LEARN CODING

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github.com/ale66/learn-coding

PRACTICE ALGORITHMICS

FORMULATION, III

Instance:

a sequence of *n* `weight/value` pairs: $w_1, \ldots w_m$ and $v_1, \ldots v_n$

an integer C

Solution:

An allocation: for each bar says what quantity is taken: $\sigma : i \rightarrow [0..w_i]$

Constraint

The sum of all taken quantities must not exceed C: $\sum_{i=1}^{n} \sigma(i) \leq C$

ALGORITHM

Think of a step-by-step process that, for any input combination, will take the most value out of the vault

Write it down in English with some Maths

Test it on some examples, e.g.

Metal	Weight avail.	Total Val.
Gold	3	162
Palladium	1	72
Silver	12	48

C = 5

IDEA

- Consider the *unit* (per Kg) value: total value / available quantity
- sort the elements by descending unit value
- start from the top, take all there is, continue below, until capacity C is reached
- the last element, and only the last, may have to be cut to measure in order not to exceed C

This is the algorithm

- Consider the *unit* (per Kg) value: total value / available quantity

Metal	Weight avail.	Total Val.	Unit val.
Gold	3	162	54
Palladium	1	72	72
Silver	12	48	4

- sort the elements by descending unit value

Metal	Weight avail.	Total Val.	Unit val.	Pos.
Palladium	1	72	72	1
Gold	3	162	54	2
Silver	12	48	48	3

- start from the top, take all there is, […]

Metal	Weight avail.	Total Val.	Unit val.	Pos.	Take
Palladium	0	72	72	1	1
Gold	3	162	54	2	
Silver	12	48	4	3	

$$\sigma(Palladium) = 1$$

C = 5-1 = 4

- [...] continue below, [...]

Metal	Weight avail.	Total Val.	Unit val.	Pos.	Take
Palladium	0	72	72	1	1
Gold	0	162	54	2	3
Silver	12	48	4	3	

$\sigma(Palladium) = 1\, \sigma(Gold) = 3$

C = 4-3 = 1

- […] until capacity C is reached

Metal	Weight avail.	Total Val.	Unit val.	Pos.	Take
Palladium	0	72	72	1	1
Gold	0	162	54	2	3
Silver	12	48	4	3	

$\sigma(Palladium) = 1 \, \sigma(Gold) = 3$

C = 1

- the last element, and only the last, may have to be cut to measure in order not to exceed C

Metal	Weight avail.	Total Val.	Unit val.	Pos.	Take
Palladium	0	72	72	1	1
Gold	0	162	54	2	3
Silver	11	48	4	3	1

$\sigma(Palladium) = 1\, \sigma(Gold) = 3\, \sigma(Silver) = 1$

C = 1-1 = 0

Total value taken: $1 \times 72 + 3 \times 54 + 1 \times 4 \;= 72 + 162 + 4 = 238$

OBERVATIONS

Theorem: our algorithm is *optimal*: it always finds the best solution

Computational cost: the costly step is sorting the table of metals

Fact: the number of basic computer steps depends on the number n of metals with law $K(n\log_2 n + n)$

- *K* depends of the details of the implementation, e.g., Python 3.12.6 on Win 11 and AMD Ryzen 9
- $n\log_2 n$ steps to sort the elements
- up to n step to select the bars and decrease the residual capacity

This problem is generally scalable to the web

INTRACTABLE PROBLEMS

Problems for which no scalable algorithm is known:

Their cost (no. of operations to complete) is expressed as $K2^n$: exponential

Only approximate solutions exist that can find a *good enough* solution with a low-growth cost function

PROBLEM VARIATION

This time the Fourty thieves’ vault contains precious artesanal objects, e.g., gold watches.

Each watch is described by weight and value

For each object, **it’s either you take it or leave it.**

KNAPSACK 0-1

Instance:

a sequence of *n* `weight/value` pairs: $w_1, \ldots w_m$ and $v_1, \ldots v_n$

an integer C

Solution:

An assignment: for each object says whether it's taken or not: $\sigma : i \rightarrow \{0, 1\}$

Constraint

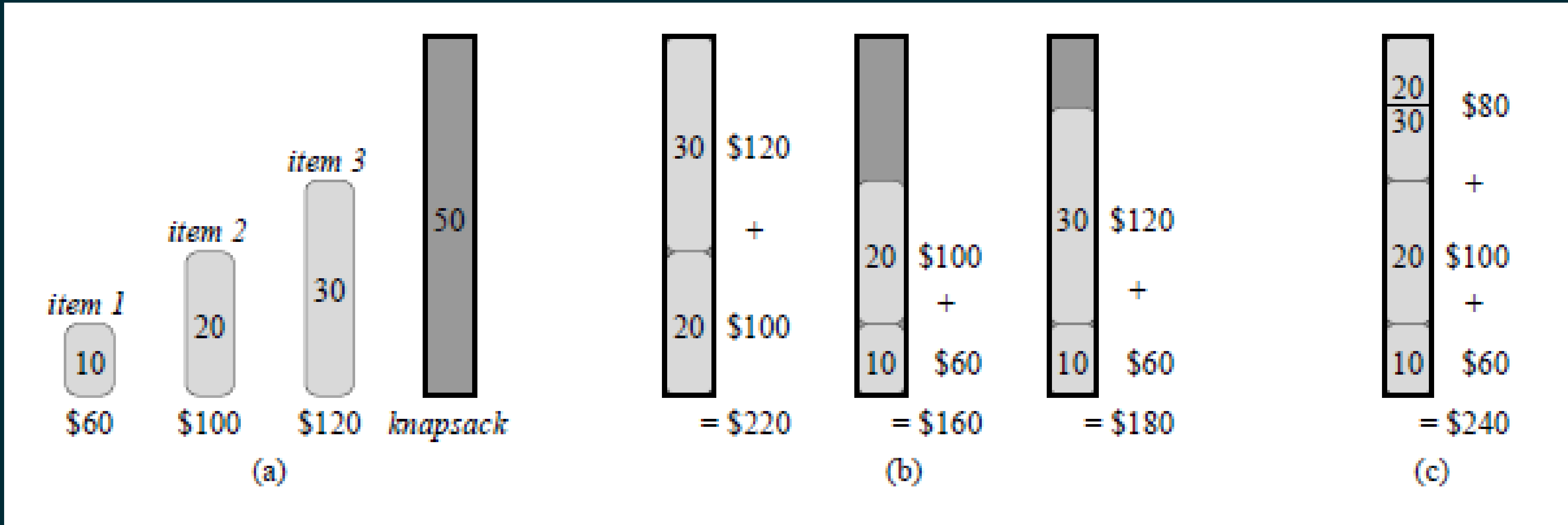
The sum of all taken weights C: $\sum_{i=1}^{n} w_i \sigma(i) \leq C$

SURPRISE WITH KNAPSACK 0-1

The algorithm seen above stops working: the computed solution can be very suboptimal

Metal	Weight	Value
Ring	10	60
Earrings	20	100
Necklace	30	120

C = 50



The returend solution (1/1/0) is not even second-best

best solution is 0/1/1 and 1/0/1 is second-best

KNAPSACK 0-1 IS INTRACTABLE

No algorithm is known that can solve it in acceptable times as n grows

The take-it-or-leave-it nature of the problem forces us to consider up to 2^n alternative solutions

this will require spending an exponential amount of operations before we could be sure that the solution at hand is the best

It is believed that no scalable algorithm will ever be found: the P vs. NP conjecture

SORTING

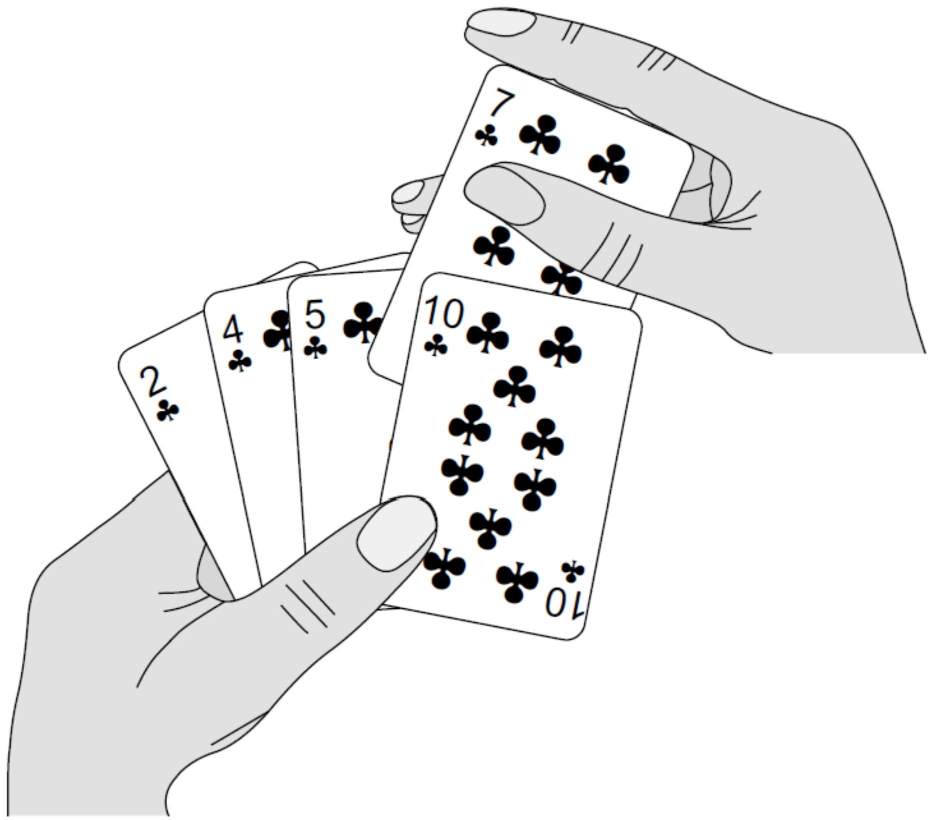


Figure 2.1 Sorting a hand of cards using insertion sort.