SLAM - Simultaneous Localization and Mapping

Group Members - Agulugaha Isuru Gamage, Aydian Brown, Alejandro Kincaid

1. Problem Description

Simultaneous Localization and Mapping (SLAM) is the problem of enabling a robot to navigate in an unknown environment while simultaneously estimating its own pose and building a map of the environment. In realistic settings, both the robot motion and sensor measurements are noisy, which makes exact state estimation impossible.

In this project, an Extended Kalman Filter (EKF) is used to implement SLAM for a robot operating in a two-dimensional continuous environment populated with static point landmarks. The robot follows a nonlinear trajectory (a circular path) and observes landmarks using noisy range–bearing measurements within a limited field of view (FOV). The objectives of this project are:

- To estimate the robot trajectory over time (localization),
- To estimate the positions of unknown landmarks (mapping),
- To demonstrate convergence of both localization and mapping using quantitative plots and animation.

Due to process noise, the true robot motion deviates from the ideal imposed trajectory. The EKF fuses noisy motion information and noisy landmark measurements to improve state estimates over time. Since multiple landmarks may be visible simultaneously and measurements are noisy, the correspondence between measurements and landmarks must be resolved using a data association strategy.

This report describes the motion model, measurement model, EKF prediction and update equations, Jacobians, landmark initialization, data association rule, and presents an analysis of convergence using plots and simulation video.

2. State Representation

The EKF-SLAM state vector consists of the robot pose and the positions of all landmarks:

$$\mathbf{x} = [x \quad y \quad \theta \quad l_{1x} \quad l_{1y} \quad \dots \quad l_{Nx} \quad l_{Ny}]^T$$

where:

- (x, y) denotes the robot position,
- θ denotes the robot heading,
- (l_{ix}, l_{iy}) are the Cartesian coordinates of landmark i.

The covariance matrix Σ represents the joint uncertainty of the robot pose and all landmark estimates.

3. Motion Model

3.1 Unicycle Motion Model

The robot motion is modeled using a unicycle kinematic model with control input:

$$\mathbf{u}_k = \begin{bmatrix} v_k \\ \omega_k \end{bmatrix}$$

where v_k is the linear velocity and ω_k is the angular velocity. The nonlinear motion equations are:

$$
\begin{aligned}
x_{k+1} &= x_k + v_k \Delta t \cos(\theta_k) \\
y_{k+1} &= y_k + v_k \Delta t \sin(\theta_k) \\
\theta_{k+1} &= \theta_k + \omega_k \Delta t
\end{aligned}
$$

Process noise is modeled as zero-mean Gaussian noise affecting the robot motion, with covariance matrix Q.

4. Measurement Model

The robot observes landmarks using noisy range–bearing measurements:

$$\mathbf{z}_k = \begin{bmatrix} r \\ \phi \end{bmatrix}$$

where:

- r is the Euclidean distance to the landmark,
- ϕ is the bearing relative to the robot heading.

For landmark i, the measurement model is:

$$
\begin{aligned}
r_i &= \sqrt{(l_{ix} - x)^2 + (l_{iy} - y)^2} \\
\phi_i &= \operatorname{atan2}(l_{iy} - y, l_{ix} - x) - \theta
\end{aligned}
$$

Measurement noise is modeled as zero-mean Gaussian with covariance:

$$R = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\phi^2 \end{bmatrix}$$

5. EKF Prediction Step

During the prediction step, only the robot pose is propagated using the nonlinear motion model:

$$\hat{\mathbf{x}}_{k|k-1} = f(\hat{\mathbf{x}}_{k-1}, \mathbf{u}_k)$$

The covariance is updated as:

$$\Sigma_{k|k-1} = F_k \Sigma_{k-1} F_k^T + Q$$

where F_k is the Jacobian of the motion model with respect to the robot pose. Landmark states remain unchanged during prediction.

6. Jacobians

6.1 Motion Model Jacobian

The Jacobian of the motion model with respect to the robot pose is:

$$F_k = \begin{bmatrix} 1 & 0 & -v_k \Delta t \sin(\theta_k) \\ 0 & 1 & v_k \Delta t \cos(\theta_k) \\ 0 & 0 & 1 \end{bmatrix}$$

6.2 Measurement Model Jacobian

For landmark i, the measurement Jacobian with respect to the robot and landmark states is:

$$H_i = \begin{bmatrix} -\frac{\Delta x}{r} & -\frac{\Delta y}{r} & 0 & \frac{\Delta x}{r} & \frac{\Delta y}{r} \\ \frac{\Delta y}{r^2} & -\frac{\Delta x}{r^2} & -1 & -\frac{\Delta y}{r^2} & \frac{\Delta x}{r^2} \end{bmatrix}$$

where $\Delta x = l_{ix} - x$, $\Delta y = l_{iy} - y$, and $r = \sqrt{\Delta x^2 + \Delta y^2}$. All other state variables have zero contribution.

7. EKF Measurement Update Step

For each measurement associated with a landmark, the EKF update equations are applied. The Kalman gain is computed as:

$$K_k = \Sigma_{k|k-1} H_k^T \left(H_k \Sigma_{k|k-1} H_k^T + R \right)^{-1}$$

The state estimate is updated using the innovation:

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k|k-1} + K_k (\mathbf{z}_k - \hat{\mathbf{z}}_k)$$

The covariance update is:

$$\Sigma_k = (I - K_k H_k) \Sigma_{k|k-1}$$

Angular residuals are wrapped to the interval $[-\pi, \pi]$to ensure consistent heading estimation.

8. Landmark Initialization

When a landmark is observed for the first time, its position is initialized using the inverse measurement model:

$$\begin{aligned} l_x &= x + r\cos(\phi + \theta) \\ l_y &= y + r\sin(\phi + \theta) \end{aligned}$$

An initial covariance is assigned to reflect uncertainty in the newly initialized landmark.

9. Data Association

The robot does not know with certainty which measurement corresponds to which landmark; therefore, data association is treated as an unknown correspondence problem. A nearest-neighbor data association strategy is employed, in which each incoming range–bearing measurement is compared against all previously initialized landmarks. For each candidate landmark, the innovation between the observed and predicted measurement is computed, and an association metric based on the measurement noise covariance is evaluated. The measurement is associated with the landmark that minimizes this distance, provided it lies below a predefined gating threshold. If no existing landmark satisfies the gating condition, the measurement is treated as originating from a new landmark and is used to initialize its state.

10. Experimental Setup

The experimental setup used in this project is summarized as follows:

- A circular desired trajectory with a radius of approximately 4 m is imposed.
- The simulation runs with a fixed discrete time step Δt.
- The robot is equipped with a range–bearing sensor with a 120° field of view.
- Gaussian process noise is applied to the motion model.
- Gaussian measurement noise is applied to landmark observations.
- Multiple static point landmarks are placed around the trajectory.
- Performance is evaluated using trajectory plots, error plots, and animation.

11. Results and Analysis

11.1 Robot Trajectory: True vs EKF vs Desired Path

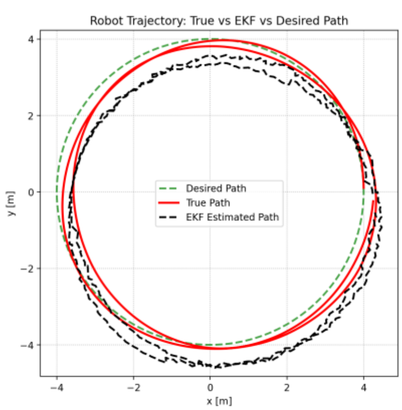

Figure 11.1 True vs EKF vs Desired Path

Figure 11.1 compares the desired circular trajectory, the true robot trajectory affected by motion noise, and the EKF-estimated trajectory. The true path deviates slightly from the desired circle due to process noise in the unicycle motion model, while the EKF estimate closely follows the true trajectory with small, bounded offsets. These deviations are most noticeable in regions with fewer landmark observations, but the estimated path remains smooth and stable throughout the simulation. Overall, this figure demonstrates that the EKF successfully fuses noisy motion and measurement data to produce an accurate and consistent estimate of the robot's trajectory.

11.2 Mean Squared Error (MSE) of Robot Position

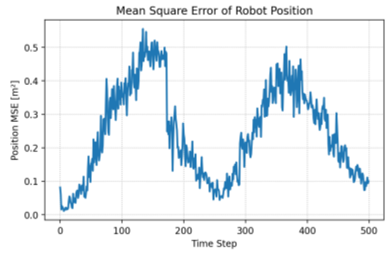

Figure 11.2 Mean Squared Error of Robot Position

Figure 11.2 shows the mean squared position error between the true robot state and the EKF estimate over time. The error is initially low but increases during early motion as uncertainty accumulates and landmarks are still being initialized. As the robot continues along the trajectory and receives additional landmark measurements, the EKF reduces the estimation error and maintains it within a bounded range. Periodic rises in error correspond to intervals with limited sensor observations, highlighting the dependence on measurement availability. This plot confirms the convergence and stability of the EKF localization performance.

11.3 True vs Estimated Heading

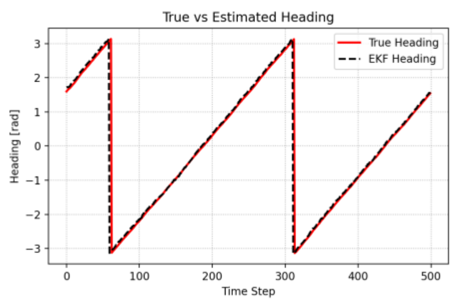

Figure 11.3 True vs Estimated Heading

Figure 11.3 compares the true robot heading with the EKF-estimated heading over time. The estimated heading closely tracks the true heading throughout the simulation, including correct handling of angular wrap-around at $\pm\pi$ radians. Minor deviations are present due to motion and measurement noise, but no long-term drift is observed. This result indicates that the EKF effectively estimates the robot's orientation and correctly handles the nonlinear nature of angular states.

11.4 True vs Estimated Landmarks (Scatter Plot)

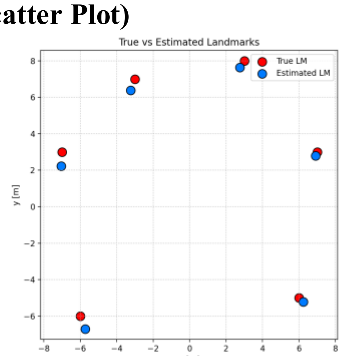

Figure 11.4 True vs Estimated Landmarks

Figure 11.4 compares the final EKF-estimated landmark positions with the true landmark locations. Most estimated landmarks lie close to their true positions, indicating successful mapping performance. Small offsets remain for some landmarks, which can be attributed to limited field-of-view, measurement noise, and uneven observation frequency during the robot's motion. Despite these factors, the EKF converges to reasonable landmark estimates, demonstrating effective simultaneous localization and mapping.

11.5 Final Landmark Estimation Error

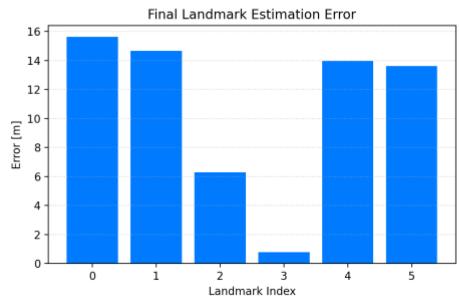

Figure 11.5 Final Landmark Estimation Error

Figure 11.5 highlights variation in landmark estimation accuracy across different landmarks. Some landmarks exhibit low error due to repeated observations from favorable viewing angles, while others show larger errors because of limited visibility or higher measurement uncertainty. Importantly, all errors remain bounded, indicating stable mapping performance and no filter divergence.

11.6 EKF-SLAM Visualization (Video)

YouTube link: https://youtu.be/xdXqOpavnHY

The video shows the evolution of the robot pose and landmark estimates over time compared to their true values. Initially, the estimates are noisy and inaccurate due to limited sensor information. As the robot continues moving and observes more landmarks, the EKF improves both localization and mapping estimates. The estimated trajectory becomes smoother and closer to the true path, and the landmark estimates converge toward their actual positions.

This video visually confirms the convergence behavior of the EKF-SLAM system and demonstrates its ability to handle noisy motion and measurement data.

12. Conclusion

This project implemented an Extended Kalman Filter (EKF)–based SLAM system for a robot navigating a two-dimensional environment with static landmarks. The robot followed a nonlinear trajectory while experiencing noisy motion and noisy range–bearing measurements. Despite these uncertainties, the EKF successfully estimated both the robot trajectory and the landmark positions. The estimated trajectory closely follows the true path while remaining smooth due to probabilistic filtering of noise. Landmark estimates gradually converge toward their true locations as repeated observations are incorporated through the EKF update step. Overall, the results demonstrate stable convergence and confirm the effectiveness of EKF-SLAM for simultaneous localization and mapping in a noisy environment.

Source code repository: https://github.com/isurugamage37/EKF-SLAM