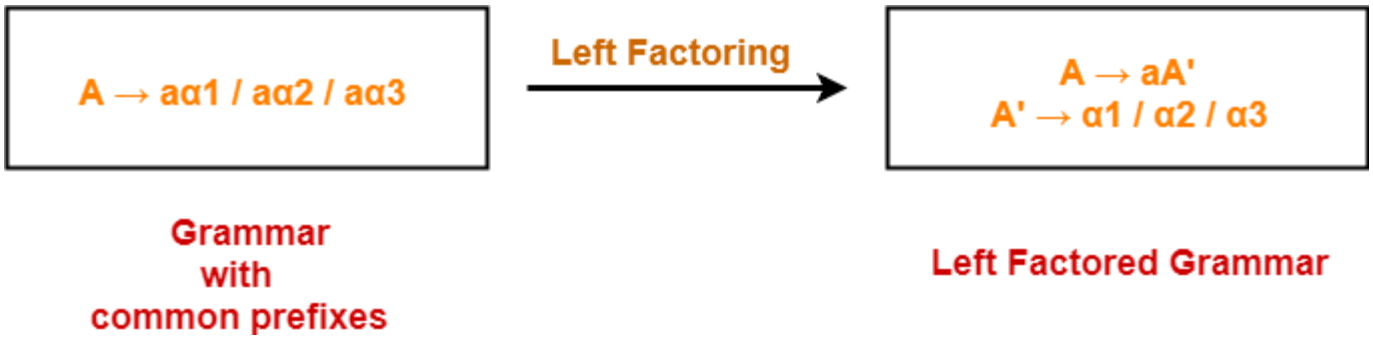
LEFT FACTORING

Example-



Problem-01:

Do left factoring in the following grammar-

S → iEtS / iEtSeS / a

E → b

Solution-

The left factored grammar is-

S → iEtSS' / a

S' → eS / ∈

E → b

Problem-02:

Do left factoring in the following grammar-

A → aAB / aBc / aAc

Solution-

Step-01:

A → aA’

A’ → AB / Bc / Ac

Again, this is a grammar with common prefixes.

Step-02:

A → aA’

A’ → AD / Bc

D → B / c

This is a left factored grammar.

Problem-03:

Do left factoring in the following grammar-

S → bSSaaS / bSSaSb / bSb / a

Solution-

Step-01:

S → bSS’ / a

S’ → SaaS / SaSb / b

Again, this is a grammar with common prefixes.

Step-02:

S → bSS’ / a

S’ → SaA / b

A → aS / Sb

This is a left factored grammar.

Problem-04:

Do left factoring in the following grammar-

S → aSSbS / aSaSb / abb / b

Solution-

Step-01:

S → aS’ / b

S’ → SSbS / SaSb / bb

Again, this is a grammar with common prefixes.

Step-02:

S → aS’ / b

S’ → SA / bb

A → SbS / aSb

This is a left factored grammar.

Problem-05:

Do left factoring in the following grammar-

S → a / ab / abc / abcd

Solution-

Step-01:

S → aS’

S’ → b / bc / bcd / ∈

Again, this is a grammar with common prefixes.

Step-02:

S → aS’

S’ → bA / ∈

A → c / cd / ∈

Again, this is a grammar with common prefixes.

Step-03:

S → aS’

S’ → bA / ∈

A → cB / ∈

B → d / ∈

This is a left factored grammar.

Problem-06:

Do left factoring in the following grammar-

S → aAd / aB

A → a / ab

B → ccd / ddc

Solution-

The left factored grammar is-

S → aS’

S’ → Ad / B

A → aA’

A’ → b / ∈

1. Left Recursion-

- A production of grammar is said to have left recursion if the leftmost variable of its RHS is same as variable of its LHS.
- A grammar containing a production having left recursion is called as Left Recursive Grammar.

Example-

$S \rightarrow Sa / \in$

(Left Recursive Grammar)

Left recursion is considered to be a problematic situation for Top down parsers.

- Therefore, left recursion has to be eliminated from the grammar.

Elimination of Left Recursion

Left recursion is eliminated by converting the grammar into a right recursive grammar.

If we have the left-recursive pair of productions-

$$A \rightarrow A\alpha / \beta$$

(Left Recursive Grammar)

where β does not begin with an A.

Then, we can eliminate left recursion by replacing the pair of productions with-

$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A' / \in$$

(Right Recursive Grammar)

This right recursive grammar functions same as left recursive grammar.

Problem-01:

Consider the following grammar and eliminate left recursion-

A → ABd / Aa / a

B → Be / b

Solution-

The grammar after eliminating left recursion is-

A → aA’

A’ → BdA’ / aA’ / ∈

B → bB’

B’ → eB’ / ∈

Problem-02:

Consider the following grammar and eliminate left recursion-

E → E + E / E x E / a

Solution-

The grammar after eliminating left recursion is-

E → aA

A → +EA / xEA / ∈

Problem-03:

Consider the following grammar and eliminate left recursion-

E → E + T / T

T → T x F / F

F → id

Solution-

The grammar after eliminating left recursion is-

E → TE’

E’ → +TE’ / ∈

T → FT’

T’ → xFT’ / ∈

F → id

Problem-04:

Consider the following grammar and eliminate left recursion-

S → (L) / a

L → L , S / S

Solution-

The grammar after eliminating left recursion is-

S → (L) / a

L → SL’

L’ → ,SL’ / ∈

Problem-05:

Consider the following grammar and eliminate left recursion-

S → S0S1S / 01

Solution-

The grammar after eliminating left recursion is-

S → 01A

A → 0S1SA / ∈

Problem-06:

Consider the following grammar and eliminate left recursion-

S → A

A → Ad / Ae / aB / ac

B → bBc / f

Solution-

The grammar after eliminating left recursion is-

S → A

A → aBA' / acA'

A' → dA' / eA' / ∈

B → bBc / f

Problem-07:

Consider the following grammar and eliminate left recursion-

A → AAα / β

Solution-

The grammar after eliminating left recursion is-

$A \rightarrow \beta A'$

$A' \rightarrow A\alpha A' / \in$

Problem-08:

Consider the following grammar and eliminate left recursion-

$A \rightarrow Ba / Aa / c$

$B \rightarrow Bb / Ab / d$

Solution-

This is a case of indirect left recursion.

Step-01:

First let us eliminate left recursion from $A \rightarrow Ba / Aa / c$

Eliminating left recursion from here, we get-

$A \rightarrow BaA' / cA'$

$A' \rightarrow aA' / \in$

Now, given grammar becomes-

$A \rightarrow BaA' / cA'$

$A' \rightarrow aA' / \in$

$B \rightarrow Bb / Ab / d$

Step-02:

Substituting the productions of A in B → Ab, we get the following grammar-

A → BaA' / cA'

A' → aA' / ∈

B → Bb / BaA'b / cA'b / d

Step-03:

Now, eliminating left recursion from the productions of B, we get the following grammar-

A → BaA' / cA'

A' → aA' / ∈

B → cA'bB' / dB'

B' → bB' / aA'bB' / ∈

This is the final grammar after eliminating left recursion.

Problem-09:

Consider the following grammar and eliminate left recursion-

X → XSb / Sa / b

S → Sb / Xa / a

Solution-

This is a case of indirect left recursion.

Step-01:

First let us eliminate left recursion from X → XSb / Sa / b

Eliminating left recursion from here, we get-

X → SaX' / bX'

X' → SbX' / ∈

Now, given grammar becomes-

X → SaX' / bX'

X' → SbX' / ∈

S → Sb / Xa / a

Step-02:

Substituting the productions of X in S → Xa, we get the following grammar-

X → SaX' / bX'

X' → SbX' / ∈

S → Sb / SaX'a / bX'a / a

Step-03:

Now, eliminating left recursion from the productions of S, we get the following grammar-

X → SaX' / bX'

X' → SbX' / ∈

S → bX'aS' / aS'

S' → bS' / aX'aS' / ∈

This is the final grammar after eliminating left recursion.

Problem-10:

Consider the following grammar and eliminate left recursion-

S → Aa / b

A → Ac / Sd / ∈

Solution-

This is a case of indirect left recursion.

Step-01:

First let us eliminate left recursion from S → Aa / b

This is already free from left recursion.

Step-02:

Substituting the productions of S in A → Sd, we get the following grammar-

S → Aa / b

A → Ac / Aad / bd / ∈

Step-03:

Now, eliminating left recursion from the productions of A, we get the following grammar-

$S \rightarrow Aa / b$

$A \rightarrow bdA' / A'$

$A' \rightarrow cA' / adA' / \in$

This is the final grammar after eliminating left recursion.