Problem Set: March 21

① ⓐ $\hat{p} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

$= 5/4$

ⓑ $\hat{p} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

$= 3/58$

② ?

③ $f(x \mid \mu, \sigma^2) = \dfrac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\dfrac{(x-\mu)^2}{2\sigma^2}\right)$

$P(X < \theta) = 0.95 = P\left(\dfrac{x-\mu}{\sigma} < \dfrac{\theta-\mu}{\sigma}\right)$

$L(\mu, \sigma^2) = -\dfrac{n}{2}\ln(2\pi\sigma^2) - \sum \dfrac{(x_i-\mu)^2}{2\sigma^2}$

$\dfrac{\partial L}{\partial \mu} = \sum \dfrac{x_i - \mu}{\sigma^2} = 0$

$\dfrac{\partial L}{\partial \sigma^2} = -\dfrac{n}{2\sigma^2} + \left(\sum \dfrac{(x_i-\mu)^2}{2}\right)\dfrac{1}{(\sigma^2)^2} = 0$

$\mu = \bar{x} = \dfrac{1}{n}\sum x_i$

$\sigma^2 = \sum \dfrac{(x_i-\bar{x})^2}{n} = \overline{(x_i-\bar{x})^2}$

$\theta = \theta_0 \sqrt{\overline{(x_i-\bar{x})^2}} + \bar{x}$

④ $P(X>2) = \cancel{1 - P(X \le 2)}$

$\cancel{= 1 - P(}$

$U = P(X>) =$

$= P\left(Z > \frac{2-\mu}{\sigma}\right)$

$= 1 - \phi\left(\frac{2-\mu}{\sigma}\right)$

⑤ $\cancel{L(\theta)}$ $f(x) = \frac{1}{\pi(1+(x-\theta)^2)}$

$L(\theta, x_1 \ldots x_n) = \frac{1}{\pi^n \prod(1+x_i-\theta))^2}$

⑥ MLE of $\mu = \frac{20 \cdot 6 + x}{21} < 15$

$= MLE(120x < 315)$

$= MLE\left(x < \frac{63}{24}\right)$

$= MLE(x < 21/8)$

$L(X \le 21/8) = 1 - P(X \ge 21/8)$

$= 1 - \lambda e^{-\lambda x}$

$\cancel{\Phi}$ ..

(7) $P(X=k|\lambda) = e^{-\lambda}\frac{\lambda^k}{k!}$

$$P(\lambda) = \prod_{i=1}^{n} e^{-\lambda}\frac{\lambda^{x_i}}{x_i} = \frac{(e^{-\lambda}\lambda^{\tilde{x}})^n}{x_1!\ldots x_n!}$$

$$g(\lambda) = e^{-\lambda}\lambda^{\tilde{x}} = \exp(-\lambda + \bar{x}\ln\lambda))$$

$$g'(\lambda) = -1 + \bar{x}/\lambda)g(x)$$

The max of g is the $\lambda = \bar{x}$. Thus MLE of SD is $\sqrt{\bar{x}}$

(8) $f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$

$$L(\lambda, x_1, \ldots, x_n) = \prod_{i=1}^{n} f(x_i, \lambda) = \prod_{i=1}^{n} \lambda e^{-\lambda x} = \lambda^n e^{-\lambda\sum_{i=1}^{n} x_i}$$

$$\frac{d\ln(L(\lambda, x_1 \ldots x_n))}{d\lambda} = 0$$

$$= \lambda = \frac{n}{\sum_{i=1}^{n} x_i}$$

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