ROYAL INSTITUTE OF TECHNOLOGY

Lab 3 tutorial Vehicle vibration control by using PD, PID, Skyhook and H_∞ controllers

SD2231 – Applied Vehicle Dynamics Control

May 2023

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Contents

1 Introduction

The following laboratory assignment is intended to give you knowledge and experience in the area of vehicle vibration control. Vibrations on vehicle directly affect passenger comfort and hence it is important to keep vibration level as low as possible. This is especially important for public transport vehicles since the passenger would like to focus on other tasks during the journey and not to be disturbed by road irregularities. The same can be said for the autonomous vehicles that are currently discussed a lot around the road bound vehicle industry. In such autonomous vehicles the driver becomes a passenger like in any other public transport (if considering fully autonomous systems) and hence the vibration control of the vehicle compartment becomes an even more important issue. To achieve required vibration level, suspension is used. Suspension system is basically compromised of springs and dampers which are passive elements. In this study potential of using active control systems for improving ride comfort will be studied and compared against traditional passive suspensions.

In this assignment you will go through simplified models of a vehicle which are subjected to vibrations excited by track or road irregularities. First control of a single degree of freedom model will be studied and results will be compared against a passive system. The same study will be carried out in the next sections of the assignment for more complicated models. The assignment is focused on using PD, PID, Skyhook and H_{∞}controllers.

1.1 Report writing

A good report includes exactly the information which is needed for the reader to understand the results and nothing more. Throughout the laboratory instruction some tasks are marked as '**(midway step)**'. These tasks should be done to follow the calculation procedure, but you do not need to reflect them in your report. Rest of the tasks should be answered in the report.

Include the following files when submitting your report:

- Your finalized report as *pdf*, please name the reports as follows:
  - **Lab3_Group?_Report.pdf**
- A zip file including all your organized MATLAB and Simulink files (Teachers should be able to run your files).

Note:

- Arrange your report hierarchy exactly based on the task arrangement in this handout.
- **You should use the template on Canvas for your final report.**

1.2 Teachers who will support this laboratory assignment

- Rocco Giossi, roccolg@kth.se
- Alireza Qazizadeh, alirezaq@kth.se

1.3 Laboratory set-up

You will solve the assignments in MATLAB and Simulink, using your own computers or the computers in the Vehicle Engineering Lab.

2 Theory

2.1 Skyhook controller

Skyhook is a method of vibration isolation where it is tried to reduce vibration transfer by decoupling effect of track or road irregularity. To make the point clear, consider the following conventional suspension model in Figure 1.

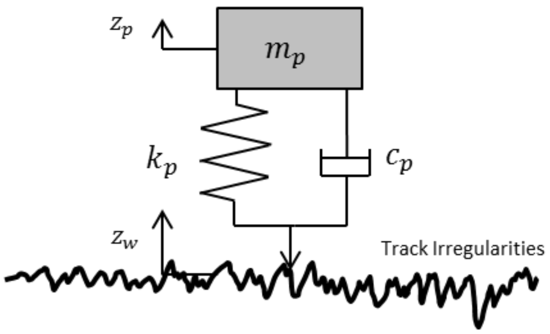

Figure 1. Conventional suspension model.

For this model the force of the damper can be calculated according to

$$F_{c_p} = -c_p.(\dot{z}_p - \dot{z}_w) \tag{1}$$

Now consider the following modified system in Figure 2 where the damper c_p is connected to the mass m_p from one end and the other end is hanged from an imaginary fixed point in the sky. The force of the damper can be calculated according to

$$F_{c_p} = -c_p.\dot{z}_p \tag{2}$$

As you can see $\dot{z}_w$ which is related to the track irregularity does not appear in the damper equation of force. This means that the damper does not transfer track irregularities to the mass which could be very useful for vehicle ride comfort improvement. This concept is called Skyhook as it functions as if the vehicle is hooked to the sky. However, it is not practical to hang one side of the damper from sky, so there is a need to find a practical method to implement this concept.

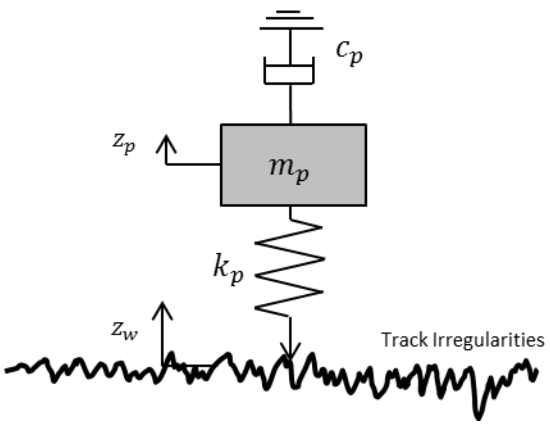

Figure 2. Modified system.

To implement Skyhook concept, one should use an actuator and a feed forward control loop as shown in Figure 3. In this way the applied force by the actuator would be the same as the damper force in Skyhook method, see Equation (2).

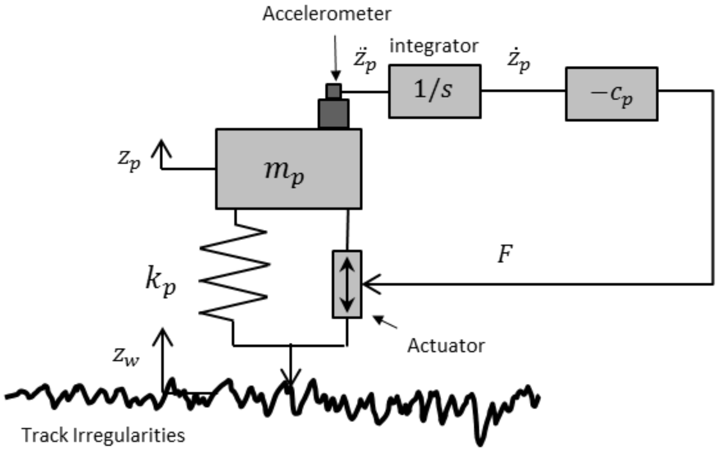

Figure 3. Actuator and feed forward control loop based on Skyhook.

2.2 $\boldsymbol{H_\infty}$ controller

Before digging into H_∞ controller principle, one needs to learn about *infinity norm* or H_∞*-norm.*

$\boldsymbol{H_\infty}$norm:
Consider a signal $z(t)$ where t is a vector of discrete time steps. If $z(t)$ at each specific time t is a column vector of dimension n, its size is measured by the usual vector norm

$$|z(t)\,|^2 = \sum_{j=1}^{n} z_j^2(t) = z^T(t)z(t) \tag{3}$$

which is the same as Euclidean norm (length of a vector). However, the signal is still time dependent and one cannot consider one scalar value as the signal size. One way for measuring the size of the entire signal $z(t)$ is to use *infinity norm*

$$\|z\|_\infty = \sup_t \; |z(t)| \tag{4}$$

where

$$\sup_t \tag{5}$$

means that the supremum (maximum) of $|z(t)|$ is to be taken over all input values t. One can use the following MATLAB syntax to calculate infinity norm of a vector (or even a matrix) A

```
n = norm(A,inf).
```

Likewise infinity norm of a SISO (single input, single output) system with the transfer function $P(s)$ is the same as the maximum magnitude of its transfer function over the whole frequency range.

$$\|P(s)\|_\infty = \sup_w \; |P(iw)| \tag{6}$$

However, in case of a MIMO (multiple input, multiple output) system the infinity norm is the maximum singular value of the system over the whole frequency range. Singular values of a linear time-invariant (LTI) system can be plotted by the following syntax

```
sigma(sys)
```

H_∞ controller principle:
Consider plant P as shown in Figure 4. The task is to find a feedback law $u_2 = Ky_2$ that gives a good controlled system. Plant P is shown in terms of its state space matrices and is partitioned. The system P is partitioned where inputs to B_1 are the disturbances, inputs to B_2 are the control inputs, outputs of C_1 are the errors to be kept small, and outputs of C_2 are the output measurements provided to the controller.

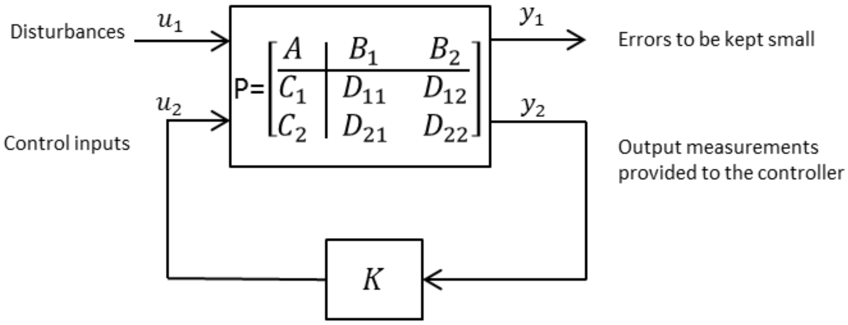

Figure 4. Plant P.

The aim is to design a controller so that the closed loop system infinity norm is minimum. Closed loop system can be calculated in MATLAB by the following syntax

```
CL= lft(P,K)
```

and its infinity norm is

$$\gamma = \|CL\|_\infty = \texttt{norm(CL,inf)} \tag{7}$$

So the problem is now reduced to find a controller K, which minimizes the value of γ. This minimization can be carried out in MATLAB automatically by using the following syntax.

```
[K,CL,GAM] = hinfsyn(P,NMEAS,NCON)
```

The controller, K, stabilizes the plant P and has the same number of states as P. `NCON` is the column size of B_2 and `NMEAS` is the row size of C_2.

`hinfsyn` uses a standard gamma-iteration technique to determine the optimal value of gamma. Starting with high and low estimates of gamma. The gamma-iteration is a bisection algorithm that iterates on the value of gamma in an effort to approach the optimal H_∞ control design. The stopping criterion for the bisection algorithm requires the relative difference between the last gamma value that failed and the last gamma value that passed be less than TOLGAM (default=.01), where TOLGAM is the relative error tolerance for `GAM`, see MATLAB help. One point to keep in mind is that in general, the solution to the infinity-norm optimal control problem is non-unique.

One can extend the model P to include weighting functions. These functions can be used to modify the output. This modification is aimed to penalize the desired signals at the desired frequencies. The extended model is shown by P_e and weighting functions by W in Figure 5. Weighting functions should be designed in a way to amplify the signals at frequency ranges where the magnitude is undesirably high. This will make the infinity norm sensitive to these frequency ranges and then minimization of γ leads to a controller that should make the output small at these frequencies. Optimization of γ is the same as running the following syntax for the extended model.

```
[Ke,CLe,GAMe] = hinfsyn(Pe,NMEASe,NCONe)
```

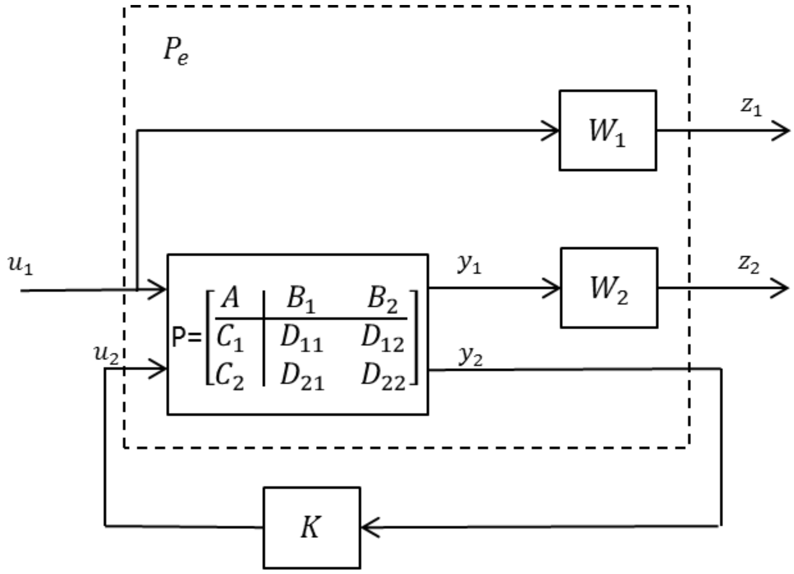

Figure 5. The extended open loop system P_e under control of K.

For example in the case of vibration of a single degree of freedom system it is desirable to reduce vibration at the natural frequency. One can penalize vibration signal with a transfer function that has a peak at the natural frequency of the system. This can be done through expanding the model to include the weighting function and applying `hinfsyn` to this new model. The weighting function penalizes vibration with natural frequency. The controller obtained through this method should be capable of controlling vibration with natural frequency.

The extended closed loop system is shown by P_{ec} in Figure 6.

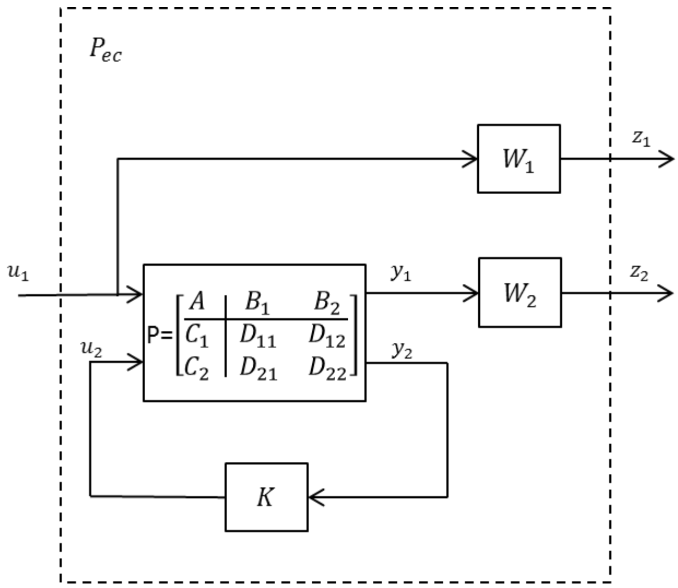

Figure 6. The extended close loop system P_{ec}.

Finally, to check the performance of the derived controller, K, it should be implemented in the control loop in Figure 4. In case the response is not satisfactory, one can change the weighing functions and look for a new controller. To learn more about the theory of H_∞ control, check [1]. For a more extensive discussion on implementation of H_∞ control, check MATLAB help.

2.3 Power spectral analysis

Track irregularities can be represented by power spectral densities (PSD). In power spectral analysis, power spectra of the vehicle response are calculated by multiplying power spectra of track irregularities with the frequency response function as follows

$$S_p(w) = |H|^2 . S_w(w) \tag{8}$$

where

w is angular frequency with dimension of $\frac{rad}{s}$

$S_w(w)$ is PSD of track irregularities

H is the transfer function between excitation and measurement point.

One has to be aware of the fact that measured power spectra of track irregularities usually are given with spatial circular frequency. Before multiplying them with the frequency response function, they have to be divided by the speed of the vehicle

$$S_w(w) = \frac{1}{v} S_w(w_{spatial}) \tag{9}$$

As example power spectra of the vertical track irregularity is given below for a standard track on a German railway network for the speed of 50 m/s.

$$S_w(w) = \frac{4.028 * 10^{-7}}{2.88 * 10^{-4} + 0.68w^2 + w^4} \tag{10}$$

3 Control of a single DOF system using PD, PID and Skyhook

In this part vibration of a single degree of freedom system will be studied.

In different parts of the exercise it will be required to study the response to different excitations of the base, by which we mean the following excitations:

Base excitations:

a. Sinusoidal excitation with amplitude of 0.05m and an arbitrary frequency e.g. the undamped natural frequency.
b. Impulse excitation with amplitude of 0.05 m and duration of 0.1 second.
c. Response to track excitation which is given in section 2.3 (Plot the PSD using the *semilogy* syntax)

Excitations a. and b. can be easily studied in Simulink environment. You can use the 'signal builder' block in Simulink to create the impulse signal. Excitation 'c' can be handled in MATLAB (as a suggestion you can use *syms* and *subs* syntaxes – make sure to plot the PSD with *semilogy* syntax and the interesting frequency range is 0-25 [rad/s]).

3.1 Damped passive system

- **Task 1.1:** Consider the single degree of freedom system shown in Figure 7 with the input disturbance $z_w(t)$. Write the equation of motion for mass m_p. Natural frequency and damping ratio of the system can be found by using the following relations.

$$w_n = \sqrt{\frac{k_p}{m_p}} \quad [\frac{rad}{s}] \qquad \zeta = \frac{c_p}{c_c} = \frac{c_p}{2\sqrt{k_p m_p}} \tag{11}$$

  Explain the physical interpretation of these two values. What is meant by underdamped, critically damped and overdamped systems? How do their responses to a step excitation differ?

- **Task 1.2:** Calculate w_n and ζ for the one degree of freedom system ($z_w(t)$ is the base excitation and hence is not a degree of freedom.) in Figure 7 considering the following parameters:

$m_p = 0.16 \quad kg$
$c_p = 0.4 \quad Ns/m$
$k_p = 6.32 \quad N/m$

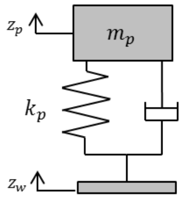

Figure 7. Single degree of freedom system. $z_w(t)$ is the input displacement.

- **Task 1.3:** Now derive the transfer function from base disturbance $z_w(s)$ to the displacement $z_p(s)$ of the mass m_p ($\frac{zp(S)}{zw(S)}$). Write the transfer function in form of natural frequency w_n and damping ratio ζ. Plot the Bode diagram of this transfer function and check the value of natural frequency on this plot. Compare the result with the bode diagram of the undamped system i.e. $c_p = 0$.

- **Task 1.4 (midway step):** Now study the response of this system to the different base excitations defined earlier in Section 3.
  - Additionally, study the **step response** considering 3 cases where $\zeta < 1$, $\zeta = 1$ and $\zeta > 1$ (this means that you should vary the value of c_p to achieve the right range for the damping ratio). Can you tell the difference between step response of underdamped, critically damped and overdamped systems?

Active control:
Now it is time to use a controller to control the vibration of the mass m_p as shown in Figure 8. The idea is to remove the damper and replace it with an ideal actuator to control vibrations of the mass. In the rest of this section you should calculate the required force for the actuator according to the three different control methods which are PD, PID and Skyhook.

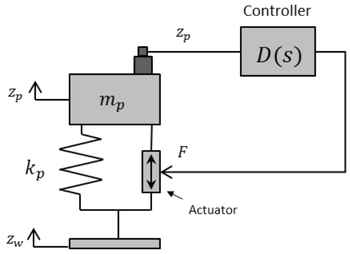

Figure 8. Single degree of freedom system with active control.

Response requirements: (on displacement)
- You should not change the undamped natural frequency of the system
- Response of the active suspension to sinusoidal excitation should show lower amplitude compared to the passive damped system.
- Impulse response of the system should
  a. show underdamped response (meaning that the response should oscillate before damping out)
  b. damp out faster than the 'damped passive system'
  c. have lower overshoot compared to 'damped passive system'

3.2 PD

In this part you will use a PD controller to control vibration of the system in Figure 8. The force calculated by this controller is

$$F(s) = D(s) * z_p(s) \tag{12}$$

and controller $D(s)$ is

$$D(s) = -(d_p + d_d s) \tag{13}$$

then the actuator force in time domain would be

$$F(t) = -d_p z_p(t) - d_d \dot{z}_p(t) \tag{14}$$

- **Task 2.1:** Derive equation of motion in time domain. Once you have found the equation of motion, use it to derive the transfer function $\frac{z_p(S)}{z_w(S)}$.

- **Task 2.2 (midway step):** After obtaining the transfer function, it is time to find the right values for d_p and d_d. Before searching for these two parameters, compare the denominators of the transfer function you just obtained with the one for the damped passive system. They are both of second order. We can get almost the same response from both of the transfer functions, if coefficients of denominator are the same. You can use this technique to get an approximate estimation of d_p and d_d.

- **Task 2.3 (midway step):** Once you have found these two parameters using the mentioned technique, you can further fine tune them to meet the response requirements to different base excitations. Keeping in mind that
    - the system should stay underdamped and
    - you should not change the undamped natural frequency of the system.

- ***Extra task 1**: Can you find the value of d_d for which the response is critically damped? (Systems that are critically or overdamped, show bumpy ride and are not very comfortable. On the other hand, systems with too little damping, damp out vibration very slowly which is not desirable either.)*

3.3 PID

In this section a PID controller will be used to control the vibrations of the mass m_p. In the case of PID, applied force is calculated by Equation (12) where $D(s)$, the controller is

$$D(s) = -(h_p + h_d s + \frac{h_i}{s}) \tag{15}$$

and actuator force in time domain would be

$$F(t) = -h_p z_p(t) - h_d \dot{z}_p(t) - h_i \int_0^t z_p\,(t) dt \tag{16}$$

- **Task 3.1 (midway step):** First derive equation of motion in time domain and then find the transfer function $\frac{z_p(S)}{z_w(S)}$ like previous part. Then try to find the right parameters for the PID controller. Try tuning parameters until you meet the response requirements to different base excitations. Keep in mind:

- h_p varies the natural frequency of the system and therefore it should have a small value.)
- At first, it may look that having a large h_i can be beneficiary. But this may cause a problem if the base excitation was a step function. What would be the problem? What is its physical meaning for the suspension system?

- ***Extra task 2:*** *Can you model this problem into Simulink by using separate blocks for the plant and the PID controller? What is the plant?*

3.4 Skyhook

Finally, the Skyhook controller will be used to control the vibration. For Skyhook controller the controller $D(s)$ is

$$D(s) = -T * s \tag{17}$$

- **Task 4.1 (midway step):** This is a simple and robust controller and the only parameter needs tuning is T. You can readily see that Skyhook controller has exactly the same structure as a *Derivative controller* (Read part 2.1 to get help on Skyhook physical meaning). Derive the equation of motion and transfer function as previous parts. Try tuning the parameter T until you meet the response requirements to different base excitations.

3.5 Chapter summary

- **Task 5.1:** Compare the **magnitude (phase diagram is not needed)** of the transfer functions for the five studied systems (undamped passive system, damped passive system, PD, PID and Skyhook - A successful active control shows lower transfer function magnitude compared to 'damped passive system'.
- **Task 5.2:** Compare also the responses to the 3 different base excitations for the damped system (for the case with $\zeta < 1$), PD, PID and Skyhook.
- **Task 5.3:** What is the problem with having a non-zero h_i in the PID control? (Hint: take a look at the step response)

4 Control of a two DOF system using Skyhook

The two degrees of freedom system shown in Figure 9 can be a simple model of a vehicle with two levels of suspension. Such a configuration is very common on a passenger train where the primary suspension connects the wheelset to the bogie and the secondary suspension connects the bogie to the carbody of the vehicle. Vibrations on the secondary mass, m_s, or vehicle carbody are of great importance as it is a measure of how comfortable the ride is.

Parameters of the system shown in Figure 9 are:

$m_p = 0.16 \quad kg$	$m_s = 0.16 \quad kg$
$c_p = 0.8 \quad Ns/m$	$c_s = 0.05 \quad Ns/m$
$k_p = 6.32 \quad N/m$	$k_s = 0.0632 \quad N/m$

4.1 Damped passive system

- **Task 6.1:** In this part you should first derive the equation of motion for the two degrees of freedom system shown in Figure 9. z_w is the base excitation and is the input to the system. Once you have found the equations of motions, use them to derive the transfer function from the base excitation to the displacement of the secondary mass i.e. $\frac{z_S(S)}{z_W(S)}$.

  Hint: write the equation of motion for the two masses and then transform them to Laplace space. This will give you an algebraic equation with two equations and two unknown as function of 's'. Use Cramer's rule to find the relation $\frac{z_S(S)}{z_W(S)}$ (To get help check chapter 5 of the reference [2] which is provided as supplementary material).

- **Task 6.2 (midway step):** Study the Bode diagram of the derived transfer function and the response to the excitations a. and b. as mentioned in Section 3.
- **Task 6.3:** Compare the amplitude of the bode diagram that you have obtained here with the bode diagram of section 3.1. What is the advantage of having two levels of suspension?

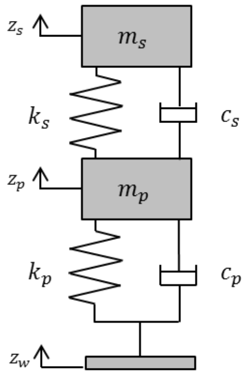

Figure 9. Two degrees of freedom system with passive suspension.

4.2 Skyhook

In the next step an actuator will be used in the secondary suspension to control the vibration of the secondary mass, m_s. In this section just the Skyhook controller will be studied as in Chapter 3 it was found that good PD and PID controllers are those with just the D part (i.e. Skyhook) being active (why?).

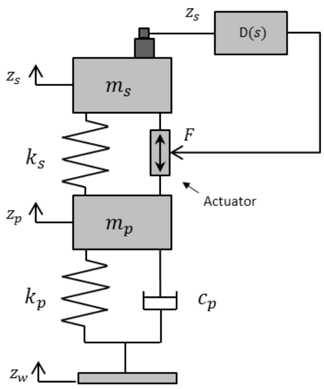

Figure 10. Two degrees of freedom system with active suspension.

- **Task 6.4:** Start with deriving the **state space model** (work with state space in this section instead of transfer function!) of the system shown in Figure 10 considering that F, z_w and $\dot{z}_w$ are inputs to the system and z_s is the output and states can be selected to be as follow:

$$X = \begin{bmatrix} z_s \\ \dot{z}_s \\ z_p \\ \dot{z}_p \end{bmatrix} \tag{18}$$

- **Task 6.5 (midway step):** Once you have derived the state space model of the system it should be easy to implement the system in Simulink and then study the system for excitations excitations a. and b. as introduced in Section 3. Try controlling the vibration of the mass m_s using a Skyhook controller. The controller $D(s)$ in Figure 10 for a Skyhook controller is

$$D(s) = -T * s \tag{19}$$

Parameter T should be selected so that the responses to excitations a. and b. introduced in Section 3 fulfil requirements mentioned in Section 3.1.

4.3 Chapter summary

- **Task 7.1:** Compare responses of the damped passive system and Skyhook controller to excitations a. and b. (Section 3) and try to fulfil the response requirements in Section 3.1.

5 Control of bounce and pitch for a simple vehicle model using Skyhook and H_∞

In this section the system is a simple vehicle model as shown in Figure 11 and the aim is to control the pitch (χ) and bounce (z) of the vehicle chassis. Mass of the chassis is shown by m and its moment of inertia by j.

In this section first you will derive the equation of motions for a passive system and then you will use Skyhook and H_∞ methods to control both pitch and bounce motions. Use the following values in your simulations:

$m = 22000 \quad kg$
$j = 700000 \quad kgm^2$

$c_1 = c_2 = c = 40000 \quad Ns/m$
$k_1 = k_2 = k = 600000 \quad N/m$

$L_1 = L_2 = L = 6 \quad m$

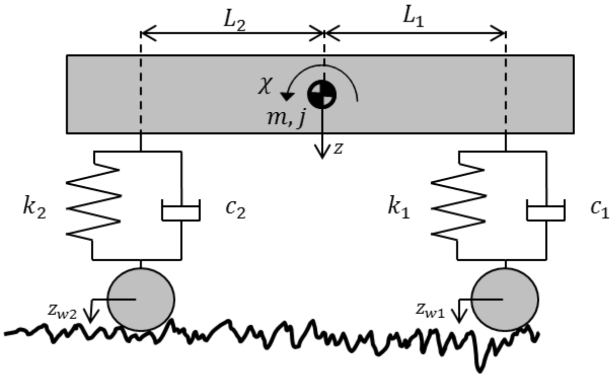

Figure 11. two degrees of freedom vehicle (bounce and pitch) – passive suspension.

5.1 Damped passive system

- **Task 8.1:** Derive both bounce and pitch equations of motion for the chassis shown in Figure 11 as function of parameters. Once you have found the equations of motion, try driving the state space model of the system where vectors of states, inputs and outputs should be as follow

$$X = \begin{bmatrix} z \\ \dot{z} \\ \chi \\ \dot{\chi} \end{bmatrix} \qquad U = \begin{bmatrix} z_{w1} \\ \dot{z}_{w1} \\ z_{w2} \\ \dot{z}_{w2} \end{bmatrix} \qquad Y = \begin{bmatrix} z \\ \chi \end{bmatrix} \tag{20}$$

- **Task 8.2 (midway step):** Once you have found the state space model, use Simulink to study the response of the model to the following excitations. Can you find the natural frequencies of the system?

Excitation 1:
Impulse

$$\begin{cases} z_{w1}(t) = 0.03 \ [m] & for\ 0 < t < 0.1, otherwise\ 0 \\ z_{w2}(t) = 0 & for\ -\infty < t < +\infty \end{cases}$$

Excitation 2:
Sinusoidal

$$\begin{cases} z_{w1}(t) = 0.01 * \sin(2\pi f t) & if\ t > 0, otherwise\ 0 \\ z_{w2}(t) = 0 & for\ -\infty < t < +\infty \end{cases}$$

where f is the frequency of excitation. Do the simulation for $f = 1\ Hz$ and $f = 8\ Hz$

5.2 Skyhook

Next, Skyhook controller should be used to control bounce and pitch. Results should be compared against those of the passive system.

In this section and next section we replace the dampers with actuators and use sensors and a controller. Thus the model will look as in Figure 12.

- **Task 9.1:** Derive equations of motion and state space model for this system. Vectors of states, inputs and outputs should be as follow:

$$X = \begin{bmatrix} z \\ \dot{z} \\ \chi \\ \dot{\chi} \end{bmatrix} \qquad U = \begin{bmatrix} z_{w1} \\ z_{w2} \\ F_{a1} \\ F_{a2} \end{bmatrix} \qquad Y = \begin{bmatrix} \dot{z} \\ \dot{\chi} \end{bmatrix} \tag{21}$$

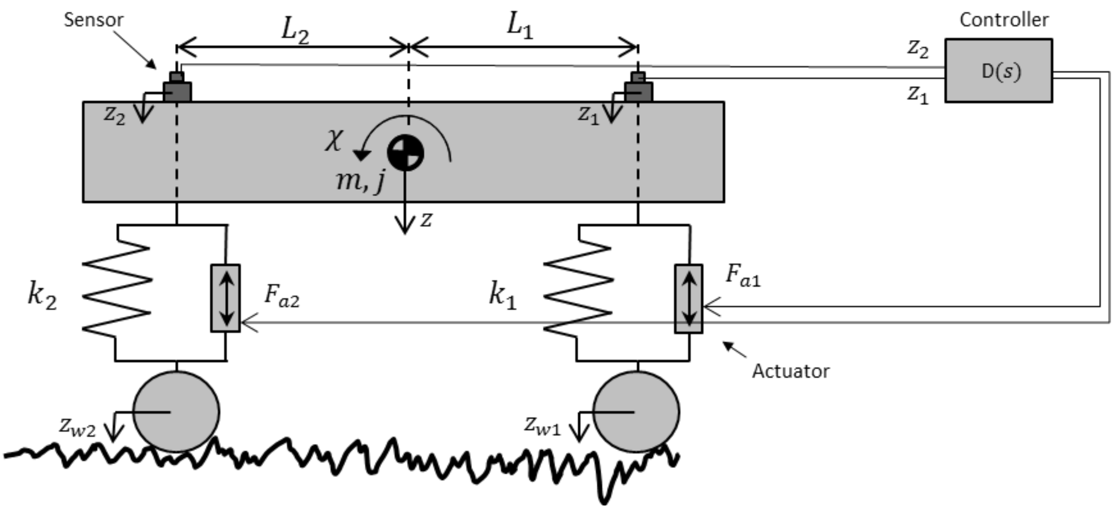

Figure 12. Two degrees of freedom system with the active suspension.

Once you have found the state space model, make the Simulink model. Then you need a controller based on Skyhook to calculate F_{a1} and F_{a2}. According to Skyhook principle, applied force or torque is proportional to velocity (output vector) which means:

$F_z = -c_z * \dot{z}$ **Force** which resists against bounce motion
$T_\chi = -c_\chi * \dot{\chi}$ **Torque** which resists against pitch motion

where c_z and c_χ are the Skyhook damping parameters.

- **Task 9.2:** To complete your Simulink model you need to find out the relation between F_z, T_χ and F_{a1}, F_{a2}. Derive this relation and implement it in Simulink. Reflect the derived relations in the report.

Now that your model is complete try to tune c_z and c_χ considering the excitations in Section5.1. Following are the response requirements.

Response requirements on $\boldsymbol{z}, \boldsymbol{\chi}$:

1- Response of the active suspension to sinusoidal excitation should show lower amplitude compared to passive damped system.
2- Response of the active suspension to impulse excitation
   a. show underdamped response (meaning that the response should oscillate before damping out)
   b. damp out faster than the 'damped passive system'
   c. have lower overshoot compared to 'damped passive system'

- **Task 9.3 (midway step):** Tune parameters c_z and c_χ and compare the results (χ and z) with the passive case. In reality it is not possible to choose very high values for c_z and c_χ, what is the limitation? (To be more realistic, you should try to limit the maximum absolute value of the force. A reasonable max value can be 10 kN.)

5.3 H_∞ Controller

Finally, H_∞ controller will be tested in this section and you will get a general idea of how this controller works. You will receive one MATLAB file and one Simulink file for this section. These files are named 'H_inf_ctrl.m' and 'Extended_model.mdl'.

The governing state space model is the same as the previous section. The aim is to control the output vector by using an H_∞ controller, K. To do so we need to design weighting functions and then form the extended plant, see section 2.2.

In this exercise we will use 4 weighting functions W_{a1}, W_{a2}, W_b and W_χ. The first two transfer functions will be the same and they penalize the controller output signals at high frequencies. Controller output signals are F_{a1} and F_{a2} (plot the bode diagram!).

$$W_{a1}(s) = W_{a2}(s) = \frac{1.75 * 10^{-3}s + 1}{2.5 * 10^{-4}s + 1} \tag{22}$$

High magnitude of these weighting functions at high frequencies mean that presence of these frequencies in F_{a1} and F_{a2} are penalized and discouraged. The reasons for penalizing high frequencies are:

1. The problem at hand is aimed for controlling two rigid body modes. These usually have low natural frequencies and therefore high frequency forces will be unnecessary.
2. Too high frequencies cut the effective life of actuators short.

W_b is the weighting function for the bounce motion, and is designed in a way to penalize the bounce motion. This transfer function can have the following structure.

$$W_b(s) = \frac{k_b s_1 s_2}{(s-s_1)(s-s_2)} \tag{23}$$

Where
k_b is the gain
$s_{1,2} = -\varepsilon \pm i\sqrt{w_{nb}^2 - \varepsilon^2}$ and ε is a small value which we select it to be 1.
w_{nb} is the frequency which should be penalized the most when considering bounce motion.

- **Task 10.1:** Calculate value of w_{nb}.

  W_χ is designed in the same way as W_b, but is used to penalize the pitch motion. Try finding the frequency which should be penalized for pitch motion, $w_{n\chi}$.

Gains for W_b and W_χ are not given, and you should try to find the proper gains yourself. Gains should be selected in a way that the designed controller keeps the system stable and provides reasonable performance compared to the other two solutions (passive and Skyhook). Start by selecting some initial values for k_b and k_χ (as a guide: k_b and k_χ can be of order 10^3 and 10^4 respectively). Once the gains are selected the extended model can be calculated as follows.

To find the extended system, we use a MATLAB syntax, `linmod`. This syntax extracts continuous- or discrete-time linear state-space model of the system around its operating point. The system can be a Simulink model of the extended system. So we form the extended model as shown in Figure 13 (refer to: 'Extended_model.mdl') and then the `linmod` syntax is used as follows

```
[A_P,B_P,C_P,D_P] = linmod('Extended_model').
```

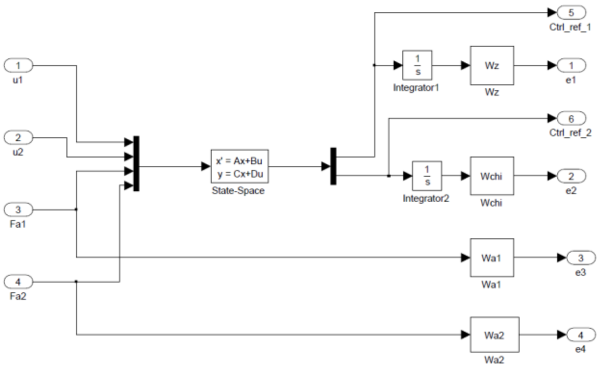

Figure 13. Extended system in Simulink.

Having the extended model, one can use `hinfsyn` to calculate the controller K. Use the provided MATLAB file to do the calculations. Once you have found the controller, implement it together with your vehicle model and study its response to **Excitation 1** defined in section 5.1. It might be hard to make this controller perform better than Skyhook, therefore a response better than the passive vehicle is considered good enough. If results are not satisfactory then you should repeat the procedure from the beginning, i.e. you should update the gains and then search for a new controller.

- **Task 10.2 (midway step):** Once a suitable controller is found study the system response to excitations defined in Section 5.1 and try to fulfil the response requirements in Section 5.2 (To be more realistic, you should try to limit the maximum absolute value of the force to 10 kN). Furthermore, plot the **magnitude** of all the weighting functions.

5.4 Chapter summary

- **Task 11.1:** Compare responses of the passive system, Skyhook and H_∞ to excitations defined in Section 5.1 and try to fulfil the response requirements in Section 5.2. (To be more realistic, you should try to limit the maximum absolute value of each actuator force to 10 kN).

H_∞ is a model based controller i.e. for obtaining the controller you need to have mathematical model of the plant. So it may be asked that what will happen if the parameters of the plant change. In our example of the vehicle, weight of the vehicle can vary due to loading and unloading of the passengers. Besides stiffness and damping parameters may also change due to temperature or aging.

- **Task 11.2:** Study the **impulse response** of the model with the original controller and vary one arbitrary parameter of the system within 15% tolerance. (Parameters you may consider are mass-inertia, damping and stiffness. Choose one as you wish!)
  - Is the system still stable?
  - Is absolute value of force still below 10 kN?
  - What challenge do you see with H_∞ control that skyhook does not have?

- ***Extra task 3:*** *Repeat task 11.2, but this time change the three parameters simultaneously to an arbitrary degree inside a 15% tolerance of the original values. Answer same questions as in task 11.2.*

6 Examination

For details on the grading process of this course please consult the SD2231_Grading_Criteria.pdf document.

7 Bibliography

[1] T. Glad and L. Ljung, Control Theory Multivariable and Nonlinear Methods.

[2] E. Andersson, M. Berg and S. Stichel, Rail Vehicle Dynamics, 2007.