

$$\angle a = -2,5$$

$$\angle a = 71,56^\circ$$

GMK

geometrijsko mjesto korijena

①

$$W(s) = \frac{k}{s(s+2)(s+3)} \quad \text{skicirajte GMK sistem}$$

led

nule - u brojniku
polovi - u nazivniku

nule - m? $\sum z_i$
nema ih $\Rightarrow m=0$

polovi - n? $\sum p_i$

$$n=3$$

$$s=0 \quad s=-2 \quad s=-3$$

\Rightarrow GMK je simetrično u odnosu na Re-osu?

* Tacka presjeka asimptota:

$$\angle a = \frac{\sum_{i=1}^n p_i - \sum_{j=1}^m z_j}{n-m}$$

$$\angle a = \frac{0-2-3-0}{3-0} = \frac{-5}{3} \left(-1 \frac{2}{3} \right)$$

* Uglovi asimptota:

$$\phi_k = \frac{(2k+1)\pi}{n-m}$$

za $k=0,1,2 \dots (n-m)$

$$\phi_0 = \frac{(2 \cdot 0 + 1)\pi}{3} = \frac{1}{3}\pi = 60^\circ$$

$$\phi_1 = \frac{(2 \cdot 1 + 1)\pi}{3} = \pi = 180^\circ$$

$$\phi_2 = \frac{(2 \cdot 2 + 1)\pi}{3} = \frac{5}{3}\pi = 300^\circ = -60^\circ$$

$\angle a = -2,5$ $\angle_{L(1)} = 71,56^\circ$
 * gledamo u kojem opsegu nam se $\angle a$ nalazi u odnosu na polove.

$0 < \angle a < -2$
 pripada

$-2 < \angle a < -3$
 ne pripada

$-3 > \angle a$

→ intervali preklapanja GPK i Re ste:
 $[0, -2]$ i $[-3, \infty)$

* Tačke grananja i sjediničenja:

$$\sum_{i=1}^n \frac{1}{\sigma_0 - p_i} - \sum_{j=1}^m \frac{1}{\sigma_0 - z_j} = 0$$

$$\frac{1}{s} + \frac{1}{s+2} + \frac{1}{s+3} = 0$$

$$3s^2 + 10s + 6 = 0$$

$$s_{1,2} = \frac{-10 \pm \sqrt{100 - 72}}{6}$$

→ ne pripada ~~GPK~~ geom. kor. sistema.

$$s_1 = -2,55$$

$$s_2 = -0,785 \checkmark$$

* izlazni uglovi:
 svi su 180°

* $f(s) = s^3 + 5s^2 + 6s + K \leftarrow$ Routhov kriterij

$$a_n = 1 (s^3)$$

$$a_{n-1} = 5 (s^2)$$

$$a_{n-2} = 6 (s)$$

$$a_{n-3} = K (s^0)$$

$$n = 3$$

s^3	1	6
s^2	5	K
s^1	$\frac{30-K}{5}$	
s^0	K	

→ to je zatvorena petlja pa:
 $1 + \frac{K}{s(s+2)(s+3)}$
 i dobijemo $f(s)$

$$b_1 = \frac{5 \cdot 6 - K \cdot 1}{5} = \frac{30 - K}{5}$$

Kad se Routhov kriterij primjenjuje kod GPK ne vrši se množenje cijele lopolne sa nekim brojem !!!

$$K > 0$$

$$\frac{30-K}{5} > 0$$

$$30-K > 0$$

$$-K > -30$$

$$K < 30$$

$$0 < K < 30$$

Za $\begin{cases} K=30 \\ K=0 \end{cases} \Rightarrow$ sistem je granično stabilan

$K=0$ odgovara graničnom, početnom stanju, tj $P_1=0$ tako da se ova vrijednost ne razmatra

pa je

$$K=30$$

$$a_0 = K = 30$$

to je s^2 (tj $R_{n-1} = a_{n-1}$, a $n=3$)

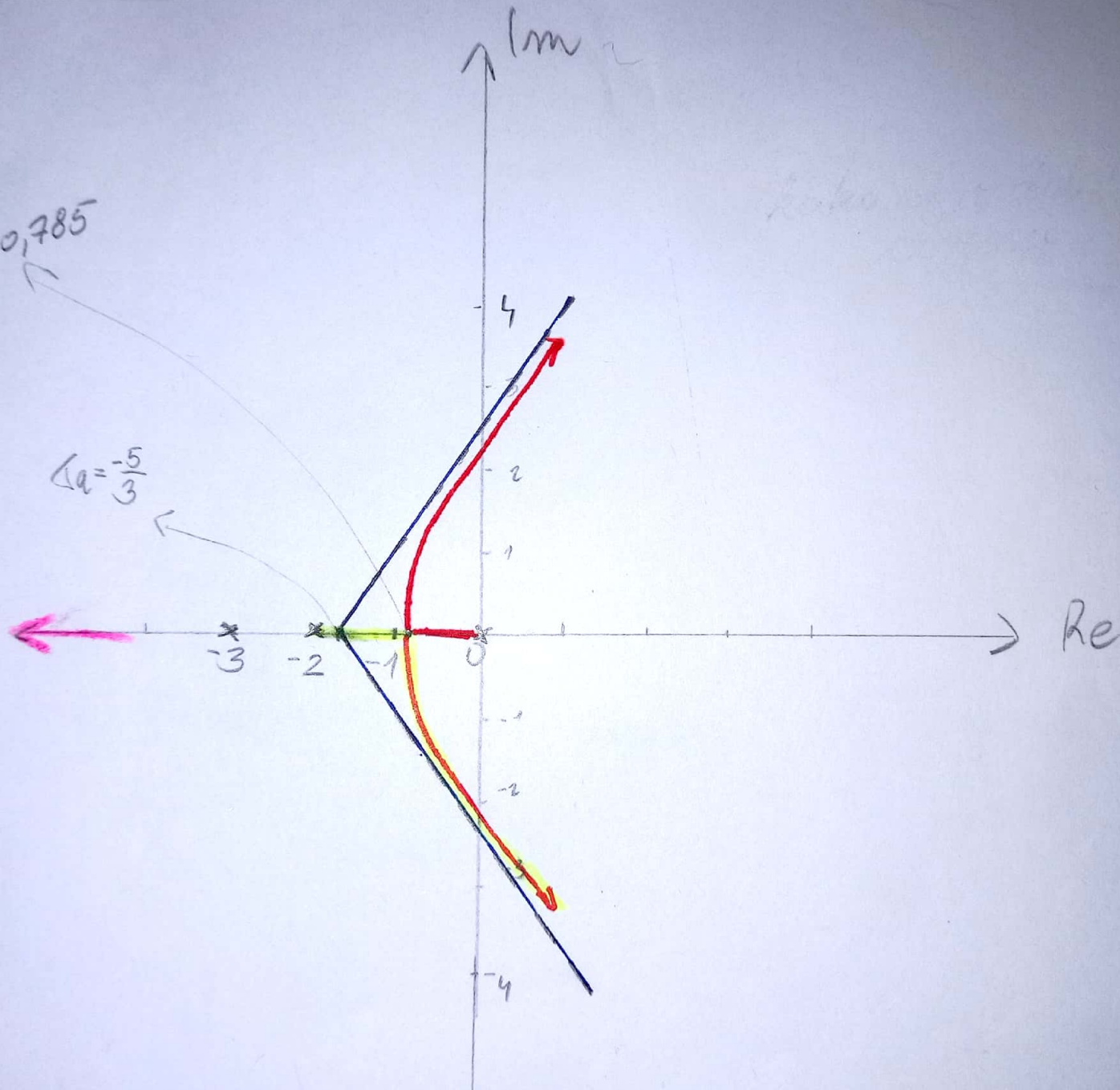
$$R_{n-1} = R_{3-1} = R_2 = (5)$$

$$\omega_{gr} = \sqrt{\frac{a_0}{R_{n-1}}}$$

$$\omega_{gr} = \sqrt{\frac{30}{5}} \approx \pm 2,46 \text{ rad/s.}$$

$$\sigma_0 = -0,785$$

$$\sigma_a = -\frac{5}{3}$$



(2) $G(s) = K \cdot \frac{s+2}{(s+1)(s^2+6s+10)}$ skicirajte GMK pistom

* nule:

$$s = -2$$

$$m = 1$$

↓

$$m = \infty = 1$$

polovi:

$$s = -1$$

$$s^2 + 6s + 10 = 0 \Rightarrow s_{1,2} = \frac{-6 \pm \sqrt{36 - 40}}{2}$$

$$s_1 = -3 - i$$

$$s_{1,2} = \frac{-6 \pm 2i}{2}$$

$$s_2 = -3 + i$$

$$s = -3 - i$$

$$s = -3 + i$$

$$n = 3$$

* tačka presjeka asimptota:

$$\sigma_a = \frac{\sum_{i=1}^n p_i - \sum_{j=1}^m z_j}{n-m} = \frac{-1 - 3 - i - 3 + i - (-2)}{3-1} = \frac{-5}{2} = -2,5$$

* uglovi asimptota:

$$\phi_k = \frac{(2k+1)\pi}{n-m}$$

$k = 0, 1, \dots$ ~~... kako smo 0 i 1? ali je to isto? ...~~

$$\phi_0 = \frac{(2 \cdot 0 + 1)\pi}{2} = \frac{\pi}{2} = 90^\circ$$

$$\phi_1 = \frac{(2 \cdot 1 + 1)\pi}{2} = \frac{3\pi}{2} = 270^\circ = -90^\circ$$

$$\phi_2 = \frac{(2 \cdot 2 + 1)\pi}{2} = \frac{5\pi}{2} = \text{a čisto?}$$

*

$$\sigma_a > -1$$

$$\sigma_a > -2$$

$$-1 > \sigma_a > -2$$

Izolarni uglovi 180° na Re osi

$$\varphi_{p1} = \arctg \frac{1}{1} = 45^\circ \quad 180^\circ - 45^\circ = 135^\circ$$

$$\varphi_{p2} = \arctg \frac{1}{2} = 26,56^\circ \quad 180^\circ - 26^\circ = 154^\circ$$

$$\varphi_p = 180^\circ + 135^\circ - 90^\circ - 154^\circ = 71^\circ$$

$$* \quad f(s) = s^3 + 7s^2 + (16+k)s + 10 + 2k$$

↙ zatvorena petlja zbog
Routhovog kriterija, tj:

$$1 + \frac{K(s+2)}{(s+1)(s^2+6s+10)}$$

$$a_n = 1$$

$$a_{n-1} = 7$$

$$a_{n-2} = 16+k$$

$$a_{n-3} = 10+2k$$

s^3	1	$16+k$
s^2	7	$10+2k$
s^1	$\frac{102+5k}{7}$	
s^0	$10+2k$	

$$b_1 = \frac{7 \cdot (16+k) - (10+2k)}{7}$$

$$b_1 = \frac{112 + 7k - 10 - 2k}{7}$$

$$b_1 = \frac{102 + 5k}{7}$$

$$102 + 5k > 0$$

$$5k > -102$$

$$k > \frac{-102}{5}$$

$$10 + 2k > 0$$

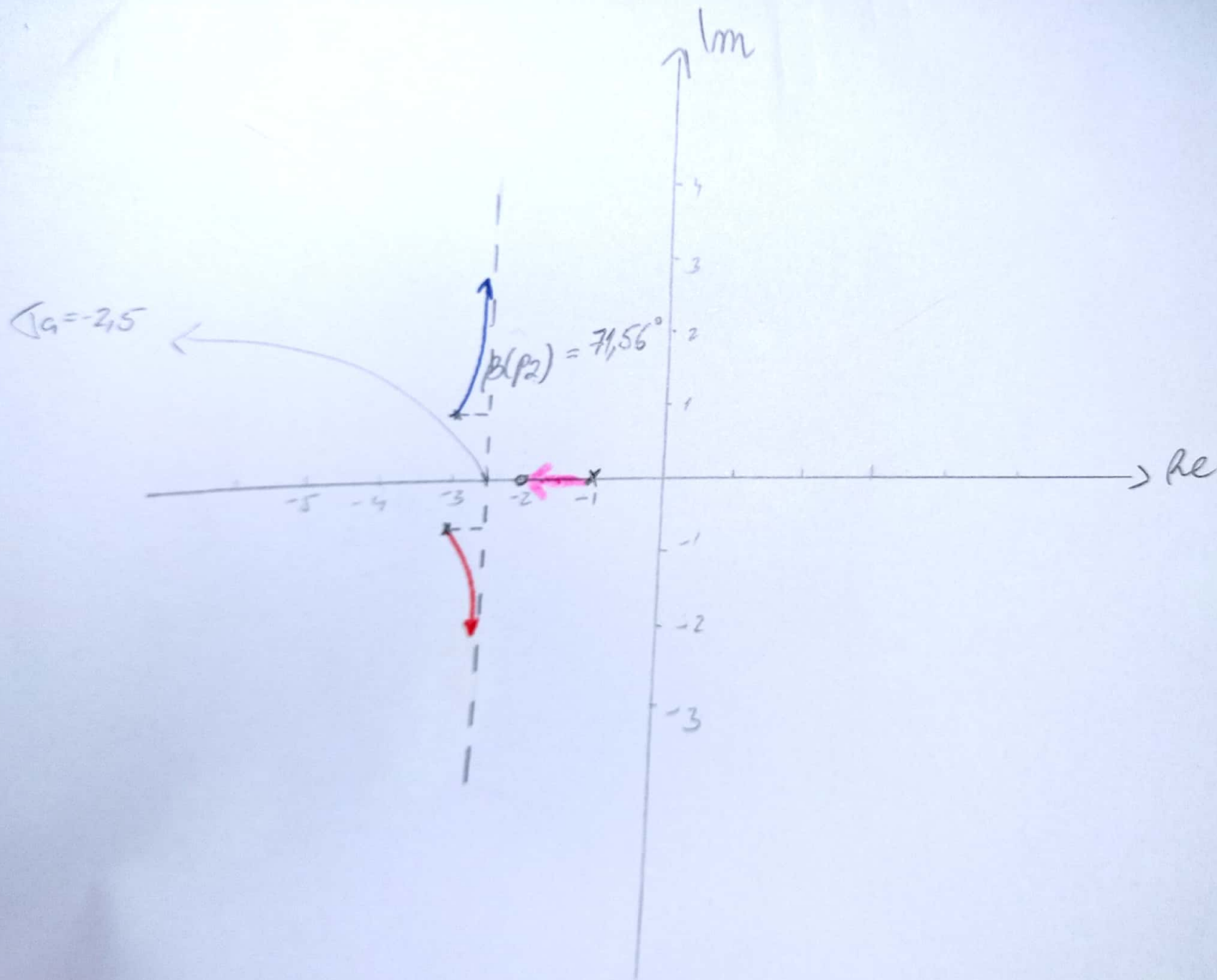
$$2k > -10$$

$$k > -5$$

$$\forall k > 0$$

\Rightarrow sistem je uvijek stabilan

o \rightarrow pole
x \rightarrow zero



(3.)

$$G_S(0) = \frac{k_0(s+1)}{s(s+2)(s^2+12s+40)}$$

→ nacrtati za
ovaj
primjer

$$m=1$$

$$n=4$$

nule:

$$s=-1$$

polovi:

$$s=0$$

$$s=-2$$

$$s^2+12s+40=0$$

$$s = -6 - 2j$$

$$s = -6 + 2j$$

* tačka presjeka asimptota

$$\sigma_a = \frac{\sum_{i=1}^n p_i - \sum_{j=1}^m z_j}{n-m} = \frac{0-2-6-2j-6+2j-(-1)}{4-1} = \frac{-14+1}{3} = \frac{-13}{3} = -4,33$$

* uglovi asimptota:

$$\phi_k = \frac{(2k+1)\pi}{n-m}$$

$$L = n-m = 4-1 = 3 \Rightarrow k = 0, 1, 2$$

$$\phi_0 = \frac{\pi}{3} = 60^\circ$$

$$\phi_1 = 180^\circ \quad \phi_2 = -60^\circ$$

*

$$-1 < \sigma_a < 0 \quad -2 > 0$$

* tačka spajanja ili gramanja je između 0 i -1 i lijevo od -2

$$\frac{1}{s} + \frac{1}{s+2} + \frac{1}{s+6-2j} + \frac{1}{s+6+2j} = \frac{1}{s+1}$$

$$3s^4 + 22s^3 + 106s^2 + 128s + 80 = 0$$

$$s_{v1} = s_{v1} = -3,68 \quad s_{v2} = -5,47$$

$$s_{v3} = -0,76 + 0,865j \quad s_{v4} = -0,76 - 0,865j$$

* izlaskni uglovi

Suma Re su 180° , pa čina se za one 'nam' - računamo pomoću:

$$\varphi_2 = \arctg \frac{2}{4} = 16,6^\circ \leftarrow \text{odakle } \frac{2}{4} ?$$

$$\varphi_1 = 153,4^\circ$$

$$\varphi_3 = -90^\circ$$

$$\varphi_4 = 158,2^\circ$$

$$G_0(s) = \frac{k(s+1)}{s(s-1)(s^2+4s+16)}$$

*

$$m = 1$$

$$s = -1$$

$$n = 4$$

$$s = 0$$

$$s = 1$$

$$s^2 + 4s + 16 = 0$$

$$s_{1,2} = \frac{-4 \pm \sqrt{16 - 64}}{2}$$

$$s_{1,2} = \frac{-4 \pm 4,28}{2}$$

$$s = -4,6 \quad s = -0,36$$

* tačka presjeka asimptota:

$$\sigma_a = \frac{\sum_{i=1}^n p_i - \sum_{j=1}^m z_j}{n-m} = \frac{0+1-4,6-0,36+1}{3} = \frac{-4,96}{3} = -1,65$$

* uglovi asimptota

$$k = 0, 1, 2$$

$$\phi_0 = \frac{(2 \cdot 0 + 1)\pi}{3} = \frac{\pi}{3} = 60^\circ$$

$$\phi_1 = \frac{(2 \cdot 1 + 1)\pi}{3} = 180^\circ$$

$$\phi_2 = -60^\circ$$