

1. a) $\{\}$ no clauses
 b) $\{\emptyset\}$ the empty clause

2. $(x \wedge y) \rightarrow z \quad \neg(x \wedge y) \vee z = \neg x \vee \neg y \vee z$

$h = \text{healthy}$

$l = \text{lazy}$

$d = \text{dancer}$

$p = \text{happy}$

$$h \wedge \bar{l} \rightarrow d$$

$$\bar{l} \wedge \bar{d} \rightarrow p$$

$$h \wedge \bar{d} \rightarrow p$$

$$p \wedge \bar{d} \rightarrow h$$

$$\bar{l} \wedge h \rightarrow \bar{p}$$

$$\bar{p} \wedge \bar{h} \rightarrow l$$

$$\bar{l} \wedge d \rightarrow h$$

$$hld \quad \checkmark$$

$$\bar{ld}p \quad \checkmark$$

$$\bar{h}dp \quad \checkmark$$

$$\bar{pd}h \quad \checkmark$$

$$\bar{lp}\bar{p} \quad \checkmark$$

$$phl \quad \checkmark$$

$$\bar{ld}h \quad \checkmark$$

$$x = \bar{d} \bar{l} p$$

dance, happy, not lazy

$$3. \sum_{d=1}^{n-1} n - (j-1)d + \sum_{d=1}^{n-1} n - (k-1)d \quad \text{with } n - (j-1)d \geq 0$$

$$\sum_{d=1}^{\lfloor \frac{n}{j-1} \rfloor} n - (j-1)d = n \left\lfloor \frac{n}{j-1} \right\rfloor - (j-1) \left[\left\lfloor \frac{n}{j-1} \right\rfloor \left(\left\lfloor \frac{n}{j-1} \right\rfloor + 1 \right) \right] / 2$$

$$n - (j-1)d \geq 0 \quad d \leq \left\lfloor \frac{n}{j-1} \right\rfloor$$

$$|\text{Waarden}(j, k; n)| = n \left[\left\lfloor \frac{n}{j-1} \right\rfloor - (j-1) \left[\left\lfloor \frac{n}{j-1} \right\rfloor \left(\left\lfloor \frac{n}{j-1} \right\rfloor + 1 \right) \right] / 2 + n \left\lfloor \frac{n}{k-1} \right\rfloor - (k-1) \left[\left\lfloor \frac{n}{k-1} \right\rfloor \times \dots \left(\left\lfloor \frac{n}{k-1} \right\rfloor + 1 \right) \right] / 2 \right]$$

$$7. \left\{ (x_i \vee x_{i+2^d} \vee x_{i+2^{d+1}}) \mid 1 \leq i \leq n - 2^{d+1}, d \geq 0 \right\} \cup \\ \left\{ (\bar{x}_i \vee \bar{x}_{i+2^d} \vee \bar{x}_{i+2^{d+1}}) \mid 1 \leq i \leq n - 2^{d+1}, d \geq 0 \right\}$$

no 3 of same color are equally spaced by powers of 2

the string 001001001... is always a solution
 - 1s are always a multiple of 3 apart
 - 0s in positions i and j , such that $i \equiv j \pmod{3}$ are always a multiple of 3 apart.

10. m clauses, n variables

↓

$m+n$ clauses, $2n$ variables

First m clauses have only negative literals

Last n clauses have only positive literals, binary

$x_{i,0}$ - the original x_i , used positively

$x_{i,1}$ - the original x_i , used as its negation

Then, $\{(\bar{x}_{i,0} \vee \bar{x}_{i,1}) \mid 1 \leq i \leq n\} \cup \{\text{the original clauses with } x_i \text{ replaced with } x_{i,0} \text{ and } \bar{x}_i \text{ replaced with } x_{i,1}\}.$

12. $S_1(y_1, \dots, y_p) = (y_1 \vee \dots \vee y_p) \wedge \bigwedge_{1 \leq j < k \leq p} (\bar{y}_j \vee \bar{y}_k)$

$$S_{\leq 1}(y_1, \dots, y_p) = \exists t (S_{\leq 1}(y_1, \dots, y_{j,t}) \wedge S_{\leq 1}(\bar{t}, y_{j+1}, \dots, y_p))$$

$$S_1(y_1, \dots, y_p) = (y_1 \vee \dots \vee y_p) \wedge S_{\leq 1}(y_1, \dots, y_p)$$

add new variables every 2 original variables, taking care of the edges with base cases.

$$\underbrace{y_1 y_2 y_3}_{S_{\leq 1}} \underbrace{t_1 y_4 y_5}_{S_{\leq 1}} \underbrace{t_2 y_6 y_7}_{S_{\leq 1}} \dots \dots \underbrace{t_k y_p}_{S_{\leq 1}} \quad \underbrace{y_1 y_2 y_3}_{S_{\leq 1}} \underbrace{t_1 y_4 y_5}_{S_{\leq 1}} \underbrace{t_2 y_6 y_7}_{S_{\leq 1}} \dots \dots \underbrace{t_k y_p}_{S_{\leq 1}}$$

13. a) $\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ d_1 s_1 s_3 & d_1 s_2 s_4 & d_1 s_3 s_5 & d_1 s_4 s_6 & d_1 s_5 s_7 & d_1 s_6 s_8 \\ d_2 s_1 s_4 & d_2 s_2 s_5 & d_2 s_3 s_6 & d_2 s_4 s_7 & d_2 s_5 s_8 \\ d_3 s_1 s_5 & d_3 s_2 s_6 & d_3 s_3 s_7 & d_3 s_4 s_8 \\ d_4 s_1 s_6 & d_4 s_2 s_7 & d_4 s_3 s_8 \end{matrix} \quad 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8$

$$\begin{aligned} & S_1(x_4, x_6, x_9, x_{13}, x_{16}) \wedge S_1(x_5, x_{10}, x_{14}, x_{17}) \wedge S_1(x_6, x_{11}, x_{15}, x_{18}) \\ & \wedge S_1(x_1, x_2, x_3, x_4, x_5, x_6) \wedge S_1(x_7, x_8, x_9, x_{10}, x_{11}) \wedge S_1(x_{12}, x_{13}, x_{14}, x_{15}) \\ & \wedge S_1(x_{16}, x_{17}, x_{18}) \wedge S_1(x_1, x_2, x_{12}, x_{16}) \wedge S_1(x_2, x_3, x_{13}, x_{17}) \\ & \wedge S_1(x_3, x_4, x_9, x_{14}, x_{18}) \wedge S_1(x_2, x_4, x_7, x_{10}, x_{15}) \wedge S_1(x_3, x_5, x_8, x_{11}, x_{12}) \end{aligned}$$

b) If the instance is represented by a disjunction of S_1 clauses, we can add each binary clause $(\bar{y}_j \vee \bar{y}_j)$ once iff. y_j and \bar{y}_j appear both in at least one of the S_1 clauses.

c) See point from Kukusut.

14. They make the instance more constrained, so that solvers might consider fewer nodes in the search tree. The additional clauses are also binary, which cascades unit propagation.

23. $n=7$ $r=4$

K
1
2
3
4
0

+ 12 variables

+ 23 clauses

clauses

$$\begin{aligned} & (\bar{S}_1^1 \vee S_2^1) \wedge (\bar{S}_2^1 \vee S_3^1) \\ \wedge & (\bar{S}_1^2 \vee S_2^2) \wedge (\bar{S}_2^2 \vee S_3^2) \\ \wedge & (\bar{S}_1^3 \vee S_2^3) \wedge (\bar{S}_2^3 \vee S_3^3) \\ \wedge & (\bar{S}_1^4 \vee S_2^4) \wedge (\bar{S}_2^4 \vee S_3^4) \\ \wedge & (\bar{x}_1 \vee S_1^1) \wedge (\bar{x}_2 \vee S_2^1) \wedge (\bar{x}_3 \vee S_3^1) \\ \wedge & (\bar{x}_4 \vee \bar{S}_1^1 \vee S_2^2) \wedge (\bar{x}_3 \vee \bar{S}_2^1 \vee S_2^2) \wedge (\bar{x}_4 \vee S_1^1 \vee S_3^2) \\ \wedge & (\bar{x}_3 \vee \bar{S}_1^2 \vee S_1^3) \wedge (\bar{x}_4 \vee \bar{S}_2^2 \vee S_2^3) \wedge (\bar{x}_5 \vee \bar{S}_3^2 \vee S_3^3) \\ \wedge & (\bar{x}_4 \vee \bar{S}_1^3 \vee S_1^4) \wedge (\bar{x}_5 \vee \bar{S}_2^3 \vee S_2^4) \wedge (\bar{x}_6 \vee \bar{S}_3^3 \vee S_3^4) \\ \wedge & (\bar{x}_5 \vee \bar{S}_1^4) \wedge (\bar{x}_6 \vee \bar{S}_2^4) \wedge (\bar{x}_7 \vee \bar{S}_3^4) \end{aligned}$$

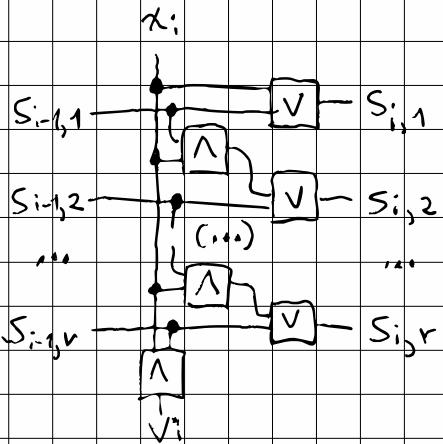
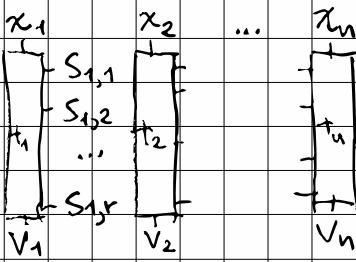
1
2
3
4
8 9 10 11 12 13 (1) (2) (3) (4) (5) (6) (7)

+ 5 variables

+ 25 clauses

$$\begin{aligned} & (\bar{x}_2 \vee b_1^4) \wedge (\bar{x}_3 \vee b_1^4) \wedge (\bar{x}_2 \vee \bar{x}_3 \vee b_2^4) \\ \wedge & (\bar{x}_4 \vee b_1^5) \wedge (\bar{x}_5 \vee b_1^5) \wedge (\bar{x}_4 \vee \bar{x}_5 \vee b_2^5) \\ \wedge & (\bar{x}_6 \vee b_1^6) \wedge (\bar{x}_7 \vee b_1^6) \wedge (\bar{x}_6 \vee \bar{x}_7 \vee b_2^6) \\ \wedge & (\bar{x}_1 \vee b_1^3) \wedge (b_1^6 \vee b_1^3) \wedge (\bar{x}_1 \vee b_1^6 \vee b_2^3) \wedge (b_2^6 \vee b_2^3) \\ \wedge & (\bar{x}_1 \vee b_2^6 \vee b_3^3) \\ \wedge & (b_1^4 \vee b_1^2) \wedge (b_1^5 \vee b_1^2) \wedge (b_2^4 \vee b_2^2) \wedge (b_2^5 \vee b_2^2) \wedge (b_1^4 \vee b_1^5 \vee b_2^2) \\ \wedge & (b_2^4 \vee b_1^5 \vee b_2^2) \wedge (b_1^4 \vee b_2^5 \vee b_3^2) \wedge (b_2^4 \vee b_2^5 \vee b_3^2) \\ \wedge & (b_2^2 \vee b_1^3) \wedge (b_3^2 \vee b_2^3) \wedge (b_2^2 \vee b_3^3) \end{aligned}$$

26.



$$S_{i,j} \rightarrow S_{i+1,j}$$

$$x_{i+r} \wedge S_{i,j} \rightarrow S_{i+r,j+1}$$

After shifting the rows:

$$S_{i,j} \rightarrow S_{i+1,j} \quad (1 \leq i < n-r, 1 \leq j \leq r) \quad (n-r)r \text{ variables namely}$$

$$x_{i+j} \wedge S_{i,j} \rightarrow S_{i,j+1}$$

$(1 \leq i \leq n-r, 0 \leq j \leq r)$ However, note that certain variables are not really needed because they can't affect the result. Only $(n-r)r$ are needed.

$$\begin{array}{|c|c|c|c|} \hline & S_{1,1} & S_{2,1} & S_{r,1} \\ \hline & S_{2,2} & \dots & S_{r,2} \\ \hline & \dots & & \dots \\ \hline & S_{r,r} & & & \end{array}$$

$$\begin{array}{|c|c|c|c|} \hline & S_{n-r,1} & & \\ \hline & \dots & & \\ \hline & S_{n-r,r-1} & \dots & S_{n-r,r} \\ \hline & S_{n-r,r} & S_{n-2,r} & S_{n-1,r} \\ \hline \end{array}$$

$x_1 \quad x_2$

(\dots)

$x_{n-1} \quad x_n$

27. b_j^K counts the number of active variables under node K. If b_j^K is set, it means there are j variables in the range covered by node K. The number of leaves t_K is clamped to r since we are interested only in whether $r+1$ are set. So $b_j^K \rightarrow [x \text{ in range of } K = j]$.

b_j^K

$$j = j' + j''$$

$b_{j'}^K$

$b_{j''}^{2k+1}$

The boundary conditions are set by asserting that the nodes 2 and 3 do not have $t+1$ leaves set between the two.

$$28. S_{\geq 1}(x_1, \dots, x_n) = S_{\leq n-1}(x_1, \dots, x_n)$$

Then, if we have a k -SAT set of clauses, we can convert each clause $C_i = (x_{i1}, x_{i2}, \dots, x_{ik})$ into a set of equivalent 3-SAT clauses with $S_{\geq 1}(C_i)$. This works for any function $S_{\geq 1}$ that uses clauses of length at most 3.

217. False: consider $A = B = \{\ell, \bar{\ell}\}$. A and B are simultaneously satisfiable by setting ℓ to any value. But $C = (A \cup B) \setminus \{\ell, \bar{\ell}\} = \emptyset$ is not satisfiable.

218. $x?B:A$

224. Add x to every leaf containing a clause originally containing x and propagate x along the tree via resolution on other variables. The final node will be either \emptyset as before, or $\{x\}$.

109. We need to find lexicographically smallest solution $x_1 \dots x_n$. We can assume that $x_0 = 1$ for simplicity. Initially, we set $K=0$ and find the largest j such that $K < j$ and there is a solution with $x_{K+1} \dots x_j = 0$. Applying this process repeatedly obtains the required solution. In the worst case, $O(n^2)$ calls to the SAT solver are required.

16. Yes. For example, in the McGregor graph of order 3, the nodes 01, 02, 12 and 32 form a 4-clique. This corner is a clique in any order as well. (e.g. 08, 09, 19 and a9 in the order 10 graph) Thus any McGregor graph requires 4 colors.

40. It is unsatisfiable whenever z is prime and the largest index is s.t. $z_i=1$ is larger than $\max(u, v)$. Otherwise, $x=z$ and $y=1$ satisfy the instance.

$7 = (1+1)_2$ is thus the largest prime which is still satisfiable.

42. $x \leftarrow t \oplus u \oplus v$

$t \ u \ v \ x$	$\bar{t} \wedge \bar{u} \wedge \bar{v} \rightarrow \bar{x}$	$(t \vee u \vee v \vee v \bar{x})$
0 0 0 0	$\bar{t} \wedge \bar{u} \wedge \bar{v} \rightarrow \bar{x}$	$(t \vee u \vee v \vee v \bar{x})$
0 0 1 1	$\bar{t} \wedge \bar{u} \wedge v \rightarrow x$	$(t \vee u \vee \bar{v} \vee v x)$
0 1 0 1	$\bar{t} \wedge u \wedge \bar{v} \rightarrow x$	$(t \vee \bar{u} \vee v \vee v x)$
0 1 1 0	$\bar{t} \wedge u \wedge v \rightarrow \bar{x}$	$(t \vee \bar{u} \vee \bar{v} \vee v \bar{x})$
1 0 0 1	$t \wedge \bar{u} \wedge \bar{v} \rightarrow x$	$(\bar{t} \vee u \vee v \vee v x)$
1 0 1 0	$t \wedge \bar{u} \wedge v \rightarrow \bar{x}$	$(\bar{t} \vee u \vee v \vee v \bar{x})$
1 1 0 0	$t \wedge u \wedge \bar{v} \rightarrow \bar{x}$	$(\bar{t} \vee \bar{u} \vee v \vee v \bar{x})$
1 1 1 1	$t \wedge u \wedge v \rightarrow x$	$(\bar{t} \vee \bar{u} \vee \bar{v} \vee v x)$

$y \leftarrow \langle t u v \rangle$

$t \ u \ v \ y$	$\bar{t} \wedge \bar{u} \wedge \bar{v} \rightarrow \bar{y}$	$(t \vee u \vee v \vee v \bar{y})$
0 0 0 0	$\bar{t} \wedge \bar{u} \wedge \bar{v} \rightarrow \bar{y}$	$(t \vee u \vee v \vee v \bar{y})$
0 0 1 0	$\bar{t} \wedge \bar{u} \wedge v \rightarrow \bar{y}$	$(t \vee u \vee \bar{v} \vee v \bar{y})$
0 1 0 0	$\bar{t} \wedge u \wedge \bar{v} \rightarrow \bar{y}$	$(t \vee \bar{u} \vee v \vee v \bar{y})$
0 1 1 1	$\bar{t} \wedge u \wedge v \rightarrow y$	$(t \vee \bar{u} \vee \bar{v} \vee v y)$
1 0 0 0	$t \wedge \bar{u} \wedge \bar{v} \rightarrow y$	$(\bar{t} \vee u \vee v \vee v \bar{y})$
1 0 1 1	$t \wedge \bar{u} \wedge v \rightarrow y$	$(\bar{t} \vee u \vee \bar{v} \vee v y)$
1 1 0 1	$t \wedge u \wedge \bar{v} \rightarrow y$	$(\bar{t} \vee \bar{u} \vee v \vee v \bar{y})$
1 1 1 1	$t \wedge u \wedge v \rightarrow y$	$(\bar{t} \vee \bar{u} \vee \bar{v} \vee v y)$

$t \wedge u \rightarrow y$	$(\bar{t} \vee \bar{u} \vee v y)$	$\bar{t} \wedge \bar{u} \rightarrow \bar{y}$	$(t \vee u \vee v \bar{y})$
$t \wedge v \rightarrow y$	$(\bar{t} \vee \bar{v} \vee v y)$	$\bar{t} \wedge \bar{v} \rightarrow \bar{y}$	$(t \vee v \vee v \bar{y})$
$u \wedge v \rightarrow y$	$(\bar{u} \vee \bar{v} \vee v y)$	$\bar{u} \wedge \bar{v} \rightarrow \bar{y}$	$(u \vee v \vee v \bar{y})$

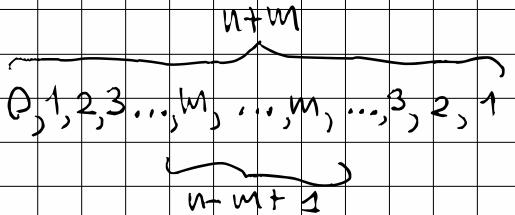
$$41. \quad 2 \leq m \leq n$$

num bins

	\wedge	\vee	\oplus	C
half-adder	1	0	1	= 2 = h
full-adder	2	1	2	= 5 = f

Now as for products $x; y_j$

$$\cos^t(b) = \left\lfloor \frac{b-1}{2} \right\rfloor f + ((b+1) \bmod e) h$$



$$\text{Total} = \sum_{i=2}^m \text{cost}(2(i-1)) + (n-m) \text{cost}(2m-1) + \text{cost}(2(m-1))$$

$$+ \sum_{i=2}^{m-1} cost(m-i+m-i+1) + mn$$

$$= (m-2)(m-1)f + (m-1)h + (n-m)(m-1)f + (m-2)f + h + mn$$

$$= 8((m-2)m + \dots + (n-m)(m-1)) + hm + mn$$

$$= 5(mn - m - n) + hm + mn$$

1

$$20 = 2$$

$$31 = 4$$

$$42 = 6$$

$$53 = 8$$

(-111)

$$m(m-2) = 2(m-1)$$

$$m - m - 1 = 2m - 1$$

$$m \cdot m-1 = 2m-1$$

$$m - m - 1 = 2m - 1$$

(1, 1)

$$\rightarrow m-1 \quad m-1 = 2(m-1)$$

$$m-2 \quad m-1 = 2(m-2) + 1$$

$$m-3 \quad m-2 = 2(m-3) + 1$$

1

$$1 \quad 2 = 3$$

$\text{cost}(2(i-1))$

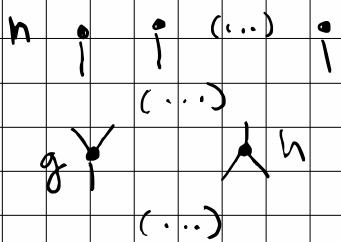
$$\text{cost}(2m-1)$$

$$c \in \mathbb{S}^4(2^{(n-1)})$$

$$c_{\alpha} s^{\frac{1}{2}} \left(z(w-i) + 1 \right)$$

$$\sum_{i=2}^m (i-2)f + h$$

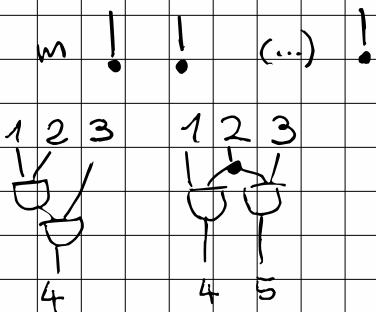
47. m outputs, n inputs
 g 2-to-1 gates
 h 1-to-2 gates (Fanout)



$$\sum_{v \in V} \deg(v) = 2|E|$$

$$|V| = m + n + g + h$$

$$m + n + (g+h)3 = 2w \quad (1)$$



regardless of how gates are interconnected, the number of wires remain constant.

$$g + m - n = h$$

$$w = 2g + m + h \quad (2)$$

$$\begin{aligned} n + m + 3(g + g + m - n) &= 2w \\ n + m + 6g + 3m - 3n &= 2w \\ 4m + 6g - 2n &= 2w \end{aligned}$$

$$w = 2m - n + 3g \quad (3)$$

119. $\{123, 234, 345, 456, 567, 678, 789, 135, 246, 357, 468, 579,$
 $147, 258, 369, 159,$
 $123, 234, 345, 456, 567, 678, 789, 135, 246, 357, 468, 579,$
 $147, 258, 369, 159\}$

$x=5$ appears 16 times

$F15 = \{123, 234, 678, 789, 246, 468, 147, 369,$
 $123, 234, 345, 456, 567, 678, 789, 135, 246, 357, 468, 579,$
 $147, 258, 369, 159\}$

- 8 clauses
- 8 literals

24 clauses

121. see implementation

122. A2: if $d = n+1$, process current assignment⁴ and go to step A6.

A2: do not use move codes for pure literals since some solutions might involve a literal whose value is redundant.

A2: do not check for $C(e) = a$ and instead always continue to step A3.

123.

$L = \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 \\ 7 & 9 & 3 & 5 & 7 & 8 & 3 & 5 & 6 & 8 & 3 & 4 & 6 & 8 & 2 & 4 & 6 & 9 & 2 & 4 & 7 \end{matrix}$

1 2 3 4 5 6 7

START = 19 16 13 10 7 4 1

LINK = 0 0 0 0 0 0 0

124. see implementation

125. B2: if $d > n$, process current assignment and go to step B6.

126.

312	315	714	915	123	135	147	159
324	624	825		234	246	258	
345	357	369		543	537	936	
456	648			546	648		
657	759			567	579		
678				867			
789				879			

(6978)	$x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9$	Units	Choice	Change
	0 0 1 1 0 - - -	9	9	639

127. 1 1 4 1 4 5 4 5 \rightarrow 1 1 4 2

130. See implementation (just select a new watchee for all clauses watching $\ell = 2k + \pi_k$, also taking care of adding the new watchees to the active ring if they were not already in it)

190. $(x_1 \vee x_2 \vee x_3 \vee x_4)$

any clause $(x_1' \vee x_2' \vee x_3')$ will be falsified by some satisfying assignment

176. J_q

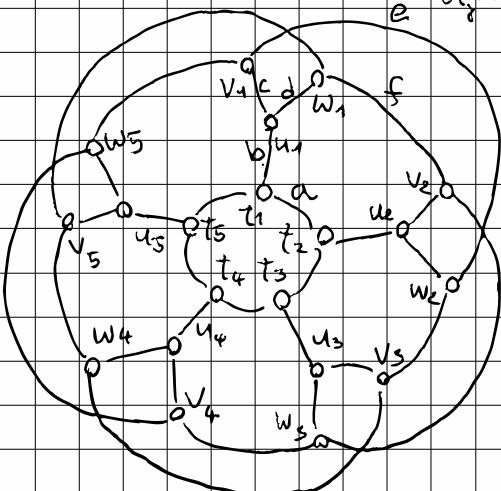
$4q$ vertices

$6q$ edges

$t_{ij}, u_{ij}, v_{ij}, w_{ij}$

$t_j - t_{j+1}, t_j - u_j, u_j - v_j$

$u_j - w_j, v_j - w_{j+1}, w_j - v_{j+1}$



a)

$$\begin{aligned} a_j &= a_{j+1} \\ a_j &= b_j \\ a_j &= b_{j+1} \\ b_j &= c_j \\ b_j &= d_j \\ c_j &= d_j \\ c_j &= e_j \\ d_j &= f_j \\ e_j &= d_{j+1} \\ f_j &= c_{j+1} \\ c_j &= f_{j+1} \\ f_j &= e_{j+1} \end{aligned}$$

b) q even

$$\chi(J_q) = 2$$

Consider the following 2-coloring, valid for even q :

$$c(t_j) = j \bmod 2$$

$$c(u_j) = (j+1) \bmod 2$$

$$c(v_j) = j \bmod 2$$

$$c(w_j) = j \bmod 2$$

$$\begin{aligned} c(t_j) &\neq c(t_{j+1}) & c(v_j) &\neq c(w_{j+1}) \\ c(t_j) &\neq c(u_j) & c(w_j) &\neq c(v_{j+1}) \\ c(u_j) &\neq c(v_j) & c(u_j) &\neq c(w_j) \\ c(v_j) &\neq c(w_j) & c(w_j) &\neq c(v_{j+1}) \end{aligned}$$

$\chi(L(J_g)) \geq 3$ because $L(J_g)$ has a 3-clique
(for example $\{a_{j_1}, a_j, b_j\}$)

Explicit 3-coloring for $L(J_g)$

$$c(a_j) = j \bmod 2$$

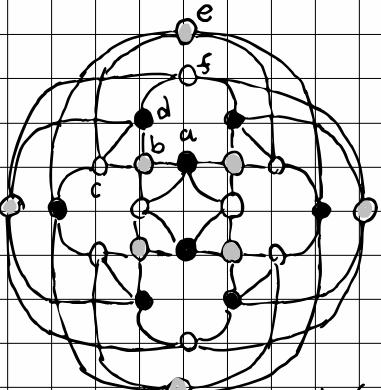
$$c(b_j) = 2$$

$$c(c_j) = j \bmod 2$$

$$c(d_j) = (j+1) \bmod 2$$

$$c(e_j) = 2$$

$$c(f_j) = j \bmod 2$$



valid for given.

$L(J_4)$

- c) For J_g , we can modify the coloring from (b) by setting $c(t_g) = c(v_g) = c(w_g) = 2$. Thus, $\chi(J_g) = 3$. A 2-coloring is not possible because J_g always contains an odd-length cycle (e.g., $t_1, t_2, \dots, t_g, t_1$).

- $L(J_g)$ can't be 2-colored because it has an odd-length cycle $(a_1, a_2, \dots, a_g, a_1)$.

- $L(J_g)$ can't be 3-colored because (b_j, c_j, d_j) must be color-wise distinct to (a_j, e_j, f_j) , because (b, c, d) is a triangle with each vertex adjacent to (a, e, f) respectively. So, each of these groups can be colored with one of the 3 arrangements of 3 elements $\{(1,2,3), (2,3,1), (3,1,2)\}$. Additionally, because $a_j - b_j$ (the arrangement used for (a_j, e_j, f_j)) must be different to the one used for $(a_{j+1}, e_{j+1}, f_{j+1})$, and since there are g of them and g is odd, this can't be done.

- Finally, $L(J_g)$ can be 4-colored because of Brook's theorem.

$$219. C = C' \diamond C''$$

$$(i) (x \vee A') \diamond (\bar{x} \vee A'') = (A' \vee A'')$$

(ii) arbitrary clauses

$$(iii) C \wedge C'' \rightarrow C' \diamond C''$$

For arbitrary clauses C' and C'' , we can define

$$C = C' \diamond C''$$

$$\exists x, x \in C', \bar{x} \in C''$$

$$C = (C' \setminus \{x\}) \cup (C'' \setminus \{\bar{x}\})$$

$$\text{where } C = \begin{cases} \emptyset & \text{otherwise} \end{cases}$$

$C' \wedge C'' \rightarrow C' \diamond C''$ because a satisfying assignment for $C' \wedge C''$ satisfies either $(C' \setminus \{x\})$ or $(C'' \setminus \{\bar{x}\})$, and if there is no such x , then \emptyset is satisfied by any assignment.

$$220. a) C \subseteq C' \wedge C \subseteq C'' \rightarrow C \subseteq C'' \quad (\text{transitivity})$$

True.

$$b) (C \vee \alpha) \diamond (C' \vee \alpha') \subseteq (C \diamond C') \vee \alpha \vee \alpha'$$

$$\text{if } C \cap C' = \emptyset: (C \vee C' \vee \alpha \vee \alpha') \subseteq (C \vee C' \vee \alpha \vee \alpha')$$

$$\text{if } \alpha' = \bar{\alpha}: (C \vee C') \subseteq \emptyset$$

$$\text{if } x \in C, \bar{x} \in C': (C \setminus \{x\}) \vee (C' \setminus \{\bar{x}\}) \vee \alpha \vee \alpha' \subseteq (C \setminus \{x\}) \vee (C' \setminus \{\bar{x}\}) \vee \alpha \vee \alpha'$$

$$\text{if } \bar{\alpha} \in C: C \vee (C \setminus \{\bar{\alpha}\}) \vee \alpha' \subseteq C \vee C' \vee \alpha \vee \alpha' = \emptyset, \quad \text{True.}$$

$$c) C \subseteq C'' \rightarrow C \diamond C' \subseteq C \diamond C''$$

$$\exists x. x \in C', \bar{x} \in C \rightarrow x \in C'' \quad (C \setminus \{\bar{x}\}) \vee (C' \setminus \{x\}) \subseteq (C \setminus \{\bar{x}\}) \vee (C'' \setminus \{x\})$$

$$\exists x. x \in C'', x \notin C, \bar{x} \in C \quad C \vee C' \subseteq (C \setminus \{\bar{x}\}) \vee (C'' \setminus \{x\})$$

but $\bar{x} \in C$, so it's false.

counter-example: $C = \bar{1}2$ $C' = 3$ $C'' = 13$

$C \subseteq C''$ is true ($3 \leq 13$)

$C \diamond C' = \bar{1}23$ $C \diamond C'' = 23$ but $\bar{1}23 \notin 23$

d) $C_1, \dots, C_m \vdash C$

$1, 2, 34 \vdash 123$

$\overbrace{1, 2, 34, 12} \subseteq 123$

In this case, $C_1 = 1$, $C_2 = 2$, $C_3 = 34$ and $C = 123$. C cannot be obtained directly from the axioms, since it's a weak form of 12 and any resolution that adds 3 , also adds 4 .

e) $C_1 \subseteq C'_1, \dots, C_m \subseteq C'_m$

$C'_1, \dots, C'_m \vdash C$

$C_1, \dots, C_m \vdash C$

$C'_i = C_{j(i)} \diamond C_{k(i)}$ $m < i \leq m+r$, $r \geq 0$

$C_j \subseteq C'_j$ $C_k \subseteq C'_k \rightarrow C_j \diamond C_k \subseteq C'_j \diamond C'_k$

$$C_j \diamond C_k = (C_j \vee \alpha_j) \diamond (C_k \vee \alpha_k) \\ \subseteq (C_j \diamond C_k) \vee \alpha_j \vee \alpha_k$$

Thus, $C_{m+r} \subseteq C'_{m+r}$ and $C_1, \dots, C_m \vdash C$

f) $C_1, \dots, C_m \vdash C \rightarrow C_1 \vee \alpha_1, \dots, C_m \vee \alpha_m \vdash C \vee \alpha_1 \vee \dots \vee \alpha_m$

$C_i \subseteq C'_i \vee \alpha_i$ $1 \leq i \leq m$

and $C_i \subseteq C'_i$ $m < i \leq m+r$ $r \geq 0$ follows inductively.

Thus, $C_r = C \subseteq C'_r = C \vee \alpha_1 \vee \dots \vee \alpha_m$

t	Lt	level	reason
0	6	1	Λ
1	4	1	46 (learned)
2	5	2	Λ
3	3	2	$\overline{3}\overline{4}\overline{5}$
4	9	2	936
5	1	2	$\overline{1}\overline{5}\overline{9}$
6	2	2	213
7	7	2	$\overline{7}\overline{5}\overline{9}$
8	8	2	$\overline{8}\overline{6}\overline{7}$
9	8	2	$\overline{8}\overline{2}\overline{5}$ (conflict)

$$C = \overline{8}\overline{2}\overline{5} \Leftrightarrow 867 = \overline{2}\overline{5}6\overline{7} \quad (l=8)$$

$$C = \overline{2}\overline{5}6\overline{7} \Leftrightarrow \overline{7}\overline{5}\overline{9} = \overline{2}\overline{5}6\overline{9} \quad (l=7)$$

$$C = \overline{2}\overline{5}6\overline{9} \Leftrightarrow 213 = 1\overline{3}\overline{5}6\overline{9} \quad (l=2)$$

$$C = 1\overline{3}\overline{5}6\overline{9} \Leftrightarrow \overline{1}\overline{5}\overline{9} = 3\overline{5}6\overline{9} \quad (l=1)$$

$$C = 3\overline{5}6\overline{9} \Leftrightarrow 936 = 3\overline{5}6 \quad (l=9)$$

$$C = 3\overline{5}6 \Leftrightarrow \overline{3}\overline{4}\overline{5} = \overline{4}\overline{5}6 \quad (l=3) \quad (\text{learn})$$

t	Lt	level	reason
0	6	1	Λ
1	4	1	46
2	5	1	$\overline{5}\overline{4}6$ (learned)
3	7	1	$\overline{7}\overline{5}\overline{6}$
4	1	1	$\overline{1}\overline{4}\overline{7}$
5	3	1	315
6	2	1	$\overline{2}\overline{3}\overline{4}$
7	9	1	$\overline{9}\overline{1}\overline{5}$
8	8	1	$\overline{8}\overline{7}\overline{9}$
9	8	1	825 (conflict)

$$C = 825 \Leftrightarrow \overline{8}\overline{7}\overline{9}$$

$$C = 2\overline{5}\overline{2}\overline{9} \Leftrightarrow \overline{9}\overline{1}\overline{5}$$

$$C = 1\overline{2}\overline{5}\overline{7} \Leftrightarrow \overline{2}\overline{3}\overline{4}$$

$$C = \overline{1}\overline{3}\overline{4}\overline{5}\overline{7} \Leftrightarrow 315$$

$$C = \overline{1}\overline{4}\overline{5}\overline{7} \Leftrightarrow \overline{1}\overline{4}\overline{7}$$

$$C = \overline{4}\overline{5}\overline{7} \Leftrightarrow 756$$

$$C = \overline{4}\overline{5}6 \Leftrightarrow \overline{4}\overline{5}6$$

$$C = 46 \Leftrightarrow 46$$

$$C = 6 \quad (\text{learned})$$

$$254. F = \{1\bar{2}, \bar{1}3, \bar{2}\bar{3}, \bar{2}\bar{4}, \bar{3}\bar{4}\}$$

t	L_t	level	reason	
0	1	1	$\bar{1}$	$c = \bar{4}\bar{3} \Leftrightarrow \bar{4}\bar{2}$
1	3	1	$\bar{3}\bar{1}$	$c = \bar{2}\bar{3} \Leftrightarrow \bar{2}\bar{3}$
2	2	1	$\bar{2}\bar{3}$	$c = \bar{3}$
3	$\bar{4}$	1	$\bar{4}\bar{2}$	
4	4	1	$\bar{4}\bar{3}$ (Conflict)	