Lab 3: Group 9

SD2231 – Applied vehicle dynamics control

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1 Introduction

The following study is aimed to understand and present different systems response in vehicle vibrations control. It is intended to study how vibrations affect the system behaviour and the effect on passengers' comfort, so it is crucial to keep the vibration level as low as possible. It is introduced a suspension subsystem that reduces the noise coming from the road and increases the comfort for the passengers. The suspension subsystem is a passive system made of one spring and one damper and it can be improved by adding an active control. This study will focus on the response of a simple passive suspension subsystem under different excitations, and it will be implemented different active controllers to better manage the vibrations understanding the effectiveness of an active controller in a suspension subsystem. Furthermore, it will be studied using a more complex two degrees of freedom system to study two different motions of the entire vehicle.

2 Control of a single DOF system using PD, PID and Skyhook

In this section, a single degree of freedom suspension system is analysed when subjected to vibrations. Figure 1 shows the analysed system:

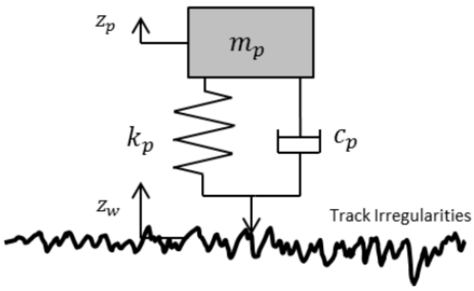

Figure 1. 1 DOF suspension model.

Where m_p is the vehicle mass, k_p the spring stiffness and c_p the damping coefficient. The only degree of freedom is the mass vertical motion z_p as function of the input disturbance z_w coming from the road.

2.1 Task 1.1: Damped passive system

Once the model is presented, it is directly possible to derive the equation of motion of the single mass system by studying its free body diagram:

$$m_p\ddot{z}_p + c_p(\dot{z}_p - \dot{z}_w) + k_p(z_p - z_w) = 0 \tag{1}$$

As visible from equation (1), the inertia force is proportional to the mass acceleration, the damping force to the relative speed between mass and road and the spring force to the relative displacement between mass and road. Once defined the equation of motion in time domain, it is time to introduce the natural frequency and the damping ratio of the system respectively in equation (2) and (3):

$$\omega_n = \sqrt{\frac{k_p}{m_p}} \tag{2}$$

$$\zeta = \frac{c_p}{c_c} = \frac{c_p}{2\sqrt{k_p m_p}} \tag{3}$$

Equation (2) defines the natural frequency of the system, being the frequency at which a system tends to oscillate in absence of external forces. In case an external force is acting on the system, at a frequency close to the natural one, the system reaches a resonance condition in which the oscillations are enlarged in amplitude with respect to other frequencies, resulting in an extremely uncomfortable status for the passengers. The damping ratio, defined in equation (3) as the ratio between the actual damping coefficient of the system and its critical damping: condition at which the system returns in its equilibrium position as quickly as possible without oscillating [2]. Depending on the damping coefficient, the system responds to the force in different ways; table 1 shows the correspondence between damping coefficient and system response:

Table 1. System response and damping ratio

Damping ratio	System response
$0< \zeta <1$	Underdamped
$\zeta = 1$	Critically damped
$\zeta > 1$	Overdamped

An underdamped suspension system tends to overshoot the equilibrium position dissipating energy and returning to the equilibrium position eliminating the oscillations smoothly. In case of a critically damped system, as priorly mentioned, the system returns as quickly as possible in the equilibrium position without oscillating and transferring all the energy to the passengers risking injuries. In case of overdamped suspension subsystems, the system tends to the equilibrium position without reaching overshoot and hence without dissipating much energy, increasing the passengers discomfort with respect to an underdamped system. The prediction of the systems behaviour is confirmed with figure 2, where it is visible the different systems response under a step input excitation of 0.05 m of amplitude:

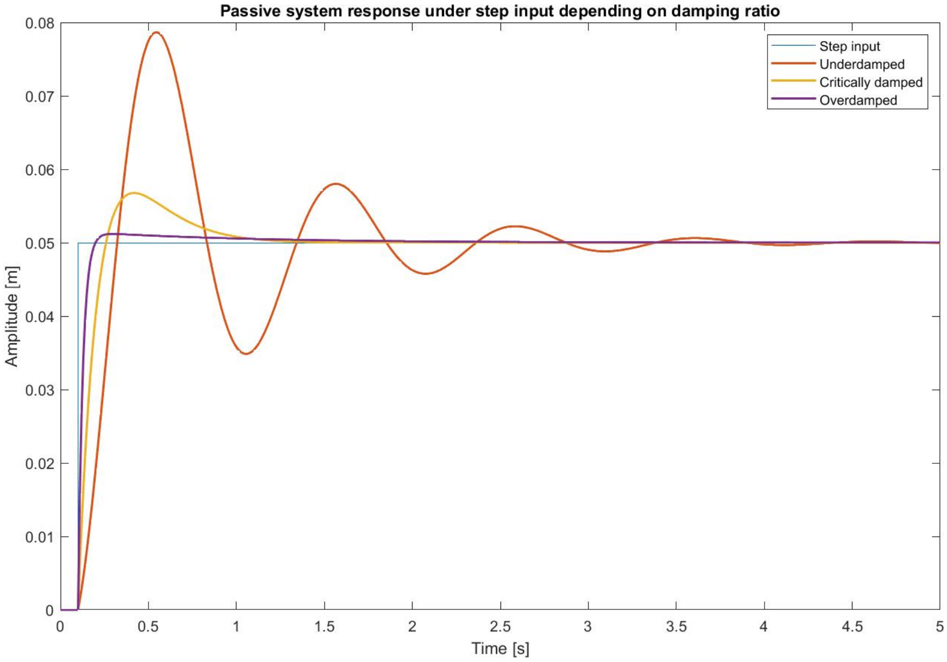

Figure 2. Step input systems response: Underdamped (red), Critically damped (yellow) and overdamped system (purple).

2.2 Task 1.2: Natural frequency and damping ratio

This section is focused on the computation of the natural frequency and damping ratio for the single degree of freedom system. The following parameters are used for the computation:

Table 2. System parameters

Mass	$m_p = 0.16$	[kg]
Damping coefficient	$c_p = 0.40$	[Ns/m]
Spring stiffness	$k_p = 6.32$	[N/m]

By substituting the parameters in equation (2) and (3), the system natural frequency and damping ratio are unequivocally determined:

Table 3. System natural frequency and damping ratio

Natural Frequency	$\omega_n = 6.2849$	[rad/s]
Damping ratio	$\zeta = 0.1989$	[-]

The results obtained suggest that the system is underdamped, to better dissipate the energy while returning to the equilibrium position aiming to increase as much as possible the passengers comfort.

2.3 Task 1.3: Damped passive system transfer function

It is now intended to derive the system transfer function in Laplace domain from the base disturbance z_w (s) to the mass displacement z_p (s). In equation (4) it the transfer function is defined as function of the vehicle parameters, while equation (5) introduces the transfer function depending on the system characteristics (natural frequency and damping ratio):

$$H(s)_{z_w(s)}^{z_p(s)} = \frac{c_p s + k_p}{m_p s^2 + c_p s + k_p} \tag{4}$$

$$H(s)_{z_w(s)}^{z_p(s)} = \frac{(2\zeta\omega_n^2)s + \omega_n^2}{s^2 + (2\zeta\omega_n^2)s + \omega_n^2} \tag{5}$$

Once defined the transfer function for the damped passive system, the undamped passive system transfer function is immediately obtained by setting the damping coefficient equal to zero, as reported in equation (6):

$$H(s)_{z_w(s)}^{z_p(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2} \tag{6}$$

Once the two systems are defined in the Laplace domain, it is possible to derive the Bode plot that shows the magnitude of the system oscillation as a function of the frequency response in decibels. Figure 3 shows the Bode plot for the damped and the undamped passive systems:

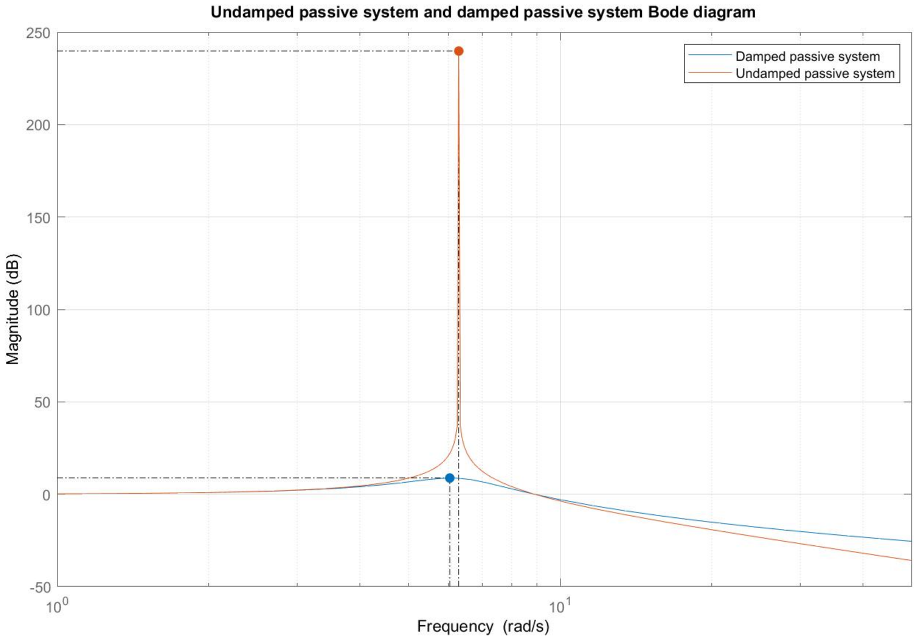

Figure 3. Magnitude of system frequency response for damped (blue line) and undamped (red line) systems.

Figure 3 shows the magnitude Bode diagram for the damped and undamped passive systems. The plot can be analysed by dividing it in three main regions according to the system response as a function of the excitation frequency. In the first region, the output of the system is equal to the input of the system as the magnitude of the system response is equal to zero. The second region, close to the natural frequency is where the oscillations are amplified and it is visible how the peak in oscillation amplitude occurs at a frequency equal to 6.03 rad/s, which is close to the undamped system natural frequency. With the introduction of the damper, the system response is extremely smoothened passing from a peak magnitude gain of 240 dB in the case of undamped system to the 8.77 dB measured in the damped system that allows the full understanding of the effectiveness of the damper in a suspension subsystem to reduce the oscillations and increase the occupants comfort. In the last area, the oscillations are reduced as visible by the negative magnitude that allows the increase of comfort for the vehicles.

2.4 Task 2.1: PD-controlled system

Having concluded the study of the passive system, it is time to include an active controller to control the vibrations coming from the road. Here it is implemented a PD-controller that aims to use a proportional and a derivative gain to fulfil the vibration control. Once the controller is introduced in the system, one can define the equation of motion in time domain:

$$m\ddot{z}_p + k_p(z_p - z_w) + d_p z_p + d_d \dot{z}_p = 0 \tag{7}$$

Where the passive damper is replaced by an active controller that relies on a proportional gain (d_p), proportional to the mass displacement, and a derivative gain (d_d) proportional to the mass speed. With the equation of motion is possible to derive the transfer function in Laplace domain from the base disturbance $z_w(s)$ to the mass displacement $z_p(s)$ as done in equation (8):

$$H(s)_{z_w(s)}^{z_p(s)} = \frac{\omega_n^2}{s^2 + \frac{d_d}{m_p} \cdot s + \frac{d_p}{m_p} + \omega_n^2} \tag{8}$$

It is possible to substitute the PD-controller with other controllers such as the PID-controller and the Skyhook-controller to study the different response of the system and compare the results in terms of vibrations, aiming to find the optimal controller to increase passengers comfort. For each controller it is necessary to derive a new transfer function as the actuator force is strictly related to the controller implemented in the system. Here for completeness are introduced the transfer functions for the PID-controller, equation (9), and for the Skyhook controller, equation (10):

$$H(s)_{z_w(s)}^{z_p(s)} = \frac{\omega_n^2 \cdot s}{s^3 + \frac{h_d}{m_p} \cdot s^2 + (\frac{h_p}{m_p} + \omega_n^2) \cdot s + \frac{h_i}{m_p}} \tag{9}$$

$$H(s)_{z_w(s)}^{z_p(s)} = \frac{\omega_n^2}{s^2 + \frac{T}{m_p} + \omega_n^2} \tag{10}$$

2.5 Extra task 1. PD-controller critical damping

Here it is intended to arithmetically determine the derivative gain of the PD-controller that provokes a critical damping condition. To do so, it is substituted in equation (3) the actuator coefficient and the damping ratio is set equal to one to univocally determine the derivative gain that allows a critical damping behaviour:

$$d_d = 2\sqrt{(k_p + d_p)m_p} = 2.0506 \tag{11}$$

Where d_d and d_p are respectively the derivative and proportional gain in the PD-controller, k_p the spring stiffness and m_p the mass of the vehicle. The value obtained here is computed with the optimised proportional gain of the PD-controller and the standard spring stiffness and mass values.

2.6 Extra task 2. PID-controller

In this section it is aimed to define the PID-controlled system by creating separate blocks for the controller and for the plant aiming to obtain the same response as the one obtained by using a single transfer function that takes care of the entire system. Figure 4 shows the Simulink schematics needed to separate the controller from the plant:

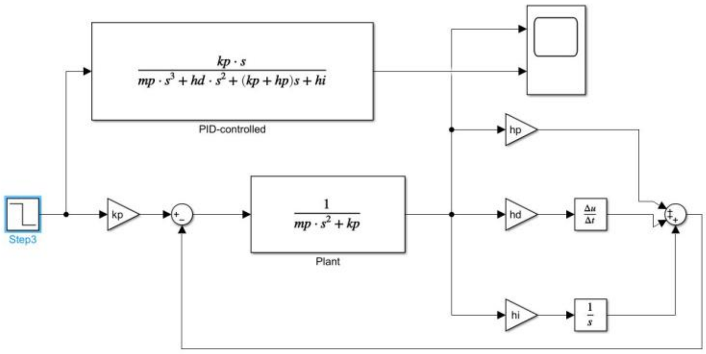

Figure 4. Simulink schematic for plant and PID-controlled system.

In figure 4, the upper block shows the transfer function for a system controlled by a PID-controller, in the lower section is visible how the system is divided into plant and controller. The plant transfer function describes a passive undamped system, as the active controller is separated from it. The PID-controller is then built by using separate branches that define the proportional (top branch), the derivative (middle branch) and the integral gain (lower branch). It is then needed to add a gain to the input impulse function equal to the spring stiffness to dimensionally get a force that is subtracted by the actuator force, output of the controller. After building the system, it is time to compare the results by analysing the response of the PID-controlled active system as a unique transfer function and the system made of plant and controller separately:

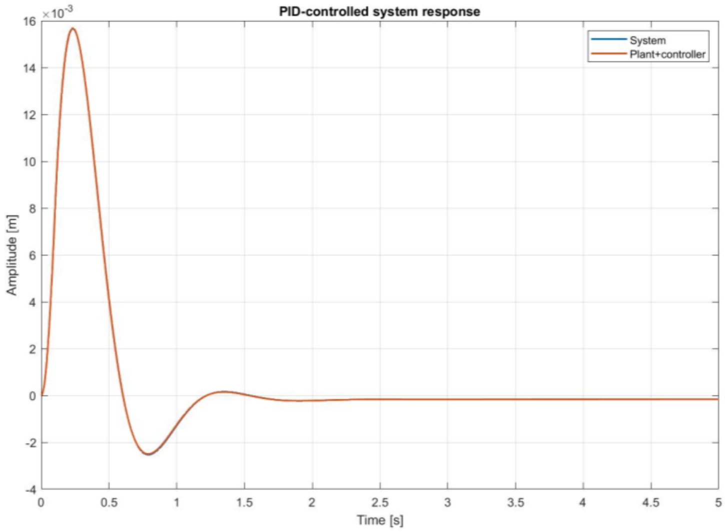

Figure 5. Step impulse excitation on the full system (blue) and plant+controller system (red).

From figure 5 it is visible how separating the plant from the controller gives the same exact result as considering the whole system in the transfer function. The advantage of splitting can be to define a simpler transfer function block to define the plant, whereas the transfer function that defines the whole system is much more complex.

2.7 Task 5.1: transfer function magnitude

To understand the effectiveness of the control systems it is beneficial to plot the magnitude of the system response as a function of the frequency. It is understandable how an active system should provide lower magnitude compared to a damped passive system. To have a full comprehension of the different active controllers response, it is decided to plot 2 passive systems: one undamped and one damped and 3 active systems controlled by a PD-controller, a PID-controller, and a Skyhook-controller respectively. Figure 6 shows the magnitude bode plot for the systems considered:

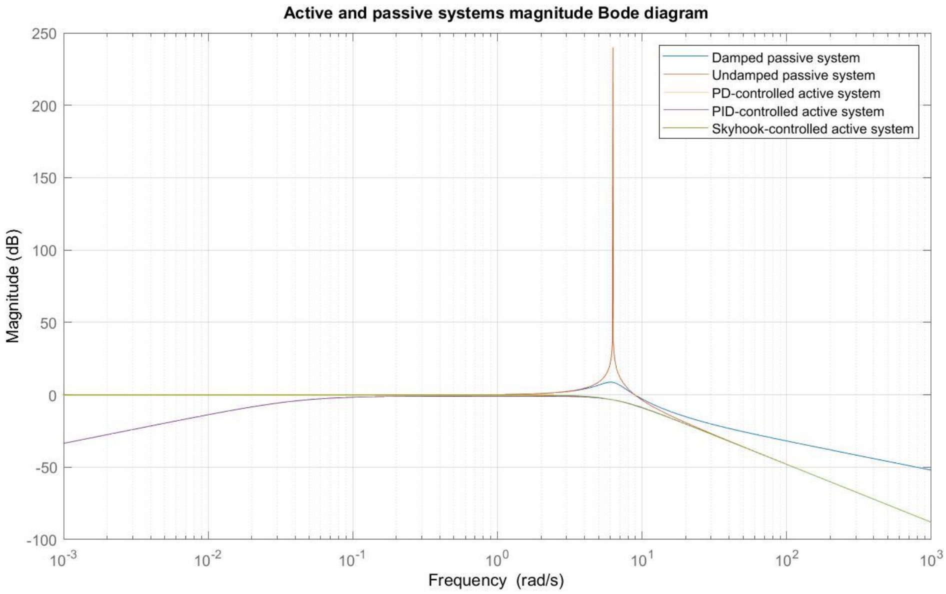

Figure 6. Magnitude bode plot: Damped passive system (blue), undamped passive system (red), PD-controlled system (yellow), PID-controlled system (purple) and Skyhook-controlled system (green).

As visible in figure 6, the active systems respond better to oscillations than the passive systems. This is clearly visible by analysing the oscillations peak magnitude in the resonance region, that is way lower in case of the implementation of active controllers. Moreover, at higher frequencies, the active systems act with a larger negative gain that is helpful to reduce the oscillations, whereas the damped passive system is less performant in this region. Having a closer look, the PD- and the Skyhook-controller system exploits the same behaviour, while the PID-controller tends to a negative gain in the low frequency region that may negatively affect the performances of the suspension.

2.8 Task 5.2: Systems response under different excitations

To have a full understanding of the systems behaviour, it is important to plot the systems response under different excitations. Here three different conditions are considered: a step impulse excitation with amplitude of 0.05 metres and of the duration of 0.1 seconds, a sinusoidal excitation with amplitude 0.05 metres at the undamped natural frequency, and lastly a road excitation modelled with the power spectral density function. Figure 7 shows the five systems response under the step impulse excitation:

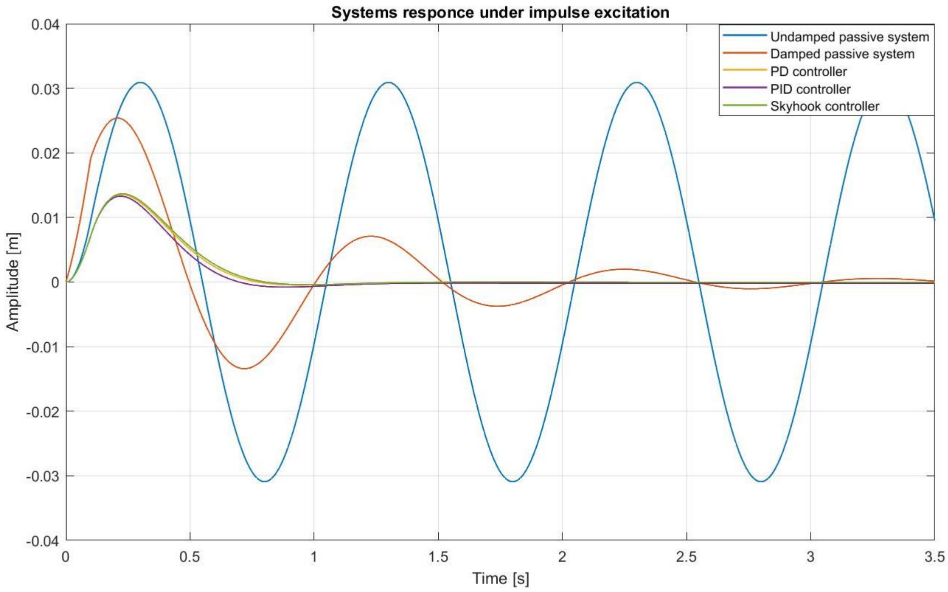

Figure 7. Step impulse excitation: Damped passive system (blue), undamped passive system (red), PD-controlled system (yellow), PID-controlled system (purple) and Skyhook-controlled system (green).

Figure 7 shows the systems response under step input excitation where the amplitude of the oscillation is plotted against time. It is decided to study the systems response for 3.5 seconds, to allow the passive undamped system to almost reach steady state condition to compare the results with respect to the active systems. As predictable, the undamped passive system does not smooth the oscillation in time and responds by keeping the oscillation amplitude constant between 0.03 and -0.03. For what concerns the active controlled systems, all of them exploit a smaller settling time with respect to the damped passive system, meaning that the passengers are exposed for a shorter time to oscillations improving the comfort. For each active controller, it is visible a smaller overshoot before reaching steady state condition, meaning that each system analysed is in underdamped conditions, allowing smooth energy dissipation to improve the passengers comfort.

Figure 8 shows the systems response under sinusoidal excitation:

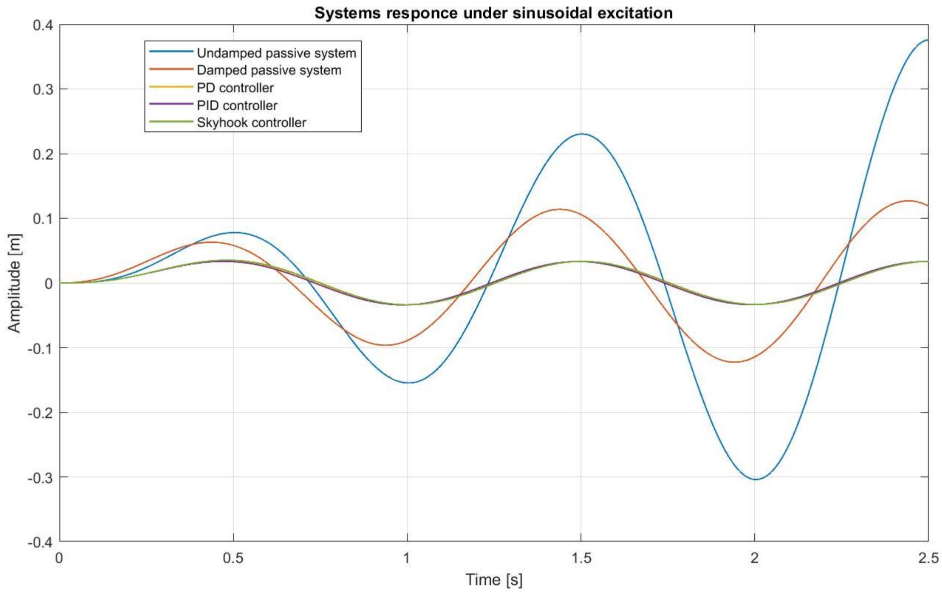

Figure 8. Sinusoidal excitation: Damped passive system (blue), undamped passive system (red), PD-controlled system (yellow), PID-controlled system (purple) and Skyhook-controlled system (green).

When exposed to a sinusoidal excitation, the undamped system tends to increase the amplitude of the oscillation as the time passes as the applied force is at the natural frequency and there is no damper to dissipate the oscillations. All three active systems provide a better response with respect to the passive damped system as the frequency amplitude is reduced with the implementation of active controllers keeping the magnitude of the oscillation close to zero. Figure 9 represents the systems response under road excitation:

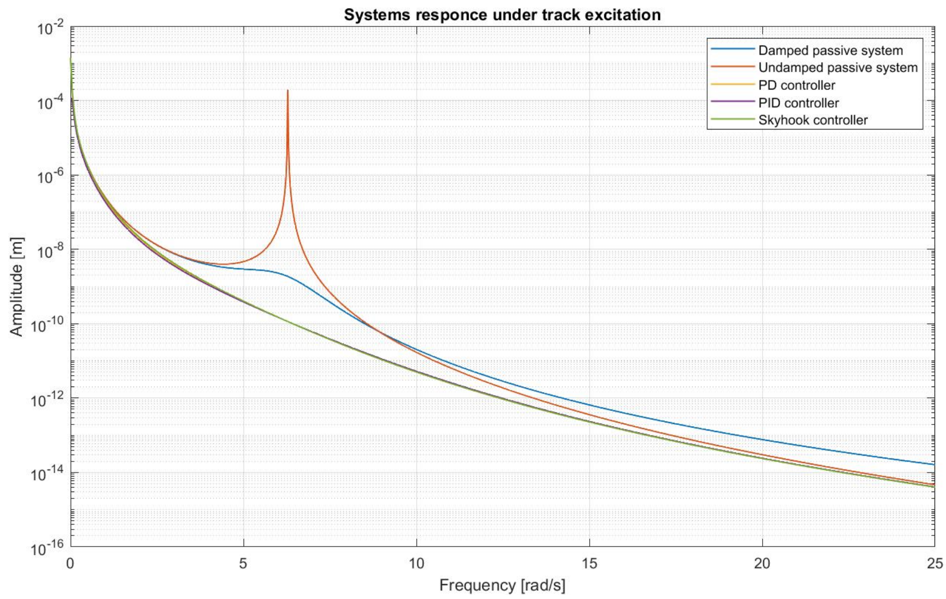

Figure 9. Track excitation: Damped passive system (blue), undamped passive system (red), PD-controlled system (yellow), PID-controlled system (purple) and Skyhook-controlled system (green).

Figure 9 shows the different systems under track excitation, as visible, the active systems ensure a lower excitation in the entire frequency domain with respect to the passive systems meaning that the controllers have a positive impact on the system particularly in the natural frequency region so that resonance is avoided.

2.9 Task 5.3: Integral gain effect

This section aims to point out the problems related to the integral gain in the PID-controlled active system. To do so, it is helpful to study the systems behaviour under a step response, as shown in figure 10:

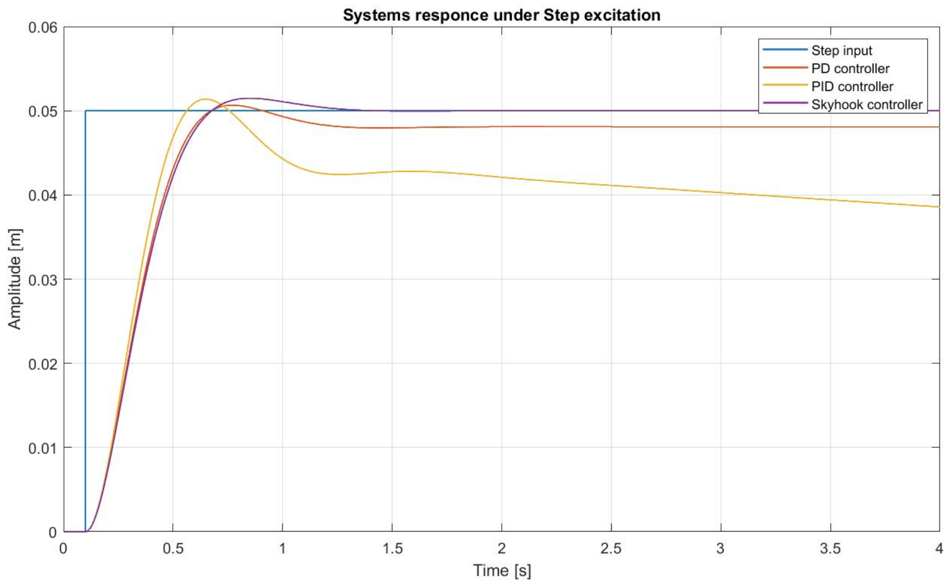

Figure 10. Step input excitation: input (blue), PD-controlled system (red), PID-controlled system (yellow) and Skyhook-controlled system (purple).

As already predicted with the analysis of the Bode plot, the integral gain causes the system to react at extremely low frequencies with a negative gain resulting in an increase of the steady state error with respect to the step input. One can affirm that the Skyhook-controlled system is the one that ensures the best behaviour between the three active systems as it cancels the steady state error, whereas the PD-controlled system conserves a small error due to the proportional gain.

3 Control of a two DOF system using Skyhook

The two degrees of freedom suspension subsystem can be modelled as a two-level suspension as visible in figure 11:

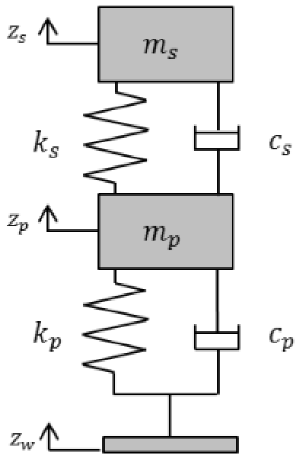

Figure 11. Two degrees of freedom system with passive suspension.

Where m_p is the unsuspended mass, m_s the suspended mass, z_p the displacement of the unsuspended mass and z_s the displacement of the suspended mass. Table 4 shows the parameters adopted to model the system:

Table 4. System parameters.

Unsuspended	**Suspended**
m_p = 0.16 [kg]	m_s = 0.16 [kg]
c_p = 0.8 [Ns/m]	c_s = 0.05 [Ns/m]
k_p = 6.32 [N/m]	k_s = 0.0632 [N/m]

As visible from the table, the suspended spring stiffness, that models the suspension stiffness, has a lower stiffness compared to the unsuspended one to ensure a larger displacement and energy dissipation than in the unsuspended spring that models the tyre stiffness.

3.1 Task 6.1: Damped passive system transfer function

To define the two degrees of freedom transfer function, it is fundamental to define firstly the two equations of motions in time domain that define the motion of the suspended and unsuspended mass. Equation (12) and equation (13) introduce suspended and unsuspended mass equation of motion respectively in time domain:

$$m_s\ddot{z}_s + c_s(\dot{z}_s - \dot{z}_p) + k_s(z_s - z_p) = 0 \tag{12}$$

$$\begin{aligned} m_p\ddot{z}_p + c_p\left(\dot{z}_p - \dot{z}_w\right) + k_p\left(z_p - z_w\right) \\ - c_s\left(\dot{z}_s - \dot{z}_p\right) - k_s(z_s - z_p) = 0 \end{aligned} \tag{13}$$

Where z_w is the excitation coming from the road. To find the transfer function, it is firstly derived the equations of motion in matrix form and then it is computed the transfer function from the road displacement to the suspended mass one. For sake of space, here only the final expression is reported in equation (14):

$$\begin{aligned} & H(s)_{z_w(s)}^{z_s(s)} \\ & = \frac{(c_p \cdot c_s)s^2 + (c_p \cdot k_s + c_s \cdot k_p)s + k_p \cdot k_s}{(m_p m_s)s^4 + \left(m_s\left(c_p + c_s\right) + m_p c_s\right)s^3 + \left(m_s\left(k_p + k_s\right) + m_p k_s + c_p c_s\right)s^2 + \left(c_p k_s + c_s k_p\right)s + k_p k_s} \end{aligned} \tag{14}$$

The transfer function is extremely more complex as the one for the single degree of freedom due to the relationship between the two masses that increases to the fourth order the transfer function.

3.2 Task 6.3: 2 DOF vs single DOF passive system

In this section it is intended to compare the two-levels passive suspension subsystem with the single level one by the analysis of the oscillations magnitude as function of the frequency. To do so, figure 12 represents the Bode diagram of the two suspensions subsystems:

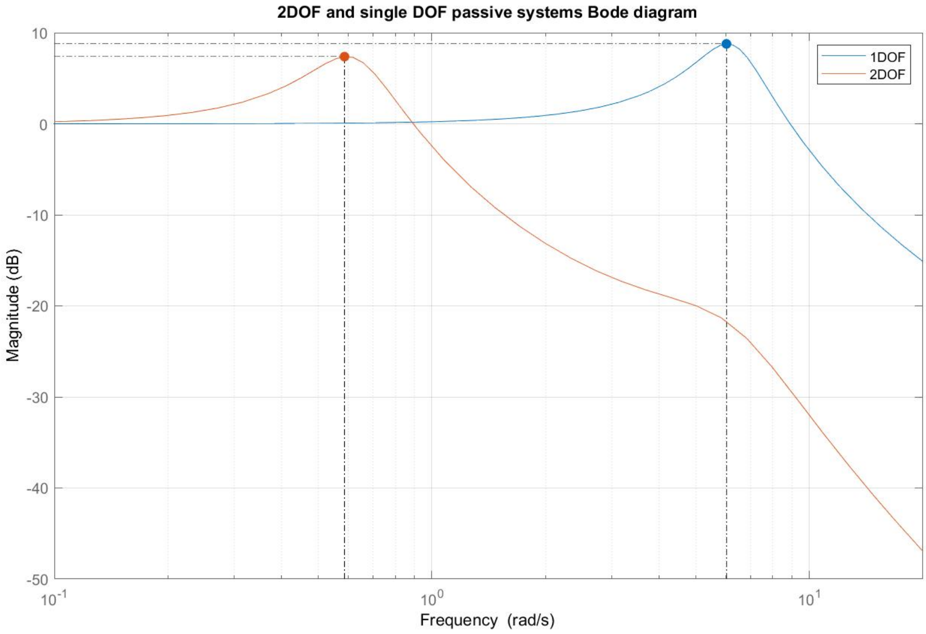

Figure 12. Bode plot for single DOF system (blue) and double DOF system (red).

As visible from figure 12, the 2-level suspension ensures a filtering of the disturbances coming from the road at much lower frequencies than the single level suspension, aiming to define a more sophisticated model. In this way, the resonance region is shifted towards a lower frequency region and the peak of amplitude is slightly reduced with respect to the single level suspension. Moreover, analysing the behaviour at higher frequencies, the 2-level suspension ensures a better suppression of the disturbances than the single level one, visible by the larger negative gain.

3.3 Task 6.4: Skyhook-controlled 2DOF system

The sky-hook controller is embedded in the two degrees of freedom system to actively control the oscillations and increase the comfort for the passengers. Being the transfer function complex to determine, it is more convenient to work with the steady state representation, where the set of differential equations is written in frequency domain. To define a two DOF suspension system controlled by a Skyhook controller, it is convenient to define the inputs as: the actuator force, the excitation displacement (z_w) and the excitation speed ($\dot{z}_w$) and as output the suspended mass displacement (z_s). In this way, by considering as state variables the two masses displacements and speeds (z_s, $\dot{z}_s$, z_p, $\dot{z}_p$) the space model can be unequivocally defined as follows:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_s} & 0 & \frac{k_s}{m_s} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_p} & 0 & -\frac{k_s+k_p}{m_p} & -\frac{c_p}{m_p} \end{pmatrix} B = \begin{pmatrix} 0 & 0 & 0 \\ -\frac{1}{m_s} & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m_p} & \frac{k_p}{m_p} & \frac{c_p}{m_p} \end{pmatrix} C = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} D = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \quad (15)$$

The A matrix represents the state variables equations, being in the first two rows the suspended mass displacement (z_s) and speed ($\dot{z}_s$) respectively, and the last two rows the unsuspended mass displacement (z_p) and speed ($\dot{z}_p$). Matrix B and C define the input and output equations respectively, as visible the system gives the output only for the suspended mass displacement. The D matrix is the feed-forward matrix and here each position is set to zero.

3.4 Task 7.1: Skyhook-controlled 2DOF vs passive system

Here is intended to analyse the systems response under two different excitations: a step impulse excitation with amplitude of 0.05 metres and of the duration of 0.1 seconds, a sinusoidal excitation with amplitude 0.05 metres at the undamped natural frequency as already done for the single degree of freedom system. Figure 13 shows the two systems response under step impulse:

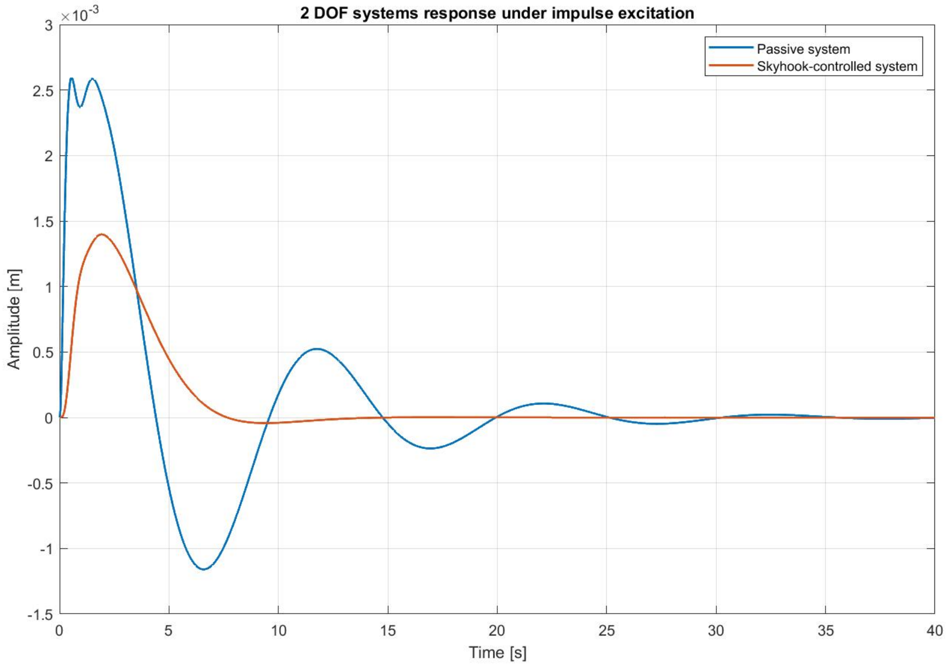

Figure 13. Impulse excitation systems response: passive damped system (blue), skyhook-controlled system (red).

After tuning the Skyhook parameter T, it is reached a satisfactory result in terms of system response. The effect of the controller is clearly visible in figure 13, where the active controller reaches the steady state condition after 12 seconds whereas the passive damped system requires more than 40 seconds, considerably reducing the time at which the passengers are exposed to oscillations. Moreover, the amplitude of the response is strongly reduced, meaning that the active controller filters the oscillations better, increasing the comfort. The active system is once again designed to be underdamped; this is clearly visible by the overshoot (negative oscillation) before reaching the steady state condition.

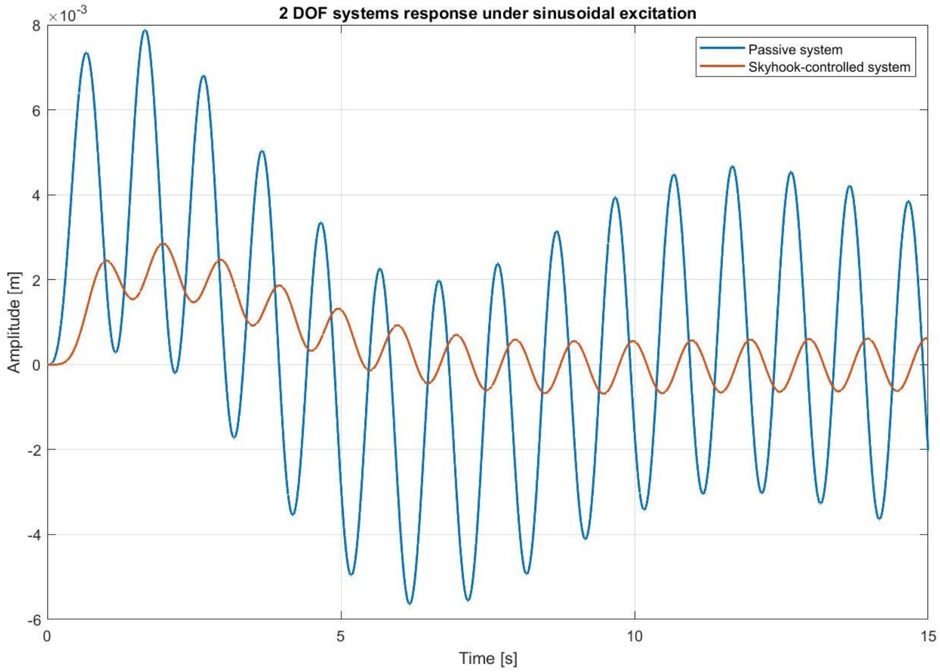

Figure 14. Sinusoidal excitation systems response: passive damped system (blue), skyhook-controlled system (red).

For what concerns the system response under sinusoidal excitation, figure 14 shows how the oscillation amplitude is reduced with the introduction of the active controller. After 10 seconds, the Skyhook-controlled system settles and oscillates around the zero following the input excitation, whereas the passive system requires more time to settle down and exploits larger oscillation peaks that affect the passengers comfort. Having analysed the tuned Skyhook-controlled system response under different excitations, it is possible to affirm that the tuning is extremely effective, and the active controller improves the performances of the suspension system in terms of comfort.

4 Control of bounce and pitch for a simple vehicle model using Skyhook and H∞

To analyse both pitch and bounce motion, it is required to introduce a new model that helps in deriving the equations of motions, starting point for the implementation of the model on Matlab.

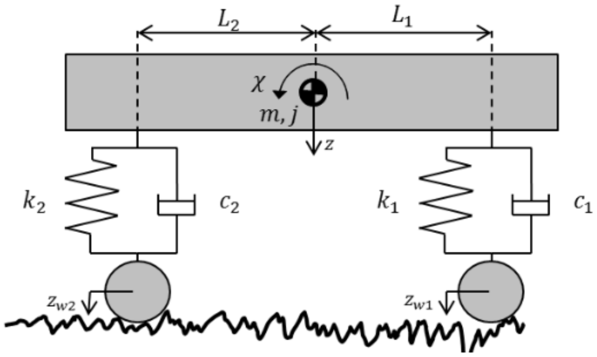

Figure 15. Two degrees of freedom vehicle (bounce and pitch) passive suspension model.

Figure 15 shows the side view of a suspension model, from which it is possible to study the free body diagram for defining the state space model of the system. The two degrees of freedom analysed are bounce, vertical motion along the z axis, and pitch, rotation around the centre of gravity. For the passive system, the following parameters are adopted:

Table 5. Model parameters

Vehicle mass	m = 22000 [kg]
Vehicle inertia moment	j =7e5 [kgm^2]
Spring stiffness	$k_1 = k_2$= 6e5 [N/m]
Damping coefficient	$c_1 = c_2$ = 4e4 [Ns/m]
CoG distance from the axles	$L_1 = L_2$ = 6 [m]

4.1 Task 8.1: Damped passive system

The damped passive system introduced in this section aims to study two different outputs, being the vertical displacement of the mass (bounce) and the rotation about the centre of gravity (pitch). To derive the equations of motion it is convenient to study the free-body diagram, where equation (16) defines the bounce motion and equation (17) the pitch motion:

$$m\ddot{z} + c_1(\dot{z} - \dot{\chi}L_1 - \dot{z}_{w1}) + c_2(\dot{z} + \dot{\chi}L_2 - \dot{z}_{w2}) + k_1(z - \chi L_1 - z_{w1}) + k_2(z + \chi L_2 - z_{w2}) = 0 \quad (19)$$

$$J\ddot{\chi} - c_1L_1(\dot{z} - \dot{\chi}L_1 - \dot{z}_{w1}) + c_2L_2(\dot{z} + \dot{\chi}L_2 - \dot{z}_{w2}) - k_1L_1(z - \chi L_1 - z_{w1}) + k_2L_2(z + \chi L_2 - z_{w2}) = 0 \quad (20)$$

Once defined the two equations of motion, it is immediate to find the state space model. For the two degrees of freedom with two outputs, the state variables are respectively bounce motion (z), bounce rate of change ($\dot{z}$), pitch motion (χ) and pitch rate of change ($\dot{\chi}$).

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{2k}{m} & -\frac{2c}{m} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{2kL^2}{J} & -\frac{2cL^2}{J} \end{pmatrix} B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{k}{m} & \frac{c}{m} & \frac{k}{m} & \frac{c}{m} \\ 0 & 0 & 0 & 0 \\ -\frac{kL}{J} & \frac{kL}{J} & -\frac{cL}{J} & \frac{cL}{J} \end{pmatrix} C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} D = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{18}$$

Equation (18) defines the state space model for the damped passive two degree of freedom with two outputs, as visible in matrix C where the first and the third state variables are required as outputs. Matrix B shows the system inputs which are the front wheel displacement (z_{w1}) and speed ($\dot{z}_{w1}$), and the rear wheel ones (z_{w2}, $\dot{z}_{w2}$) respectively.

4.2 Task 9.1: Skyhook-controlled system

After defining the damped passive system equations of motion and state space model, it is time to introduce the active skyhook-controller to understand the differences between a passive and an active system, this process is done by substituting the damping coefficient with the actuator forces. Equation (19) and (29) defines the equation of motion for the bounce and pitch respectively:

$$m\ddot{z} + k_1(z - \chi L_1 - z_{w1}) + k_2(z + \chi L_2 - z_{w2}) = F_{a1} + F_{a2} \tag{16}$$

$$J\ddot{\chi} - k_1 L_1(z - \chi L_1 - z_{w1}) + k_2 L_2(z + \chi L_2 - z_{w2}) = F_{a1}L_1 + F_{a2}L_2 \tag{17}$$

It can be visible how the damping forces are replaced by the actuator forces of the active controller. Having defined the equations of motion, the state space model is then obtained as follows:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{2k}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{2kL^2}{J} & 0 \end{pmatrix} B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{k}{m} & \frac{k}{m} & \frac{1}{m} & \frac{1}{m} \\ 0 & 0 & 0 & 0 \\ -\frac{kL}{J} & \frac{kL}{J} & -\frac{L}{J} & \frac{L}{J} \end{pmatrix} C = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} D = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{21}$$

As already mentioned, to get the steady state model for the active system it is only required to remove from the equations the damping coefficients. The actuators are implemented externally on Simulink and act as a feedback loop.

4.3 Task 9.2: Relationship between actuator forces and bounce force and pitch torque

To define the Skyhook-active controller it is needed to replace the damping coefficient in the passive system state space model with an external relationship between the actuator forces (F_{a1} and F_{a2}) and the force that resists to the bounce motion (F_z) and the torque that resists against pitch motion (T_χ). Equation (22) and (23) shows the relations between the forces:

$$F_{a1} = \frac{F_z L - T_\chi}{2L} \tag{22}$$

$$F_{a2} = \frac{F_z L + T_\chi}{2L} \tag{23}$$

To obtain equation (22) and (23) it is necessary to study the moment equilibrium around the rear axle and front axle respectively. For this reason, the force acting against the bounce motion needs to be multiplied by half the length of the wheelbase as it acts in the vehicle centre of gravity. After performing the momentum equilibrium, it is necessary to divide by the entire length of the wheelbase to obtain the actuator forces. One can see a negative sign for the pitch moment in equation (22) since the actuator force in the front generates a moment in the same direction of the pitch reactive torque. Whereas the actuator force acting on the rear axle acts both against the bounce reactive force and pitch reactive torque.

4.4 Task 10.1: H∞ controlled system

The H∞-controlled system is defined with the same state space model as the Skyhook-controlled one, but the actuator requires massive modifications to work properly. To ensure an effective functioning of the H∞ controller, 4 weighting functions are defined to modify the state space model output to get a good functioning of the controller. The first two weighting functions (W_{a1} and W_{a2}) act to reduce the controller output signals at high frequencies to smoothen oscillations at high frequencies on the front and rear axle, while the other two weighting functions (W_b and W_χ) act to penalise the bounce and pitch motion respectively. To have an effective functioning of the bounce and pitch weighting functions, it is needed to define the frequencies at which penalise the most the two motions. It is demonstrated that it is convenient to compute the eigenfrequencies of the state space model input matrix to determine the frequencies and assign the first eigenfrequency for the bounce weighting function and the third one to the pitch weighting function, respecting the order of the state variables.

4.5 Task 11.1: Systems response under impulse and sinusoidal excitations

After defining the two degrees of freedom systems for bounce and pitch motion, it is time to compare the three systems response to understand the effectiveness of the controllers. Two different excitations are considered, an impulse and a sinusoidal excitation on the front axle and null excitation on the rear axle. The systems will be compared in terms of response magnitude amplitude and time required to reach a steady state condition for the impulse excitation, both for bounce and pitch motion. Figure 16 and 17,shows the bounce and pitch systems response respectively, under impulse excitation:

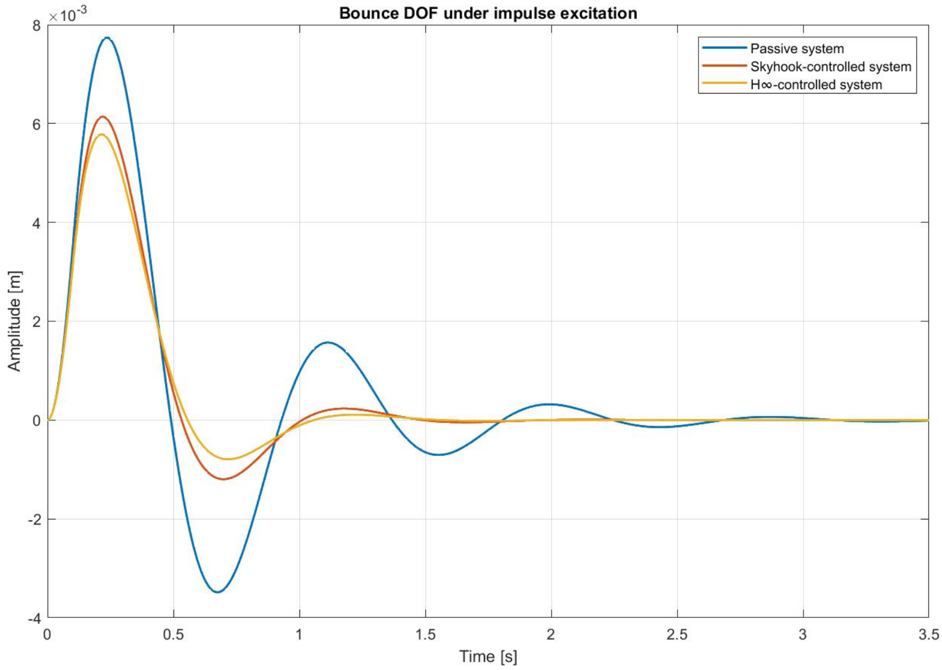

Figure 16. Bounce motion response under impulse excitation for passive (blue), Skyhook-controlled (red) and H∞-controlled system (yellow).

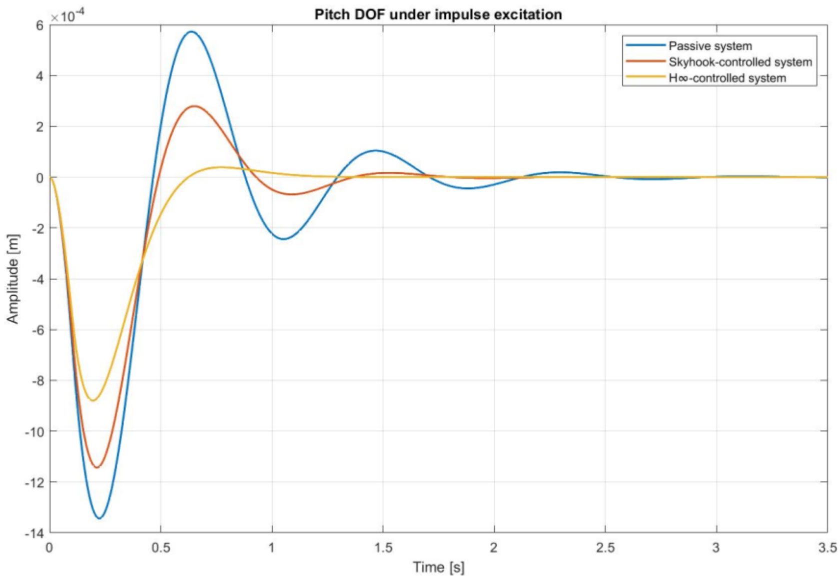

Figure 17. Pitch motion response under impulse excitation for passive (blue), Skyhook-controlled (red) and H∞-controlled system (yellow).

As visible from figure 16 and 17, both the active systems reach steady state before the passive system, ensuring a lower exposure time to excitations for the passengers. The H∞-controlled system is the one that ensures the best oscillation suppression, as both for the bounce and the pitch, the magnitude of the oscillation is lower than the other controllers. It is then visible how both the active systems are underdamped, due to the overshoot region before reaching steady state, a fundamental condition to dissipate the energy in the suspension while ensuring good comfort for the passengers. Figure18 and 19 represents the systems response under sinusoidal excitation:

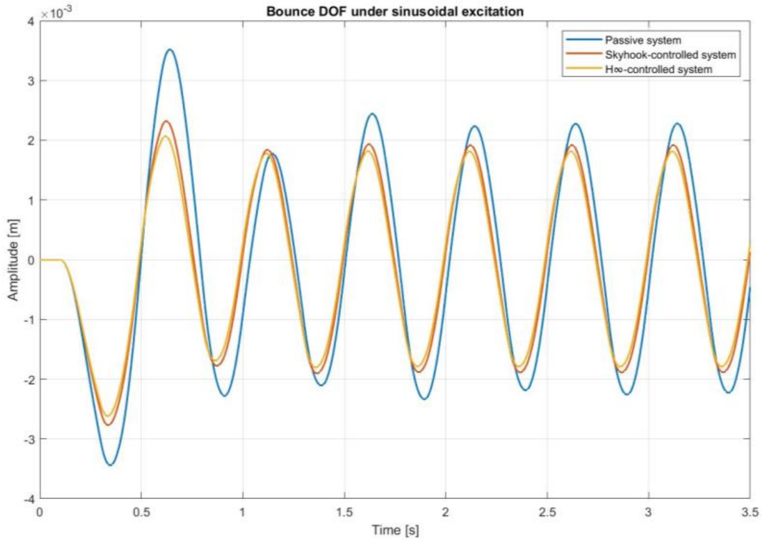

Figure 18. Bounce motion response under sinusoidal excitation for passive (blue), Skyhook-controlled (red) and H∞-controlled system (yellow).

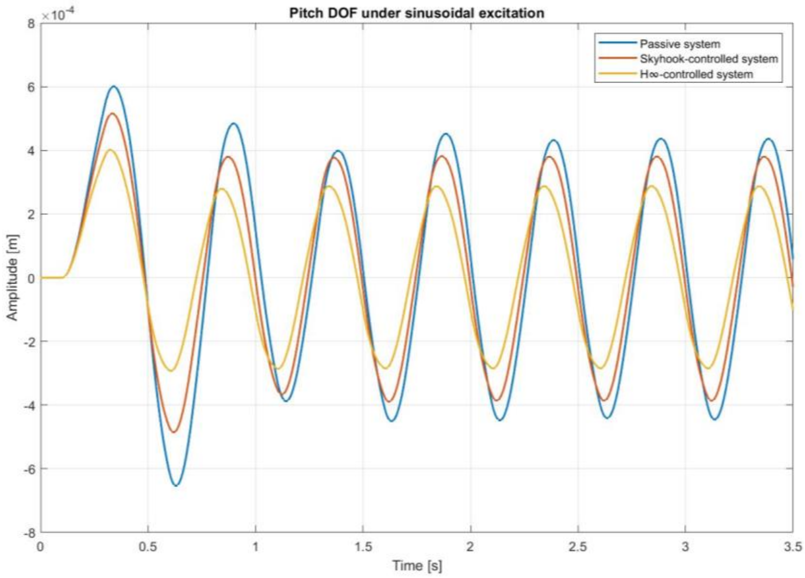

Figure 19. Pitch motion response under sinusoidal excitation for passive (blue), Skyhook-controlled (red) and H∞-controlled system (yellow).

The analysis of the system response under sinusoidal excitation allows to understand that both the active systems are stable in time and smoothen better the oscillations coming from the road. Also in this situation, the H∞-controller ensures the best behaviour as the magnitude of the oscillations is reduced as much as possible. Moreover, it is considered the magnitude of the actuator forces of the two active systems as it is hard to design suspension subsystems able to exploit huge forces. It is demonstrated that for each excitation, the actuator forces never overcome the threshold value of 10kN confirming that the actuators are suitable for the design constraints.

4.6 Task 11.2: Parameters influence on systems response under impulse excitation

After having analysed the performances of the three suspension systems depending on the road excitation, it can be useful to study the influence of the vehicle parameters on the systems response, being spring stiffness, mass and inertia moment and damping coefficient. In this section it is decided to act on the mass and inertia of the vehicle as it would modify the behaviour of the suspension system the most. The systems are compared based on an impulse excitation only, as performed in section 4.5. Table 6 shows the parameters adopted for studying the impact of the mass on the systems response:

Table 6. Vehicle parameters with mass-inertia variation

Standard values	**+15%**	**-15%**
m = 22000 kg	m = 25300 kg	m = 18700 kg
k = 6e5 N/m	k = 6e5 N/m	k = 6e5 N/m
c = 4e4 Ns/m	c = 4e4 Ns/m	c = 4e4 Ns/m
J = 7e5 kg/m^2	J = 8.05 e5 kg/m^2	J = 5.95e5 kg/m^2

It is decided to study two different conditions: an increase of mass and inertia of 15% due to an overload of the vehicle, and a decrease of the same amount representing a limit condition. Figure 20 shows the bounce and pitch response for the passive damping system and the two active systems controlled by the Skyhook controller and the H∞ respectively with a mass-inertia increasement of 15%:

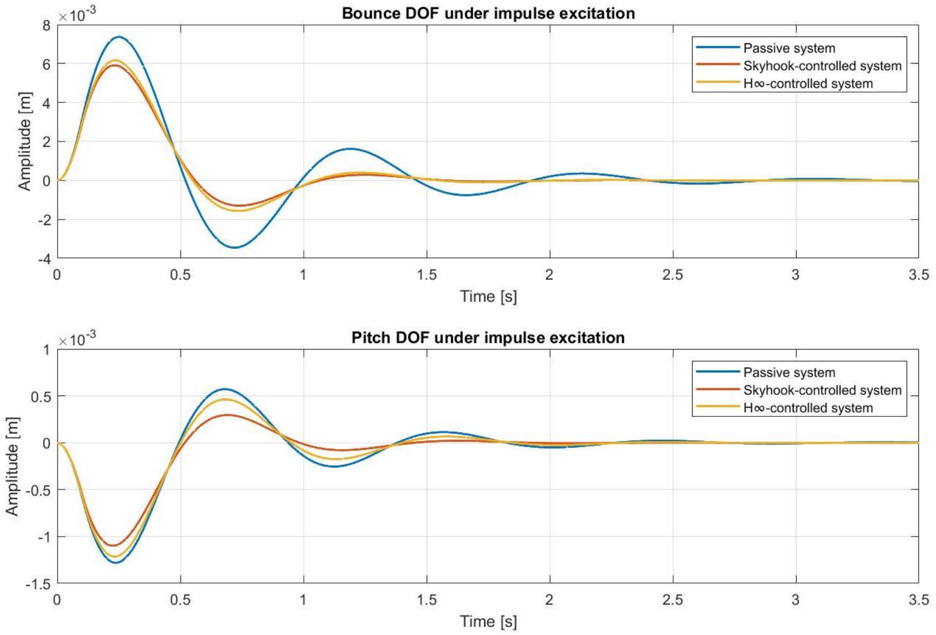

Figure 20. Bounce (top) and pitch (bottom) motion under impulse excitation with mass-inertia increasement by 15%.

With an increasement of the mass-inertia, each system remains stable as the responses converge to steady state, both active systems have a slower response with respect to the one at standard parameters as the controllers are not any longer optimised for the mass increasement, but they still ensure a good behaviour of the suspension without reaching the actuator force threshold. It is also visible how the H∞-controlled system now has higher peak oscillations than the Skyhook-controlled one, meaning that the former is more affected by system parameters variation. If one considers a mass-inertia reduction of 15% with respect to the standard value, bounce and pitch response are represented in figure 21 below:

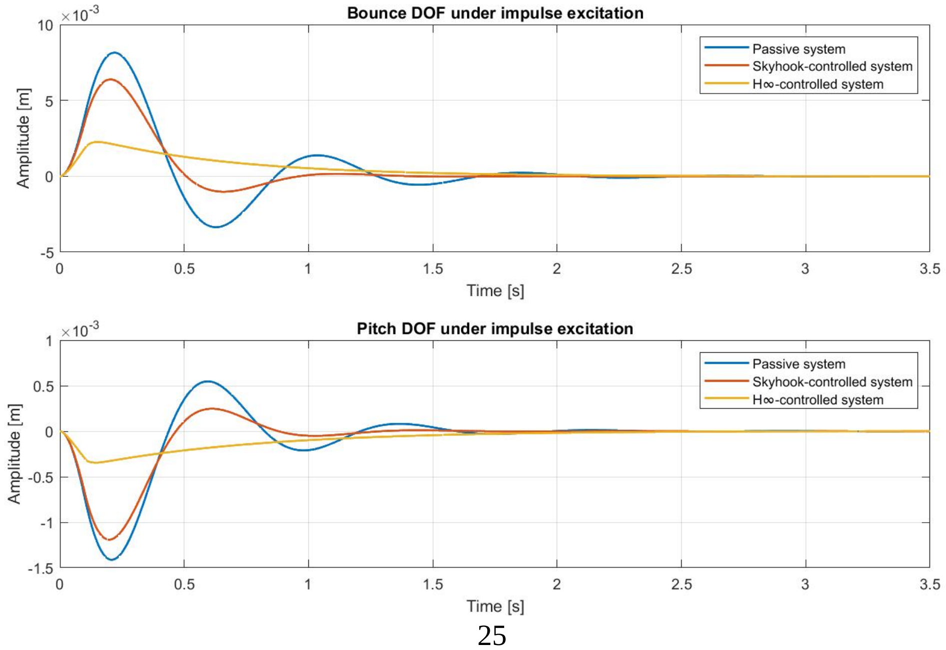

Figure 21. Bounce (top) and pitch (bottom) motion under impulse excitation with mass-inertia decrease by 15%.

As visible from the systems response, the mass reduction negatively affects particularly the H∞-controlled system as it now provides an overdamped behaviour, extremely dangerous for the passengers that may suffer from huge acceleration on the seat as the suspension system does not dissipate much energy transferring it to the passengers. All the systems are stable as they reach steady state conditions, but it is noticed that the actuator forces overcome the 10kN threshold, so it would be required to retune the weighting function gains to stay below the thresholds. Once again it is demonstrated how the H∞ controller behaviour is strongly related to the vehicle parameters, particularly to the vehicle mass, but with the perfect tuning it is reached a better response than the one obtained by the more stable Skyhook-controller.

4.7 Extra task 3:

In this section it is intended to study the systems response when all three parameters are changing simultaneously within the range of ±15% of their original value. Table 7 shows the values adopted in the conditions analysed in this section:

Table 7. Vehicle parameters

Standard parameters	+15%	-15%
m = 22000 kg	m = 25300 kg	m = 18700 kg
k = 6e5 N/m	k = 6.9e5 N/m	k = 5.1e5 N/m
c = 4e4 Ns/m	c = 4.6e4 Ns/m	c = 3.4e4 Ns/m
J = 7e5 kg/m^2	J = 8.05 e5 kg/m^2	J = 5.95e5 kg/m^2

Once defined the vehicle parameters for the different conditions, figure 22 shows the systems response when each parameter is increased of 15%:

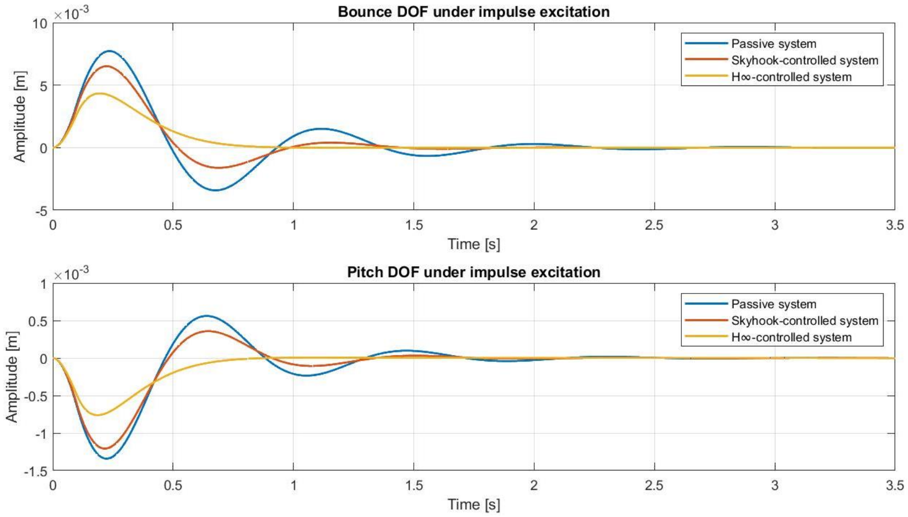

Figure 22. Bounce (top) and pitch (bottom) motion under impulse excitation with vehicle parameters increasement by 15%.

If simultaneously mass-inertia, damping coefficient and spring stiffness are increased by 15%, each system shows a lower amplitude of the oscillation due to the increasement of the mass-inertia and the difference between the damped passive system and the active ones is lowered due to the larger damping coefficient. Due to the simultaneous change of the vehicle parameters, the H∞ controller is characterised once again by an overdamping behaviour due to the high gains for the weighting function gains that regulate pitch and bounce, while the Skyhook-controller maintains a good response even after the changes meaning that it is a more suitable application for vehicles as it is less affected by changes in vehicle parameters. Figure 23 shows the system response when each parameter is lowered of 15% with respect to the standard values for which the controllers are optimised:

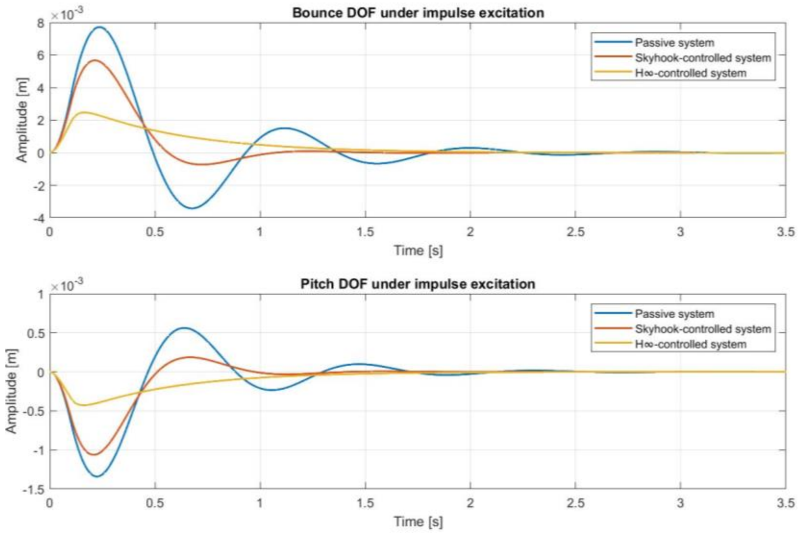

Figure 23. Bounce (top) and pitch (bottom) motion under impulse excitation with vehicle parameters decrease by 15%.

As visible from figure 23, the lower damping coefficient increases the difference between the damped passive system amplitude response and the active systems one. The H∞ again works in overdamping behaviour being extremely uncomfortable for passengers, while the Skyhook controller clearly guarantees the best behaviour between the three systems ensuring that the actuator forces are inside the threshold. To reach better performances with the H∞ controller it would be convenient to have gains that are dependent from the vehicle parameters to always ensure an underdamped behaviour and obtain smoother response than the skyhook controller for several parameters combinations as it happens for the optimised one.

5 References

[1] D. Wanner, C. Casanueva, M. Mehdi Davari, and J. Edrén, “Lab 1 tutorial Longitudinal Dynamics – Slip Control.” pp. 1–15, 12-Mar-2014.

[2] P. Wang, Q. Wang, X. Xu, N. Chen, “Fractional Critical Damping Theory and Its Application in Active Suspension Control” 19-Nov_2017.