2.5.1)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A \to A}{\neg A, A \to}\,[\neg\mathrm{L}]}{(\neg A \wedge \neg B), A \to}\,[\wedge\mathrm{L}]}{A \to \neg(\neg A \wedge \neg B)}\,[\neg\mathrm{R}] \quad \dfrac{\dfrac{\dfrac{B \to B}{\neg B, B \to}\,[\neg\mathrm{L}]}{(\neg A \wedge \neg B), B \to}\,[\wedge\mathrm{L}]}{B \to \neg(\neg A \wedge \neg B)}\,[\neg\mathrm{R}]}{(A \vee B) \to \neg(\neg A \wedge \neg B)}\,[\vee\mathrm{L}]}{\to ((A \vee B) \supset \neg(\neg A \wedge \neg B))}\,[\supset\mathrm{R}] \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A \to A}{A \to (A \vee B)}\,[\vee\mathrm{R}]}{\to (A \vee B), \neg A}\,[\neg\mathrm{R}] \quad \dfrac{\dfrac{B \to B}{B \to (A \vee B)}\,[\vee\mathrm{R}]}{\to (A \vee B), \neg B}\,[\neg\mathrm{R}]}{\to (A \vee B), (\neg A \wedge \neg B)}\,[\wedge\mathrm{R}]}{\neg(\neg A \wedge \neg B) \to (A \vee B)}\,[\neg\mathrm{L}]}{\to (\neg(\neg A \wedge \neg B) \supset (A \vee B))}\,[\supset\mathrm{R}]}{\to ((A \vee B) \equiv \neg(\neg A \wedge \neg B))}\,[\wedge\mathrm{R}]$$

2.5.2)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A \to A}{\to A, \neg A}\,[\neg\mathrm{R}]}{\to A, (\neg A \vee B)}\,[\vee\mathrm{R}]}{\to (\neg A \vee B), A}\,[\mathrm{XR}] \quad \dfrac{B \to B}{B \to (\neg A \vee B)}\,[\vee\mathrm{R}]}{(A \supset B) \to (\neg A \vee B), (\neg A \vee B)}\,[\supset\mathrm{L}]}{(A \supset B) \to (\neg A \vee B)}\,[\mathrm{CR}]}{\to ((A \supset B) \supset (\neg A \vee B))}\,[\supset\mathrm{R}] \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A \to A}{\neg A, A \to}\,[\neg\mathrm{L}]}{\neg A, A \to B}\,[\mathrm{WR}]}{A, \neg A \to B}\,[\mathrm{XL}]}{\neg A \to (A \supset B)}\,[\supset\mathrm{R}] \quad \dfrac{\dfrac{B \to B}{A, B \to B}\,[\mathrm{WL}]}{B \to (A \supset B)}\,[\supset\mathrm{R}]}{(\neg A \vee B) \to (A \supset B)}\,[\vee\mathrm{L}]}{\to ((\neg A \vee B) \supset (A \supset B))}\,[\supset\mathrm{R}]}{\to ((A \supset B) \equiv (\neg A \vee B))}\,[\wedge\mathrm{R}]$$

2.5.3)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{F(a) \to F(a)}{\neg F(a), F(a) \to}\,[\neg\mathrm{L}]}{\forall y \neg F(y), F(a) \to}\,[\forall\mathrm{L}]}{F(a) \to \neg\forall y \neg F(y)}\,[\neg\mathrm{R}]}{\exists x F(x) \to \neg\forall y \neg F(y)}\,[\forall\mathrm{R}]}{\to (\exists x F(x) \supset \neg\forall y \neg F(y))}\,[\supset\mathrm{R}] \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{F(a) \to F(a)}{\to F(a), \neg F(a)}\,[\neg\mathrm{R}]}{\to \neg F(a), F(a)}\,[\mathrm{XR}]}{\to \neg F(a), \exists x F(x)}\,[\exists\mathrm{R}]}{\to \exists x F(x), \neg F(a)}\,[\mathrm{XR}]}{\to \exists x F(x), \forall y \neg F(y)}\,[\forall\mathrm{R}]}{\neg\forall y \neg F(y) \to \exists x F(x)}\,[\neg\mathrm{L}]}{\to (\neg\forall y \neg F(y) \supset \exists x F(x))}\,[\supset\mathrm{R}]}{\to (\exists x F(x) \equiv \neg\forall y \neg F(y))}\,[\wedge\mathrm{R}]$$

2.5.4)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{F(a) \to F(a)}{\to F(a), \neg F(a)}\,[\neg\mathrm{R}]}{\to F(a), \exists x \neg F(x)}\,[\exists\mathrm{R}]}{\to \exists x \neg F(x), F(a)}\,[\mathrm{XR}]}{\to \exists x \neg F(x), \forall y F(y)}\,[\forall\mathrm{R}]}{\neg\forall y F(y) \to \exists x \neg F(x)}\,[\neg\mathrm{L}]}{\to (\neg\forall y F(y) \supset \exists x \neg F(x))}\,[\supset\mathrm{R}] \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{F(a) \to F(a)}{\forall y F(y) \to F(a)}\,[\forall\mathrm{L}]}{\neg F(a), \forall y F(y) \to}\,[\neg\mathrm{L}]}{\forall y F(y), \neg F(a) \to}\,[\mathrm{XL}]}{\neg F(a) \to \neg\forall y F(y)}\,[\neg\mathrm{R}]}{\exists x \neg F(x) \to \neg\forall y F(y)}\,[\forall\mathrm{R}]}{\to (\exists x \neg F(x) \supset \neg\forall y F(y))}\,[\supset\mathrm{R}]}{\to (\neg\forall y F(y) \equiv \exists x \neg F(x))}\,[\wedge\mathrm{R}]$$

2.5.5)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A \to A}{\to A, \neg A}\,[\neg\mathrm{R}]}{\to A, (\neg A \vee \neg B)}\,[\vee\mathrm{R}]}{\to (\neg A \vee \neg B), A}\,[\mathrm{XR}] \quad \dfrac{\dfrac{\dfrac{B \to B}{\to B, \neg B}\,[\neg\mathrm{R}]}{\to B, (\neg A \vee \neg B)}\,[\vee\mathrm{R}]}{\to (\neg A \vee \neg B), B}\,[\mathrm{XR}]}{\to (\neg A \vee \neg B), (A \wedge B)}\,[\wedge\mathrm{R}]}{\neg(A \wedge B) \to (\neg A \vee \neg B)}\,[\neg\mathrm{L}]}{\to (\neg(A \wedge B) \supset (\neg A \vee \neg B))}\,[\supset\mathrm{R}] \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A \to A}{(A \wedge B) \to A}\,[\wedge\mathrm{L}]}{\to A, \neg(A \wedge B)}\,[\neg\mathrm{R}]}{\to \neg(A \wedge B), A}\,[\mathrm{XR}]}{\neg A \to \neg(A \wedge B)}\,[\neg\mathrm{L}] \quad \dfrac{\dfrac{\dfrac{B \to B}{(A \wedge B) \to B}\,[\wedge\mathrm{L}]}{\to B, \neg(A \wedge B)}\,[\neg\mathrm{R}]}{\to \neg(A \wedge B), B}\,[\mathrm{XR}]}{\neg B \to \neg(A \wedge B)}\,[\neg\mathrm{L}]}{(\neg A \vee \neg B) \to \neg(A \wedge B)}\,[\vee\mathrm{L}]}{\to ((\neg A \vee \neg B) \supset \neg(A \wedge B))}\,[\supset\mathrm{R}]}{\to (\neg(A \wedge B) \equiv (\neg A \vee \neg B))}\,[\wedge\mathrm{R}]$$

2.6.1)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A \to A}{A \to A, B(a)}\,[\mathrm{WR}]}{A \to B(a), A}\,[\mathrm{XR}] \qquad B(a) \to B(a)}{(A \supset B(a)), A \to B(a), B(a)}\,[\supset\mathrm{L}]}{(A \supset B(a)), A \to B(a)}\,[\mathrm{CR}]}{(A \supset B(a)), A \to \exists x B(x)}\,[\exists\mathrm{R}]}{A, (A \supset B(a)) \to \exists x B(x)}\,[\mathrm{XL}]}{(A \supset B(a)) \to (A \supset \exists x B(x))}\,[\supset\mathrm{R}]}{\exists x(A \supset B(x)) \to (A \supset \exists x B(x))}\,[\forall\mathrm{R}]}{\to (\exists x(A \supset B(x)) \supset (A \supset \exists x B(x)))}\,[\supset\mathrm{R}] \quad \dfrac{\dfrac{A \to A \qquad \exists x B(x) \to \exists x B(x)}{(A \supset \exists x B(x)) \to (\neg A \vee \exists x B(x))}\,[2.5.2.\mathrm{L}] \quad \dfrac{\dfrac{\dfrac{\dfrac{A \to A}{\neg A, A \to}\,[\neg\mathrm{L}]}{\neg A, A \to B(a)}\,[\mathrm{WR}]}{A, \neg A \to B(a)}\,[\mathrm{XL}]}{\neg A \to (A \supset B(a))}\,[\supset\mathrm{R}]}{\neg A \to \exists x(A \supset B(x))}\,[\exists\mathrm{R}] \quad \dfrac{\dfrac{\dfrac{B(a) \to B(a)}{A, B(a) \to B(a)}\,[\mathrm{WL}]}{B(a) \to (A \supset B(a))}\,[\supset\mathrm{R}]}{B(a) \to \exists x(A \supset B(x))}\,[\exists\mathrm{R}]}{\exists x B(x) \to \exists x(A \supset B(x))}\,[\forall\mathrm{R}]}{(\neg A \vee \exists x B(x)) \to \exists x(A \supset B(x))}\,[\vee\mathrm{L}]}{(A \supset \exists x B(x)) \to \exists x(A \supset B(x))}\,[\mathrm{Cut}]}{\to ((A \supset \exists x B(x)) \supset \exists x(A \supset B(x)))}\,[\supset\mathrm{R}]}{\to (\exists x(A \supset B(x)) \equiv (A \supset \exists x B(x)))}\,[\wedge\mathrm{R}]$$

2.6.2)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A(a) \to A(a)}{A(a) \to A(a), B}\,[\mathrm{WR}]}{A(a) \to B, A(a)}\,[\mathrm{XR}] \qquad B \to B}{(A(a) \supset B), A(a) \to B, B}\,[\supset\mathrm{L}]}{(A(a) \supset B), A(a) \to B}\,[\mathrm{CR}]}{A(a), (A(a) \supset B) \to B}\,[\mathrm{XL}]}{\forall x A(x), (A(a) \supset B) \to B}\,[\forall\mathrm{L}]}{(A(a) \supset B) \to (\forall x A(x) \supset B)}\,[\supset\mathrm{R}]}{\exists x(A(x) \supset B) \to (\forall x A(x) \supset B)}\,[\forall\mathrm{R}]}{\to (\exists x(A(x) \supset B) \supset (\forall x A(x) \supset B))}\,[\supset\mathrm{R}] \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A(a) \to A(a)}{A(a) \to A(a), B}\,[\mathrm{WR}]}{\to A(a), (A(a) \supset B)}\,[\supset\mathrm{R}]}{\to A(a), \exists x(A(x) \supset B)}\,[\exists\mathrm{R}]}{\to \exists x(A(x) \supset B), A(a)}\,[\mathrm{XR}]}{\to \exists x(A(x) \supset B), \forall x A(x)}\,[\forall\mathrm{R}]}{\neg\forall x A(x) \to \exists x(A(x) \supset B)}\,[\neg\mathrm{L}] \quad \dfrac{\dfrac{\dfrac{B \to B}{A(a), B \to B}\,[\mathrm{WL}]}{B \to (A(a) \supset B)}\,[\supset\mathrm{R}]}{B \to \exists x(A(x) \supset B)}\,[\exists\mathrm{R}]}{(\neg\forall x A(x) \vee B) \to \exists x(A(x) \supset B)}\,[\vee\mathrm{L}]}{(\forall x A(x) \supset B) \to \exists x(A(x) \supset B)}\,[\mathrm{Cut}]}{\to ((\forall x A(x) \supset B) \supset \exists x(A(x) \supset B))}\,[\supset\mathrm{R}]}{\to (\exists x(A(x) \supset B) \equiv (\forall x A(x) \supset B))}\,[\wedge\mathrm{R}]$$

2.6.3)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A(a)\to A(a)\qquad B(a)\to B(a)}{A(a),(A(a)\supset B(a))\to B(a)}\,[\text{detachment}]}{\forall xA(x),(A(a)\supset B(a))\to B(a)}\,[\forall\text{L}]}{\forall xA(x),(A(a)\supset B(a))\to \exists xB(x)}\,[\exists\text{R}]}{(A(a)\supset B(a))\to(\forall xA(x)\supset\exists xB(x))}\,[\supset\text{R}]}{\exists x(A(x)\supset B(x))\to(\forall xA(x)\supset\exists xB(x))}\,[\forall\text{R}]}{\to(\exists x(A(x)\supset B(x))\supset(\forall xA(x)\supset\exists xB(x)))}\,[\supset\text{R}]$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A(a)\to A(a)}{A(a)\to A(a),B(a)}\,[\text{WR}]}{\to A(a),(A(a)\supset B(a))}\,[\supset\text{R}]}{\to A(a),\exists x(A(x)\supset B(x))}\,[\exists\text{R}]}{\to\exists x(A(x)\supset B(x)),A(a)}\,[\text{XR}]}{\to\exists x(A(x)\supset B(x)),\forall xA(x)}\,[\forall\text{R}]}{\neg\forall xA(x)\to\exists x(A(x)\supset B(x))}\,[\neg\text{L}]\qquad \dfrac{\dfrac{\dfrac{B(a)\to B(a)}{A(a),B(a)\to B(a)}\,[\text{WL}]}{B(a)\to(A(a)\supset B(a))}\,[\supset\text{R}]}{B(a)\to\exists x(A(x)\supset B(x))}\,[\exists\text{R}]}{\exists xB(x)\to\exists x(A(x)\supset B(x))}\,[\forall\text{R}]$$

$$\dfrac{\dfrac{\forall xA(x)\to\forall xA(x)\qquad\exists xB(x)\to\exists xB(x)}{(\forall xA(x)\supset\exists xB(x))\to(\neg\forall xA(x)\vee\exists xB(x))}\,[2.5.2.\text{L}]\qquad (\neg\forall xA(x)\vee\exists xB(x))\to\exists x(A(x)\supset B(x))}{(\forall xA(x)\supset\exists xB(x))\to\exists x(A(x)\supset B(x))}\,[\text{Cut}]$$

$$\dfrac{(\forall xA(x)\supset\exists xB(x))\to\exists x(A(x)\supset B(x))}{\to((\forall xA(x)\supset\exists xB(x))\supset\exists x(A(x)\supset B(x)))}\,[\supset\text{R}]$$

$$\to(\exists x(A(x)\supset B(x))\equiv(\forall xA(x)\supset\exists xB(x)))\,[\wedge\text{R}]$$

2.6.4)

$$\dfrac{\dfrac{\neg A\to\neg A\qquad B\to B}{(\neg A\supset B)\to(\neg\neg A\vee B)}\,[2.5.2.\text{L}]\qquad (\neg\neg A\vee B)\to(\neg B\supset A)}{(\neg A\supset B)\to(\neg B\supset A)}\,[\text{Cut}]$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A\to A}{\neg B,A\to A}\,[\text{WL}]}{A\to(\neg B\supset A)}\,[\supset\text{R}]}{\to(\neg B\supset A),\neg A}\,[\neg\text{R}]}{\neg\neg A\to(\neg B\supset A)}\,[\neg\text{L}]\qquad \dfrac{\dfrac{\dfrac{B\to B}{B\to B,A}\,[\text{WR}]}{B\to A,B}\,[\text{XR}]}{\neg B,B\to A}\,[\neg\text{L}]}{B\to(\neg B\supset A)}\,[\supset\text{R}]}{(\neg\neg A\vee B)\to(\neg B\supset A)}\,[\vee\text{L}]$$

2.6.5)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A\to A}{B,A\to A}\,[\text{WL}]}{A\to(B\supset A)}\,[\supset\text{R}]}{\to(B\supset A),\neg A}\,[\neg\text{R}]}{\neg\neg A\to(B\supset A)}\,[\neg\text{L}]\qquad \dfrac{\dfrac{\dfrac{\dfrac{B\to B}{B\to B,A}\,[\text{WR}]}{B\to A,B}\,[\text{XR}]}{\neg B,B\to A}\,[\neg\text{L}]}{B,\neg B\to A}\,[\text{XL}]}{\neg B\to(B\supset A)}\,[\supset\text{R}]}{(\neg\neg A\vee\neg B)\to(B\supset A)}\,[\vee\text{L}]$$

$$\dfrac{\dfrac{\neg A\to\neg A\qquad\neg B\to\neg B}{(\neg A\supset\neg B)\to(\neg\neg A\vee\neg B)}\,[2.5.2.\text{L}]\qquad (\neg\neg A\vee\neg B)\to(B\supset A)}{(\neg A\supset\neg B)\to(B\supset A)}\,[\text{Cut}]$$

2.7)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A(a)\to A(a)}{A(a)\to A(a),B}\,[\text{WR}]}{\to A(a),(A(a)\supset B)}\,[\supset\text{R}]}{\to A(a),\exists x(A(x)\supset B)}\,[\exists\text{R}]}{\to\exists x(A(x)\supset B),A(a)}\,[\text{XR}]}{\to\exists x(A(x)\supset B),\forall xA(x)}\,[\forall\text{R}]\qquad \dfrac{\dfrac{\dfrac{B\to B}{A(a),B\to B}\,[\text{WL}]}{B\to(A(a)\supset B)}\,[\supset\text{R}]}{B\to\exists x(A(x)\supset B)}\,[\exists\text{R}]}{(\forall xA(x)\supset B)\to\exists x(A(x)\supset B),\exists x(A(x)\supset B)}\,[\supset\text{L}]$$

$$\dfrac{(\forall xA(x)\supset B)\to\exists x(A(x)\supset B),\exists x(A(x)\supset B)}{(\forall xA(x)\supset B)\to\exists x(A(x)\supset B)}\,[\text{CR}]$$

Detachment Rule)

$$\dfrac{\dfrac{\dfrac{\dfrac{A \to A}{A \to A, B}\,[\mathrm{WR}]}{A \to B, A}\,[\mathrm{XR}] \qquad B \to B}{(A \supset B), A \to B, B}\,[\supset \mathrm{L}]}{(A \supset B), A \to B}\,[\mathrm{CR}]}{A, (A \supset B) \to B}\,[\mathrm{XL}]$$

Double Negation)

$$\dfrac{\dfrac{\dfrac{\dfrac{A \to A}{\neg A, A \to}\,[\neg \mathrm{L}]}{A \to \neg\neg A}\,[\neg \mathrm{R}]}{\to (A \supset \neg\neg A)}\,[\supset \mathrm{R}] \qquad \dfrac{\dfrac{\dfrac{A \to A}{\to A, \neg A}\,[\neg \mathrm{R}]}{\neg\neg A \to A}\,[\neg \mathrm{L}]}{\to (\neg\neg A \supset A)}\,[\supset \mathrm{R}]}{\to (A \equiv \neg\neg A)}\,[\wedge \mathrm{R}]$$