Lab 2: Lateral Dynamics - State Estimation

SD2231– Applied vehicle dynamics control

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1 Task 1. Washout filtering approach of side-slip estimation

1.1 Task 1.a

1.1.1 Model Estimator

In this approach, the body side-slip is estimated through a model of the vehicle. Here, a bicycle model is used, which has the following simplifications.

- Y-axis simmetry
- no lateral, no longitudinal load transfer
- no roll pitch or jump movements
- no aerodynamic effect, no chassis suspension

The model equations are reported in the Handout for this lab:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{C_{12}+C_{34}}{v_x} & mv_x + \frac{l_f C_{12} - l_r C_{34}}{v_x} \\ \frac{l_f C_{12} - l_r C_{34}}{v_x} & \frac{l_f^2 C_{12} + l_r^2 C_{34}}{v_x} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} C_{12} \\ l_f C_{12} \end{bmatrix} u \tag{1}$$

where $x_1 = v_y, x_2 = \dot{\psi}, u = \delta$ tyre-to-road angle. We can express the relationship between δ and the steering wheel angle (SWA) as $\delta = K_s \cdot SWA$. From the model, it is possible to find an expression for $x_1(v_y)$ under the assumption that v_x is constant, and by assuming steady-state motion, meaning that $\dot{x}_1 =, \dot{x}_2 = 0$. The Simulink implementation of the model estimator is done through the layout presented in figure 1

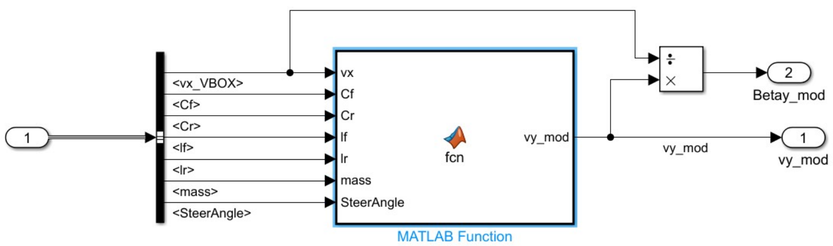


Figure 1: Model estimator implementation

The Matlab function contains the expression for v_y, obtained under the assumptions stated above.

$$v_y^{mod}(SWA, v_x) = \frac{v_x(l_r(l_r+l_f)C_{12}C_{34} - l_f - C_{12}mv_x^2)}{K_s((l_f+l_r)^2 C_{12}C_{34} + mv_x^2(l_r C_{34} - l_f C_{12}} \cdot SWA \tag{2}$$

In the implementation, we receive the signal SWA already from the VBOX (channel 39), then we define the signal `SteerAngle` as $\frac{SWA}{K_s}$.

Once v_y is estimated, since we expect $v_y^{mod} << v_x$ we can estimate $\beta^{mod} = atan(\frac{v_y^{mod}}{v_x}) \approx \frac{v_y^{mod}}{v_x}$

So, in conclusion, the model estimation technique works good in those cases in which the car is undergoing a steady-state maneuver, which is compliant with the assumption and simplifications needed for the bicycle model.

1.1.2 Integration method

An alternative way in which the estimation can be carried out is through integration method, which relies on utilizing kinematic relationships and data from the inertial measurement unit (IMU) of the vehicle. The IMU captures the accelerations and angular velocities of the car's body. From the kynematic relationship of a rigid body in lateral direction, we can obtain an expression for v_y^{kin}:

$$v_y^{kin} \approx \int_0^T (a_y(1-K_{roll}) - \dot{\psi}_z v_x)dt \tag{3}$$

Where K_{roll} is the roll gradient. The underlying assumptions made here to obtain such expression are:

- vertical velocity and pitch Euler Angle are assumed to be small
- body roll from suspension is assumed to be linearly depended on lateral acceleration
- road banking negligible

here is the layout of the implementation of the integration estimation.

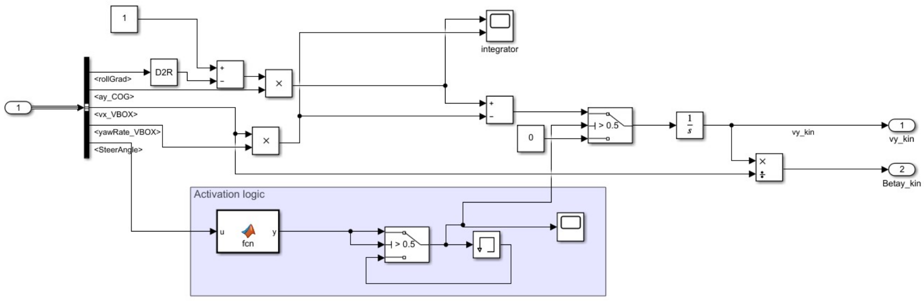


Figure 2: integration estimation method

One feature that we decided to implement in an activation logic based on the value of the steering angle. By inspecting the recorded data from the maneuvers, we noticed that in some simulations the recording starts before the meaningful maneuver is performed. Therefore the integrator will start to integrate the variables before the interesting section, increasing the integral charge.

This is an undesired charge, because we would like to integrate only from the moment the maneuver starts. To fix this we construct an activation logic, that integrates the measured variable from the moment that `steerAngle` is above a certain threshold (we chose 0.03) for the first time. Then, with the aid of the *memory* block, we make sure that the integration cannot be de-actived once it's been activated.

1.1.3 Washout filter

Because of the assumptions made in each of the two technique used, we have that:

- model estimation works best in steady state conditions, but during transient motion there will be errors
- Integration estimation are more suitable during transient phase of the motion, but it is subjected to drift

So, when facing a general maneuver, we can expected neither estimators to always produce an accurate estimation. By deploying a wash-out filter, we're able to combine the two estimators, by weighting them with a low-pass filter and a high-pass filter.

$$v_y^{washout} = \frac{1}{1+sT} v_y^{mod} + \frac{sT}{1+sT} v_y^{kyn} \tag{4}$$

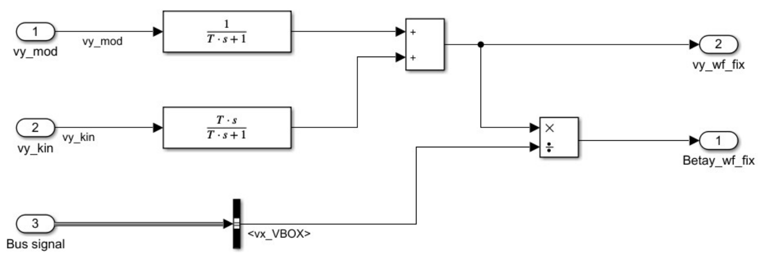


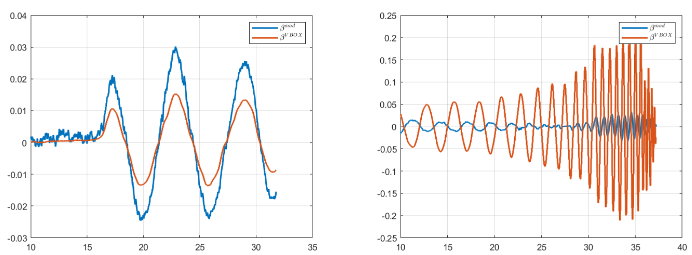
Figure 3: washout filter layout

The value T is a tunable parameter that select the cutoff frequency of the low-pass filter, so it determines how to fuse the two estimations together. With low value of T, the washout filter weights more the model estimation, disregarding the integration one. As T increases, the filter removes the high frequency components from the model estimation, and "replace" them with the one coming from the integral estimation. Once again, at first, the estimation of $v_y^{washout}$ is carried out, then on that the estimation of $\beta^{washout}$ is based.

1.2 Task 1.b

The values of cornering stiffness are tuned to improve the model in comparison to the test drives. The reasoning behind the tuning was trying to make the model estimation β^{mod} close to the real β, for small values of slip α. In the bicycle model, the lateral forces are modelled like $F = C \cdot (-\alpha)$, which is a valid approximation only for small values of α (slip angle of the tyre). Therefore, we focused our attention on the simulations in which the true value of α_{12}, α_{34} was small, namely test drive n°2 (*Slalom at 30 km/h*) and test drive n°4 (*Frequency sweep at 50 km/h*).

With the default values provided ($C_f = C_r = 100000$), the model estimation was not accurate enough

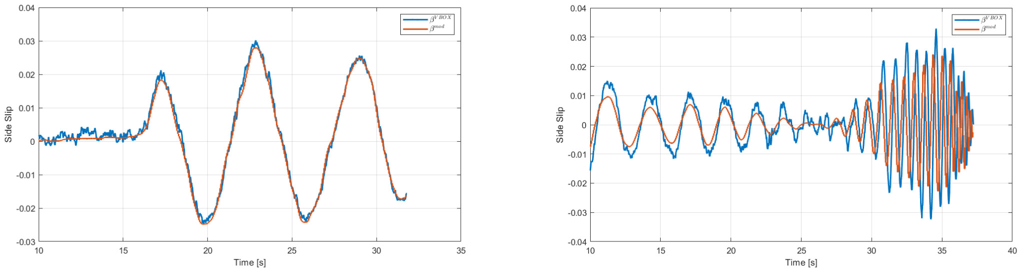


(a) sim2 with $C_r = C_f = 100000$ (b) sim4 with $C_r = C_f = 100000$

Figure 4: New values for cornering stiffness

In fig:4a we can see how the model of the estimation is wrong, while in fig:4b, both the model and the phase of the estimation are not correct.

In the tuning procedure, we noticed that the model estimation for sim2 was able to match nicely the true β without requiring to much extensive tuning, so we focused more on tuning the parameters until the model gave satisfactory values for the test drive with frequency sweep. The tuning was done manually until the amplitude was equally large and the oscillation was in phase. We settled on $C_f = 200000$ and $C_r = 250000$ and in figure 4 shows the improvement.



(a) sim2 with new values of C_f, C_r

(b) sim2 with new values of C_f, C_r

Figure 5: Default values of cornering stiffness

With the tuned values, in sim4 the model estimation is still slightly off-phased, while in sim2 the estimation matches nicely the real β. This can be explained by noticing that in sim2, v_x remains constant, as required by the assumptions of the model-based estimation, while that is not true for *sim*4, therefore we expect the model estimation to be more accurate in sim2. Moreover, in sim4 values of α_{12}, α_{34} are bigger than the ones in sim2. Therefore, the linear tyre model will be a better approximation of the real system for sim2 than sim4, justifying again the inaccuracy that we still have on sim4 with the newly tuned cornering stiffness.

The estimation for the model was also doubled checked for the other simulations. However, there were no big improvements in the model after tuning for the other test drives.

In our bicycle model, the lateral movements that can occur in the axle will affect the dynamics of the car and are accounted for by the lateral tyre stiffness parameter. The front wheels will have more movable parts and linkages to allow the wheels to turn. The rear wheels likely don't have that many different parts and will have a relatively higher tire stiffness. Due to these differences between them, the rear tyres will have a higher tyre stiffness compared to the front wheels.

Usually, high tire stiffness gives good linear approximations when the slip is low. We did not bother too much to tune the parameters to improve for the third simulation as that simulation has too high slip when the linear model would not be good.

1.3 Task 1.c

In the tuning procedure of T for the wash-out filter, all maneuvers recorded need to be taken into consideration and a trade-off needed to be made. Depending on the nature of the maneuver, one estimation technique can result in a more suitable value than the other due to how large the slip is and if it's a case when the model could be valid.

For instance, the first test (*Constant radius cornering*) is a steady-state cornering maneuver, which fits the hypothesis of the model estimation ($\dot{v}_y = 0$). For this test, β^{mod} is a better estimation than β^{kin}, which is subject to drift, and therefore it will be better to have a small T, such that the wash-out filter's estimation is mostly β^{mod}.

A similar analysis can be conducted for simulation 2 (*slalom at slow speed*, in which v_x remains constant and the maneuver is a slalom, which has a steady-state motion. Therefore using low T together with β^{mod} would give a good estimation in this case.

On the other hand, simulation 3 (*High speed step steer*), represents an abrupt steer maneuver, in which lateral speed will change very fast, steady-state assumption is not met. Here you are also having a large slip as a result of the lateral speed, which is a situation when the linear tire model would not be suitable for our bicycle model. This is what we also saw in the simulation with the model's estimation being very off from ground truth. In this scenario, the estimation performed by integrating the lateral acceleration (β^{kin}) is more appropriate. This translates to a desire to have a high T to make β^{kin} predominant in the output of the wash-out filter.

The frequency sweep is a tricky situation. You have constant speed during the test with increasing frequency on the steering input. In the beginning, the scenario is very similar to the slalom test drive and the situation can be approximated to steady-state when there are no large lateral velocities. In the end, however, the lateral velocity increases which can be explained by the car's momentum during cornering. In that situation, the linear tire model and transient motion are not suitable assumptions for estimating the slip. In the test drive, we did notice that the model was good in the beginning compared to the end. For this reason, our T needed to be set to a sweet spot between the two estimators for our final washout filter.

In the end, a common value needs to be selected, that ensures that the estimation in each test case is acceptable. Through trial and error, we selected T = 0.8 for the washout filter.

1.4 Task 1.d

To evaluate the quality of the estimation, the values of Mean Squared error and Max error are computed for each test case. Before performing this analysis, Data needs to be slightly pre-processed.

`Beta_VBOX` will have clear spikes whenever the longitudinal speed $v_x = 0$, causing therefore problems with the measurements. These portions of data are irrelevant and need to be cut out from the recorded data. The moments in which v_x might be 0 are the beginning and the end of recording: so we constrained the simulation time of the Simulink model to avoid these regions.

Secondly, when evaluating the quality of the estimators, is important also to define when such estimation is required to be accurate. To understand that, we looked at the `SteerAngle` and we selected a meaningful interval of time in which either the maneuver is performed, or in which the "transient phase" of the maneuver itself has terminated. An example of this is during the first simulation when the point of the test was to observe the slip during a steady-state motion. A steady-state motion happened between times t=14-47.28 seconds when the steering input is constant and there are fewer transient motions.

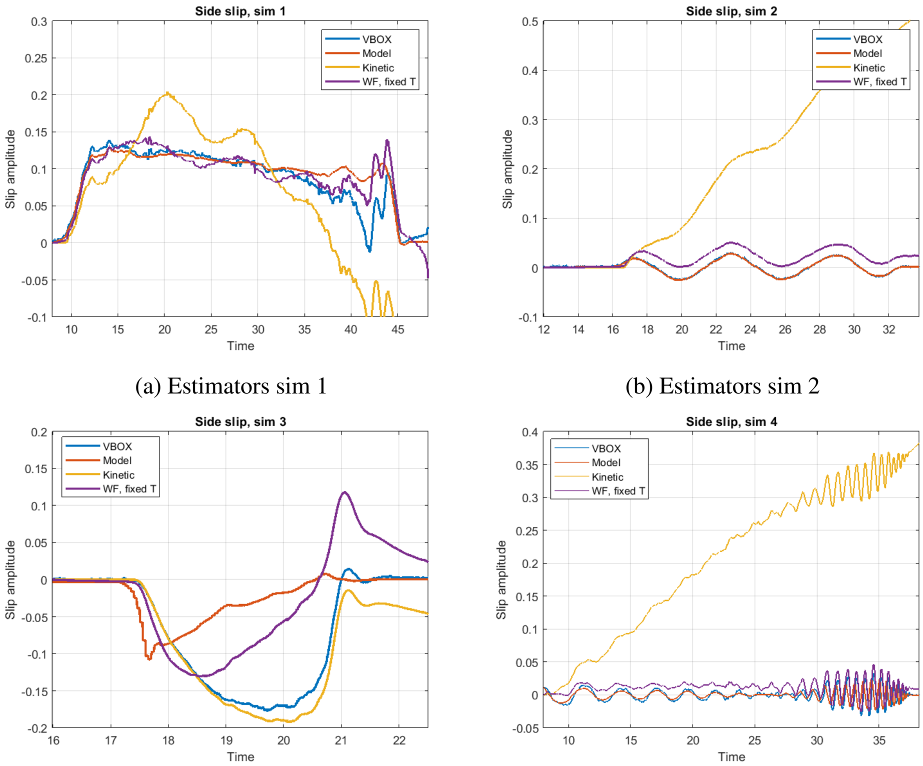
The recorded performance for each simulation is reported in the table 1 below.

	Range	MSE mod	MAX mod	MSE kin	MAX kin	MSE wf	MAX wf
sim 1	14 : 47.28	5.20e-04	9.37e-02	9.28e-03	4.85e-01	4.07e-04	7.5e-02
sim 2	15 : 31.78	1.37e-06	3.40e-03	6.66e-02	4.65e-01	5.2e-04	2.85e-02
sim 3	17 : 21.52	9.29e-03	1.54e-01	4.58e-04	5.98e-02	5.78e-03	1.31e-01
sim 4	10 : 37.13	4.66e-05	2.38e-02	5.08e-02	3.65e-01	2.5e-04	4.66e-02

Table 1: Performance of filters

Secondly, the recording of data starts before the maneuver of interest is performed, When performing the estimation, we decided to simulate the model only in a window of time around the maneuver of interest.

1.5 Task 1.e



(a) Estimators sim 1 (b) Estimators sim 2

(c) Estimators sim 3 (d) Estimators sim 4

Figure 6: All the different estimators.

The estimation of the lateral speed using the simplified bicycle model could do well when the longitudinal speed or the lateral speed is constant as the model is derived from constant states. In the first simulation with the circular motion, the lateral speed is constant during the test drive. This is a clear situation of when the model will do well, which was seen in the estimation for the side slip. However, at the end of first simulation, the lateral velocity changes abruptly together with the longitudinal speed when the driver is gaining control of the car. In this situation, the velocities are not constant and the model would not be valid. For this reason, we see a drift in the estimation for the model, which makes sense.

During the transient part of the first simulation, the integral is better at capturing the change in the side slip, which can be seen in the figure. The integral having better estimations for the transient phase can also be confirmed by the third simulation with the steep steering. This is a scenario where the velocities are changing abruptly and the model estimation drifts compared to the integral which is great at capturing the change even though it's not perfect. The model estimation also drifts in this scenario as this is likely a situation when the linear tire model might not hold when the slip angle is greater than 4 degrees, which is approximately the upper limit of the linear part of the curve for the slip.

The integral has the disadvantage of integrating from the beginning of time and will accumulate all the previous calculations, even if the values are small. If the sensor reading is bad, the integral will easily drift the longer it integrates. This was noticeable in all test drives that the integral would drift already from the beginning. We did try to let the integral stay deactivated until there was a large steering input, but the drifting still occurred.

This drawback of the integral is clearly an issue for the washout filter which adds the calculations from the model together with the result from the integral. With our choice T = 0.8. We could for example see an offset in the side slip during the fourth simulation that is added on top of the model. We saw that if we reduced the variable T, the offset would reduce. The washout filter would for that reason drift early in this test drive. On one hand, the choice of the T meant that we still were leaning more toward the model's estimation. The washout filter could be good at estimating the slip during the first, but it would also drift at the end of the simulation due to the issues the model has. The advantage the washout filter had was that it was better at capturing the transient motion at the end thanks to the integral.

1.6 Task 1.f

To improve the performance of the wash-out filter, we investigate now an implementation with a variable T.

The drawback that the previous implementation of the washout filter had, was that the value T, which determines how the two estimations are weighted together, was fixed, and could not adjust dynamically with the maneuver. This caused poor performance in those instances where the behaviour of the vehicle changed from transient to steady-state or viceversa.

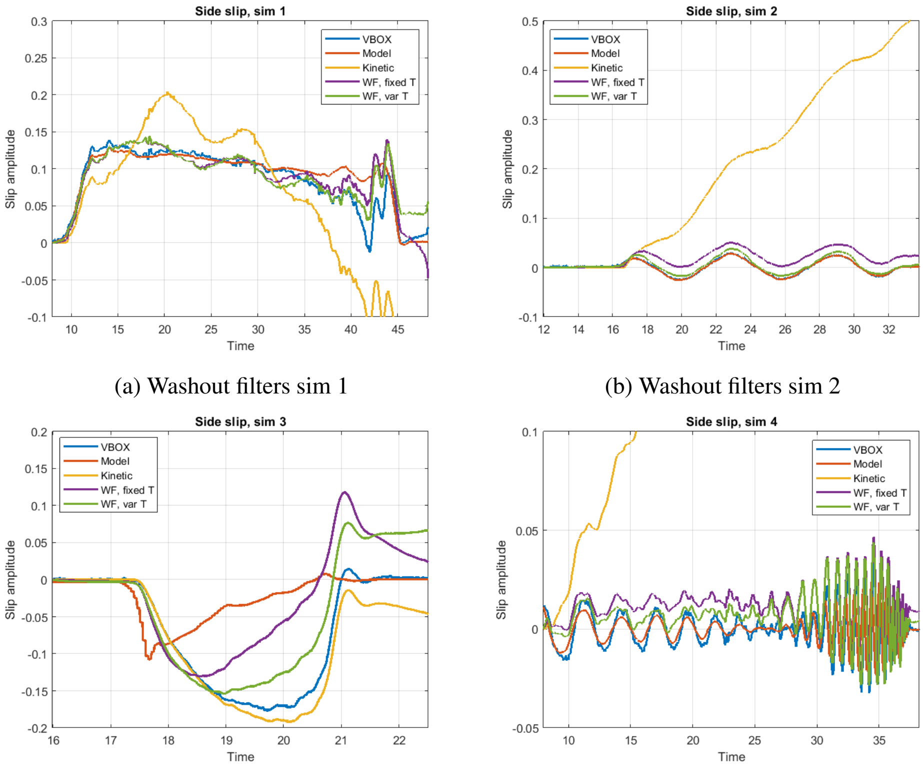
As a matter of fact, It can happen that a given maneuver has both a phase of transient motion and one of steady-state motion. The benefit of having a variable t is that it allows to dynamically adjust as the maneuver changes during the simulation. As suggested by the assignment, we designed a variable T that assigns more weight to the model estimation whenever the maneuver is less transient. To determine the such gain, we used a linear combination of two signals that describe "how transient the maneuver is". The signals involved are : yaw rate $\dot{\psi}$ and longitudinal acceleration a_x (suitably de-noised).

The first one represents how quickly a car is turning or changing direction of motion. The second one instead represents the change in longitudinal velocity, which also cause the motion to be transient (the steady-state motion, with the bicycle model, has as assumption a constant longitudinal velocity v_x). 7 Both variables are taken in their absolute value, because any changes (regardless of the sign) in yaw angle and longitudinal speed result in the motion to be transient, so positive and negative increments should be treated in the same way. Finally, we select a low threshold to avoid T being 0;

The computation of the varibale T is :

$$T_{variable} = min(\infty, max(0.01, |\dot{\psi}| + 1.5|a_x|) \tag{5}$$

So, as the motion is transient(meaning low value of $\dot{\psi}, a_x$), T is small, and the washout filter assigns more weight on the model estimation. The reported values of MSE and MAXERR between the fixed and variable washout filter can be seen in table 2.



(a) Washout filters sim 1

(b) Washout filters sim 2

(c) Washout filters sim 3

(d) Washout filters sim 4

Figure 7: All the different estimators compared with the improved washout filter.

	MSE fix	MAX fix	MSE var	MAX var
sim1	4.08e-04	7.50e-02	3.66e-04	5.55e-02
sim2	5.20e-04	2.85e-02	3.34e-05	1.18e-02
sim3	5.78e-03	1.31e-01	1.25e-03	8.10e-02
sim4	2.50e-04	4.66e-02	1.70e-04	4.37e-02

Table 2: The table shows the MSE and maximum error for washout filter with fixed T and changing T.

The implementation of a variable T makes the washout filter able to adjust to the specific maneuver, therefore reducing the trade off that we had to make in the design of the fixed washout filter before. As we can see in Figure 7, green plot represent the variable washout filter, while purple lines are the estimations done with the constant washout filter with T = 0.8.

The fixed washout filter has a worse estimation in both cases, because when designing it, we had to select a value that was acceptable for all simulations. Instead, the improved filter is able to assign higher values to transient motions when needed, as well as weighting more the model estimation when the motion is steady state.

We are now comparing Figure 7b (steady-state motion) and Figure 7c (highly transient motion). By involving a variable T, we can adjust the value according to each simulation: as it appears, in simulation 2, T is much lower than 0.8, since the steady-state motion performed (slalom with constant speed) makes the model estimation better. On the other hand, in simulation 3, Equation 5 is able to give values of T higher than 0.8, preferring here the integration estimation. These improvements on performance indicates that Equation 5 fits the purposes for which it was designed.

2 Task 2. Unscented Kalman Filter estimation

2.1 Task 2.a

In the function `Vehicle_state_eq`, the equations for calculating $\dot{v}_x$, $\dot{v}_y$, and $\ddot{\psi}_z$ were added to calculate the predicted state of v_x, v_y, and $\dot{\psi}_z$ for the sigma points by using Runge-Kutta. The equations we added for the derivatives were

$$\dot{v}_{x,t} = \frac{-F_{12,t-1}\sin\delta}{m} + \dot{\psi}_{z,t-1}v_{y,t-1} \tag{6}$$
$$\dot{v}_{y,t} = \frac{F_{34,t-1} + F_{12,t-1}\cos\delta}{m} - \dot{\psi}_{z,t-1}v_{x,t-1} \tag{7}$$
$$\ddot{\psi}_{z,t} = \frac{l_f F_{12,t-1}\cos\delta - l_r F_{34,t-1}\cos\delta}{I_z} \tag{8}$$
$$\tag{9}$$

where linear tire model is used to calculate the lateral forces with

$$F_{12} = -C_{12}\alpha_{12} \tag{10}$$
$$F_{34} = -C_{34}\alpha_{34} \tag{11}$$
$$\alpha_{12} = \arctan\left(\frac{v_y + \dot{\psi}_z l_f}{v_x}\right) - \delta \tag{12}$$
$$\alpha_{34} = \arctan\left(\frac{v_y - \dot{\psi}_z l_f}{v_x}\right). \tag{13}$$

Our measurement equations, implemented inside `Vehicle_meas_eq`, used the predicted state from the sigma points. The measurement variables are v_x, a_y, and $\dot{\psi}_z$ using the predicted states v_x, v_y, and $\dot{\psi}_z$ that are outputted from `ukf_predict1`. The measurement equations are

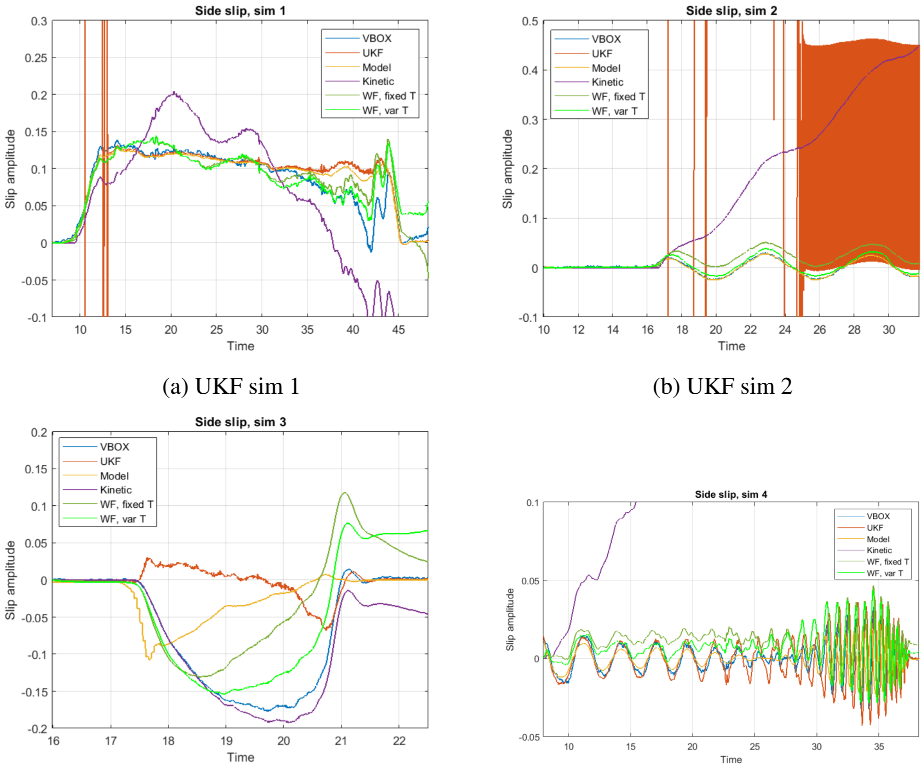
$$v_x = v_x \tag{14}$$
$$a_y = \frac{F_{34} + F_{12}\cos\delta}{m} \tag{15}$$
$$\dot{\psi}_z = \dot{\psi}_z. \tag{16}$$

The forces here are the same as equations 10 and 11.

The for-loop inside `UKF_start`, contains `ukf_predict1` that takes in previous time step's states, covariance matrix, `Vehicle_state_eq` as function handler, process noise Q, and the steering angle at that time step. The function returns a matrix with all the sigma points propagated through the dynamic model together with the covariance matrix P. They are sent into `ukf_update1` together with the measurements with the measurement variables from `VBOX`, the measurement function as function handler, measurement noise R, and the steering angle at that time instance.

The initial state that was chosen from the beginning was $x_0 = [0.01, 0, 0]^T$ and the initial covariance matrix with diagonal elements $\mathrm{diag}([0.01, 0.1, 0.1])$ on the diagonal in the 3x3 matrix. Process noise Q had $\sigma^2 = 0.1$ in its diagonal elements and measurement noise R had $\sigma^2 = 0.01$ in its diagonal elements.

2.2 Task 2.b



(a) UKF sim 1

(b) UKF sim 2

(c) UKF sim 3

(d) UKF sim 4

Figure 8: Base UKF compared to the other estimators.

When comparing the base UKF with the other estimators from task 1, we saw that the UKF was good when the motion was close to steady-state which is at the beginning of the first simulation, and beginning of the fourth simulation in figure 8a and 8d. The UKF did well on the transient part of the frequency sweep by having a reasonable amplitude in figure 8d. This is much better than the model that was bad during the transient part and the washout filter that had an offset caused by the drift in the integral.

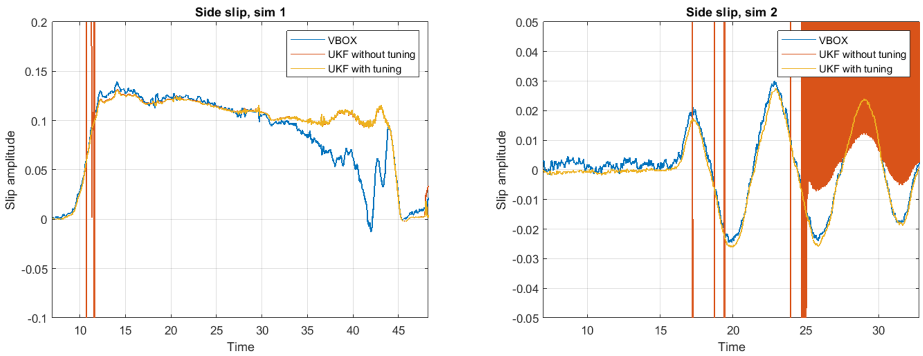
We also saw that the UKF was good at the beginning of the simulation for the circular motion when it is steady-state. The UKF was also very close to the estimation from the model. However, in the end during the transient part, both the model and UKF are bad. A reason why this happens is likely due to modelling with a linear tyre model that is not good for large side slips, which causes both estimators to fail. As previously mentioned, this is the case when the washout filter would be better than the UKF thanks to accounting for the transient motion from the integral estimator. This guess for the issues of using the linear tyre model could also be confirmed by the third simulation in figure 8c when the UKF does significantly worse than the other estimators. In the third simulation, there are certainly large side slips that would affect the forces calculated in the UKF which clearly affected the UKF. To avoid the issue of the slip being positive instead of negative, a lower tyre stiffness could be used which we saw improved the result. As there is a linear relation between the force and the slip, having lower tyre stiffness would mean that the force would be lower with greater slip and the same results would not have been seen. In the end, one should remember that there are fundamental issues with using this linear tyre model for this simulation.

Regarding simulation 2 in figure 8b, the estimation was not good with our initial covariance where there are large values on the slip. We saw that if we reduced the initial variance on the yaw rate to 0.01, the estimation became better. This tells us it's important to also choose a good initial state and covariance as the estimation starts with these values to predict the state and create the sigma points and the uncertainty matrix. Having a lower uncertainty on the yaw rate could make sense for simulation 2 as there is a more constant longitudinal speed and predictable yaw rate during the simulation, which gives room to have lower uncertainty on the yaw rate. These states are also estimated which could be affected by the initial covariance. The errors achieved from using UKF can be seen in table 3.

	MSE of UKF	MAX of UKF
sim1	2.48e-03	1.43e-01
sim2	2.00e0	1.60e0
sim3	1.45e-02	2.24e-01
sim4	2.46e-04	7.47e-02

Table 3: Performance of the UKF

2.3 Task 2.c



(a) UKF sim 1 (b) UKF sim 2

Figure 9: Base UKF before and after tuning.

One of the big changes after adding α, β, κ is that we were able to remove the spikes that occurred before in simulation 1 as seen in figure 9a. In simulation 2 we had bad estimation with our previous tuning and initial covariance, which also improved as seen in figure 9b. The rest had similar estimations as before. This is a sign that the estimation is better. The purpose of these parameters is they are used to determine how the sigma points should spread out from the sigma point in the center, which is the state from the previous time step, or the mean when working with Gaussian distribution. We saw that slightly bringing the sigma points closer improves the result. The closer we bring the sigma points to the mean, the higher probability is assumed that the next state will be close to the mean (previous time step). This change reduces the chance that the prediction is wrong and cannot be corrected that well by the update step. The values we chose was $\alpha = 0.8, \beta = 2, \text{and} \kappa = 0$. We set kappa to zero to avoid the scaling from kappa affecting the scaling of the α, which is one common method to bring the sigma points closer. Setting $\beta = 2$ is done as we want to assume there is a Gaussian distribution, which is reasonable in the sense that the next state will not be far away from the previous state.

We tried improving the estimation by changing the process noise Q and measurement noise R. Increasing the variance is done when you are more uncertain about the predicted estimation or the measurement from the IMU. The IMU can contain some noisy measurements as most of the variables are calculated from acceleration. When studying the acceleration, one could see that measurement is noisy and would also affect the final estimation such that it diverged. Therefore, to reduce how much the measurement will correct the predicted estimation, a higher variance helps with that so one gets a lower Kalman gain, which is used to correct the predicted estimation. All the variances in R were increased to reduce the effect of the noise. The final measurement noise was R = diag(0.1,0.2,0.08) where the diagonal elements in the matrix correspond to the measurement variable v_x, a_y, and $\dot{\psi}_z$.

We also chose to slightly decrease the variances in process noise but kept the variance for lateral velocity higher as this is estimated and the model is not perfect. Reducing the variances for the other states also helped because the resulting ratio between Q and R will decide the Kalman gain. If we increased the process noise, you would come closer to a Kalman gain similar to before tuning and the estimation would be slightly more noisy as it depends more on the measurement again when increasing the process and measurement noise together. The process noise we settled on was Q = diag(0.04,0.07,0.03).

One should therefore remember that it's possible to have different tuning on the noises and achieve similar results because the Kalman gain could be similar. The noises we chose happened to be those presented here because of our manual tuning with trial and error rather than using an exhaustive algorithm to find the optimal process and measurement noise. When tuning using Q and R-matrix, there were no significant changes as mentioned earlier, but we did see some improvement in the MSE and maximum error.

In the end, most of the improvements were achieved thanks to the tuning of $\alpha, \beta, \text{and} \kappa$ which brought the sigma points closer to the mean in order to avoid diverging estimations. The process and measurement noise did help a little bit, but no significant changes were seen than some small changes in the error. The big improvements were seen in simulations 1 and 2 which are seen in figure 9 and the errors for all simulations are seen in table 4.

	MSE no tuning	MAX no tuning	MSE tuning	MAX tuning
sim1	2.48e-03	1.43e-01	2.48e-03	1.43e-01
sim2	2.00e+0	1.60e0	4.99e-04	5.75e-02
sim3	1.45e-02	2.24e-01	1.44e-02	2.23e-01
sim4	2.46e-04	7.47e-02	2.46e-04	7.36e-02

Table 4: The table shows the MSE and maximum error for washout filter with fixed T and changing T.

2.4 Task 2.d

As previously mentioned there were some issues with UKF compared to the other estimators that likely were due to using a linear tyre model. To handle this issue, we changed to working with the Brush tyre model which is non-linear and would model the transient dynamics better than the linear model.

The equations to estimate the states $\dot{v}_x$, $\dot{v}_y$, and $\ddot{\psi}_z$ are the same from previous task. The only change is how the forces are calculated which follows

$$F_y = -C_\alpha \tan(\alpha) f(\lambda) \tag{17}$$

$$f(\lambda) = \begin{cases} (2-\lambda)\lambda & \text{if } \lambda \leq 1 \\ 1 & \text{if } \lambda > 1 \end{cases} \qquad \text{where } \lambda = \frac{\mu F_z}{2C_\alpha |tan(\alpha)|}. \tag{18}$$

and the equations are taken from the chapter about tyre and tyre models from the course literature in Vehicle Dynamics. The new brush model can be seen as an extension of the linear model: for small values of α, (which corresponds to $\lambda > 1$), the model is indeed the same as before. The difference is for high values of $alpha$ ($\lambda \leq 1$), for which now, a new equation is provided, that represents better the saturation of the lateral force.

The assumption of the bicycle model will still be valid, the only difference will be in the computations of the lateral forces that are derived from the equilibrium equation and equation for torque. For F_z, we computed them in the standing still configuration: so the vertical load is distributed between the front tyre and the rear one proportionally to l_f, l_r which expresses the position of the axles from the car's center of gravity. So, we will have

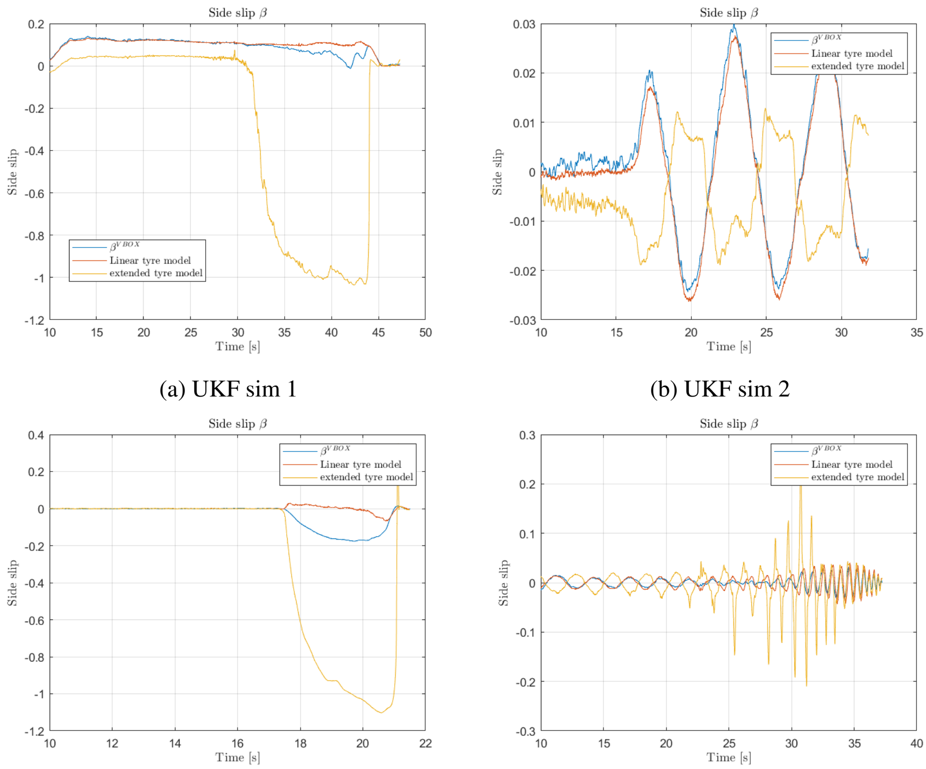
$$F_{z12} = \frac{mass \cdot g \cdot l_r}{L} \tag{19}$$

$$F_{z34} = \frac{mass \cdot g \cdot l_f}{L} \tag{20}$$

The lateral slip for each wheel with a lateral tyre stiffness C_α is calculated according to equations 12 and 13.

Since we're implementing a new tyre model, the values of cornering stiffness we used before might not be the best fit now: that is because the tuning was aimed to obtain the best representation of lateral forces with a linear model of the tyre.

The value of cornering stiffness is kept the same as the previous tuning, because in the new model, for the low value of slip, the forces are still computed with the linear model, therefore it seemed reasonable to maintain the values that we tuned before. $C_f = 200000$ N/rad, $C_r = 250000$ N/rad.



(a) UKF sim 1 (b) UKF sim 2

(c) UKF sim 3 (d) UKF sim 4

Figure 10: UKF with brush model for tyre

By running the simulations, we noticed that the new estimations coming from the extended brush model for the tyre were incorrect: By switching to a more extensive model we obtained the counter-intuitive result that the estimation of the side slip angle is wrong.

The conclusions that we can draw from here are that probably the implementation of the new extended model is probably wrong, causing this unexpected result in the simulation.

Another possible source of error could be coming from the tuning that we chose for the cornering stiffness since these values are involved in the computation of λ, a too aggressive tuning of these variables can propagate the errors into the new model.

Secondly, again, in the computation of λ we assumed that the vertical forces can be computed from the standing still configuration and that the weight of the vehicle is distributed among the tyre based on the position of the center of gravity. This assumption might be too strict, since during any maneuver, there will be inevitably load transfer, therefore changing the value of F_z throughout the simulation.