1 Formulari electromagnetisme

1.1 Electrostàtica

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon \vec{E}$$

$$\int_{\mathcal{S}} \vec{D} \cdot \mathrm{d}\vec{\mathcal{S}} = \int \rho \, \mathrm{d}\mathcal{S} \qquad \text{(Gauss)}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \rho(\vec{r}') \frac{\vec{r} - \vec{r}'}{\|\vec{r} - \vec{r}'\|^3} \, \mathrm{d}^3 r'$$

$$\phi = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}')}{\|\vec{r} - \vec{r}'\|^3} \, \mathrm{d}^3 r', \quad \Delta\phi = -\int_{\infty} \vec{E} \cdot \mathrm{d}\vec{l}$$

$$\vec{P} = \int r' \rho(\vec{r}') \, \mathrm{d}^3 r'$$

$$C = \frac{Q}{\Delta\phi}, \quad W_E = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} Q \Delta\phi$$

$$\vec{\nabla} \cdot \vec{E} = 0, \quad \vec{\nabla} \cdot \vec{D} = \rho, \quad -\vec{\nabla} \cdot \vec{P} = \rho_p, \quad \vec{P} \cdot \hat{n} = \sigma_p$$

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}, \quad \varepsilon = \varepsilon_0 (1 + \chi_e) \qquad \text{(LIU)}$$

Condicions de contorn

$$\hat{n}_{21} \cdot \left(\vec{D}_1 - \vec{D}_2\right) = \sigma \iff D_{1n} - D_{2n} = \sigma$$
$$\hat{n}_{21} \times \left(\vec{E}_1 - \vec{E}_2\right) = 0 \iff E_{1t} = E_{2t}$$

1.2 Magnetostàtica

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu \vec{H}$$

$$\oint_{\mathcal{C}} \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 \int_{\mathcal{S}} \vec{J} \cdot \mathrm{d}\vec{\mathcal{S}} \qquad \text{(Ampère)}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \vec{J}(\vec{r}') \times \frac{\vec{r} - \vec{r}'}{\|\vec{r} - \vec{r}'\|^3} \, \mathrm{d}^3 r' \qquad \text{(Biot–Savart)}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \rho_m(\vec{r}') \frac{\vec{r} - \vec{r}'}{\|\vec{r} - \vec{r}'\|} \, \mathrm{d}^3 r' + \mu_0 M' \qquad \text{(Per un mat. mag.)}$$

$$\vec{J} = g \vec{E} \qquad \text{(Ohm)}$$

$$\vec{J} = \rho \vec{v}, \quad J \, \mathrm{d}^3 r = I \, \mathrm{d}l$$

$$\vec{F}_{1 \to 2} = \int \vec{J}_2(\vec{r}_2) \times \vec{B}_1(\vec{r}_2) \, \mathrm{d}V_2$$

$$\vec{H}(\vec{r}) = \frac{1}{4\pi} \int \rho_m(\vec{r}') \frac{\vec{r} - \vec{r}'}{\|\vec{r} - \vec{r}'\|^3} \, \mathrm{d}^3 r'$$

$$\vec{m} = \frac{1}{2} \int \vec{r}' \times \vec{J}(\vec{r}') \, \mathrm{d}^3 r'$$

$$\vec{M} = \chi_m \vec{H}, \quad \mu = \mu_0 (1 + \chi_m) \qquad \text{(LIU)}$$

$$\vec{M} \cdot \hat{n} = \sigma_m, \quad \vec{M} \times \hat{n} = \vec{K}_m$$
$$\vec{\nabla} \cdot \vec{B} = 0, \quad \vec{\nabla} \cdot \vec{H} = -\vec{\nabla} \cdot \vec{M} = \rho_m$$
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_T, \quad \vec{\nabla} \times \vec{H} = \vec{J}, \quad \vec{\nabla} \times \vec{M} = \vec{J}_m$$

Condicions de contorn

$$\hat{n}_{21} \cdot \left(\vec{B}_1 - \vec{B}_2\right) = 0 \iff B_{1n} = B_{2n}$$
$$\hat{n}_{21} \times \left(\vec{H}_1 - \vec{H}_2\right) = \vec{K} \iff H_{1t} - H_{2t} = K$$

1.3 Camps de variació lenta

$$\Phi = \int_{\mathcal{S}} \vec{B} \cdot \mathrm{d}\vec{\mathcal{S}}$$

$$\xi = \oint_{\mathcal{C}} \vec{E} \cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} \qquad \textit{(fem)}$$

$$\Phi_{21} = M_{21} I_1, \quad M_{21} = \frac{\mu_0}{4\pi} \oint_{\mathcal{C}_1} \oint_{\mathcal{C}_2} \frac{\mathrm{d}\vec{l}_2 \cdot \mathrm{d}\vec{l}_1}{\|\vec{r}_2 - \vec{r}_1\|}$$

$$W_M = \begin{cases} I\Phi & (\vec{B} \text{ extern}) \\ \frac{1}{2} L I^2 & (\vec{B} \text{ propi}) \end{cases}$$

1.4 Lleis de Maxwell

$$\vec{\nabla} \cdot \vec{D} = \rho \qquad \text{(Gauss E)}$$
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{(Faraday)}$$
$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \text{(Gauss M)}$$
$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad \text{(Ampère)}$$

$$\vec{F}(\vec{r}) = \int \left[\rho(\vec{r}')\vec{E}(\vec{r}') + \vec{J}(\vec{r}') \times \vec{B}(\vec{r}')\right] \mathrm{d}^3 r' \qquad \text{(Força de Lorentz)}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon \vec{E}$$
$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu \vec{H}$$

$$\vec{S} = \vec{E} \times \vec{H}, \quad \frac{\mathrm{d}}{\mathrm{d}t}(W_{EM} + E_{cin}) = -\oint \vec{S} \cdot \mathrm{d}\mathcal{S}$$

$$W_{EM} = \frac{1}{2} \int (\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B}) \, \mathrm{d}^3 r'$$

1.5 Dimensions

$$[E] = \mathrm{V\,m^{-1}}, \quad [D] = [P] = \mathrm{C\,m^{-2}}, \quad [p] = \mathrm{C\,m}$$
$$[B] = \mathrm{T} = \mathrm{Wb\,m^{-2}}, \quad [H] = [M] = \mathrm{A\,m^{-1}}, \quad [m] = \mathrm{A\,m^2}$$
$$[\xi] = \mathrm{V}, \quad [\Phi] = \mathrm{Wb} = \mathrm{V\,s}, \quad [L] = \mathrm{H}$$

$$[\rho] = \mathrm{C\,m^{-3}}, \quad [\sigma] = \mathrm{C\,m^{-3}}, \quad [J] = \mathrm{A\,m^{-2}}, \quad [K] = \mathrm{A\,m^{-1}}$$
$$[\varepsilon] = \mathrm{C\,V^{-1}\,m^{-1}}, \quad [\mu] = \mathrm{Wb\,A^{-1}\,m^{-1}}, \quad [\chi] = 1$$