1 Quantum optics formulae

1.1 Lorentz classical model

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = f\frac{eE_0}{m}\cos(\omega t), \qquad \begin{aligned} f > 0 &\Rightarrow \text{ absorption} \\ f < 0 &\Rightarrow \text{ st. emission} \end{aligned}$$

1.2 Classical coherence

$$g^{(1)}(\tau) = \frac{\left\langle E^{(-)}(t) + E^{(+)}(t+\tau)\right\rangle}{\langle E^{(-)}(t)E^{(+)}(t)\rangle}$$

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \left|g^{(1)}(\tau)\right| \begin{cases} = 0 & \Rightarrow \text{incoh. field} \\ \in (0,1) & \Rightarrow \text{part. coh. field} \\ = 1 & \Rightarrow \text{fully coh. field} \end{cases}$$

$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle^2}$$

$$g^{(2)}(\tau)\begin{cases} \text{stationary field} & g^{(2)}(\tau) = 1 \\ \text{fluctuating field} & \begin{cases} g^{(2)}(0) \geq 1 \\ g^{(2)}(\tau) \leq g^{(2)}(0) \end{cases} \quad \Rightarrow \text{bunching} \end{cases}$$

1.3 Susceptibility

$$\chi = \frac{Nex}{\varepsilon E} \equiv \chi' + i\chi'', \quad \alpha \equiv \frac{ik\chi}{2}$$

1.4 Phase and group velocities

$$v_{ph} \equiv \frac{\mathrm{d}z}{\mathrm{d}t} = \frac{c}{1 + \dfrac{\chi'}{2}} = \frac{c}{n(\omega)}, \quad v_g = \left.\frac{\Delta z}{\Delta t}\right|_{max} = \frac{c}{n + \omega\dfrac{\partial n}{\partial\omega}}$$

- Superluminal light: $\dfrac{\partial n}{\partial\omega} < 0$.
- Slow light: $\dfrac{\partial n}{\partial\omega} > 0$.

1.5 Einstein's rate equations

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = \underbrace{-B_{12}N_1\rho(\omega)}_{\text{absorption}} + \underbrace{B_{21}N_2\rho(\omega)}_{\text{st. emission}} + \underbrace{A_{21}N_2}_{\text{sp. em.}}, \quad \frac{\mathrm{d}N_2}{\mathrm{d}t} \equiv -\frac{\mathrm{d}N_1}{\mathrm{d}t}$$

1.6 Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\left|\psi(t)\right\rangle = (H_0 + V)\left|\psi(t)\right\rangle, \quad \left|\psi(t)\right\rangle = \sum_i a_i(t)e^{-i\omega_i t}\left|i\right\rangle$$

$$\Rightarrow \dot{a}_k = -\frac{i}{\hbar}\sum_{i=1}^{n}\langle k|V|i\rangle\, a_i e^{-i(\omega_i - \omega_k)t}$$

1.7 Rabi oscillations

$$\Omega \equiv \frac{\vec{\mu}_0\cdot\vec{E}_0}{\hbar} \propto I_{laser}, \quad \Delta \equiv \omega_0 - \omega, \quad \Omega' \equiv \sqrt{\Omega^2 + \Delta^2}$$

$$\begin{cases} \dot{a}_1 = i\frac{\Omega}{2}e^{-i\Delta t}a_2, \quad \dot{a}_2 = i\frac{\Omega}{2}e^{i\Delta t}a_1 \\ a_1(t) = Ae^{-i(\Delta-\Omega')t/2} + Be^{-i(\Delta+\Omega')t/2} \\ a_2(t) = Ce^{+i(\Delta-\Omega')t/2} + De^{+i(\Delta+\Omega')t/2} \end{cases}$$

$$\begin{cases} a_1(0) = 1 \\ a_2(0) = 0 \end{cases} \Rightarrow P_2(t) = a_2 a_2^\star = \left(\frac{\Omega}{\Omega'}\right)^2\sin^2\left(\frac{\Omega'}{2}t\right)$$

1.8 AC Stark splitting and dressed atom

$$\exists\vec{E} = \vec{E}_0\cos(\omega t) \Rightarrow \begin{aligned} E_1^{\pm} &= E_1 + \hbar\frac{\Delta}{2} \pm \hbar\frac{\Omega'}{2} \\ E_2^{\pm} &= E_2 - \hbar\frac{\Delta}{2} \pm \hbar\frac{\Omega'}{2} \end{aligned}$$

1.9 Equation of motion of the density matrix

$$\dot{\bar{\rho}} = -\frac{i}{\hbar}\left[\bar{H} - H_{IP}, \bar{\rho}\right] + L\bar{\rho}$$

1.10 Temporal evolution of a closed atom

$$\dot{\rho}_{ii} = -\frac{i}{\hbar}[H,p]_{ii} + \sum_k \underbrace{(\Gamma_{ki} + \Lambda_{ki})\rho_{kk}}_{\text{add to } |i\rangle} - \sum_k \underbrace{(\Gamma_{ik} + \Lambda_{ik})\rho_{ii}}_{\text{extract from } |i\rangle}$$

$$\dot{\rho}_{ij} = -\frac{i}{\hbar}[H,p]_{ij} - \frac{1}{2}\underbrace{\left[\sum_{i,j}\Gamma_{(i\vee j)k} + \Lambda_{(i\vee j)k}\right]}_{\text{extract from } |i\rangle \text{ or } |j\rangle}\rho_{ij}$$

1.11 Density matrix for a closed two-level atom

$$\begin{aligned} \dot{\rho}_{11} &= \Omega\, y_{12}, \quad \dot{\rho}_{22} = -\dot{\rho}_{11} \\ \dot{\rho}_{12} &= i\Delta\rho_{12} + i\frac{\Omega}{2}(\rho_{22} - \rho_{11}) \\ \dot{x}_{12} &= -\Delta y_{12} \\ \dot{y}_{12} &= \Delta x_{12} + \frac{\Omega}{2}(\rho_{22} - \rho_{11}) \end{aligned}$$

where $\rho_{12} = x_{12} + iy_{12}$.

1.12 Optical Bloch equations

$$\begin{aligned} u &= \rho_{21} + \rho_{12} = 2\,\mathrm{Re}\{\rho_{12}\} \\ v &= i(\rho_{21} - \rho_{12}) = 2\,\mathrm{Im}\{\rho_{12}\} \\ w &= \rho_{22} - \rho_{11} \end{aligned} \quad \Rightarrow \quad \begin{aligned} \dot{u} &= -\Delta v \\ \dot{v} &= \Delta u + \Omega w \\ \dot{w} &= -\Omega v \end{aligned}$$