Tails of natural hazards

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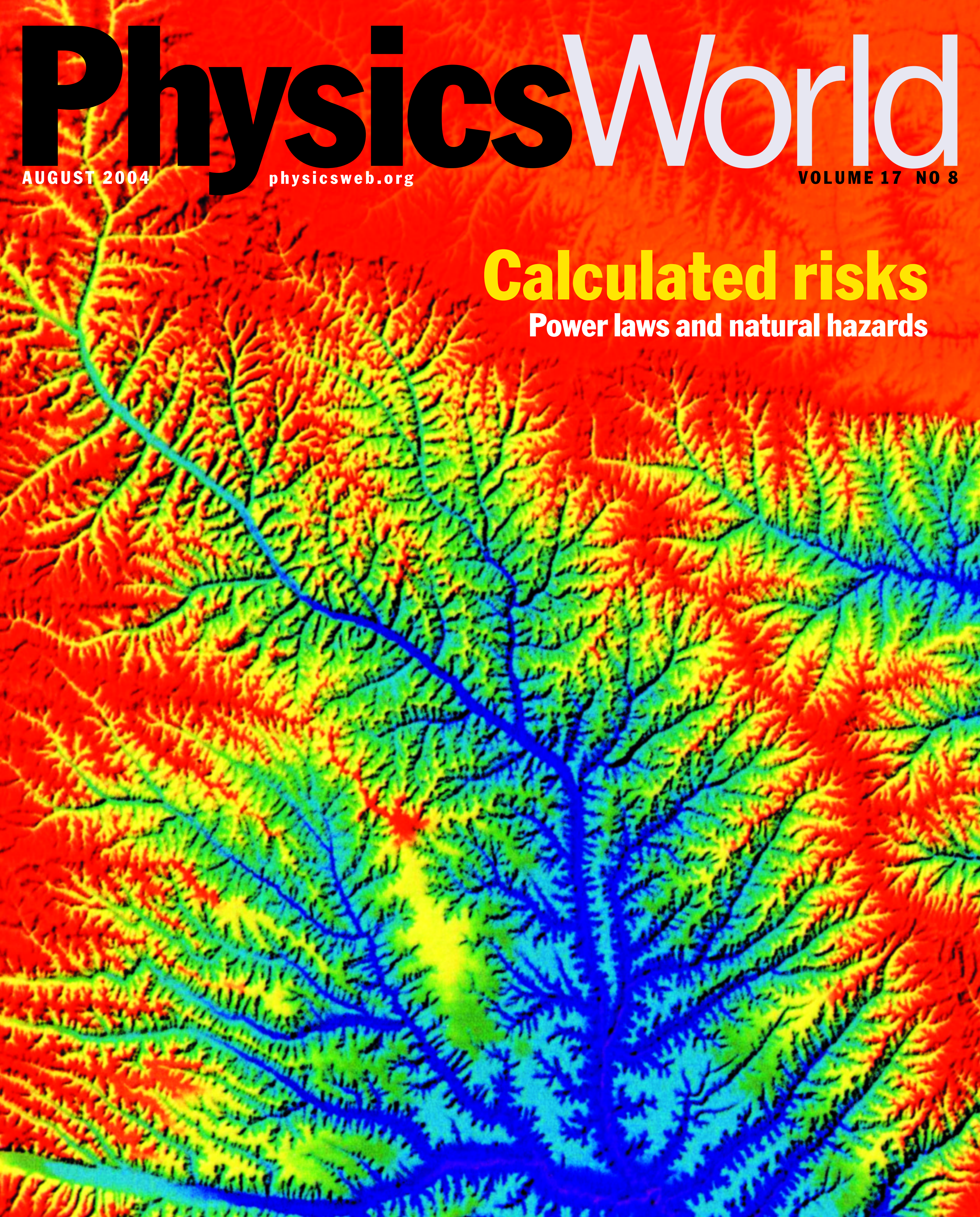

Calculated risks

Power laws and natural hazards

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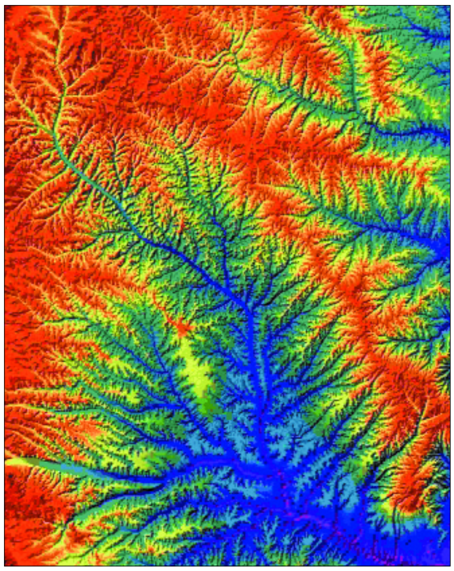
Power laws are helping researchers assess the risks posed by extreme natural hazards, while statistical physics may reveal an underlying universality in nature

Tails of natural hazards

Bruce D Malamud

SEVERE storms, floods and earthquakes affect the lives of millions of people each year, and can have a devastating effect on the environment. Understanding when and where these and other natural hazards – such as landslides, extreme waves and forest fires – will strike next is therefore an outstanding challenge in the earth sciences. One way to tackle this problem is to figure out what actually causes extreme natural hazards. However, another fruitful approach is to examine the laws of chance.

Over the last couple of decades researchers have been applying techniques from statistical physics and complexity theory to natural hazards. One striking characteristic of these studies is the prevalence of power laws, in which the probability $p(x)$ of an event of magnitude x or greater occurring is given by the equation $p(x) \sim x^{-c}$, where c is a constant that can be a fraction. In such distributions, large values of x are much rarer than smaller ones. It has been known for some time, for example, that the number of earthquakes of size E or greater – where E is the energy released – in different geographic regions or even across the world as a whole is proportional to $E^{-(0.8 \text{ to } 1.0)}$. Power-law distributions are increasingly being used by reinsurance companies and governments to assess the risks posed by natural hazards, and it is clear that power laws are going to play an important role in the environmental and social-policy decisions of the 21st century.



The power of nature – the false-colour topography of a drainage network in the Loess Plateau, Shanxi province, China. The same approximate pattern is repeated at multiple scales and there is no inherent scale in the picture to give an idea of its size. The image actually represents an area of approximately 220 km by 300 km. This is a ninth-order drainage network, which means that eight different levels of side branches feed into more principle branches. Scale invariance and associated power-law statistics such as this appear to underlie many natural phenomena.

What's in a tail?

Ideally we would like to know the largest event that will occur for a specific location in a given period of time. This is called a probabilistic hazard assessment. If records are available for a long period, these can be used to estimate the average likelihood of an event. For instance, if we have 1000 years of flow data – volume of water per unit time – for a particular location along a river, then we have a relatively good idea of the peak flow that will occur, on average, every 100 years. However, if such long records are not available, we have to estimate the size of the largest event that might occur over a given period of time, based on the occurrence of smaller events over a shorter period of time. In other words, we rely on assessments based on frequency–size probability distributions.

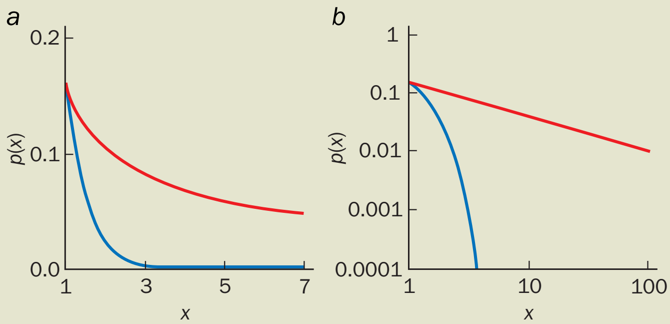
In these distributions the probability that a particular value will occur is plotted against the value itself, which allows one to assess the probability of a particular event occurring in a given period, or its "recurrence interval". The question for people who study natural hazards, then, is what kind of probability distribution do these events follow? Unfortunately there is no simple answer to this question, not least because extreme events are very rare. Remarkably, however, an increasing number of natural hazards are being shown to follow "heavy tailed" distributions such as power-law distributions.

The tail of a distribution refers to its most extreme values. To illustrate this, consider a Gaussian, or normal, distribution that is symmetrical about a central peak. The values of the distribution fall away on both sides of the peak, and the "tails" are those values to the far left and far right. In the Gaussian distribution these tails are exponentials, while a distribution with a heavy tail decays much more slowly (figure 1).

A heavy tail is an attribute of a phenomenon called scale invariance, which appears to underlie many natural systems. For example, when we look at a photograph of a mountain range, a drainage network or ripples of sand in a river, we need some familiar object in the photograph to indicate the scale because the shapes appear approximately similar at each scale. In the image shown on this page, for instance, we have no way of telling how wide the structure is: it could be as narrow as 1 mm, like the features in a dendritic tree in mineralogy, or as large as 1000 km. In fact, this is a digital elevation image of a drainage network in China, and shows an area of 220 km by 300 km.

Scale invariance and power-law statistics are also true of fractals – the term coined by Benoît Mandelbrot to describe a geometric shape that can be subdivided in parts, each of which is a reduced-size copy of the whole. The characteristic

1 Normal versus heavy tails



The shape of the tails in a statistical distribution – in which the probability, *p*, of a particular value, *x*, occurring is plotted against that value – gives an indication of how often the most extreme values occur. (*a*) The tails of two different distributions are shown: the first is an exponential tail from a normal or Gaussian distribution with a mean value 0 and variance 1 (blue), while the second is a power-law distribution with exponent −0.6 (red). The tail of the power law decays much more slowly than that of the exponential, which means that extreme values occur much more often. (*b*) When logarithmic axes are used, the power law becomes a straight line.

measure of any fractal is its dimension, D, which is the exponent in the power-law relationship that describes the fractal (figure 2).

The shape of natural hazards

Many natural systems have a frequency–size distribution that follows a power law, including the frequency–area of soil particles and the frequency–volume of clouds. It is therefore reasonable to consider that the same might be true for the size distribution of natural hazards over time. Here, it is the extreme tails that are important because they give an idea as to how often, on average, the largest events might occur. The decisions that might be made based on the shape of these tails are numerous: how high a bridge should be built, how deep electricity pylons need to be anchored, and so on. With very little historical data to play with, however, we have to make our best guess as to what the tails actually look like. If we pick a distribution where the tail decays quickly, then large events are very rare. In a distribution with heavy tails, on the other hand, large events are much more likely.

One of the first people to apply power-law distributions was the Italian economist Vilfredo Pareto. In the late 19th century Pareto worked out that the distribution of income and wealth has the form $N \sim x^{-m}$, where N is the number of earners who receive incomes higher than x, and m is a constant. This is the origin of the "80:20 rule": a small number of people have a disproportionate amount of wealth or influence. Today, Pareto distributions are used to model systems as diverse as Internet traffic and financial risk.

Another early proponent of power laws was George Kingsley Zipf of Harvard University, who showed in the 1930s that many social and linguistic systems follow a frequency–rank distribution that is an inverse power law: $f_n \sim n^{-b}$, where f_n is the frequency of items with rank n. Both Pareto and Zipf distributions are power laws. However, the Pareto function describes frequencies "greater than or equal to", while Zipf functions describe frequencies "equal to". For example, when we count the number of times that different words appear in *Sidelights on Relativity* – an 8454 word treatise that Einstein wrote in 1921 – we find that "the" is the most common word (rank $n = 1$) with $f_1 = 689$ appearances. This is followed by "of" (rank $n = 2$), which occurs $f_2 = 512$ times, and so on through to "relativity" (rank $n = 40$), which appears just 28 times. It ends with those 798 different words that appear just once, such as "abandon", "mensuration" and "yes". The Zipf frequency–rank distribution follows an inverse power law with exponent $b = 0.9$, and even though there are 1536 different words in Einstein's treatise, just 50 of these account for 50% of all of the words written.

Similarly, seismologists have known for decades that large earthquakes are rare, while small ones are very frequent. Today, it is generally accepted by the scientific community that the frequency–size distribution of small, medium and large earthquakes follows a power law. Indeed, this information is regularly combined with other details about the local geology and building materials to estimate the relative impact of earthquakes in a particular region. However, the size distribution of the very rare, extremely big earthquakes is much less well understood. For a start, we need to know whether the power-law behaviour itself ceases to hold at the very largest earthquake sizes, or whether the exponent of the distribution might change slightly in a given region as one moves from one class of earthquake sizes to another.

Didier Sornette of the University of California at Los Angeles and Vladlen Pisarenko of the Russian Academy of Sciences have recently applied what is called extreme-value theory to characterize the size distribution of earthquakes. First proposed by Ronald Aylmer Fisher at Cambridge University in the 1920s, this branch of statistics provides a limiting distribution for the largest events in the distribution (i.e. those in the tails). Extreme-value theory therefore attempts to do for the extremes of a distribution what the central-limit theorem – which states that given enough values, the values tend towards a normal distribution – does for all values in a distribution.

Another popular approach, in which researchers attempt to model the complex system that causes earthquakes, is to use a cellular automaton "slider–block" model. Here, blocks with equal mass are connected to each other by springs and each block is connected to a single driving plate (figure 3). This system evolves through the interaction of blocks via the springs, friction with the "ground", and the input of energy through the movement of the driver plate. A block remains stationary as long as the net force on it is less than the static resisting force. However, if the net force on a block exceeds this resisting force it begins to slip. This can trigger the slip of adjacent blocks, and result in "earthquakes" that have power-law frequency–size statistics.

John Rundle of the University of California at Davis has recently shown that models based on these principles can successfully reproduce many of the statistics of real earthquakes. Furthermore, his group has shown that the statistical distributions for the occurrence of earthquakes can be computed from such simulations, thereby eliminating the need for assumptions about the form of the appropriate statistical distribution.

Fiery Earth

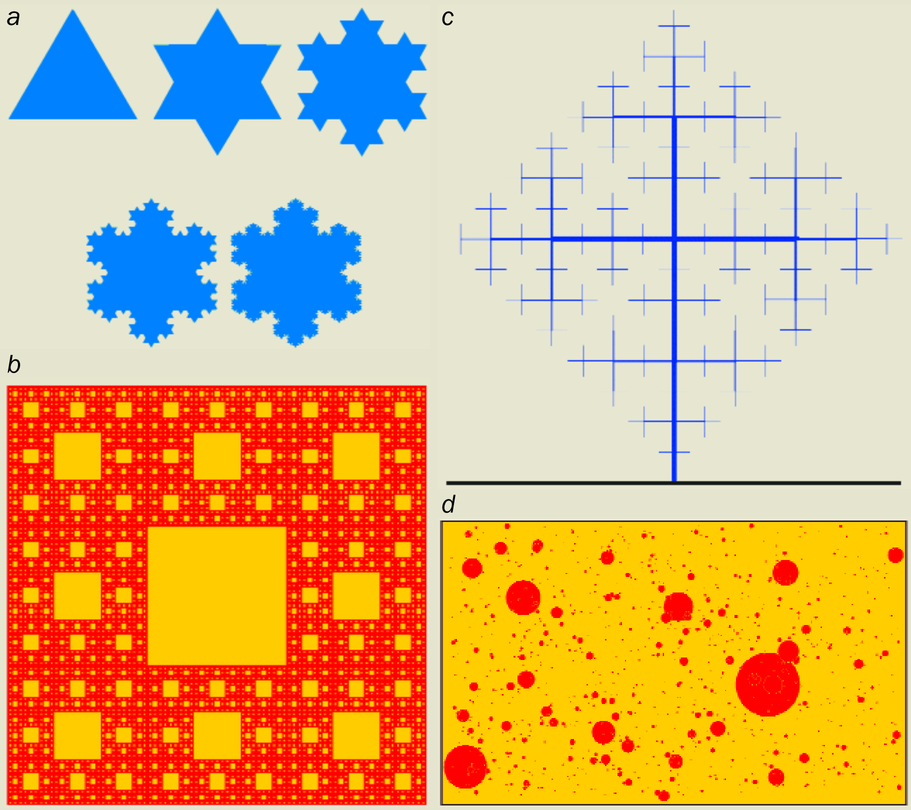
In the last 12 months countries such as Brazil, France and the US have witnessed devastating wildfires, which have caused fatalities and a loss of infrastructure. The ways in which wildfires begin and spread are very complex, and the proximity of combustible materials varies widely. Furthermore, the behaviour of an individual wildfire depends strongly on meteorological conditions, topography, vegetation and, of course, efforts to extinguish it. Despite all this complexity, however, the frequency–area distribution of wildfires is remarkably similar in a wide variety of environments.

In 1998 the present author along with Donald Turcotte and Gleb Morein, then at Cornell University, studied wildfire records from Alaska, the western US and in the Australian Capital Territory. We found that the number of wildfires, n_F, scales as an inverse power law with the burned area, A_F: $n_F \sim A_F^{-(1.3 \text{ to } 1.5)}$. This simple relationship was found to hold for the three different regions over 2–5 orders of magnitude in A_F.

Earlier this year James Millington, George Perry and the author at King's College London extended this study by examining records of 88 000 wildfires in the US between 1970 and 2000. With so many data we were able to study the statistics of wildfires in 17 different eco-regions – large land units, each with distinct climates, vegetation and topography – and also as a function of the cause of the fire (i.e. whether it was started by humans or by lightning).

Each of the eco-regions shows excellent power-law behaviour over more than five orders of magnitude in A_F, with the number of fires $n_F \sim A_F^{-(1.3 \text{ to } 1.8)}$ (figure 4). The data also reveal that the power-law exponent decreases from east to west across the US, and we have found fundamental differences in anthropogenic-versus natural-wildfire distributions. This is the first time that the changes of wildfire statistics as a function of eco-region and cause have been examined, and it could allow researchers to assess the risks posed by wildfires in the same way that earthquake hazards are assessed.

2 Fractals and self-similarity



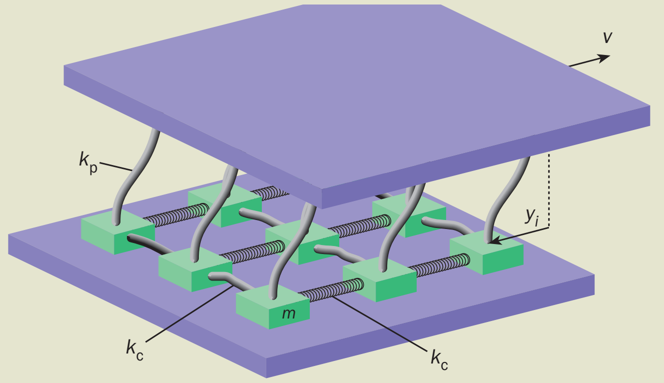
Fractals can be constructed by repeating the same pattern at different scales (the number of repetitions is called the order of the fractal). Three examples of these deterministic constructions are shown. (*a*) The zeroth order of a "Koch snowflake" is a triangle, and in each successive order triangles with sides 1/3 the size are added to the middle of each side, increasing the perimeter of the snowflake by a factor of 4/3. The Koch snowflake has fractal dimension $D = \log(4)/\log(3) \approx 1.262$. (*b*) In the "Sierpinski carpet", pieces are removed rather than added. The zeroth order is a solid square divided into nine smaller squares, the central of which has been removed (resulting in the large yellow square). This construction has a fractal dimension $D \approx 1.893$ and is shown here at order five. (*c*) In this simplistic "drainage model" we begin with a cross and then add other crosses that are 1/3 the size at each successive order. This example is shown here at order 3 and has a fractal dimension $D = 2.0$. (*d*) The fractals (*a*)–(*c*) contrast with our final example, which is a stochastic construction: 1000 circles with a probability–size distribution that is an inverse power law are randomly placed inside a rectangle. The resulting fractal dimension is $D = 1.8$ and the same *approximate* pattern is repeated at multiple scales.

Another active area of power-law research involves volcanic eruptions. Because volcanoes erupt in different ways, there are various approaches that can be used to quantify the size of the eruption. One is to use estimates for the total volume of ash produced during the eruption. In 1997 Turcotte used volcanic-eruption data from the period 1785 to 1985 to build a frequency–volume distribution. He showed that for volumes of ash between 0.001 and 100 km^3, the number of volcanic eruptions, N, greater than a given volume, V, over this 200 year period follows an inverse power-law relationship: $N \sim V^{-2/3}$. He concluded that volcanic eruptions are scale invariant over a significant range of sizes.

David Pyle of Cambridge University has found similar frequency–size relationships for volcanic eruptions based on thousands of years of records, and used them to project the likely scale of future extreme volcanic events. For instance, his probability assessments suggest that a volcanic event at least as big as the Minoan eruption of Santorini in 1645 BC should occur several times every 1000 years. Pyle's research has also been used by others to estimate the potential total gas emissions from volcanoes over time and the potential probability of "super volcanoes" existing on other Earth-like planets.

The last natural hazard I would like to mention is landslides, which constitute a serious threat in many countries and are commonly associated with a trigger such as an earthquake. A triggered landslide event can consist of anything from a single landslide up to tens of thousands of individual landslides occurring over periods of minutes to weeks after the original trigger. Over the last decade about a dozen groups of researchers have examined both freshly triggered and historical landslide "inventories" – databases based on information about individual landslides in a given locality obtained from photos, maps and fieldwork – in many different parts of the world. They have generally found that the frequency–area distribution of medium and large landslides in each inventory decays as an inverse power of the landslide area, with exponents –2.2 to –2.8 over 2–4 orders of magnitude of landslide area. Other examples of "mass wasting" that have been shown to exhibit power-law statistics include snow avalanches, rockfalls and possibly sediment gravity flows in the oceans (see figure 4*b*).

3 Slip and slide



Researchers are using cellular automata "slider–block" models to model earthquakes. Shown is a 2D model in which an array of blocks – each with mass m – is pulled across a surface by a driver plate at a constant velocity v. The horizontal displacement for each block in the grid is given by y_i. Each block is coupled to the adjacent blocks with either leaf or coil springs (spring constant k_c) and to the driver plate with leaf springs (spring constant k_p). If the net force on a block exceeds the static resisting force (friction with the ground) it begins to slip, which can trigger "earthquakes" that have a power-law frequency–size distribution similar to that of real earthquakes.

Fausto Guzzetti and Paola Reichenbach at the Istituto di Ricerca per la Protezione Idrogeologica in Perugia, Italy, along with Turcotte and the present author, have recently examined three well documented landslide events from Italy, Guatemala and the US. Each event had a different triggering mechanism (rapid snowmelt, heavy rainfall and an earthquake), and we found that the landslide areas for all three are well approximated by the same probability distribution: an exponential "rollover" for small landslide areas and an inverse power law with exponent –2.4 for medium and large areas. This general distribution is observed despite large differences in landslide types, patterns and triggering mechanisms, and this is the first time that the same probability distribution has been proposed for freshly triggered landslide events.

Meanwhile, Colin Stark of Columbia University is working with stochastic differential equations to create a model that reproduces the landslide statistics we see in nature. Approaches like this could have a large impact in current and future research into natural hazards.

Universal laws

There are also other natural hazards that appear to follow heavy-tailed distributions, including the size of hailstones, the heights of waves and the diameters of asteroids that impact the Earth. One question that many researchers are trying to answer is whether this scale-invariant behaviour – which holds over many orders of magnitude for many different natural hazards – represents some sort of universality in nature, and, if so, what might explain it? A number of theoretical and philosophical models have been proposed that produce power-law behaviour in various systems in nature.

One of the best known of these ideas is self-organized criticality, which was proposed in 1987 by Per Bak, Chao Tang and Kurt Wiesenfeld while at the Brookhaven National Laboratory. With self-organized criticality, the interaction of components in a system causes it to settle into a state where a "critical" – i.e. scale invariant – spatial or temporal structure appears. The "input" to the system is small and constant (for example the steady addition of trees or grains of sand), whereas the "output" is a series of events (for example forest fires or avalanches) that follow power-law statistics. Many researchers believe that self-organized criticality or similar ideas such as self-organized complexity might be unifying themes in many natural and human-made processes.

In another model, Turcotte has proposed that small clusters of, say, trees on a grid coalesce to form larger clusters, and clusters are lost in fires that occur randomly. The result is a self-similar inverse-cascade that satisfies an inverse power-law distribution of cluster sizes. Turcotte has related this inverse-cascade model to the results of several cellular-automata models and also to real data for different natural hazards, as a way of exploring why there are so few large events in nature and so many more smaller ones.

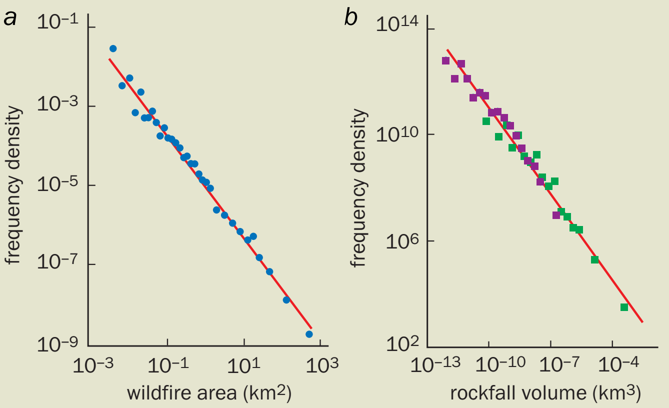
But what does all this imply for risk? One can certainly explore theoretical constructs like self-organized criticality and cascading clusters in an effort to explain whether the power laws in natural hazards and other systems reveal an underlying universality in nature. However, the bottom line is that the power-law behaviour we observe means that large natural hazards are not as infrequent as we might have originally thought. Although some government agencies and reinsurance companies are calculating risk using heavy-tailed distributions, others still use thinner-tailed distributions that are much less conservative in their estimation of risk. One example of this lies in flood assessment in the US, where a distribution called the log-Pearson type III is routinely used to calculate flood recurrence intervals. Within the US, this distribution, which has a federal mandate, applies well to many locations, but there are certainly other locations where heavy-tailed distributions are much more appropriate.

Critical points

We have seen many examples of natural systems with frequency–size distributions that show heavy-tailed or power-law behaviour. However, I should stress that there are many others that do not. For instance, the distribution of human heights and weights for a given ethnic group, sex and age tends to follow a Gaussian distribution. In other words, not everything in nature is scale invariant! Furthermore, although a large variety of natural hazards appear to follow scale-invariant distributions over many orders of magnitude, there are upper and lower limits to this behaviour. It is therefore important to find these limits, and also to establish over how many orders of magnitude the power-law behaviour holds. What happens at the very largest, most extreme sizes is currently very unclear, and a source of vigorous debate. Most extreme-event experts would, however, agree that they do not have sufficient data to make accurate assessments for natural hazards. On the other hand, society needs some sort of answer, so they do the best they can.

Lastly, it is important to be aware that by creating a frequency–size distribution and using it to assess risks, one is assuming that the events themselves are uncorrelated in time (i.e. that they are Poissonian). However, in reality it is rare to find a time series of events that is not connected temporally in some way. If large events do in fact cluster together significantly, then it is important to include this in a risk analysis. The datasets I have considered in this article are approximately Poissonian for the largest events, so frequency–size distributions can be presented with at least some validity. Meanwhile, researchers are continuing to develop techniques

4 Examples of power-law distributions



Power laws have been found to describe the frequency–size distributions of many natural hazards. (*a*) Wildfires in the Mediterranean eco-region of the US. Frequency densities, f, (i.e. the number of fires per unit area "bin" per year per eco-region area) are plotted as a function of the area of the wildfire, A_F. Fitting the data with a power law gives excellent agreement with $f = 1.0 \times 10^{-5} A_F^{-1.3}$ (i.e. a straight line on logarithmic axes) for wildfire areas between about 0.01 to 1000 km^2. (*b*) Rockfalls also follow such power-law behaviour. Here the number of rockfalls per unit volume bin is plotted as a function of their volume, V_R, for two different datasets: an earthquake-triggered rockslide event in Umbria, Italy, in 1997 (purple) and historical data from Yosemite between 1980 and 2002 (green). Despite taking place under very different conditions, the datasets follow a power law of the form $2.34 V_R^{-1.07}$ remarkably well for rock volumes between 0.001 to 1000 000 m^3.

that combine the clustering (i.e. the persistence, or memory) of the events with the statistical distribution of their sizes, be it heavy tailed or not.

There are certainly no easy answers to the question of which distribution to use to estimate the risks posed by nature's hazards. But power laws do allow us to make conservative and realistic estimates of these risks. Furthermore, since power laws are the only statistical distributions that are completely scale invariant, they offer a unique way to explore the possibility of an underlying universality in nature.

Further reading

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