# Conformal prediction.
# **Exercises**

1. Write a function to implement Minmax conformal prediction with $\hat{\mu} =$ ridge regression (add $\lambda$ as an argument of the function)

**Algorithm 1** Minmax conformal prediction

**Require:** $(x_i, y_i)$, $i = 1, \ldots, n$, $\alpha$, regression algorithm $\hat{\mu}$, new test point $x_{n+1}$

**for** $i = 1, \ldots, n$ **do**
  $\hat{\mu}^{-i}(x) = \hat{\mu}(x; (x_l, y_l), l \in \{1, \ldots, n\} \setminus \{i\})$
  $T_i^{-i} = |y_i - \hat{\mu}^{-i}(x_i)|$
**end for**
$W_\alpha$ is the $k$th ordered value among $T_1^{-1}, \ldots, T_n^{-n}$ and $k = \lceil (1-\alpha)(n+1) \rceil$.
Compute $\underline{\mu}(x_{n+1}) = \min\{\hat{\mu}^{-1}(x_{n+1}), \ldots, \hat{\mu}^{-n}(x_{n+1})\}$
Compute $\bar{\mu}(x_{n+1}) = \max\{\hat{\mu}^{-1}(x_{n+1}), \ldots, \hat{\mu}^{-n}(x_{n+1})\}$
$C_\alpha(x_{n+1}) = [\underline{\mu}(x_{n+1}) - W_\alpha, \bar{\mu}(x_{n+1}) + W_\alpha]$

2. Simulate $X \sim \text{Uniform}([0,1]^p)$, for $p = 100$, and $Y$ from:

$$Y = f(\beta^T X) + \epsilon\sqrt{1 + (\beta^t X)^2}$$

where $f(x) = 2\sin(\pi x) + \pi x$, $\beta^t = (1, 1, 1, 1, 1, 0, \ldots, 0)$ and $\varepsilon$ is independent standard Gaussian noise.

Simulate training data in setting i) $n = 50$ and ii) $n = 5000$ and for each setting compute average width and coverage of the conformal prediction intervals using an algorithm of your choice. Compare to the expected width of the oracle prediction interval of 8.91. Comment the results.