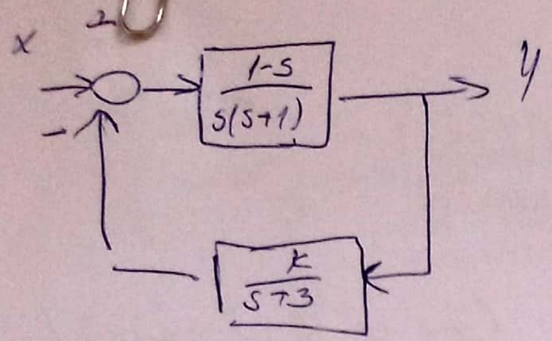
Routhov i Nyquistov k.



Routhov k.

-> zatvorena petlja

-> pov. veza

$$G(s) = \frac{\frac{1-s}{s(s+1)}}{1+\frac{1-s}{s(s+1)}\cdot\frac{k}{s+3}} = \frac{\frac{1-s}{\cancel{s(s+1)}}}{\frac{s(s+1)(s+3)+k(1-s)}{\cancel{s(s+1)}(s+3)}} = \frac{(s+3)(1-s)}{s(s^2+4s+3)+k(1-s)}$$

$$= \frac{(s+3)(1-s)}{s^3+4s^2+3s+k-ks} = \frac{(s+3)(1-s)}{s^3+4s^2+3s-ks+k}$$

$s^3 \Rightarrow 1$ (a_n)

$s^2 \Rightarrow 4$ (a_{n-1})

$s \rightarrow 3-k$ (a_{n-2})

$s^0 \Rightarrow k$ (a_{n-3})

a_n a_{n-2} a_{n-4}

a_{n-1} a_{n-3} a_{n-5}

s^3	1	3-k	0
s^2	4	k	0
s^1			
s^0	k		

$$b_1 = \frac{4(3-k)-k}{4} = \frac{12-4k-k}{4} = \frac{12-5k}{4}$$

$12-5k > 0$

$-5k > -12$

$5k < 12$

$k < \frac{12}{5}$

$k \in [0, \frac{12}{5}]$

za $k=1$

Nyquistov k.

=> serijska veza

-> otvorena petlja

$$G(s) = \frac{(1-s)1}{s(s+1)(s+3)}$$

$s \Rightarrow j\omega$

$$G(j\omega) = \frac{(1-j\omega)}{j\omega(j\omega+1)(j\omega+3)} \cdot \frac{-j\omega(j\omega-1)(j\omega-3)}{-j\omega(j\omega-1)(j\omega-3)}$$

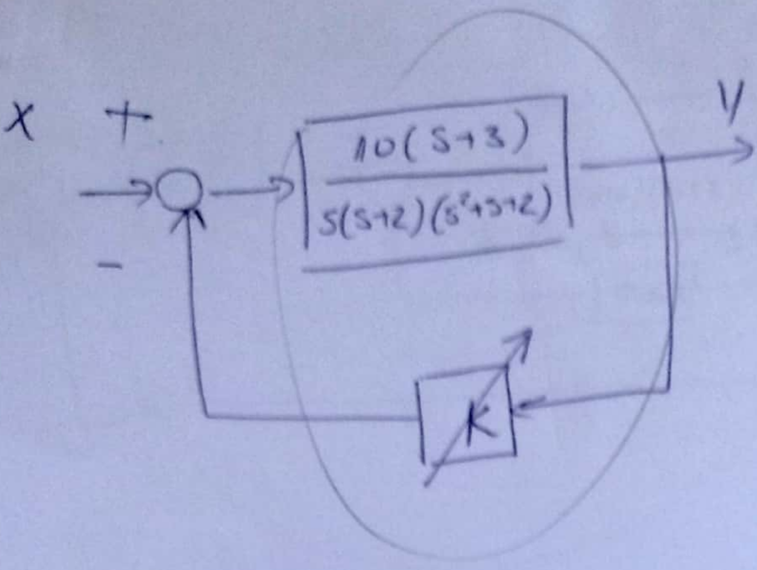
$$G(j\omega) = \frac{(1-j\omega)[(j\omega)^2-3j\omega-j\omega+3]\cdot(-j\omega)}{-(j\omega)^2((j\omega)^2-1^2)((j\omega)^2-3^2)} = \frac{(-j\omega-\omega^2)(-\omega^2-4j\omega+3)}{\omega^2(-\omega^2-1)(-\omega^2-9)}$$

$$G(j\omega) = \frac{j\omega^3+4(j\omega)^2-3j\omega+\omega^4+4j\omega^3-3\omega^2}{-\omega^2(\omega^2+1)(\omega^2+9)}$$

$$G(j\omega) = \frac{j\omega^3-4\omega^2-3j\omega+\omega^4+4j\omega^3-3\omega^2}{-\omega^2(\omega^2+1)(\omega^2+9)} = \frac{\omega^4+5j\omega^3-7\omega^2-3j\omega}{-\omega^2(\omega^2+1)(\omega^2+9)}$$

$$= \frac{\cancel{\omega}(-\omega^3-5j\omega^2+7\omega+3j)}{-\omega^2(\omega^2+1)(\omega^2+9)} = \underbrace{\frac{-\omega^3+7\omega}{\omega(\omega^2+1)(\omega^2+9)}}_{Re} + \underbrace{\frac{3-5\omega^2}{\omega^2(\omega^2+1)(\omega^2+9)}j}_{Im}$$

ROUTHOV KRITERIJ



pov. v

$$W(s) = \frac{10(s+3)}{s^4+3s^3+4s^2+s(4+10K)+30K}$$

$s^4 \Rightarrow 1 \quad (a_n)$

$s^3 \Rightarrow 3 \quad (a_{n-1})$

$s^2 \Rightarrow 4 \quad (a_{n-2})$

$s \Rightarrow 4+10K \quad (a_{n-3})$

$s^0 \Rightarrow 30K \quad (a_{n-4})$

$n=4$

$a_n \quad a_{n-2} \quad a_{n-4}$

$a_{n-1} \quad a_{n-3} \quad a_{n-5}$

s^4	1	4	$30K$
s^3	3	$4+10K$	0
s^2	$\cancel{8-10k}$ $\frac{8-10K}{3}$	$\cancel{90K}$ $30K$	$0/3$
s^1	$\frac{-100K^2-230K+32}{8-10K}$		
s^0	$30K$		

$$b_1 = \frac{3\cdot 4 - 4 - 10K}{3} = \frac{8-10K}{3}$$

$8-10K > 0$

$-10K > 8$

$K < \frac{4}{5}$

$$b_2 = \frac{90K - 0}{3} = 30K$$

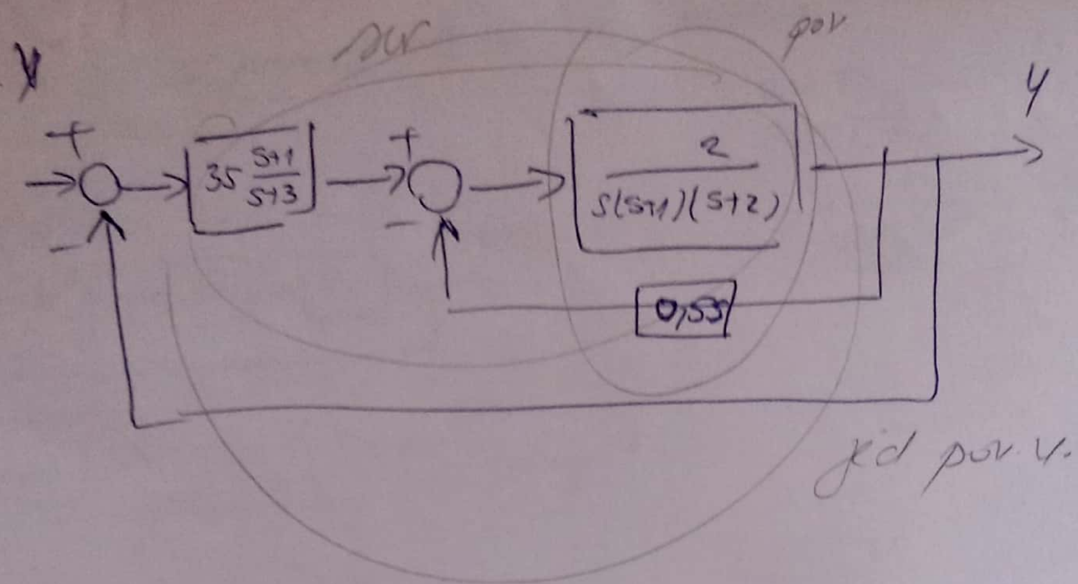
$$c_1 = \frac{(8-10K)(4+10K) - 3\cdot 90K}{\frac{8-10K}{3}} \Rightarrow c_1 = \frac{-100K^2-230K+32}{8-10K}$$

$-100k^2 - 230k + 32 > 0$

$$k_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$k_1 = -0{,}1488 \quad k_2 = -2{,}152$

ROUTHOV KRITERIJ



$$W(s) = \frac{70(s+1)}{s^4+6s^3+6s^2+79s+70}$$

$s^4 \Rightarrow 1 \quad a_n \quad (s^4)$

$s^3 \rightarrow 6 \quad a_{n-1} \quad (s^3)$

$s^2 \Rightarrow 6 \quad a_{n-2} \quad (a_2)$

$s^1 \rightarrow 79 \quad a_{n-3}$

$s^0 \rightarrow 70 \quad a_{n-4}$

a_n	a_{n-2}	a_{n-4}
a_{n-1}	a_{n-3}	a_{n-5}

s^4	1	6	70
s^3	6	79	0
s^2	$-\frac{43}{6}$ -4,3	70	1.6
s	15,76		
s^0	70		

↓ nestabilan

$$b_1 = \frac{6\cdot 6-79\cdot 1}{6} = \frac{36-79}{6} = \frac{-43}{6}$$

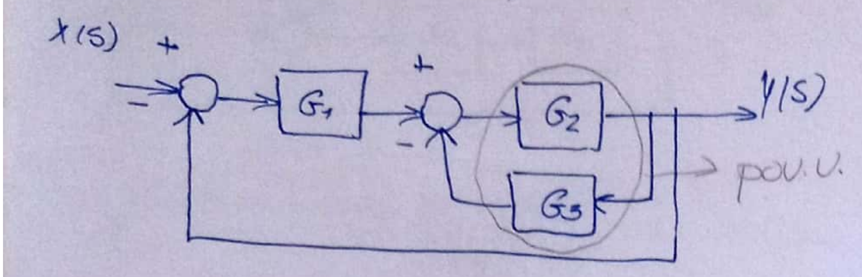
$$b_2 = \frac{6\cdot 70-0\cdot 1}{6} = 70$$

$$c_1 = \frac{-\frac{43}{6}\cdot 79-70\cdot 6}{-\frac{43}{6}} = 15,76$$

KRITERIJI

→ ROUTHOV

① Odrediti stabilnost pomoći Routhovog kriterija



$$G_1(S) = K_1 \cdot \frac{S+T_1}{S+T_2} \qquad K_1 = 35$$

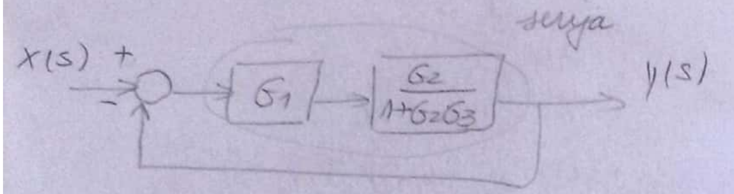
$$G_2(S) = \frac{K_2}{S(S+T_1)(S+T_3)} \qquad K_2 = 2$$

$$G_3(S) = K_3 S \qquad K_3 = 0{,}5$$

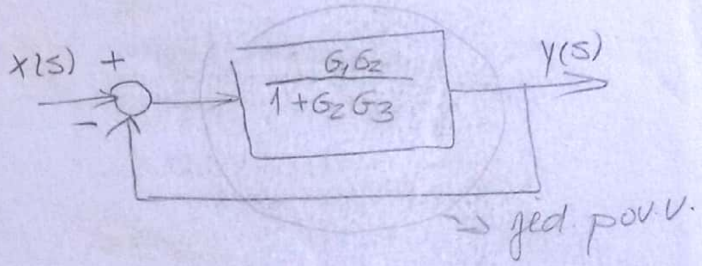
$T_1 = 1$

$T_2 = 3$

$T_3 = 2$



⇒



$$G_0(S) = \frac{\frac{G_1G_2}{1+G_2G_3}}{1+\frac{G_1G_2}{1+G_2G_3}} = \frac{\frac{G_1G_2}{1+G_2G_3}}{\frac{1+G_2G_3+G_1G_2}{1+G_2G_3}} = \frac{G_1G_2}{1+G_2G_3+G_1G_2} = \frac{G_1G_2}{1+G_2(G_1+G_3)}$$

$$G_0(S) = \frac{\frac{35(S+1)}{S+3}\cdot\frac{2}{S(S+1)(S+2)}}{1+\frac{2}{S(S+1)(S+2)}\cdot\left(\frac{35(S+1)}{S+3}+0{,}5S\right)} = \frac{\frac{70(S+1)}{S(S+1)(S+2)(S+3)}}{1+\frac{2(35(S+1)+0{,}5S(S+3))}{S(S+1)(S+2)(S+3)}}$$

$$= \frac{\frac{70(S+1)}{S(S+1)(S+2)(S+3)}}{\frac{S(S+1)(S+2)(S+3)+70(S+1)+S(S+3)}{S(S+1)(S+2)(S+3)}} = \frac{70(S+1)}{(S^2+S)(S^2+5S+6)+70S+70+S^2+3S}$$

$$= \frac{70(S+1)}{S^4+5S^3+6S^2+S^3+5S^2+6S+70S+70+S^2+3S} = \frac{70(S+1)}{S^4+6S^3+9S^2+79S+70}$$

$a_n(S^4) = 1$

$a_{n-1}(S^3) = 6$

$a_{n-2}(S^2) = 9$

$a_{n-3}(S) = 79$

$a_{n-4}(S^0) = 70$

S^4	1	9	70
S^3	6	79	0
S^2	−4,16	70	
S^1	180,09		
S^0	70		

$$b_1 = \frac{6\cdot 9 - 79}{6} = \frac{-25}{6} = -4{,}16$$

$$b_2 = \frac{6\cdot 70 - 0}{6} = 70$$

$$c_1 = \frac{-4{,}16\cdot 79 - 70\cdot 6}{-4{,}16} = \frac{-329{,}16-420}{-4{,}16} = 180{,}09$$

⇒ sistem je nestabilan ima dva prijelaza u desnoj poluravni

→ HURWITZOV

② Odrediti stabilnost pomoću Hurwitzovog kriterija

$G_1 = Ts \qquad G_2 = \dfrac{K_1}{1+T_1 s} \qquad G_3 = \dfrac{K_2}{1+T_2^2 s^2}$

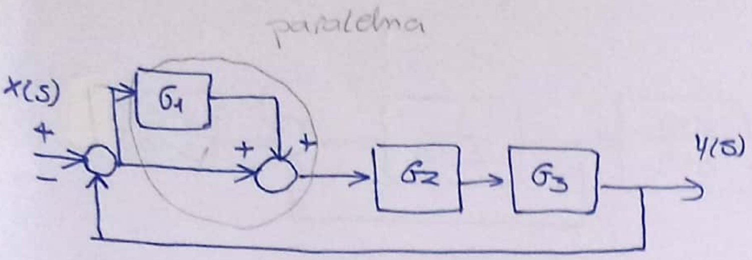
$K_1 = 10$

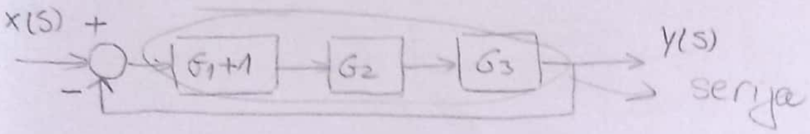
$K_2 = 1$

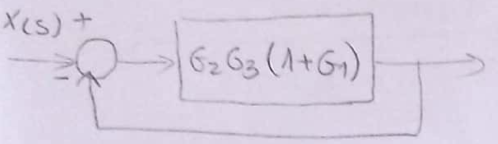
$T_1 = 0{,}05s$

$T_2 = 2s$

$T = 0{,}1s$







$$\Rightarrow \boxed{\frac{G_2G_3(1+G_1)}{1+G_2G_3(1+G_1)}}$$

$$G_0(s) = \frac{G_2G_3(1+G_1)}{1+G_2G_3(1+G_1)} = \frac{\frac{10}{1+0{,}05s}\cdot\frac{1}{1+4s^2}(1+0{,}1s)}{1+\frac{10}{1+0{,}05s}\cdot\frac{1}{1+4s^2}\cdot(1+0{,}1s)} = \frac{\frac{10\cdot(1+0{,}1s)}{\cancel{(1+0{,}05s)(1+4s^2)}}}{\frac{(1+0{,}05s)(1+4s^2)+10(1+0{,}1s)}{\cancel{(1+0{,}05s)(1+4s^2)}}}$$

$$= \frac{10(1+0{,}1s)}{(1+0{,}05s)(1+4s^2)+10(1+0{,}1s)} = \frac{10(1+0{,}1s)}{1+4s^2+0{,}05s+0{,}2s^3+10+1s}$$

$$= \frac{10(1+0{,}1s)}{0{,}2s^3+4s^2+1{,}05s+11}$$

$a_n(s^3) = 0{,}2$

$a_{n-1}(s^2) = 4$

$a_{n-2}(s) = 1{,}05$

$a_{n-3}(s^0) = 11$

$D_1 = a_{n-1}$

$D_1 = 4$

$$D_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} = \begin{vmatrix} 4 & 11 \\ 0{,}2 & 1{,}05 \end{vmatrix} = 4{,}2 - 2{,}2 = 2$$

$D_3 = a_0 \cdot D_{n-1}$

$D_3 = 11 \cdot 2 = 22$

$D_1 > 0$

$D_2 > 0$

$D_3 > 0$

$\dfrac{D_2}{D_1} > 0 \quad \Rightarrow$ sistem je stabilan

$\dfrac{D_3}{D_2} > 0$

2.

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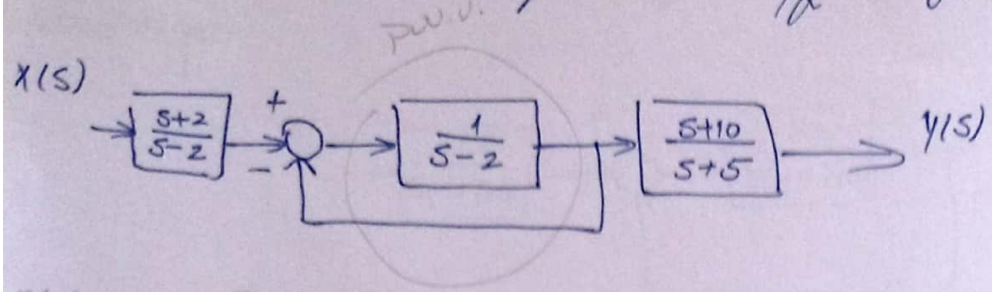
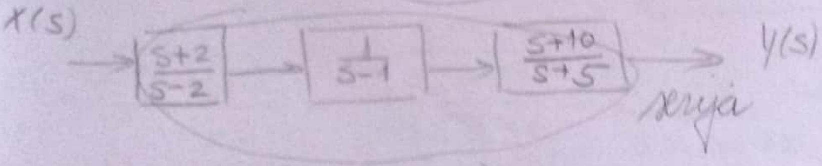
NYQUIST

(3.) Odrediti stabilnost upotrebom Nyqistovog kriterija



$$\frac{\frac{1}{s-2}}{1+\frac{1}{s-2}} = \frac{\frac{1}{s-2}}{\frac{s-2+1}{s-2}} = \frac{1}{s-1}$$



$$G_o(s) = \frac{(s+2)(s+10)}{(s-2)(s-1)(s+5)}$$

$s \to j\omega$

$$G(j\omega) = \frac{(j\omega+2)(j\omega+10)}{(j\omega-2)(j\omega-1)(j\omega+5)} = \frac{(j\omega)^2+10j\omega+2j\omega+20}{((j\omega)^2-j\omega-2j\omega+2)(j\omega+5)} = \frac{-\omega^2+12j\omega+20}{(-\omega^2-3j\omega+2)(j\omega+5)}$$

$$= \frac{-\omega^2+12j\omega+20}{-j\omega^3-5\omega^2-3(j\omega)^2-15j\omega+2j\omega+10} = \frac{-\omega^2+12j\omega+20}{-j\omega^3-5\omega^2+3\omega^2-13j\omega+10} = \frac{-\omega^2+12j\omega+20}{-j\omega^3-2\omega^2-13j\omega+10}$$

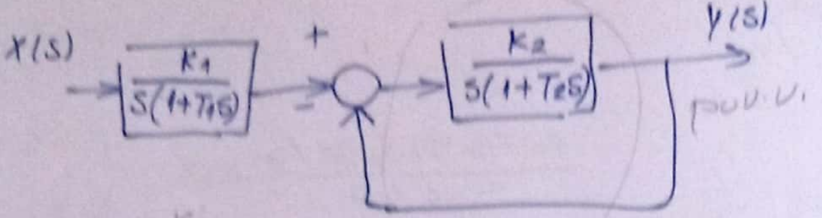
$$= \frac{-\omega^2+12j\omega+20}{10-2\omega^2-j(\omega^3+13\omega)} = \frac{-\omega^2+12j\omega+20}{(-j\omega^3+13j\omega)+(10-2\omega^2)} \cdot \frac{(-j\omega^3-13j\omega)+(10-2\omega^2)}{(-j\omega^3-13\omega)-(10-2\omega^2)}$$

ROUTHOV K.

(5.) Odrediti K_1 i K_2 za koje je sistem stabilan upotrebom Routhovog kriterija.

$T_1 = 0{,}027$

$T_2 = 0{,}13$



$$= \frac{\frac{K_2}{s(1+0{,}13s)}}{1+\frac{K_2}{s+0{,}13s^2}} = \frac{\frac{K_2}{s+0{,}13s^2}}{\frac{s+0{,}13s^2+K_2}{s+0{,}13s^2}} = \frac{K_2}{s+0{,}13s^2+K_2}$$

$$= \frac{K_1K_2}{(s+0{,}027s^2)(s+0{,}13s^2+K_2)}$$ ← serija

$$G_0(s) = \frac{\frac{K_2K_1}{(s+0{,}027s^2)(0{,}13s^2+s+K_2)}}{1+\frac{K_2K_1}{(s+0{,}027s^2)(0{,}13s^2+s+K_2)}} = \frac{K_1K_2}{(s+0{,}027s^2)(0{,}13s^2+s+K_2)+K_1K_2}$$

$$= \frac{K_1K_2}{0{,}13s^3+s^2+K_2s+0{,}0035s^4+0{,}027s^3+0{,}027K_2s^2+K_1K_2}$$

$$f(s) = 0{,}0035s^4+0{,}157s^3+s^2(1+0{,}027K_2)+K_2s+K_1K_2 \quad / \cdot 285$$

$$f(s) = s^4+44{,}7s^3+s^2(285+7{,}7K_2)+285K_2s+285K_1K_2$$

$a_n(s^4) = 1$

$a_{n-1}(s^3) = 44{,}7$

$a_{n-2}(s^2) = 285+7{,}7K_2$

$a_{n-3}(s) = 285K_2$

$a_{n-4}(s^0) = 285K_1K_2$

s^4	1	$285+7{,}7K_2$	$285K_1K_2$
s^3	$44{,}7$	$285K_2$	0
s^2	$\frac{12\,739+59{,}19K_2}{44{,}7}$	$285K_1K_2$	
s^1			
s^0	$285K_1K_2$		

$$b_1 = \frac{44{,}7\cdot(285+7{,}7k_2)-285k_2}{44{,}7} = \frac{12739+344{,}19k_2-285k_2}{44{,}7}$$

$$b_1 = \frac{12739+59{,}19k_2}{44{,}7} \quad \begin{matrix}/:10\\ /:10\end{matrix}$$

$$b_1 = \frac{1273{,}9+5{,}92k_2}{4{,}47}$$

$$\frac{1273{,}9+5{,}92k_2}{4{,}47} > 0$$

$$1273{,}9+5{,}9k_2 > 0$$

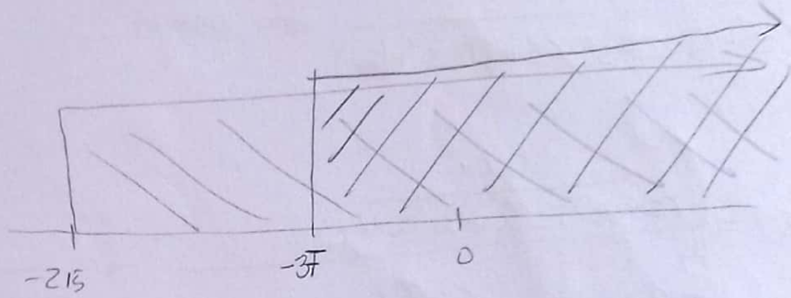
$$5{,}92k_2 > -1273{,}9$$

$$k_2 > -215{,}185$$

$$b_2 = \frac{44{,}7\cdot 285k_1k_2-0}{\cancel{44{,}7}}$$

$$b_2 = 285k_1k_2$$

$$285k_1k_2 > 0$$



$$285+7{,}7k_2 > 0$$

$$7{,}7k_2 > -285$$

$$k_2 > -37{,}013$$

$$k_2 \in [-37;\ 37]$$

$$k_2 \approx 10$$

$$c_1 = \frac{\frac{1273{,}9+5{,}92k_2}{4{,}47}\cdot 285k_2 - 285k_1k_2\,44{,}7}{\frac{1273{,}9+5{,}9k_2}{4{,}47}}$$

$$c_1 = \frac{\frac{362805k_2+1687{,}2k_2^2}{4{,}47}-12739{,}5k_1k_2}{\frac{1273{,}9+5{,}92k_2}{4{,}47}} = \frac{\frac{362805k_2+1687{,}2k_2^2-56945{,}5k_1k_2}{\cancel{4{,}47}}}{\frac{1273{,}9+5{,}92k_2}{\cancel{4{,}47}}}$$

$$c_1 = \frac{362805k_2+1687{,}2k_2^2-56945{,}5k_1k_2}{1273{,}9+5{,}92k_2} > 0 \qquad c_1 = \frac{362{,}8k_2+1{,}68k_2^2-56{,}9k_1k_2}{1{,}274+0{,}0059k_2}$$

$$362805k_2+1687{,}2k_2^2-56945{,}5k_1k_2 > 0 \quad /:1000$$

$$362{,}8k_2+1{,}68k_2^2-56{,}9k_1k_2 > 0$$

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za $k_2 = 10$

$3628 + 168 - 569k_1 > 0$

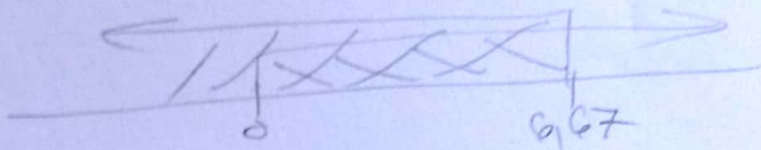
$-569k_1 > -3796$

$\boxed{k_1 < 6,67}$

$285k_1k_2 > 0$

$2850k_1 \geq 0$

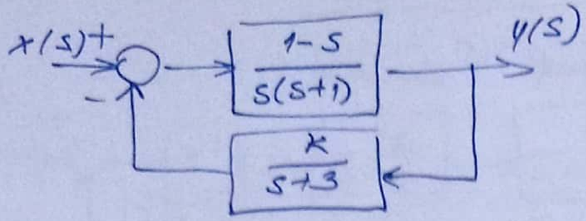
$\boxed{k_1 \geq 0}$



$k_1 \in [0; 6,67]$

(5.) ROUTH. I NYQUIST. K.

Odrediti stabilnost sistema



* Routhov k.

⇒ zatvorena petlja

⇒ pov. v.

$$G_o(s)=\frac{\frac{1-s}{s(s+1)}}{1+\frac{1-s}{s(s+1)}\cdot\frac{K}{s+3}}=\frac{\frac{1-s}{s(s+1)}}{\frac{s(s+1)(s+3)+K(1-s)}{s(s+1)(s+3)}}=\frac{(1-s)(s+3)}{s(s+1)(s+3)+K(1-s)}$$

$$f(s)=(s^2+s)(s+3)+K-Ks=s^3+3s^2+s^2+3s$$

$$f(s)=s^3+3s+s^2+3s+K-Ks$$

$$f(s)=s^3+4s^2+3s-Ks+K$$

$$f(s)=s^3+4s^2+s(6-K)+K$$

$a_n(s^3)=1$

$a_{n-1}(s^2)=4$

$a_{n-2}(s)=6-K$

$a_{n-3}(s^0)=K$

s^3	1	$6-K$
s^2	4	K
s^1	$\frac{24-5K}{4}$	
s^0	K	

$$b_1=\frac{4\cdot(6-K)-K}{4}=\frac{24-4K-K}{4}=\frac{24-5K}{4}$$

$24-5K>0$

$-5K>-24$

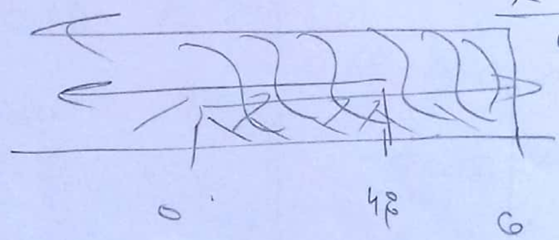
$K<4{,}8$

$K>0$

$6-K>0$

$-K>-6$

$K<+6$



$K\in(0;4{,}8)$ ✓

* Nyquistov k.

⇒ otvorena petlja za $K=1$

$$G_o(s)=\frac{(1-s)\cdot K}{s(s+1)(s+3)}=\frac{1-s}{s(s+1)(s+3)}\qquad s\to j\omega$$

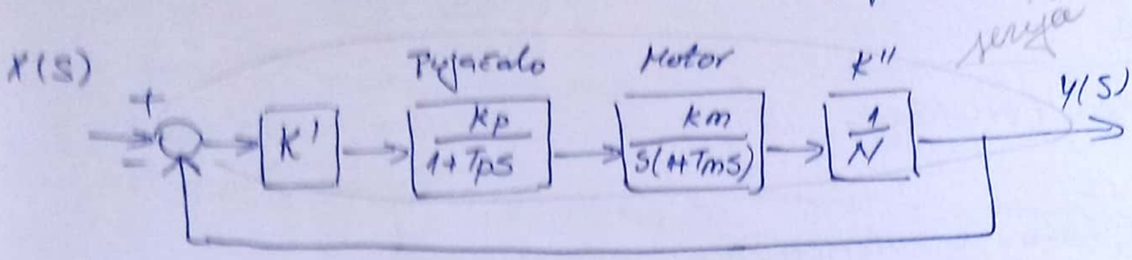
$$G(j\omega)=\frac{1-j\omega}{j\omega(j\omega+1)(j\omega+3)}=\frac{1-j\omega}{(-\omega^2+j\omega)(j\omega+3)}=\frac{1-j\omega}{-j\omega^3-3\omega^2-\omega^2+3j\omega}$$

$$=\frac{1-j\omega}{-4\omega^2-(j\omega^3-3j\omega)}\cdot\frac{-4\omega^2+(j\omega^3-3j\omega)}{-4\omega^2+(j\omega^3-3j\omega)}$$

⑤ a) odrediti $G_0(S)$ zatvorenog kruga HURWITZOV K.

b) stabilnost Hurwitzovim k. i odrediti K za koje je sis. stabilan

c) za $K=80$ nacrtati bodeove dijagrame



$T_p = 0{,}02s$

$T_m = 0{,}16s$

a) $$\frac{K}{(1+T_p s)s(1+T_m s)}$$

p.w.v.

$$G_0(S) = \frac{\frac{K}{(1+0{,}02S)(S+0{,}16S^2)}}{1+\frac{K}{(1+0{,}02S)(S+0{,}16S^2)}} \Rightarrow G_0(S) = \frac{K}{(1+0{,}02S)(S+0{,}16S^2)+K}$$

$$G_0(S) = \frac{K}{S+0{,}16S^2+0{,}02S^2+0{,}0032S^3+K} = \frac{K}{0{,}0032S^3+0{,}18S^2+S+K}$$

$a_n\ (S^3) = 0{,}0032$

$a_{n-1}(S^2) = 0{,}18$

$a_{n-2}(S^1) = 1$

$a_{n-3}(S^0) = K$

$D_1 = a_{n-1}$

$D_1 = 0{,}18$

$$D_2 = \begin{vmatrix} 0{,}18 & K \\ 0{,}0032 & 1 \end{vmatrix} = 0{,}18 - 0{,}0032K > 0$$

$-0{,}0032K > -0{,}18$

$\boxed{K < 56{,}25}$

$D_3 = a_0 \cdot D_{n-1}$

$D_3 = K\,(0{,}18 - 0{,}0032K) > 0$

$K > 0 \qquad K < 56{,}25$

$K \in (0; 56{,}25)$

$K = 56{,}25 \Rightarrow$ sistem granično stabilan

$W_0(S) = \frac{20}{\quad}$

c) $K = 80$

$$G = \frac{K}{s(1+0{,}02s)(1+0{,}16s)} = \frac{80}{s(1+0{,}02s)(1+0{,}16s)} \Rightarrow \frac{80}{(s+0{,}02s^2)(1+0{,}16s)}$$

$s + 0{,}16s^2 + 0{,}02s^2 + 0{,}0032s^2$

$0{,}0032s^3 + 0{,}18s^2 + s$

$s \to j\omega$

$$G(j\omega) = \frac{80}{j\omega(1+0{,}02j\omega)(1+0{,}16j\omega)}$$

AMPL

$$L(j\omega) = 20\log 80 - 20\log\frac{1}{j\omega} - 20\log(1+0{,}02j\omega) - 20\log(1+0{,}16j\omega)$$

$$L(j\omega) = L(A) - L(B) - L(C) - L(D)$$

FAZA

$$\varphi(j\omega) = \operatorname{arctg} 80 - \operatorname{arctg}\frac{1}{j\omega} - \operatorname{arctg}(1+0{,}02s) - \operatorname{arctg}(1+0{,}16s)$$

$$\varphi(j\omega) = \varphi(A) - \varphi(B) - \varphi(C) - \varphi(D)$$

OSNOVNI ČLANOVI	AMPLITUDNI D. NAGIB		LOMNA F.	FAZNI DIJAG.	
$A = 80$, $A = 38$	/		/	$\varphi = 0°$	
$B = \frac{1}{j\omega}$	-20		$\omega_B = 1$	$\varphi = -90°$	
$C = \frac{1}{1+0{,}02j\omega}$	0	$\omega < \omega_C$	$\omega_C = 50$	$\varphi = 0°$	$\omega_C \le 5$
	-20	$\omega > \omega_C$		$\varphi = -90°$	$\omega_C \ge 500$
$D = \frac{1}{1+0{,}16s}$	0	$\omega < \omega_D$	$\omega_D = 6{,}25$	$\varphi = 0°$	$\omega_D \le 0{,}6$
	-20	$\omega > \omega_D$		$\varphi = -90°$	$\omega_D \ge 62{,}5$

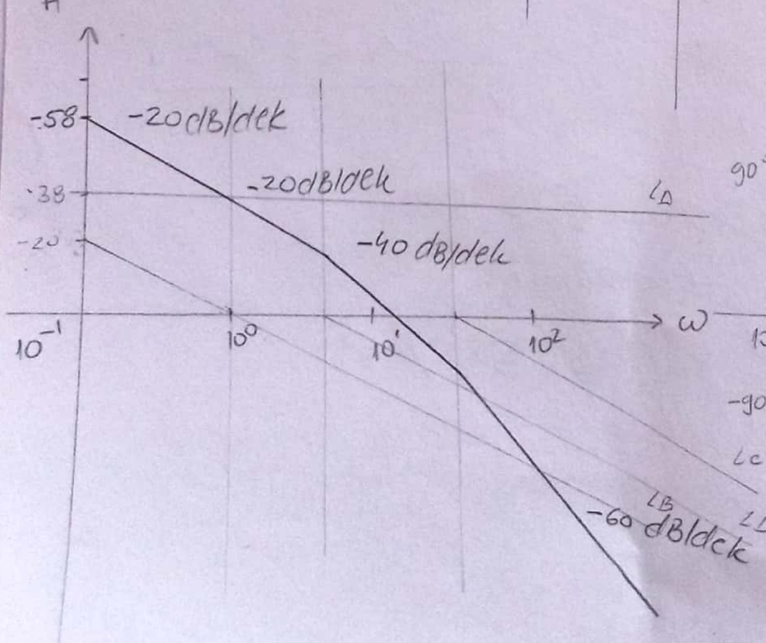
Bodeov u m fajlu

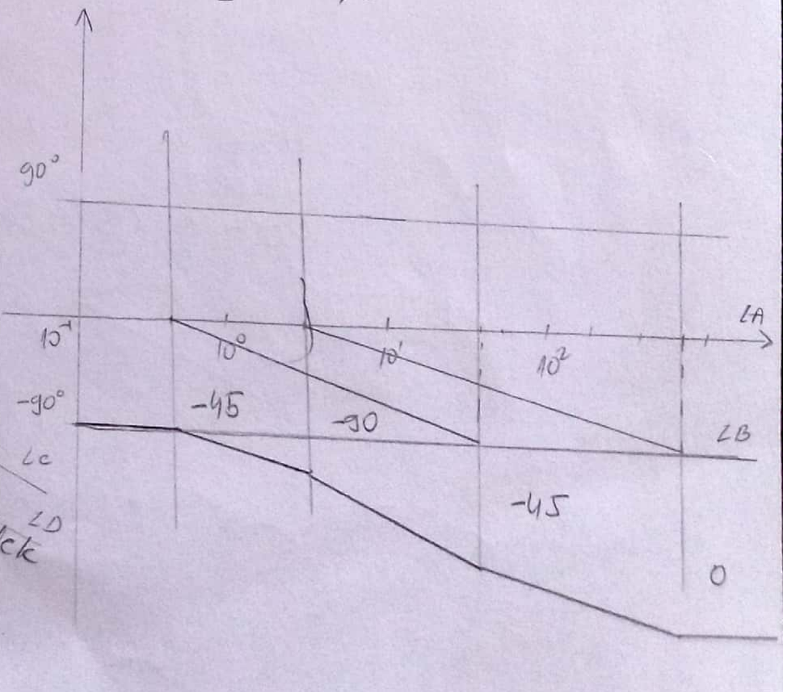
brojnik=[0 0 0 80];

nazivnik=[0,0032 0,18 1 0];

H=tf(brojnik,nazivnik);

bode(H);





HURWITZOV K.

7) Odrediti stab. sistema i nacrtati Bodeove dijagrame otvorenog sistema

$$W_o(s) = \frac{20}{s(s+2)(s+5)}$$

Hurwitzov k. ⇒ zatvorena petlja

$$G_o(s) = \frac{20}{s(s+2)(s+5)+20} = \frac{20}{(s^2+2s)(s+5)+20} = \frac{20}{s^3+7s^2+10s+20}$$

$$s^3+5s^2+2s^2+10s$$

$a_n(s^3) = 1$

$a_{n-1}(s^2) = 7$

$a_{n-2}(s^1) = 10$

$a_{n-3}(s^0) = 20$

$D_1 = a_0$

$D_1 = 20$

$$D_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} = \begin{vmatrix} 7 & 20 \\ 1 & 10 \end{vmatrix} = 70-20 = 50$$

$D_3 - a_0 \cdot D_{n-1}$

$D_3 = 20 \cdot 50 = 100$

$D_1 > 0$

$D_2 > 0$

$D_3 > 0$

$\frac{D_2}{D_1} > 0$

⇒ sistem je stabilan!

$\frac{D_3}{D_2} > 0$

Scanned with CamScanner

8) Odrediti KR za koje će sistem biti stabilan upotrebom Nyquist. kriterija.

$$G(s) = \frac{KR}{(s^3+3s^2+2s)}$$

Routhov k.
⇒ zatvorena petlja

$$Go(s) = \frac{\frac{KR}{s^3+3s^2+2s}}{1+\frac{KR}{s^3+3s^2+2s}} \Rightarrow Go(s) = \frac{KR}{\underbrace{s^3+3s^2+2s+KR}}$$

$f(s) = s^3+3s^2+2s+KR$

$a_n(s^3)=1$

$a_{n-1}(s^2)=3$

$a_{n-2}(s^1)=2$

$a_{n-3}(s^0)=KR$

s^3	1	2	0
s^2	3	KR	0
s^1	$\frac{6-KR}{3}$		
s^0	KR		

$b_1 = \frac{6-KR}{3}$

$6-KR>0$

$-KR>-6$

$KR<6$

$KR>0$

$KR \in [0,6]$

$KR=6$ sistem je gran. stab.

Nyquistov k. ⇒ OTVORENA PETLJA

za $KR=1$

$$G(s) = \frac{1}{s^3+3s^2+2s} \quad s \Rightarrow j\omega$$

$j^2=-1 \quad j^4=1$

$j^3=-j \quad j^1=j$

$$G(j\omega) = \frac{1}{(j\omega)^3+3\cdot(j\omega)^2+2j\omega}$$

$$G(j\omega) = \frac{1}{-j\omega^3-3\omega^2+2j\omega} = \frac{1}{(2j\omega-j\omega^3)+3\omega^2} \cdot \frac{(2j\omega-j\omega^3)-3\omega^2}{(2j\omega-j\omega^3)-3\omega^3}$$

$$= \frac{2j\omega-j\omega^3-3\omega^2}{(2j\omega-j\omega^3)^2-(3\omega^2)^2} = \frac{2j\omega-j\omega^3-3\omega^2}{(2j\omega)^2-2j\omega j\omega^3+(j\omega^3)^2-9\omega^4} = \frac{2j\omega-j\omega^3-3\omega^2}{-4\omega^2+6\omega^2-\omega^6-9\omega^4}$$

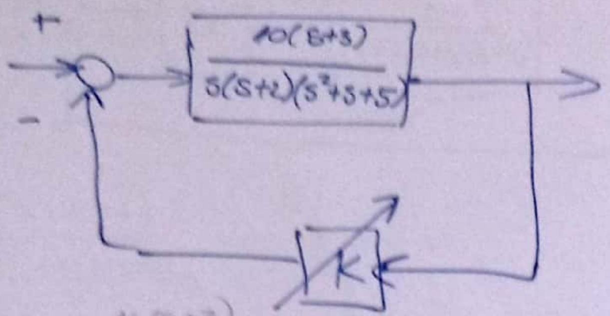
$$= \frac{2j\omega-j\omega^3-3\omega^2}{2\omega^2-\omega^6-9\omega^4} = \frac{-3\cancel{\omega^2}}{\cancel{\omega^2}(2-\omega^4-9\omega^2)} + \frac{\cancel{\omega}(2j-j\omega^2)}{\cancel{\omega}(2\omega-\omega^5-9\omega^3)}$$

$$= \underbrace{\frac{-3}{2-\omega^4-9\omega^2}}_{Re(j\omega)} + \underbrace{\frac{2-\omega^2}{2\omega-\omega^5 9\omega^3}j}_{Im(j\omega)}$$

(5)

ROUTHOV K.

Odredite parametar k za koji je sistem stabilan i granično stabilan. Odredite periodu oscilacija koje pri tome nastaju. Routhov k.



$$G_0 = \frac{\frac{10(s+3)}{s(s+2)(s^2+s+5)}}{1+\frac{10(s+3)}{s(s+2)(s^2+s+5)}\cdot k} = \frac{10(s+3)}{s(s+2)(s^2+s+5)+10k(s+3)} = \frac{10(s+3)}{(s^2+s2)(s^2+s+5)+10ks+30k}$$

$$= \frac{10(s+3)}{s^4+s^3+5s^2+2s^3+2s^2+10s+10ks+30k} = \frac{10(s+3)}{s^4+3s^3+7s^2+s(10+10k)+30k}$$

$$f(s) = s^4+3s^3+7s^2+(10+10k)s+30k$$

$a_n(s^4) = 1$

$a_{n-1}(s^3) - 3$

$a_{n-2}(s^2) = 7$

$a_{n-3}(s) = 10+10k$

$a_{n-4}(s^0) = 30k$

s^4	1	7	30k
s^3	3	10+10k	0
s^2	$\frac{25-10k}{3}$	30k	
s^1	$\frac{-100k^2-80k+250}{25-10k}$		
s^0	30k		

$$b_1 = \frac{35-10-10k}{3} = \frac{25-10k}{3}$$

$$b_2 = \frac{\cancel{3}\,30k-0}{\cancel{3}} = 30k$$

$$c_1 = \frac{\frac{25-10k}{3}(10+10k)-90k}{\frac{25-10k}{3}}$$

$$= \frac{\frac{(25-10k)(10+10k)-270k}{3}}{\frac{25-10k}{3}}$$

$$= \frac{250+250k-100k-100k^2-270k}{25-10k}$$

$$= \frac{-100k^2-80k+250}{25-10k}$$

$$-100k^2-80k+250>0 \quad /\cdot(-10)$$

$$10k^2+8k-25>0$$

$10+10k>0$

$10k>-10$

$k>-1$

$30k>0$

$k>0$

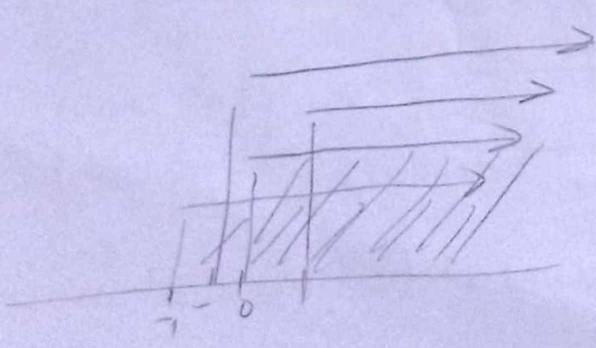
$$10k^2-8k-25=0$$

$$k_{1,2} = \frac{8\pm\sqrt{64+100}}{20}$$

$$= \frac{8\pm\sqrt{164}}{20}$$

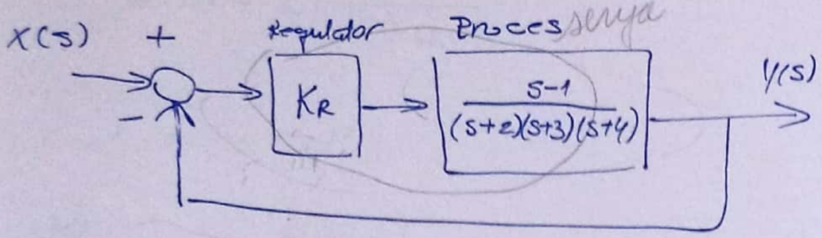
$k>1{,}04$

$k>-0{,}2$



(10.) **HURWITZOV K.**

Odrediti K_R za koje je sistem stabilan



$$\frac{K_R(s-1)}{(s+2)(s+3)(s+4)} \leftarrow + \text{pov.v.}$$

$$G_o(s) = \frac{K_R(s-1)}{(s+2)(s+3)(s+4)+K_R(s-1)} = \frac{K_R(s-1)}{s^3+9s^2+26s+24+K_R s-K_R}$$

$(s^2+5s+6)(s+4)$

$s^3+4s^2+5s^2+20s+6s+24$

$s^3+9s^2+s(26+K_R)_s+24-K_R$

$a_n(s^3)=1$

$a_{n-1}(s^2)=9$

$a_{n-2}(s^1)=26+K_R$

$a_{n-3}(s^0)=24-K_R$

s^3	1	$26+K_R$	0
s^2	9	$24-K_R$	0
s^1	$\frac{10K_R-210}{9}$		
s^0	$24-K_R$		

$b_1 = \frac{9\cdot(26+K_R)-24+K_R}{9}$

$b_1 = \frac{234+9K_R-24+K_R}{9}$

$b_1 = \frac{10K_R+210}{9}$

$10K_R-210>0$

$10K_R>210$

$K_R>21$

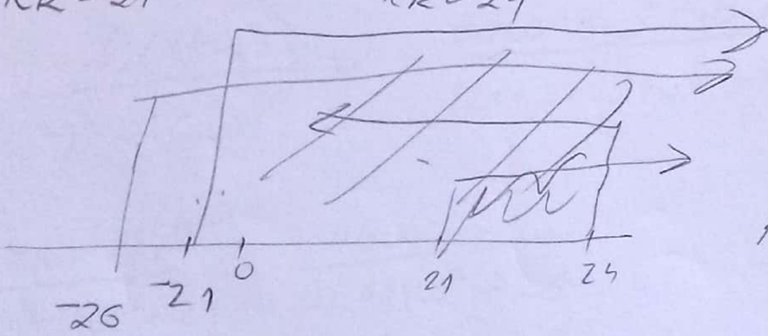
$24-K_R>0$

$-K_R>24$

$K_R<24$

$26+K_R>0$

$K_R>-26$



$K_R \in [-21, 24]$

$D_1 = a_0 \Rightarrow D_1 = 24-K_R>0$

$-K_R>-24$

$\underline{K_R<24}$

$$D_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} = \begin{vmatrix} 9 & 24-K_R \\ 1 & 26+K_R \end{vmatrix} = 234+9K_R-24+K_R = 210+10K_R>0$$

$10K_R>-210$

$\underline{K_R>-21}$

$D_3 = a_0 \cdot D_2 = (24-K_R)\cdot(210+10K_R)$

$=$

znam dalje...

11

(11)

NYQUIST K.

Odrediti K za koji je sistem stabilan upotrebom Nyquistovog k.

$$G(s) = K \cdot \frac{8s-3}{s^2-6s+11}$$

⇒ Routhov k

zatvorena petlja

$$G_0(s) = \frac{K(8s-3)}{s^2-6s+11+K(8s-3)} \leftarrow \frac{\frac{K(8s-3)}{s^2-6s+11}}{1+\frac{K(8s-3)}{s^2-6s+11}} = \frac{K(8s-3)}{s^2-6s+11+K(8s-3)}$$

$$G_0(s) = \frac{K(8s-3)}{s^2-6s+11+8Ks-3K} = \frac{K(8s-3)}{s^2+s(8K-6)+11-3K}$$

$a_n(s^2) = 1$

$a_{n-1}(s^1) = 8K-6$

$a_{n-2}(s^0) = 11-3K$

s^2	1	$11-3K$
s^1	$8K-6$	
s^0	$11-3K$	

$8K-6>0$

$8K>6$

$K>0{,}75$

$11-3K>0$

$-3K>-11$

$K<3{,}66$

$K \in (0{,}75;\ 3{,}66)$

za $K=1$

$$G_0 = \frac{8s-3}{s^2-6s+11} \qquad s \to j\omega$$

$$G(j\omega) = \frac{8j\omega-3}{-\omega^2-6j\omega+11} = \frac{8j\omega-3}{(11-\omega^2)-6j\omega} \cdot \frac{(11-\omega^2)+6j\omega}{(11-\omega^2)+6j\omega}$$

$$= \frac{(8j\omega-3)(11-\omega^2+6j\omega)}{(11-\omega^2)^2-(6j\omega)^2} = \frac{88j\omega-8j\omega^3-48\omega^2}{121-22\omega^2+\omega^4+36\omega^2} = \frac{88j\omega-8j\omega^3-48\omega^2}{\omega^4+11\omega^2+121}$$

$$= \underbrace{\frac{88j\omega-8j\omega^3}{\omega^4+11\omega^2+121}}_{Im} - \underbrace{\frac{48\omega^2}{\omega^4+11\omega^2+121}}_{Re}$$

NYQUIST / BODE

(12.)

$$W_o(s) = \frac{3(s+2)^2}{s(s+10)(s+1)} \quad \frac{3(s^2+4s+4)}{(s^2+10s)(s+1)} = \frac{3s^2+8s+12}{s^3+s^2+10s^2+10s} = \frac{3s^2+8s+12}{s^3+11s^2+10s}$$

$s \Rightarrow j\omega$

$$W(j\omega) = \frac{3(j\omega+2)^2}{j\omega(j\omega+10)(j\omega+1)} = \frac{3(-\omega^2+4j\omega+4)}{(-\omega^2+10j\omega)(j\omega+1)} = \frac{-3\omega^2+12j\omega+12}{-j\omega^3-\omega^2-10\omega^2+10\omega}$$

$$= \frac{-3\omega^2+12j\omega+12}{(9\omega-10\omega^2)-j\omega^3} \cdot \frac{(9\omega-10\omega^2)+j\omega^3}{(9\omega-10\omega^2)+j\omega^3}$$

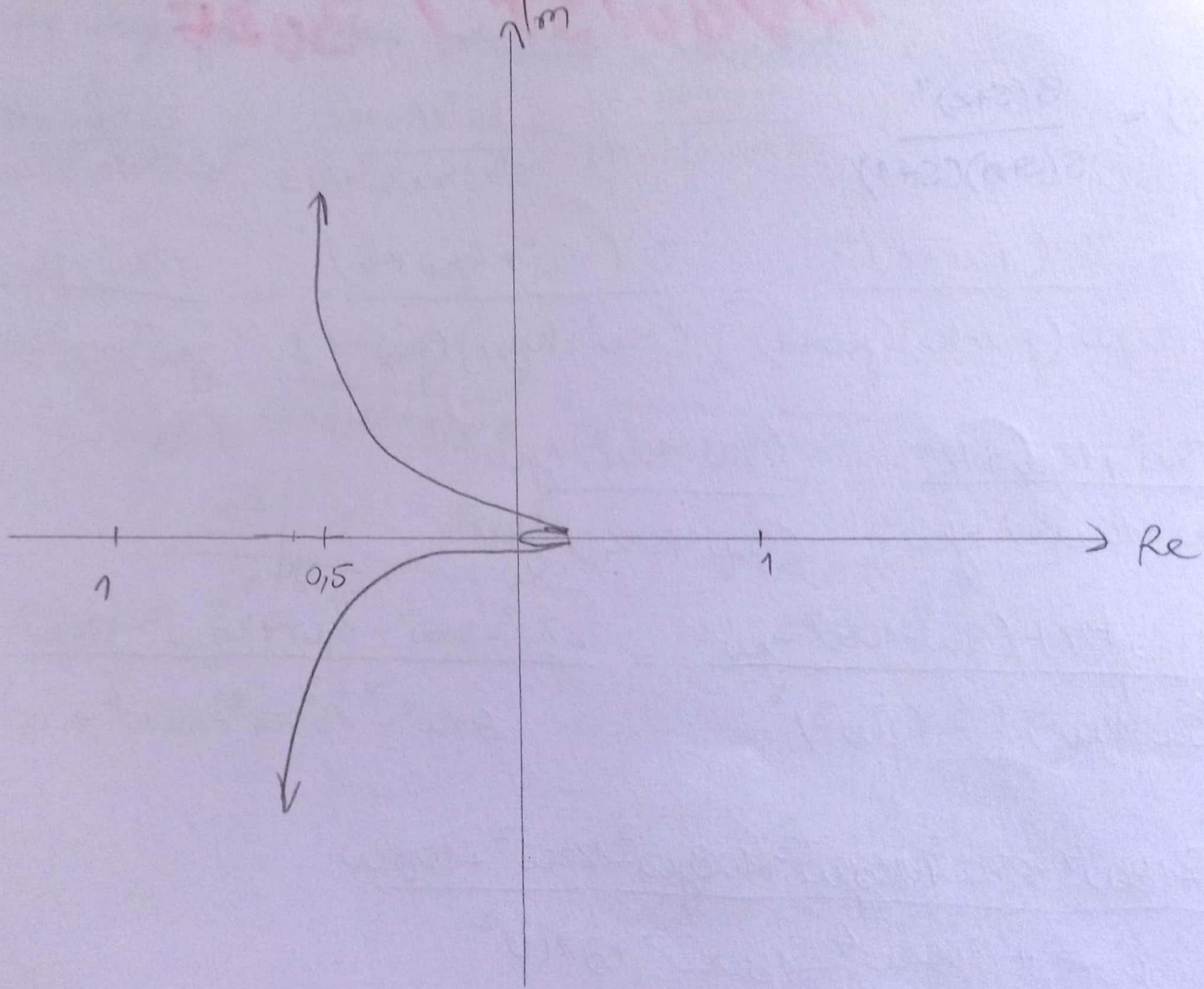
$$= \frac{(-3\omega^2+12j\omega+12)\cdot(9\omega-10\omega^2+j\omega^3)}{(9\omega-10\omega^2)^2-(j\omega^3)^2} = \frac{-27\omega^3+30\omega^4-3j\omega^5+108j\omega^2-120j\omega^3-120\omega^4+108\omega}{81\omega^2-180\omega^3+100\omega^4+\omega^6}$$

$$= \frac{-3j\omega^5+18\omega^4-27\omega^3+108j\omega^3+108j\omega^2-120\omega^2+108\omega}{\omega^6+100\omega^4-180\omega^3+81\omega^2}$$

$$= \frac{\cancel{\omega}(18\omega^3-27\omega^2-120\omega+108)}{\cancel{\omega}(\omega^5+100\omega^3-180\omega^2+81\omega)} + \frac{\cancel{\omega}(108\omega^2+108\omega-3\omega^4)}{\cancel{\omega}(\omega^5+100\omega^3-108\omega^2+81\omega)}j$$

$$= \underbrace{\frac{18\omega^3-27\omega^2-120\omega+108}{\omega^5+100\omega^3-180\omega^2+81\omega}}_{Re} + \underbrace{\frac{-3\omega^3+108\omega+108}{\omega^4+100\omega^2-108\omega+81}j}_{Im}$$

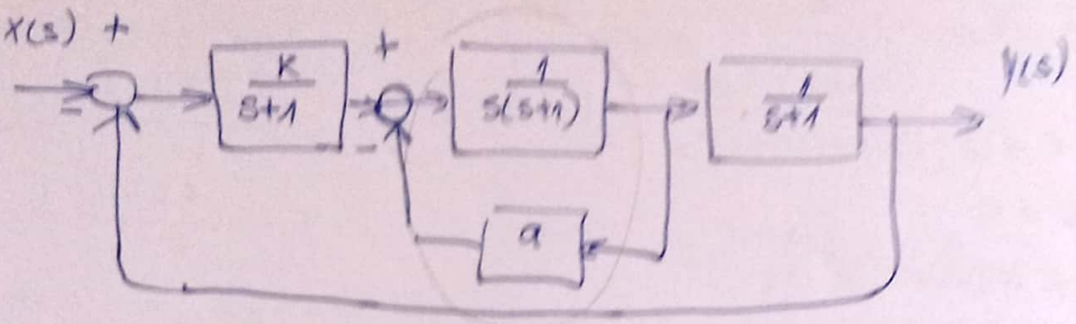
ω	-1	-0,5	0	0,5	1
Re					
Im					


Im
Re
1
0,5
1

(14.)

ROUTHOV K.

Odredite oblast u kojoj je sistem stabilan. Nacrtati graf i označite područje stabilnosti.



$$\frac{\frac{1}{s(s+1)}}{1+\frac{1}{(s+1)s}a} = \frac{\frac{1}{s(s+1)}}{\frac{s(s+1)+a}{(s+1)s}} = \frac{1}{s(s+1)+a}$$

$$\frac{K}{(s+1)\cdot(s+1)\left[(s+1)\cdot s+a\right]} = \frac{K}{s^4+3s^3+s^2(3+a)+s(1+2a)+a+K}$$

$(s^2+2s+1)(s^2+s+a)$

$s^4+s^3+as^2+2s^3+2s^2+2as+s^2+s+a$

$s^4+3s^3+s^2(3+a)+s(1+2a)+a$

$a_n(s^4)=1$

$a_{n-1}(s^3)=3$

$a_{n-2}(s^2)=3+a$

$a_{n-3}(s^1)=1+2a$

$a_{n-4}(s^0)=a+K$

s^4	1	$3+a$	$a+K$
s^3	3	$1+2a$	0
s^2	$\frac{8-5a}{3}$	$a+K$	
s^1			
s^0	$a+K$		

$3+a>0$

$a>-3$

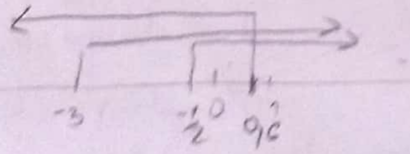
$1+2a>0$

$2a>-1$

$a>-\frac{1}{2}$

a:

$$b_1=\frac{3\cdot(3+a)-1-2a}{3}=\frac{9-3a-1-2a}{3}=\frac{8-5a}{3}$$



$8-5a>0$

$-5a>-8$

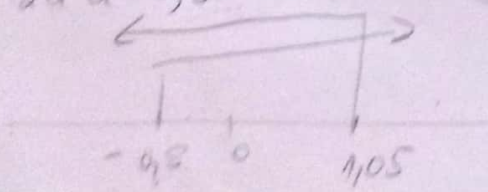
$a<0,6$

$a\in[-0,5;0,6]$

K:

za $a=0,5$

$$b_2=\frac{3a+3k}{3}=a+k$$



$0,5+k>0$

$k>-0,5$

$K\in(-0,5;1,05)$

$$c_1 = \frac{\frac{8-5a}{3}(1+2a) - (3a+3k)}{\frac{8-5a}{3}} = \frac{\frac{(8-5a)(1+2a) - 3\cdot(3a+3k)}{\cancel{3}}}{\frac{8-5a}{\cancel{3}}}$$

$$c_1 = \frac{8-16a-5a-10a^2-9a-9k}{8-5a} = \frac{8-30a-10a^2-9k}{8-5a}$$

$$8-30a-10a^2-9k \geq 0$$

$$8-15-2{,}5-9k \geq 0$$

$$-9{,}5-9k \geq 0$$

$$-9k \geq 9{,}5$$

$$k < 1{,}05$$

BODEOV D.

15.

$$G(s) = \frac{43{,}2\,(s+20)}{s^2(s+1)(s^2+12s+35)}$$

$$\frac{43{,}2s + 864}{(s^3+s^2)(s^2+12s+35)}$$

$$s^5+12s^4+35s^3+s^4+12s^3+32s^2$$

$$s^5+13s^4+47s^3+32s^2$$

$$s^2+12s+35=0$$

$$s_{1,2} = \frac{-12 \pm \sqrt{144-140}}{2} = \frac{-12 \pm 2}{2}$$

$$s_1=-7 \quad s_2=-5$$

$$(s+7)(s+5)$$

$$G(s) = \frac{43{,}2\,(s+20)}{s^2(s+1)(s+5)(s+7)}$$

$$G(s) = 43{,}2 \cdot \frac{1}{s^2} \cdot 20\cdot\left(\frac{1}{20}s+1\right) \cdot \frac{1}{(s+1)} \cdot \frac{1}{5\cdot\left(\frac{1}{5}s+1\right)} \cdot \frac{1}{7\left(\frac{1}{7}s+1\right)}$$

$$G(s) = \frac{43{,}2\cdot 20}{5\cdot 7} \cdot \frac{1}{s^2} \cdot (0{,}05s+1) \cdot \frac{1}{s+1} \cdot \frac{1}{0{,}2s+1} \cdot \frac{1}{0{,}14s+1}$$

$$G(s) = \frac{24{,}7\cdot(0{,}05s+1)}{s^2(s+1)(0{,}2s+1)(0{,}14s+1)}$$

$$s \rightarrow j\omega$$

$$G(j\omega) = \frac{24{,}7\cdot(1+0{,}05j\omega)}{(j\omega)^2\cdot(1+j\omega)(1+0{,}2s)(1+0{,}14s)}$$

$$L(j\omega) = 20\log 24{,}7 + 20\log(1+0{,}05j\omega) - 20\log\left(\frac{1}{j\omega}\right)^2 - 20\log(1+j\omega) - 20\log(1+0{,}2s) - 20\log(1+0{,}14s)$$

$$L(j\omega) = L(A)+L(B)-L(C)-L(D)-L(E)-L(F)$$

$$\varphi(j\omega) = \varphi(A)+\varphi(B)-\varphi(C)-\varphi(D)-\varphi(E)-\varphi(F)$$

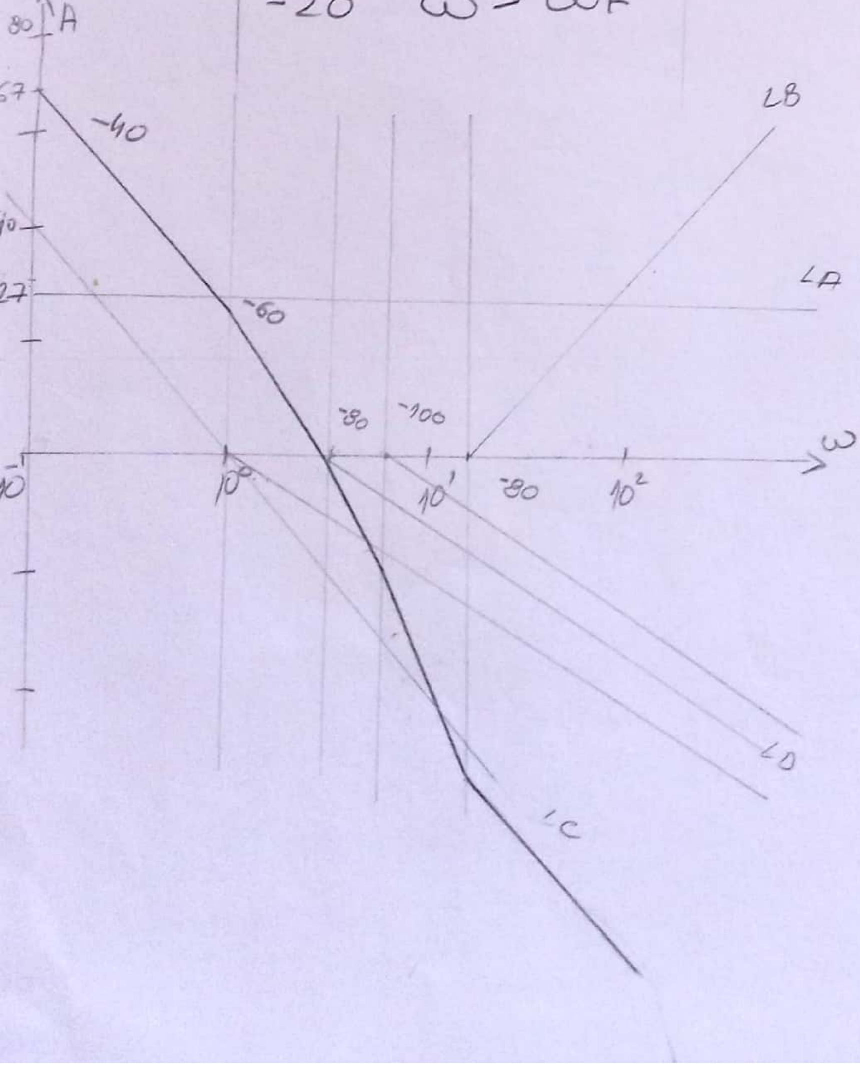
m fajl

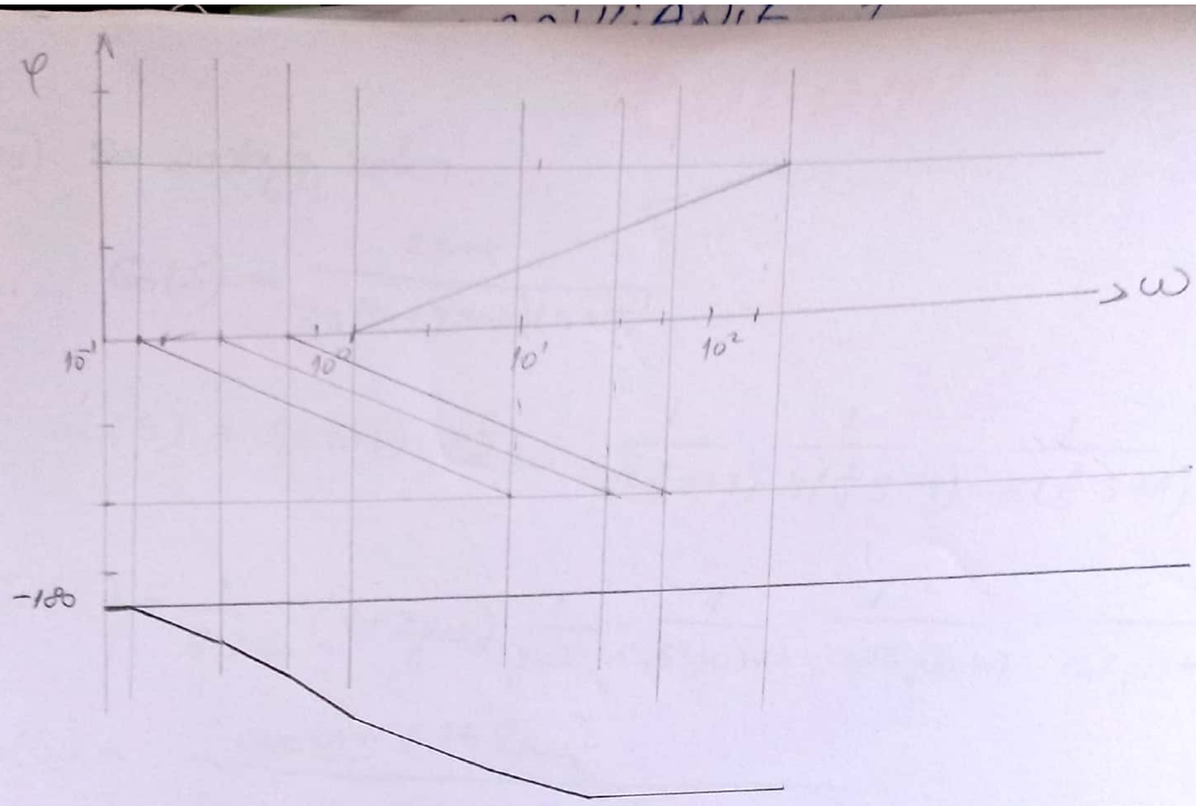
```
s = tf('s');
G = (43.2*(20+s))/((s^3+s^2)*(s^2+12*s+35));
bode(G)
```

ili

```
syms
brojnik = [        ];
nazivnik = [        ];
H = tf(brojnik, nazivnik);
bode(H);
```

OSN. Č.	AMPL. D. NAGIB		LOMNA F	FAZ. D.	
$A=29,7$ $A=27,8$	/		/	$\varphi=0°$	
$B=1+0,05j\omega$	0 20	$\omega<\omega_c$ $\omega>\omega_c$	$\omega_B=20$	$\varphi=0°$ $\varphi=90°$	$\omega\leq 2$ $\omega\geq 200$
$C=\left(\frac{1}{j\omega}\right)^2$	-40		$\omega_C=1$	$\varphi=-180°$	
$D=\frac{1}{1+j\omega}$	0 -20	$\omega<\omega_D$ $\omega>\omega_D$	$\omega_D=1$	$\varphi=0°$ $\varphi=-90°$	$\omega\leq 0,1$ $\omega\geq 10$
$E=\frac{1}{1+0,2s}$	0 -20	$\omega<\omega_E$ $\omega>\omega_E$	$\omega_E=5$	$\varphi=0°$ $\varphi=-90°$	$\omega\leq 0,5$ $\omega\geq 50$
$F=\frac{1}{1+0,14s}$	0 -20	$\omega<\omega_F$ $\omega>\omega_F$	$\omega_F=7$	$\varphi=0°$ $\varphi=-90°$	$\omega\leq 0,7$ $\omega\geq 70$





φ
ω
10^{-1}
10^{0}
10^{1}
10^{2}
-180

BODEOV D.

(16) Sa zadnjeg roka

$$G_0(S) = \frac{2s+1}{2s(s+3)(s+4)(s+5)}$$

$$G_0(S) = (2s+1)\cdot\frac{1}{2j\omega}\cdot\frac{1}{3(\frac{1}{3}s+1)}\cdot\frac{1}{4(\frac{1}{4}s+1)}\cdot\frac{1}{5(\frac{1}{5}s+1)}$$

$$G_0(S) = \frac{1}{60\cdot 2}(1+2j\omega)\cdot\frac{1}{j\omega}\cdot\frac{1}{0{,}33j\omega+1}\cdot\frac{1}{0{,}25j\omega+1}\cdot\frac{1}{0{,}2j\omega+1}$$

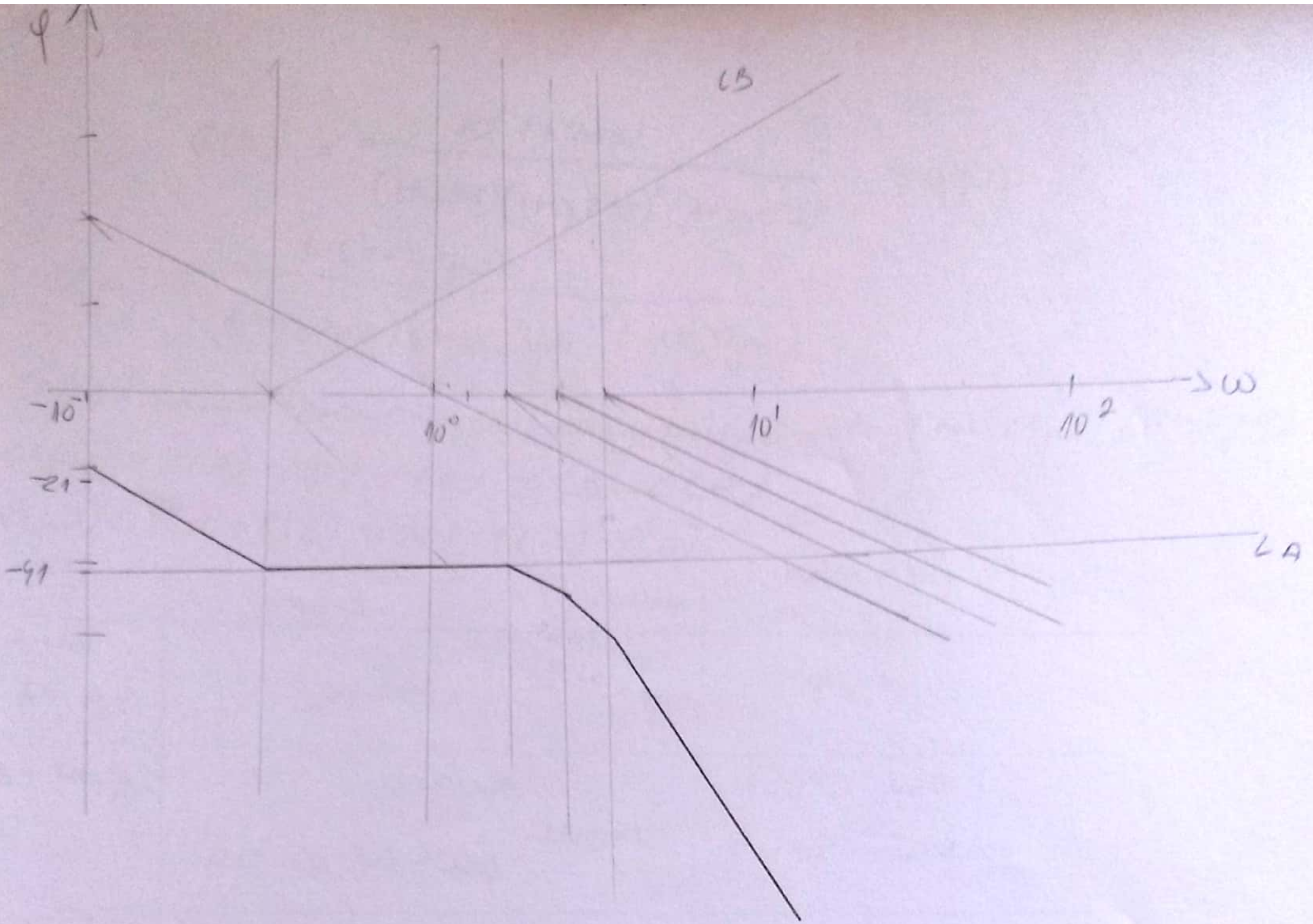
$$G_0(S) = \frac{0{,}0083\,(1+2j\omega)}{j\omega(1+0{,}33j\omega)(1+0{,}25j\omega)(1+0{,}2j\omega)}$$

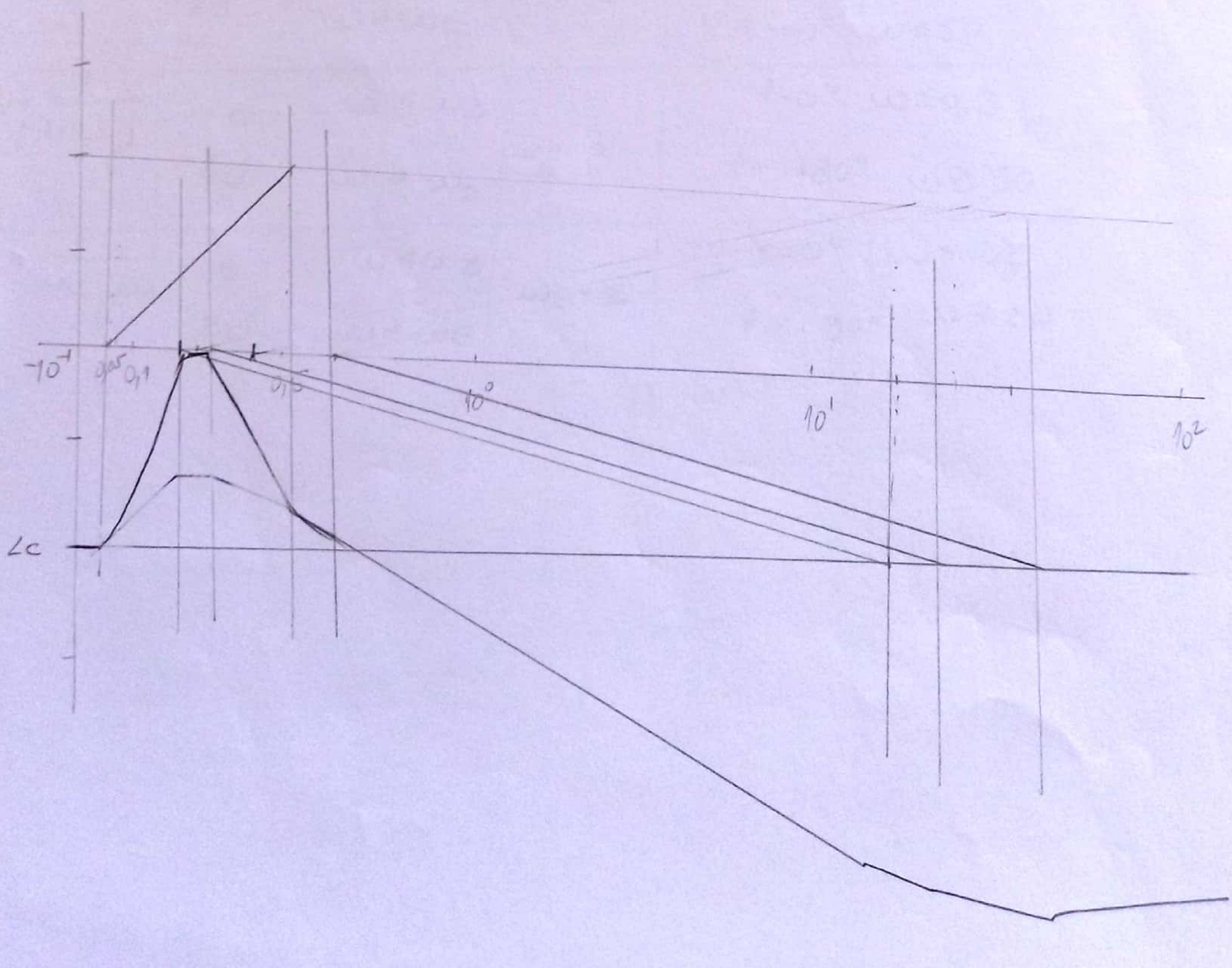
$$L(j\omega) = 20\log 0{,}0083 + 20\log(1+2j\omega) + 20\log\frac{1}{j\omega} - 20\log\frac{1}{1+0{,}33j\omega} - 20\log\frac{1}{1+0{,}25j\omega} - 20\log\frac{1}{1+0{,}2j\omega}$$

$$L(j\omega) = L(A) + L(B) - L(C) - L(D) - L(E) - L(F)$$

$$\varphi(j\omega) = \varphi(A) + \varphi(B) - \varphi(C) - \varphi(D) - \varphi(E) - \varphi(F)$$

OSN. E.	AMPL. D NAGIB		LOMNA F	FAZNI D.	
$A = 0{,}0083$ $A = -41{,}5$	/		/	$\varphi = 0°$	
$B = 1+2j\omega$	0 20	$\omega < \omega_B$ $\omega > \omega_B$	$\omega_B = 0{,}5$	$\varphi = 0$ $\varphi = 90°$	$\omega < 0{,}05$ $\omega > 0{,}5$
$C = \frac{1}{j\omega}$	-20		$\omega_C = 1$	$\varphi = -90°$	
$D = \frac{1}{1+0{,}33j\omega}$	0 -20	$\omega < \omega_D$ $\omega > \omega_D$	$\omega_D = 3$	$\varphi = 0$ $\varphi = -90°$	$\omega \leq 0{,}3$ $\omega \geq 30$
$E = \frac{1}{1+0{,}25j\omega}$	0 -20	$\omega < \omega_E$ $\omega > \omega_E$	$\omega_E = 4$	$\varphi = 0$ $\varphi = -90°$	$\omega \leq 0{,}4$ $\omega \geq 40$
$F = \frac{1}{1+0{,}2j\omega}$	0 -20	$\omega < \omega_F$ $\omega > \omega_F$	$\omega_F = 5$	$\varphi = 0$ $\varphi = -90°$	$\omega \leq 0{,}5$ $\omega \geq 50$





Scanned with CamScanner

(17)

BODEOV D.

$$G(s) = \frac{10(1+0,1s)}{(1+0,2s)(1+0,33s)^2(1+0,5s)} \qquad s \to j\omega$$

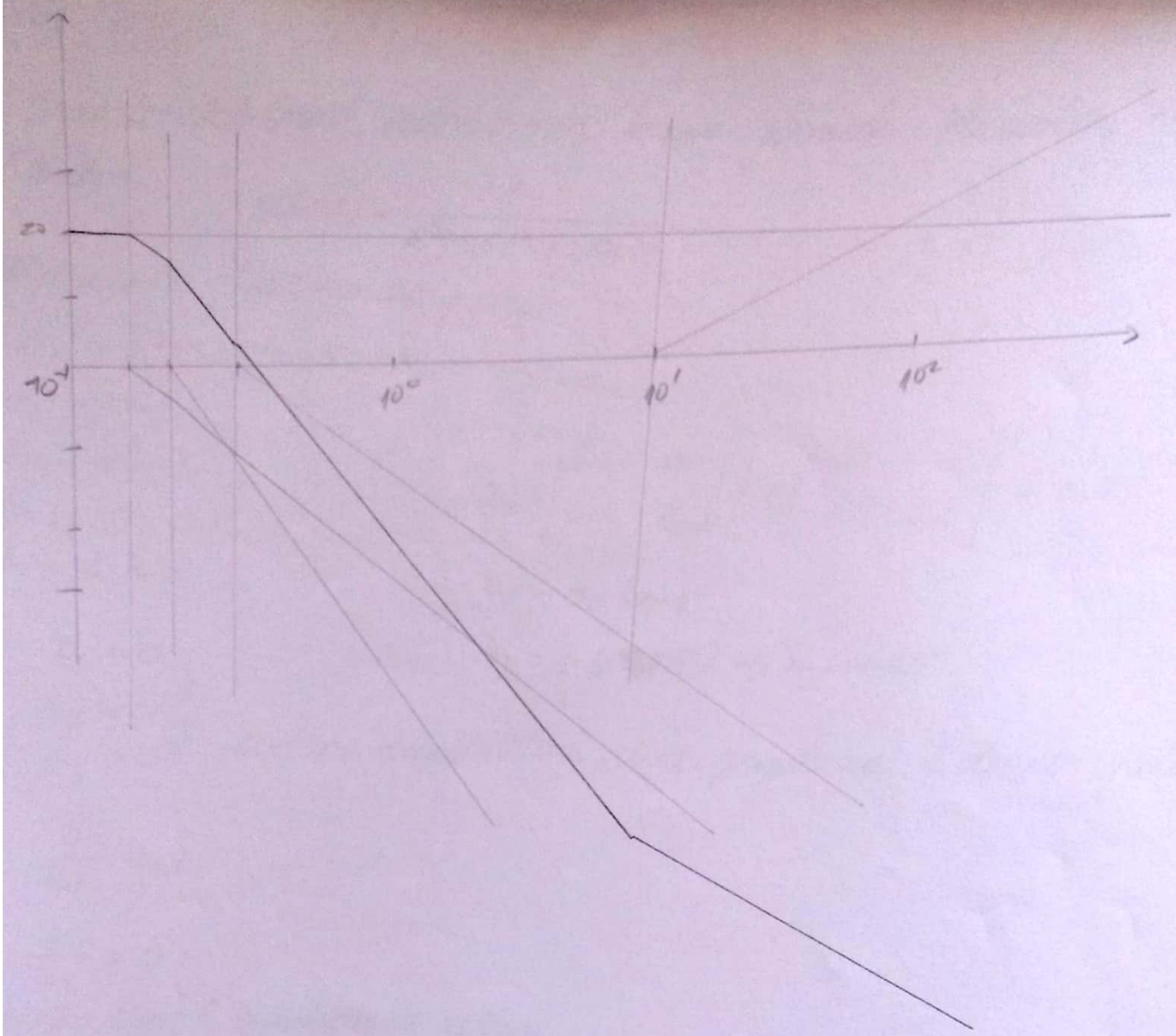
$$G(j\omega) = \frac{10(1+0,1j\omega)}{(1+0,2j\omega)(1+0,33j\omega)^2(1+0,5j\omega)}$$

$$L(j\omega) = 20\log 10 + 20\log(1+0,1j\omega) - 20\log(1+0,2j\omega) - 20\log(1+0,33j\omega)^2(1+0,5j\omega)$$

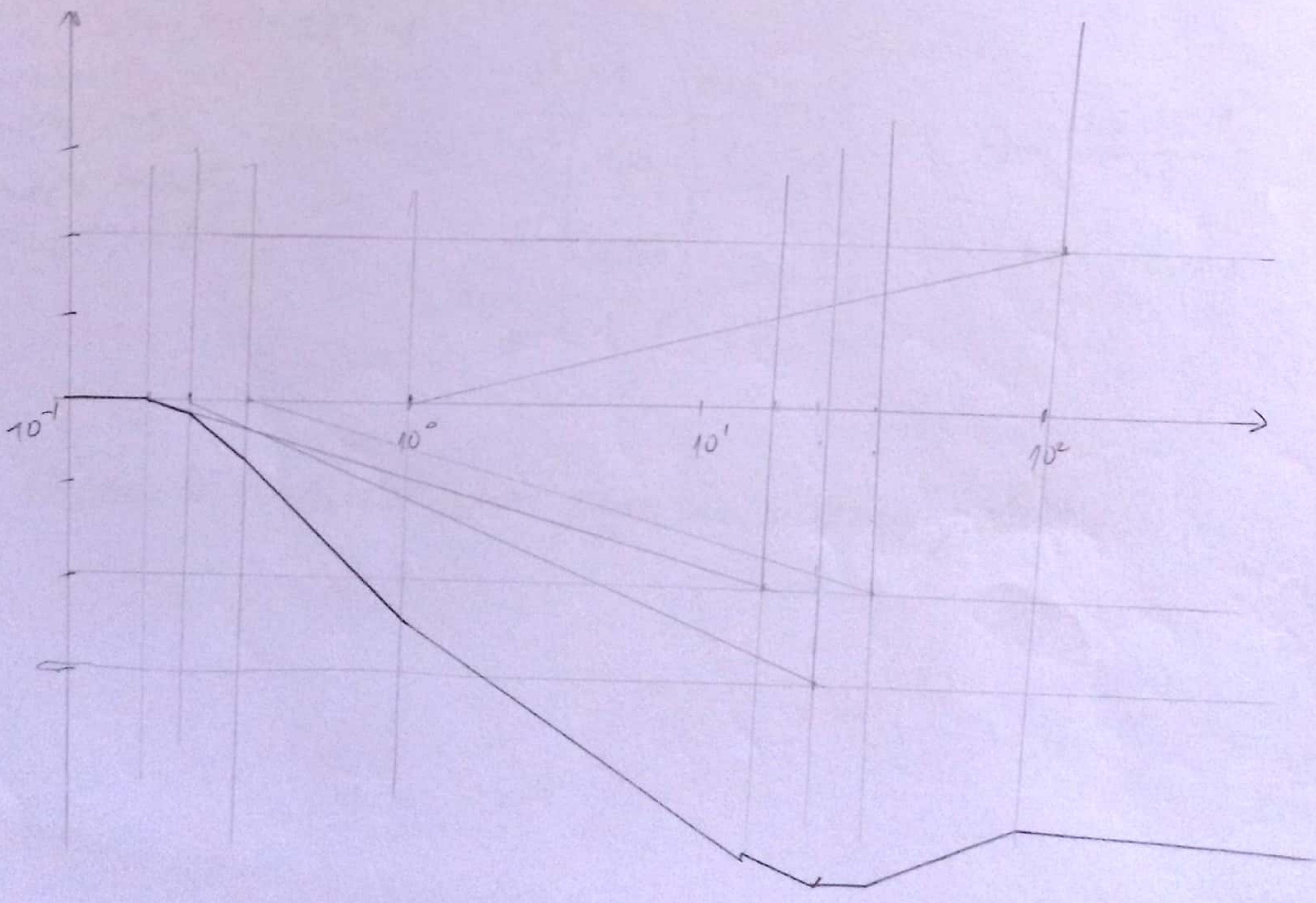
$$L(j\omega) = L(A) + L(B) - L(C) - L(D) - L(E)$$

$$\varphi(j\omega) = \varphi(A) + \varphi(B) - \varphi(C) - \varphi(D) - \varphi(E)$$

OSN. Č.	AMPL. D NAGIB		AMPL. D LOMNA	FAZNI D	
$A = 10$ $A = 20$	/		/	$\varphi = 0°$	
$B = 1+0,1j\omega$	0 20	$\omega < \omega_B$ $\omega > \omega_B$	$\omega_B = 10$	$\varphi = 0°$ $\varphi = 90°$	$\omega \leq 1$ $\omega \geq 100$
$C = \frac{1}{1+0,2j\omega}$	0 -20	$\omega < \omega_c$ $\omega > \omega_c$	$\omega_C = 5$	$\varphi = 0°$ $\varphi = -90°$	$\omega \leq 0,5$ $\omega \geq 50$
$D = \frac{1}{(1+0,33j)^2}$	0 -40	$\omega < \omega_D$ $\omega > \omega_D$	$\omega_D = 3$	$\varphi = 0°$ $\varphi = -180°$	$\omega \leq 0,3$ $\omega \geq 30$
$E = \frac{1}{1+0,5j\omega}$	0 -20	$\omega < \omega_E$ $\omega > \omega_E$	$\omega_E = 2$	$\varphi = 0°$ $\varphi = -90°$	$\omega \leq 0,2$ $\omega \geq 20$



20
10^{-1}
10^{0}
10^{1}
10^{2}



10^{-1}
10^{0}
10^{1}
10^{2}

HURWITZ, BODE, ROUTH

Ispitati stabilnost zatvorenog kruga pomoću Hurwitza, Routha i Bodea.

$$W_o(s) = \frac{1}{s^3+1{,}5s^2+0{,}5s+1}$$

⊗ Hurwitz ⇒ zatvorena petlja

$f(s) = s^3+1{,}5s^2+0{,}5s+1$

$a_n(s^3) = 1$

$a_{n-1}(s^2) = 1{,}5$

$a_{n-2}(s^1) = 0{,}5$

$a_{n-3}(s^0) = 1$

$D_1 = a_0$

$D_1 = 1$

$$D_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_{n} & a_{n-2} \end{vmatrix} = \begin{vmatrix} 1{,}5 & 1 \\ 1 & 0{,}5 \end{vmatrix} = 0{,}75 - 1 = -0{,}25$$

$D_3 = a_0 \cdot D_{n-1}$

$D_3 = 1 \cdot (-0{,}25) \Rightarrow D_3 = -0{,}25$

$D_1 = 0$

$D_2 < 0$

$D_3 > 0$ sistem ⇒ nestabilan, dva prijelaza u desnoj poluravni

$\frac{D_2}{D_1} < 0$

$\frac{D_3}{D_2} > 0$

⊗ Routhov k → zatvorena petlja

$f(s) = s^3+1{,}5s^2+0{,}5s+1$

$a_n(s^3) = 1$

$a_{n-1}(s^2) = 1{,}5$

$a_{n-2}(s^1) = 0{,}5$

$a_{n-3}(s^0) = 1$

s^3	1	0,5	0
s^2	1,5	1	0
s^1	−0,166		
s^0	1		

$$b_1 = \frac{1{,}5 \cdot 0{,}5 - 1}{1{,}5}$$

$b_1 = -0{,}166$

sistem ⇒ nestabilan, dva prijelaza u desnoj poluravni

(*) Bodeov → OTVORENA PETLJA

$$W(s) = \frac{1}{s^3+1,5s^2+0,5s+1} = \frac{\frac{1}{s^3+1,5s^2+0,5s}}{1+\frac{1}{s^3+1,5s^2+0,5s}}$$

$$W_0(s) = \frac{1}{s^3+1,5s^2+0,5s} \Rightarrow \text{OTVORENA PETLJA}$$

$$s^3+1,5s^2+0,5s = s(s^2+1,5s+0,5)$$

$$s^2+1,5s+0,5 = 0$$

$$s_{1,2} = \frac{-1,5 \pm \sqrt{2,25-2}}{2} = \frac{-1,5 \pm 0,5}{2} \qquad s_1 = -1 \qquad s_2 = -0,5$$

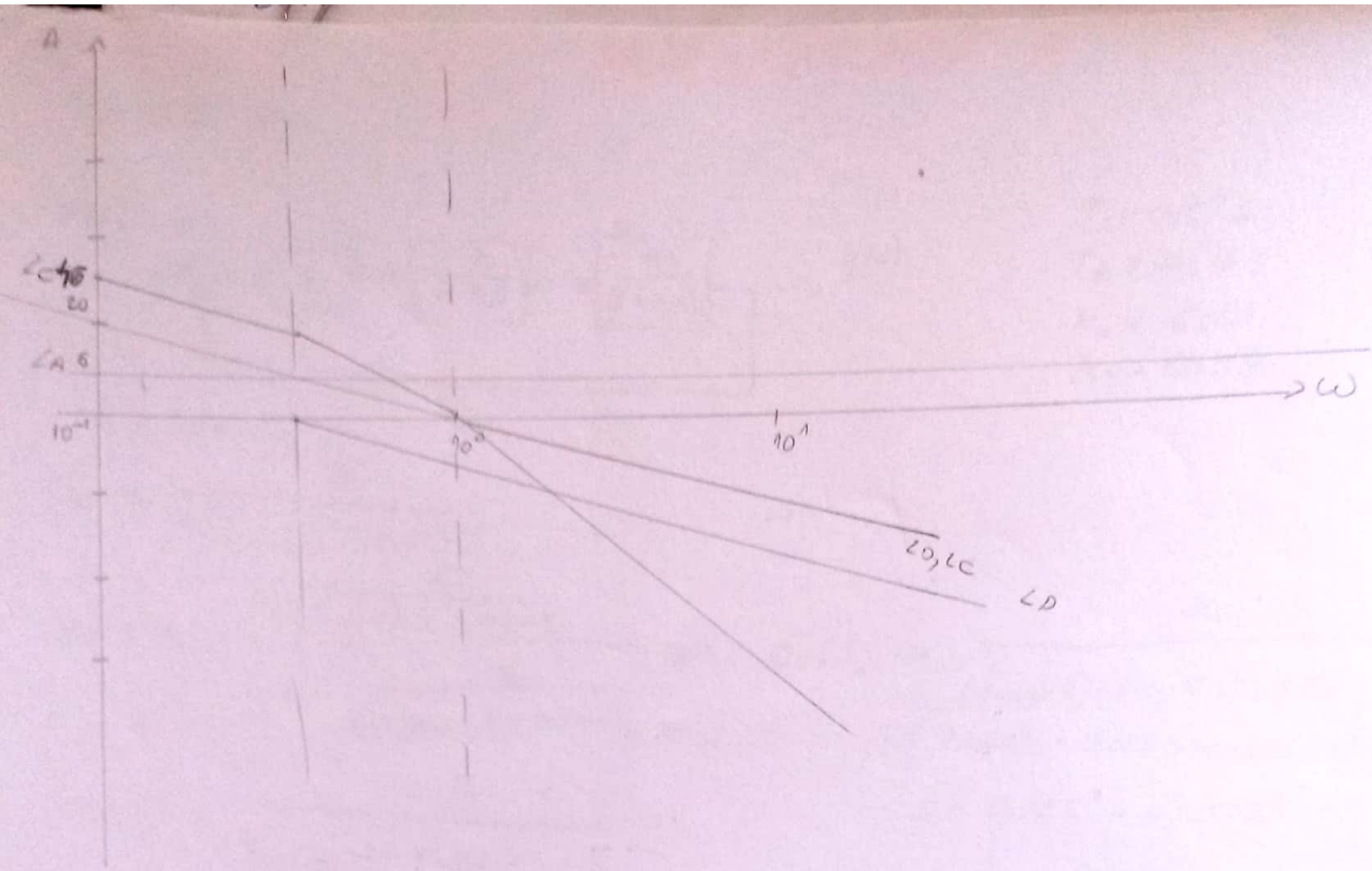
$$(s+1)(s+0,5)$$

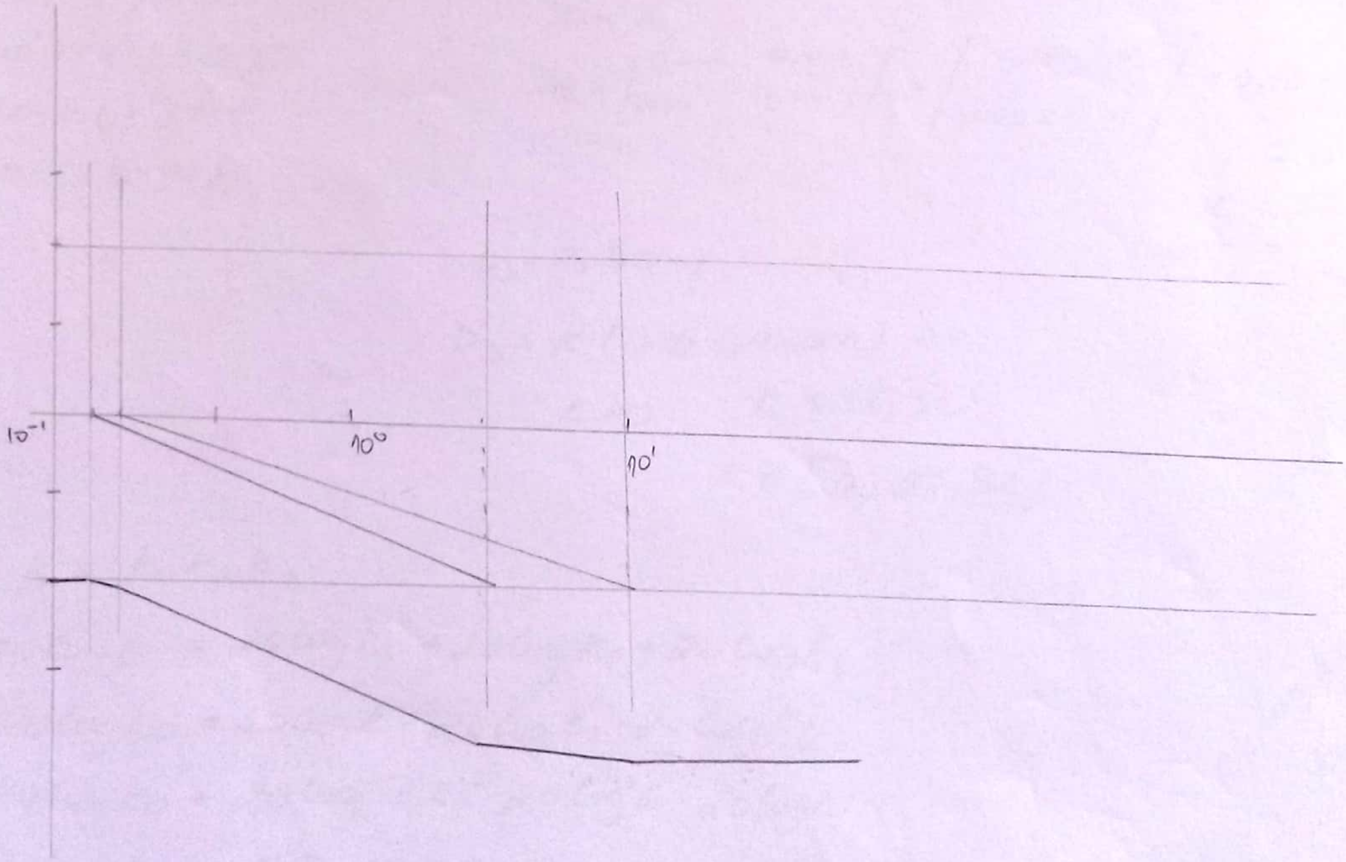
$$W_0(s) = \frac{1}{s(s+1)(s+0,5)} \Rightarrow W_0(j\omega) = \frac{1}{j\omega(j\omega+1)(j\omega+0,5)}$$

$$W_0(j\omega) - \frac{1}{0,5} \cdot \frac{1}{j\omega} \cdot \frac{1}{(j\omega+1)} \cdot \frac{1}{(2j\omega+1)}$$

$$\angle(j\omega) = \angle(A) - \angle(B) - \angle(C) - \angle(D) \qquad \varphi(j\omega) = \varphi(A) - \varphi(B) - \varphi(C) - \varphi(D)$$

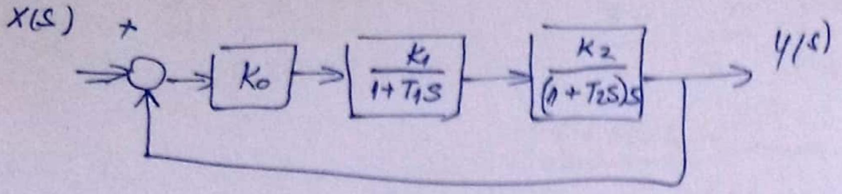
OSN. ČLAN	NAGIB		LOMNA F	FAZNI D.	
$A = 2$ $A = 6dB$	/		/	$\varphi = 0°$	
$B - \frac{1}{j\omega}$	-20		$\omega_B - 1$	$\varphi = -90°$	
$C = \frac{1}{1+j\omega}$	0 -20	$\omega < \omega_C$ $\omega > \omega_C$	$\omega_C = 1$	$\varphi = 0$ $\varphi = -90°$	$\omega \leq 0,1$ $\omega \geq 10$
$D = \frac{1}{2j\omega+1}$	0 -20	$\omega < \omega_D$ $\omega > \omega_D$	$\omega_D = 0,5$	$\varphi = 0$ $\varphi = -90°$	$\omega \leq 0,05$ $\omega \geq 5$


A
∠C 45
20
∠A 6
ω
10^{-1}
10^{0}
10^{1}
∠D, ∠C
∠D


10^{-1}
10^{0}
10^{1}

(19.) **HURWITZOV K.**

Odrediti: k_0



$T_1 = 0{,}02\,s$
$T_2 = 0{,}16\,s$
$K_1 = 6\,dB$
$K_2 = 10\,dB$

$K = k_0 k_1 k_2$

$$G_0 = \frac{K}{(1+0{,}02s)(1+0{,}16s)\cdot s}$$

$$G(s) = \frac{\frac{K}{(1+0{,}02s)s(0{,}16s+1)}}{1+\frac{K}{(1+0{,}02s)(0{,}16s+1)s}} \Rightarrow G_0(s) = \frac{K}{(1+0{,}02s)(1+0{,}16s)s+K}$$

$$s(1+0{,}16s+0{,}02s+0{,}0032s^2)+K$$

$$s+0{,}18s^2+0{,}0032s^3+K$$

$$G_0(s) = \frac{K}{0{,}0032s^3+0{,}18s^2+s+K}$$

$a_n(s^3) = 0{,}0032$
$a_{n-1}(s^2) = 0{,}18$
$a_{n-2}(s^1) = 1$
$a_{n-3}(s^0) = K$

$D_1 = K$

$$D_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} = \begin{vmatrix} 0{,}18 & K \\ 0{,}0032 & 1 \end{vmatrix} = 0{,}18 - 0{,}0032K$$

> 0

$K < 56{,}25$

$D_3 = a_0 \cdot D_{n-1}$

$D_3 = K\cdot(0{,}18 - 0{,}0032K) > 0$

$K > 0 \qquad K < 56{,}32$

$K \in [0;\ 56{,}32]$

$K = k_0 k_1 k_2$

$20\log K = 20\log k_0 + 20\log k_1 + 20\log k_2$

$20\log k_0 = 20\log K - 20\log k_1 - 20\log k_2$

$20\log k_0 = 20\log 56{,}25 - 20\log 6 - 20\log 10$

$20\log k_0 = 35 - 15{,}56 - 20$

$20\log k_0 = -0{,}56$

$$k_0 = 10^{\frac{-0{,}56}{20}} \Rightarrow k_0 = 10^{-0{,}028}$$

$$\boxed{k_0 = 0{,}938}$$

$20\log k_0 = 0{,}6$
$k_0 = 10^{0{,}6/20}$

[illegible]

$$W_o(s) = \frac{3(s+2)^2}{s(s+10)(s+1)}$$

$$W_o(s) = \frac{3\cdot 4(\frac{1}{2}s+1)^2}{s\cdot 10(\frac{1}{10}s+1)(1+s)} = \frac{3\cdot 4}{10}\cdot\frac{1}{s}\cdot\frac{(0{,}5s+1)^2}{(0{,}1s+1)(s+1)} \qquad s \to j\omega$$

$$W_o(j\omega) = 1{,}2\cdot\frac{1}{j\omega}\cdot\frac{(1+0{,}5j\omega)^2}{(1+0{,}1j\omega)(1+j\omega)}$$

$$L(j\omega) = 20\log 1{,}2 + 20\log(1+0{,}5j\omega)^2 - 20\log\frac{1}{j\omega} - 20\log(1+j\omega) - 20\log(1+0{,}1j\omega)$$

$$L(j\omega) = L(A) + L(B) - L(C) - L(D) - L(E)$$

$$\varphi(j\omega) = \operatorname{arctg} 1{,}2 + \operatorname{arctg}(1+0{,}5j\omega)^2 - \operatorname{arctg}\frac{1}{j\omega} - \operatorname{arctg}(1+j\omega) - \operatorname{arctg}(1+0{,}1j\omega)$$

$$\varphi(j\omega) = \varphi(A) + \varphi(B) - \varphi(C) - \varphi(D) - \varphi(E)$$

OSN. C.	AMPL. D NAGIB		COMNA F	FAZ. D.
$A=1{,}2$ $A=1{,}6$	/		1 /	$\varphi=0^\circ$
$B=(1+0{,}5j\omega)^2$	0 40	$\omega\leq\omega_B$ $\omega>\omega_B$	$\omega_B=2$	$\varphi=0 \quad \omega\leq 0{,}2$ $\varphi=180^\circ \quad \omega\geq 20$
$C=\frac{1}{j\omega}$	-20		$\omega_C=1$	$\varphi=-90^\circ$
$D=\frac{1}{1+j\omega}$	0 -20	$\omega<\omega_D$ $\omega>\omega_D$	$\omega_D=1$	$\varphi=0^\circ \quad \omega\leq 0{,}1$ $\varphi=-90 \quad \omega\geq 10$
$E=\frac{1}{1+0{,}1j\omega}$	0 -20	$\omega<\omega_E$ $\omega>\omega_E$	$\omega_E=10$	$\varphi=0^\circ \quad \omega\leq 1$ $\varphi=-90^\circ \quad \omega\geq 100$

s^2+2s+4

$= \quad 3s^2+6s+8$

(10) Routhov kriterij

Ispitati stabilnost sis.

$$f(s) = s^5 + 2s^4 + s^3 + 3s^2 + 4s + 5$$

$a_n = 1$
$a_{n-1} = 2$
$a_{n-2} = 1$
$a_{n-3} = 3$
$a_{n-4} = 4$
$a_{n-5} = 5$

s^5	1	1	4
s^4	2 +	3	5
s^3	$-\frac{1}{2}$ -	$\frac{3}{2}$	0 / (2)
s^2	9 +	5	
s^1	$\frac{31}{18}$		
s^0	5		

$$c_1 = \frac{-\frac{1}{2} \cdot 3 \cdot \frac{3}{2} \, 2}{-\frac{1}{2}}$$

$$c_1 = \frac{-\frac{3}{2} - 3}{-\frac{1}{2}}$$

$$c_1 = \frac{\frac{-3-6}{\cancel{2}}}{-\frac{1}{\cancel{2}}} = 9$$

$$c_2 = \frac{-\frac{5}{2} - 0}{-\frac{1}{2}}$$

$$c_2 = 5$$

$$d_1 = \frac{\frac{27}{2} + \frac{5}{2}}{\frac{9}{1}}$$

$$d_1 = \frac{31}{18}$$

(11.) $f(s) = s^4 + 2s^3 + s^2 + 2s + 1$

$a_n = 1$
$a_{n-1} = 2$
$a_{n-2} = 1$
$a_{n-3} = 2$
$a_{n-4} = 1$

s^4	1	1	1
s^3	2	2	0
s^2	ε	1	
s^1	$\frac{2\varepsilon - 2}{\varepsilon}$		
s^0	1		

$$c_1 = \frac{\varepsilon \cdot 2 - 2}{\varepsilon}$$

sis. ⇒ nestabilan
→ dva prijelaza u desnoj poluravni

$$b_1 = \frac{2 \cdot 1 - 2 \cdot 1}{2} = \frac{0}{2} = \infty$$

$$b_2 = \frac{2 \cdot 1 - 0}{2} = 1$$

(12.) $f(s) = s^5 + 2s^4 + s^3 + 2s^2 + s + 2$

$a_n = 1 \quad (s^5)$
$a_{n-1} = 2 \quad (s^4)$
$a_{n-2} = 1 \quad (s^3)$
$a_{n-3} = 2 \quad (s^2)$
$a_{n-4} = 1 \quad (s^1)$
$a_{n-5} = 2 \quad (s^0)$

s^5	1	1	1
s^4	2	2	2
s^3	0	0	0
s^2			
s^1			
s^0			

$\Rightarrow$ $2s^4 + 2s^2 + 2 \,/'$

$8s^3 + 4s$ → članovi idućeg reda

$b_1 = \frac{2-2}{2} = 0$

s^5	1	1	1
s^4	2	2	2
s^3	$\overset{\circ}{8}$	4	0
s^2	1	2	0
s^1	-12		
s^0	2		

sistem je nestabilan ⇒ 2 prijelaz u desnoj poluravni

(13) Hurwitzov kriterij

$f(s) = s^3 + 8s^2 + 14s + 24$

$a_3 = 1$

$a_2 = 8$

$a_1 = 14$

$a_0 = 24$

$D_1 = a_{n-1}$

$D_1 = 8$

$$D_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} = \begin{vmatrix} 14 & 24 \\ 1 & 8 \end{vmatrix} = 14 \cdot 8 - 24 \cdot 1 = 88$$

$D_1 = a_{n-1}$

$$D_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix}$$

$$D_3 = \begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} \\ a_n & a_{n-2} & a_{n-4} \\ 0 & a_{n-1} & a_{n-3} \end{vmatrix}$$

ili

$D_3 = a_0 \cdot D_{n-1}$

$D_1 = a_{n-1}$

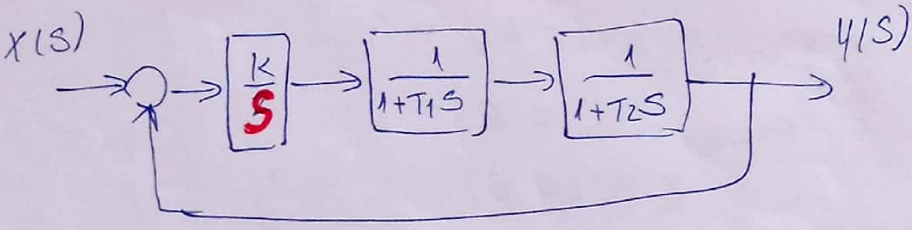
$D_1 = 8$

$$D_2 = \begin{vmatrix} 8 & 24 \\ 1 & 14 \end{vmatrix} = 88$$

$D_3 = a_0 \cdot D_{n-1} = 24 \cdot 88 \Rightarrow D_3 = 2112$

(14) T_1, T_2 su poznate konstante

Odrediti K za koji je sis. stabilan (koristiti Routha i Hurwitza) $T_1 = 0{,}5$ $T_2 = 0{,}25$



$$G(s) = \frac{K}{s(1+0{,}5s)(1+0{,}25s)}$$

$$G_0(s) = \frac{\frac{K}{s(1+0{,}5s)(1+0{,}25s)}}{1 + \frac{K}{s(1+0{,}5s)(1+0{,}25s)}}$$

$$G_0(s) = \frac{K}{s(1+0{,}5s)(1+0{,}25s) + K}$$

$$G_0(s) = \frac{K}{s(1+0,25s+0,5s+0,125s^2)+K}$$

$$G_0(s) = \frac{K}{s+0,75s^2+0,125s^3+K}$$

$$G_0(s) = \frac{K}{0,125s^3+0,75s^2+s+K}$$

Rathov k.

$$b_1 = \frac{0,75-0,125K}{0,75}$$

$a_3(a_n) = 0,125$

$a_2(a_{n-1}) = 0,75$

$a_1(a_{n-2}) = 1$

$a_0(a_{n-3}) = K$

s^3	0,125	1
s^2	0,75	K
s^1	$\frac{0,75-0,125K}{0,75}$	
s^0	K	

$K>0 \qquad 0,75-0,125K>0$

$-0,125K>-0,75$

$K<6$

$K\in(0,6)$

Hurwitzov k.

$D_1 = a_{n-1} \Rightarrow D_1 = 0,75$

$$D_2 = \begin{vmatrix} 0,75 & K \\ 0,125 & 1 \end{vmatrix} = 0,75 - 0,125K = 0$$

$0,75-0,125K>0$

$K<6$

$D_3 = a_0 \cdot D_{n-1} = K\cdot(0,75-0,125K) = 0$

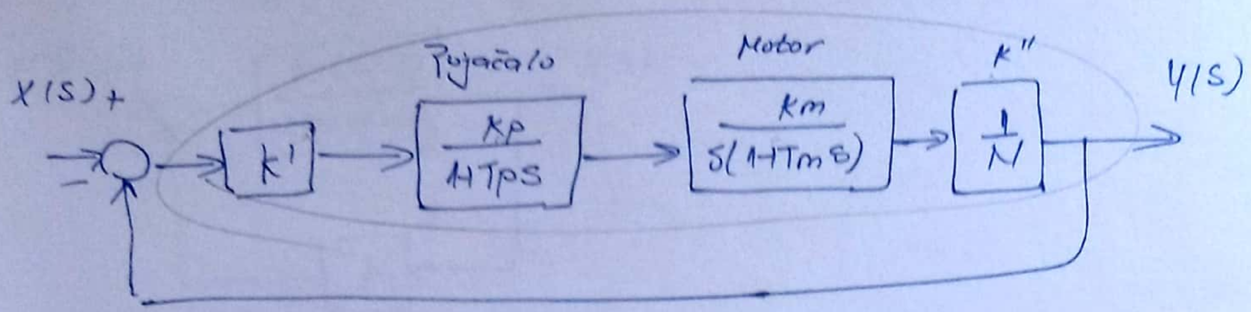
$K>0 \qquad K<6$

$K\in(0,6)$

?

(4.)

Upotreba Hurwitzovog kriterija



$T_p = 0{,}02\ s \quad T_m = 0{,}16$

$$K = k' k_p k_m k''$$

$$G_0(s) = \frac{K}{(1+T_p s)s(1+T_m s)}$$

$$G(s) = \frac{\frac{K}{(1+T_p s)s(1+T_m s)}}{1 + \frac{K}{(1+T_p s)s(1+T_m s)}} = \frac{\frac{K}{\cancel{(1+T_p s)s(1+T_m s)}}}{\frac{s(1+T_p s)(1+T_m s)+K}{\cancel{(1+T_p s)s(1+T_m s)}}} = \frac{K}{s(1+0{,}02s)(1+0{,}16s)+K}$$

$$G(s) = \frac{K}{s(1+0{,}16s+0{,}02s+0{,}0032s^2)+K} = \frac{K}{\underbrace{0{,}0032s^3+0{,}18s^2+s+K}}$$

$a_n(a_3) = 0{,}0032$

$a_{n-1}(a_2) = 0{,}18$

$a_{n-2}(a_1) = 1$

$a_{n-3}(a_0) = K$

$$D_1 = a_{n-1} \Rightarrow D_1 = 0{,}18$$

$$D_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} = \begin{vmatrix} 0{,}18 & K \\ 0{,}0032 & 1 \end{vmatrix} = 0{,}18 - 0{,}0032K > 0$$

$$-0{,}0032K > -0{,}18$$

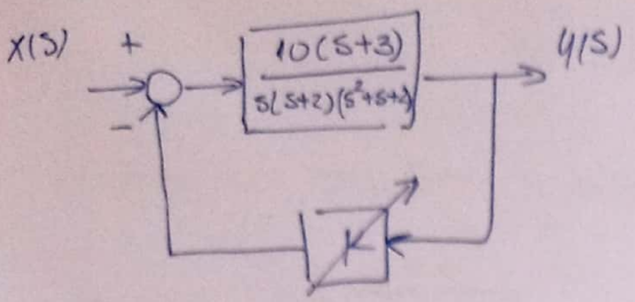
$$K < 56{,}25$$

$$D_3 = a_0 \cdot D_{n-1} = K \cdot (0{,}18 - 0{,}0032K) > 0$$

$$K > 0 \qquad K < 56{,}25$$

$$K \in (0; 56{,}25)$$

15. Odrediti k da bi sistem bio stabilan (Routhov kriterij)



$$G(s) = \frac{\frac{10(s+3)}{s(s+2)(s^2+s+2)}}{1+\frac{10(s+3)}{s(s+2)(s^2+s+2)}k} = \frac{\frac{10(s+3)}{s(s+2)(s^2+s+2)}}{\frac{s(s+2)(s^2+s+2)+10k(s+3)}{s(s+2)(s^2+s+2)}} = \frac{10(s+3)}{(s^2+2s)(s^2+s+2)+10ks+30k}$$

$$= \frac{10(s+3)}{s^4+s^3+2s^2+2s^3+2s^2+4s+10ks+30k} = \frac{10(s+3)}{s^4+3s^3+4s^2+s(4+10k)+30k}$$

$a_4 (a_n) = 1$

$a_3 (a_{n-1}) = 3$

$a_2 (a_{n-2}) = 4$

$a_1 (a_{n-3}) = 4+10k$

$a_0 (a_{n-4}) = 30k$

s^4	1	4	$30k$
s^3	3	$4+10k$	0
s^2	$\frac{8-10k}{3}$	$30k$	
s^1	$\frac{-100k^2-230k+32}{8-10k}$		
s^0	$30k$		

$$b_1 = \frac{12-4-10k}{3} = \frac{8-10k}{3}$$

$230k$

$$c_1 = \frac{\frac{8-10k}{3}\cdot 4+10k - 90k}{\frac{8-10k}{3}} = \frac{\frac{32+80k-40k-100k^2}{3} - 90k}{\frac{8-10k}{3}} = \frac{\frac{-100k^2-230k+32}{3}}{\frac{8-10k}{3}}$$

$$= \frac{-100k^2-203k+32}{8-10k} > 0$$

$-100k^2-230k+32=0$

$k_1 = -0{,}1488$, $k_2 = -2{,}152$

$\frac{8-10k}{3} = 0$

$8-10k = 0$

$-10k = -8$

$k = \frac{4}{5}$

16. Zadana je prijenosna fun. otvorenog sis. Odrediti stabilnost pomoću Hurwitza

$$W(s) = \frac{20}{s(s+2)(s+5)}$$

Zatvoren sistem ⇒ jedinična pov. v.

$$W_0(s) = \frac{\frac{20}{s(s+2)(s+5)}}{1+\frac{20}{s(s+2)(s+5)}} = \frac{\frac{20}{\cancel{s(s+2)(s+5)}}}{\frac{s(s+2)(s+5)+20}{\cancel{s(s+2)(s+5)}}} = \frac{20}{s(s+2)(s+5)+20}$$

$$W_0(s) = \frac{20}{s(s^2+5s+2s+10)+20} = \frac{20}{s^3+5s^2+2s^2+10s+20}$$

$$W_0(s) = \frac{20}{s^3+7s^2+10s+20}$$

$a_3 (a_n) = 1$

$a_2 (a_{n-1}) = 7$

$a_1 (a_{n-2}) = 10$

$a_0 (a_{n-3}) = 20$

$D_1 = a_{n-1}$

$D_1 = 7$

$$D_2 = \begin{vmatrix} 7 & 20 \\ 1 & 10 \end{vmatrix} = 70 - 20 = 50$$

$$D_3 = \begin{vmatrix} 7 & 20 & 0 \\ 1 & 10 & 0 \\ 0 & 7 & 20 \end{vmatrix} \begin{matrix} 7 & 20 \\ 1 & 10 \\ 0 & 7 \end{matrix} = 1400 - 400 = 1000$$

$$D_3 = a_0 \cdot D_{n-1} = 20 \cdot 50 = 1000$$

$D_1 > 0$ ✓

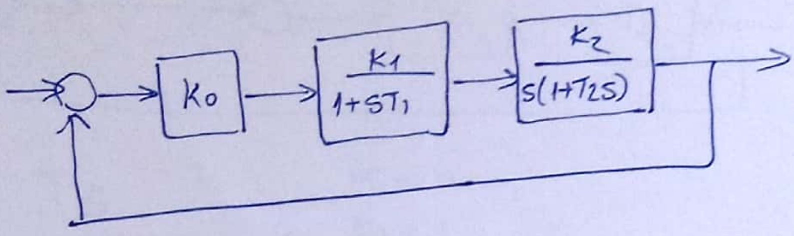
$\frac{D_2}{D_1} > 0 \quad \frac{50}{7} > 0$

$\frac{D_3}{D_2} > 0 \quad \frac{1000}{50} > 0$

sistem je stabilan

7) Hurwitzov kriterij

$T_1 = 0,02 \quad T_2 = 0,16 \quad K_1 = 6\ dB \quad K_2 = 10\ dB$



$$G(S) = \frac{K_0 K_1 K_2}{(1+ST_1)s(1+T_2S)} \Rightarrow K$$

$$G(S) = \frac{K}{(1+ST_1)s(1+T_2S)} \qquad G_0(S) = \frac{\frac{K}{(1+ST_1)s(1+T_2S)}}{1+\frac{K}{s(1+T_1s)(1+T_2S)}}$$

$$G_0(S) = \frac{K}{S(1+T_1S)(1+T_2S)+K}$$

$$G_0(S) = \frac{K}{S(1+0,02S)(1+0,16S)+K} = \frac{K}{S(1+0,16S+0,02S+0,0032S^2)+K}$$

$$G_0(S) = \frac{K}{0,0032S^3+0,18S^2+S+K}$$

$a_3\ (a_n) = 0,0032$

$a_2\ (a_{n-1}) = 0,18$

$a_1\ (a_{n-2}) = 1$

$a_0\ (a_{n-3}) = K$

$D_1 = a_{n-1}$

$D_1 = 0,18$

$$D_2 = \begin{vmatrix} 0,18 & K \\ 0,0032 & 1 \end{vmatrix} = 0,18 - 0,0032K > 0$$

$K < 56,25$

$D_3 = a_0 D_{n-1} = K(0,18 - 0,0032K) > 0$

$K > 0 \quad K < 56,25$

$K = k_0 \cdot k_1 \cdot k_2$

$20\log k = 20\log k_0 + 20\log k_1 + 20\log k_2$

$20\log k_0 = 20\log k - 20\log k_1 - 20\log k_2$

$20\log k_0 = 20\log 56 - 20\log 6 - 20\log 10$

$20\log k_0 = 34,96 - 15,563 - 20$

$20\log k_0 = -0,603$

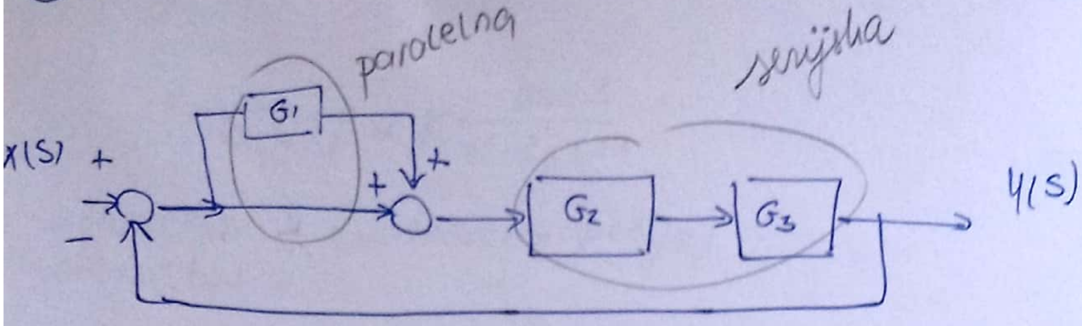
$K_0 = 10^{\frac{-0,603}{20}}$

$K_0 = 10^{-0,03015}$

$k_0 = 0,9329$

16.

(18.) Hurwitzov i Routhov k.



$G_1 = T s$

$G_2 = \frac{K_1}{1+T_1 s}$

$G_3 = \frac{K_2}{1+T_2^2 s^2}$

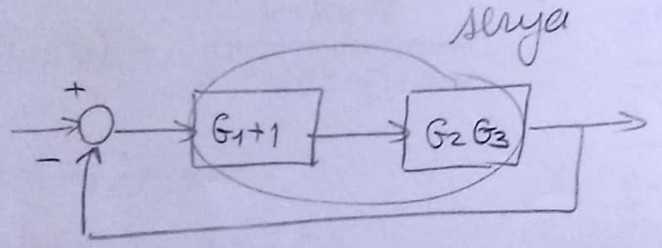
$K_1 = 10$

$K_2 = 1$

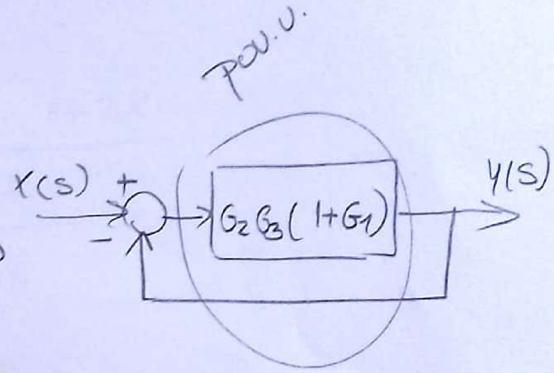
$T = 0{,}1\ s$

$T_1 = 0{,}05\ s$

$T_2 = 2\ s$



⇒



$$G_0(s) = \frac{G_2G_3(1+G_1)}{1+G_2G_3(1+G_1)} = \frac{\frac{10}{1+0{,}1s}\cdot\frac{1}{1+4s^2}\cdot(1+0{,}1s)}{1+\frac{10}{1+0{,}1s}\cdot\frac{1}{1+4s^2}(1+0{,}1s)}$$

$$G_0(s) = \frac{\frac{10}{(1+0{,}1s)(1+4s^2)}\cdot(1+0{,}1s)}{1+\frac{10(1+0{,}1s)}{(1+0{,}1s)(1+4s^2)}} = \frac{\frac{10(1+0{,}1s)}{\cancel{(1+0{,}1s)(1+4s^2)}}}{\frac{(1+0{,}1s)(1+4s^2)+10(1+0{,}1s)}{\cancel{(1+0{,}1s)(1+4s^2)}}}$$

$$= \frac{10(1+0{,}1s)}{1+4s^2+0{,}1s+0{,}4s^3+10+s} = \frac{10(1+0{,}1s)}{0{,}4s^3+4s^2+1{,}1s+11}$$

$a_3(a_n) = 0{,}4$

$a_2(a_{n-1}) = 4$

$a_1(a_{n-2}) = 1{,}1$

$a_0(a_{n-3}) = 11$

$D_1 = a_{n-1}$

$D_1 = 4$

$D_2 = \begin{vmatrix} 4 & 11 \\ 0{,}4 & 1{,}1 \end{vmatrix} = 4{,}4 - 4{,}4 = 0$

$D_3 = a_0 D_{n-1} \Rightarrow D_3 = 0$

$$\left.\begin{aligned} D_1 &> 0 \\ \frac{D_2}{D_1} &= 0 \\ \frac{D_3}{D_2} &= 0 \end{aligned}\right\}$$ s.s. granično stabilan

17.

(20) Nyquistov kriterij

Pri izradi Nyquista prvo odrediti k preko nekog kriterija pa tek onda raditi Nyquista!

$$G(s) = K\frac{8s-3}{s^2-6s+11}$$

→ Routhov k (zatvorena petlja)
- Nyquistov k

Routhov k:

$$G_o(s) = \frac{\frac{k(8s-3)}{s^2-6s+11}}{1+\frac{k(8s-3)}{s^2-6s+11}} = \frac{\frac{k(8s-3)}{\cancel{s^2-6s+11}}}{\frac{s^2-6s+11+k(8s-3)}{\cancel{s^2-6s+11}}} = \frac{k(8s-3)}{s^2-6s+11+8ks-3k}$$

$$G_o(s) = \frac{k(8s-3)}{s^2+s(8k-6)+11-3k}$$

$a_n(a_2) = 1$

$a_{n-1}(a_1) = 8k-6$

$a_{n-2}(a_0) = 11-3k$

s^2	1	$11-3k$
s^1	$8k-6$	
s^0	$11-3k$	

$8k-6>0$ $\quad$ $11-3k>0$

$8k>6$ $\quad$ $-3k>-11$ $\quad \Rightarrow k\in\left(\frac{3}{4}, \frac{11}{3}\right)$

$k>\frac{3}{4}$ $\quad$ $k<\frac{11}{3}$

→ uzimam neko k iz intervala $k\in\left(\frac{3}{4}, \frac{11}{3}\right)$ i radim Nyquista

$K=1$

$$G(s) = \frac{8s-3}{s^2-6s+11}$$

$s=j\omega$

$$G(j\omega) = \frac{8j\omega-3}{(j\omega)^2-6j\omega+11}$$

$$G(j\omega) = \frac{8j\omega-3}{-\omega^2-6j\omega+11}$$

$$G(j\omega) = \frac{8j\omega-3}{(11-\omega^2)-6j\omega}\cdot\frac{(11-\omega^2)+6j\omega}{(11-\omega^2)+6j\omega}$$

$$G(j\omega) = \frac{(8j\omega-3)(11-\omega^2+6j\omega)}{(11-\omega^2)^2-(6j\omega)^2}$$

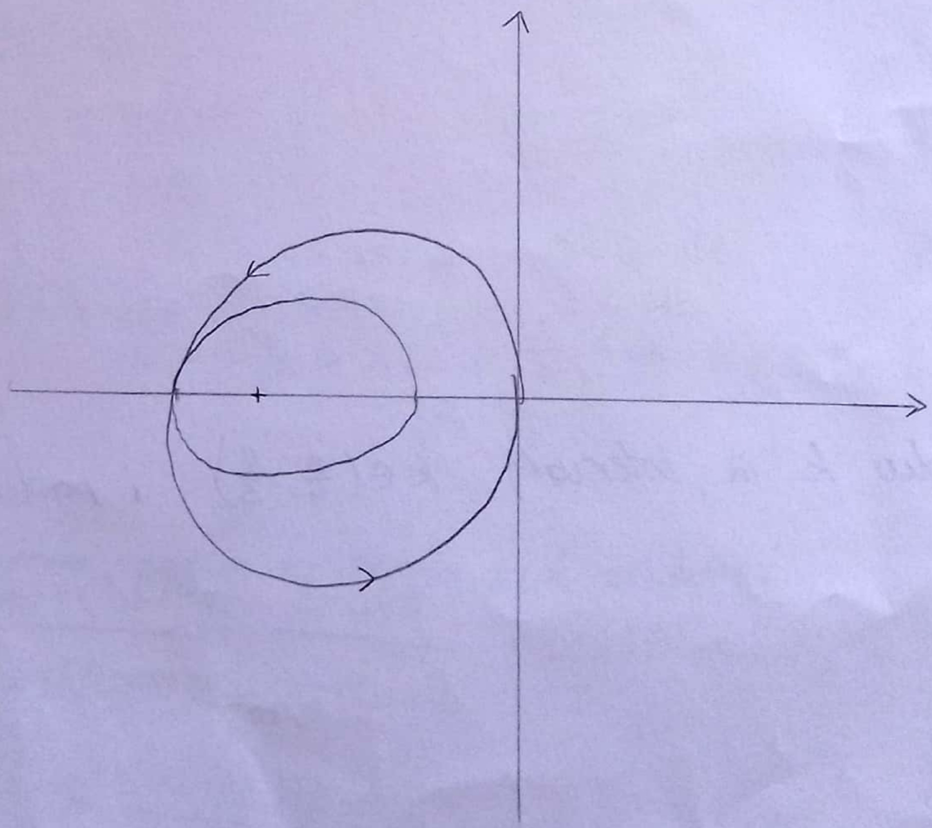
$$G(j\omega) = \frac{88j\omega-8j\omega^3+48(j\omega)^2-33+3\omega^2-18j\omega}{11^2-2\cdot 11\omega^2+\omega^4+36\omega^2}$$

$$G(j\omega) = \frac{88j\omega-8j\omega^3-48\omega^2-33+3\omega^2-18j\omega}{121-22\omega^2+\omega^4+36\omega^2}$$

$$G(s) = \frac{-45\omega^2 - 33}{\omega^4 + 14\omega^2 + 121} + \frac{70\omega - 8\omega^3}{\omega^4 + 14\omega^2 + 121} \cdot j$$

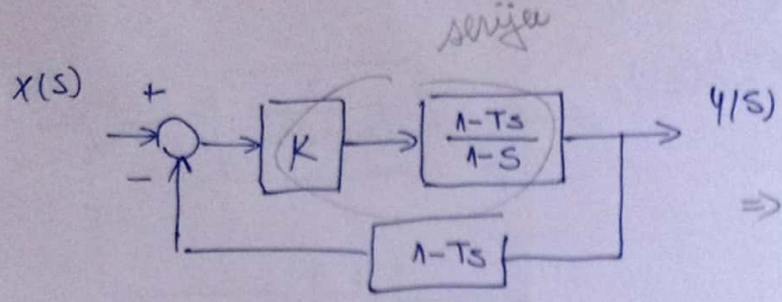
ω	0	1	2
Re	-0,27	-0,57	0,39
Im	0	0,45	0,39

```
s = tf('s');
G1 = (8s-3)/(s^2-6s+11);
nyquist(G1);
```

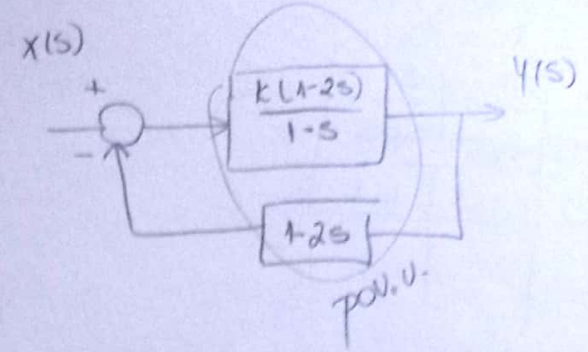


Scanned with CamScanner

(21) Nyquistov kriterij



$\Rightarrow$



$T = 2$

$$G_0(S) = \frac{\frac{K(1-2S)}{1-S}}{1 + \frac{K(1-2S)}{1-S}\cdot(1-2S)} = \frac{\frac{K(1-2S)}{\cancel{1-S}}}{\frac{1-S+K(1-2S)(1-2S)}{\cancel{1-S}}} = \frac{K(1-2S)}{1-S+K(1-2S-2S+4S^2)}$$

$$G_0(S) = \frac{K(1-2S)}{1-S+K-2KS-2KS+4KS^2} = \frac{K(1-2S)}{4KS^2+S(-4K-1)+1+K}$$

$a_2(a_n) = 4K$

$a_1(a_{n-1}) = -4K-1$

$a_0(a_{n-2}) = 1+K$

S^2	$4K$	$1+K$
S^1	$-4K-1$	
S^0	$1+K$	

$1+K>0 \quad K>-1$

$-4K-1>0 \quad -4K>1 \quad K<-\frac{1}{4}$

$\Rightarrow K \in \left(-1, \frac{1}{4}\right)$

$4K>0 \quad K>0$

Za $K = 0{,}25 \left(\frac{1}{4}\right)$

$\Rightarrow$ otvorena petlja $\Rightarrow$ serija svih blokova!

$$G(S) = \frac{K(1-2S)(1-2S)}{1-S} = \frac{0{,}25(1-4S+4S^2)}{(1-S)}$$

$S \to j\omega$

$$G(j\omega) = \frac{0{,}25(1-4j\omega-4\omega^2)}{1-j\omega}\cdot\frac{1+j\omega}{1+j\omega}$$

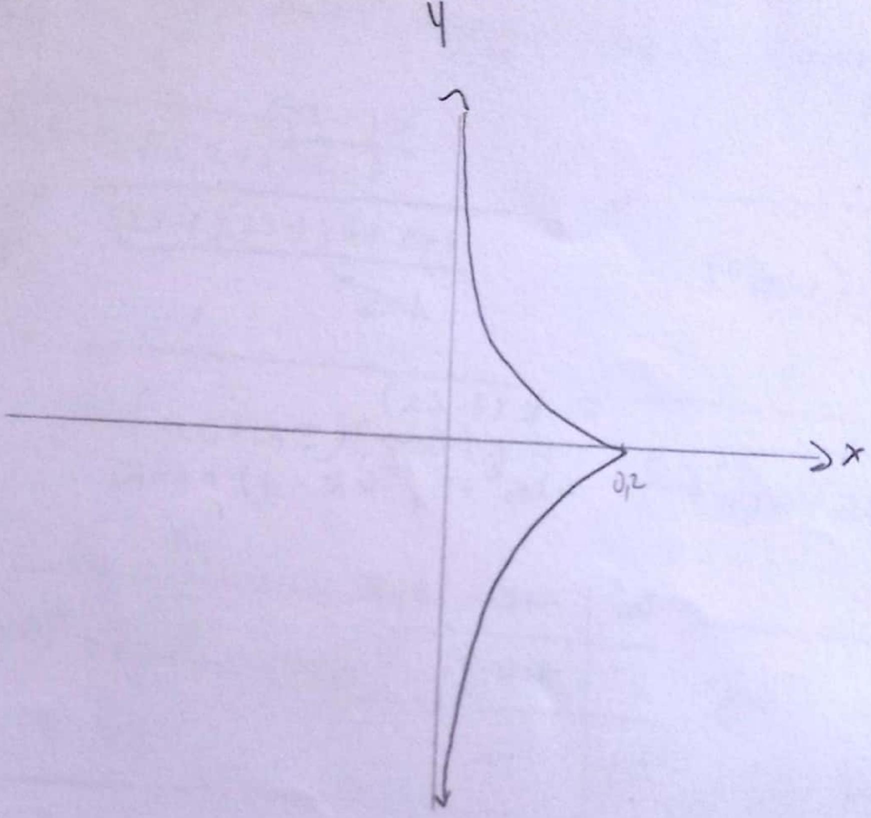
$$G(j\omega) = \frac{(0{,}25-2j\omega-2\omega^2)(1+j\omega)}{1^2-(j\omega)^2} = \frac{0{,}25-0{,}25j\omega-2j\omega-\cancel{2(j\omega)^2}-\cancel{2\omega^2}-2j\omega^3}{1^2+\omega^2}$$

$$G(j\omega) = \frac{-2j\omega^3-2{,}25j\omega+0{,}25}{1+\omega^2} = \frac{0{,}25}{1+\omega^2} + \frac{-2\omega^3-2{,}22\omega}{1+\omega^2}\,j$$

$$G(s) = \frac{0,25}{1+\omega^2} + \frac{-2\omega^3 - 2,25\omega}{1+\omega^2} j$$

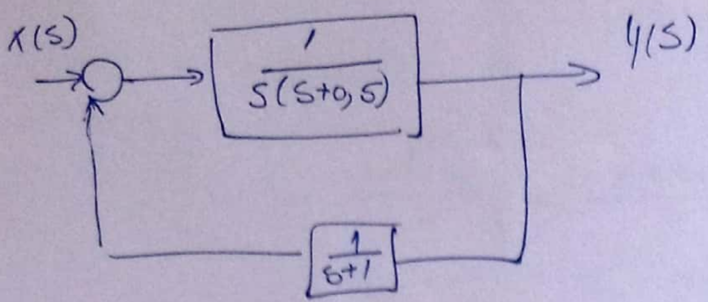
$16 - 4,5$

ω	0	1	2	
Re	0,25	0,125	0,05	
Im	0	2,125	4,1	



(22.) Nyquistov k.

$i^1 = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$



$\Rightarrow$ otvorena petlja kod Nyquista $\Rightarrow$ serijska veza blokova

$$G(s) = \frac{1}{s(s+0,5)(s+1)}$$

$s \Rightarrow j\omega$

$$G(j\omega) = \frac{1}{j\omega(j\omega+0,5)(j\omega+1)} = \frac{1}{j\omega((j\omega)^2+j\omega+0,5j\omega+0,5)}$$

$$G(j\omega) = \frac{1}{(j\omega)^3+(j\omega)^2+0,5(j\omega)^2+0,5j\omega} = \frac{1}{-j\omega^3-\omega^2-0,5\omega^2+0,5j\omega}$$

$$G(j\omega) = \frac{1}{(-\omega^2-0,5\omega^2)+(0,5j\omega-j\omega^3)} = \frac{1}{-1,5\omega^2+(0,5j\omega-j\omega^3)} \cdot \frac{-1,5\omega^2-(0,5j\omega-j\omega^3)}{-1,5\omega^2-(0,5j\omega-j\omega^3)}$$

$$G(j\omega) = \frac{-1,5\omega^2-0,5j\omega+j\omega^3}{(-1,5\omega^2)^2-(0,5j\omega-j\omega^3)^2} = \frac{-1,5\omega^2-0,5j\omega+j\omega^3}{2,25\omega^4-(-0,25\omega^2+0,5\omega^4+\omega^6)}$$

$$G(j\omega) = \frac{-1,5\omega^2-0,5j\omega+j\omega^3}{2,25\omega^4+0,25\omega^2+0,5\omega^4+\omega^6} = \frac{-1,5\omega-0,5j+j\omega^2}{2,25\omega^3+0,25\omega+0,5\omega^3+\omega^5}$$

$$G(j\omega) = \frac{-1,5\omega}{2,75\omega^3+\omega^5+0,25\omega} + \frac{\omega^2-0,5}{2,75\omega^3+\omega^5+0,25\omega} j \quad \ldots$$

$$G(j\omega) = \frac{1,5}{\omega^4+2,75\omega^2+0,25} + \frac{\omega^2-0,5}{\omega^5+2,75\omega^3+0,25} j$$

ω	0	0,4	1
Re	-6	-3,15	-0,6
Im	$-\infty$	-1,78	0,2