Build and evaluate state estimation Models using EKF and UKF

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Last but not least, I would like to thank all my friends from KTH, for their encouragement and support.

Abstract

In vehicle control practice, there are some variables, such as lateral tire force, body slip angle and yaw rate, that cannot or is hard to be measured directly and accurately. Vehicle model, like the bicycle model, offers an alternative way to get them indirectly, however due to the widely existent simplification and inaccuracy of vehicle models, there are always biases and errors in prediction from them. When developing advanced vehicle control functions, it is necessary and significant to know these variables in relatively high precision. Kalman filter offers a choice to estimate these variables accurately with measurable variables and with vehicle model together. In this thesis, estimation models based on Extended Kalman Filter (EKF) and Uncented Kalman Filter (UKF) are built separately to evaluate the lateral tire force, body slip angel and yaw rate of two typical passenger vehicles. Matlab toolbox EKF/UKF developed by Simo Särkkä, et al. is used to implement the estimation models. By comparing their principle, algorithm and results, the better one for vehicle state estimation will be chosen and justified.

The thesis is organized in the following 4 parts:

First, EKF and UKF are studied from their theory and features.

Second, vehicle model used for prediction in Kalman filter is build and justified.

Third, algorithms of EKF and UKF for this specific case are analysed. EKF and UKF are then implemented based on the algorithms with the help of Matlab toolbox EKF/UKF.

Finally, comparisons between EKF and UKF are presented and discussed.

Contents

1. Introduction

This chapter gives the reader a short introduction to the background and aim of the master thesis.

1.1. Background

Driving safety has always been a hot topic for many car manufactures and researchers. It is one of the most important factors that are of concern during the design and manufacturing process of vehicles. Evidences show that a large proportion of accidents happen because of the loss of control of vehicles [1]. With the development of technology, today many different new things such as ABS, ESP, DSC and VSC are introduced to the vehicle industrial to improve the control performance. In order to fulfil the function of these systems, a lot of vehicle information is needed to support them. Some of them like longitudinal speed and steering angle are easy to measure with sensors, but some others like lateral force and body slip angle are hard to measure because of immature technology or high cost of sensors. For example, a non-contact optical sensor used to measure rear sideslip angle may cost 15,000 €. A dynamometric hub used to measure wheel force in real time and moments acting at the wheel centre may cost 100,000 € [2]. It is quite uneconomical and even impossible to equip such kinds of sensors on most vehicles, since their price is usually several ten thousands Euro. However, these kinds of information may greatly improve the functionality of advanced vehicle control system. The demands are increasing with the growth of the modern vehicle industry. Vehicle models, like quarter car model and bicycle model, provide an alternative to *predict* these "immeasurable" parameters from measured ones. Variables, which are hard to measure, can be calculated with dynamic equations from variables easy to measure. However, there are always simplifications and assumptions during modelling, which makes the prediction from models often biased and inaccurate. For the measured parameters, there is always noise or error caused by sensors and environment during measurement, making the results noisy and inaccurate as well. A combination of measurement and model prediction could be a possible solution for these problems to estimate unmeasured variables and filter measured ones. Since Kalman filter was originally developed for use in spacecraft navigation, it has been extended to adapt different kinds of usages and conditions, especially for complex and nonlinear systems. Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF) and augmented Kalman filter were developed one after another. Their usage has been extended to various fields in many applications as well. In vehicle related researched, a lot of studies have been done with Kalman filter and derivations. Rezaei and Sengupta [4] used dynamic bicycle model as process model and data from GPS and vehicle sensor as input to estimate vehicle location. They were fused in an EKF. The results compared favourably with position estimation by fusing Global Positioning System (GPS) and inertial navigation system (INS) through a kinematic model. Larsen et al [5] used an augmented Kalman Filter to estimate the vehicle location and auto calibrate the odometry. Venhovens & Naab [6], Wenzel et al [7] and Antonov et al [8] implemented different kinds of Kalman filters to estimate vehicle states including force, movement and slip angles. Dakhlallah et al [9] and Doumiati et al [2] separately used four-wheel vehicle model as process model to estimate vehicle state such as slip angle and tyre force. Comparison between EKF and UKF has been made to reveal the superiority of UKF over EKF. At the present stage, Kalman filter is a mature technique widely used in vehicle industry providing credible estimation for vehicle state and location information to advanced vehicle control systems.

1.2. Problem description

In preparation for coming Applied Vehicle Dynamics course at KTH, it is of interest to build and evaluate two different designs of state estimation models for estimating longitudinal and lateral vehicle velocity and yaw rate, which use existing vehicle model and easy-to-measure parameters to estimate the unmeasured parameters and make measured parameters less noisy.

1.3. Aim

The major aim of this study is to design EKF and UKF to provide credible estimation of vehicle states. There are two goals to reach in this thesis. First is to implement EKF/UKF to estimate vehicle's longitudinal, lateral vehicle velocity and yaw rate. Second is to compare the results from EKF and UKF to reveal their strength and weaknesses.

2. Method

This chapter explains the methods used to reach the result and the analysis.

CarSim is a simulation software that is used to handle vehicle dynamic problems. It is mainly used to predict and simulate important factors of vehicles such as handling stability, braking ability, ride comfort, and vehicle dynamics. CarSim can define experiment condition, experiment process, characteristic parameters of the vehicle system in detail, and provide reliable results as reference for a vehicle dynamic design. CarSim includes many of the tools that are needed in vehicle dynamic simulation and observation.

In this study, EKF/UKF toolbox is used to implement the Kalman filters, and data from CarSim is used as input and the reference value of output as well. It consists of three main parts:

First, the 4-wheel vehicle model from [2] and [3] are modified and validated for this specific case. Pre-calculated data from CarSim software, which is reliable and relatively accurate, is used as input. It is used as an output reference to be compared with for validation as well.

Second, Gaussian transfer and unscented transfer are done to develop the EKF and UKF with the help of EKF/UKF toolbox.

Finally, the results from EKF and UKF are compared to each other and CarSim data. Analysis of the strength and weaknesses is also done according to the comparison.

3. Vehicle model and its validation

This chapter presents the development of the vehicle model used in the Kalman filter and its validation.

3.1. Vehicle model

The Kalman filter combines prediction that is calculated from a vehicle model and measurement from sensors to find the optimal estimation for vehicle running states. To get the most reliable estimation from Kalman filter, a correct and precise model is as significant as the credible measurements. In this problem, what is of most interest is the velocity and yaw rate of the vehicle. Four-wheel vehicle model (FWVM) [2] shown in Figure 1 is chosen for its simplicity and sufficient representation of a vehicle. Only steering angle, forces applied on each axle and normal tire force are used as input. Forces, motion and slip angle of vehicle body and wheels can all be calculated from the model.

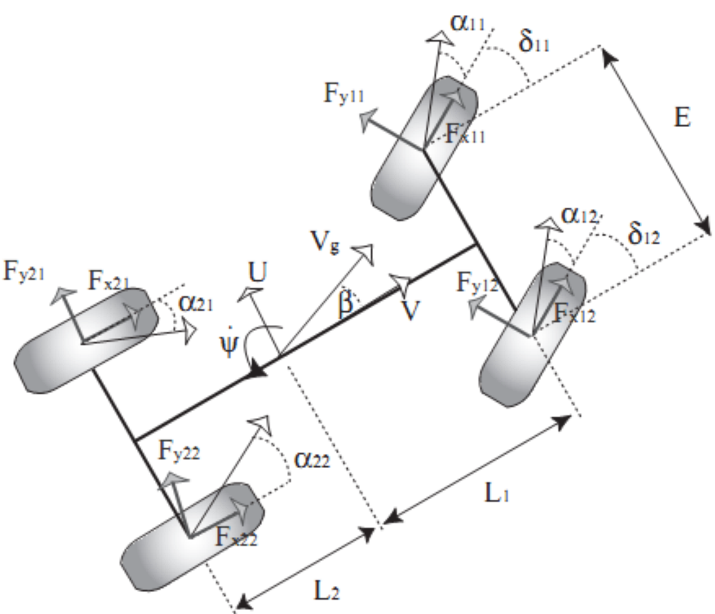


Figure 1. Four-wheel vehicle model [2].

To make the calculation simple, in this study, it is assumed that:

1. The difference between two wheels in one axle is negligible, i.e. $F_{x1} = F_{x11} + F_{x12}$, $F_{x2} = F_{x21} + F_{x22}$.

2. Front steering angles are equal, rear steering angles are negligible, i.e. $\delta_{11} = \delta_{12} = \delta$, $\delta_{21} = \delta_{22} = 0$.

According to Figure 1, the dynamic relationship can be formulated as:

$$\dot{V}_x = \frac{1}{m}\left[F_{x11}\cos(\delta) + F_{x12}\cos(\delta) - F_{y11}\sin(\delta) - F_{y12}\sin(\delta) + F_{x21} + F_{x22}\right] + V_y * \dot{\psi} \tag{1}$$

$$\dot{V}_y = \frac{1}{m}\left[F_{x11}\sin(\delta) + F_{x12}\sin(\delta) + F_{y11}\cos(\delta) + F_{y12}\cos(\delta) + F_{y21} + F_{y22}\right] - V_x * \dot{\psi} \tag{2}$$

$$\ddot{\psi} = \frac{1}{I_z}\Big[0.5 * \big(2L_1(F_{y11} + F_{y12}) - E(F_{x11} - F_{x12})\cos(\delta)\big) + 0.5 * \big(2L_1(F_{x11} + F_{x12}) - E(F_{y12} - F_{y11})\big) * \sin(\delta) - (F_{y21} + F_{y22}) * L_2 + F_{x22} - F_{x21}\Big] \tag{3}$$

Where m is vehicle mass, I_z is yaw moment of inertia, E is wheel track, L_1 and L_2 are the distance from the centre of gravity to the front and rear axles respectively, $\dot{\psi}$ is yaw rate, β is vehicle slip angle, V_x V_y is centre of gravity velocity, F_{xij} is longitudinal tire force, F_{yij} is lateral tire force, δ is steering angle and $\alpha_{i,j}$ is tyre slip angle [2]. The lateral force F_{yij} can be calculated according to relaxation model. When the tyre slip angles change, the corresponding lateral tire forces are built up with a time lag. This transient behaviour of tires can be formulated by a relaxation length σ. Then the lateral force yields [4]:

$$\dot{\mathbf{F}}_{\mathbf{yij}} = \frac{\mathbf{V_g}}{\boldsymbol{\sigma}_\mathbf{i}}\left(-\mathbf{F}_{\mathbf{yij}} + \overline{\mathbf{F}_{\mathbf{yij}}}\right) \tag{4}$$

$\overline{F_{yij}}$ is reference tire-force calculated from Dugoff's empirical model [5]:

$$\overline{\mathbf{F}_{\mathbf{yij}}} = \mathbf{C}_{\boldsymbol{\alpha}\mathbf{i}}\tan\boldsymbol{\alpha}_{\mathbf{ij}} \cdot \mathbf{f}(\boldsymbol{\lambda}) \tag{5}$$

Where $C_{\alpha i}$ is the lateral stiffness, α_{ij} is the slip angle and $f(\lambda)$ is given by:

$$f(\lambda) = \begin{cases} (2-\lambda)\lambda, & if\ \lambda < 1 \\ 1, & if\ \lambda \geq 1 \end{cases} \qquad \text{where } \lambda = \frac{\mu F_{zij}}{2C_{\alpha i}|\tan\alpha_{ij}|} \tag{6}$$

Where α_{ij} are front and rear tire slip angles, which can be calculated by:

$$\boldsymbol{\alpha}_{11} = \boldsymbol{\delta} - \mathbf{arctan}\left[\frac{\mathbf{V_g} + \mathbf{L_1}\dot{\boldsymbol{\psi}}}{\mathbf{V_g} - \mathbf{E}\dot{\boldsymbol{\psi}}/2}\right] \tag{7}$$

$$\boldsymbol{\alpha}_{12} = \boldsymbol{\delta} - \mathbf{arctan}\left[\frac{\mathbf{V_g}\boldsymbol{\beta} + \mathbf{L_1}\dot{\boldsymbol{\psi}}}{\mathbf{V_g} + \mathbf{E}\dot{\boldsymbol{\psi}}/2}\right] \tag{8}$$

$$\boldsymbol{\alpha}_{21} = -\mathbf{arctan}\left[\frac{\mathbf{V_g}\boldsymbol{\beta} - \mathbf{L_2}\dot{\boldsymbol{\psi}}}{\mathbf{V_g} - \mathbf{E}\dot{\boldsymbol{\psi}}/2}\right] \tag{9}$$

$$\alpha_{22} = -\mathbf{arctan}\left[\frac{V_g\beta - L_2\dot{\psi}}{V_g + E\dot{\psi}/2}\right] \tag{10}$$

To build a Kalman filter, a system given in state space form is required. The nonlinear stochastic state space system can be described as:

$$\begin{cases} \dot{\mathbf{X}}(\mathbf{t}) = \mathbf{f}\big(\mathbf{X}(\mathbf{t}), \mathbf{U}(\mathbf{t})\big) + \mathbf{q}(\mathbf{t}) \\ \mathbf{Y}(\mathbf{t}) = \mathbf{h}\big(\mathbf{X}(\mathbf{t}), \mathbf{U}(\mathbf{t})\big) + \mathbf{r}(\mathbf{t}) \end{cases} \tag{11}$$

The input vector U includes the steering angle:

$$\mathbf{U} = [\boldsymbol{\delta}] = [\mathbf{u_1}] \tag{12}$$

According to the dynamic functions above, the state vector includes yaw rate, vehicle speed, vehicle body slip angle and lateral force from each tire:

$$\mathbf{X} = \left[\mathbf{V_x}, \mathbf{V_y}, \ddot{\psi}\right] = [\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}] \tag{13}$$

The measurements are vehicle velocity and yaw rate. The measure vector Y is:

$$\mathbf{Y} = [\mathbf{V_x}, \dot{\psi}] = [\mathbf{y_1}, \mathbf{y_2}] \tag{14}$$

$q(t)$ and $r(t)$ are process and measurement noise vectors respectively, which are both assumed to be zero mean and uncorrelated Gaussian distributions.
Consequently, the state space functions of the continuous nonlinear system are:

$$\dot{x}_1 = \frac{1}{m}\left[F_{x11}\cos(u_1) + F_{x12}\cos(u_1) - F_{y11}\sin(u_1) - F_{y12}\sin(u_1) + F_{x21} + F_{x22}\right] + x_2 * x_3 \tag{15}$$

$$\dot{x}_2 = \frac{1}{m}\left[F_{x11}\sin(u_1) + F_{x12}\sin(u_1) + F_{y11}\cos(u_1) + F_{y12}\cos(u_1) + F_{y21} + F_{y22}\right] - x_1 * x_3 \tag{16}$$

$$\dot{x}_3 = \frac{1}{I_z}\Big[0.5 * \Big(2L_1(F_{y11} + F_{y12}) - E(F_{x11} - F_{x12})\cos(u_1)\Big) + 0.5 * \Big(2L_1(F_{x11} + F_{x12}) - E\big(F_{y12} - F_{y11}\big)\Big) * \sin(u_1) - \big(F_{y21} + F_{y22}\big) * L_2 + F_{x22} - F_{x21}\Big] \tag{17}$$

(18)

$$Y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot X + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot U \tag{19}$$

3.2. Parameters and data

Due to limited conditions, there is no real vehicle equipped with velocity, yaw or slip angle sensors available in this study. All vehicle parameters and data presented in this thesis are acquired from CarSim software. Two vehicles and situations are chosen. In Scenario 1, the vehicle is a D-Class four-wheel drive SUV. Its parameters are shown in Table 1.

Table 1. Vehicle parameters of the SUV in Scenario 1.

Parameter	Value	Parameter	Value
m	1609 kg	*Steer Gear Ratio*	20
I_z	1765 kg·m²	$C_{\alpha 1}$	53025
L_1	1.050 m	$C_{\alpha 2}$	53025
L_2	1.569 m	σ_1	0.565
E	1.565 m	σ_2	0.565
$\boldsymbol{\mu}$	0.85	*Speed*	120 km/h

In Scenario 2, the vehicle is a front-wheel drive hatchback. Its parameters are shown in Table 2.

Table 2. Vehicle parameters of the hatchback in Scenario 2.

Parameter	Value	Parameter	Value
m	830 kg	*Steer Gear Ratio*	20
I_z	1111 kg·m²	$C_{\alpha 1}$	34400
L_1	1.103 m	$C_{\alpha 2}$	34400
L_2	1.244 m	σ_1	0.565
E	1.416 m	σ_2	0.565
$\boldsymbol{\mu}$	0.85	*Speed*	120 km/h

CarSim can calculate the most common used vehicle variables according to a given driving event or procedure. The data used in following chapters as inputs, measurements and comparisons are all done by CarSim. For both scenarios, the vehicle does double lane change with 120 km/h speed on flat road.

3.3. Model validation

Although the Kalman filter can combine model prediction and sensors measurements to get optimal estimation, the model precision is still very important in order to improve the estimation. In every model, there may be errors that are introduced by simplification, unmodelled properties, parameter variations and other factors. However, to ensure filtering effect, the result from the vehicle model should be roughly consistent with the actual situation. Before implementing the Kalman filters, it is necessary to check the correctness and accuracy of the model.

In this study, the vehicle model is nonlinear and continuous. A Simulink model shown in Figure 2 is built.

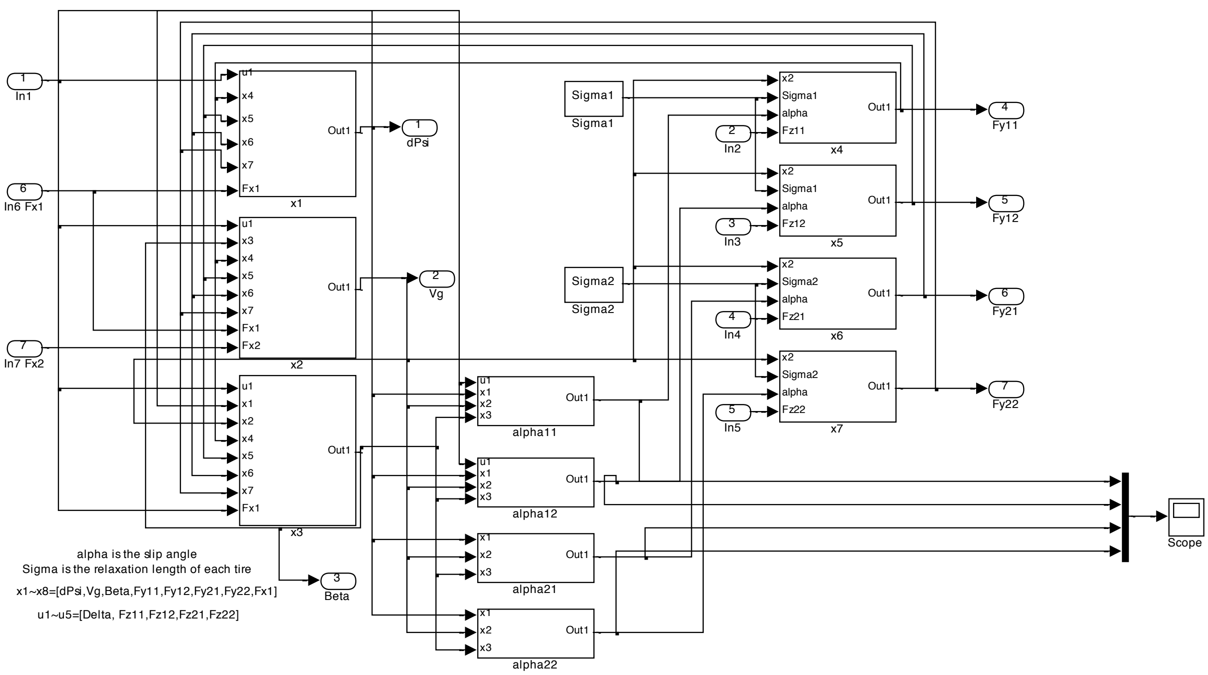


Figure 2. Continuous non-linear model built in Matlab/Simulink.

Scenario 1 is used in the Simulink simulation. Data from CarSim is regarded as “real values”. Figure 3 shows yaw rate, vehicle speed, body slip angle and lateral tire force from the Simulink simulation and CarSim. The difference is quite small, which means the model is correct and precise enough to be used in Kalman filter.

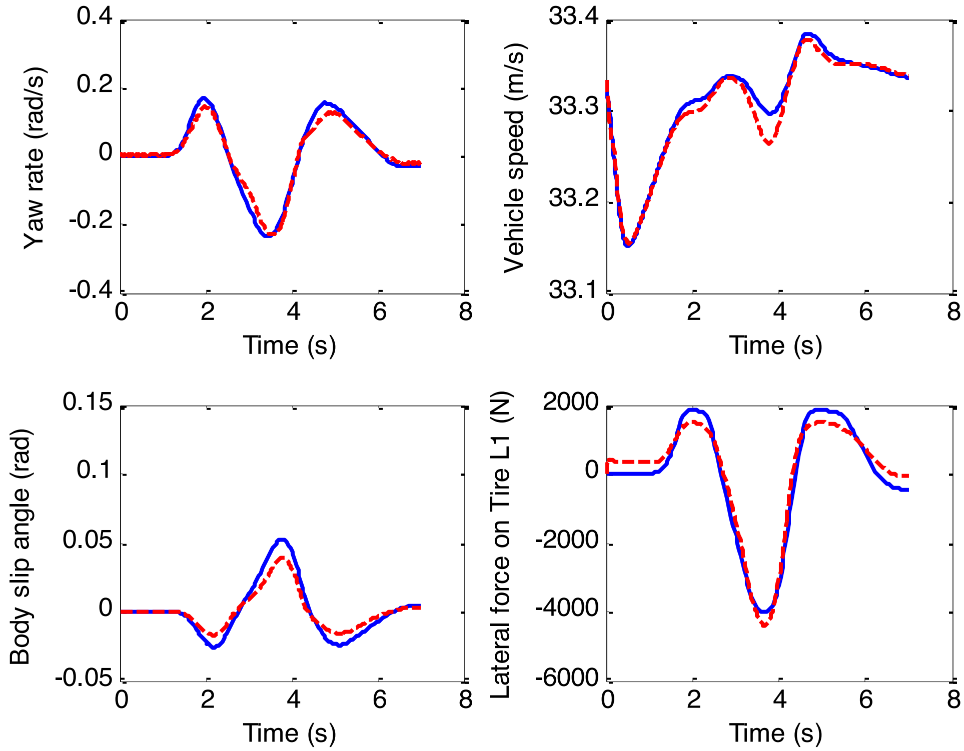


Figure 3. Comparison between CarSim data and vehicle model simulation result (dashed: CarSim, solid: FWVM).

4. Development of EKF and UKF

This chapter describes the development of EKF and UKF, including their principle, algorithm and implementation.

In this study, because of the highly nonlinear properties of the system, Kalman filter cannot be used directly. To overcome the linear assumption, EKF and UKF can be used.

4.1. Development of EKF

The fundamental requirement of Kalman filter is that the system should be linear. However, it is quite hard to find a linear system in real world. For complex system like road vehicle, it is common that the system is highly non-linear, which can be typically formulated as:

$$\mathbf{x_k} = \mathbf{f}(\mathbf{x_{k-1}}, \mathbf{k-1}) + \mathbf{q_{k-1}} \tag{20}$$

$$\mathbf{y_k} = \mathbf{h}(\mathbf{x_k}, \mathbf{k}) + \mathbf{r_k} \tag{21}$$

where $\mathrm{x_k} \in \mathbb{R}^n$ is state, $y_k \in \mathbb{R}^m$ is measurement, $q_{k-1} \sim N(0, Q_{k-1})$ is process noise, $r_k \sim N(0, R_k)$ is measurement noise, f is (possibly nonlinear) dynamic model function and h is the (again possibly nonlinear) measurement model function [4].

EKF is one approach to apply Kalman filter on non-linear system. The idea for EKF is to linearize the model before the implementation of the Kalman filter. Taylor expansion is used as a tool for linearization. The system can be extended to any orders, but to make the calculation simple and save computing resources, usually only first (Jacobian) and second order (Hessian) Taylor expansion is used.

Similar to Kalman filter, EKF has two steps as well. For the first order EKF, the steps are:

Prediction:

$$\mathbf{m_k^-} = \mathbf{f}(\mathbf{m_{k-1}}, \mathbf{k-1})$$

$$\mathbf{P_k^-} = \mathbf{F_x}(\mathbf{m_{k-1}}, \mathbf{k-1})\mathbf{P_{k-1}}\mathbf{F_x^T}(\mathbf{m_{k-1}}, \mathbf{k-1}) + \mathbf{Q_{k-1}} \tag{22}$$

Update:

$$\mathbf{v_k} = \mathbf{y_k} - \mathbf{h}(\mathbf{m_k^-}, \mathbf{k}) \tag{23}$$

$$\mathbf{S_k = H_x(m_k^-, k)P_k^- H_x^T(m_k^-, k) + R_k} \tag{24}$$

$$\mathbf{K_k = P_k^- H_x^T(m_k^-, k)S_k^{-1}} \tag{25}$$

$$\mathbf{m_k = m_k^- + K_k v_k} \tag{26}$$

$$\mathbf{P_k = P_k^- - K_k S_k K_k^T} \tag{27}$$

where the matrices $F_x(m, k-1)$ and $H_x(m, k)$ are the Jacobians of f_i and h_i with elements

$$[\boldsymbol{F_x(m, k-1)}]_{j,j'} = \left.\frac{\partial f_j(x, k-1)}{\partial x_{j'}}\right|_{x=m} \tag{28}$$

$$[\boldsymbol{H_x(m, k)}]_{j,j'} = \left.\frac{\partial h_j(x, k)}{\partial x_{j'}}\right|_{x=m} \tag{29}$$

For the second order EKF, the corresponding steps are:

Prediction:

$$\mathbf{m_k^- = f(m_{k-1}, k-1) + \frac{1}{2}\sum_i e_i\, tr\left\{F_{xx}^{(i)}(m_{k-1}, k-1)P_{k-1}\right\}} \tag{30}$$

$$\mathbf{P_k^- = F_x(m_{k-1}, k-1)P_{k-1}F_x^T(m_{k-1}, k-1)}$$

$$\mathbf{+\frac{1}{2}\sum_{i,i'} e_i e_{i'}^T\, tr\left\{F_{xx}^{(i)}(m_{k-1}, k-1)P_{k-1}F_{xx}^{(i')}(m_{k-1}, k-1)P_{k-1}\right\} + Q_{k-1}} \tag{31}$$

Update:

$$\mathbf{v_k = y_k - h(m_k^-, k) - \frac{1}{2}\sum_i e_i\, tr\left\{H_{xx}^{(i)}(m_k^-, k)P_k^-\right\}} \tag{32}$$

$$\mathbf{S_k = H_x(m_k^-, k)P_k^- H_x^T(m_k^-, k)}$$

$$\mathbf{+\frac{1}{2}\sum_{i,i'} e_i e_{i'}^T\, tr\left\{H_{xx}^{(i)}(m_k^-, k)P_k^- H_{xx}^{(i')}(m_k^-, k)P_k^-\right\} + R_k} \tag{33}$$

$$\mathbf{K_k = P_k^- H_x^T(m_k^-, k)S_k^{-1}} \tag{34}$$

$$\mathbf{m_k = m_k^- + K_k v_k} \tag{35}$$

$$\mathbf{P_k = P_k^- - K_k S_k K_k^T} \tag{36}$$

where the matrices $F_x(m, k-1)$ and $H_x(m, k)$ are the Jacobians of f and h given by Equations (28) and (29). The matrices $F_{xx}(m, k-1)$ and $H_{xx}(m, k)$ are Hessian matrices of f and h:

$$[\boldsymbol{F}_{\boldsymbol{xx}}^{(\boldsymbol{i})}(\boldsymbol{m}, \boldsymbol{k}-\mathbf{1})]_{\boldsymbol{j},\boldsymbol{j}'} = \left.\frac{\boldsymbol{\partial^2 f_j(x,k-1)}}{\boldsymbol{\partial x_j \partial x_{j'}}}\right|_{\boldsymbol{x}=\boldsymbol{m}} \tag{37}$$

$$[\boldsymbol{F}_{\boldsymbol{xx}}^{(\boldsymbol{i})}(\boldsymbol{m}, \boldsymbol{k})]_{\boldsymbol{j},\boldsymbol{j}'} = \left.\frac{\boldsymbol{\partial^2 h_j(x,k)}}{\boldsymbol{\partial x_j \partial x_{j'}}}\right|_{\boldsymbol{x}=\boldsymbol{m}} \tag{38}$$

$e_i = (0 \cdots 010 \cdots 0)^{\mathrm{T}}$ is a unit vector in direction of the coordinate axis i , that is, it has a 1 at position i and 0 at other positions.

It is also noticed that both EKF methods are derived discretely. However, most vehicle models are derived in continuous form. Classical Runge-Kutta method (RK4) is used to approximate solutions of ordinary differential equations from step *n* to step *n+1*. For the differential function:

$$\boldsymbol{y}' = \boldsymbol{f}(\boldsymbol{t}, \boldsymbol{y}) \tag{39}$$

Given *y-value* at the initial time t_0 is y_0: $\boldsymbol{y}(\boldsymbol{t_0}) = \boldsymbol{y_0}$, *y-value* at step *n+1* can be approximated by *y-value* at step *n* as:

$$\boldsymbol{y_{n+1}} = \boldsymbol{y_n} + \frac{\boldsymbol{h}}{\mathbf{6}}(\boldsymbol{k_1} + \mathbf{2}\boldsymbol{k_2} + \mathbf{2}\boldsymbol{k_3} + \boldsymbol{k_4}) \tag{40}$$

where

$$\mathbf{k_1} = \mathbf{f}(\mathbf{t_n}, \mathbf{y_n}) \tag{41}$$

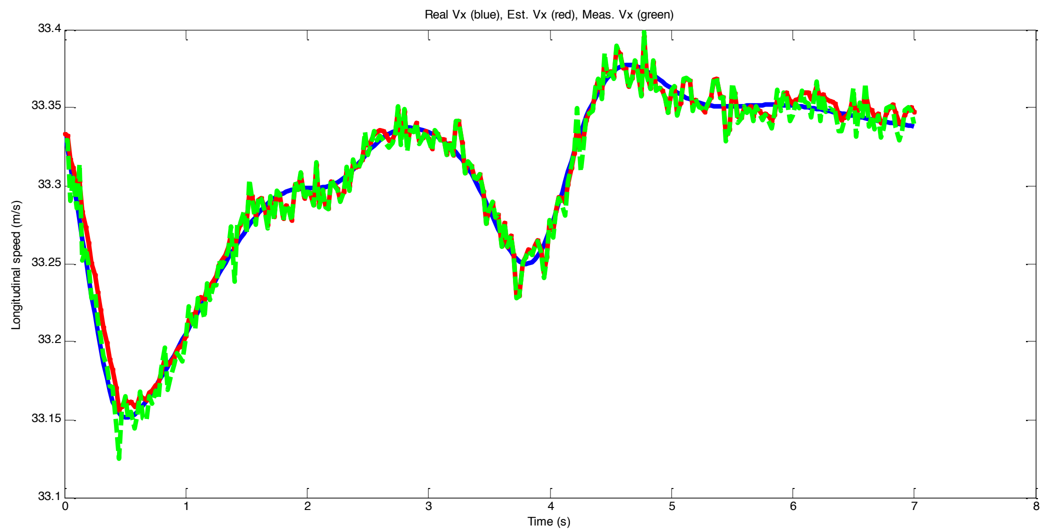
$$\boldsymbol{k_2} = \boldsymbol{f}\left(\boldsymbol{t_n} + \frac{\boldsymbol{h}}{\mathbf{2}}, \boldsymbol{y_n} + \frac{\boldsymbol{h}}{\mathbf{2}}\boldsymbol{k_1}\right) \tag{42}$$

$$\boldsymbol{k_3} = \boldsymbol{f}\left(\boldsymbol{t_n} + \frac{\boldsymbol{h}}{\mathbf{2}}, \boldsymbol{y_n} + \frac{\boldsymbol{h}}{\mathbf{2}}\boldsymbol{k_2}\right) \tag{43}$$

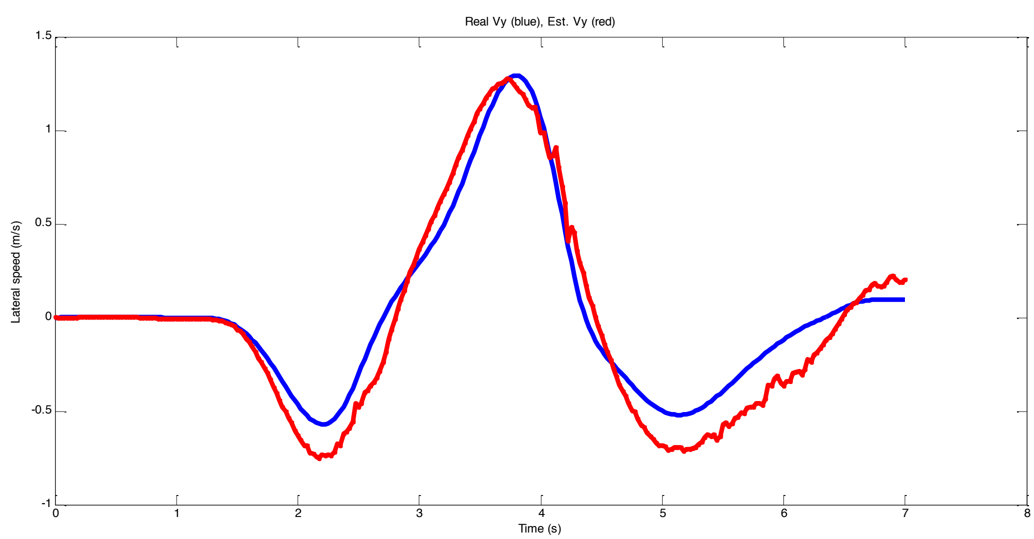
$$\boldsymbol{k_4} = \boldsymbol{f}(\boldsymbol{t_n} + \boldsymbol{h}, \boldsymbol{y_n} + \boldsymbol{h}\boldsymbol{k_3}) \tag{44}$$

h is the fixed time interval between two adjacent steps.

The vehicle model given in Section 3.1 is linearized and built as EKF with the help of Matlab toolbox. The estimations of longitudinal, lateral speed, vehicle slip angle, yaw-rate, tire forces and used friction for Scenario 1 are shown below in Figure 4-Figure 7. The estimations coincide with the real value in principle.

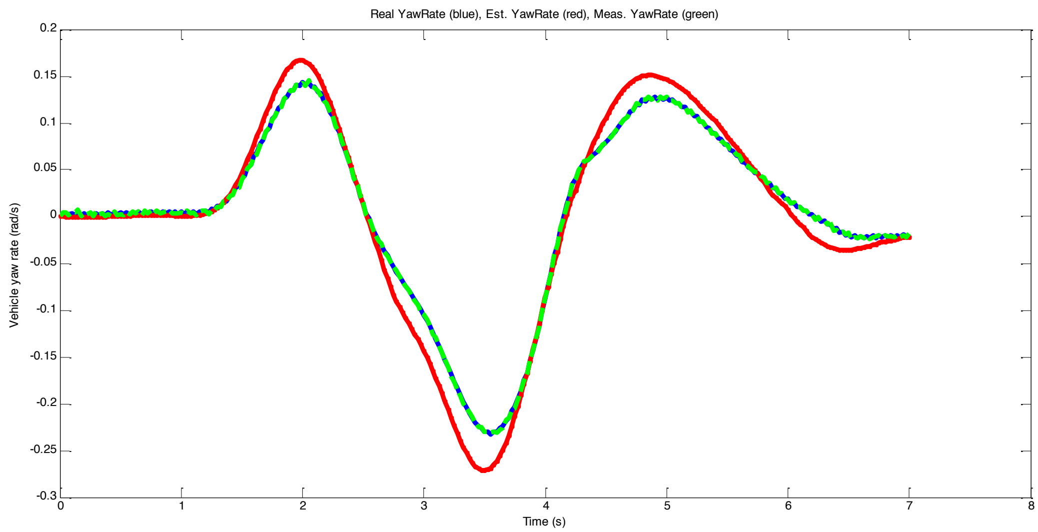


(a)

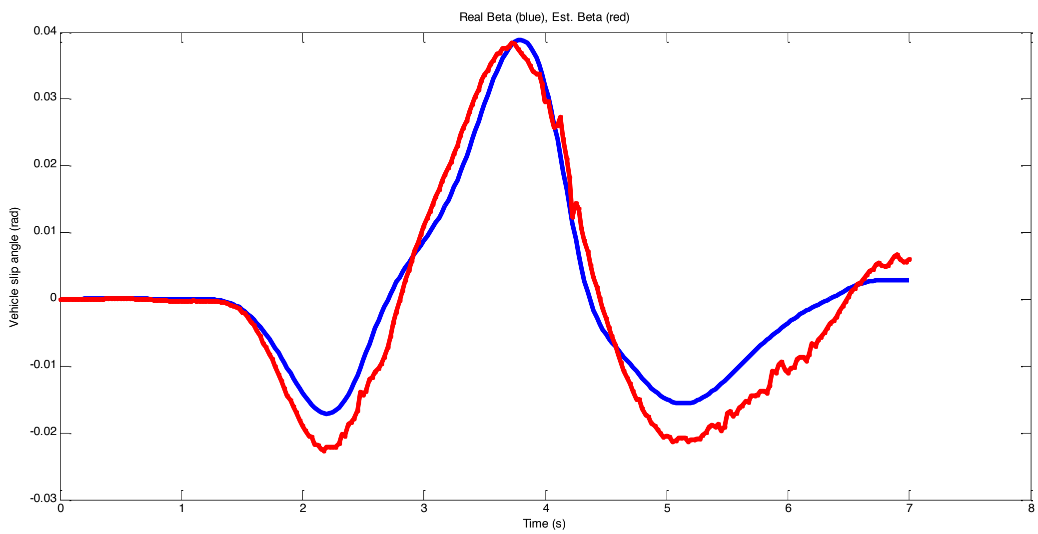


(b)

Figure 4. EKF estimations for lateral and longitudinal vehicle velocity (a) longitudinal (b) lateral. Estimated: red, Real: blue.



(a)



(b)

Figure 5. EKF estimations for vehicle yaw rate and slip angle (a) yaw rate (b) vehicle slip angle β . Estimated: red, Real: blue.

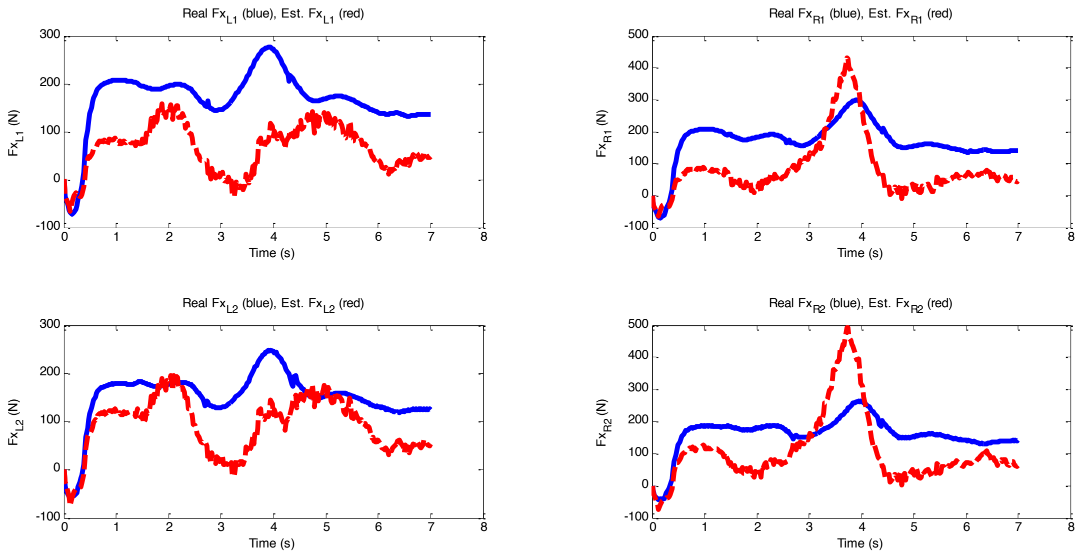


Figure 6. Longitudinal force on each tire. Estimated: red, Real: blue.

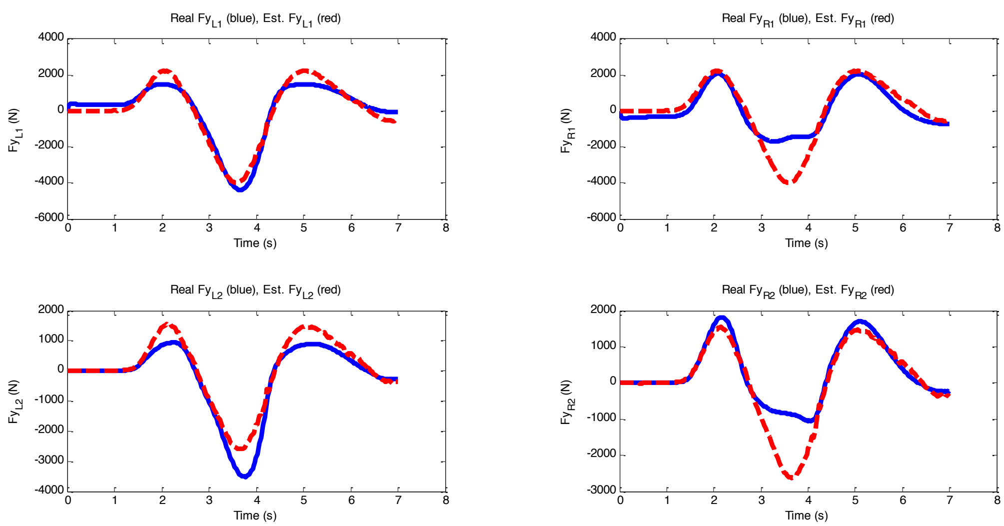


Figure 7. Lateral force on each tire. Estimated: red, Real: blue.

4.2. Development of UKF

EKF can provide estimation for non-linear continuous system according to model prediction and measurement, but it still has quite a lot of limitations and disadvantages. EKF needs linearized system function. For complex system like vehicle and aeroplane, linearization has to be done for each step. The linearized equations are even more complicated, and it takes a lot of time for linearization and calculation. In this study, it's found that it may take several seconds or even minutes for each step, this implementation makes EKF unpractical in real time estimation used in vehicles. Linearization and approximation from continuous to discrete states both introduces errors, which makes the estimation imprecise. The errors may increase with the system complexity. Low efficiency and low precision make it improper for EKF to be used in the vehicle, especially in real time estimation, so there is an urgent need for a quicker and more precise method to estimate the vehicle states.

Unscented Kalman Filter (UKF) is a great alternative of EKF. Instead of linearizing differential equations and handling the linearized equations with classical Kalman filter, UKF uses Unscented Transform (UT) to propagate mean and covariance directly.

By forming an augmented state variable, which concatenates the state and noise components together, augmented UKF can be obtained to better capture the odd-order moment information [2]. This requires that the sigma points generated during the predict step are also used in the update step, so that the effect of these terms are truly propagated through the nonlinearity [8]. The prediction and update steps of the augmented UKF in matrix form are:

Prediction:

$\tilde{\mathrm{x}}_{\mathrm{k-1}} = [x_{k-1}^T \; q_{k-1}^T \; r_{k-1}^T]$ as

$$\widetilde{\mathbf{X}}_{\mathbf{k-1}} = [\widetilde{\mathbf{m}}_{\mathbf{k-1}} \cdots \widetilde{\mathbf{m}}_{\mathbf{k-1}}] + \sqrt{\mathbf{c}}\left[\mathbf{0} \; \sqrt{\widetilde{\mathbf{P}}_{\mathbf{k-1}}} - \sqrt{\widetilde{\mathbf{P}}_{\mathbf{k-1}}}\right] \tag{45}$$

where

$$\widetilde{\mathbf{m}}_{\mathbf{k-1}} = [\mathbf{m^T}_{\mathbf{k-1}} \; \mathbf{0} \; \mathbf{0}]^{\mathbf{T}} \tag{46}$$

$$\widetilde{\mathbf{P}}_{\mathbf{k-1}} = \begin{bmatrix} \boldsymbol{P}_{\boldsymbol{k-1}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{Q}_{\boldsymbol{k-1}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{R}_{\boldsymbol{k-1}} \end{bmatrix} \tag{47}$$

Then the predicted state mean $\mathrm{m}_{\mathrm{k}}^{-}$ and the predicted covariance $\mathrm{p}_{\mathrm{k}}^{-}$ are:

$$\widehat{\mathbf{X}}_{\mathbf{k}} = \mathbf{f}(\mathbf{X}_{\mathbf{k-1}}^{\mathbf{X}}, \mathbf{X}_{\mathbf{k-1}}^{\mathbf{q}}, \mathbf{k-1}) \tag{48}$$

$$\mathbf{m_k^-} = \widehat{\mathbf{X}}_\mathbf{k}\mathbf{w_m} \tag{49}$$

$$\mathbf{P_k^-} = \widehat{\mathbf{X}}_\mathbf{k}\mathbf{W}\left[\widehat{\mathbf{X}}_\mathbf{k}\right]^\mathbf{T} \tag{50}$$

Update:

$$\mathbf{Y_k^-} = \mathbf{h}\left(\widehat{\mathbf{X}}_\mathbf{k}, \mathbf{X_{k-1}^r}, \mathbf{k}\right) \tag{51}$$

$$\boldsymbol{\mu}_\mathbf{k} = \mathbf{Y_k^-}\mathbf{w_m} \tag{52}$$

$$\boldsymbol{S_k} = \mathbf{Y}_{\boldsymbol{k}}^{-}\boldsymbol{W}[\boldsymbol{Y}_{\boldsymbol{k}}^{-}]^{\boldsymbol{T}} \tag{53}$$

$$\mathbf{C_k} = \widehat{\mathbf{X}}_\mathbf{k}\mathbf{W}[\mathbf{Y_k^-}]^\mathbf{T} \tag{54}$$

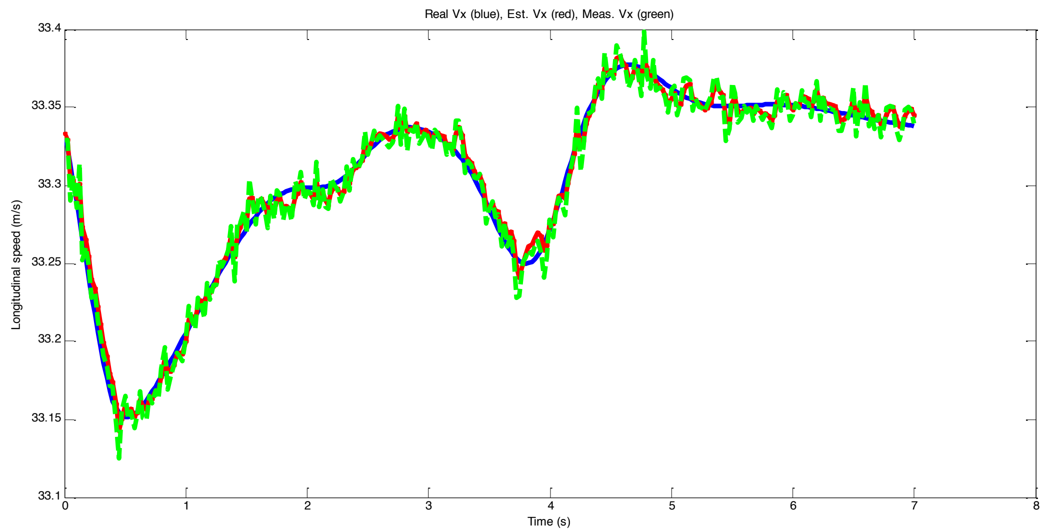
$$\mathbf{K_k} = \mathbf{C_k}\mathbf{S_k^{-1}} \tag{55}$$

$$\mathbf{m_k} = \mathbf{m_k^-} + \mathbf{K_k}[\mathbf{y_k} - \boldsymbol{\mu}_\mathbf{k}] \tag{56}$$

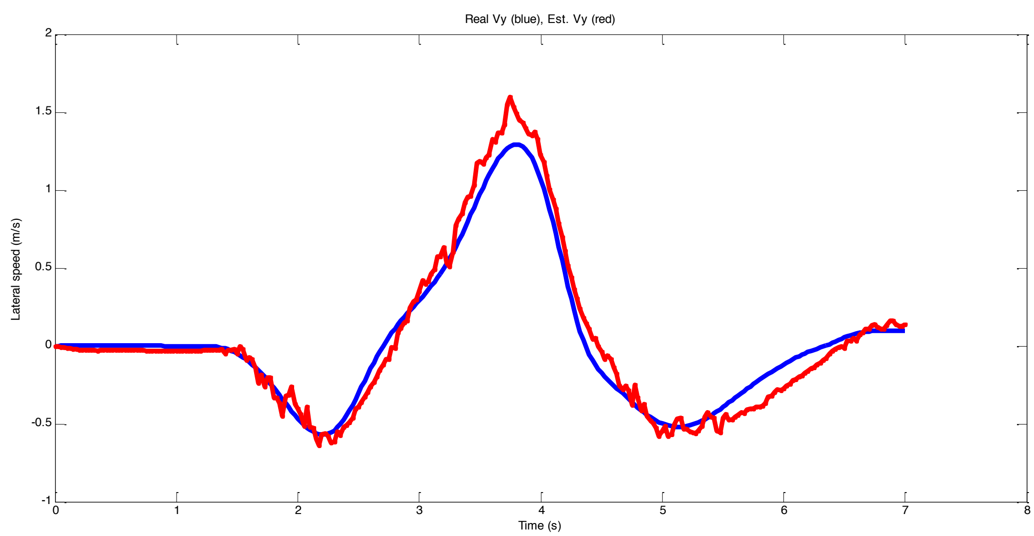
$$\mathbf{P_k} = \mathbf{P_k^-} - \mathbf{K_k}\mathbf{S_k}\mathbf{K_k^T} \tag{57}$$

where μ_k is predicted mean, S_k is covariance of the measurement, C_k is cross-covariance of the state and measurement, K_k is filter gain, m_k is updated state mean and P_k is updated state covariance.

The same vehicle model given in Section 3.1 is then built as UKF with the help of Matlab toolbox as well. The estimations of longitudinal, lateral speed, vehicle slip angle, yaw-rate, tire forces and used friction for Scenario 1 are shown below in Figure 8-11.

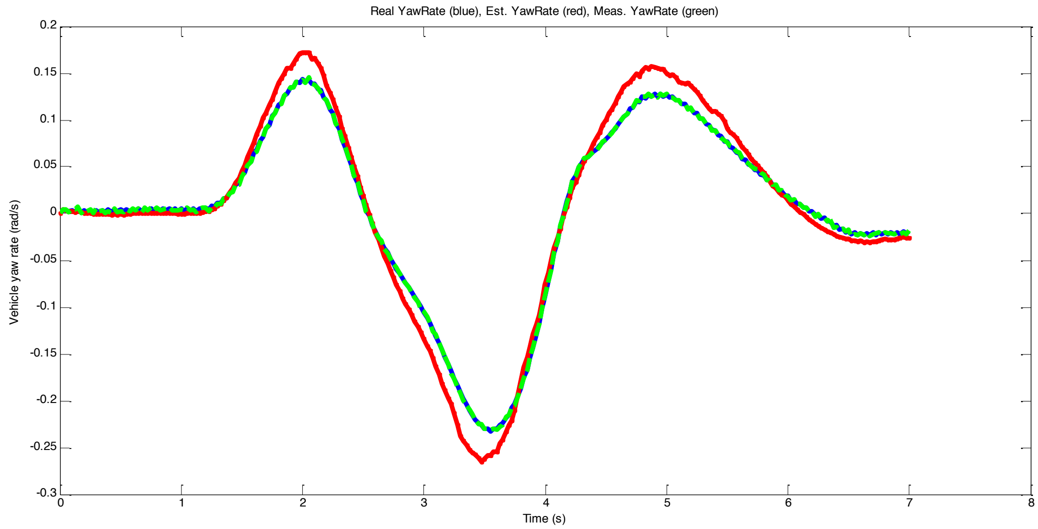


(a)

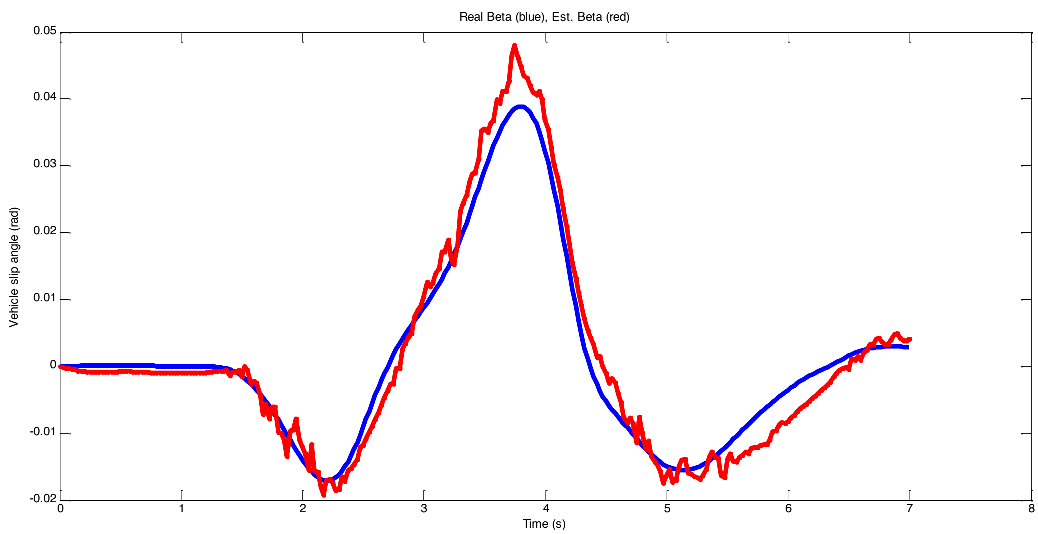


(b)

Figure 8. UKF estimations for lateral and longitudinal vehicle velocity (a) longitudinal (b) lateral. Estimated: red, Real: blue.



(a)



(b)

Figure 9. UKF estimations for vehicle yaw rate and slip angle (a) yaw rate (b) vehicle slip angle β . Estimated: red, Real: blue.

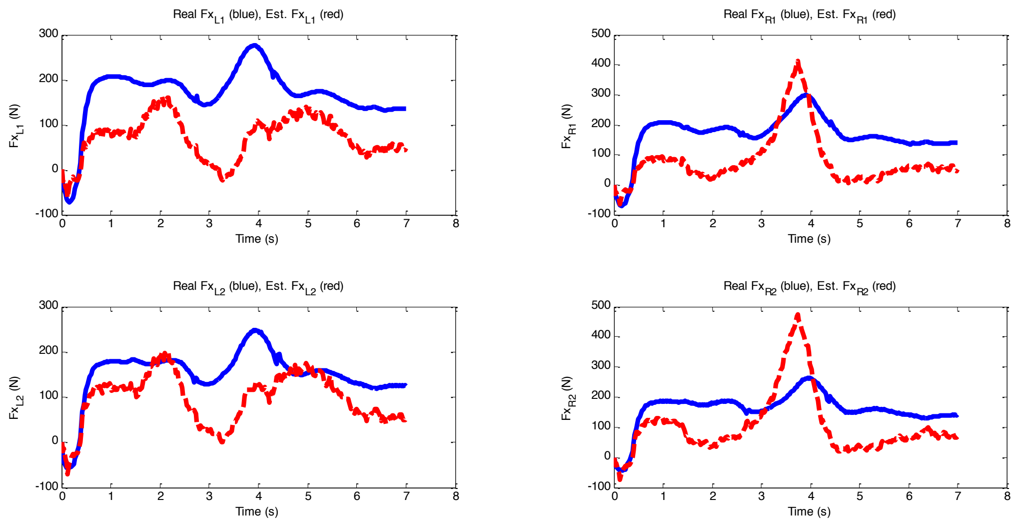


Figure 10. Longitudinal force on each tire. Estimated: red, Real: blue.

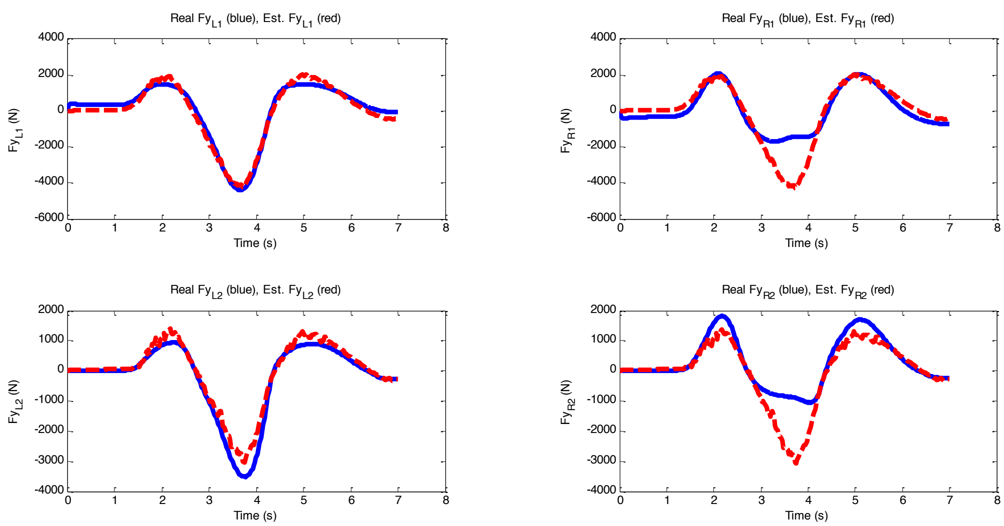


Figure 11. Lateral force on each tire. Estimated: red, Real: blue.

5. Results and analysis

This chapter shows the estimations from both EKF and UKF, compares them with the "real value", analyses their strengths and weaknesses, and discusses the reasons

Now there are two kinds of Kalman filters designed for the same vehicle model. However, due to their different algorithms, the computing speed and how much the estimations are close to the real value varies. In this chapter, EKF and UKF are compared under two scenarios given in Section 3.2. Vehicle velocity, yaw rate, vehicle body slip angle, and tire forces are of concern and chosen to be estimated in this study. Among them, longitudinal speed and yaw rate can be measured directly. They constitute the measurement. The other variables are predicted according to the vehicle model and adjusted by EKF or UKF to form estimations. The results for Scenario 1 are shown in Figure 12-Figure 17, the results for Scenario 2 are shown in Figure 18-Figure 23.

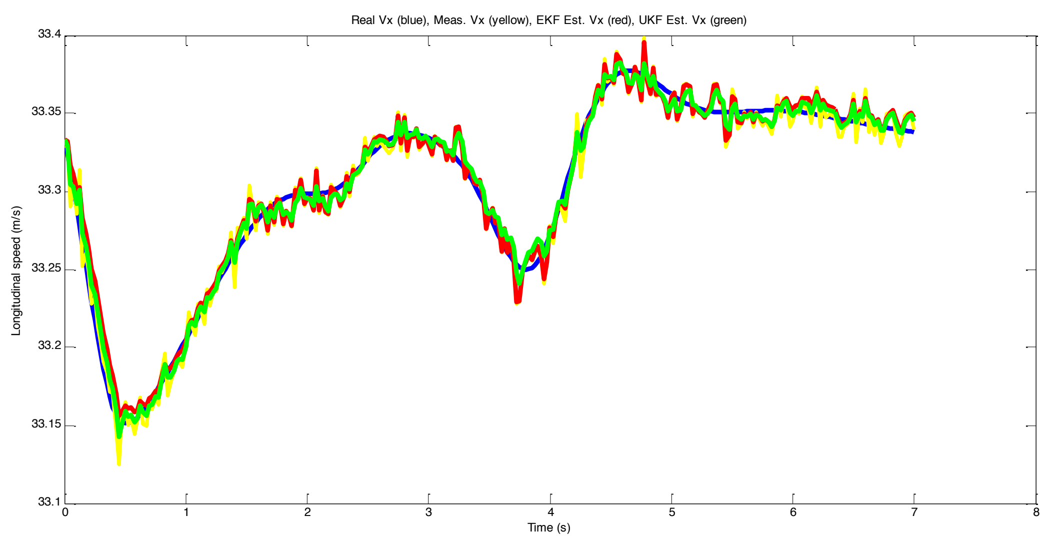


Figure 12. Estimations for longitudinal vehicle velocity Vx in Scenario 1. Estimated EKF: red, Estimated UKF: green, Real: blue.

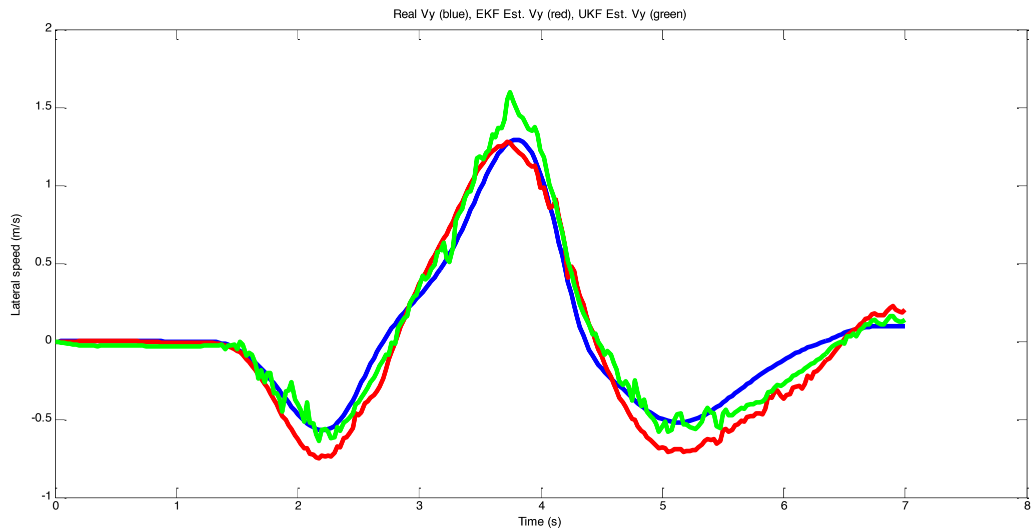


Figure 13. Estimations for lateral vehicle velocity Vy in Scenario 1. Estimated EKF: red, Estimated UKF: green, Real: blue.

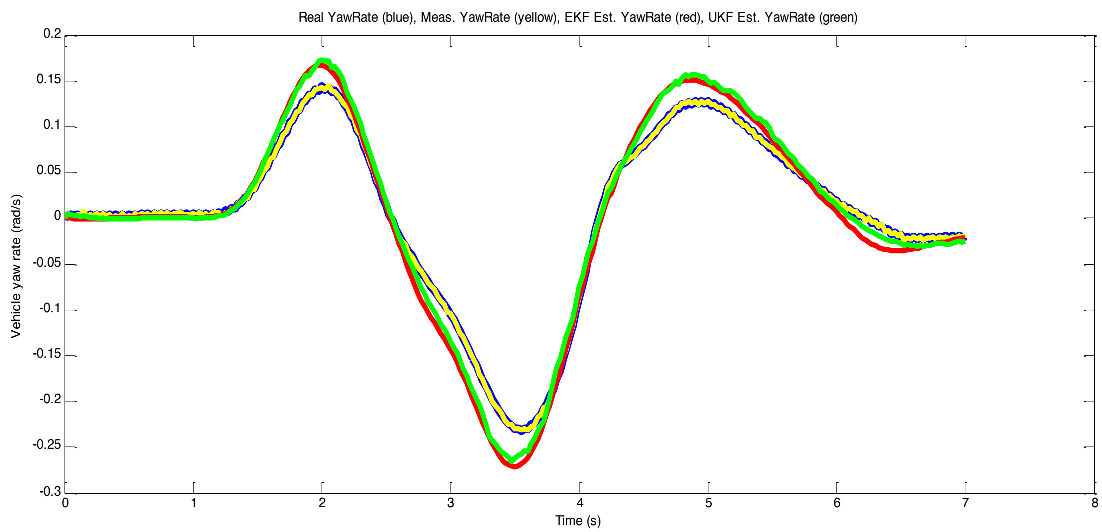


Figure 14. Estimations for vehicle yaw rate in Scenario 1. Estimated EKF: red, Estimated UKF: green, Real: blue.

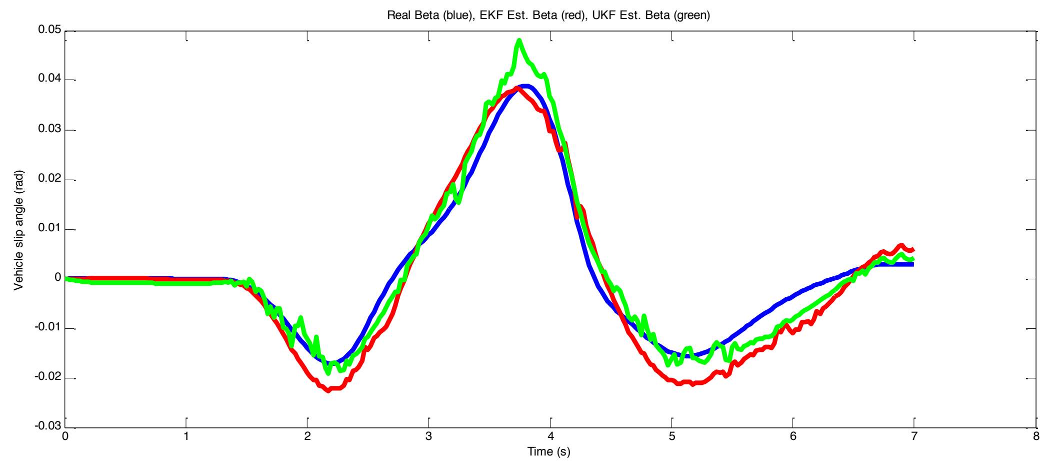


Figure 15. Estimations for vehicle slip angle Beta in Scenario 1. Estimated EKF: red, Estimated UKF: green, Real: blue.

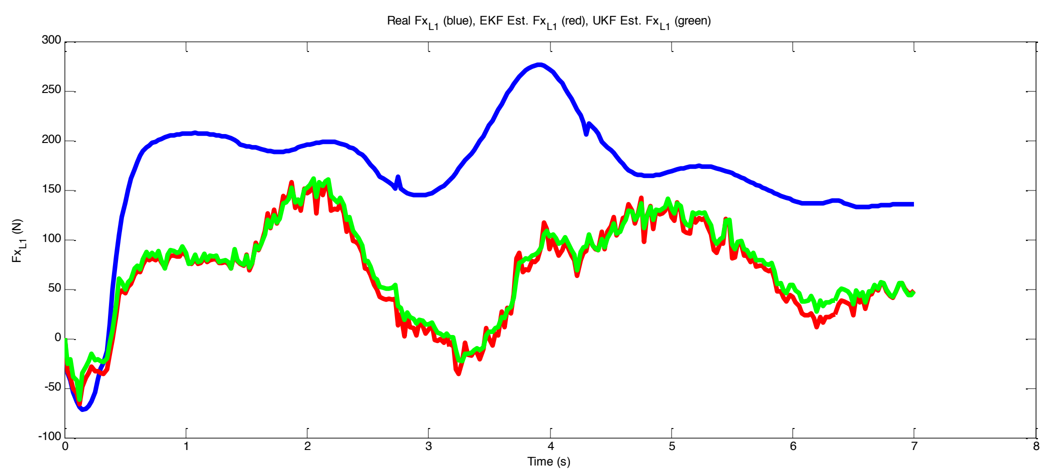


Figure 16. Estimation of longitudinal force of front left wheel in Scenario 1. Estimated EKF: red, Estimated UKF: green, Real: blue.

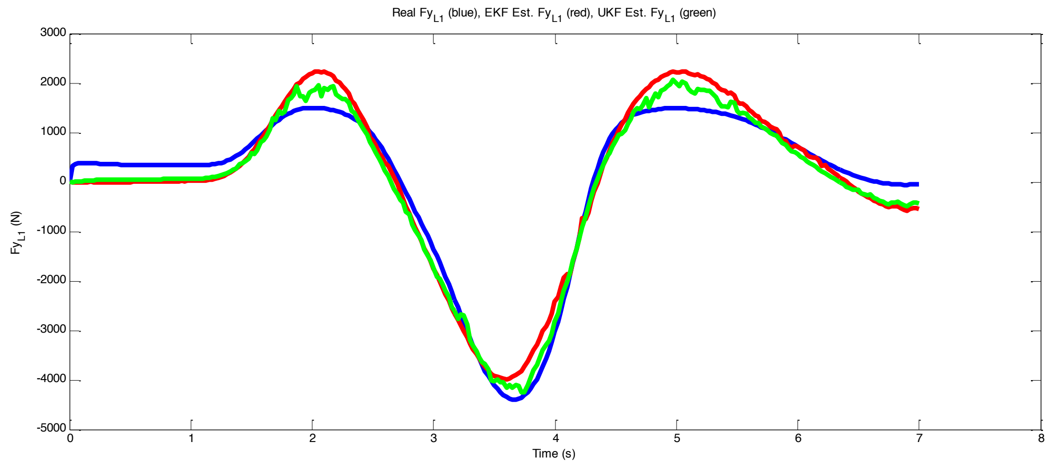


Figure 17. Estimation of lateral force of front left wheel in Scenario 1. Estimated EKF: red, Estimated UKF: green, Real: blue.

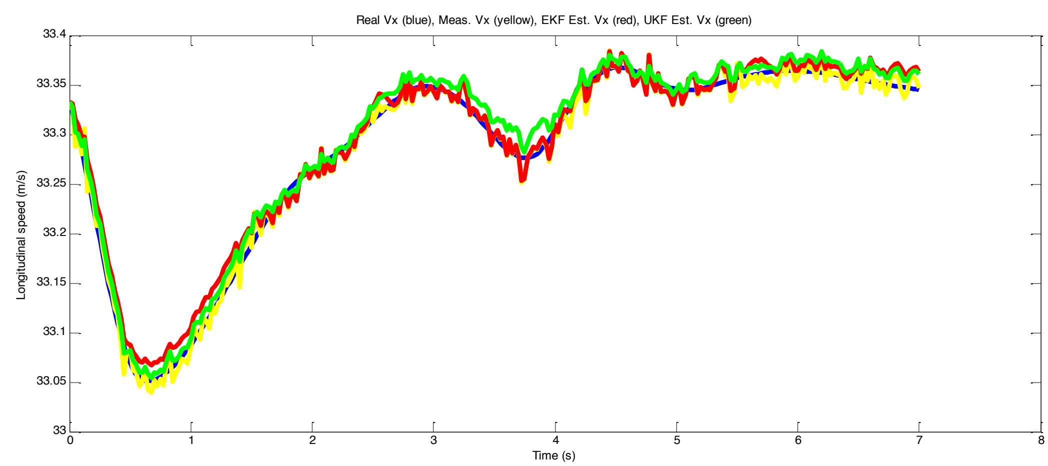


Figure 18. Estimations for longitudinal vehicle velocity Vx in Scenario 2. Estimated EKF: red, Estimated UKF: green, Real: blue.

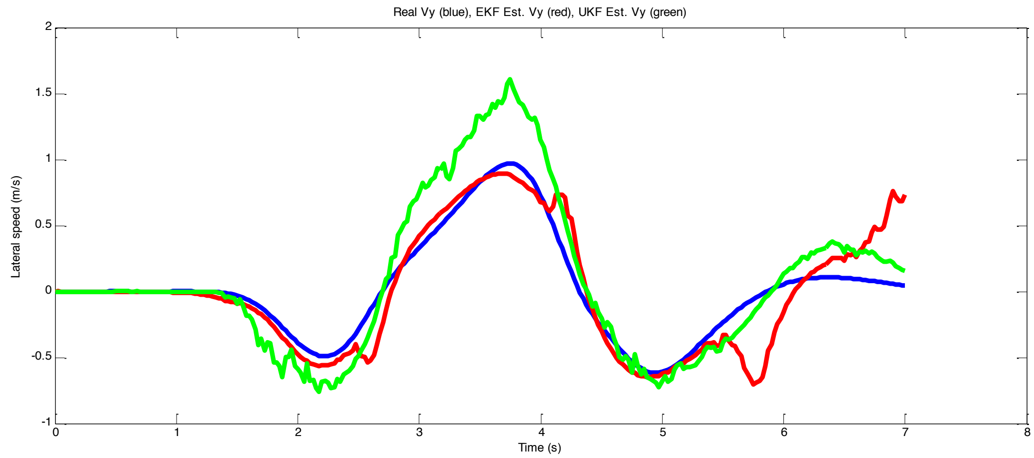


Figure 19. Estimations for lateral vehicle velocity Vy in Scenario 2. Estimated EKF: red, Estimated UKF: green, Real: blue.

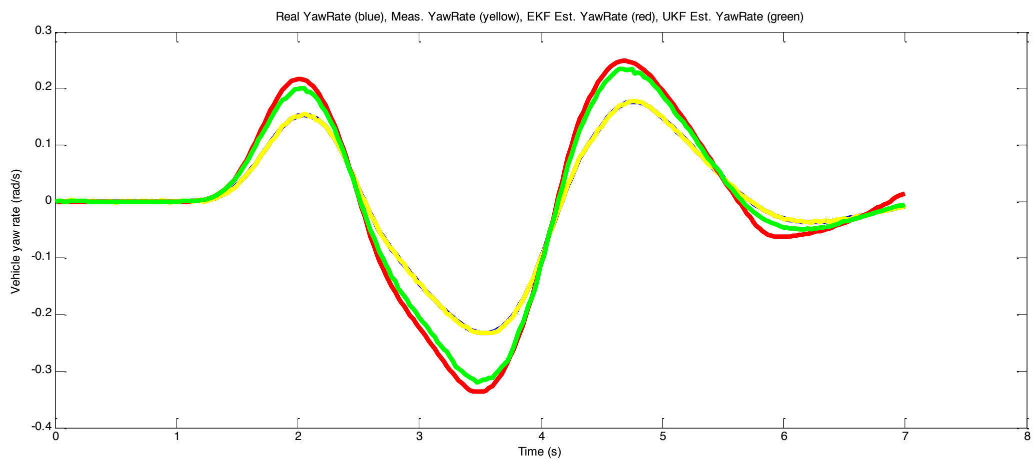


Figure 20. Estimations for vehicle yaw rate in Scenario 2. Estimated EKF: red, Estimated UKF: green, Real: blue.

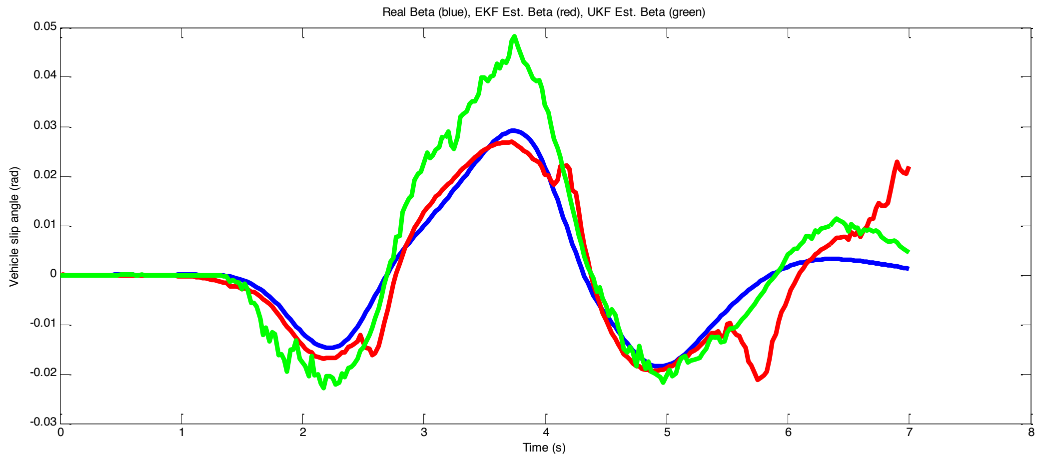


Figure 21. Estimations for vehicle slip angle Beta in Scenario 2. Estimated EKF: red, Estimated UKF: green, Real: blue.

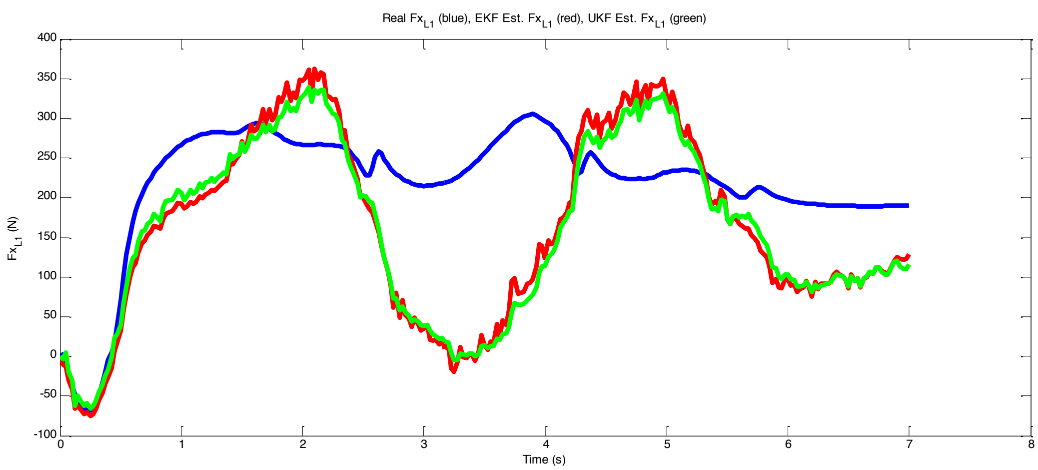


Figure 22. Estimation of longitudinal force of front left wheel in Scenario 2. Estimated EKF: red, Estimated UKF: green, Real: blue.

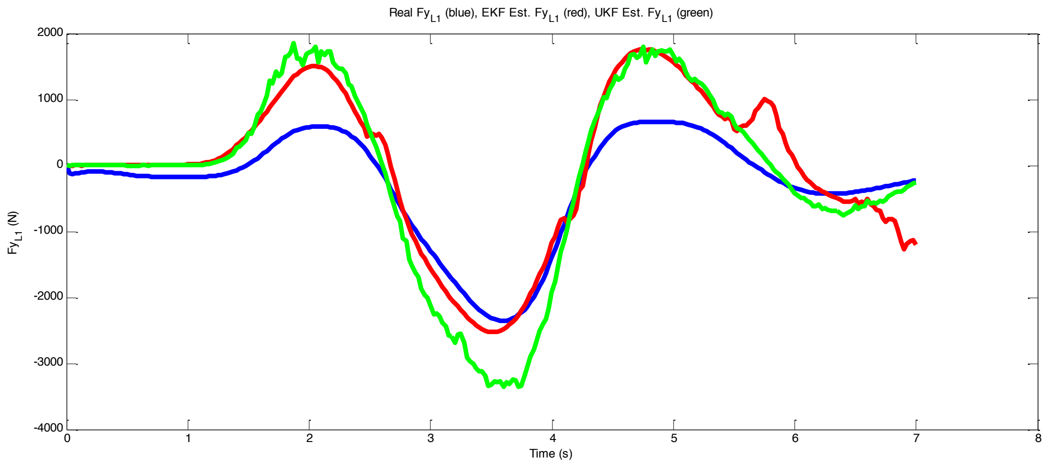


Figure 23. Estimation of lateral force of front left wheel in Scenario 2. Estimated EKF: red, Estimated UKF: green, Real: blue.

From the figures above, it can be seen that for both scenarios, estimations of Vx, Vy and yaw-rate from EKF and UKF are similar to its real value. However, estimations of wheel force, especially longitudinal wheel force, are different from real one. This may be introduced by the large simplification of tire force model.

Comparing estimations from UKF to those from EKF, EKF gives out results closer to the real value, while the results from UKF are more noisy and different from the real one. However, the last part of Figure 19, 20 and 23 show that UKF is more robust to interference compared to EKF.

The computation time of UKF is apparently longer than EKF, which is caused by more complex algorithm of UKF and UT. However, in this case, UKF does not give out more accurate estimations.

6. Conclusions

This chapter concludes the whole procedure of this study and reveal the found results. Conclusions are extracted and presented from the results.

In this study, the history and usage of Kalman filter and its derivations are investigated. Principles of Kalman filter, Extended Kalman Filter and Unscented Kalman Filter are revised. Four-wheel vehicle model (FWVM) is chosen as the vehicle model used in the filters. Its differential equations are then derived and simplified. Vehicle data from CarSim software are chosen as the input to the model and 'real value', which is then compare to. A Simulink model was then built based on the differential equations of FWVM to validate it. Its results coincide with value from CarSim well.

EKF and UKF based on the FWVM have been built. Their results are compared to the value from CarSim, and it is found out that:

Both EKF and UKF can provide estimation to the vehicle velocity, yaw-rate and tire force. The estimation of vehicle velocity and yaw-rate are quite close to the real one, but there are deviations between the estimation and real value of tire force. UKF uses more time to compute. However, in this case, its results deviate more from real value compared with EKF. UKF is more robust to interference.

7. Recommendations and future work

In this chapter, suggestions for future work on usage of Kalman filter and its derivations for vehicle applications are given.

In this study, it took a quite long time (more than 1 second per step) for both Kalman filters. The complexity of the vehicle model resulted in the very complicated inline functions used to predict the means. To compute them takes too much time, which makes the filters unpractical to be used in real time cases. If available, better vehicle model, or a better way to simplify it could help to reduce the computing time dramatically. But it may be hard to find such a simple yet precise vehicle model.

Because of limited conditions, no data from real vehicle is available during this study. Data from CarSim is used as substitution. CarSim can provide rough vehicle response to given conditions. However, there could be difference between CarSim value and real value. There are quite a lot of details or conditions that were not included in the CarSim algorithm and model. If condition allows, a vehicle equipped with speed, yaw-rate and tire force sensors, which can provide the variables discussed, will be a better choice.

8. References

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