MA 677 HW 6 Hypothesis Testing I

4/5/19

(1) (a) If $\beta = 1$

power $= \alpha(\beta) = P(X \geq 1) = 1 - (1 - e^{-\lambda}) = e^{-\lambda} = e^{-\frac{1}{\beta}}$

If $\beta < 1$

power $= 1 - \beta(\beta) = 1 - P(X < 1) = e^{-\frac{1}{\beta}}$

power function $= e^{-\frac{1}{\beta}}$

(b) size of test is probability of type I error

$\alpha = \beta \geq 1 \; e^{-\frac{1}{\beta}} \geq 1$

(2) If $p = 0.2$

power $= \alpha(p) = P(Y \geq 7) + P(Y \leq 1) = 1 - P(Y < 7) + P(Y \leq 1) \approx 0.136$

If $p \neq 0.2$

power $= \beta(p) = 1 - P(1 < Y \leq 7) = 1 - \sum_{i=1}^{6} C_{20}^{i} p^{i} (1-p)^{20-i}$

Calculated by R: $p = 0$, $\beta(p) = 1$; $p = 0.1$, $\beta(p) = 0.392$;

$p = 0.2$, $\beta(p) = 0.136$

$p = 0.3$, $\beta(p) = 0.3996$, $p = 0.4$, $\beta(p) = 0.75$

$p = 0.5$, $\beta(p) = 0.94$

$p = 0.981$, $p = 1$, $\beta(p) = 1$

size of test is prob of type I error, $\alpha = 0.136$

(3) Size of test is prob of type I error.

$\alpha(\mu) = P(T(x) > c) = P(|\bar{x}_n - \mu_0| > c) = 0.05$

$\Rightarrow P\left(\frac{|\bar{x}_n - \mu_0|}{\sqrt{\frac{\sigma^2}{n}}} > \sqrt{n} c\right) = 0.5$

$\sqrt{n} c = 1.96$, $n = 25$

$\therefore c = 0.392$

④ ⓐ $P(Y \le c_1 | p0.4) + P(Y \ge c_2 | \phi = 0.4) \approx 0.1$

$$= \sum_{i=0}^{c_1} C_9^i \, 0.4^i \, 0.6^{9-i} + 1 - \sum_{i=0}^{c_2-1} C_9^i \, 0.4^i \, 0.6^{9-i}$$

$$= 1 - \sum_{i=c_1+1}^{c_2-1} C_9^i \, 0.4^i \, 0.6^{9-i}$$

when $c_1 = 1$ and $c_2 = 7$

$P(Y \le c_1 | \phi = 0.4) + P(Y \ge c_1 | \phi = 0.4) = 0.0956$

which is closest to 0.1

ⓑ size of test is type I error $\alpha (0.6 = 0.0956$