In [2]:

```
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as sps
import math
%matplotlib inline
```

In [135]:

```
alpha = 0.95
theta = 1
num_tests = 1000
N = 100
```

In [136]:

```
def test_probability(llvl,rlvl,num,distr_gen):
    qnt = 0.0
    for i in range(num):
        X = distr_gen()
        if llvl(X,alpha) < theta < rlvl(X,alpha) :
            qnt+=1

    return round(qnt / num,3)
```

Доверительные интеравалы уровня α равномерного распределения :

$$\left(\frac{2\bar{X}}{1+\frac{1}{\sqrt{3n(1-\alpha)}}}, \frac{2\bar{X}}{1-\frac{1}{\sqrt{3n(1-\alpha)}}} \right)$$

$$\left(X_{(1)}, \frac{X_{(1)}}{1-\sqrt[n]{\alpha}} \right)$$

$$\left(X_{(n)}, \frac{X_{(n)}}{\sqrt[n]{1-\alpha}} \right)$$

In [37]:

```
def trust_level_uniform_average_l(X, alpha) :
    l = 2 * np.mean(X)/(1 + 1/(3 * len(X) * (1-alpha))**0.5)
    return l
def trust_level_uniform_average_r(X, alpha) :
    r = 2 * np.mean(X)/(1 - 1/(3 * len(X) * (1-alpha))**0.5)
    return r

def trust_level_uniform_min_l(X, alpha) :
    l = np.min(X)
    return l
def trust_level_uniform_min_r(X, alpha) :
    r = np.min(X)/(1 - alpha**(1/len(X)))
    return r

def trust_level_uniform_max_l(X, alpha) :
    l = np.max(X)
    return l
def trust_level_uniform_max_r(X, alpha) :
    r = np.max(X)/((1 - alpha)**(1/len(X)))
    return r
```

In [73]:

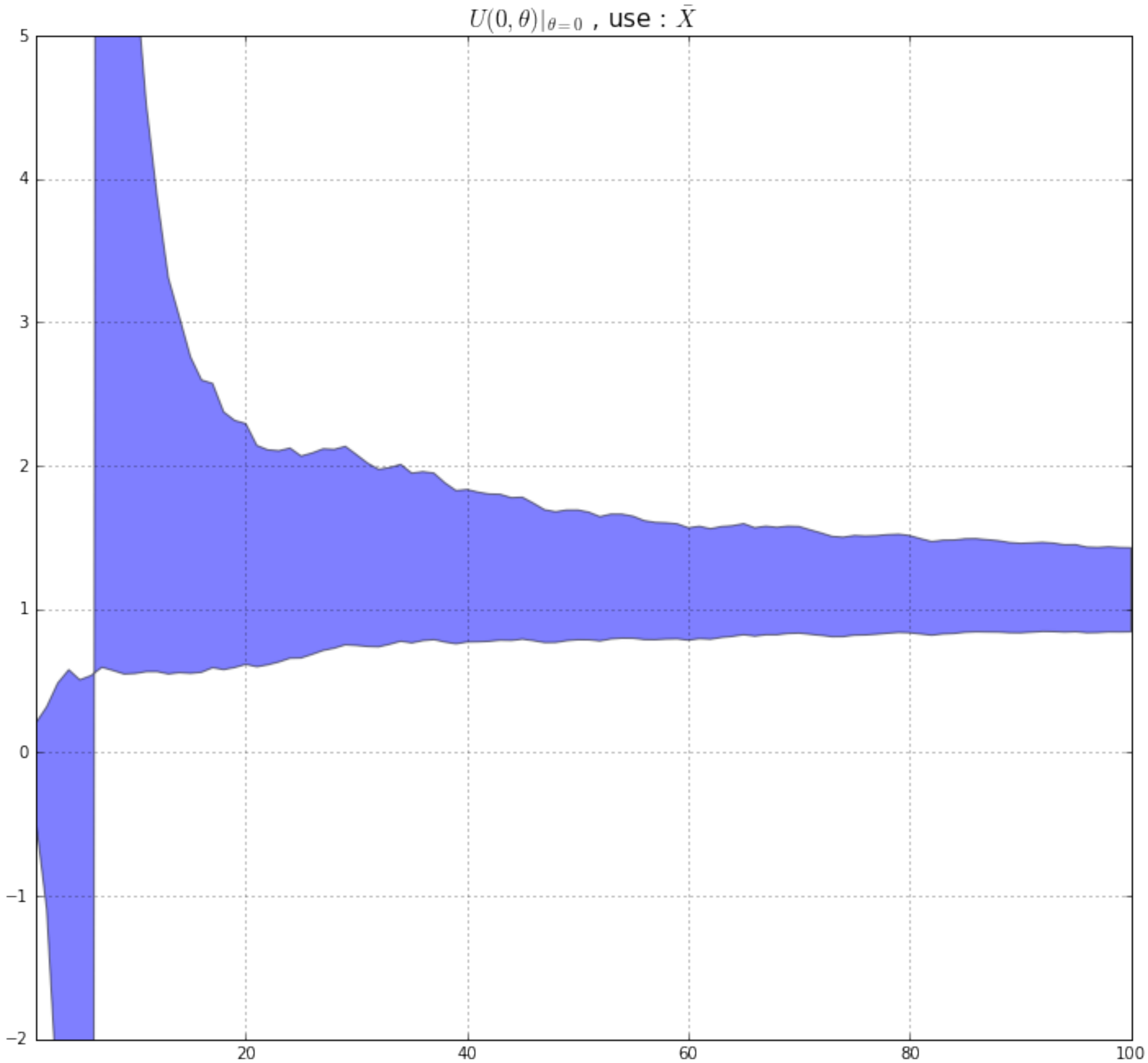
```
def build_plot(OX,low,high,distr,method,xlim=[],ylim=[]):
    title = distr + ' , use : ' + method
    plt.figure(figsize=(12,11))
    plt.fill_between(n,low,high,alpha=0.5)
    plt.grid(True)
    plt.title(title,fontsize=15)
    if len(ylim) == 2 :
        plt.ylim(ylim)
    if len(xlim) == 2 :
        plt.xlim(xlim)
    plt.show()
```

In [97]:

```
X = sps.uniform.rvs(size=N)
n = np.arange(1,N+1,1)
```

In [140]:

```
trust_avg_l = [trust_level_uniform_average_l(X[0:k],alpha) for k in n]
trust_avg_r = [trust_level_uniform_average_r(X[0:k],alpha) for k in n]

build_plot(n,trust_avg_l,trust_avg_r,
           r'$U(0,\theta)|_{\theta = 0}$',r'$\bar{X}$',
           [1,100],[-2,5])
for k in {10,100} :
    print('n = ' + str(k) + ', P = '
          + str(test_probability(trust_level_uniform_average_l,
                trust_level_uniform_average_r,
                num_tests,
                lambda : sps.uniform.rvs(size=k))))
```

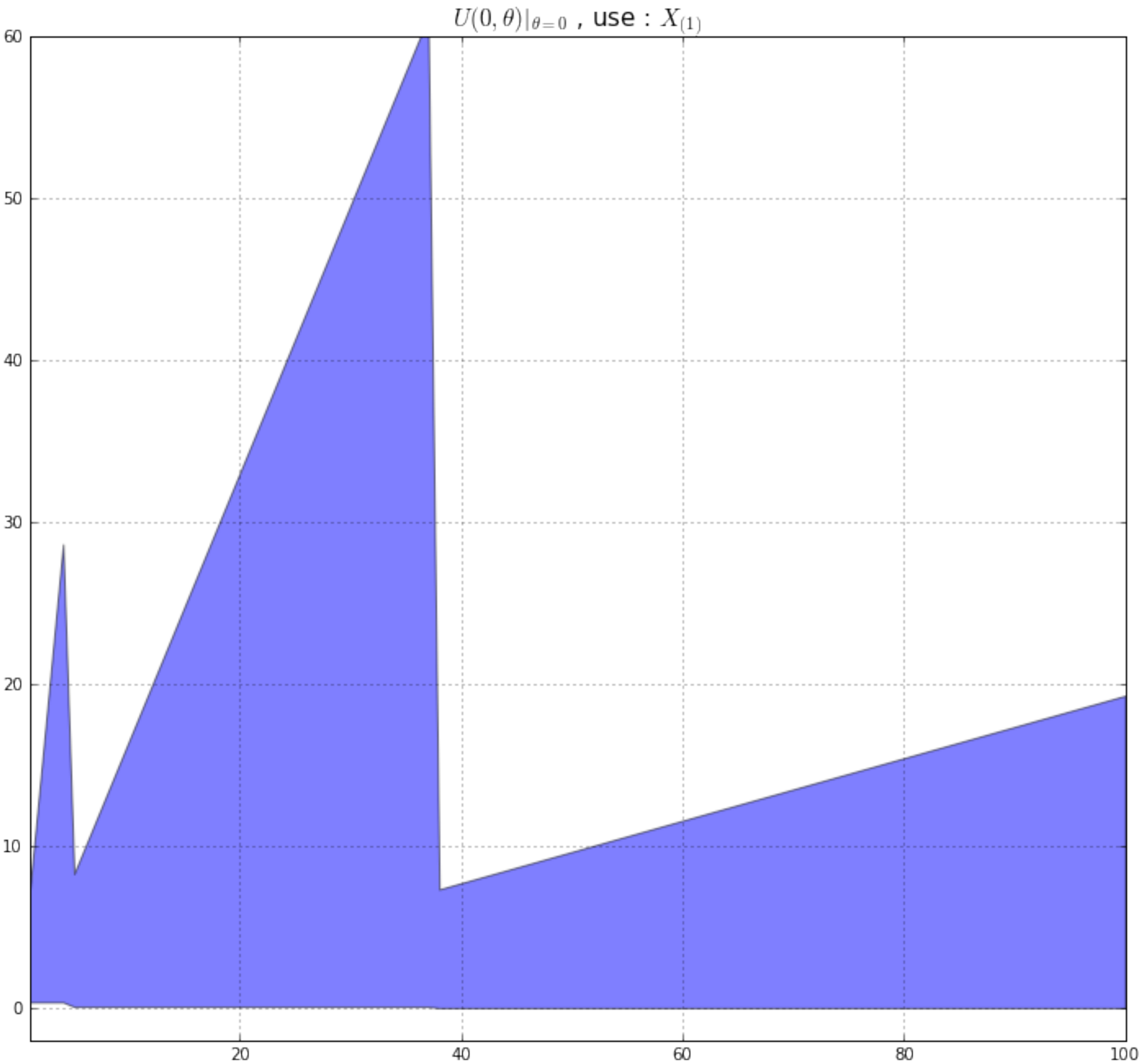



```
n = 10, P = 1.0
n = 100, P = 1.0
```

Этот интервал не является точным, т.к. получен с сипользованием неравенства Чебышева.

In [141]:

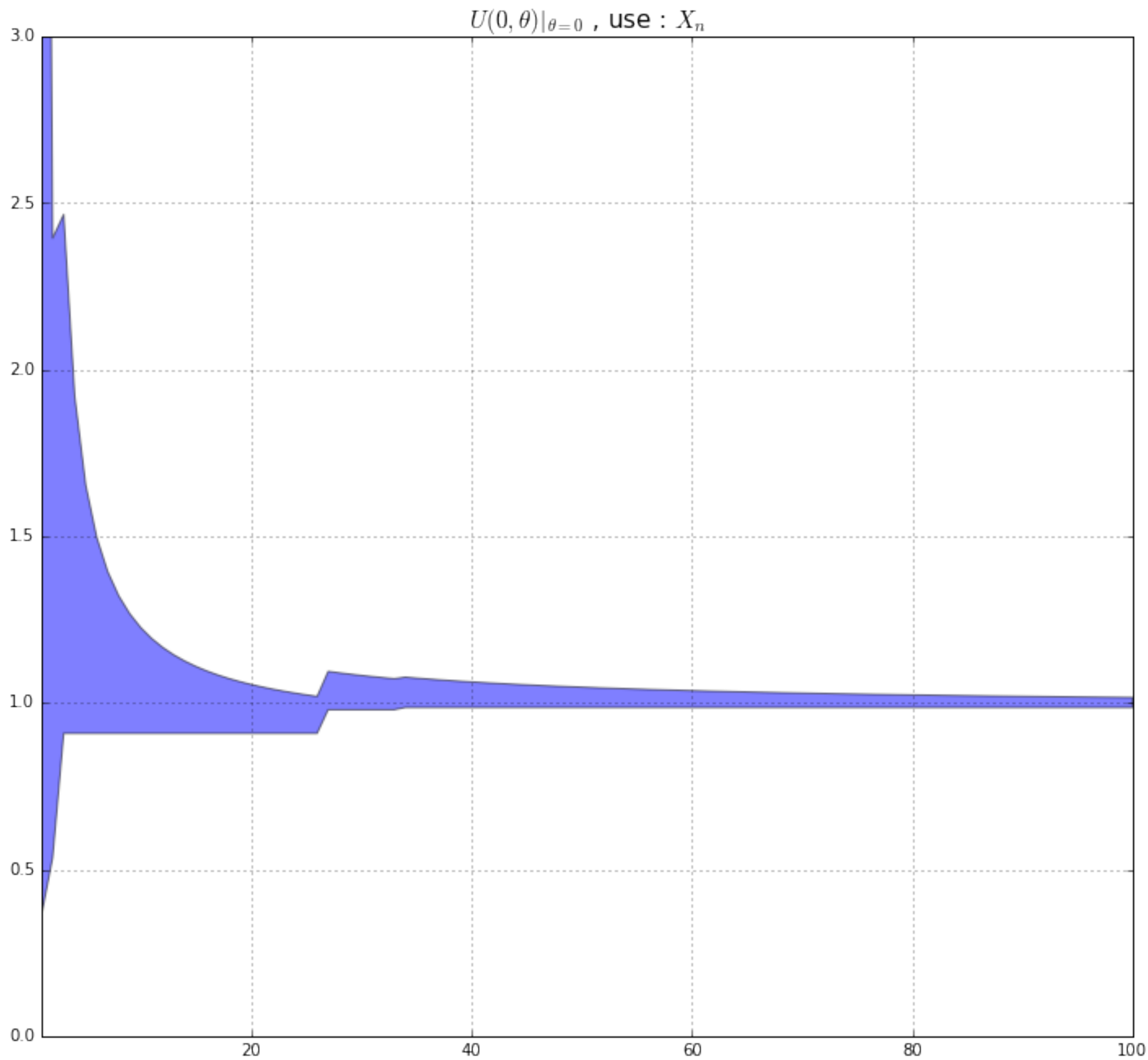
```
trust_min_l = [trust_level_uniform_min_l(X[0:k],alpha) for k in n]
trust_min_r = [trust_level_uniform_min_r(X[0:k],alpha) for k in n]
build_plot(n,trust_min_l,trust_min_r,
           r'$U(0,\theta)|_{\theta = 0}$',r'$X_{(1)}$',[1,100],[-2,60])

for k in {10,100} :
    print('n = ' + str(k) + ', P = '
          + str(test_probability(trust_level_uniform_min_l,
                trust_level_uniform_min_r,
                num_tests,
                lambda : sps.uniform.rvs(size=k))))
```




```
n = 10, P = 0.945
n = 100, P = 0.952
```

In [142]:

```
trust_max_l = [trust_level_uniform_max_l(X[0:k],alpha) for k in n]
trust_max_r = [trust_level_uniform_max_r(X[0:k],alpha) for k in n]

build_plot(n,trust_max_l,trust_max_r,
           r'$U(0,\theta)|_{\theta = 0}$',r'$X_{n}$',[1,100],[0,3])

for k in {10,100} :
    print('n = ' + str(k) + ', P = '
          + str(test_probability(trust_level_uniform_max_l,
                trust_level_uniform_max_r,
                num_tests,
                lambda : sps.uniform.rvs(size=k))))
```




```
n = 10, P = 0.942
n = 100, P = 0.949
```

Интервал (1) дает хорошее приближение (лучше чем (2)), но является избыточным, т.к. использует нижнюю оценку вероятности, а не точное значение.

Интервал (2) не сходятся к θ

Интервал (3) сходятся к θ и дает хорошую точность

Асимптотический доверительный интервал уровня α для $cauchy(\theta, 1)$:

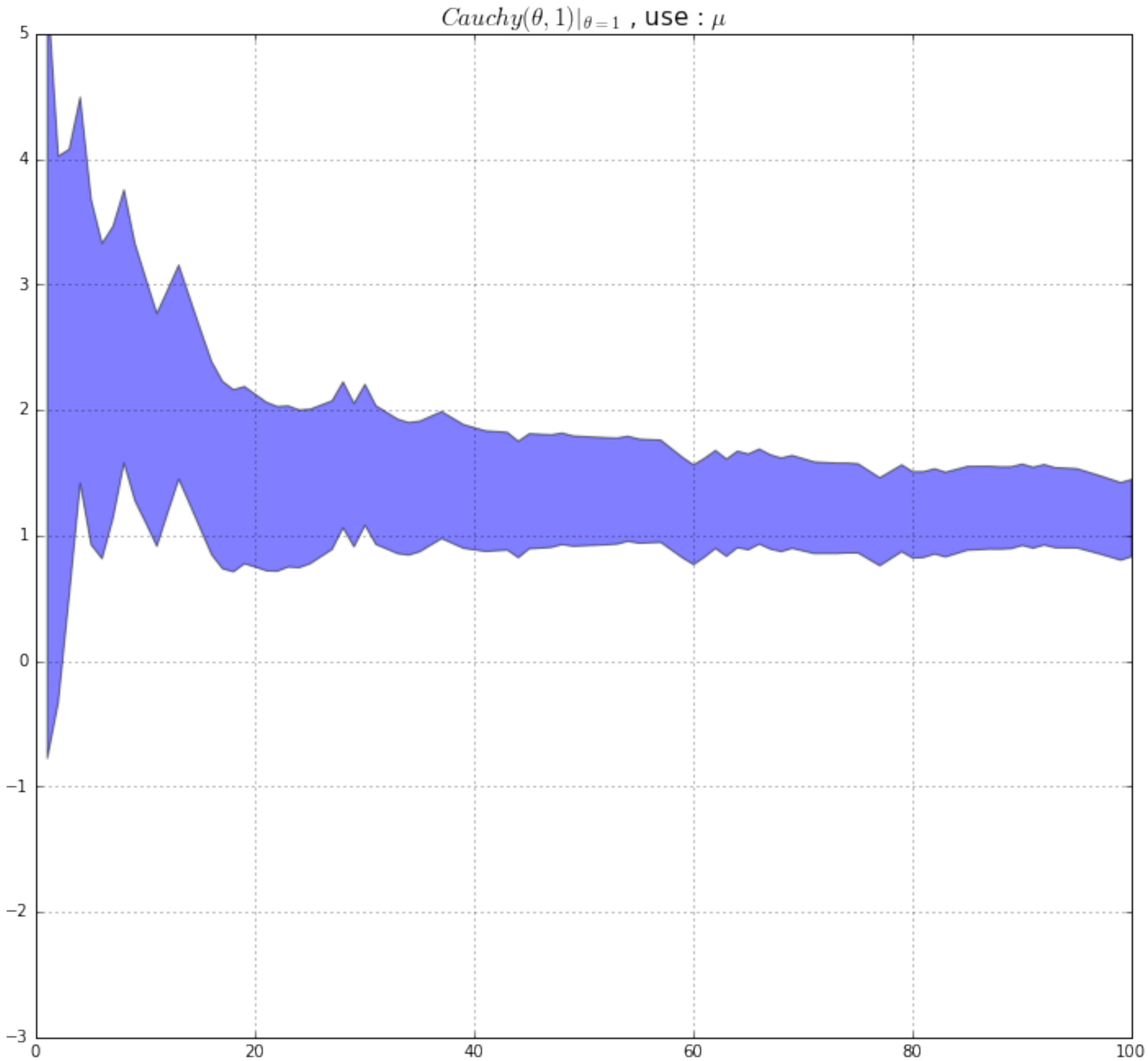
$(\mu - Z_{\frac{\alpha+1}{2}} \frac{\pi}{2\sqrt{n}}, \mu + Z_{\frac{\alpha+1}{2}} \frac{\pi}{2\sqrt{n}})$, где Z_x - квантиль уровня x для нормального распределения, μ - выборочная медиана.

In [155]:

```
def trust_level_cauchy_l(X,alpha):
     return np.median(X)
        - sps.norm.ppf((1 + alpha)/2) * np.pi /(2 * (len(X) ** 0.5))

def trust_level_cauchy_r(X,alpha):
     return np.median(X)
        + sps.norm.ppf((1 + alpha)/2) * np.pi /(2 * (len(X) ** 0.5))
```

In [181]:

```
theta = 1
X = sps.cauchy.rvs(theta,scale=1,size=N)
trust_max_l = [trust_level_cauchy_l(X[0:k],alpha) for k in n]
trust_max_r = [trust_level_cauchy_r(X[0:k],alpha) for k in n]

build_plot(n,trust_max_l,trust_max_r,
           r'$Cauchy(\theta,1)|_{\theta = 1}$',r'$\mu$',[],[-3,5])

for k in {10,100} :
    print('n = ' + str(k) + ', P = '
          + str(test_probability(trust_level_cauchy_l,
                    trust_level_cauchy_r,
                    num_tests,
                    lambda : sps.cauchy.rvs(theta,scale=1,size=k))))
```




```
n = 10, P = 0.93
n = 100, P = 0.944
```

Асимптотический доверительный интервал для $Pois(\theta)$ уровня α:

$$\left(\bar{X} - Z_{\frac{1+\alpha}{2}}\sqrt{\frac{\bar{X}}{n}}, \bar{X} + Z_{\frac{1+\alpha}{2}}\sqrt{\frac{\bar{X}}{n}}\right)$$

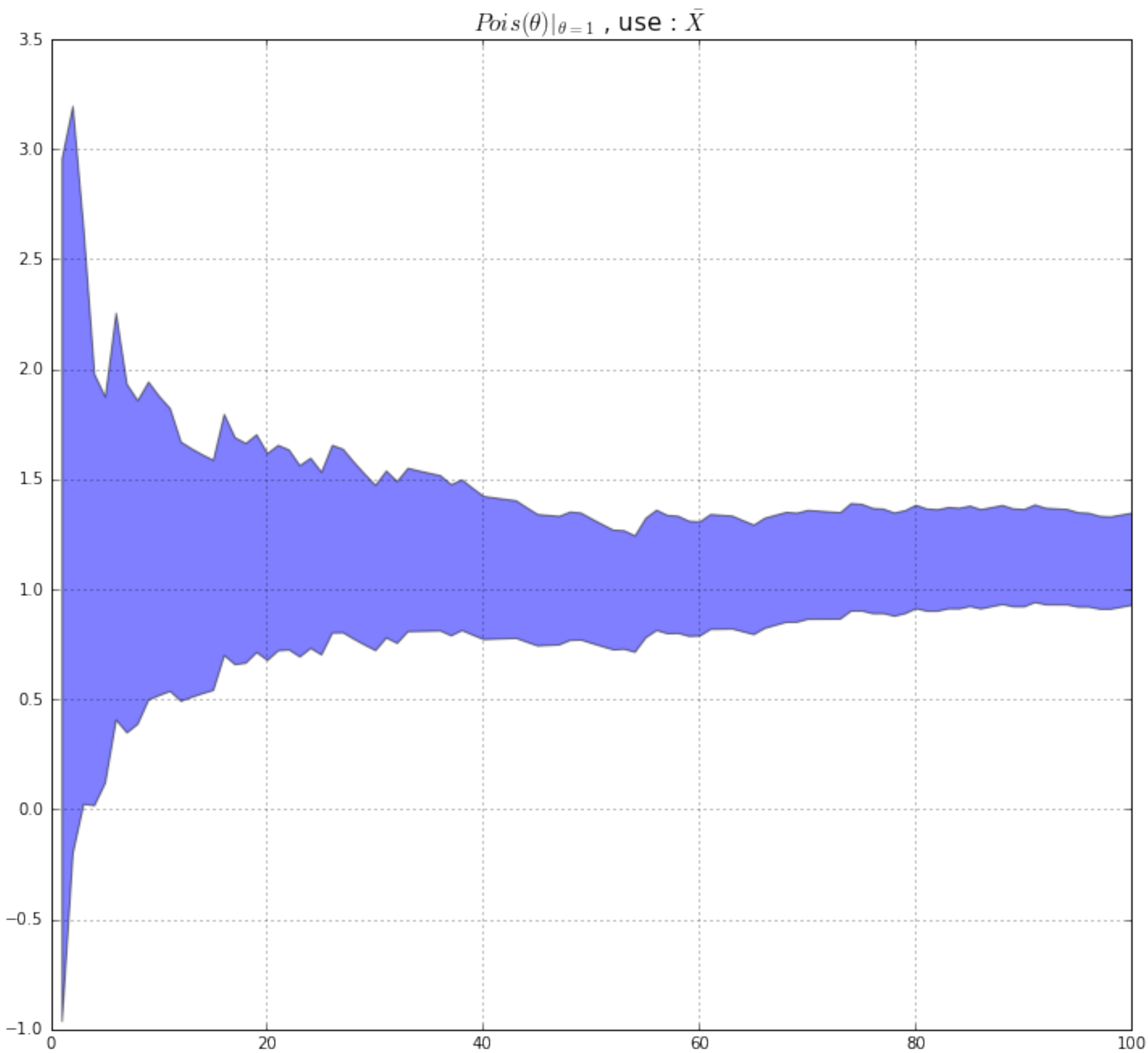
где Z_x - квантиль уровня x для стандартного нормального распределения $\mathcal{N}(0, 1)$.

In [194]:

```
def trust_level_pois_l(X,alpha):
    return np.mean(X)
        - sps.norm.ppf((1 + alpha)/2)*((np.mean(X)/len(X)) ** 0.5)

def trust_level_pois_r(X,alpha):
    return np.mean(X)
        + sps.norm.ppf((1 + alpha)/2)*((np.mean(X)/len(X)) ** 0.5)
```

In [189]:

```
theta = 1
X = sps.poisson.rvs(mu=theta,size=N)
trust_max_l = [trust_level_pois_l(X[0:k],alpha) for k in n]
trust_max_r = [trust_level_pois_r(X[0:k],alpha) for k in n]

build_plot(n,trust_max_l,trust_max_r,
           r'$Pois(\theta)|_{\theta = 1}$',r'$\bar{X}$',[],[])

for k in {10,100} :
    print('n = ' + str(k) + ', P = '
          + str(test_probability(trust_level_pois_l,
                       trust_level_pois_r,
                       num_tests,
                       lambda : sps.poisson.rvs(mu=theta,size=k))))
```




```
n = 10, P = 0.938
n = 100, P = 0.941
```

Асимптотический доверительный интервал для $\Gamma(\theta, \lambda)$ уровня α с неизвестным θ:

$$\left(\frac{\lambda - Z_{\frac{1+\alpha}{2}} \sqrt{\frac{\lambda}{n}}}{\bar{X}}, \frac{\lambda + Z_{\frac{1+\alpha}{2}} \sqrt{\frac{\lambda}{n}}}{\bar{X}} \right)$$

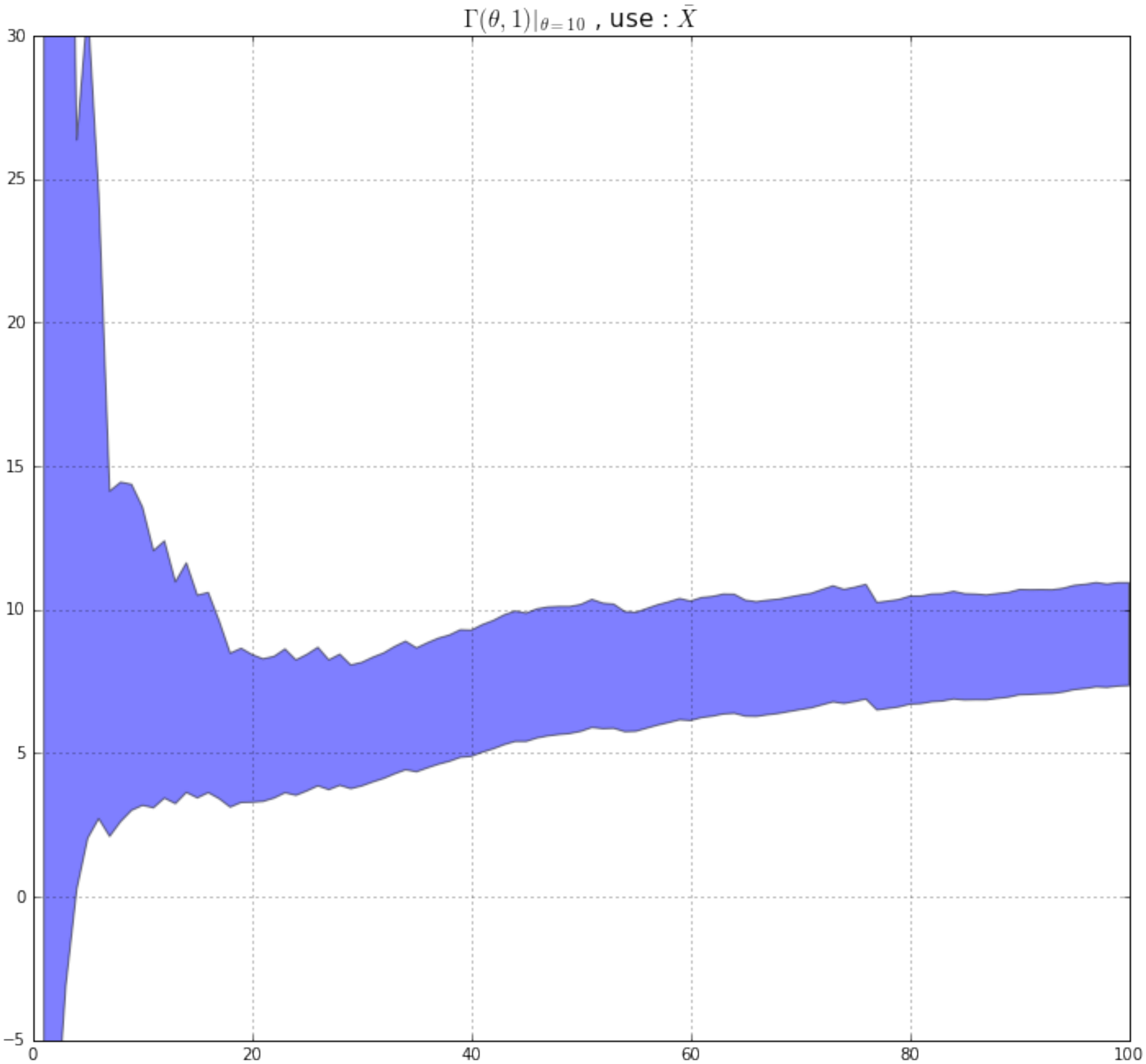
где Z_x - квантиль уровня x для стандартного нормального распределения $\mathcal{N}(0, 1)$.

In [190]:

```
lmb = 1
def trust_level_gamma_l(X,alpha):
    return (lmb - sps.norm.ppf((1 + alpha)/2) * (lmb/len(X))**0.5)/np.mean(X)

def trust_level_gamma_r(X,gamma):
    return (lmb + sps.norm.ppf((1 + alpha)/2) * (lmb/len(X))**0.5)/np.mean(X)
```

In [196]:

```
theta = 10
X = sps.gamma.rvs(a=lmb,scale=1/theta,size=N)
trust_max_l = [trust_level_gamma_l(X[0:k],alpha) for k in n]
trust_max_r = [trust_level_gamma_r(X[0:k],alpha) for k in n]

build_plot(n,trust_max_l,trust_max_r,
           r'$\Gamma(\theta,1)|_{\theta = 10}$',r'$\bar{X}$',[],[-5,30])

for k in {10,100} :
    print('n = ' + str(k) + ', P = '
          + str(test_probability(trust_level_pois_l,
                trust_level_pois_r,
                num_tests,
                lambda : sps.poisson.rvs(mu=theta,size=k))))
```




```
n = 10, P = 0.957
n = 100, P = 0.951
```