Ondas gravitacionales: obtención de la ecuación y simulación

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Trabajo Fin de Grado

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Indice

1. Abstract

- In this work, the theoretical study of Einstein's field equations has been approached, particularising them for the vacuum in order to obtain non-zero solutions that propagate at the speed of light. Just as electromagnetic waves are found in electromagnetism, gravitational waves are found in the theory of general relativity.
- These waves will propagate throughout the Universe undisturbed, and can transmit a great deal of information about the source from which they come. The only problem with these waves is that they are associated with the weakest force in the Cosmos and are very difficult to detect. Despite this, they can nowadays be measured quite accurately and are compatible with theoretical results, supporting Einstein's theory of general relativity.

1. Abstract

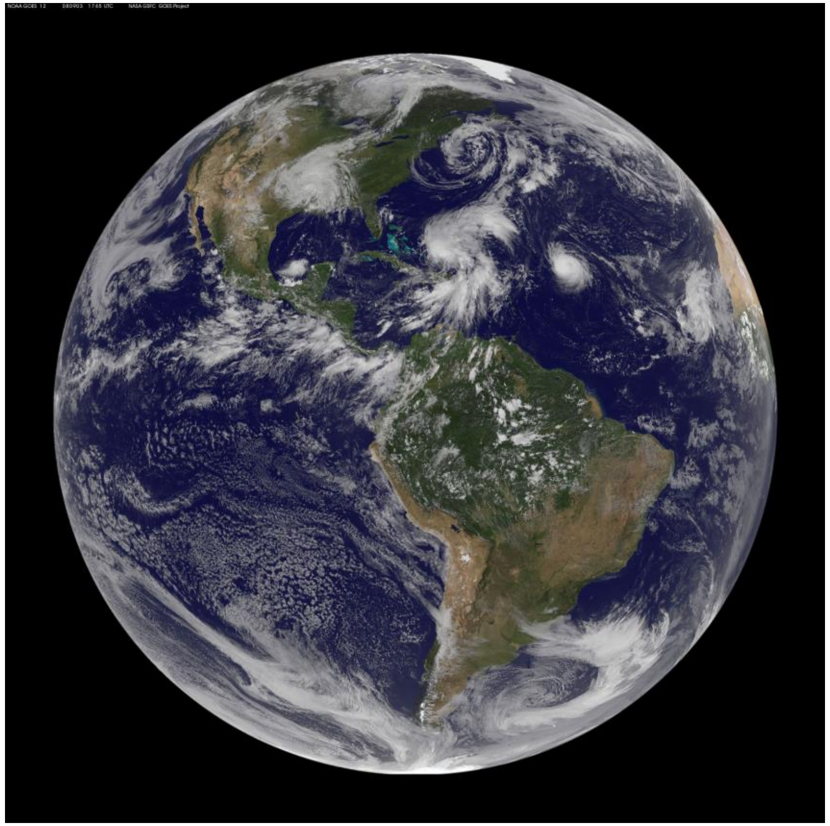
- We present a study of gravitational waves: how they arise in the vacuum that produces them, how they propagate, what associated power they carry, and finally we present a practical application in which we study the gravitational waves produced by the collision of two black holes.

2. Introducción

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + k\,\Lambda g_{\mu\nu} = \frac{8\,\pi G}{c^4} T_{\mu\nu}$$

$$\nabla_\mu T^{\mu\nu} = 0$$



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3. Objetivos

- Aproximación en campo débil para obtener la ecuación de unas ondas gravitatorias en distintos casos

- Descripción de la onda

- Potencia radiada por un sistema gravitatorio

- Descripción y aplicación de un sistema binario

4. Fundamento teórico: Newton

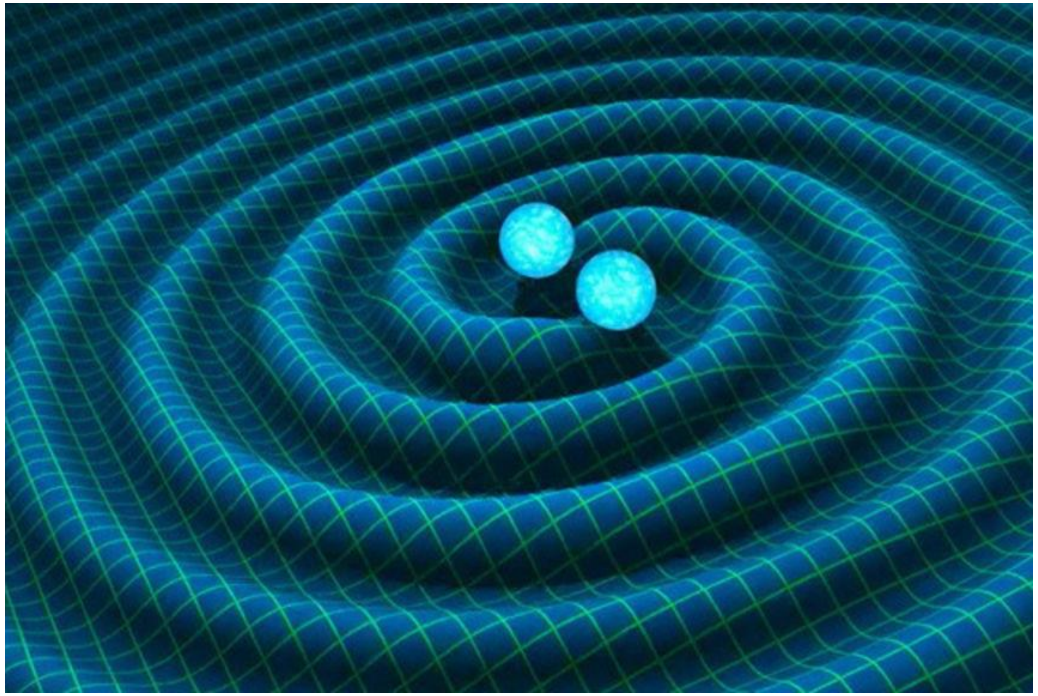
- Minkowski: $R_{\mu\nu} = 0$ y $\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

- Métrica: $g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu}$

- Gauge armónico de Donder: $\nabla_\mu h^{\mu\nu} - \frac{1}{2}\partial^\nu h = 0$

$$\boxed{\frac{1}{2}\epsilon\partial^2 h_{\mu\nu} = \frac{\epsilon}{2}\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) h_{\mu\nu} \approx -\kappa\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T\right)}$$

4. Fundamento teórico: Newton



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4. Fundamento teórico: Arbritario

- Métrica: $\tilde{g}_{\mu\nu} = g_{\mu\nu} + \epsilon\, h_{\mu\nu}$
- Tensor energía-impulso: $\tilde{T}_{\mu\nu} \approx T_{\mu\nu} + \epsilon\, t_{\mu\nu}$
- Cambio general de coordenadas: $h'_{\mu\nu} = h_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$

$$\boxed{\begin{aligned}&\frac{1}{2}\left(\nabla^2 h_{\mu\nu} + R_\mu^\rho h_{\nu\rho} + R_\nu^\rho h_{\mu\rho}\right) - R_{\mu\rho\nu\lambda} h^{\rho\lambda} \\ &\approx -k\left(t_{\mu\nu} - \frac{1}{2} g_{\mu\nu} t + \frac{1}{2} g_{\mu\nu} h^{\rho\lambda} T_{\rho\lambda} - \frac{1}{2} h_{\mu\nu} T\right)\end{aligned}}$$

4. Fundamento teórico: Polarización

- $h_{\mu\nu} = C_{\mu\nu} e^{i\, k_\lambda x^\lambda}$
- $C_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & C_+ & C_X & 0 \\ 0 & C_X & -C_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
- Oscilación +:

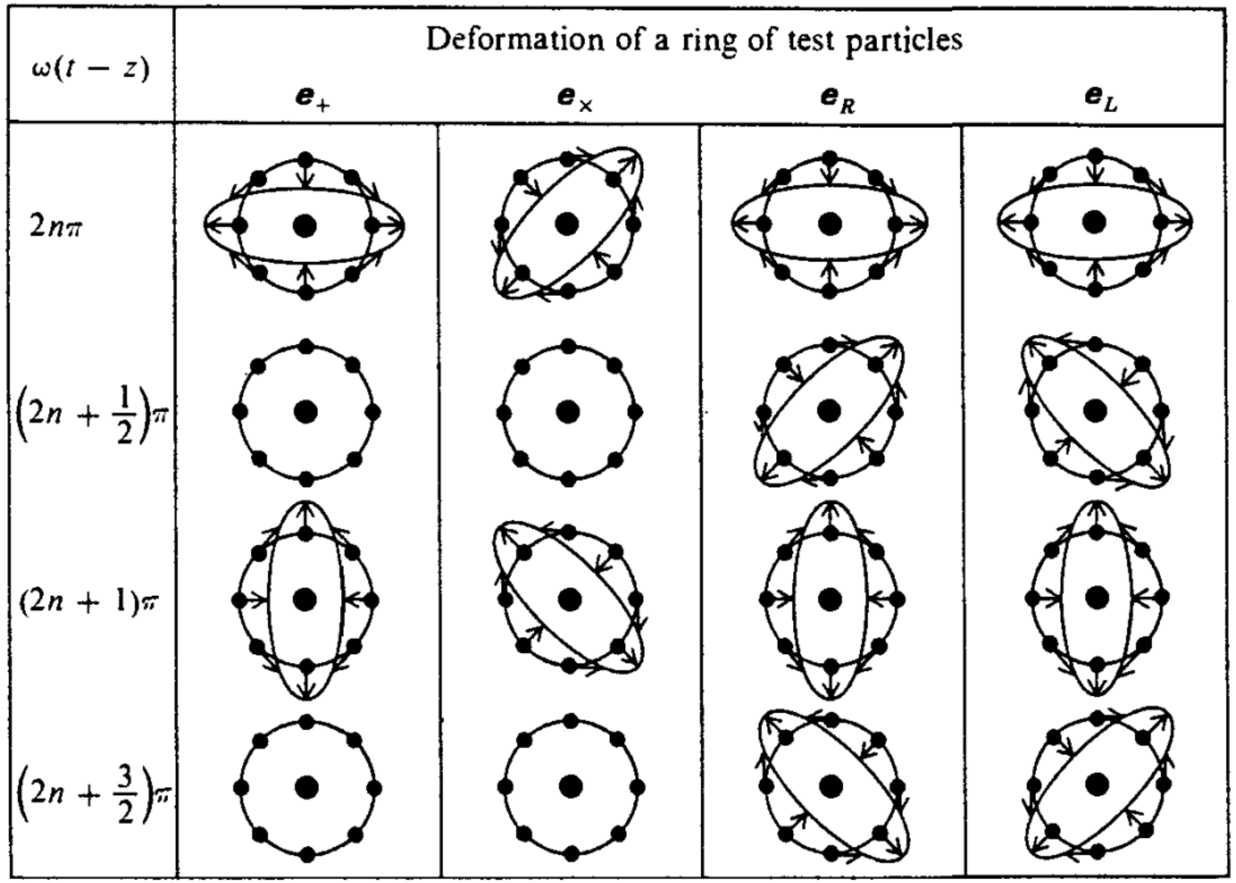
$$ds^2 = dt^2 - (1 + \epsilon C_+ \cos[k(t-z)])dx^2 - (1 - \epsilon C_+ \cos[k(t-z)])dy^2 - dz^2$$

- Oscilación x:

$$ds^2 = dt^2 - dy^2 - dz^2 + 2\epsilon\, C_\times\, cos[k(t-z)]\, dx\, dy$$

- Oscilación circular:

$$C_R = \frac{1}{2}(C_+ + i\, C_\times) \text{ y } C_L = \frac{1}{2}(C_+ - iC_\times)$$



Charles W. Misner, Kip S. Thorne, and John A. Wheeler. Gravitation. W. H. Freeman and Company, 1973.

4. Fundamento teórico: Fuentes

- Redefiniendo $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$

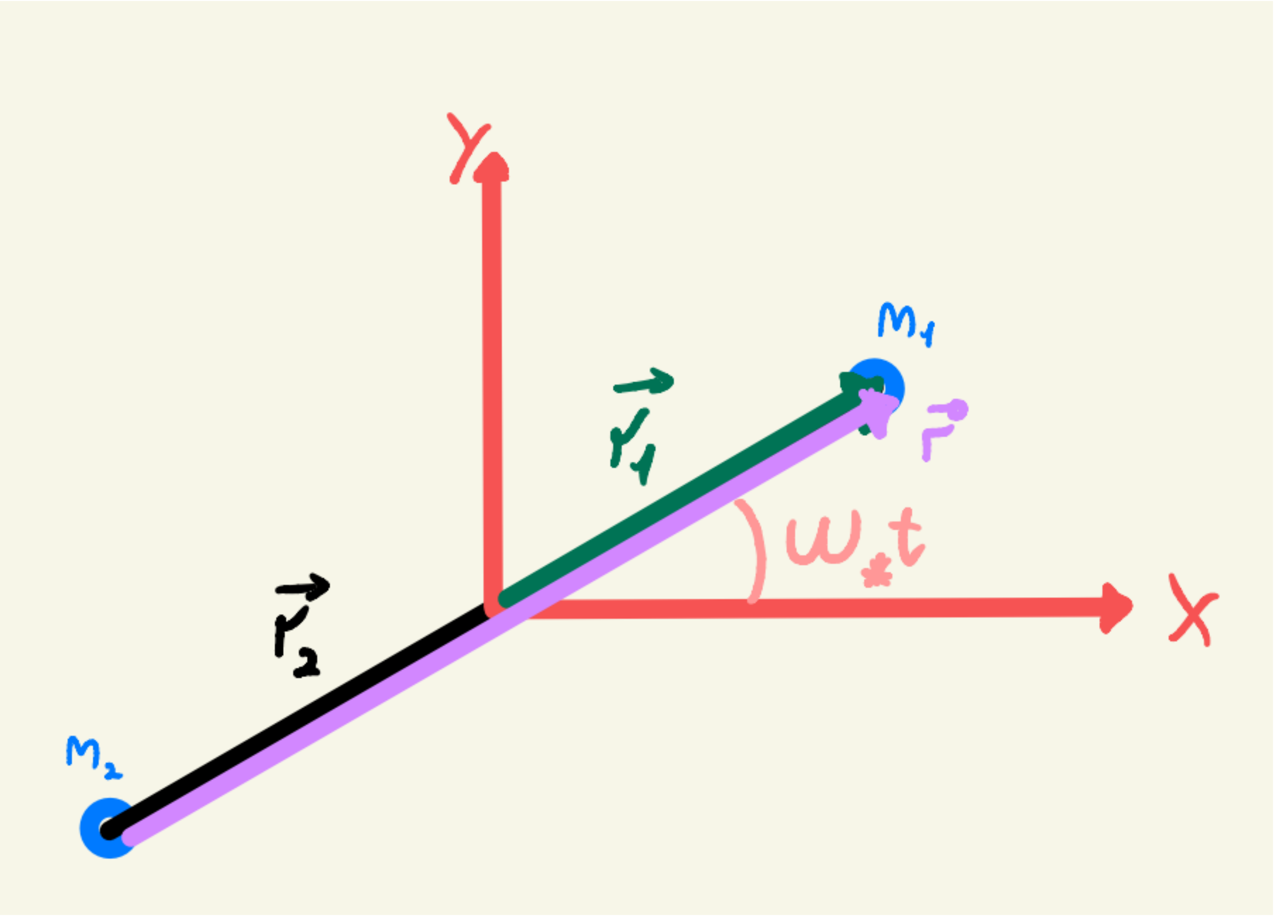
- Gauge armónico de Donder: $\partial_t \bar{h}^{t\nu} + \partial_i \bar{h}^{i\nu} = 0$

$$\boxed{\bar{h}^{ij} \longrightarrow \bar{h}^{tj} \longrightarrow \bar{h}^{tt}}$$

- $\bar{h}_{ij}(t, \vec{y}) \approx \frac{k}{12\pi r}\left[\partial_t^2 Q_{ij} + \delta_{ij}\partial_t^2 \int r^2 t^{tt}(t, \vec{y}) d^3 y\right]_{\{t=t_R\}}$

- Potencia radiada: $< P >= \frac{G_N}{45c^5} \sum_{i,j} |\dddot{Q}_{ij}\dddot{Q}_{ij}|$

5. Caso práctico: sistema binario



$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\vec{r}_1 = \frac{\mu a}{m_1}\begin{pmatrix} \cos(\omega_* t) \\ sen(\omega_* t) \\ 0 \end{pmatrix}$$

$$\vec{r}_2 = -\frac{\mu a}{m_2}\begin{pmatrix} \cos(\omega_* t) \\ sen(\omega_* t) \\ 0 \end{pmatrix}$$

$$\vec{r} = a\begin{pmatrix} \cos(\omega_* t) \\ sen(\omega_* t) \\ 0 \end{pmatrix}$$

5. Caso práctico: sistema binario

$$t_{tt} = \mu\delta(x)\delta(y)\delta(z)$$

$$h_0 = \frac{8G_N^2 m_1 m_2}{2a}$$

$$< \dot{a} > = -\frac{64G_N^3}{5a^3} m_1\, m_2 (m_1 + m_2)$$

$$< P > = \frac{32G_N^4 m_1^2 m_2^2}{5a^5}(m_1 + m_2)$$

$$f = \frac{1}{\pi}\sqrt{\frac{G_N(m_1 + m_2)}{a^3}} = \frac{1}{T}$$

$$< \frac{dL}{dt} > = -\frac{32G_N^{\frac{7}{2}}}{5a^{\frac{7}{2}}} m_1^2\, m_2^2 \sqrt{m_1 + m_2}$$

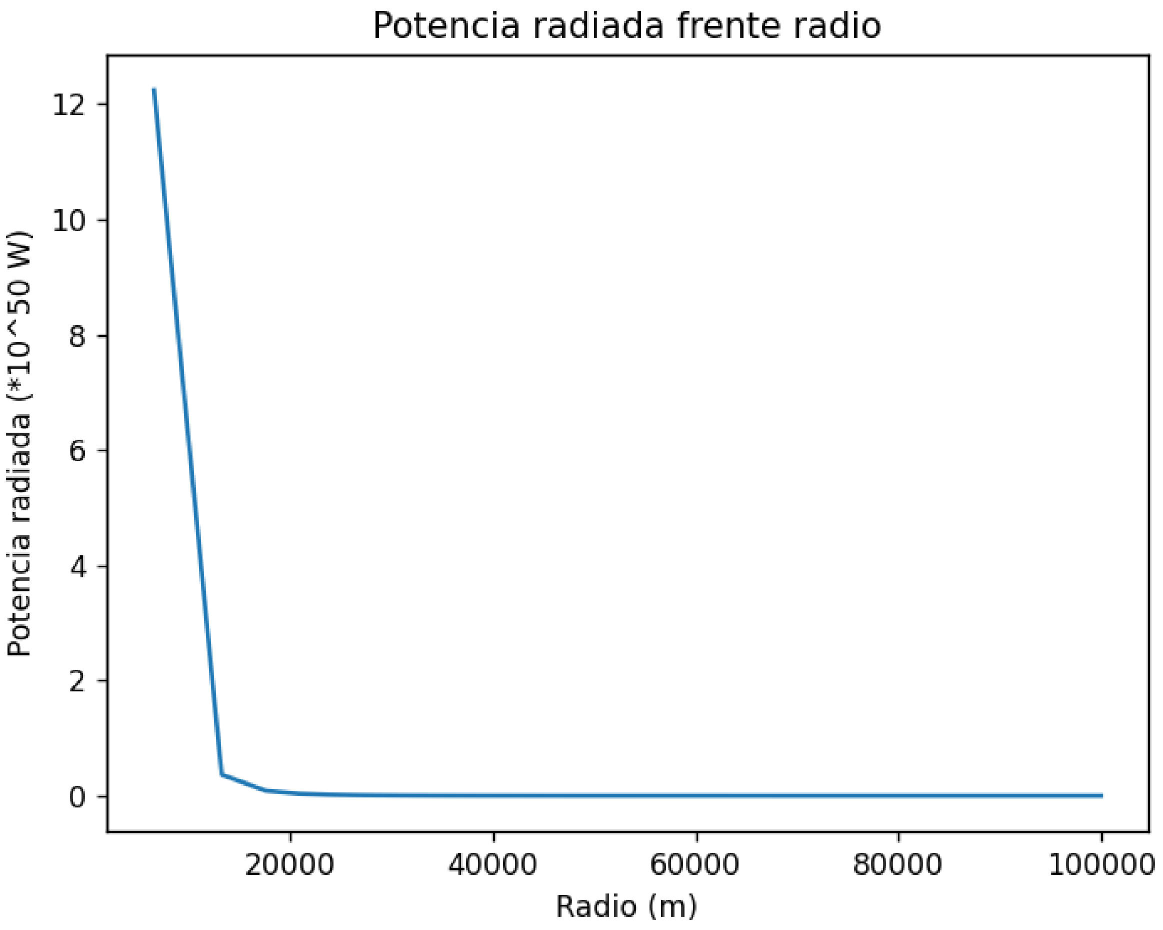
$$\tau = \frac{5a_0^4}{256G_N^3 m_1 m_2 (m_1 + m_2)}$$

5. Caso práctico: sistema binario: ejemplo concreto

- $m_1 = 1{,}4\ M_{\odot}$
- $m_2 = 1{,}3\ M_{\odot}$
- $T = 150\ min$

- $a_0 = 5{,}69\ \cdot 10^8\ m$
- $\tau = 1{,}36\ \cdot 10^7\ years$
- $< P_0 > = 1{,}39\ \cdot 10^{25}\ W$

$$a = a_0 + < \dot{a} > T$$



6. Conclusions

- We began with an introduction to general relativity in Chapter 1, and gave a general idea of the objectives of the present work in Chapter 2, the key points to be addressed in the following chapters. In Chapter 3, the gravitational wave equation was obtained under different general conditions, where the oscillation modes and the radiated power, among others, of the gravitational waves were described. Finally, Chapter 4, where the practical case of a binary system composed of black holes was studied.
- The waves obtained are, in general, not very energetic, taking into account the order of magnitude of the rest of the forces, except at the moment of a collision of the black holes when they are very close, where we started to obtain huge values of the radiated power. At these moments, huge amounts of emitted energy are reached, among the largest in the Universe, and this is one reason why, experimentally, when the mass of the resulting object is measured, it has a mass less than the sum of the initial constituents: because the missing mass has been emitted by gravitational waves.

6. Conclusions

- From these gravitational waves, from the profile of the radiated power, we can simulate the value of the initial masses, for example, and the amount of energy that the system has lost, and the data we have used as a starting point can be obtained from what we have used as a result.
- These waves provide a lot of information about the system, and will be further explored for the ongoing study of the stochastic gravitational-wave background.

Muchas gracias por su atención