



Lonardi et al. Phys. Rev. E 107, 024302  
(2023)

# Infrastructure adaptation and emergence of loops in network routing with time-dependent loads

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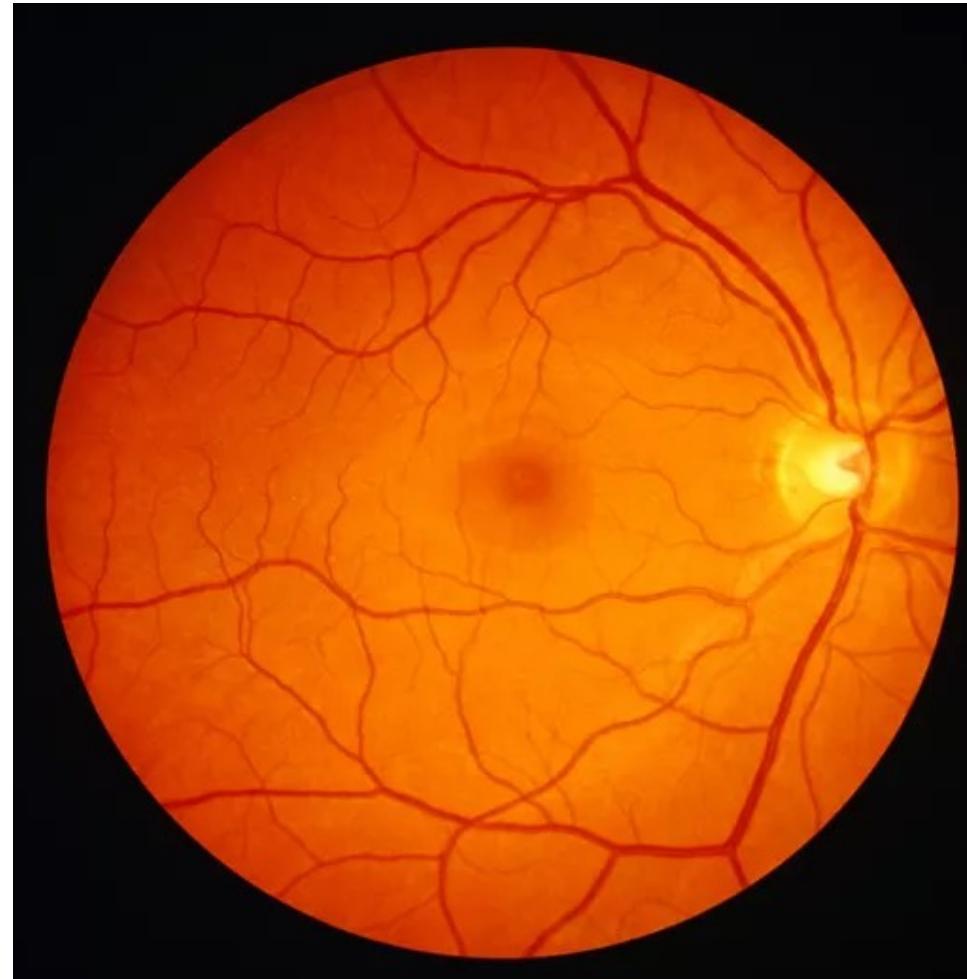


imprs-is

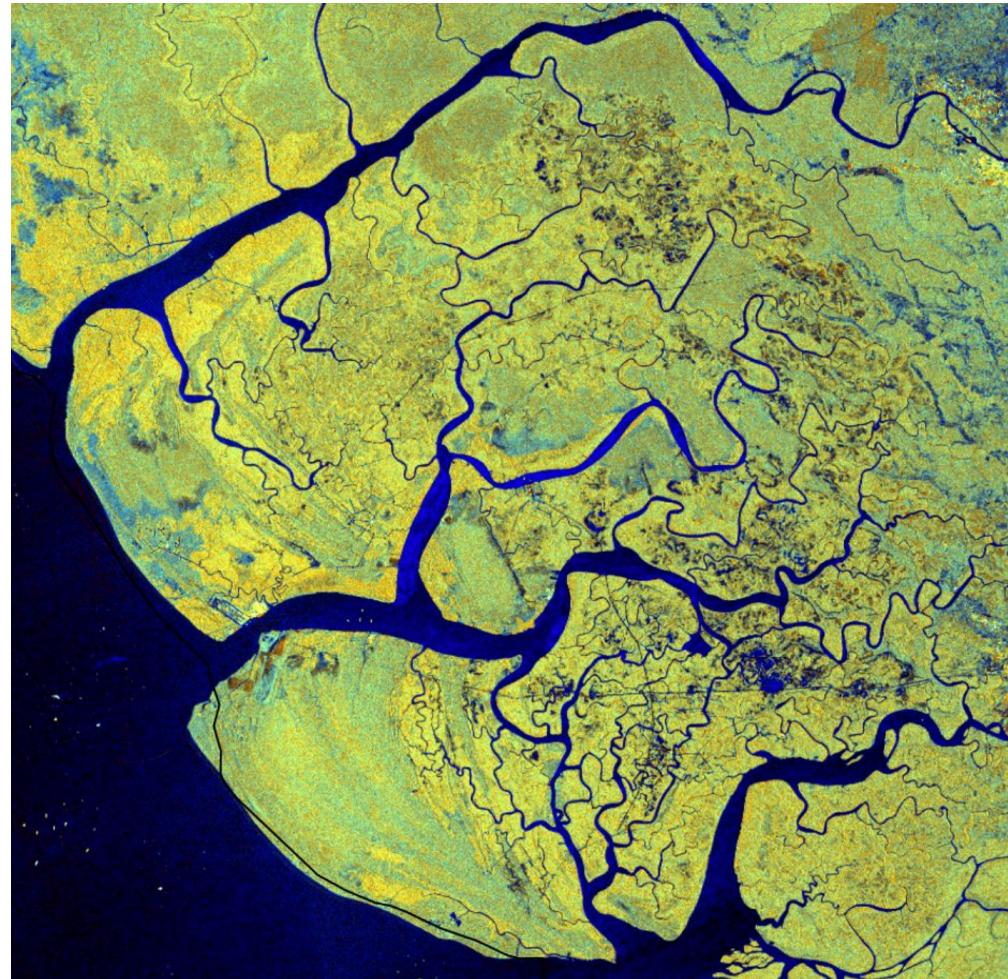


# Motivation

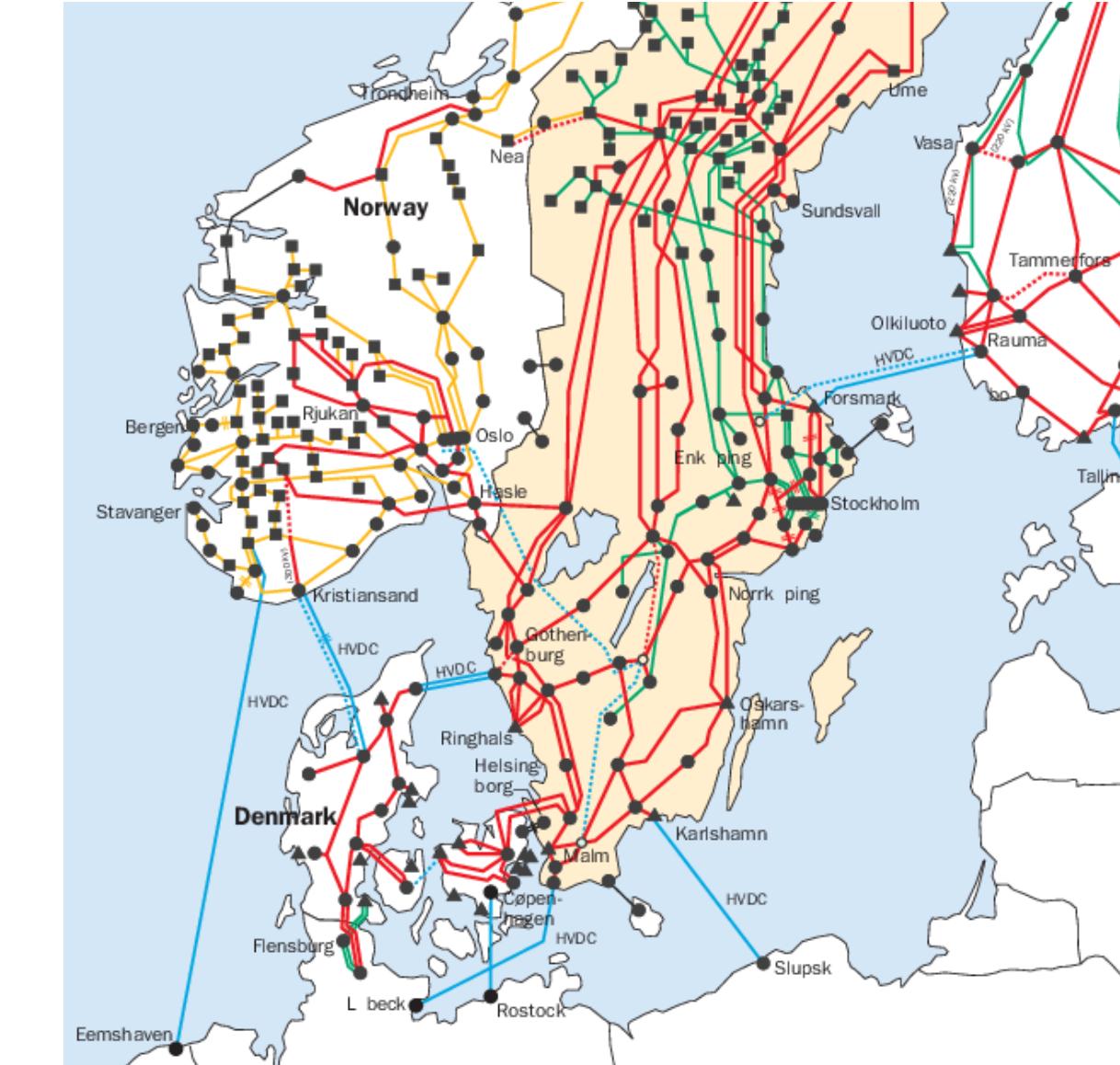
Transport networks are pervasive at different scales



UHB Trust



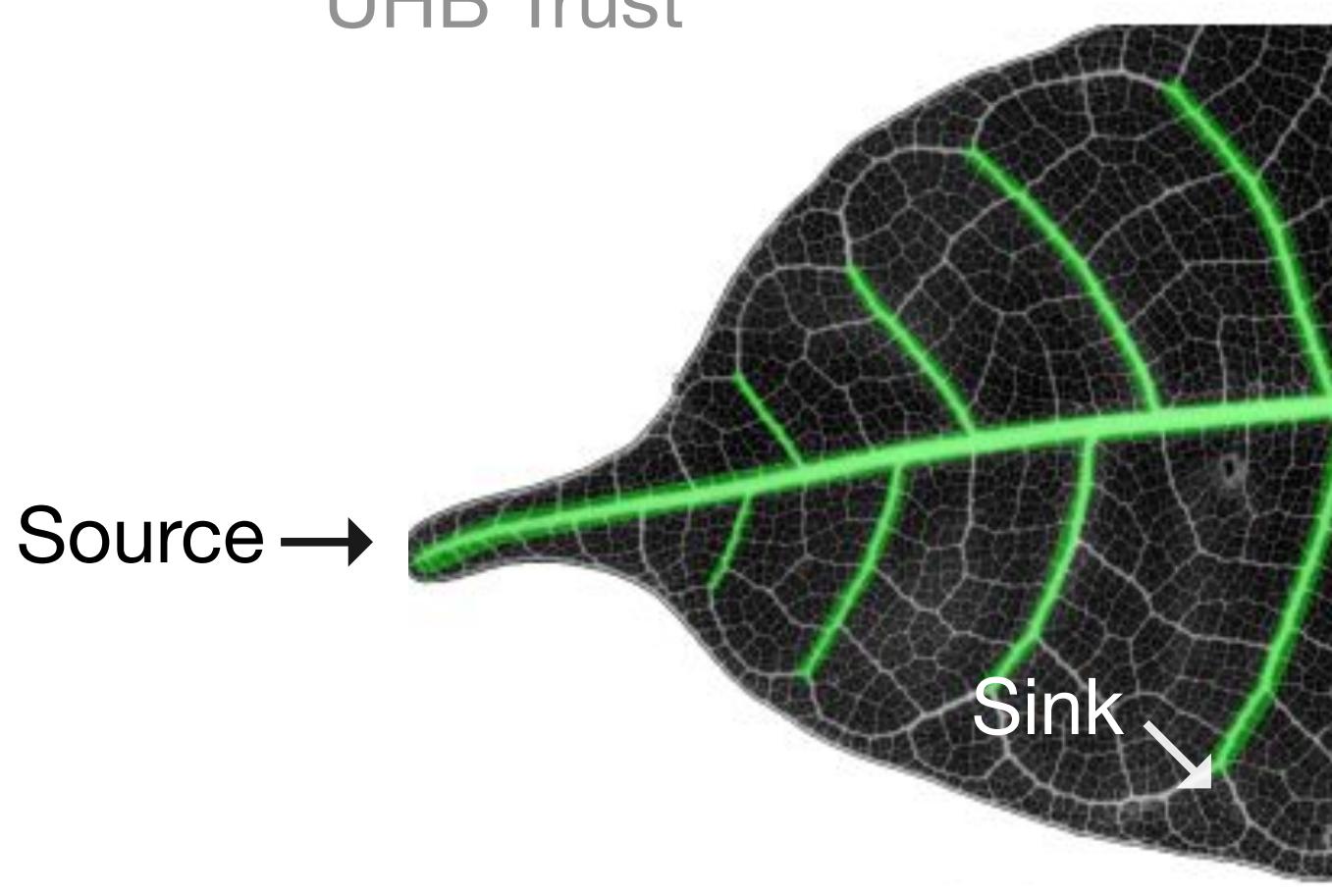
ESA



Perninge  
KTH (2011)



Transport for  
London

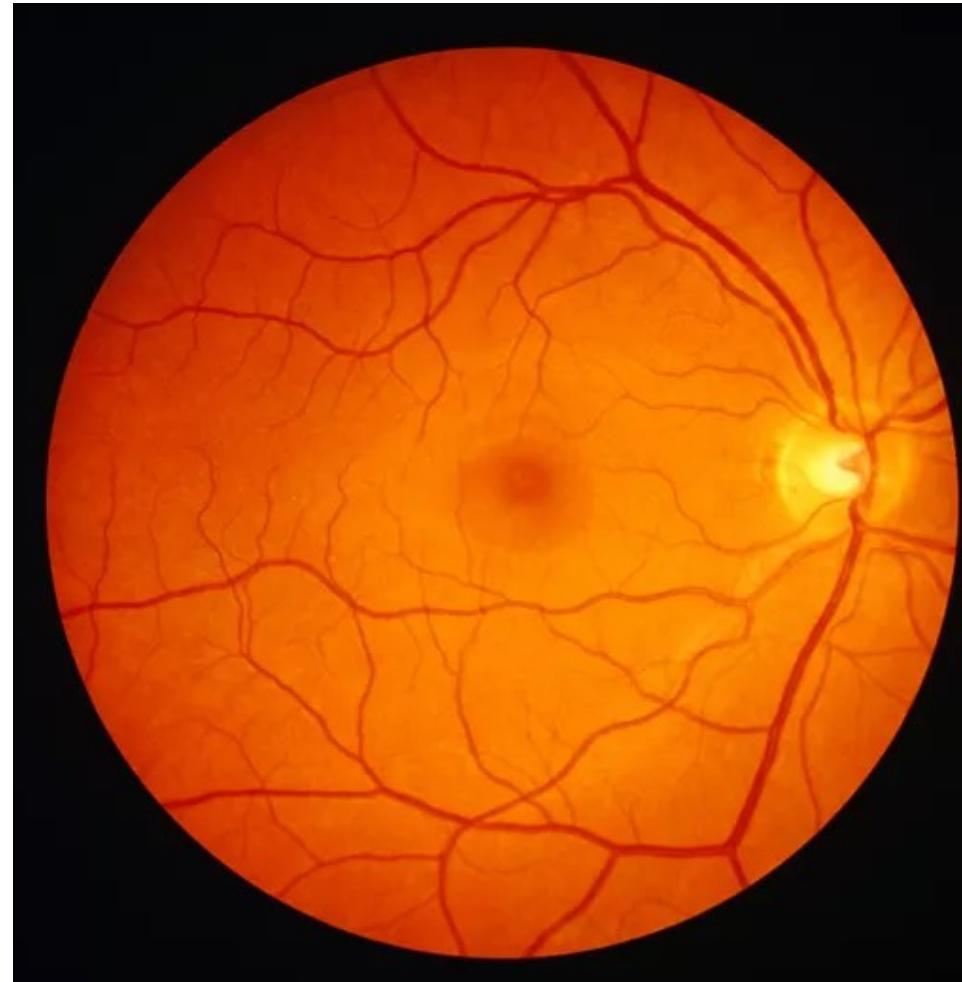


Ronellenfitsch and Katifori  
PRL (2016)

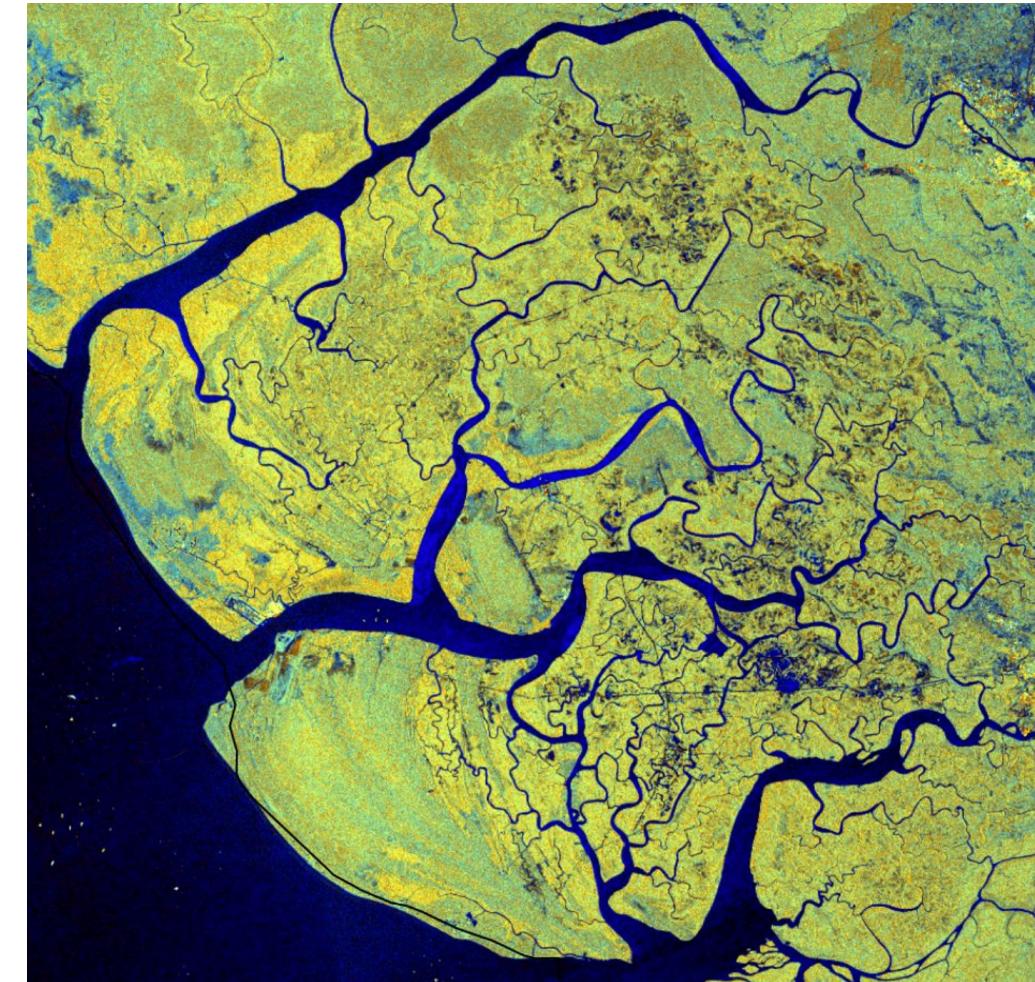
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Transport networks are pervasive at different scales

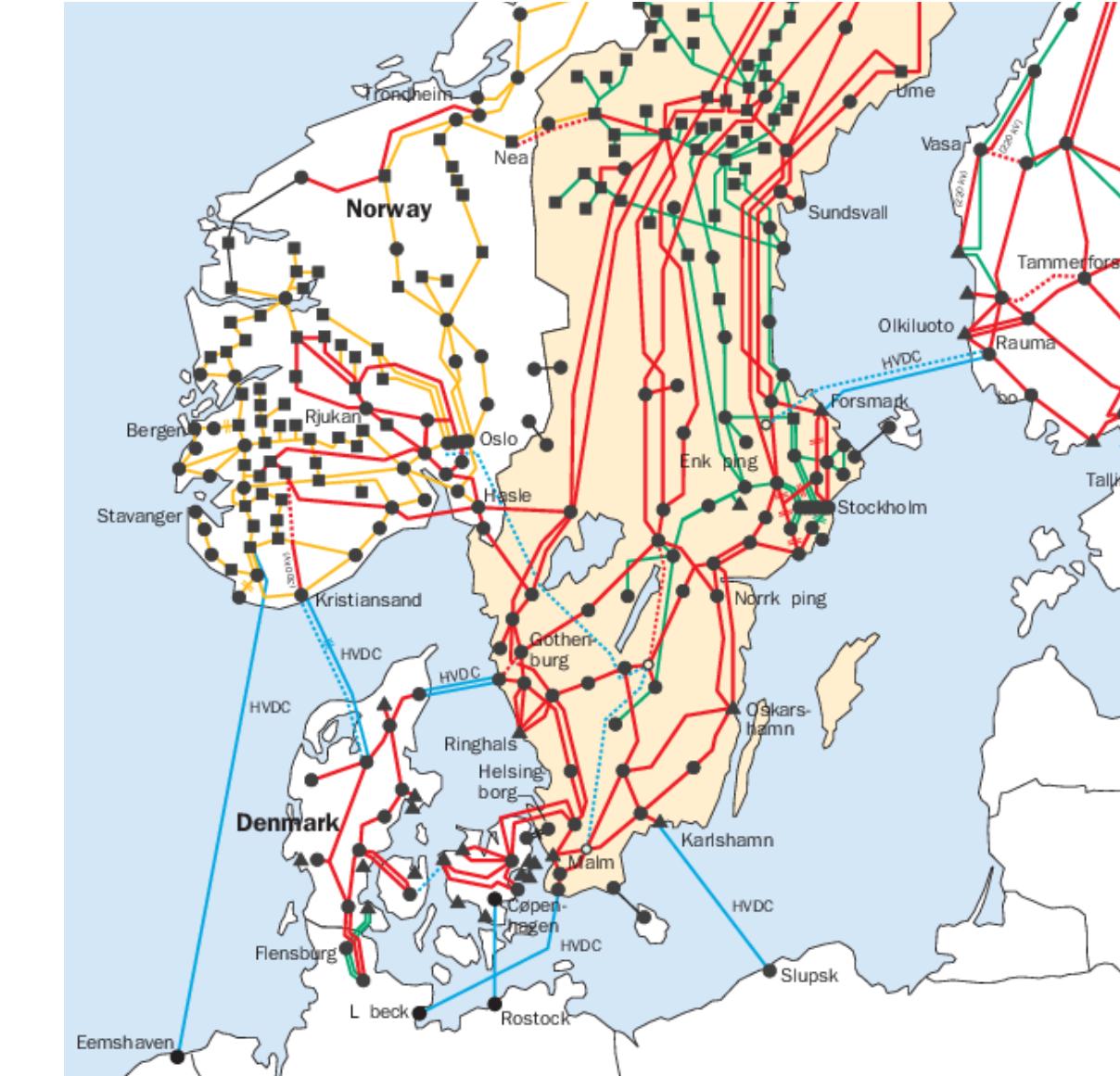
## Natural systems



UHB Trust



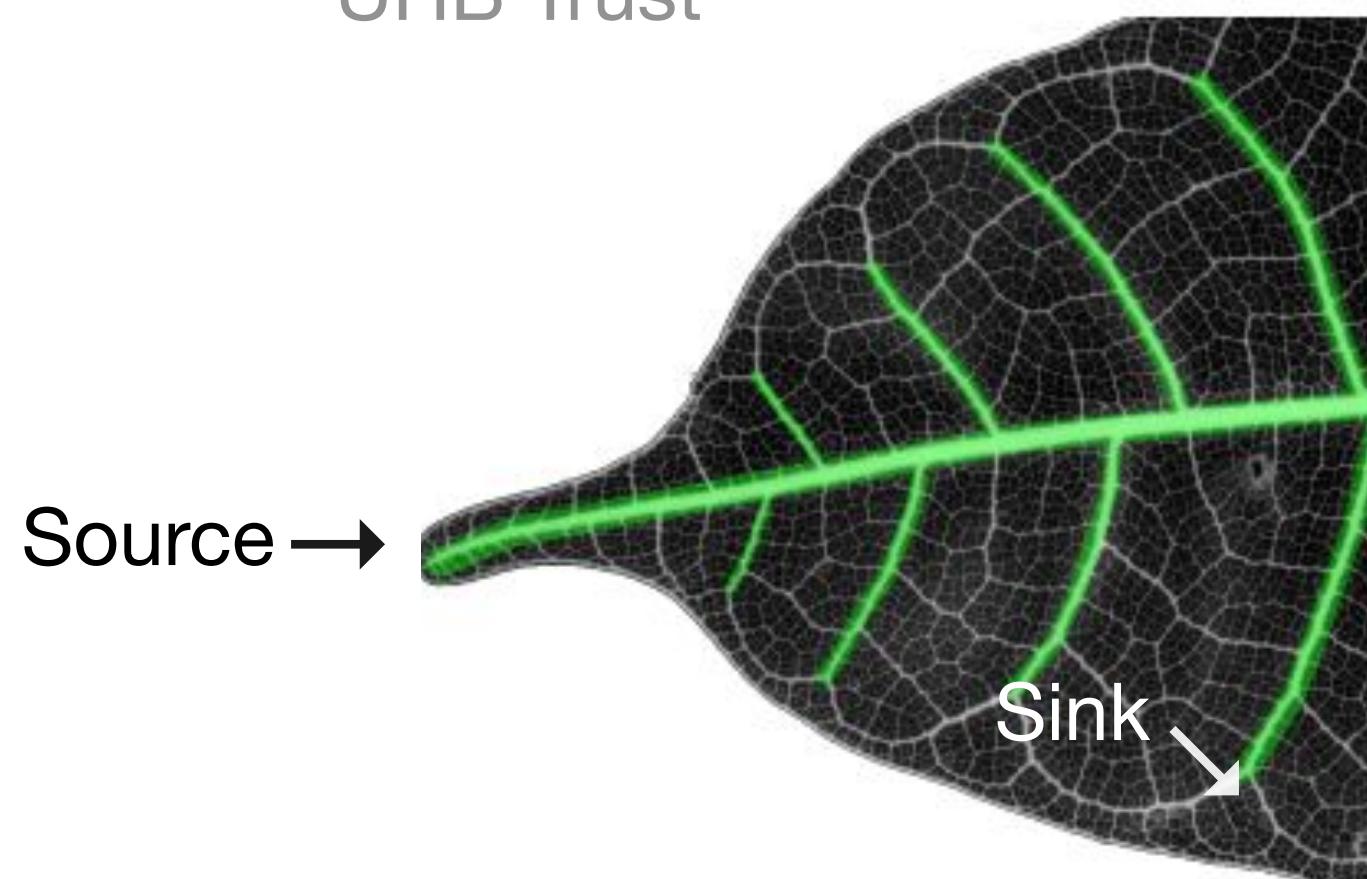
ESA



Perninge  
KTH (2011)

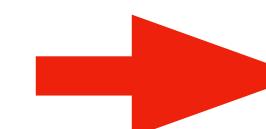


Transport for  
London



Source →

Ronellenfitsch and Katifori  
PRL (2016)

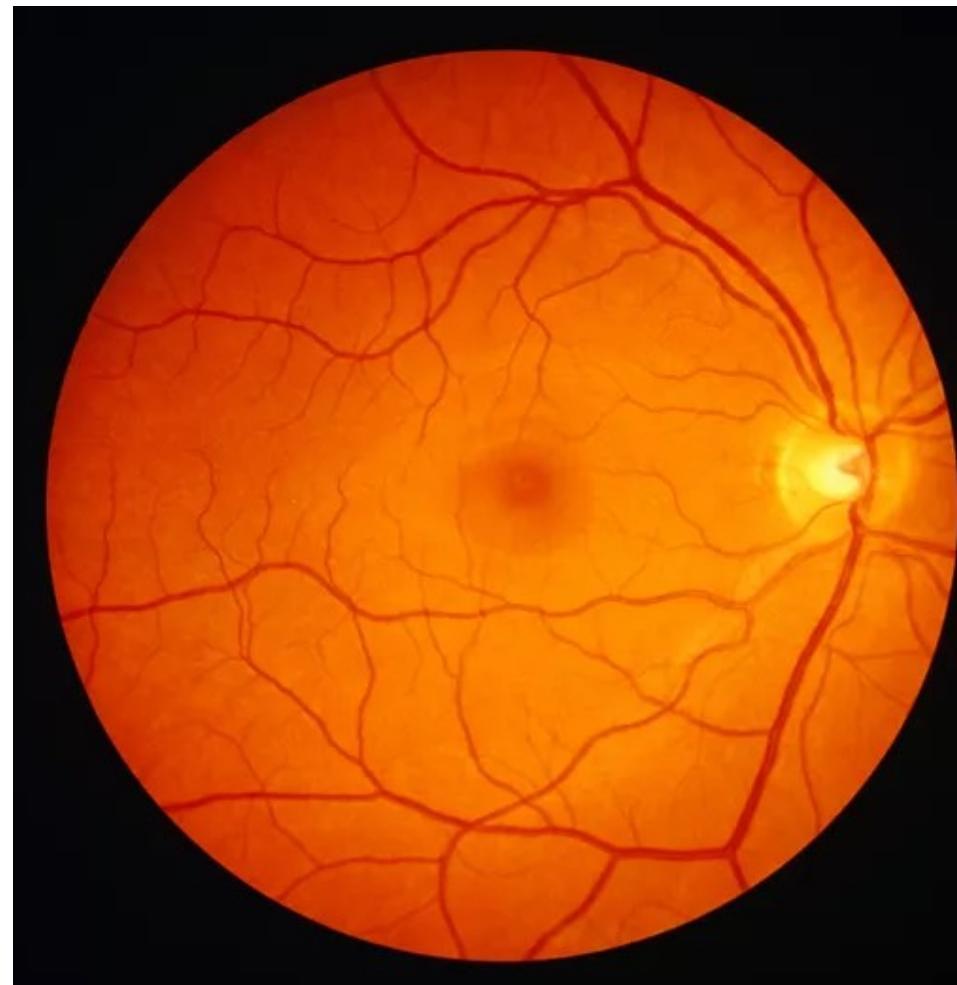


Adaptation leads to the emergence of  
macroscopic properties

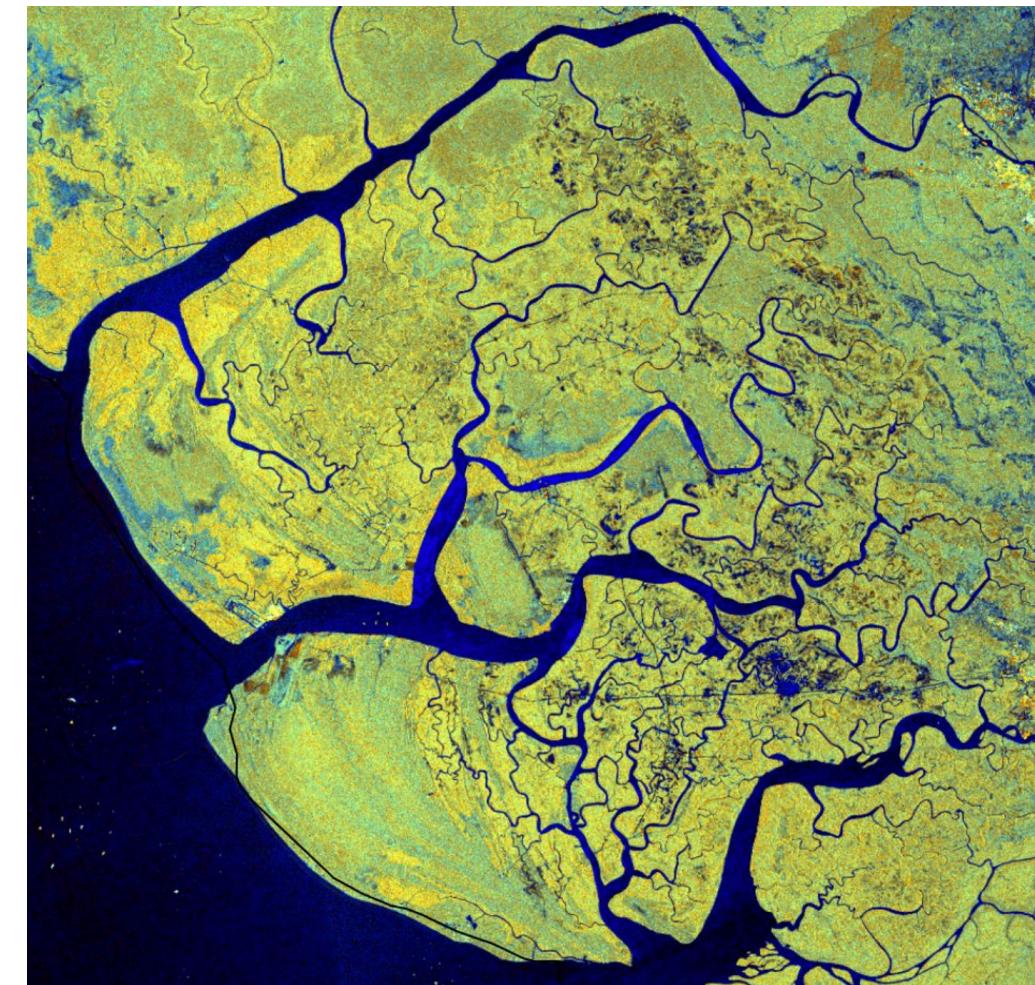
# Motivation

Transport networks are pervasive at different scales

## Natural systems

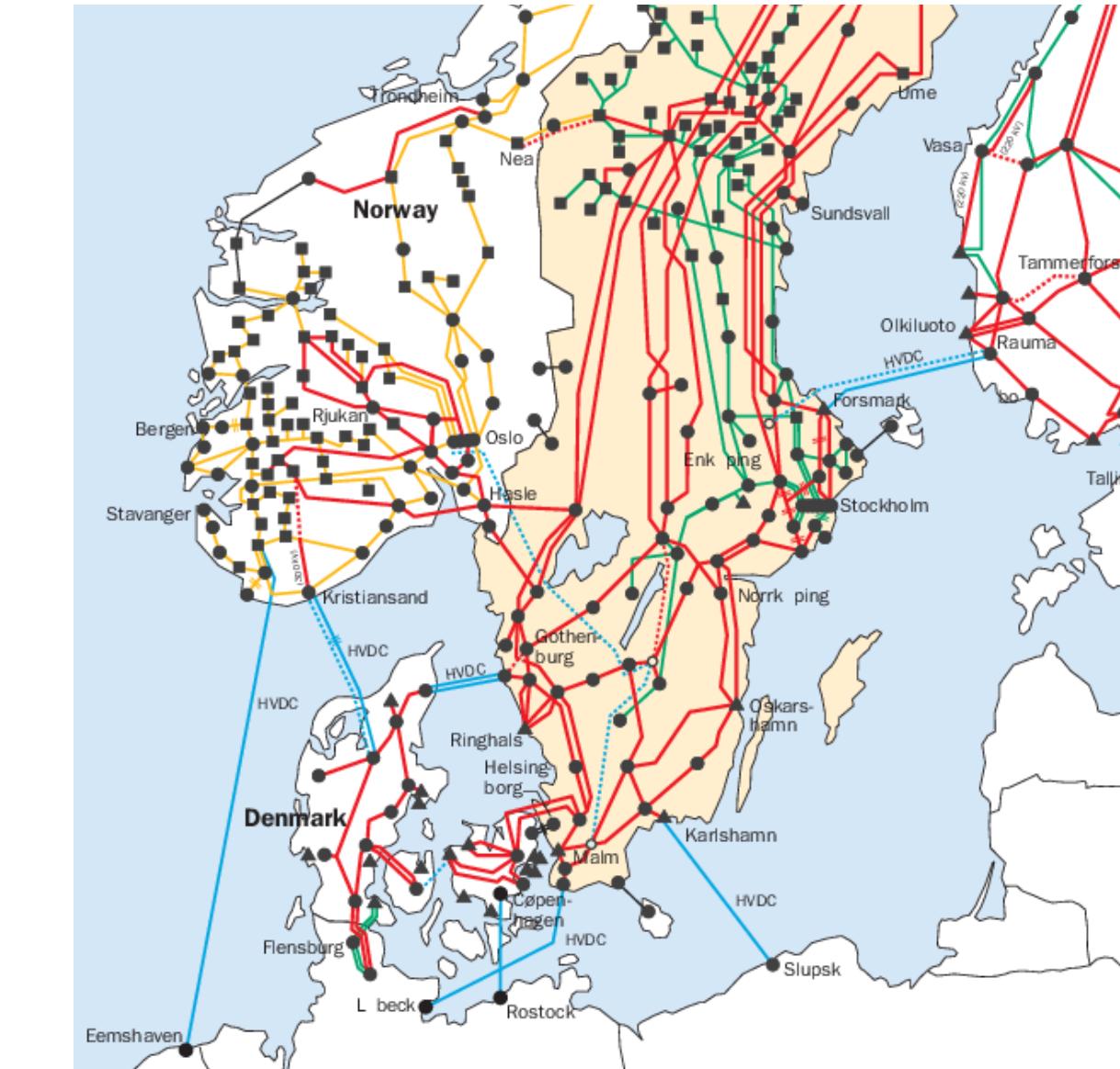


UHB Trust



ESA

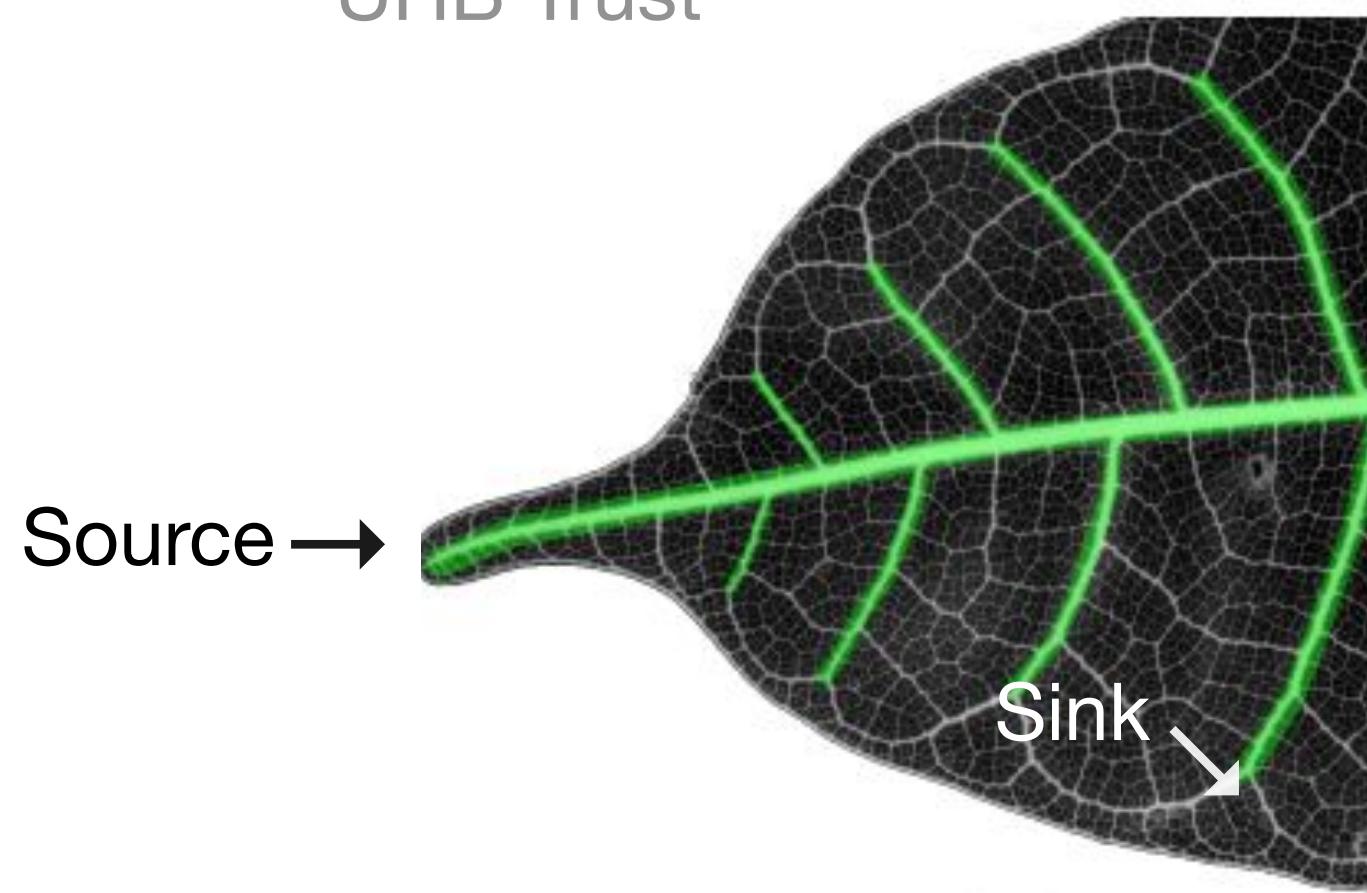
## Artificial systems



Perninge  
KTH (2011)



Transport for  
London



Source →

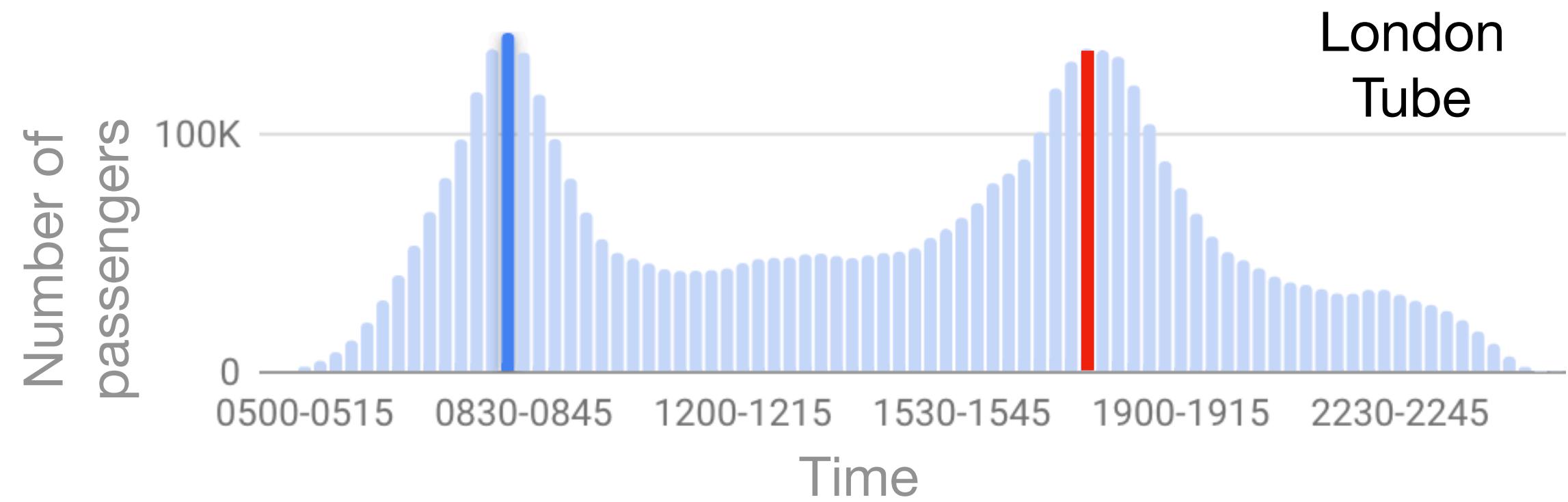
Ronellenfitsch and Katifori  
PRL (2016)

→ Adaptation leads to the emergence of macroscopic properties

→ Idea: leverage adaptation to design urban transportation

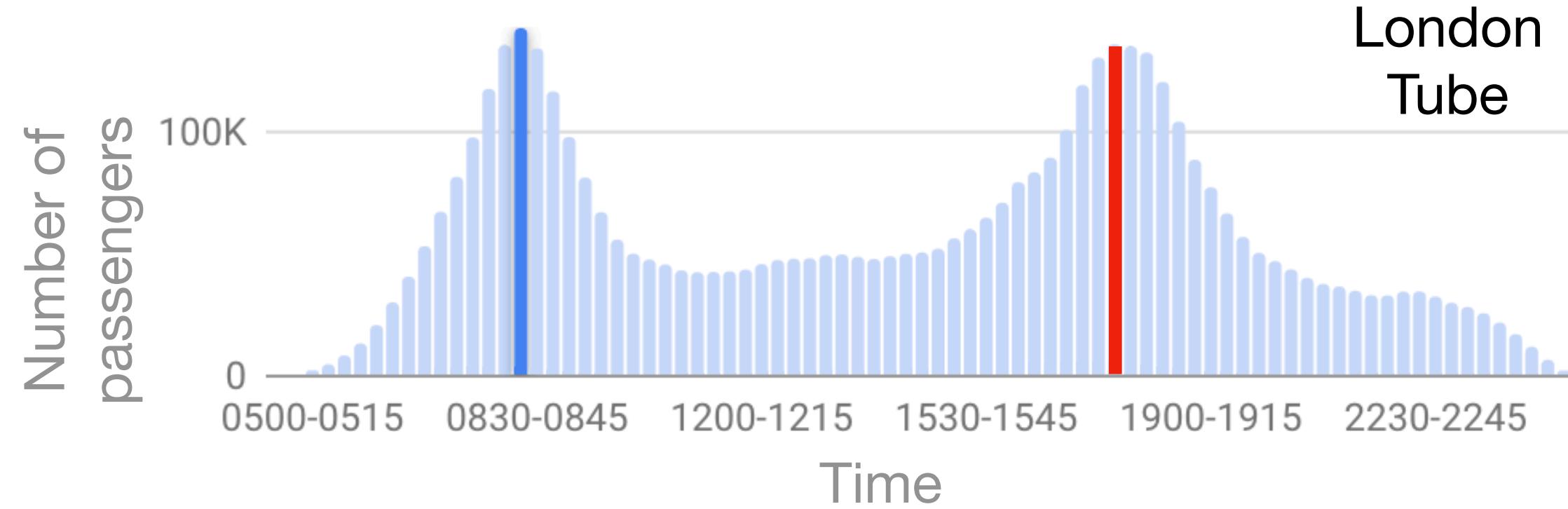
# Modeling assumption

Node loads are dynamic!



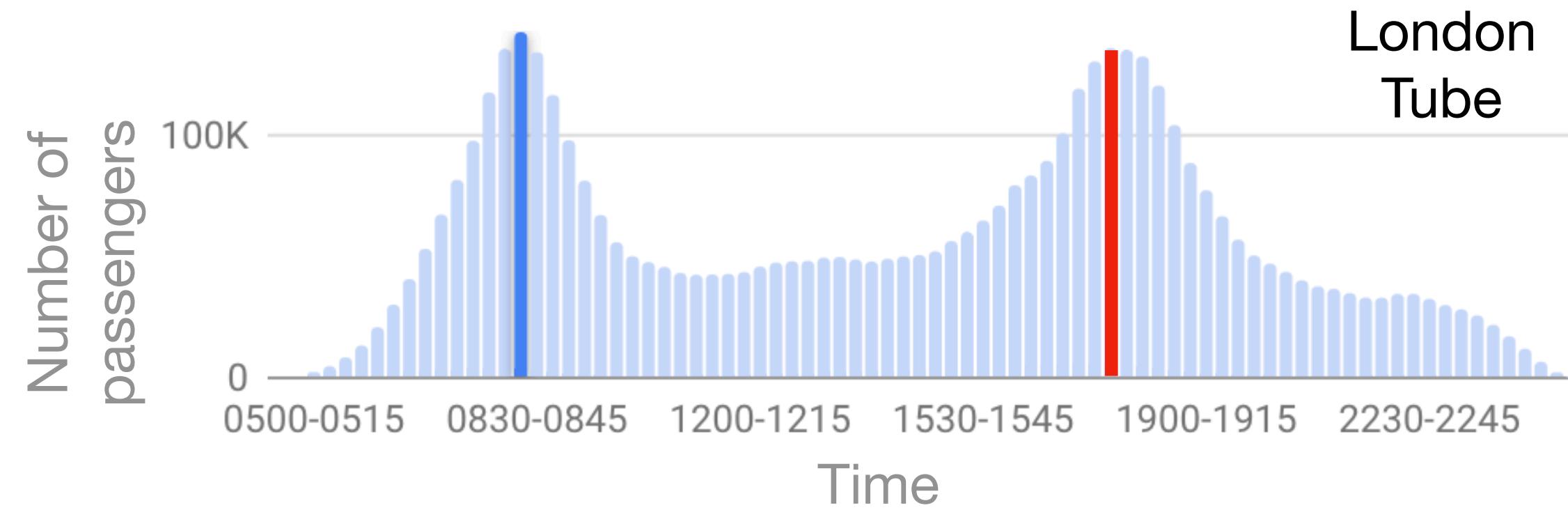
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# Modeling assumption

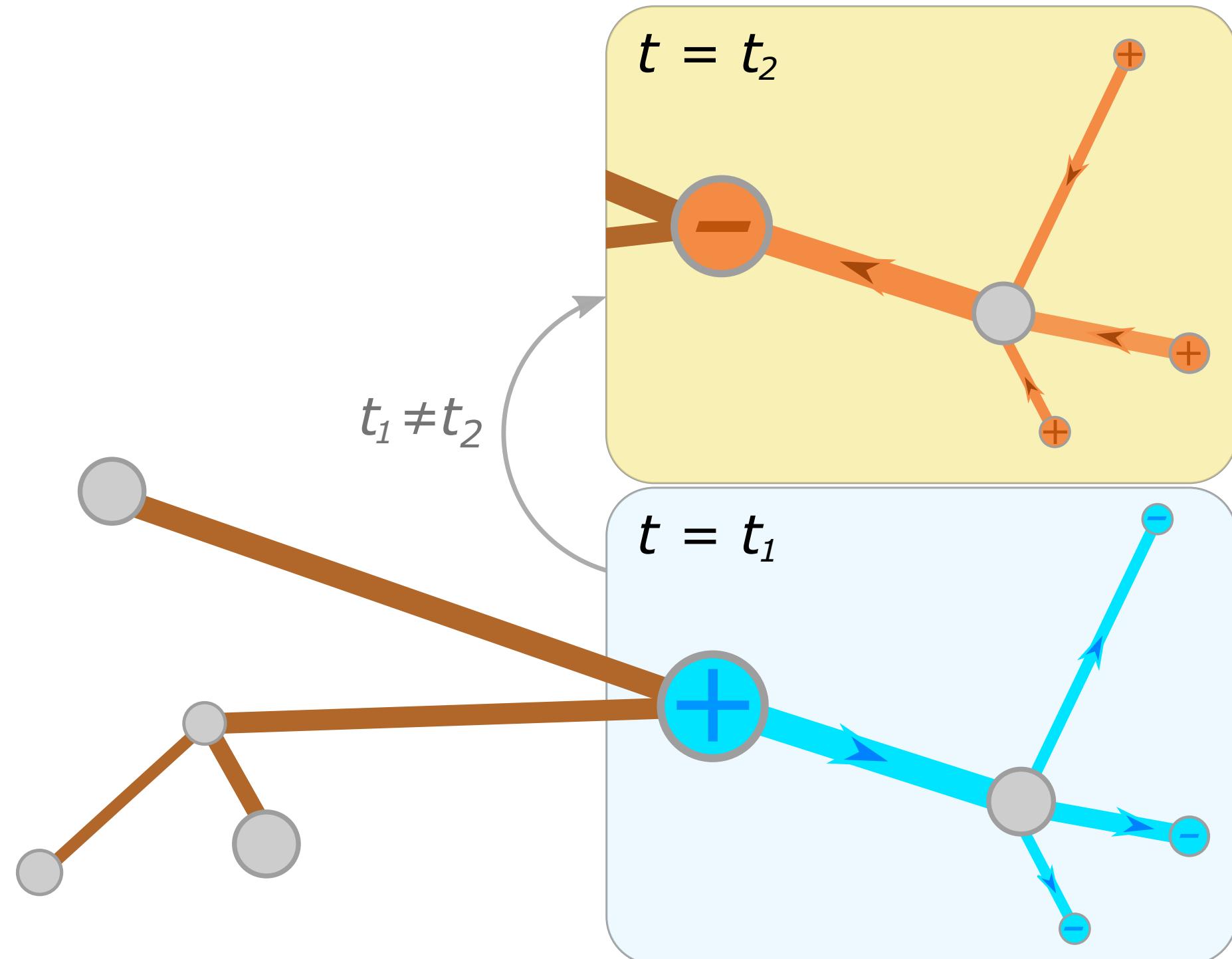
Node loads are dynamic!



Often neglected by  
adaptation models!



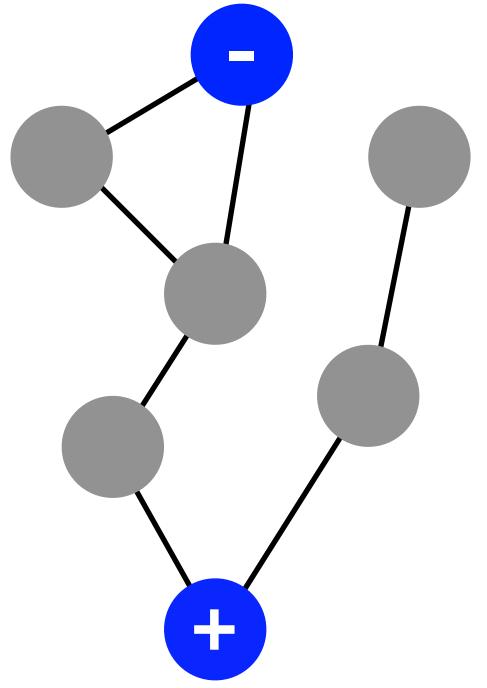
# Research questions



Adapted from  
Lonardi PRE (2023)

- 1) Can we find adaptation rules for time-dependent node loads?
- 2) Does adaptation shed light on transport network properties?

# Background

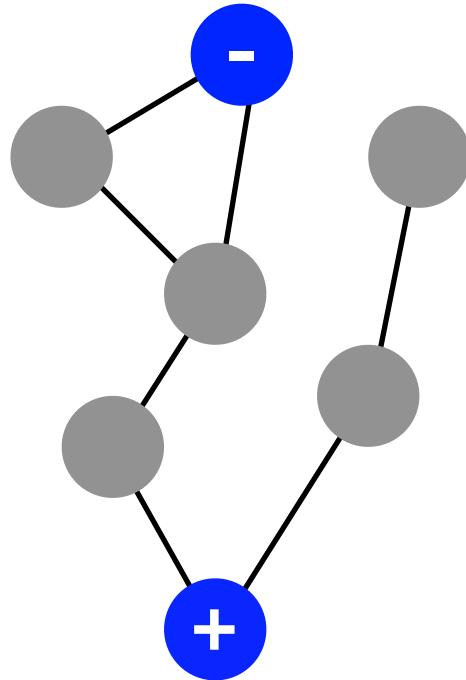


$\mu_e$  : road capacity

$F_e$  : load displacement

$$\left\{ \begin{array}{l} \frac{d\mu_e}{dt} = \frac{f(|F_e|)}{w_e} - \mu_e \\ \text{Kirchhoff's law} \end{array} \right.$$

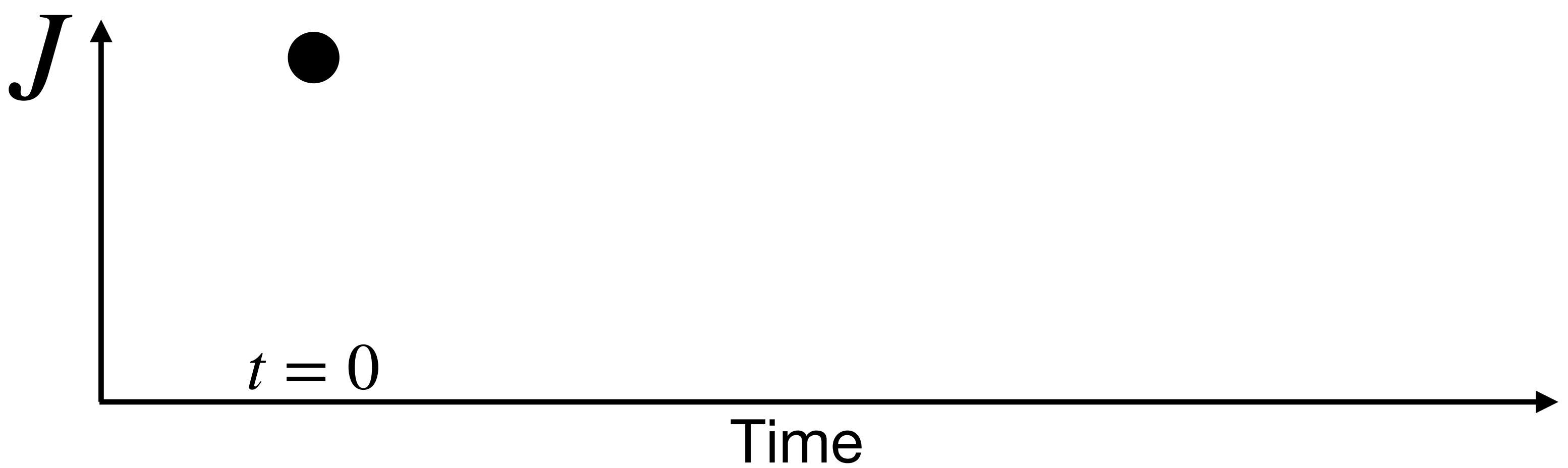
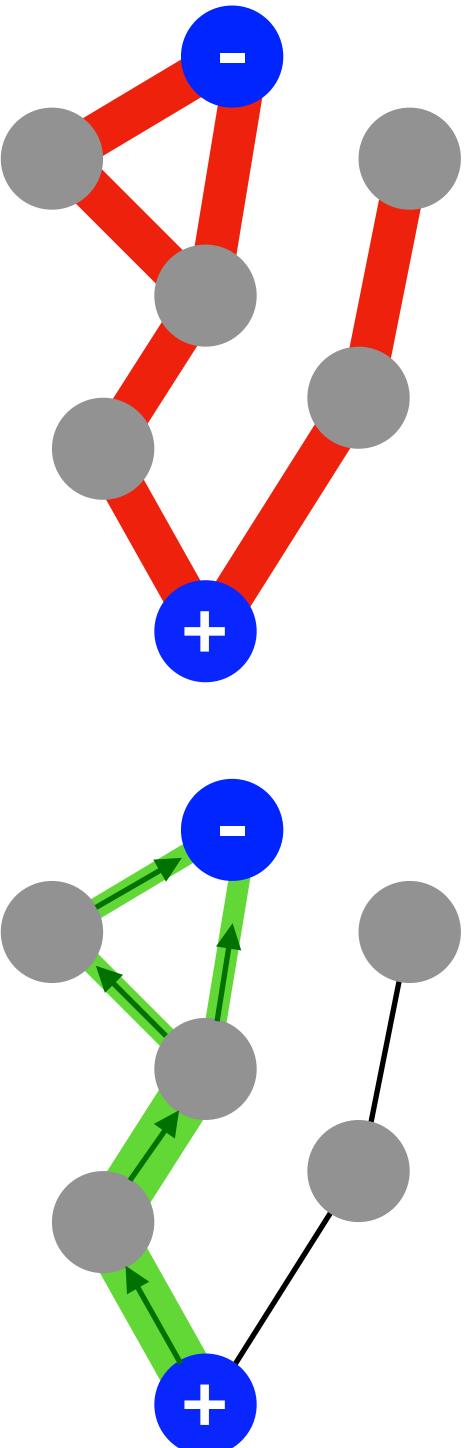
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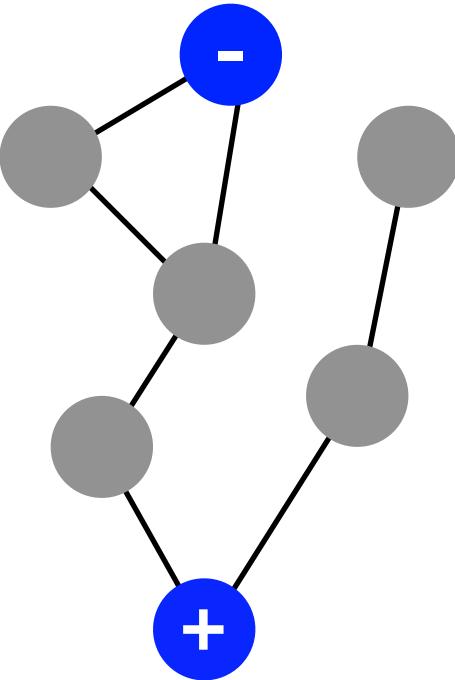
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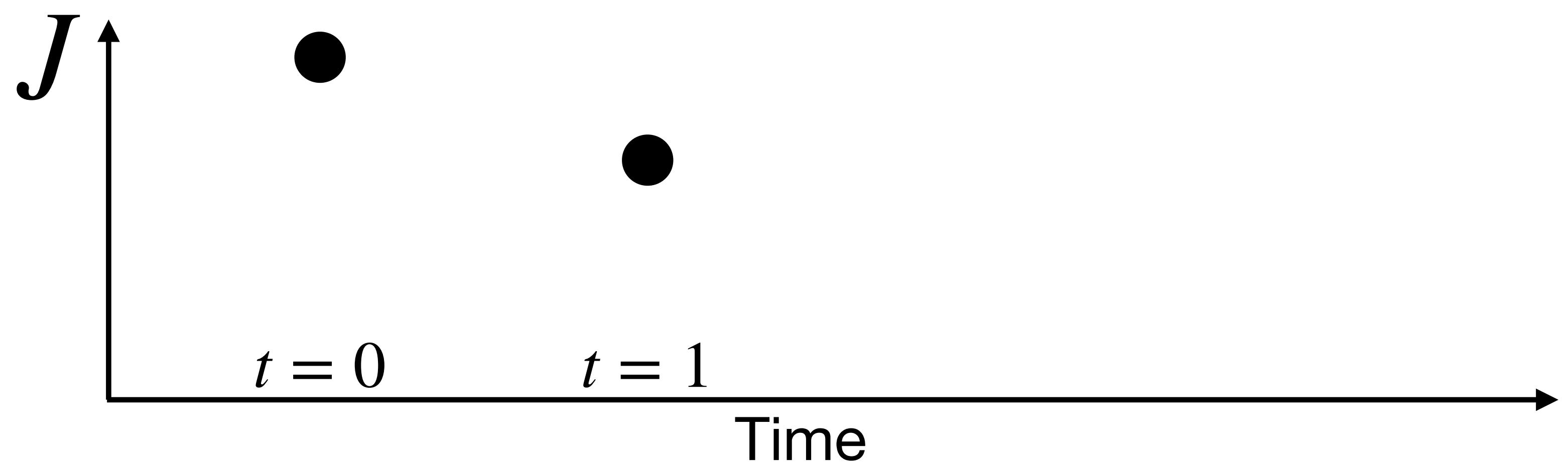
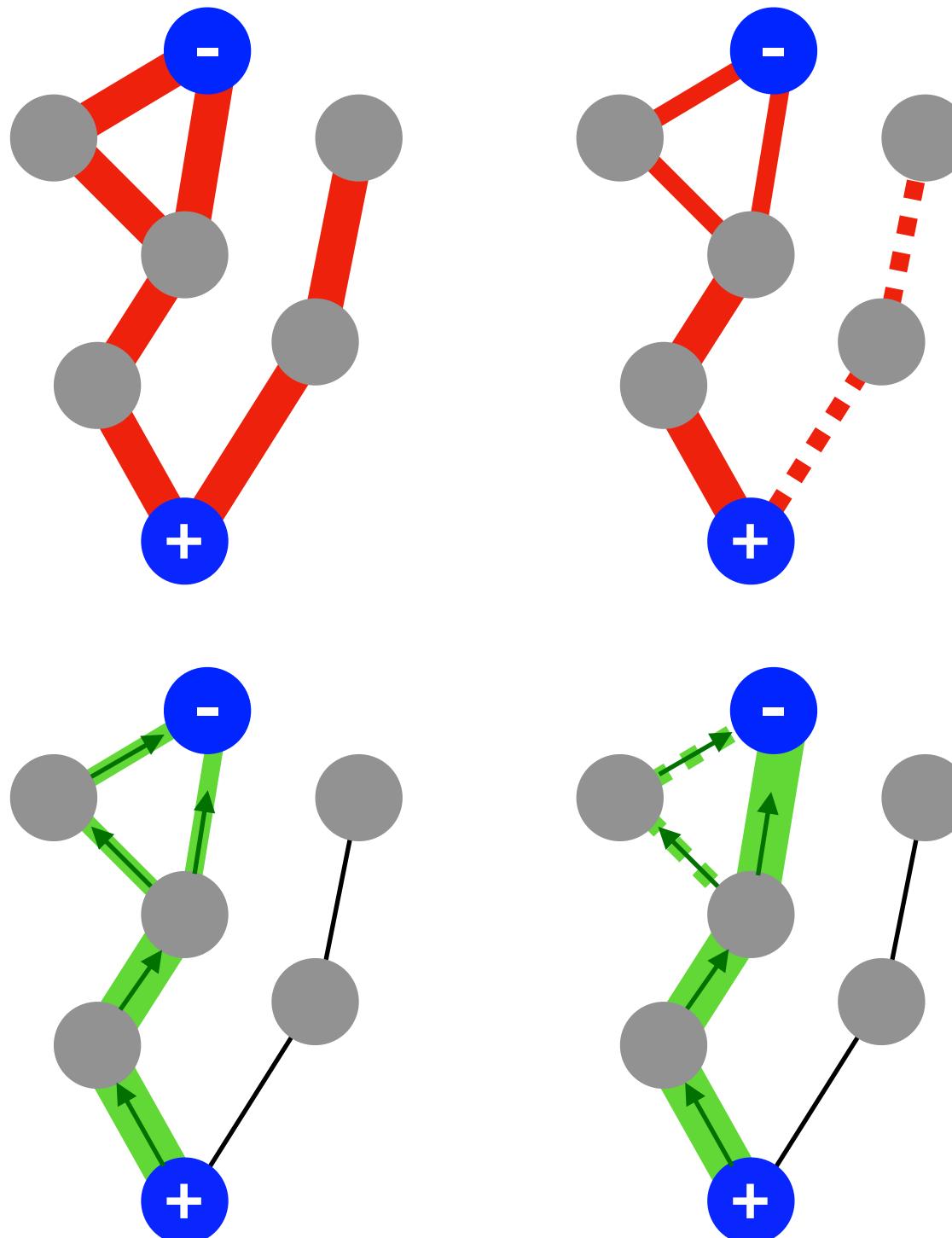
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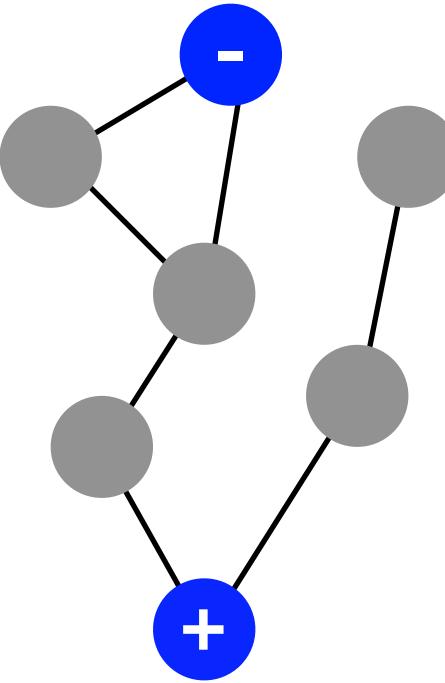
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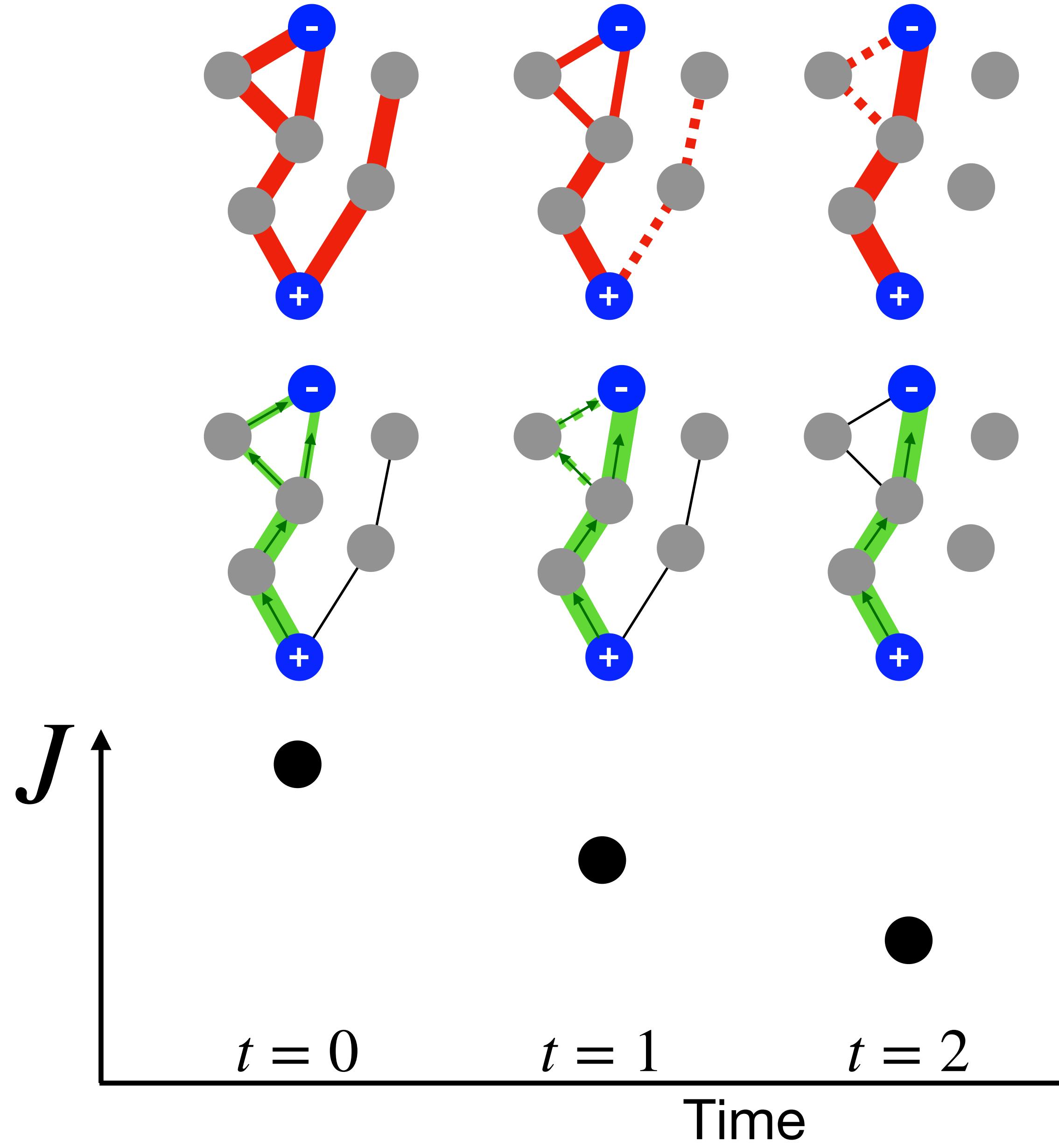
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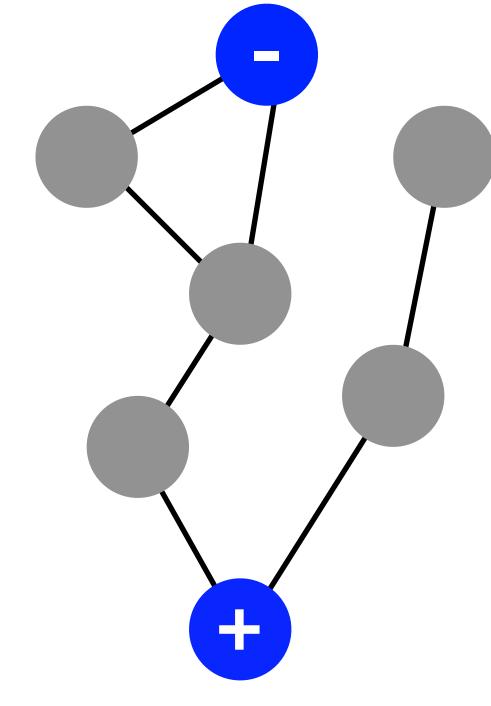
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# Background

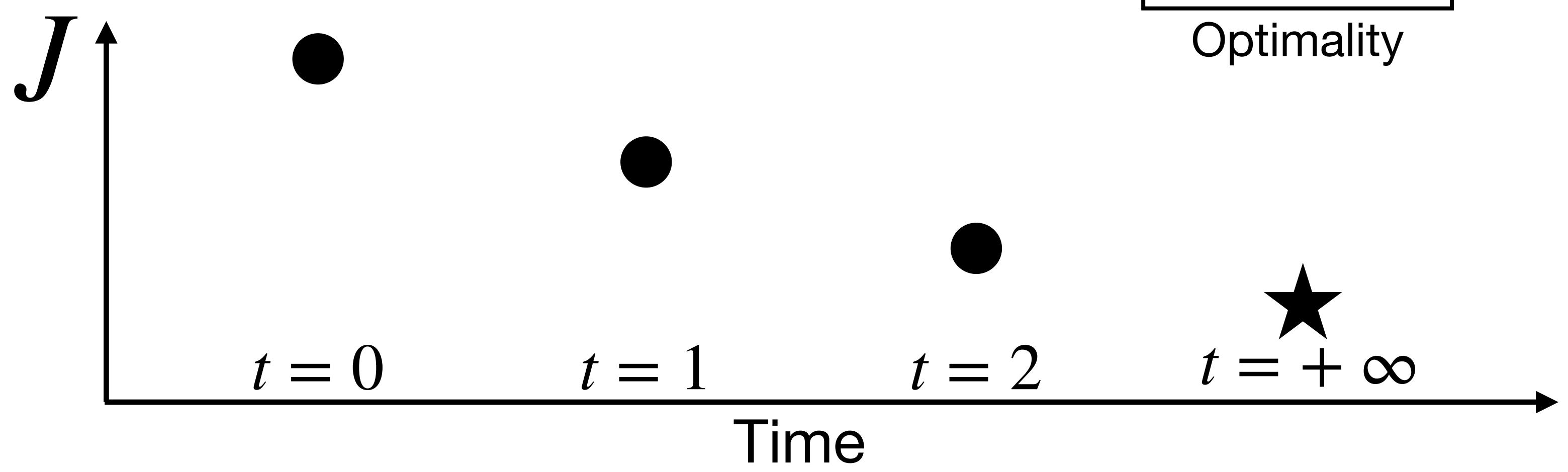


$\mu_e$  : road capacity

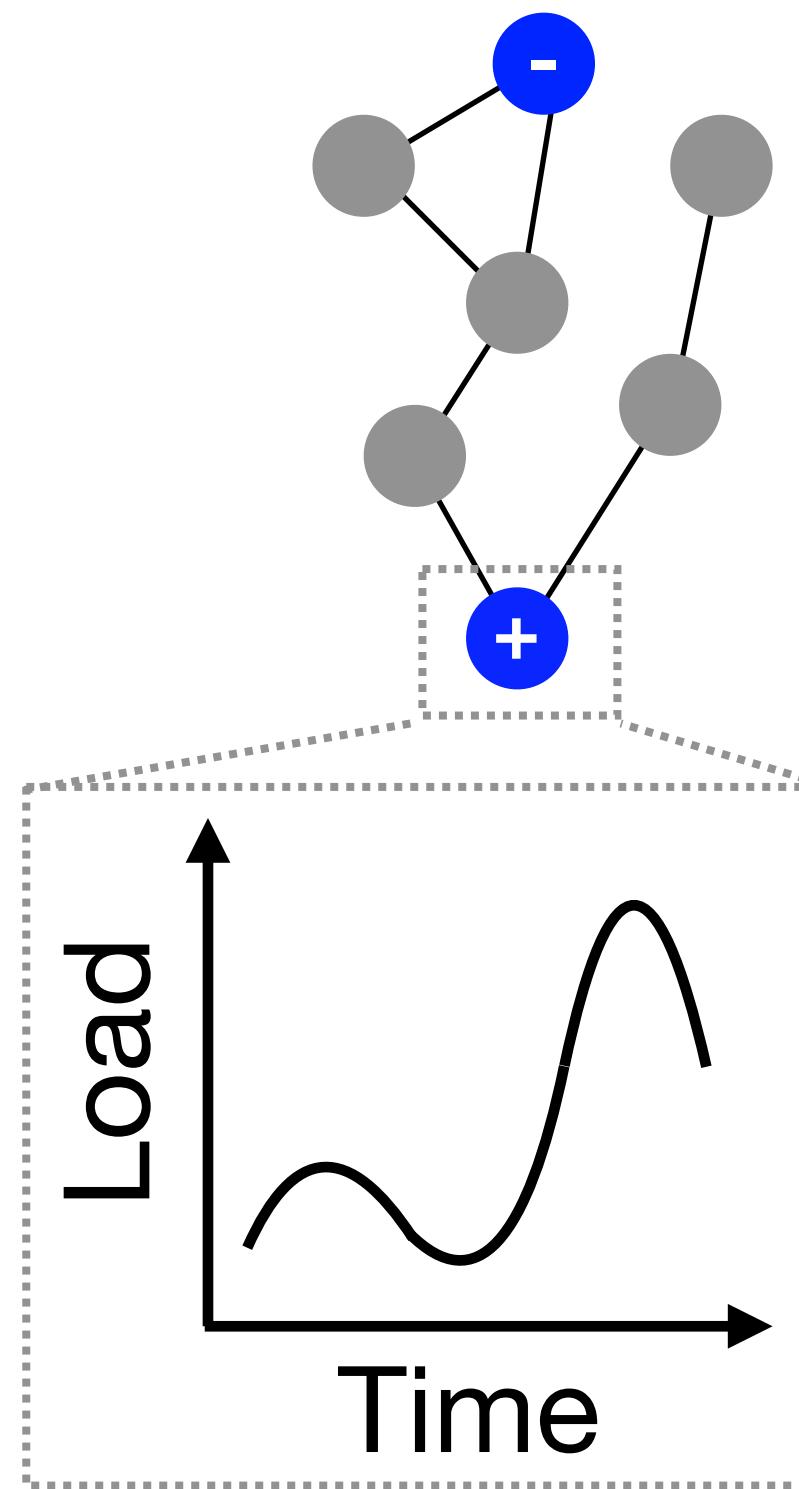
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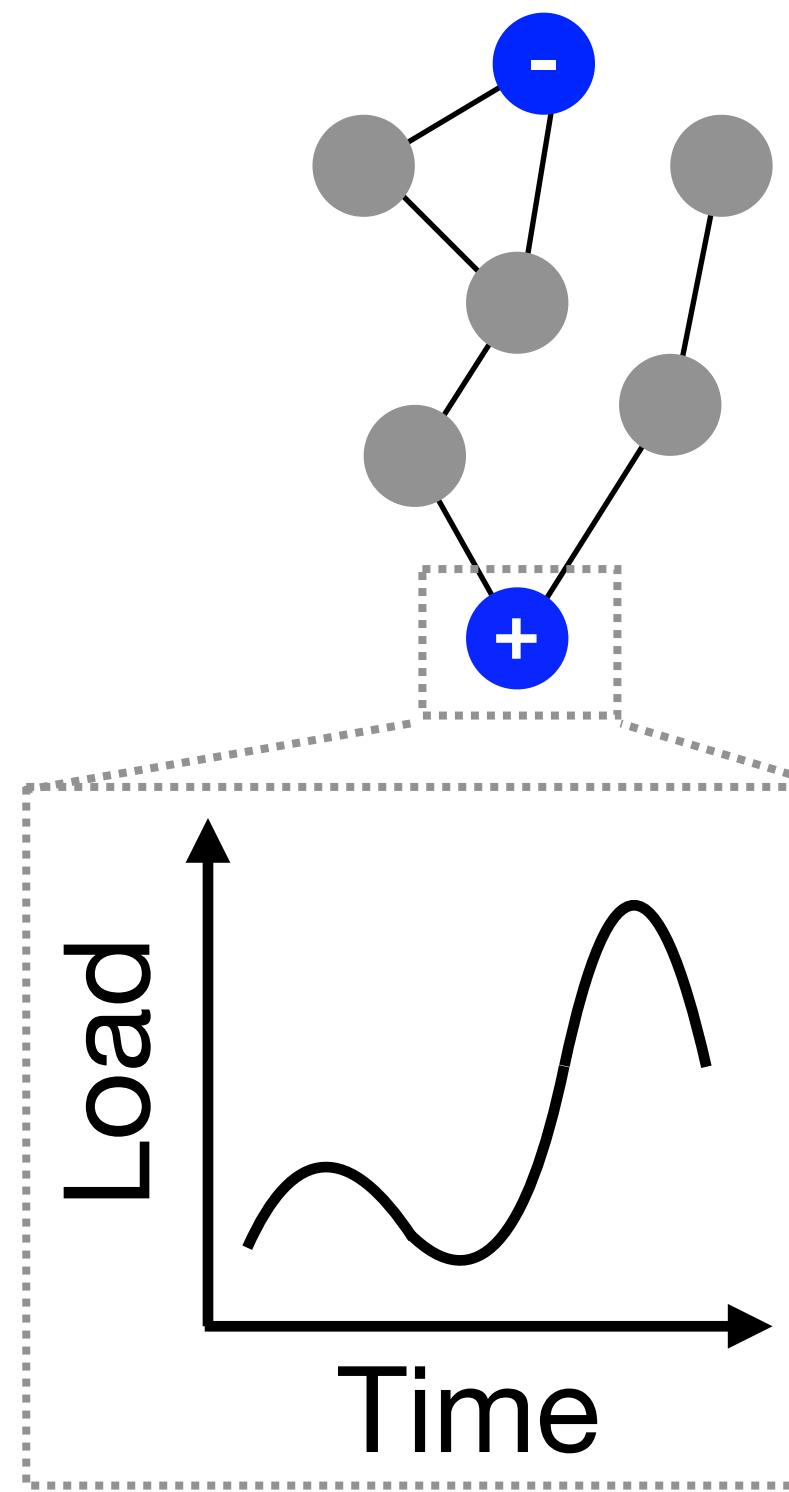
Kirchhoff's law



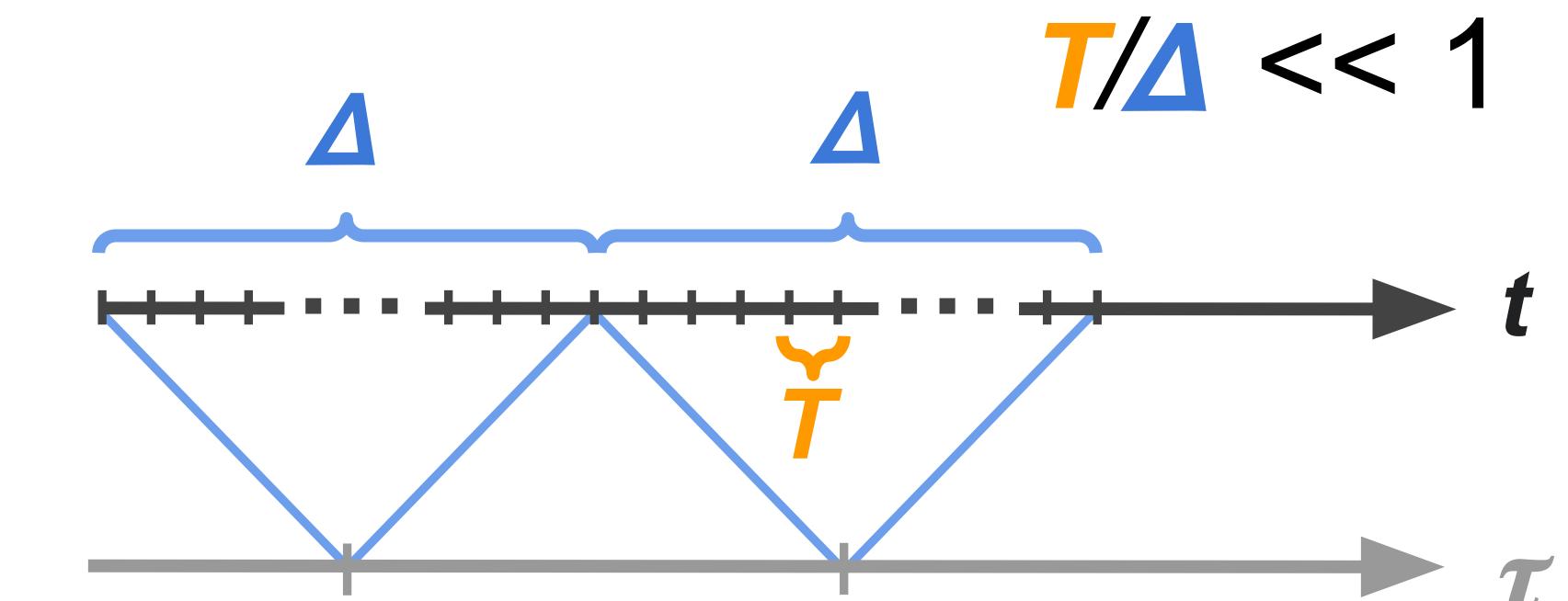
# Method



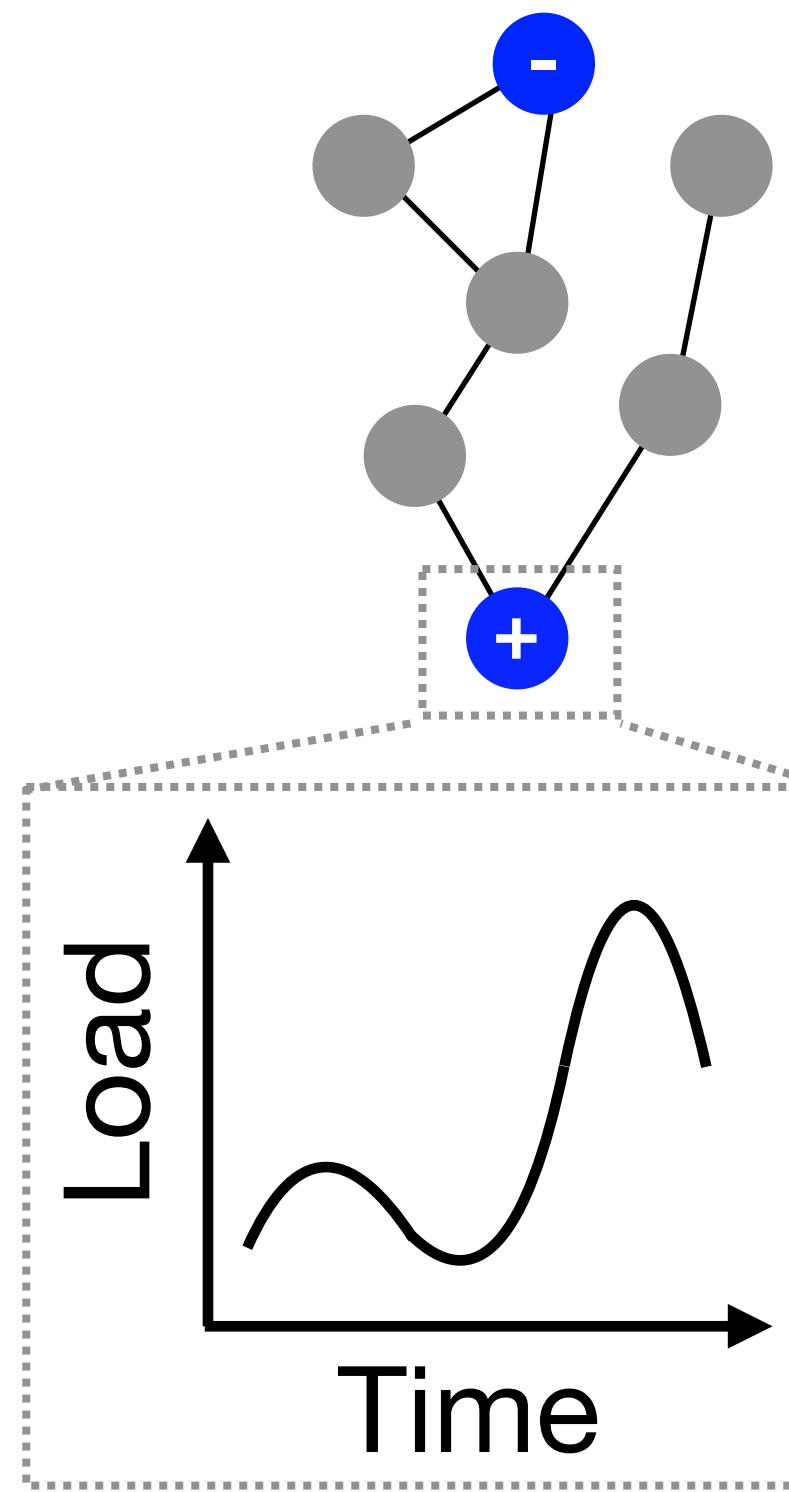
# Method



- 1) Slow time scale ( $\tau$ ) and coarse observation window ( $\Delta$ ) for the network manager  
Fast time scale ( $t$ ) for the passengers



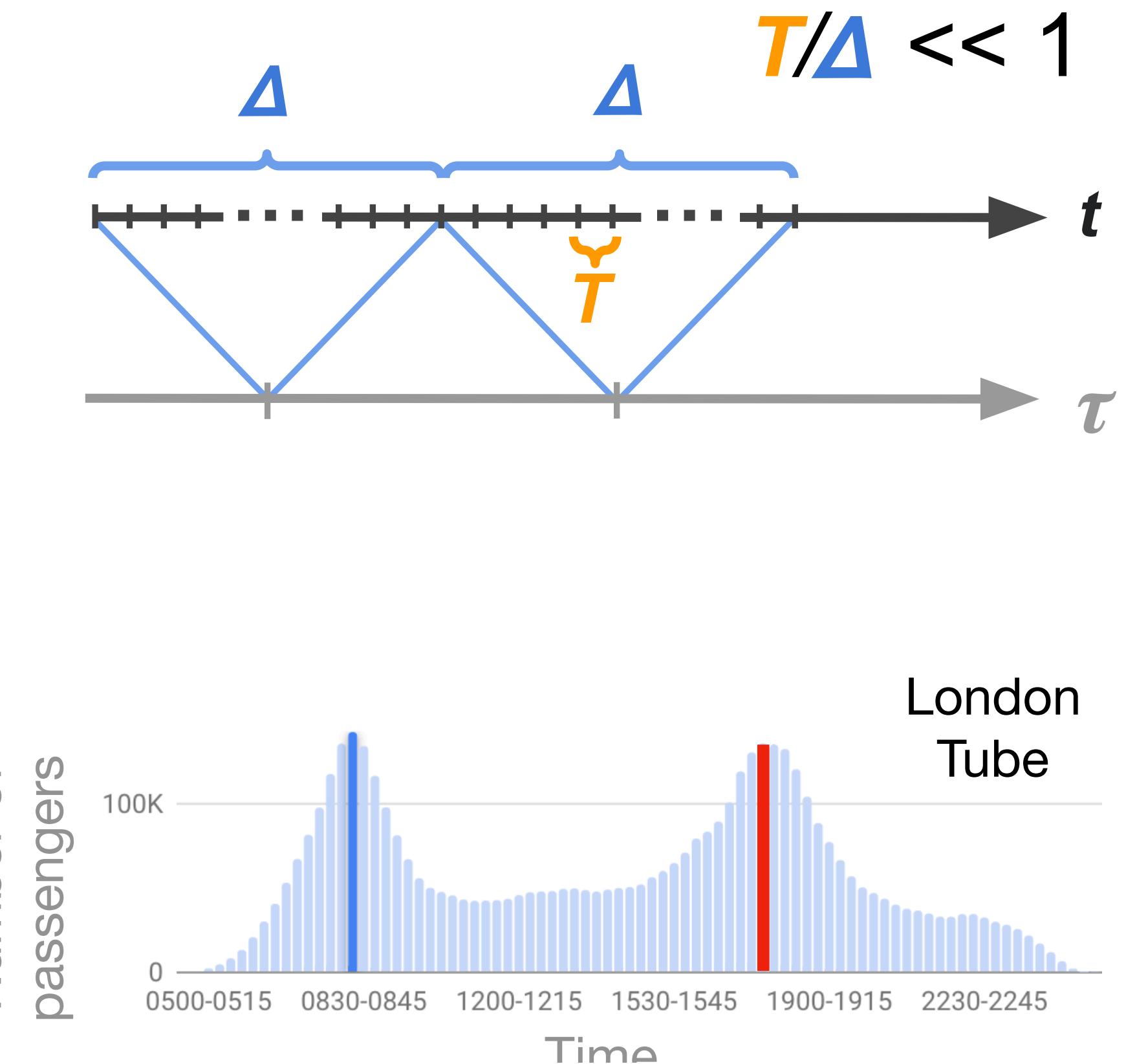
# Method



- 1) Slow time scale ( $\tau$ ) and coarse observation window ( $\Delta$ ) for the network manager

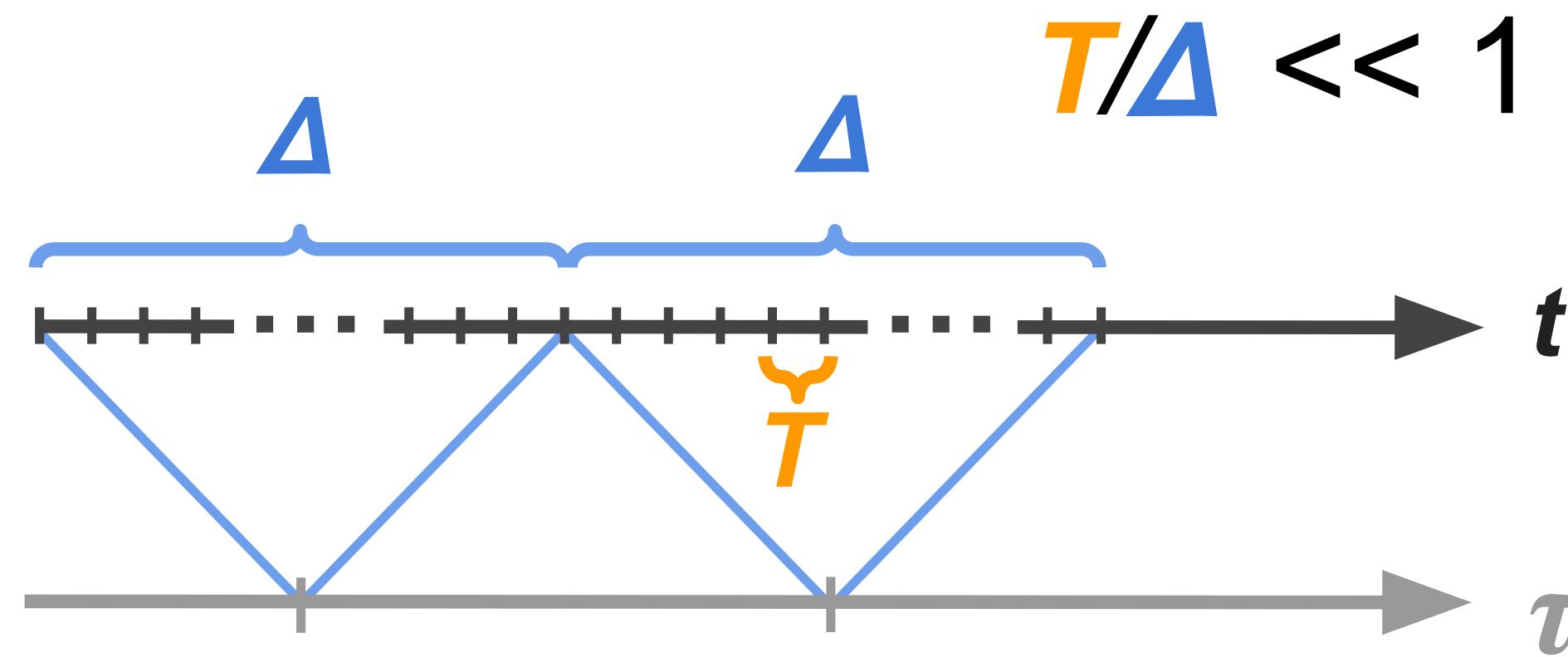
Fast time scale ( $t$ ) for the passengers

- 2) Periodic fast time loads
- $$S(t) = S(t+T)$$



# Results

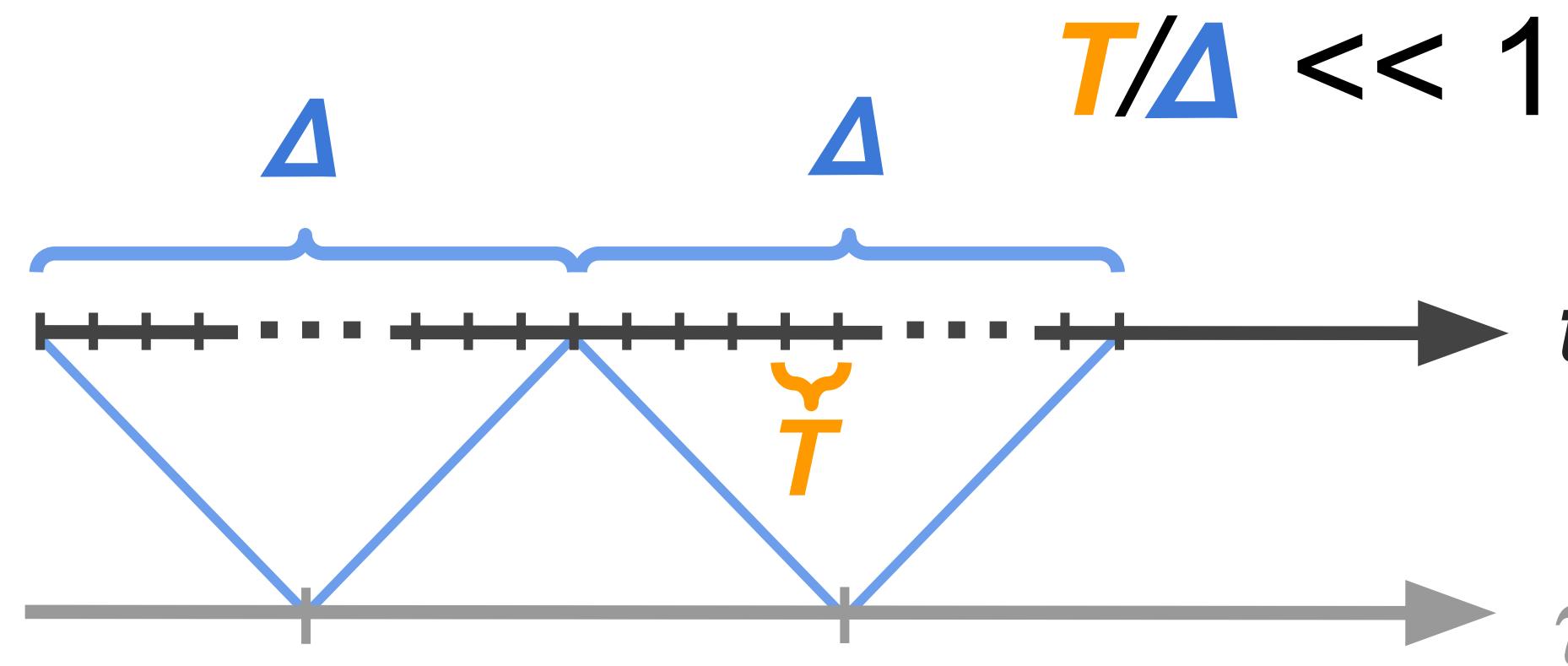
**Contribution 1:** Closed-form adaptation rules



$$S_v(t) = \sum_{n_v} c_v^{n_v} \exp\left(i \frac{2\pi}{T} n_v t\right)$$

# Results

**Contribution 1:** Closed-form adaptation rules



$$S_v(t) = \sum_{n_v} c_v^{n_v} \exp\left(i \frac{2\pi}{T} n_v t\right)$$

$$\frac{d\mu_e}{d\tau} = \Psi(\mu_e, C) - \mu_e$$

$C$  “=” combination of loads’ Fourier coefficients over  $T$

# Results

## Contribution 2: Proxy for robustness of networks



# Results

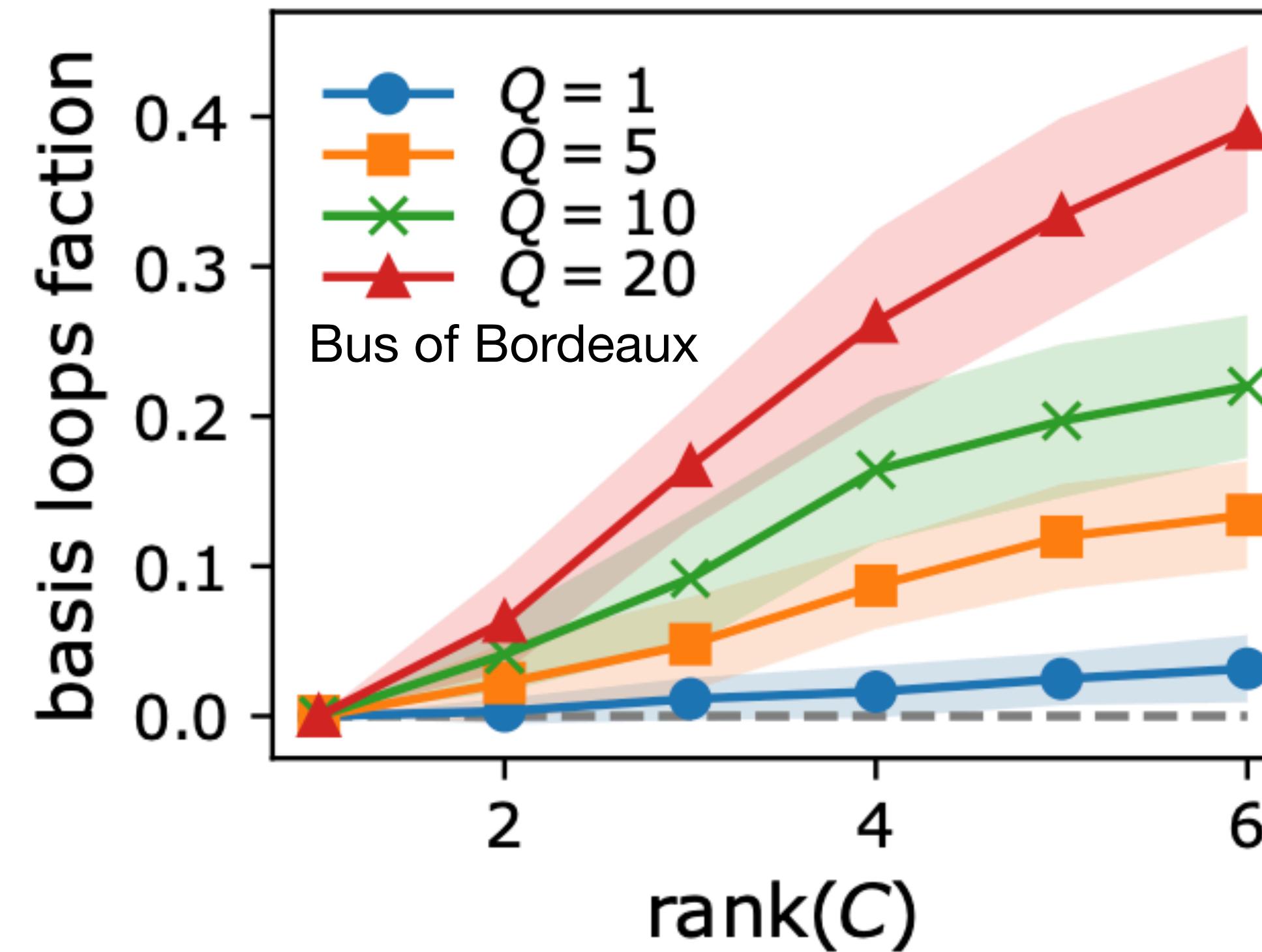
## Contribution 2: Proxy for robustness of networks



Contribution 3: Scalable method →  $C$  is only computed once

# Results

## Contribution 2: Proxy for robustness of networks



$Q$  = number of origins and destinations

# Take aways

## Questions

- 1) Can we find adaptation rules for time-dependent node loads?
- 2) Does adaptation shed light on transport network properties?

## Answers

- Contribution 1:** Closed-form adaptation rules
- Contribution 2:** Proxy for robustness of networks
- Contribution 3:** Scalable method

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Mario Putti  
(Uni Padova)



Enrico Facca  
(Uni Bergen)



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# Thank you!



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[aleable.github.io](https://github.com/aleable)



imprs-is

