Class 6 - Important Distributions and Densities Homework

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Section 5.1 # 7, 10, 16, 18, 25, 27, 28, 38

⑦ ⓐ Let's think about each possible value of T in isolation

$T=1 \Rightarrow$ got a 6 on the first roll $\Rightarrow P(T=1) = 1/6$

$T=2 \Rightarrow$ got anything other than 6 on first roll, and 6 on second roll

$P(T=2) = (1-\frac{1}{6})(\frac{1}{6}) = 5/36$

$T=3 \Rightarrow P(T=3) = (1-\frac{1}{6})^2(\frac{1}{6}) = \frac{25}{216}$

$P(T=t) = 1/6(1-\frac{1}{6})^{t-1}$

$\boxed{\text{Geometric distribution w/ } p = 1/6}$

ⓑ $P(T=1) = 1/6$, $P(T=2) = 5/36$, $P(T=3) = \frac{25}{216}$

$P(T>3) = 1 - P(T=1) - P(T=2) - P(T=3) = 1 - \frac{1}{6} - \frac{5}{36} - \frac{25}{36} = \boxed{0.5787}$

ⓒ $P(T=4) = (\frac{5}{6})^3(\frac{1}{6}) = 0.0965$

$P(T=5) = (\frac{5}{6})^4(\frac{1}{6}) = 0.0804$

$P(T=6) = (\frac{5}{8})^5(\frac{1}{6}) = 0.0670$

so $P(T>6) = P(T>3) - P(T=4) - P(T=5) - P(T=6) =$
$0.5787 - 0.0965 - 0.0804 - 0.0670 = 0.3348$

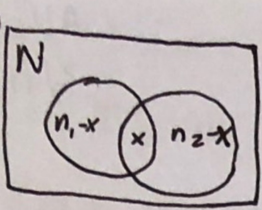
We know $P(T>6 \mid T>3) = \frac{P(T>6 \cap T>3)}{P(T=3)}$ but $T>6 \Rightarrow$

thus $P(T>6 \cap T>3) = P(T>6) = 0.3348$

$P(T>6 \mid T>3) = \frac{0.3348}{0.5787} = 0.5787$

$P(X=0) = e^{-3} = 0.0498$

10 (a)



How many ways can x people be seclected? $\Rightarrow \binom{N}{x}$

For remainder of n_1, $\binom{N-x}{n_1-x}$ ways they could be seleted

For the n_2 count, there is $\binom{N-x-(n_1-x)}{n_2-x} = \binom{N-n_1}{n_2-x}$ ways they could be selected.

Number of ways intersection of size x could be seleted is equal to

$$\binom{N}{x}\binom{N-x}{n_1-x}\binom{N-n_1}{n_2-x}$$

The total number of ways n_1 and n_2 can be selected from population of size N is equal to $\binom{N}{n_1}\binom{N}{n_2}$

Therefore $$P(X=x) = \frac{\binom{N}{x}\binom{N-x}{n_1-x}\binom{N-n_1}{n_2-x}}{\binom{N}{n_1}\binom{N}{n_2}}$$

(b) If we set $x = n_{12}$, then $$P(X=n_{12}) = \frac{\binom{N}{n_{12}}\binom{N-x}{n_1-n_{12}}\binom{N-n_1}{n_2-n_{12}}}{\binom{N}{n_1}\binom{N}{n_2}}$$

Let's define as a function of N

$$f(N) = \frac{\binom{N}{n_{12}}\binom{N-x}{n_1-n_2}\binom{N-n_1}{n_2-n_{12}}}{\binom{N}{n_1}\binom{N}{n_2}}$$

Now we can examine $$\frac{f(N+1)}{f(N)} = \frac{\binom{N+1}{n_{12}}\binom{N+1-x}{n_1-n_{12}}\binom{N+1-n_1}{n_2-n_{12}}}{\binom{N+1}{n_1}\binom{N+1}{n_2}} \cdot \frac{\binom{N}{n_1}\binom{N}{n_2}}{\binom{N}{n_{12}}\binom{N-x}{n_1-n_{12}}}$$

We can now break this into parts:

$$\frac{\binom{N}{n_1}}{\binom{N+1}{n_1}} = 1 - \frac{n_1}{N+1}$$

$$\frac{\binom{N}{n_2}}{\binom{N+1}{n_2}} = 1 - \frac{n_2}{N+1}$$

(16) $n = 60 \cdot 5 = 300$; $P = 0.01 \Rightarrow \lambda = 3$

$P(X=0) = e^{-3} = 0.0498$

$P(X=1) = 3e^{-3} = 0.1494$

$P(\text{Miss at most one call}) = P(X=0) + P(X=1) = 0.0498 + 0.1494 = 0.1992$

(18) (a) $P_{\text{raisin}} = \frac{600}{500} = 1.2$; $n=1$, $\lambda = 1.2$

$P(X=0) = e^{-1.2} = 0.301$

(b) $P_{\text{chip}} = \frac{400}{500} = 0.8$; $n = 1 \Rightarrow \lambda = 0.8$

$P(X=2) = \frac{0.8^2}{2!} e^{-0.8} = 0.14379$

(c) $P_{\text{chip or raisin}} = \frac{600+400}{500} = 2 \Rightarrow \lambda = 2$

$P(X \geq 2) = 1 - P(X=0) - P(X=1) = 1 - e^{-2} - 2e^{-2} = 0.594$

(25) Cost of paying each time $= 100 \cdot 0.10 = 10$

So if he gets caught more than three times, he'll pay more

$P = 0.05$; $n = 100 \Rightarrow \lambda = 5$

$P(X=0) = e^{-5} = 0.00674$

$P(X=1) = 5e^{-5} = 0.0337$

$P(X=2) = \frac{25}{2} e^{-5} = 0.0842$

$P(X=3) = \frac{125}{6} e^{-5} = 0.1404$

$P(X > 2) = 1 - 0.00674 - 0.0337 - 0.0842 - 0.1404 = 0.73496$

$\hookrightarrow$ 73% chance he'll pay more by skipping meter.

(27) $m = 100 \times (.001) = .1$ Thus $p(\text{at least one accident}) = 1 - e^{-.1} = 0.0952$

(28) Random variable x represents number of accidents for 100 plants in a given year.

$p = 0.001$ for an individual plant, and $n = 100 \Rightarrow \lambda = 0.01$

$P(X=0) = e^{-0.1} = 0.905 \Rightarrow P(X>0) = 1 - P(X=0) = 1 - 0.905$
$= 0.095$

(38) (a) Let x represent the number of defective items in the sample.

We know $P(X=d) = \frac{\binom{D}{d}\binom{N-D}{n-d}}{\binom{N}{n}}$ and in this case $D=5, N=20, n=5$

So $P(X=1) = \frac{\binom{5}{1}\binom{20-5}{5-1}}{\binom{20}{5}} = 5 \cdot \frac{\binom{15}{4}}{\binom{20}{5}} = 5 \cdot \frac{1365}{15{,}504} = 0.4402$

(b) Since we're sampling with replacement, the probability of finding a defective item is the same each time:

$p = \frac{15}{20} = 0.75 \Rightarrow P(X=1) = b(5, 0.75, 1) = 5 \cdot (0.75^4)(0.25)$
$= 0.39551$

Section 5.2 # 1, 17, 21, 37

(1) (a) $F_U(u) = P(U \le u) = u$ for u in $[0, 1)$

$f_u(u) = \frac{d}{du} F_U(u) = 1$

$F_Y(y) = P(Y \le y) = P(U + 2 \le y) = y - 2$ for y in $[2, 3]$

$f_Y(y) = \frac{d}{dy} F_Y(y) = 1$

(b) $F_Y(y) = P(Y \le y) = P(U^3 \le y) = P(U \le y^{\frac{1}{3}}) = y^{\frac{1}{3}} \Rightarrow$

$f_Y(y) = \frac{1}{3} y^{-}$

(17) (a) $f(x) = \frac{d}{dx} F(x) = \frac{d}{dx} \sin^2\left(\frac{\pi x}{2}\right) = \pi \cos\left(\frac{\pi x}{2}\right) \sin\left(\frac{\pi x}{2}\right)$

(b) $P(X < \frac{1}{4}) = F(\frac{1}{4}) = \sin^2\left(\frac{\pi}{8}\right) = 0.146$

(21) $P(Y \leq y) = P(F(X)) \leq y) = P(X \leq F^{-1}(y)) = F(F^{-1}(y)) = y$ on $[0,1]$

(37) $F_Y(y) = \frac{1}{\sqrt{2\pi y}} e^{-\frac{\log^2(y)}{2}}$, for $y > 0$