Productes entre vectors

$$\vec{A}\cdot\vec{B}\wedge\vec{C}=\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}=\vec{B}\cdot\vec{C}\wedge\vec{A}=\vec{C}\cdot\vec{A}\wedge\vec{B} \tag{1}$$

$$\vec{A}\wedge(\vec{B}\wedge\vec{C})=\vec{B}(\vec{A}\cdot\vec{C})-\vec{C}(\vec{A}\cdot\vec{B}) \tag{2}$$

$$(\vec{A}\wedge\vec{B})\wedge\vec{C}=\vec{B}(\vec{A}\cdot\vec{C})-\vec{A}(\vec{B}\cdot\vec{C}) \tag{3}$$

$$(\vec{A}\wedge\vec{B})\cdot(\vec{C}\wedge\vec{D})=(\vec{A}\cdot\vec{C})(\vec{B}\cdot\vec{D})-(\vec{A}\cdot\vec{D})(\vec{B}\cdot\vec{C})=\vec{A}\cdot\big[\vec{B}\wedge(\vec{C}\wedge\vec{D})\big] \tag{4}$$

$$(\vec{A}\wedge\vec{B})\wedge(\vec{C}\wedge\vec{D})=\vec{C}\big[\vec{A}\cdot(\vec{B}\wedge\vec{D})\big]-\vec{D}\big[\vec{A}\cdot(\vec{B}\wedge\vec{C})\big]=\vec{B}\big[\vec{A}\cdot(\vec{C}\wedge\vec{D})\big]-\vec{A}\big[\vec{B}\cdot(\vec{C}\wedge\vec{D})\big] \tag{5}$$

Operador nabla

$$\vec{\nabla}(fg)=f\vec{\nabla}g+g\vec{\nabla}f \tag{6}$$

$$\vec{\nabla}(\vec{A}\cdot\vec{B})=(\vec{B}\cdot\vec{\nabla})\vec{A}+(\vec{A}\cdot\vec{\nabla})\vec{B}+\vec{B}\wedge(\vec{\nabla}\wedge\vec{A})+\vec{A}\wedge(\vec{\nabla}\wedge\vec{B}) \tag{7}$$

$$\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}_1|}=-\frac{\vec{r}-\vec{r}_1}{|\vec{r}-\vec{r}_1|^3};\qquad \vec{\nabla}_1\frac{1}{|\vec{r}-\vec{r}_1|}=\frac{\vec{r}-\vec{r}_1}{|\vec{r}-\vec{r}_1|^3} \tag{8}$$

$$\vec{\nabla}\cdot(f\vec{A})=(\vec{\nabla}f)\cdot\vec{A}+f(\vec{\nabla}\cdot\vec{A}) \tag{9}$$

$$\vec{\nabla}\cdot(\vec{A}\wedge\vec{B})=\vec{B}\cdot(\vec{\nabla}\wedge\vec{A})-\vec{A}\cdot(\vec{\nabla}\wedge\vec{B}) \tag{10}$$

$$\vec{\nabla}\cdot(\vec{\nabla}\wedge\vec{A})=0\quad\Rightarrow\quad\vec{\nabla}\cdot\vec{B}=0\Leftrightarrow\vec{B}=\vec{\nabla}\wedge\vec{A} \tag{11}$$

$$\vec{\nabla}\cdot\vec{\nabla}f\equiv\nabla^2 f \tag{12}$$

$$\vec{\nabla}\cdot\vec{r}=3;\qquad\vec{\nabla}\cdot\frac{\vec{r}-\vec{r}_1}{|\vec{r}-\vec{r}_1|}=\frac{2}{|\vec{r}-\vec{r}_1|};\qquad\nabla^2\frac{1}{|\vec{r}-\vec{r}_1|}=-4\pi\delta(\vec{r}-\vec{r}_1) \tag{13}$$

$$\vec{\nabla}\wedge(f\vec{A})=(\vec{\nabla}f)\wedge\vec{A}+f(\vec{\nabla}\wedge\vec{A}) \tag{14}$$

$$\vec{\nabla}\wedge(\vec{A}\wedge\vec{B})=(\vec{B}\cdot\vec{\nabla})\vec{A}-(\vec{A}\cdot\vec{\nabla})\vec{B}+(\vec{\nabla}\cdot\vec{B})\vec{A}-(\vec{\nabla}\cdot\vec{A})\vec{B} \tag{15}$$

$$\vec{\nabla}\wedge\vec{\nabla}f=0\quad\Rightarrow\quad\vec{\nabla}\wedge\vec{A}=0\Leftrightarrow\vec{A}=\vec{\nabla}f \tag{16}$$

$$\vec{\nabla}\wedge\vec{\nabla}\wedge\vec{A}=\vec{\nabla}(\vec{\nabla}\cdot\vec{A})-\nabla^2\vec{A} \tag{17}$$

$$\vec{\nabla}\wedge\vec{r}=0;\qquad\vec{\nabla}\wedge\frac{\vec{r}-\vec{r}_1}{|\vec{r}-\vec{r}_1|}=0 \tag{18}$$

$$\text{Si:}\quad\vec{n}\equiv\frac{\vec{r}-\vec{r}_1}{|\vec{r}-\vec{r}_1|}\qquad(\vec{A}\cdot\nabla)\vec{n}=\frac{1}{|\vec{r}-\vec{r}_1|}\big[\vec{A}-\vec{n}(\vec{A}\cdot\vec{n})\big]\equiv\frac{\vec{A}_\perp}{|\vec{r}-\vec{r}_1|} \tag{19}$$

Teoremes d'integrals vectorials

$$\int_v\vec{\nabla}\cdot\vec{A}\,dv=\oint_S\vec{A}\cdot\vec{n}\,ds\qquad\text{(teorema de la divergència)} \tag{20}$$

$$\int_v\vec{\nabla}\wedge\vec{A}\,dv=\oint_S\vec{n}\wedge\vec{A}\,ds\qquad\text{(teorema del rotacional)} \tag{21}$$

$$\int_v\vec{\nabla}f\,dv=\oint_S f\,\vec{n}\,ds\qquad\text{(teorema del gradient)} \tag{22}$$

$$\int_S(\vec{\nabla}\wedge\vec{A})\cdot\vec{n}\,ds=\oint_C\vec{A}\cdot d\vec{l}\qquad\text{(teorema de Stokes)} \tag{23}$$

$$\int_S\vec{n}\wedge\vec{\nabla}f\,ds=\oint_C f\,d\vec{l} \tag{24}$$

Coordenades rectangulars: $dl = \sqrt{dx^2 + dy^2 + dz^2}$; $dv = dx\, dy\, dz$

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\vec{e}_x + \frac{\partial f}{\partial y}\vec{e}_y + \frac{\partial f}{\partial z}\vec{e}_z \tag{25}$$

$$\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \tag{26}$$

$$\vec{\nabla} \wedge \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{e}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{e}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{e}_z \tag{27}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \tag{28}$$

Coordenades cilíndriques: $dl = \sqrt{d\rho^2 + (\rho\, d\varphi)^2 + dz^2}$; $dv = \rho\; d\rho\, d\varphi\, dz$

$$\vec{\nabla} f = \frac{\partial f}{\partial \rho}\vec{e}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\vec{e}_\varphi + \frac{\partial f}{\partial z}\vec{e}_z \tag{29}$$

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z} \tag{30}$$

$$\vec{\nabla} \wedge \vec{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\vec{e}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\vec{e}_\varphi + \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}(\rho A_\varphi) - \frac{\partial A_\rho}{\partial \varphi}\right)\vec{e}_z \tag{31}$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2} = \frac{\partial^2 f}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial f}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2} \tag{32}$$

Coordenades esfèriques: $dl = \sqrt{dr^2 + (r\, d\theta)^2 + (r \sin\theta\, d\varphi)^2}$; $dv = r^2 \sin\theta\; dr\, d\theta\, d\varphi$

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\vec{e}_\varphi \tag{33}$$

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi} \tag{34}$$

$$\begin{aligned} \vec{\nabla} \wedge \vec{A} \quad = \quad & \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\varphi \sin\theta) - \frac{\partial A_\theta}{\partial \varphi}\right]\vec{e}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r}(r A_\varphi)\right]\vec{e}_\theta \\ & + \frac{1}{r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial \theta}\right]\vec{e}_\varphi \end{aligned} \tag{35}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2} \tag{36}$$

Coordenades curvilínies: $dl = \sqrt{(h_1\, du_1)^2 + (h_2\, du_2)^2 + (h_3\, du_3)^2}$; $dv = h_1 h_2 h_3\; du_1 du_2 du_3$

$$\vec{\nabla} f = \frac{1}{h_1}\frac{\partial f}{\partial u_1}\vec{e}_1 + \frac{1}{h_2}\frac{\partial f}{\partial u_2}\vec{e}_2 + \frac{1}{h_3}\frac{\partial f}{\partial u_3}\vec{e}_3 \tag{37}$$

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{h_1\, h_2\, h_3}\left[\frac{\partial}{\partial u_1}(h_2 h_3 A_1) + \frac{\partial}{\partial u_2}(h_3 h_1 A_2) + \frac{\partial}{\partial u_3}(h_1 h_2 A_3)\right] \tag{38}$$

$$\vec{\nabla} \wedge \vec{A} = \frac{1}{h_1\, h_2\, h_3}\begin{vmatrix} h_1\vec{e}_1 & h_2\vec{e}_2 & h_3\vec{e}_3 \\ \partial/\partial u_1 & \partial/\partial u_2 & \partial/\partial u_3 \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix} \tag{39}$$

$$\nabla^2 f = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2 h_3}{h_1}\frac{\partial f}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_3 h_1}{h_2}\frac{\partial f}{\partial u_2}\right) + \frac{\partial}{\partial u_3}\left(\frac{h_1 h_2}{h_3}\frac{\partial f}{\partial u_3}\right)\right] \tag{40}$$