Sufficiency theorem

① $\prod_{i=1}^{n} p^{x_i}(1-p)^{1-x_i} = p^{\sum_{i=1}^{n} x_i}(1-p)^{n-\sum_{i=1}^{n} x_i}$

$f(x_1, x_2 \ldots x_n; t; p) = p^t(1-p)^{n-t}$ where

$\sum_{i=1}^{n} x_i = t$

$g(t; p) = \binom{n}{t} p^t (1-p)^{n-t}$ for $t = 0, 1, \ldots n$

$P(x_1, x_2 \ldots x_n \mid T = t; p) = \frac{f(x_1, x_2, \ldots x_n; t; p)}{g(t; p)}$

$= \frac{p^t(1-p)^{n-t}}{\binom{n}{t} p^t (1-p)^{n-t}}$

$= \frac{1}{\binom{n}{t}}$

② Let $x \in \{1, 2, \ldots\}^n$ and $t = t(x) = x_1 + \ldots + x_n$.
we have that

$L(x; p) = \prod_{i=1}^{n} p(1-p)^{x_i - 1} = p^n (1-p)^{t-n}$

from which sufficiency is obvious.

③ Negative binomial with fixed r is an exponential family:

$$f_\theta(x) = \binom{r+x-1}{r-1} \theta^r (1-\theta)^x$$

$$= \exp\left\{ \log(1-\theta)\, x + \log\binom{r+x-1}{r-1} + r\log\theta \right\}$$

Since $K(x) = x$ in this case, the complete sufficient stat for θ based on an iid sample $x_1, \ldots, x_n$ is

$$T = \sum_{i=1}^{n} x_i$$

④ ?

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⑤ $f(x_1, \dots, x_n \mid \alpha) = \dfrac{\beta^{n\alpha}}{\Gamma(\alpha)^n} \left(\prod_{i=1}^{n} x_i^{\alpha-1} \right) \exp\left(-\beta \sum_{i=1}^{n} \right.$

$$\prod_{i=1}^{n} x_i^{\alpha-1} = \exp\left((\alpha-1) \sum_{i=1}^{n} \ln(x_i) \right)$$

$$T = \sum_{i=1}^{n} \ln(x_i)$$