Lab Assignment C
Multivariable Control

Examination: Submitted report in CANVAS.

Preparation:

- Read the manual and plan for your work
- Perform Task 1 prior to the first lab session

1 Introduction

In this lab assignment we will explore multivariable control and robustness issues. Further, a comparison of the multivariable control scheme with a decentralized control scheme will be conducted. The lab assignment will be conducted in simulation on an aircraft model.

2 Process model

The process model is given as a linearized model of the vertical-plane dynamics of an aircraft, and has three inputs, three outputs, and five states as follows:

Inputs:

- u_1: spoiler angle $[deg/10]$
- u_2: forward acceleration $[m\ s^{-2}]$
- u_3: elevator angle $[deg]$

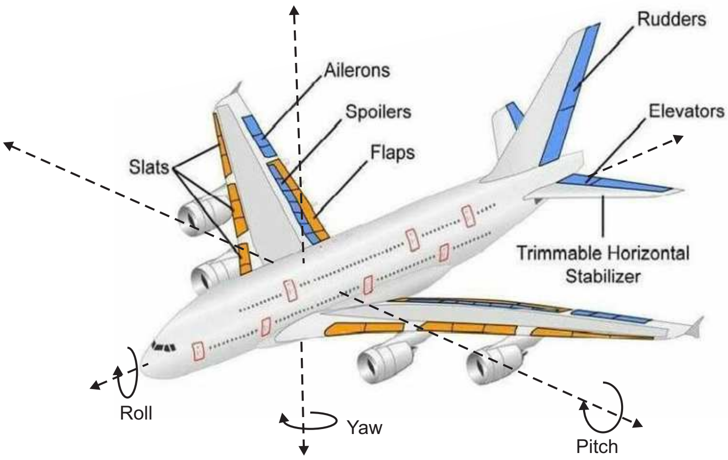
States:

- x_1: relative altitude $[m]$
- x_2: forward speed $[m\ s^{-1}]$
- x_3: pitch angle $[deg]$
- x_4: pitch rate $[deg\ s^{-1}]$
- x_5: vertical speed $[m\ s^{-1}]$

Outputs:

- y_1: relative altitude $[m]$
- y_2: forward speed $[m\ s^{-1}]$
- y_3: pitch angle $[deg]$

In Fig. 1, the notation and control surfaces for an aircraft are shown.



Figur 1: Control surfaces and notation. The picture is an adaptation of the aircraft control graphics from ScienceDirect.

2.1 Model parameters

The system matrices for the state space model are given as follows:

$$A = \begin{bmatrix} 0 & 0 & 1.1320 & 0 & -1.0000 \\ 0 & -0.0538 & -0.1712 & 0 & 0.0705 \\ 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0.0485 & 0 & -0.8556 & -1.0130 \\ 0 & -0.2909 & 0 & 1.0532 & -0.6859 \end{bmatrix} \tag{1}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ -0.1200 & 1.0000 & 0 \\ 0 & 0 & 0 \\ 4.4190 & 0 & -1.6650 \\ 1.5750 & 0 & -0.0732 \end{bmatrix} \tag{2}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{3}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4}$$

The transfer function for the aircraft model is then given by

$$G = C(sI - A)^{-1}B + D \tag{5}$$

2.2 Uncertainties

The aircraft models is affected by uncertainty. We assume that the actuator dynamics is currently neglected but will be reflected as uncertainties in the form of first and second order dynamics as follows:

$$G_{u1} = \frac{k_1}{\tau_1 s + 1} \tag{6}$$

$$G_{u2} = \frac{k_2}{\tau_2^2 s^2 + 2\zeta\tau_2 s + 1} \tag{7}$$

$$G_{u3} = \frac{k_3}{\tau_3 s + 1} \tag{8}$$

$$\tag{9}$$

There, G_{u1} acts on u_1, G_{u2} acts on u_2, and G_{u3} acts on u_3. For the parameters the following uncertainty ranges are considered:

- $k_1 = 1$
- $0.5 < \tau_1 < 1.2$
- $0.9 < k_2 < 1.1$
- $0.5 < \tau_2 < 2.5$
- $0.9 < \zeta < 1$
- $k_3 = 1$
- $1 < \tau_3 < 1.8$

3 Problem description

Your task is now to design different controllers for the process and perform a comparative analysis of the designs. In the comparison you have to consider a metric for the performance of the closed loop system, but also how complicated it was to achieve a feasible solution. The analysis should also discuss the robustness of the design.

The specification for the design is as follows:

- The bandwidth for the closed loop system should be $10\ rad\ s^{-1}$,
- The desired output should be tracked with zero steady state error.

NOTES:

- If you are not able to achieve the desired performance specification, explain the hinder and do a design for a possible specification
- Remember that the analysis is for a MIMO system and that the input directions affect the analysis. Choose appropriate reference signal directions to assess the design in simulation.

4 Design and test of different control schemes

The problem can be solved by designing different control schemes. All of them have pros and cons in design and also in term of the performance that can be achieved. At least two control schemes will be designed and compared.

Task 1: Development environment

The lab assignment will be performed in a simulated environment, where the designed controller will be tested and evaluated. For this end a development environment is needed. Remember that you need to be able to add the uncertain dynamics and disturbances at the input side to study the effects on the performance. Prepare the Simulink diagram such that you are able to plot and log signals that are needed to the later tasks.

The uncertainties should be switchable (on/off) and the parameters should be freely selectable.

Create a script that makes use of the parameters and generates the linear models and also the uncertain dynamics for the process.

Simulate the system to get acquainted with the aircraft model.

Task 2: Decentralized control for MIMO systems

Design a decentralized controller for y_1, y_2 and y_3. You may use your preferred design scheme for the individual controllers, but we suggest to try the IMC design methodology as described in section 8.3 of *Control Theory*.

1. Perform a control structure selection with I/O pairing. Use the RGA and one other method to select the I/O pairs. Are there contradiction?
2. Design the individual controllers
3. Assess the design on the MIMO process using the transfer functions matrices and their singular values.
4. Determine the extreme case command input directions and disturbance directions.
5. Simulate the closed loop system.

For the simulation. Implement the controller in the development environment and test the control scheme for the following scenarios:

- Step changes in the command input r_1, r_2, and r_3. Select different directions.
- Steps in the disturbances. Even here directions can be chosen!

Finally, assess the performance of the closed loop system including the interactions!

Task 3: Multivariable control using IMC

You can choose to either perform task 3 or task 4. It is optional to perform both.

Design a multivariable controller for y_1, y_2 and y_3 using the IMC design methodology as described in section 8.3 of *Control Theory*.

1. Design the MIMO IMC controller and calculate the 1-DOF Controller
2. Assess the design on the MIMO process using the transfer functions matrices and their singular values.
3. Determine the extreme case command input directions and disturbance directions.
4. Simulate the closed loop system.

Test the control system for the same scenarios as for Task 2.

Task 4: Multivariable control using state-feedback

You can choose to either perform task 3 or task 4. It is optional to perform both.

Design a multivariable state-feedback controller where you assume that the states can be measured without error. The following steps that need to be performed during the design.

1. Augment the state space model such that the resulting state feedback will contain integral action for each controlled variable y_i.
2. Derive the state feedback gain using pole placement.
3. Implement controller and simulate the closed loop system.

Test the control system for the same scenarios as for Task 2.

5 Analysis of the designs

The designed control schemes need to be compared with each other in terms of performance, engineering effort/complexity, and also in terms of the robustness.

Task 5: Performance analysis

The performance of the control schemes from task 2 and either 3 or 4 can be evaluated using the RMSE on the control error, as well as settling time and overshoots. For this end you have to log the control error for the different scenarios. In addition, the closed loop sensitivity transfer function matrix need to be analysed and compared with the specification.

Draw conclusions on the performance of the different control schemes and compare them. Reflect on the complexity of the design procedure and the effort that is needed to achieve a feasible design.

Task 6: Robustness analysis

In accordance with section 6.3 an uncertainty model has to be derived, which takes the following form

$$G_0 = (I + \Delta_G)G$$

There an upper bound for Δ_G has to be found which captures all possible model errors. For this end a large number of G_0^i has to be defined where the parameters $k_2,\ \tau_1,\ \tau_2,\ \tau_3$, and ζ are randomly picked from the respective interval.

Now a Δ_G is designed which creates an upper bound for the four bode magnitude plots of all the $(G_0^i - G)G^{-1}$. The chosen transfer functions should have a low order.

Perform the robustness analysis for the multivariable control schemes in task 3 or 4, depending on which one you have chosen, by evaluating

$$||\Delta_G T||_\infty < 1$$

Draw conclusions closed loop system's robustness towards uncertainties.

Task 7: Practical robustness analysis (Optional)

For each of the randomly picked parameter sets in task 4, a simulation run can be made and the performance can be evaluated. Create a script which enables you to do the following in an automated way:

- Randomly pick a parameter value for each parameter and store them. The number of picks need to be adjustable.
- Run a simulation in Simulink with the closed loop system for the currently picked parameters
- Quantify the performance of the closed loop systems from task 2 and 3, and store it
- Determine the worst case performance and the best case performance
- Generate plots for the performance in relation to the parameter variations

Note: In case that the closed loop system looses stability, then this need to be detected and stored.

Do the results reflect the outcome from task 6?

Task 7: Analysis of the engineering process

You have now designed at least two control schemes. Beside their differences in performance and robustness, the engineering of these control schemes is different in terms of efforts that need to be placed, the complexity of achieving a feasible design. Compare the engineering process of the cases that you have explored and highlight the pros and cons for each. Also reflect on the engineering process if the system would have 20 inputs and 50 outputs. Which control design approach do you prefer?

6 Reporting

Create a report for the tasks that you have performed and provide evidence for your claims and results.