4077 HW 1/24/19

MLE of proportion p

$f(x_i;p) = p^{x_i}(1-p)^{1-x_i}$

$L(p) = \prod_{i=1}^{n} f(x_i;p)$

$= p^{\Sigma x_i}(1-p)^{n-\Sigma x_i}$

$\log L(p) = \Sigma x_i \log(p) + (n-\Sigma x_i)\log(1-p)$

Multiplying through we get

$(\Sigma x_i - np) = 0$ thus

$\hat{p} = \frac{\sum_{i=1}^{n} x_i}{n}$

$= \frac{12}{70}$

2. ~~[illegible]~~

Σ(all probabilities) = 1

This can't be true if $\theta = 1$ or 0.

3. $P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$

$L(\lambda)' = \sum_{i=1}^{n} (x_i \log \lambda - \lambda - \log x_i!)$

$= \log \lambda \sum_{i=1}^{n} x_i - n\lambda - \sum_{i=1}^{n} \log x_i!$

$\hat{\lambda} = \bar{x}$

This can't be true because $\hat{\lambda} = 0$ and $\Sigma(p)$ must equal 1.

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4. $f(x_i;\theta_1,\theta_2) = \frac{1}{\sqrt{\theta_2}\sqrt{2\pi}} \exp\left[-\frac{(x_i-\theta_1)^2}{2\theta_2}\right]$

$L(\theta_1,\theta_2) = \prod_{i=1}^{n} (x_i;\theta_1,\theta_2)$

$= \theta_2^{-n/2}(2\pi)^{-n/2} \exp\left[-\frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2\right]$

$\log L(\theta_1,\theta_2) = -\frac{n}{2}\log\theta_2 - \frac{n}{2}\log(2\pi) - \frac{\sum(x_i-\theta_1)^2}{2\theta_2}$

$\sum x_i - n\theta_1 = 0$

$\hat{\theta} = \hat{\mu} = \frac{\sum x_i}{n} = \bar{x}$

$\hat{\theta}^2 = \frac{\sum(x_i-\bar{x})^2}{n}$