 5\omega^4}$$

$$= \frac{\cancel{\omega}(2-\omega^2)}{\cancel{\omega}(-4\omega - \omega^4 - 5\omega^3)} j + \frac{3\omega^2}{\cancel{\omega}(-4\omega - \omega^4 - 5\omega^3)}$$

$$\boxed{= \frac{2-\omega^2}{-4\omega - \omega^4 - 5\omega^3} j + \frac{3\omega}{-4\omega - \omega^4 - 5\omega^3}}$$