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The influence of sea surface temperature on tropical-cyclone intensity

Alfredo Hernández[1], Isabel Serra[1, 2] and Álvaro Corral[1, 3]

[1]Centre de Recerca Matemàtica, [2]Barcelona Supercomputing Center, and [3]Complexity Science Hub Vienna

Introduction

For the North Atlantic (N. Atl.) basin, it has been shown [1] that the probability distribution of the so-called power-dissipation index (PDI, a rough estimation of released energy) is indeed affected by the annual and basin-wide averaged sea surface temperature (SST), displacing towards more extreme values on warm years (high-SST). As the PDI integrates (cubic) wind speed over tropical-cyclone (TC) lifetime, it is an open question where the PDI increase comes from (higher speed, longer lifetime, or both).

Data

To characterise a TC one needs to define a physically relevant measure of released energy. The released energy of each TC is summarised as

$$PDI = \sum_t v_t^3 \Delta t. \tag{1}$$

The raw hurricane best track data (HURDAT2) is provided by the National Hurricane Center. We intentionally limit this study to the satellite era (1966–2016), as it is the most reliable.

Then, the hurricane observational data is classified into occurrences in low-SST and high-SST years depending on whether they are lower or greater than

$$\langle \mathrm{SST} \rangle = \sum_{y \in Y} \frac{\mathrm{SST}(y)}{Y}, \tag{2}$$

where $\mathrm{SST}(y)$ is the mean SST of the year y, and Y is the total number of years studied. The SST data (HadISST1) is provided by the Met Office Hadley Centre.

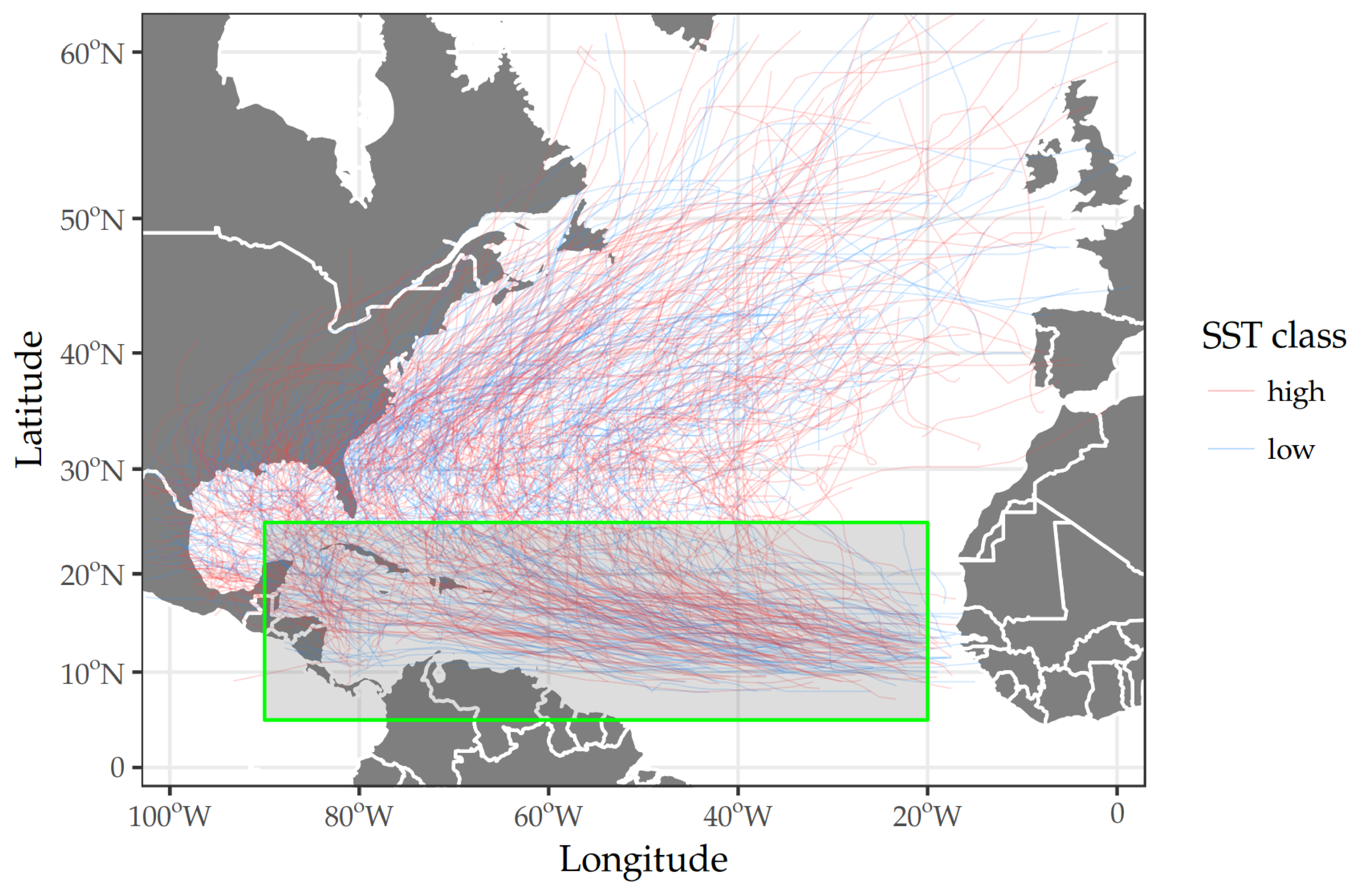

Figure 1: Tropical-cyclones best tracks for the North Atlantic basin

Hypothesis

The hypothesis is that the SST does not directly affect the maximum wind speed of a TC: storms of equal duration should, in theory, have the same wind speed and PDI, and have the same joint distribution:

$$f(Y \mid X = x)_{\mathrm{low}} = f(Y \mid X = x)_{\mathrm{high}}. \tag{3}$$

The physical reasoning behind this is that once the cyclone is activated, the wind speed should not depend on its underlying SST.
Instead of working with the exact joint distributions f, we study the expected value of the distributions:

$$E(Y \mid X = x)_{\mathrm{low}} = E(Y \mid X = x)_{\mathrm{high}}, \tag{4}$$

where $E(Y \mid X = x)$ is estimated by performing a ordinary least squares (OLS) regression analysis on the data sets.

Data shows

Our empirical results show a remarkable correlation in the joint distribution of lifetime and wind speed, as seen in Figure 2.

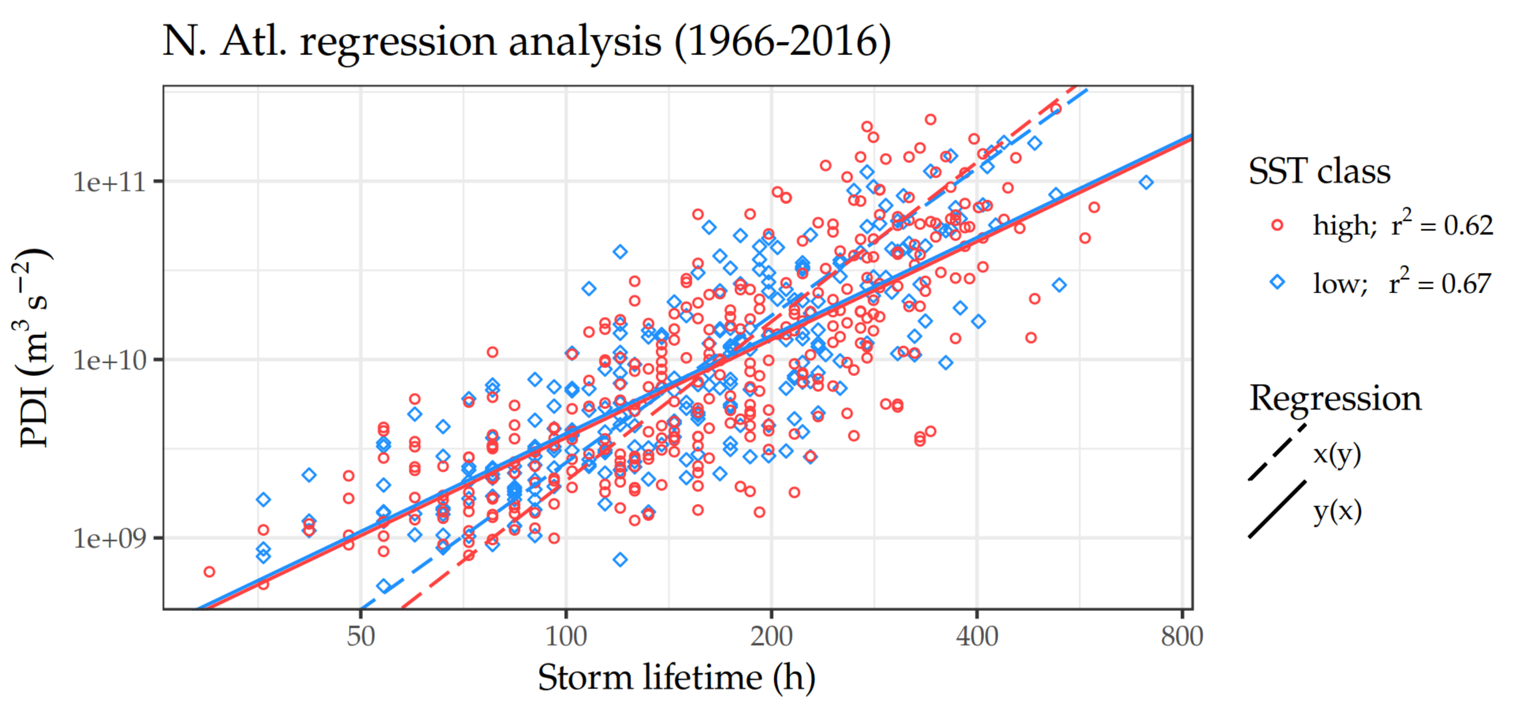

Figure 2: PDI vs lifetime regression analysis for tropical-cyclones (excluding tropical depressions) on the North Atlantic basin

It is important to notice that the relationships between the TC variables are of non-linear nature. This naturally means that our regressions need to follow a so-called log–log model:

$$\log \Psi = \alpha + \beta \log \Phi + \epsilon, \tag{5}$$

where $\log \Psi \equiv Y$ and $\log \Phi \equiv X$.

Methodology

In essence, our methodology will consist in comparing the low-SST and high-SST regression models using non-parametric permutation tests.

For the permutation tests, we use bootstrap in order to get a better estimation of the value of the different studied statistical coefficients (such as $\hat{\alpha}$, $\hat{\beta}$, r^2) and their standard errors than the one obtained using the value obtained using the OLS method. Withal, we compare the results obtained using both methods.

The following null hypothesis is proposed to test the data:

$$H_0 : \hat{\alpha}_h = \hat{\alpha}_l \wedge \hat{\beta}_h = \hat{\beta}_l \tag{6}$$

We use the following statistics:

$$T^{(1)} = |\hat{\alpha}_h - \hat{\alpha}_l|, \quad T^{(2)} = |\hat{\beta}_h - \hat{\beta}_l|, \quad T^{(3)} = |r_h^2 - r_l^2|. \tag{7}$$

From [3], better statistics are proposed that take into account the standard error of the coefficient estimates:

$$T^{(4)} = \frac{|\hat{\alpha}_h - \hat{\alpha}_l|}{\widehat{\mathrm{se}}(\hat{\alpha}_h - \hat{\alpha}_l)}, \quad T^{(5)} = \frac{|\hat{\beta}_h - \hat{\beta}_l|}{\widehat{\mathrm{se}}(\hat{\beta}_h - \hat{\beta}_l)}, \quad T^{(6)} = T^{(4)} + T^{(5)} \tag{8}$$

where $\widehat{\mathrm{se}}(\hat{\alpha}_h - \hat{\alpha}_l) = \sqrt{\widehat{\mathrm{se}}(\hat{\alpha}_h)^2 + \widehat{\mathrm{se}}(\hat{\alpha}_l)^2}$ and $\widehat{\mathrm{se}}(\hat{\beta}_h - \hat{\beta}_l) = \sqrt{\widehat{\mathrm{se}}(\hat{\beta}_h)^2 + \widehat{\mathrm{se}}(\hat{\beta}_l)^2}$.

Results

The value of the studied statistics $T^{(i)}$ obtained from the N. Atl. data set can be seen in Table 1.

X	Y	$T^{(1)}$	$T^{(2)}$	$T^{(3)}$	$T^{(4)}$	$T^{(5)}$	$T^{(6)}$
PDI	lifetime	0.025	0.001	0.051	0.101	0.010	0.111
lifetime	PDI	0.299	0.028	0.051	1.388	1.295	2.683

Table 1: Value of the studied statistics for North Atlantic basin data set

The results of the permutation tests performed on the N. Atl. data can be seen in Tables 2 and 3.

X	Y	$p(T^{(1)})$	$p(T^{(2)})$	$p(T^{(3)})$	$p(T^{(4)})$	$p(T^{(5)})$	$p(T^{(6)})$
PDI	lifetime	0.176	0.154	0.740	0.174	0.148	0.162
lifetime	PDI	0.990	0.930	0.766	0.990	0.926	0.972

Table 2: p-values of the standard (OLS) permutation test for the North Atlantic basin data set

X	Y	$p(T^{(1)})$	$p(T^{(2)})$	$p(T^{(3)})$	$p(T^{(4)})$	$p(T^{(5)})$	$p(T^{(6)})$
PDI	lifetime	0.146	0.121	0.711	0.137	0.117	0.128
lifetime	PDI	0.870	0.806	0.705	0.864	0.795	0.830

Table 3: p-values of the bootstrap-powered permutation test test for the North Atlantic basin data set

The results show that none of the performed permutation tests is able to reject the null hypothesis H_0.

Conclusion

Some remarkable results can be extracted from the methodology:

- Taking into account the standard error does not change the p-value significantly.
- Although the bootstrap-powered permutation test does not give a significantly different result than using the OLS-powered permutation test, it is more sensible to hypothesis testing.

Our conclusions are compatible with the view of tropical cyclones as an activation process, in which, once the event has started, its intensity is kept in critical balance between attenuation and intensification (and so, higher SST does not trigger more intensification).

References

[1] Á. Corral, A. Ossó, and J. E. Llebot. Scaling of tropical-cyclone dissipation. *Nature Physics*, 6(9):693–696, sep 2010. ISSN 1745-2473. doi: 10.1038/nphys1725. URL `http://www.nature.com/articles/nphys1725`.

[2] P. Good. *Permutation, Parametric and Bootstrap Tests of Hypotheses*, volume 53 of *Springer Series in Statistics*. Springer-Verlag, New York, 2005. ISBN 0-387-20279-X. doi: 10.1007/b138696. URL `http://link.springer.com/10.1007/b138696`.

[3] D. Polko-Zając. Application of the permutation test for comparing regression models. In *10th Professor Aleksander Zelias International Conference on Modelling and Forecasting of Socio-Economic Phenomena*, 2016.

[4] P. J. Webster, G. J. Holland, J. A. Curry, and H.-R. Chang. Changes in tropical cyclone number, duration, and intensity in a warming environment. *Science (New York, N.Y.)*, 309(5742):1844–6, 2005. ISSN 1095-9203. doi: 10.1126/science.1116448. URL `http://www.ncbi.nlm.nih.gov/pubmed/16166514`.

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