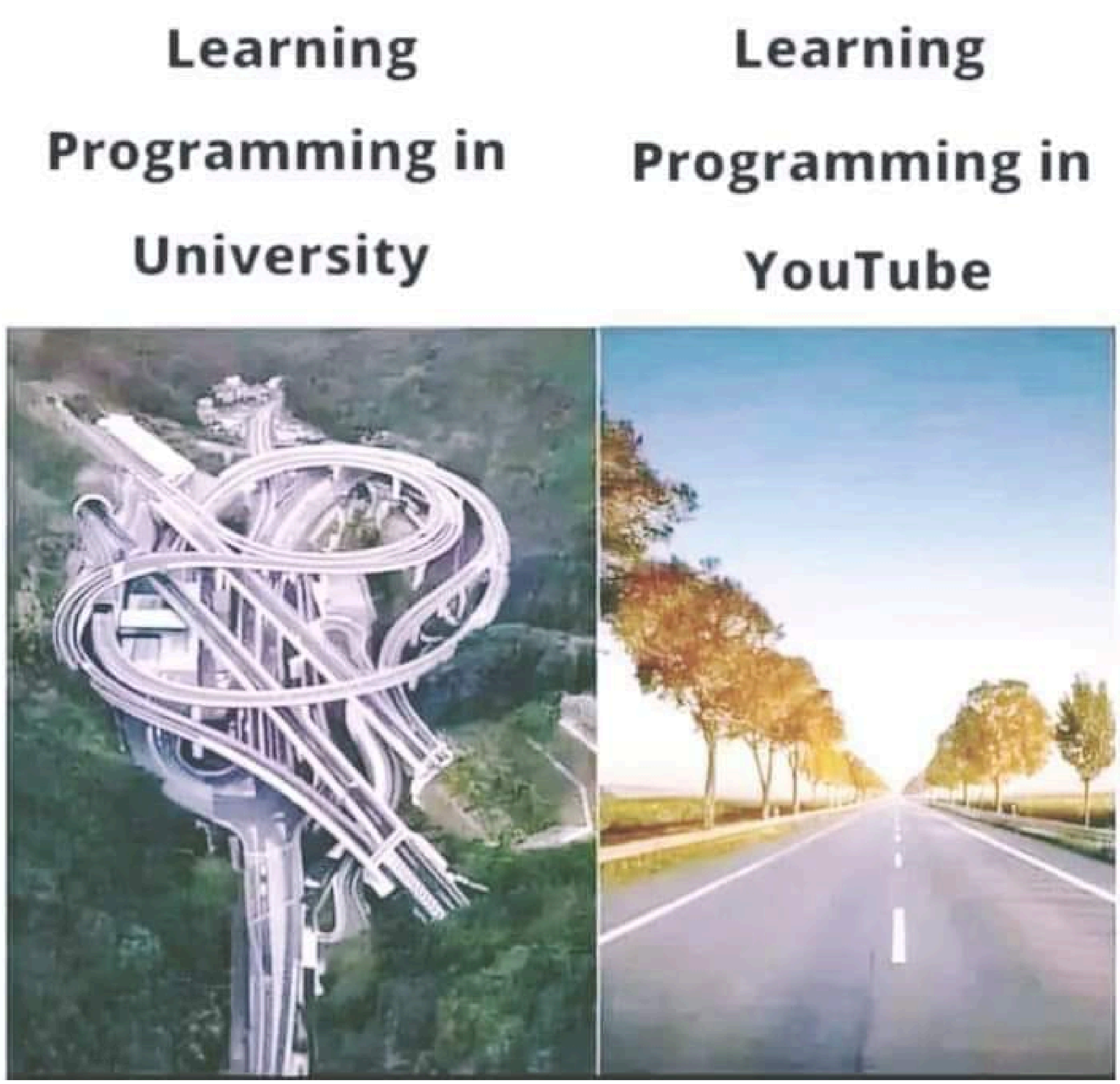
Tree-based sorting

AP



- Mock lecture on Tree-based sorting
- I will explain my pegagogy as I go through

Objectives

We continue work on the sorting problem

We start learning a new, powerful data structure: trees

Trees in CS were developed rather early to manage *scarce* dynamic memory

The grow *downward*

Concept Checking

Concept check: Sorting

input: a sequence of integers

output: a reorganisation such that each element will be less than or equal the next

$a \ = \ [5, 0, 2, 11, 18, 11, 6, 36]$

$a.sorted() \ = \ [0, 2, 5, 6, 11, 11, 18, 36]$

“easy to check, not so easy to establish’ ’

Q: sorting might in fact destroy some information. What might it be?

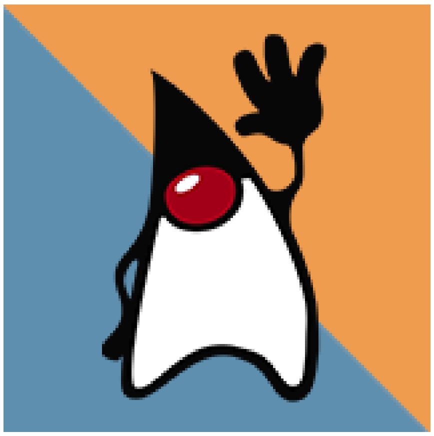
- min, max and median are available in constant time: $a[0]$, $a[n-1]$ and $a[\frac{n}{2}]$, respectively.
- membership can be checked with about $\log_2 n$ comparisons
- *stability:* multiple copies of the same number should keep their original ordering

$a \ = \ [5, 0, 2, 11', 18, 11'', 6, 36]$

$a.sorted() \ \Rightarrow [0, 2, 5, 6, 11', 11'', 18, 36]$

Concept check: sorting in Java

```java
import java.util.Arrays;

int[] myArray = { 5, 0, 2, 11, 18, 11, 6, 36 };

Arrays.sort(myArray);

System.out.println(Arrays.toString(myArray));
```



CC: build your arrays class

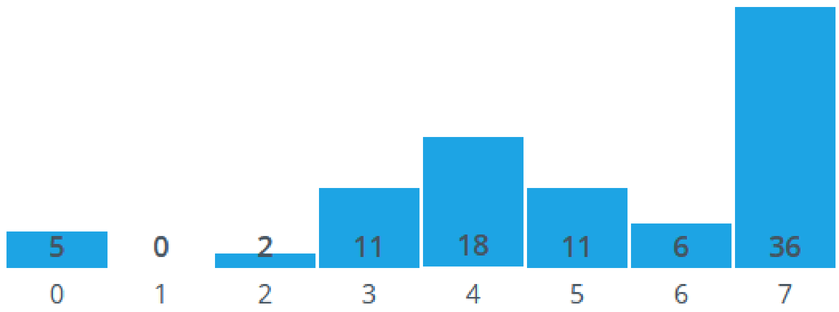
```java
public class MyArray {
    private int[] arrayData; // Internal array to store elements
    private int size; // Number of actual elements in the array

    // Constructor to initialize the internal array
    // capacity is the maximun allowed number of elements
    public MyArray(int capacity) {
        arrayData = new int[capacity];
        size = 0;
    }
```

See the class file from last week

Sorting

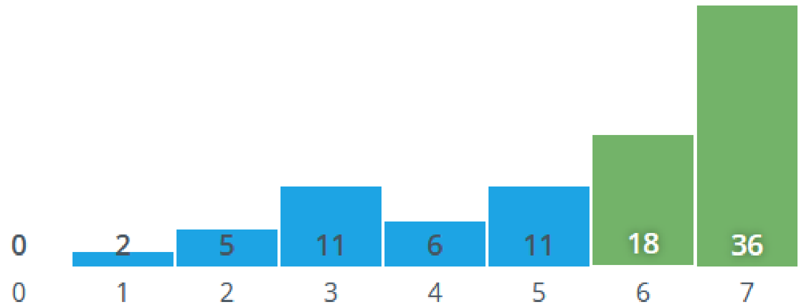
Sorting by pairwise comparison

```
    // Method to sort the array
    public void sort() {
        // Simple implementation of the Bubble Sort algorithm
        for (int i = 0; i < size - 1; i++) {
            for (int j = 0; j < size - i - 1; j++) {
                if (arrayData[j] > arrayData[j + 1]) {
                    // Swap arrayData[j] and arrayData[j+1]
                    int temp = arrayData[j];
                    arrayData[j] = arrayData[j + 1];
                    arrayData[j + 1] = temp;
                }
            }
        }
    }
```



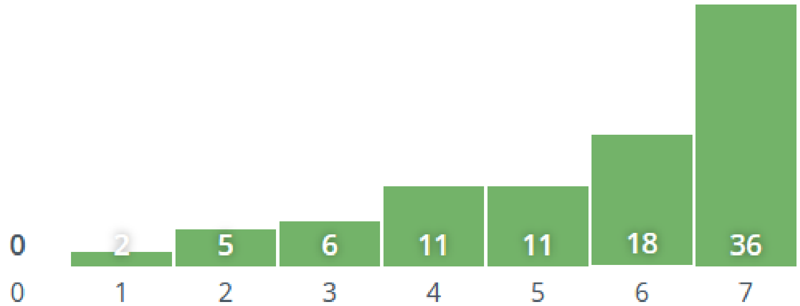
```
// Method to sort the array
public void sort() {
    // Simple implementation of the Bubble Sort algorithm
    for (int i = 0; i < size - 1; i++) {
        for (int j = 0; j < size - i - 1; j++) {
            if (arrayData[j] > arrayData[j + 1]) {
                // Swap arrayData[j] and arrayData[j+1]
                int temp = arrayData[j];
```

```
                arrayData[j] = arrayData[j + 1];
                arrayData[j + 1] = temp;
    ...
```



- values in green are in their final position
- *all blue elements have been seen already and we have ideas about where they will likely end up…*

```
// Method to sort the array
public void sort() {
    // Simple implementation of the Bubble Sort algorithm
    for (int i = 0; i < size - 1; i++) {
        for (int j = 0; j < size - i - 1; j++) {
            if (arrayData[j] > arrayData[j + 1]) {
                // Swap arrayData[j] and arrayData[j+1]
                int temp = arrayData[j];
                arrayData[j] = arrayData[j + 1];
                arrayData[j + 1] = temp;
            }
        }
    }
}
```



- green elem. have indices corresponding to their ranking: the no. of $\leq$ elements.

- only contigous elements will ever be swapped
- all pairwise comparisons are attempted, often several times: is it really needed?
- what if the data is already half-sorted?

  Sorting often takes place after an update to one or more values destroys the sorted property of the array. So, sorting is called to re-establish the property.

Example:

$b = [0, 2, 6, 5, 11, 11, 18, 36]$

Cost analysis

- when `i=0` the inner cycle on `j` executes n-1 times,
- then `i=1` and the inner cycle on `j` executes n-2 times, and so on.
- all in all, the innermost code will execute about $\frac{n(n-1)}{2} \approx n^2$ times
- our BubbleSort algorithm won't scale up to web data, log analysis, machine learning etc.
- we seek algorithms that *look at data* and carry out only as many comparisons/swaps as needed.

The tree abstraction

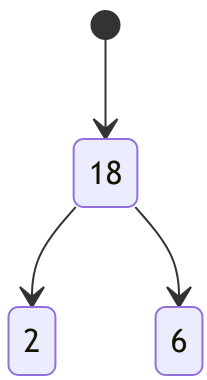
Idea

Idea: a data structure that stores values in a way that *represents* what is known about its *rank* in the final version of the sequence.

It will reduce unnecessary comparisons.

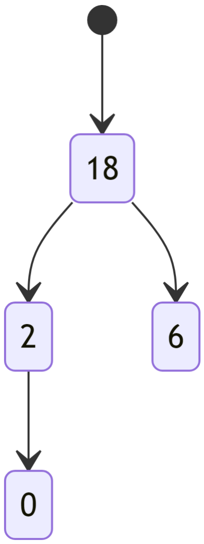
The new structure has visual properties that simplify algorithm design and analysis: it's everywhere in CS!

A tree



- a special *root* element which is directly accessible
- each element has access to 0..k elements, called *children*
- siblings are not connected to each other directly

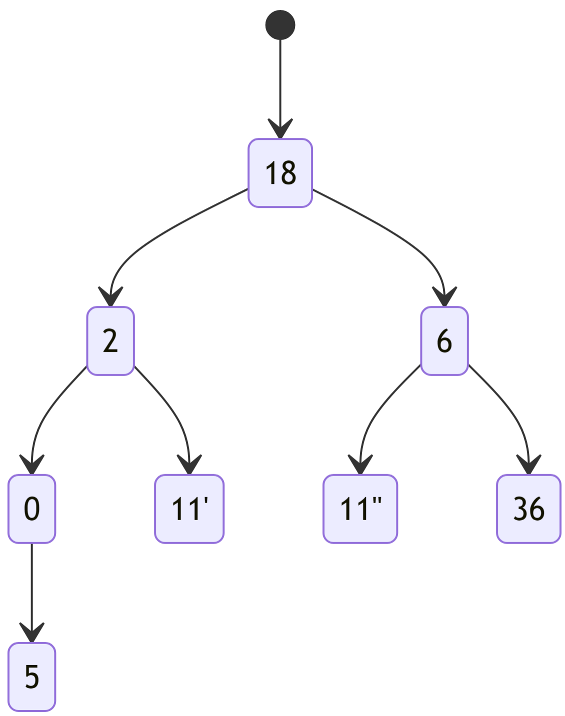
A tree, cont'd



- childless elements are called leaves
- the *height* of the tree is defined as the *longest* root-to-leaf path.

A Binary tree: k=2.

Children elements will be *left* and *right*, resp.

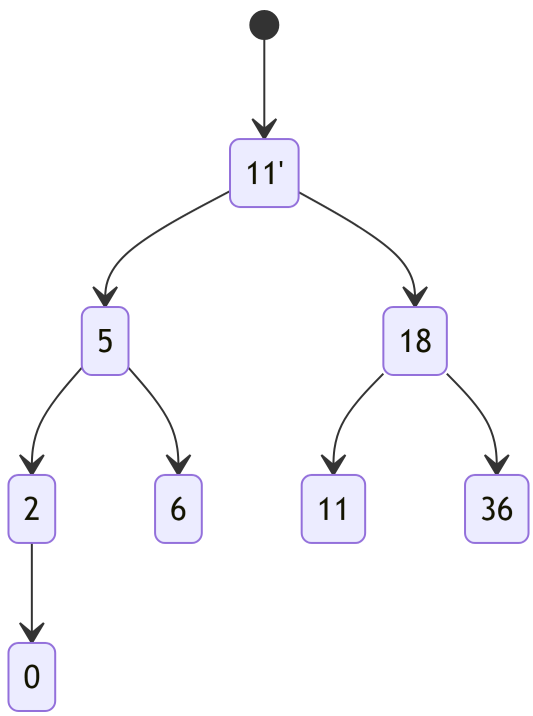


- Complete left-to-right: a BT with no ‘holes’
- never a right leaf node without its left sibling.

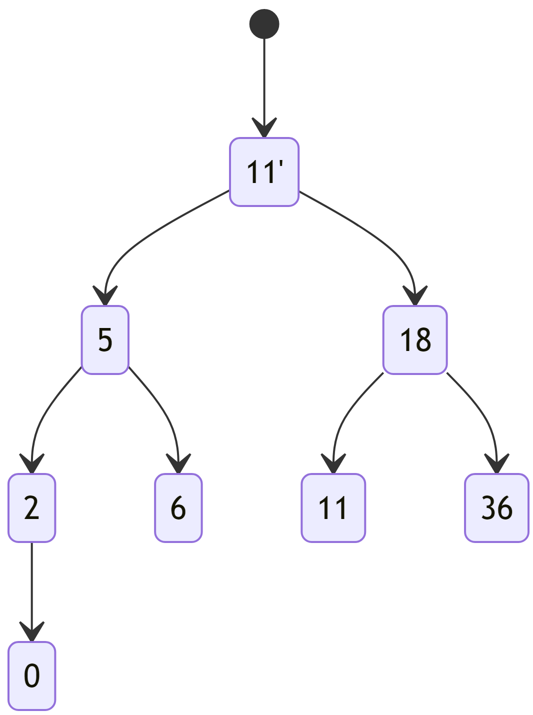
A Binary Search Tree (BST)

At each level

- the left child never exceed ($\leq$) its parent
- the right child always does


11'
5
18
2
6
11
36
0

Position on the BST relates to ranking



- Q: where are min, max and median elements?
- Q: Can you think of an algo. that prints out the values in sorted fashion?

Rethinking arrays

- each element, say `a[i]` is *next* to two (at most): `a[i-1]` and `a[i+1]`
- real physical memory or the simplest of abstractions?

Bin. trees as a data structure

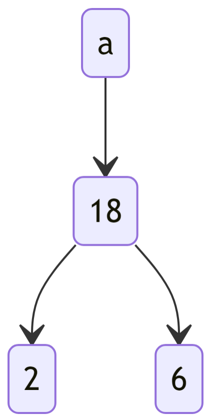
each `tree[i]` element is *next* to its parent, `tree[parent(i)]` and one or two children: `tree[left(i)]` and `tree[right(i)]`.

- Fact: complete BTs can be implemented in RAM with *no extra space* and minimal time overhead to compute the `parent()`, `left()` and `right()` functions.
- elegant functions will implement ordered BTs/BSTs: sorting accelerates!

BST as abstraction over RAM

The BST is a *view* of, essentially, an array:

```
int[] a = {18, 2, 6};
```



Assume indexing from 1 and try these functions:

```
int left(int i) {return 2*i;}

int left(int i) {return 2*i+1;}

int parent(int i) {return (int) i/2;}
```

In depth: tree navigation

Concept checking: tree navigation

left(1) $\rightarrow$ 2

right(1) $\rightarrow$ 3

left(2) $\rightarrow$ 4

right(2) $\rightarrow$ 5

left(3) $\rightarrow$ 6

- left(52) $\rightarrow$?
- right(52) $\rightarrow$?

- parent(52) $\rightarrow$?

efficiency

- Thanks to binary representation, division/multiplication by 2 can be done in one CPU cycle

```
// implements left()
byte originalByte = 0b0011_0100; // 52 in binary

int shiftedByte = originalByte << 1; // Shift left by 1 positions
```

- visiting a complete BST is very efficient!

```
// implements parent()
byte originalByte = 0b0011_0100; // 52 in binary

int shiftedByte = originalByte >> 1; // Shift right by 1 positions
```

We now leave the idealised vision of complete BST to look at *general* BST whose shape could be *irregular.*

In Depth: The BST class

BST as a class, I

```
// Define a class for the nodes of the tree
class Node {
    int value;
    Node left, right;

    public Node(int item) {
        value = item;
        // at 'birth,' nodes are childless
        left = right = null;
    }
}
```

BST as a class, II

```
// Define the Binary Search Tree (BST) class
class BinarySearchTree {
    // Root of BST
    Node root;

    // Constructor
    BinarySearchTree() {
        root = null;
    }
```

BST as a class, III

```
 // Method to insert a new key
    void insert(int value) {
        root = insertRec(root, value);
    }

    // Recursive insert function
    Node insertRec(Node root, int value) {
        // If the tree is empty, return a new node
        if (root == null) {
            root = new Node(value);
            return root;
        }

        // Otherwise, recur down the tree
        if (value < root.value)
            root.left = insertRec(root.left, value);
        else if (value > root.value)
            root.right = insertRec(root.right, value);

        // Return the (unchanged) node pointer
        return root;
    }
```

My first BST

```
    // Main method to test the BinarySearchTree class
    public static void main(String[] args) {
        BinarySearchTree bst = new BinarySearchTree();

        // Insert values into BST
        bst.insert(11);
        bst.insert(5);
        bst.insert(18);
        bst.insert(2);
        bst.insert(6);
        bst.insert(11);
        bst.insert(36);
        bst.insert(0);

        // Print the inorder traversal of the BST
        System.out.println("Inorder traversal of the BST:");
        bst.inorder();
    }
}
```

Live Coding

Tree transversal

Given a reference to the root node, print its content in ascending order.

Exploit the recursive BST property: all sub-trees are BST themselves.

```
    // Method to conduct inorder traversal of the tree
    void inorder() {
        inorderRec(root);
    }

    // Visit the BT and print out the values
    // in ascending order
```

```java
    // Method to conduct inorder traversal of the tree
    void inorder() {
        inorderRec(root);
    }

    // Recursive function for inorder traversal
    void inorderRec(Node root) {
        if (root != null) {
            inorderRec(root.left);
            System.out.print(root.value + " ");
            inorderRec(root.right);
        }
    }
```

Analysis

Good properties

- in-order transversal of the BST corresponds to sorting.
- if the BST is *balanced:* no. of left successors and no. of right successor is roughly equal:
- height, i.e., the longest root-to-leaf possible visit, is going to be about $\log_2 n$
- finding max and min will require only $\log_2 n$ accesses.
- in general, we can find the element of a given rank with only $\log_2 n$ accesses.

Bad properties

- if the BST is *unbalanced,* it could end up, e.g., with all left successors and no right successor
- finding max or max would then take n accesses: no better than with an unsorted array.

Conclusions

BubbleSort: easy to visualise+implement but costly

Many comparisons are pointless; costs grow with n^2

Arrays: a straightforward abstraction of RAM

- Tree: a new abstration, relatively easy to code and lightweight
- Binary Search Trees: a type of tree that makes values easy to sort/search
- However BST may degenerate into (costly) *straight lists*
- We need techniques to turn arbitrary sequences into balanced BST