# Exercitii - 1

**I.** Calculati urmatoarele integrale curbilinii folosind definitia integralei curbilinii de speta a doua.

(1) $\int\limits_C (x+2y)dx + x^2ydy$ unde curba C este frontiera domeniului $D = \{(x,y) \in \mathbb{R} | x^2 + y^2 \leq 4, x \leq 0\}$, parcursa in sens invers acelor de ceasornic.

(2) $\int\limits_C xdx + (x+y)dy$ unde curba C este frontiera domeniului $D = \{(x,y) \in \mathbb{R} | x^2 + y^2 \leq 4, x \leq 0, y \geq 0\}$, parcursa in sens trigonometric.

(3) $\int\limits_C ydx + (2x-y)dy$ unde curba C este frontiera domeniului $D = \{(x,y) \in \mathbb{R} | x^2 + y^2 \leq 1, y \leq 0\}$, parcursa in sens trigonometric.

(4) $\int\limits_C (x+y)dx - xydy$ unde curba $C = [AB] \cup [BC] \cup [CA]$ unde $A(1,-1)$, $B(1,3)$ si $C(4,3)$.

(5) $\int\limits_C (2x+y)dx - 2xdy$ unde curba $C = [AB] \cup [BC] \cup [CA]$ unde $A(0,0)$, $B(3,0)$ si $C(0,6)$.

**II.** Calculati integralele de la exercitiul anterior folosind formula lui Green.

**III.** Calculati lucrul mecanic al fortei $\overline{F}(x,y,x) = y\bar{i} + x^2\bar{j} - (x+y+z)\bar{k}$ al carei punct de aplicatie descrie curba

$$C: \begin{cases} x = 2\cos t \\ y = 3\sin t \quad , \quad t \in [0, 2\pi]. \\ z = t \end{cases}$$

**IV.** Calculati urmatoarele integrale duble

(1) $\iint_D (x+xy)dxdy, \;\; D = \{(x,y) \in \mathbb{R}^2 : x^2 + \frac{y^2}{4} \leq 9, \; x \geq y\}$

(2) $\iint_D (x^2+y)dxdy, \quad D$ este limiat de curbele $y = x^2, x = y^2$.

(3) $\iint_D \operatorname{arctg}\frac{y}{x} dxdy, \quad D = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \le 9,\ x \le \sqrt{3}y \le 3x\}$

(4) $\iint_D \left(1 + \sqrt{x^2 + y^2}\right) dexdy, \ D = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 - y \le 0, x \ge 0\}$

(9) $\iint_D \frac{1}{(y+x)^2},$ $D$ este limitat de dreptele $y - 2x = 0, y + 2x = 0,\ y = 1,\ y = 2$

**V.** Calculati aria urmatoarelor suprafate

1) $S = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 4,\ x \ge 0,\ z \ge 0,\ x \le y \le x\sqrt{3}\}$

2) $S = \{(x,y,z) \in \mathbb{R}^3 : 2x + y + 2z = 1,\ x^2 + y^2 \le 1\}$

**VI.** Calculati urmatoarele integrale de suprafata

1) $\displaystyle\iint_S z d\sigma,\ S = \{(x,y,z) \in \mathbb{R}^3 : 3x + 2y + z = 6,\ x^2 + y^2 \le 4,\ y \ge 0\}$

2) $\displaystyle\iint_S y\sqrt{x} d\sigma \quad S = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = 4,\ 0 \le z \le 2\}$

3) $\displaystyle\iint_S z d\sigma,\ S = \{(x,y,z) \in \mathbb{R}^3 : z^2 = x^2 + y^2,\ x^2 + y^2 \le 2y,\ z \ge 0\}$

4) $\displaystyle\iint_S (x - y) d\sigma, \quad = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \le 9,\ x \ge 0,\ y \ge 0,\ z \le 0\}$

**VII.** Calculati fluxul campului vectorial $\overline{F}$ prin suprafata $S$ orientata dupa normala interioara

1) $\overline{F} = y^2\overline{i} - x\overline{i} + z\overline{j} + (x+y)\overline{k}, \quad S = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 4, x, y, z \ge 0\}$

2) $\overline{F} = y^2\overline{i} - x\overline{i} + z\overline{j} + (x+y)\overline{k},\ S = \{(x,y,z) \in \mathbb{R}^3 : z = x^2 + y^2, 0 \le z \le 1\}$

3) $\overline{F} = x\overline{i} + y\overline{i} + z\overline{j} + (x+y)\overline{k},\ S = \{(x,y,z) \in \mathbb{R}^3 : z^2 = x^2 + y^2, 0 \le z \le 4\}$

**VIII.** Calculati fluxul campului vectorial $\overline{F} = z\overline{i} + x\overline{j} + z\overline{k}$ prin fata exterioara a tetraedrului limitat de planele $x = 0,\ y = 0,\ z = 0,\ x + y + 3z = 6$