# RISK MODELS FROM TREE-STRUCTURED MRFS FOLLOWING MULTIVARIATE POISSON DISTRIBUTIONS

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# TWO MAJOR INSURANCE SYSTEMS

Traditional Insurance:

- Centralized risk-sharing mechanism;
- Insurer = intermediary;
- Standardized contracts $\Rightarrow$ portfolio expansion.

Decentralized Insurance (?):

- Removal of a central intermediary;
- Reform of mutualization principles;
- Use of modern technologies.

Common point:

- Based on risk pooling;
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  $\downarrow$ individual contributions through risk sharing.

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# OVERVIEW

# INDIVIDUAL RISK MODEL - CLASSICAL FRAMEWORK

Definition:

- Portfolio of $n$ participants $\boldsymbol{X} = (X_i)_{i=1}^n$.
- Each has a non-negative monetary loss $X_i, i \in [n]$.
- $\mu_i = \mathbb{E}[X_i] < \infty$, $\sigma_i^2 = \mathrm{Var}(X_i) < \infty$.
- Aggregate portfolio loss $S_n$:

$$S_n = X_1 + X_2 + \cdots + X_n.$$

Assumptions about the random variables $X_i$:

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## RISK SHARING: DEFINITION AND OPTIMAL RULE

- Fair risk sharing rule: a function $h : \boldsymbol{X} \mapsto (h_{i,n}(S_n))_{i\in[n]}$ that allocates the total cost $S_n = \sum_{i=1}^n X_i$ among $n$ participants such that:

$$\sum_{i=1}^{n} h_{i,n}(S_n) = S_n \quad \text{and} \quad \mathbb{E}[h_{i,n}(S_n)] = \mathbb{E}[X_i] \quad \text{(fairness).}$$

- Optimal rule: the conditional expectation

$$h^{\star}_{i,n}(S_n) = \mathbb{E}[X_i \mid S_n]$$

minimizes the mean squared error:

$$\mathbb{E}\left[(X_i - h(S_n))^2\right],$$

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## RELAXING THE CLASSICAL FRAMEWORK ASSUMPTIONS

- Portfolio of $n$ heterogeneous risks: $\boldsymbol{X} = (X_v)_{v=1}^{n}$ with $X_v = \sum_{i=1}^{N_v} B_{v,i}$.

- Each risk $X_i$ in $\boldsymbol{X}$ follows a compound Poisson distribution:

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## APPROACHES TO MULTIVARIATE POISSON MODELING

- Copula-based methods:
  - Allow separate modeling of marginals and dependence
  - Often avoided in discrete contexts due to theoretical and computational challenges

- Common-Shock Models (MPCS):
  - Provide clear stochastic interpretation of dependence mechanisms
  - One shock event can trigger claims across multiple risks
  - Parameter count grows exponentially with dimension
  - Becomes computationally intractable for large portfolios

# TREE-STRUCTURED MRFS WITH POISSON MARGINALS

- Heterogeneity: assumed in many works (see e.g. **?**);
- Dependence: Relaxing the independence assumption:
  -

## TREE-STRUCTURED MRFS WITH POISSON MARGINALS

Let

- Tree $\mathcal{T} = (\mathcal{V}, \mathcal{E})$ with root $r \in \mathcal{V}$;
- Parameters: means $\boldsymbol{\lambda} = (\lambda_v)_{v \in \mathcal{V}}$ and dependencies $\boldsymbol{\alpha} = (\alpha_e)_{e \in \mathcal{E}}$;
- Constraint: $\alpha_{(\mathrm{pa}(v),v)} \in [0, \min(\sqrt{\lambda_v/\lambda_{\mathrm{pa}(v)}}, \sqrt{\lambda_{\mathrm{pa}(v)}/\lambda_v})]$.
- Recursive construction:

$$N_v = \begin{cases} L_r, & \text{if } v = r, \\ \alpha_v \sqrt{\dfrac{\lambda_v}{\lambda_{\mathrm{pa}(v)}}} \circ N_{\mathrm{pa}(v)} + L_v, & \text{otherwise,} \end{cases}$$

  where $\boldsymbol{L} = (L_v)_{v \in \mathcal{V}}$ are independent Poisson variables with $\lambda_{L_v} = \lambda_v - \alpha_{(\mathrm{pa}(v),v)} \sqrt{\lambda_{\mathrm{pa}(v)} \lambda_v}$.

$\Rightarrow$ $\boldsymbol{N} \sim \mathrm{MPMRF}(\boldsymbol{\lambda}, \boldsymbol{\alpha}, \mathcal{T})$ with marginals $N_v \sim \mathrm{Poisson}(\lambda_v)$.

# TREE-BASED MRF WITH POISSON MARGINAL DISTRIBUTIONS

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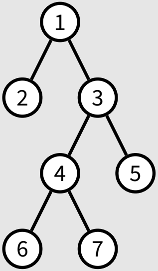


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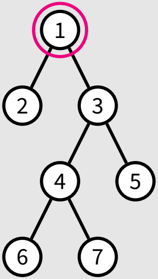


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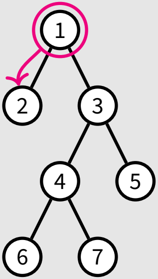


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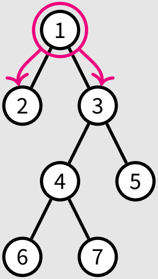


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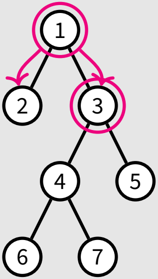


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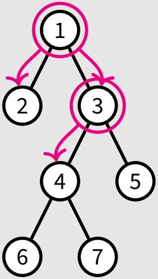

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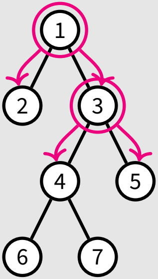


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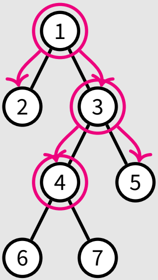


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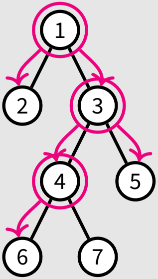


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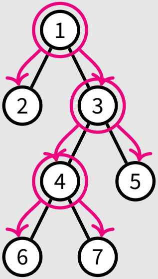


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