$$L\frac{di}{dt} + Ri_a = e_m(t) - K_b\frac{d\theta_m}{dt}$$

$$L\frac{di}{dt} + Ri_a = e_m(t) - \frac{K_b}{K_R}\frac{dx_m}{dt}$$

$$L\frac{di}{dt} + Ri_a = e_m(t) - \frac{K_b}{K_R}\left(\frac{N_2}{N_1}\right)\dot{x}_L$$

$$(Ls + R)I_a(s) = E_m(s) - \frac{K_b}{K_R}\frac{N_2}{N_1}sX_L(s)$$

Eq — 1

$$J_m + J_P = J$$

$$J\frac{d\theta_m^2}{dt^2} + D_1\frac{d\theta_m}{dt} = \tau_m$$

$$\frac{J}{K_R}\ddot{x}_m + \frac{D_1}{K_R}\dot{x}_m = K_m I_a$$

$$\frac{J}{K_R}\frac{N_2}{N_1}\ddot{x}_L + \frac{D_1}{K_R}\frac{N_2}{N_1}\dot{x}_L = K_m I_a$$

$$\left(\frac{J}{K_R}\frac{N_2}{N_1}s^2 + \frac{D_1}{K_R}\frac{N_2}{N_1}s\right)X_L(s) = K_m I_a(S)$$

Eq — 2

$$J_2\ddot{\theta}_L + D_2\dot{\theta}_L = \tau_L$$

(2)

$$J_2 = J_g + J_3$$

Where,

$$\tau_L = K_s f_L(t)$$

$$\tau_L = K_s [m_L \ddot{x}_L + b\dot{x}_L + d(t)]$$

So,

$$J_2\ddot{\theta}_L + D_2\dot{\theta}_L = K_s[m_L\ddot{x}_L + b\dot{x}_L + d(t)]$$

$$\frac{J_2}{K_R}\ddot{x}_L + \frac{D_2}{K_R}\dot{x}_L - K_s m_L\ddot{x}_L - K_s b\dot{x}_L - K_s d(t) = 0,$$

$$\left(\frac{J_2}{K_R} - K_s m_L\right)\ddot{x}_L + \left(\frac{D_2}{K_R} - K_s b\right)\dot{x}_L - K_s d(t) = 0$$

$$\left[\left(\frac{J_2}{K_R} - K_s m_L\right)s^2 + \left(\frac{D_2}{K_R} - K_s b\right)s\right] X_L(s) = K_s D(s)$$

eq — (3)

eq (2) − eq (3)

$$\left[\underbrace{\left(\frac{J}{K_R}\frac{N_2}{N_1} - \frac{J_2}{K_R} + K_s m_L\right)}_{C_1}s^2 + \underbrace{\left(\frac{D_1}{K_R}\frac{N_2}{N_1} - \frac{D_2}{K_R} + K_s b\right)}_{C_2}s\right] X_L(s)$$

$$= K_m I_a(s) - K_s D(s)$$

For $\frac{X_L(s)}{E_m(s)} \rightarrow D(s) = 0$

$$I_a(s) = \frac{1}{K_m}\left[C_1 s^2 + C_2 s\right] X_L(s) \quad \text{— (4)}$$

Put eq. 4 in 1

$$(Ls+R)\frac{1}{K_m}(C_1 s^2 + C_2 s)\; X_L(s) = E_m(s) - \frac{K_b N_2}{K_R N_1}$$

$$E_m(s) = \left[(Ls+R)\frac{1}{K_m}(C_1 s^2 + C_2 s) + \frac{K_b}{K_R}\frac{N_2}{N_1} s\right] X_L(s)$$

$$E_m(s) = \left[\frac{(Ls+R)(C_1 s^2 + C_2 s) + K_m (K_b/K_R)(N_2/N_1) s}{K_m}\right] X_L(s)$$

$$E_m(s) = \left[\frac{s\left[(Ls+R)(C_1 s + C_2) + K_m\left(\frac{K_b}{K_R}\right)\left(\frac{N_2}{N_1}\right)\right]}{K_m}\right] X_L(s)$$

$$\frac{X_L(s)}{E_m(s)} = \frac{K_m}{s\left[(Ls+R)(C_1 s + C_2) + K_m K_b \left(\frac{N_2}{N_1}\right)\left(\frac{1}{K_R}\right)\right]}$$

For $\frac{X_L(s)}{D(s)} \rightarrow E_m(s) = 0$

$$(C_1 s^2 + C_2 s)\, X_L(s) + K_s D(s) = K_m I_a \quad (5)$$

from eq 1

$$I_a(s) = \frac{-\frac{K_b}{K_R}\frac{N_2}{N_1} s \quad X_L(s)}{Ls + R}$$

Put in 5

$$(c_1 s^2 + c_2 s) X_2(s) + K_s D(s) = K_m \frac{\frac{-K_b}{K_R}\frac{N_2}{N_1} s X_2(s)}{Ls+R}$$

$s X_2(s)$

$$\left[c_1 s^2 + c_2 s + \frac{K_m K_b \left(\frac{1}{K_R}\frac{N_2}{N_1}\right) s}{Ls+R}\right] X_2(s) + K_s D(s) = 0$$

$$\frac{s\left\{(Ls+R)(c_1 s + c_2) + K_m K_b \left(\frac{1}{K_R}\frac{N_2}{N_1}\right)\right\}}{Ls+R} X_2(s) = -K_s D(s),$$

$$\frac{X_2(s)}{D(s)} = \frac{-K_s (Ls+R)}{s\left[(Ls+R)(c_1 s + c_2) + K_m K_b \left(\frac{1}{K_R}\frac{N_2}{N_1}\right)\right]}$$

$$= \frac{-K_s (Ls+R)}{s\left[(Ls+R)\left(\left(\frac{J_{mot}}{K_R} r - \frac{J_{lead}}{K_R} + K_s m_L\right) s + \left(\frac{D_1 r - D_2}{K_R} + K_s b\right)\right) + \frac{K_m K_b}{K_R} r\right]}$$

$$\frac{-K_s K_R (Ls+R)}{s\left[(Ls+R)\left(\left(J_{mot} r - J_{lead} + K_s K_R m_L\right) s + \left(D_1 r - D_2 + K_s K_R b\right)\right) + K_m K_b r\right]}$$

$$\frac{K_m K_R}{s\left[(Ls+R)\left(\left(J_{mot} r - J_{lead} + K_s K_R m_L\right) s + \left(D_1 r - D_2 + K_s K_R b\right)\right) + K_m K_b r\right]}$$