The LGP method proposed for solving the STMP problem is described. The main objective is to achieve the plants' grades since the iron ore price is proportional to the required grade targets for the products of the plants. Therefore, the LGP method is appropriate for solving this problem, as it is possible to choose the priority order of the objectives. Algorithm 1 shows its pseudo-code.

Algorithm 1: LGP

Input : Vector of objective functions $Z = \left(z_1, z_2, \cdots, z_{|Z|}\right)$,
set TOL of tolerances ϵ to be used in Eqs. 14 and 15,
TL for solving each MILP model

Output: Set Sol of MILP solutions from TOL

1 $i \leftarrow 1$ {Index for each tolerance value $\epsilon_i \in TOL$}
2 $Sol \leftarrow \emptyset$ {Set of solutions returned by the LGP method}
3 **foreach** $\epsilon_i \in TOL$ **do**
4     $l \leftarrow 1$ {Index for the objective functions}
5     $C \leftarrow \emptyset$ {Set containing the values returned by the MILP solver}
6   **while** $l \leq |Z|$ **do**
7   *model* $\leftarrow$ MILP model generated having z_l as objective function ($l \in \{1, 2, \cdots, |Z|\}$), $\epsilon = \epsilon_i$ as the tolerance used in Eqs. (14)-(15), and constraints $z_k \leq c_k \, \forall k = 1, \cdots, l-1$ added to the model
8         $c_l \leftarrow$ MILP-Solver(*model*, *TL*) {Value of the l-th objective function returned by the MILP solver after TL seconds at most}
9         $C \leftarrow C \cup \{c_l\}$
10         $l \leftarrow l + 1$
11   **end**
12     $s \leftarrow$ solution returned by the MILP solver concerning the tolerance ϵ_i
13     $Sol \leftarrow Sol \cup \{s\}$
14     $i \leftarrow i + 1$
15 **end**
16 **Return** Sol {Set of solutions for all tolerances $\epsilon_i \in TOL$}

According to Algorithm 1, initially, three entries are provided: vector Z containing the objective functions (four in this study), set TOL with the admissible values for the tolerances ϵ adopted in Equations (14) and (15) (in this study, $TOL = \{0, 0.01, 0.02, 0.03, 0.04, 0.05\}$), and run time TL for solving each MILP model. The output is the set Sol of solutions generated by the MILP solver.

Lines 1 and 2 initialise the counter of the elements of the set TOL of tolerances ϵ and the set Sol of solutions generated by the MILP solver, respectively. Next, a loop is executed in lines 3-15 to generate a set Sol of solutions returned by the LGP method for all tolerances of the set TOL.

Lines 4 and 5 initialise the index for the objective functions and the set C containing the values returned by the MILP solver, respectively.

Then, a loop in lines 6-11 is realised to obtain a set C containing the values of all objective functions $z_l \in Z$ returned by the solver. In line 7, a model containing the objective function z_l is generated. In line 8, this model is executed by the MILP solver, returning its optimum value c_l to z_l or the best value found in TL seconds at most. When $l \geq 2$, the solver uses the values $c_k \in C \; \forall k < l$ as constraints for each objective function z_k. In line 9, the value c_l is included in the set C. This loop ends when all models are solved.

In line 12, the solution returned by the MILP solver concerning the tolerance ϵ_i is assigned to the variable s. In line 13, this solution is inserted into the set Sol of solutions. Next, in line 14, the index for the tolerance $\epsilon_i \in TOL$ is incremented. Finally, the method returns the set Sol containing all solutions returned from the LGP method for all tolerances.

To show how the LGP algorithm works, suppose that a mine has three mine fronts with data input informed in table above. In addition, considering that the plants' iron grade target is equal to 60% iron, the target for the ore proportion in particle size range 1 (according to Table 4 of the manuscript) is equal to 40%, the stripping ratio WT is equal to 2, and the target for the mass required by the plant is equal to 500 t. Additionally, consider that there is only a single tolerance ϵ equal to 5%, there is only one type of ore, and there are three objectives z_1, z_2, and z_3.

Example of input data from a mine

Front #	Ore mass (tonnes)	Iron grade (%)	Size range 1 (%)	Waste (tonnes)
1	500	55	43	1000
2	600	60	45	800
3	700	63	40	700

The highest priority goal (z_1) is to minimise deviations from the grade target of the plant. With this tolerance, the lower bound for the grade target is 57%, and the upper bound is 63%. Table above shows that it is enough to feed the plant with 500 t of ore from fronts 2 or 3 to obtain positive and negative deviations equal to zero for the plant's grade target within this tolerance. In this case, $c_1 = 0$ is the value of the objective function z_1. Now, the algorithm reruns the MILP solver, this time to find the solution that minimises the second objective z_2: that is, it seeks the solution with the smallest deviation from the target for the ore proportion in the particle size range 1 with a tolerance of 5% from the plant's grade target. In this case, the MILP model has $z_1 \leq c_1 = 0$ as an additional constraint to the set of constraints (5)-(17). From table above, it can be seen that front 3 satisfies the constraint $z_1 \leq 0$ and is the only one with particle size range deviation equal to zero; that is, $c_2 = 0$. Thus, the model returns as the best solution the extraction of 500 t from front 3, in which the ore proportion in particle size range 1 (40%) is the same as that required by the plant; thus, $z_2 = c_2 = 0$. Then, the LGP algorithm advances to the third objective (z_3) to minimise the deviation from the stripping ratio target. Next, constraints $z_1 \leq c_1 = 0$ and $z_2 \leq c_2 = 0$ are incorporated into the original set of model constraints. From table above, it can be seen that front 1 contains the waste mass necessary to reach the stripping ratio target. However, the extraction of this front does not satisfy the two added constraints. Consequently, front 3 remains the one that contains material that satisfies all the constraints. Thus, $z_3 = c_3 = 300$ ($= WT \times$ 500 - 700 = 2 $\times$ 500 - 700). As there are no more objectives to be analysed, the LGP algorithm returns as a solution with a tolerance of 5% of the plant's grade target, the extraction of 500 t from front 3.