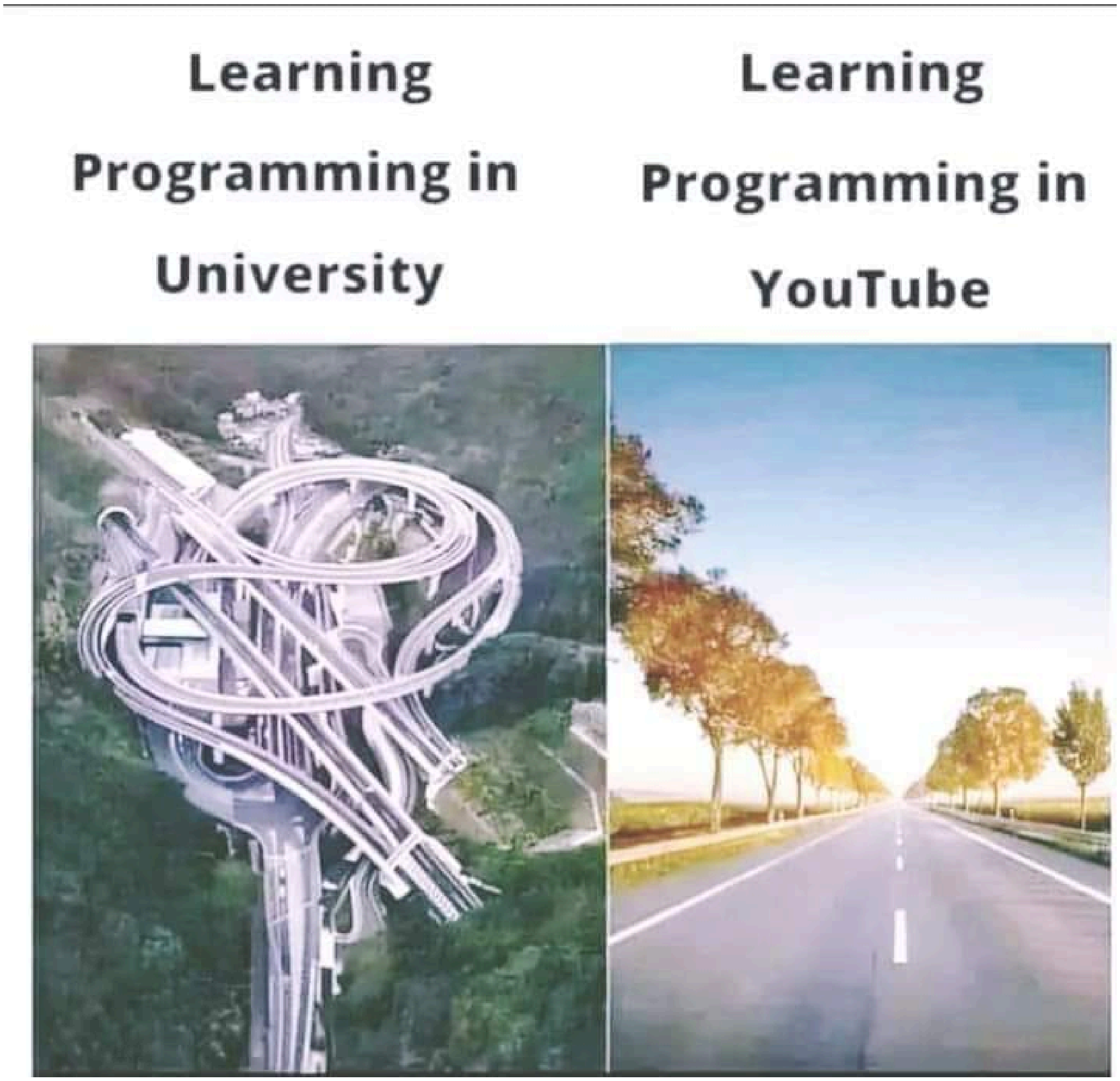
Tree-based sorting

AP



- Mock lecture on Tree-based sorting
- I will explain my pegagogy as I go through

Concept check: Sorting

input: a sequence of integers

output: a reorganisation such that each element will be less than or equal the next

$a \;=\; [5, 0, 2, 11, 18, 11, 6, 36]$

$a.sorted() \;=\; [0, 2, 5, 6, 11, 11, 18, 36]$

"easy to check, not so easy to establish''

Q: sorting might in fact destroy some information. What might it be?

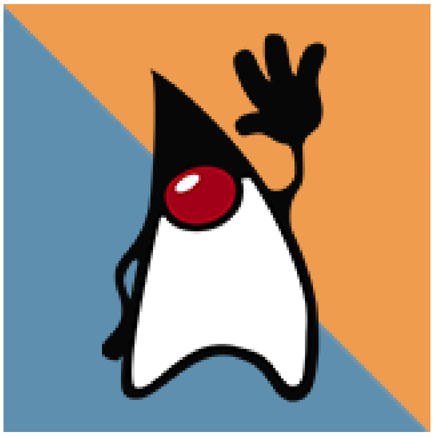
- min, max and median are available in constant time: $a[0]$, $a[n-1]$ and $a[\frac{n}{2}]$, respectively.
- membership can be checked with $\log_2 n$ comparisons at most
- *stability:* multiple copies of the same number should keep their original ordering

$a \;=\; [5, 0, 2, 11', 18, 11'', 6, 36]$

$a.sorted() \;\Rightarrow [0, 2, 5, 6, 11', 11'', 18, 36]$

Concept check: sorting in Java

```java
import java.util.Arrays;

int[] myArray = { 5, 0, 2, 11, 18, 11, 6, 36 };

Arrays.sort(myArray);

System.out.println(Arrays.toString(myArray));
```



CC: build your arrays class

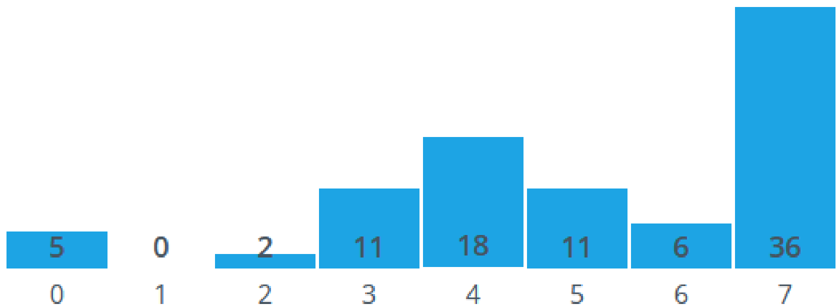
```java
public class MyArray {
    private int[] arrayData; // Internal array to store elements
    private int size; // Number of actual elements in the array

    // Constructor to initialize the internal array
    // capacity is the maximun allowed number of elements
    public MyArray(int capacity) {
        arrayData = new int[capacity];
        size = 0;
    }
```

See the class file from last week

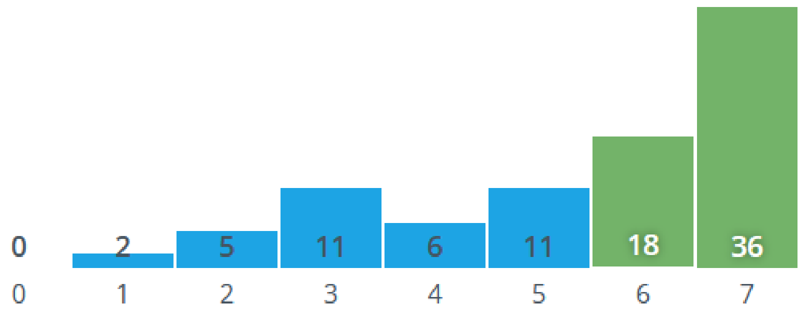
Sorting by pairwise comparison

```java
    // Method to sort the array
    public void sort() {
        // Simple implementation of the Bubble Sort algorithm
        for (int i = 0; i < size - 1; i++) {
            for (int j = 0; j < size - i - 1; j++) {
                if (arrayData[j] > arrayData[j + 1]) {
```

```
                    // Swap arrayData[j] and arrayData[j+1]
                    int temp = arrayData[j];
                    arrayData[j] = arrayData[j + 1];
                    arrayData[j + 1] = temp;
                }
            }
        }
    }
```

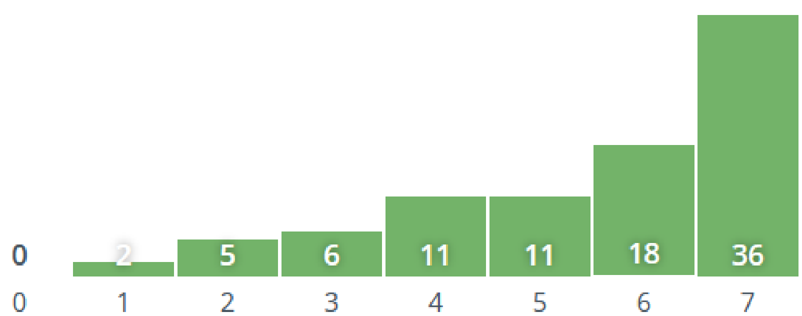


```
// Method to sort the array
public void sort() {
    // Simple implementation of the Bubble Sort algorithm
    for (int i = 0; i < size - 1; i++) {
        for (int j = 0; j < size - i - 1; j++) {
            if (arrayData[j] > arrayData[j + 1]) {
                // Swap arrayData[j] and arrayData[j+1]
                int temp = arrayData[j];
                arrayData[j] = arrayData[j + 1];
                arrayData[j + 1] = temp;
    ...
```



- values in green are in their final position
- *all blue elements have been seen already and we have ideas about where they will likely end up…*

```java
// Method to sort the array
public void sort() {
    // Simple implementation of the Bubble Sort algorithm
    for (int i = 0; i < size - 1; i++) {
        for (int j = 0; j < size - i - 1; j++) {
            if (arrayData[j] > arrayData[j + 1]) {
                // Swap arrayData[j] and arrayData[j+1]
                int temp = arrayData[j];
                arrayData[j] = arrayData[j + 1];
                arrayData[j + 1] = temp;
            }
        }
    }
}
```



- green elem. have indices corresponding to their ranking: the no. of $\leq$ elements.

- only contigous elements will ever be swapped
- all pairwise comparisons are attempted, often several times: is it really needed?
- what if the data is already half-sorted?

  Sorting often takes place after an update to one or more values destroys the sorted property of the array. So, sorting is called to re-establish the property.

Example:

$b = [0, 2, 6, 5, 11, 11, 18, 36]$

Cost analysis

- when `i=0` the inner cycle on `j` executes n-1 times,
- then `i=1` and the inner cycle on `j` executes n-2 times, and so on.
- all in all, the innermost code will execute about $\frac{n(n-1)}{2} \approx n^2$ times
- our BubbleSort algorithm won't scale up to web data, log analysis, machine learning etc.
- we need algorithms that looks at data and carry out only as many comparisons/swaps as needed.

The tree abstraction

Idea: a data structure that stores values in a way that *represents* what is known about its *rank* in the final version of the sequence.

It will reduce unnecessary comparisons.

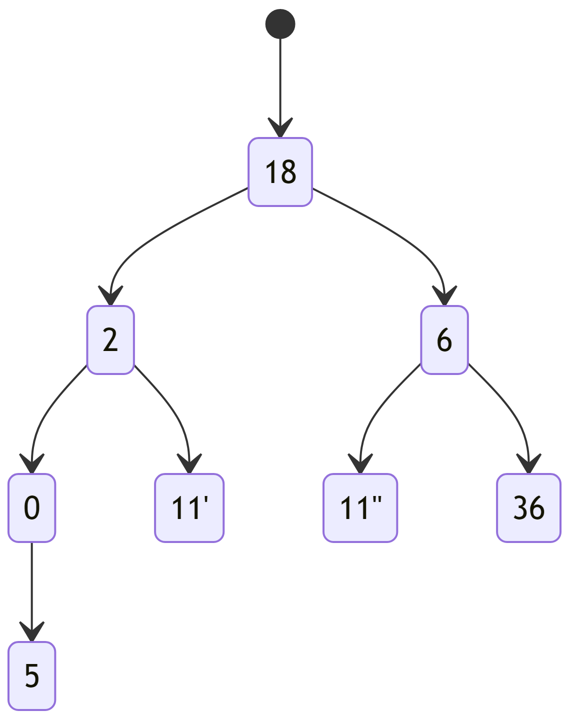
The new structure has visual properties that simplify algorithm design and analysis: it's everywhere in computer science.

A tree

- a special *root* element which is directly accessible
- each element has access to 0..k elements, called *children*
- siblings are not connected to each other directly
- childless elements are called leaves
- the *height* of the tree is defined as the *longest* root-to-leaf path.

A Binary tree: k=2.

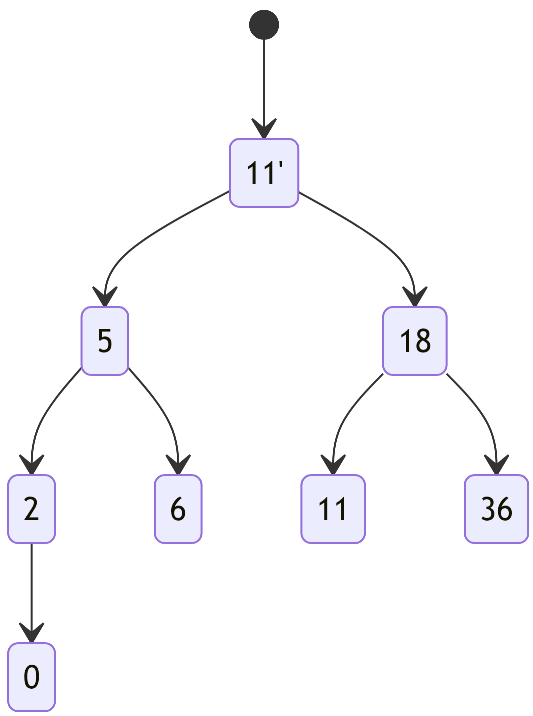
Children elements will be *left* and *right*, resp.



- Complete left-to-right: the binary tree has no 'holes'
- never a right leaf node without its left sibling.

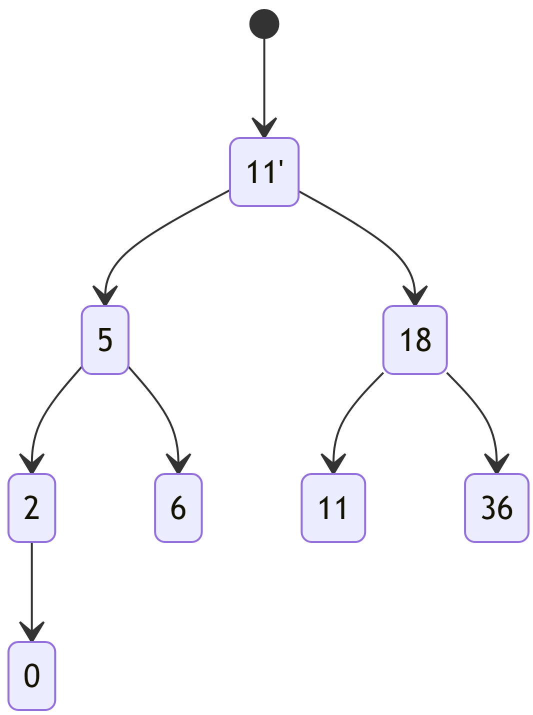
A Binary Search Tree (BST)

- recursively, left children never exceed ($\leq$) their anchestors
- right children always do.



11'
5
18
2
6
11
36
0

Position on the BST relates to ranking



- Q: where are min, max and median elements?
- Q: Can you think of an algorithm that will print out the values in sorted fashion?

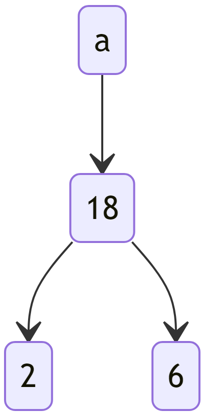
Tree as a data structure

- in arrays, each element, say *a[i]* is 'next' to two (at most): *a[i-1]* and *a[i+1]*
- in binary tree, *tree[i]* is 'next' to its parent, *tree[parent(i)]* and one or two children: *tree[left(i)]* and *tree[right(i)]*.
- fact: the binary tree organization can be implemented in RAM with no extra space and minimal time overhead to compute the *parent(), left()* and *right()* functions.
- elegant functions will implement ordered trees and make sort and in general accessing the sequence quick.

BST serialization

The BST is a *view* over an array:

```
int[] a = {18, 2, 6};
```



Assume indexing from 1 and try these functions:

```
int left(int i) {return 2*i;}

int left(int i) {return 2*i+1;}

int parent(int i) {return (int) i/2;}
```

Efficency

- Thanks to binary representation, division/multiplication by 2 can be done in one machine instruction
- visiting a complete BST is very efficient!

```
// implements left()
byte originalByte = 0b0011_0100; // 52 in binary

int shiftedByte = originalByte << 1; // Shift left by 1 positions
```

```
// implents parent()
byte originalByte = 0b0011_0100; // 52 in binary

int shiftedByte = originalByte >> 1; // Shift right by 1 positions
```

- we now abandon the idealised vision of complete BST to look at BST whose shape could be *irregular*

Create a BST from output, I

```
// Define a class for the nodes of the tree
class Node {
    int value;
    Node left, right;

    public Node(int item) {
        value = item;
        // at 'birth,' nodes are childless
        left = right = null;
    }
}
```

```
// Define the Binary Search Tree (BST) class
class BinarySearchTree {
    // Root of BST
    Node root;

    // Constructor
    BinarySearchTree() {
        root = null;
    }
```

Create a BST from output, 2

```
 // Method to insert a new key
    void insert(int value) {
        root = insertRec(root, value);
    }
```

```java
// Recursive insert function
Node insertRec(Node root, int value) {
    // If the tree is empty, return a new node
    if (root == null) {
        root = new Node(value);
        return root;
    }

    // Otherwise, recur down the tree
    if (value < root.value)
        root.left = insertRec(root.left, value);
    else if (value > root.value)
        root.right = insertRec(root.right, value);

    // Return the (unchanged) node pointer
    return root;
}
```

Live coding

```java
// Method to conduct inorder traversal of the tree
void inorder() {
    inorderRec(root);
}

// Visit the BT and print out the values
// in ascending order
```

```java
// Method to conduct inorder traversal of the tree
void inorder() {
    inorderRec(root);
}

// Recursive function for inorder traversal
void inorderRec(Node root) {
    if (root != null) {
        inorderRec(root.left);
```

```
            System.out.print(root.value + " ");
            inorderRec(root.right);
        }
    }
```

```
    // Main method to test the BinarySearchTree class
    public static void main(String[] args) {
        BinarySearchTree bst = new BinarySearchTree();

        // Insert values into BST
        bst.insert(11);
        bst.insert(5);
        bst.insert(18);
        bst.insert(2);
        bst.insert(6);
        bst.insert(11);
        bst.insert(36);
        bst.insert(0);

        // Print the inorder traversal of the BST
        System.out.println("Inorder traversal of the BST:");
        bst.inorder();
    }
}
```

Good properties

- in-order transversal of the BST corresponds to sorting.
- if the BST is *balanced:* no. of left successors and no. of right successor is roughly equal:
- height, i.e., the longest root-to-leaf possible visit, is going to be about $\log_2 n$
- finding max and min will require only $\log_2 n$ accesses.
- in general, we can find the element of a given rank with only $\log_2 n$ accesses.

Bad properties

- if the BST is *unbalanced*, it could end up, e.g., with all left successors and no right successor
- finding max or max would then take n accesses: no better than with an unsorted array.