

Overeducation Over the Lifecycle: Disentangling Frictions, Innate Ability, and Job-Specific Experience

Aleandro Palazzo

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Abstract

This paper studies overeducation persistence using a directed-search model where workers differ by education, field, innate occupational ability, job-specific experience, and age, whereas occupations differ by complexity and educational requirements. Calibrated to NLSY79 and O*NET, the model reproduces empirical patterns and decomposes persistence: frictions matter early, while accumulation of non-transferable job-specific experience (specialization) dominates long-run persistence; age and apparent overeducation play a minor role; slower learning amplifies persistence. Policies that speed early learning and reduce frictions are most effective. Education is exogenous since I focus on post-schooling dynamics. Selection is captured via different ability distributions across groups.

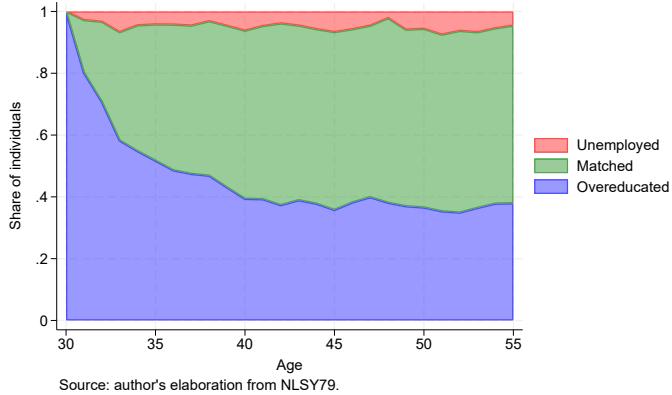
1 Introduction

Roughly 20–35% of workers globally hold jobs for which they are formally overqualified (Leuven and Oosterbeek (2011), McGuinness et al. (2018)). This phenomenon, known as overeducation, raises important questions about labor market efficiency, the returns to education, and the effective utilization of skills. Although overeducation has been extensively studied (e.g., Alba-Ramirez (1993), Duncan and Hoffman (1981), Verdugo and Verdugo (1989)), little is known about its dynamics at the individual level and how it evolves over the lifecycle. This paper investigates the mechanisms behind overeducation persistence, aiming to explain why some workers remain overeducated for many years. Using the 1979 National Longitudinal Survey of Youth (NLSY79), I find that, particularly in the early stage of the career, overeducation is partly a temporary condition driven by labor market frictions. However, for a substantial share of workers, overeducation persists. This persistence is primarily explained by a specialization

mechanism: workers accumulate job-specific experience in overeducated roles that is not transferable to alternative occupations where they would be better matched.

To illustrate the relevance of this research question, Figure 1 tracks the overeducation status of individuals who were overeducated at age 30. Many initially transition into better matches: after one year, around 20% exit overeducation, and after five years, roughly half are no longer overeducated. However, after ten years about 40% remain overeducated, a share that changes little even twenty years later.

Figure 1: Evolution of the status of overeducated workers at age 30.



Note: This figure follows all the individuals in the sample who are classified as *overeducated* at age 30 (NLSY79). The x-axis reports the age starting from 30; the y-axis shows the fraction of the original subsample (normalized to 100%) in each subsequent status. Series shown are: (i) **remaining overeducated** (blue area), that is the share still employed in occupations whose typical educational requirement (from O*NET) is lower than the worker's attainment; (ii) **transitioned to matched** (green area), that is the share employed in occupations whose typical requirement matches the worker's education; (iii) **transitioned to unemployment** (red area), that is the share currently unemployed. Overeducation is defined as having attained a higher education level than the occupation's typical requirement. See Section 2.1 for sample restrictions and further details. See Section 2.3 for details on the overeducation definition.

This paper addresses the issue of overeducation persistence by developing a directed search model, in the spirit of Menzio et al. (2016) and Menzio and Shi (2011), in which workers differ in educational level and field, innate ability across occupations, job-specific experience, and age. On the firm side, occupations vary in complexity, defined as the minimum skill level required to perform the job effectively.

I calibrate the framework using US individual-level data from the NLSY79 and information on occupational requirements from O*NET¹. The model successfully replicates the empirical patterns of overeducation persistence observed in the data. I then use the calibrated model to decompose persistence into four theoretical channels. Labor market frictions, the “temporary” channel, account for a significant share of persistence early in

¹See Baley et al. (2022), Lise and Postel-Vinay (2020), and Guvenen et al. (2020) for similar strategies.

the career, explaining about 25 percentage points of the total persistence, but its contribution declines with age, falling to roughly 10 points later in life. The most important driver is the specialization channel, which account for 65–70 percentage points early in the lifecycle: workers accumulate job-specific experience in their current overeducated occupation, which is not transferable to alternative positions. In this case, remaining in a formally overqualified job is individually optimal. By contrast, the “age” channel, capturing skill depreciation and a shorter time horizon, plays only a minor role for younger workers. However, later in the lifecycle, persistence is best explained by a combination of age and accumulated experience. Finally, a small share reflects “apparent overeducation” (Chevalier (2003)), where workers appear overeducated but are actually in their best feasible match given idiosyncratic characteristics, for example, holding a degree that is less valued by the labor market. A key factor shaping these results is the role of learning frictions: the slower the process of learning about workers’ ability, the more persistent overeducation becomes, as the incentive to switch occupations diminishes when returns are uncertain.

This paper contributes mainly to three strands of the literature. First, it adds to the literature on overeducation, dating back to Freeman (1975), who argued that returns to higher education declined in the 1970s. Much of the empirical work that followed, inspired by Duncan and Hoffman (1981), focused on the effects of overeducation on wages and employment (e.g., Verdugo and Verdugo (1989)). Given concerns about omitted variable bias (see Leuven and Oosterbeek (2011)), several studies attempted to address this issue using instrumental variables (e.g., Korpi and Tåhlin (2009)) or fixed effects approaches (e.g., Bauer (2002), Dolton and Vignoles (2000)). However, the accuracy of these results is often questioned due to the difficulty of finding credible instruments and the possibility that workers sort into occupations based on unobserved characteristics (Leuven and Oosterbeek (2011)). Importantly, most of this literature has emphasized cross-sectional aspects (e.g., Alba-Ramirez (1993), Hartog (2000)), with relatively little attention to dynamics. Notable exceptions include Rubb (2003) and Clark et al. (2017), who provide empirical evidence of substantial persistence in overeducation. This observed persistence is hard to reconcile with existing theories. Human capital theory implies that all the investment in human capital sustained by individuals will eventually be rewarded by firms. The career mobility view (Sicherman and Galor (1990), Sicherman (1991)) suggests that workers accept initially mismatched jobs as stepping stones to better-matched positions, implying a temporary phenomenon. In a similar spirit, Dolado et al. (2009) develop a random search model in which workers, endowed with two skill levels and the option to search while employed (unlike in Gautier (2002)), may temporarily accept an overqualified position due to labor market frictions, but eventually transition to a matched job. However, none of these models can explain the

long-term persistence documented in this paper.

Second, this paper contributes to the literature on directed search theory, initiated by Montgomery (1991) and further developed by Moen (1997). The structure of the model builds closely on Menzio et al. (2016), but introduces additional layers of heterogeneity. In particular, workers differ not only in general experience, age, and employment status, as in Menzio et al. (2016), but also in education, job-specific experience, and innate occupational ability.

Lastly, this paper is closely related to the literature on skills mismatch (e.g., Baley et al. (2022), Guvenen et al. (2020), Lise and Postel-Vinay (2020)). From this literature, I borrow the empirical measures of job complexity and workers' ability. Additionally, although the focus of this paper is on overeducation, the model can be easily used to study skills mismatch as well, since workers differ in their skill level (which is determined by their education and innate ability).

The remainder of the paper is organized as follows. Section 2 describes the data and provides descriptive evidence. Section 3 presents the theoretical framework. Section 4 details the calibration, validates the model, and presents the decomposition results. Section 5 compares the baseline findings with those from the younger NLSY97 cohort. Section 6 concludes.

2 Data and Descriptive Evidence

2.1 Data and Sample

For the empirical part of the paper, the primary dataset is the National Longitudinal Survey of Youth 1979 (NLSY79), a well-known panel study that has followed a nationally representative sample of 12,686 U.S. citizens born between 1957 and 1964, interviewed annually from 1979 to 1994 and biannually thereafter. The longitudinal nature of this dataset is fundamental to this research, as it allows us to track individuals' career paths over their working lives, which is crucial for understanding the long-term dynamics of overeducation.

Moreover, I use the O*NET database for data on occupations, which contains hundreds of standardized and occupation-specific descriptors on almost 1,000 occupations covering the entire U.S. economy.

The analysis focuses on prime-age individuals between 25 and 55. This age range is particularly relevant for the study of overeducation as it covers the period when most workers have completed their formal education and are fully integrated into the labor market. The relevant sample is defined as standard (see Baley et al. (2022) for example): by dropping individuals with more than 2 years of military service, weak labor

attachment (more than 10 years out of the labor market), those who were already working at the beginning of the sample, and those with no ability scores. The final sample consists of 3,297 individuals, accounting for a total of 1,005,947 monthly observations.

To validate the robustness of my findings, I also conducted a comparative analysis using the more recent National Longitudinal Survey of Youth 1997 (NLSY97). This comparison allows us to verify whether the dynamics of overeducation have remained consistent across different generations.

2.2 Measuring Job Complexity and Workers’ Innate Ability

To measure the complexity of each occupation, I follow the literature on skill mismatch (Baley et al. (2022), Lise and Postel-Vinay (2020), Guvenen et al. (2020)).

First, the complexity of each occupation is measured using detailed descriptors from the O*NET database. These descriptors provide comprehensive information on the skills and knowledge required for various jobs. Using a principal component analysis (PCA), the numerous raw descriptors are systematically reduced into four primary dimensions of complexity: verbal, mathematical, social, and technical skills. Then, each occupation is assigned to one of four complexity levels (r_1, r_2, r_3, r_4) based on the skill requirements in these four dimensions.

Second, worker skills are measured based on six Armed Services Vocational Aptitude Battery (ASVAB) scores contained in the NLSY79 dataset, individual scores on the Rotter locus of control scale, and the Rosenberg self-esteem scale. Following a similar procedure as for skill requirements, these scores are reduced into four dimensions of worker abilities in mathematics, verbal, social, and technical skills. Finally, each worker has been assigned an “innate ability” in each occupation (high or low)² based on the distance between the skills of the workers and the skills required by the job.

2.3 Measuring Education, Fields, and Overeducation Status

Individuals are first divided into two educational groups: E^L , representing those with a high school degree or less, and E^H , for those with a college degree or higher (2 or more years of college completed). Among those with a college degree or higher, I further distinguish between two fields of study: humanities-related fields, E_{hum}^H , and non-humanities fields, E_{other}^H . I explain the rationale for this distinction below.

First, I assigned to each graduated worker one of 4 possible fields: medical, humanities, social sciences and Law, STEM. Similarly, I assigned a field to each occupation based on the job family indicated in O*NET. As can be seen from Table 1, humani-

²Look at Section 3 for more details on innate ability definition

ties is the only field that does not have any occupations in the most complex subset, r_4 . Therefore, to simplify the analysis, I group medical, social sciences and law, and STEM into a single field, assuming they have similar characteristics in terms of access to complex jobs.

Therefore I will use three types of individuals: low-educated, E^L , high-educated in some fields related to humanities, E_{hum}^H , and high-educated in fields not related to humanities, E_{other}^H . As it will become clear in the model, E_{hum}^H have a disadvantage compared to E_{other}^H , since they cannot access the most complex jobs in r_4 ; at the same time they have an advantage compared to E^L since in intermediate-complexity jobs (complexity r_2, r_3) they have a higher skill level (on average) and a different signal for innate ability.

Finally, workers are classified as overeducated if their educational attainment is higher than the typical requirement for their occupation, as specified in the O*NET database. Similarly, workers are classified as undereducated (matched) if their education is lower than (equal to) the occupation's typical requirement.

Table 1: Number of jobs by field and complexity level.

Job Field	Complexity Level (r)			
	1	2	3	4
Medical	1	3	10	11
Humanities	0	16	13	0
Social Science and Law	7	34	42	4
STEM	0	3	5	34
None	23	64	47	0
Total	31	120	117	49

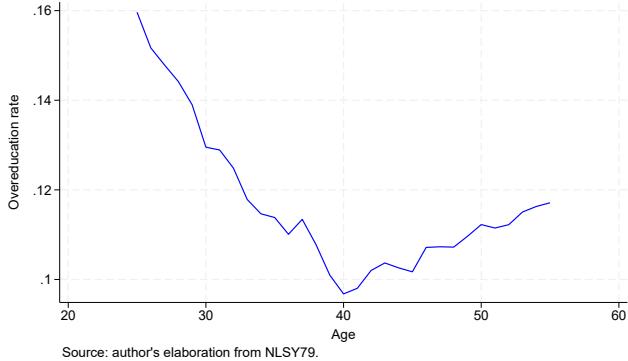
Note: This table reports the number of occupations by job field and by complexity level (r_1, r_2, r_3, r_4). Complexity levels are based on Baley et al. (2022), who compute complexity scores in 4 dimensions (verbal, mathematical, social, technical) for each occupation using O*NET skill descriptors. Each cell contains the count of distinct occupations that belong to the indicated field and complexity level. The “Total” row reports column sums, that is the total number of occupations with a given complexity. A zero entry means no occupations of that field are classified at that complexity. The “None” field groups occupations not assigned to one of the four main fields. See Section 2.2 for details on the complexity classification.

2.4 Incidence of Overeducation

As shown in Figure 2, approximately 16% of workers are overeducated at the beginning of their careers. This initial incidence gradually declines to about 10% by age 40 before slightly rising again later in life. In general, the incidence of overeducation is lower than found in other papers. This is likely to be a lower bound since only 2 educational levels (any college degree vs high school diploma or less) are used.

The decreasing pattern in the first half of the lifecycle is coherent with a model of frictions, learning and sorting (like the one in this paper), where it takes time for workers to find the right job. As far as the slight increase in the second part of the lifecycle is concerned, this paper provides evidence that this is primarily caused by workers specializing in their current occupation, rather than due to factors related to age per se.

Figure 2: Share of overeducated workers by age.



Note: This figure reports the share of workers classified as overeducated at each age in the NLSY79 sample. Overeducation is defined by comparing individual education to the occupation's typical requirement (O*NET). Series are unconditional cohort shares. See Section 2.1 for sample restrictions and further details. See Section 2.3 for details on the overeducation definition.

2.5 Overeducation and Complexity in the Data

To understand which types of jobs are most affected by overeducation, the relationship between complexity and educational requirements is presented here. First, notice that occupations in the highest complexity level (r_4) strictly require a college degree (see Section 3.4 for details), making it impossible for them to employ overeducated workers by construction.

For occupations at lower complexity levels (r_1, r_2, r_3), a college degree is not a universal requirement, meaning that these roles can be filled by overeducated workers. Table 2 illustrates the clear relationship between job complexity and educational requirements: relatively less complex jobs require a college degree much more rarely, with only about 6% of complexity level 1 jobs and less than 17% of complexity level 2 jobs having this requirement. In contrast, more than half of the jobs in complexity level 3 require a college degree.

Table 3 shows where overeducated workers are actually employed, revealing that the majority of overeducation is concentrated in jobs with complexity level 2, which accounts for nearly 70% of all overeducated workers. A smaller fraction of overeducated

workers are found in jobs with complexity level 3 (around 22%) and complexity level 1 (approximately 9%).

This distribution suggests that overeducation is not a random phenomenon but is instead concentrated in jobs that offer a degree of challenge (r_2) but do not strictly require a college degree.

Table 2: Education requirements and complexity levels.

Complexity (r)	# occupations $E_j = E^H$	# occupations	Share $E_j = E^H$
1	2	31	6.45%
2	20	120	16.67%
3	63	117	53.85%
4	49	49	100%

Note: This table shows, for each complexity level r , the total number of occupations (# occupations) and the subset that formally requires a college degree ($E_j = E^H$). The percentages are computed as the ratio between occupations with a college requirement and the total occupations at the corresponding complexity level. Complexity levels are based on Baley et al. (2022), who compute complexity scores in 4 dimensions (verbal, mathematical, social, technical) for each occupation using O*NET skill descriptors. See Section 2.2 for details on the complexity classification.

Table 3: Overeducated workers and complexity.

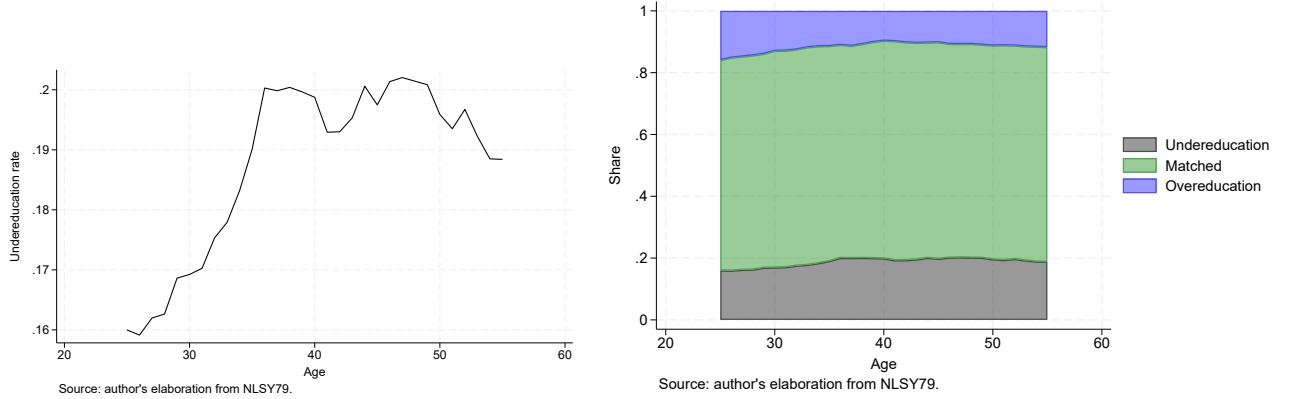
Complexity (r)	# overeducated	Share out of total overeducated
1	10,533	8.88%
2	82,098	69.24%
3	25,937	21.88%
4	0	0%

Note: This table reports the number and share of overeducated workers by occupation complexity (r_1, r_2, r_3, r_4), computed from the NLSY79 sample. Complexity levels are based on Baley et al. (2022), who compute complexity scores in 4 dimensions (verbal, mathematical, social, technical) for each occupation using O*NET skill descriptors. See Section 2.2 for details on the complexity classification. See Section 2.3 for details on the overeducation definition.

2.6 Incidence of Undereducation

The incidence of undereducation is significant, ranging from 16% to more than 20% as shown in figure 3 (left panel). This magnitude is in line with the findings of other studies (Leuven and Oosterbeek (2011)). This supports the idea that education is not the only factor that is relevant when hiring a worker. Interestingly, the share of undereducated workers increases during the lifecycle, providing evidence for a learning mechanism that matches workers and firms based on factors not related to education (e.g., innate ability). Finally, figure 3 (right panel) displays the share of workers in each status to give a complete picture of each condition, over the lifecycle.

Figure 3: Share of undereducated workers by age (left panel). Share of workers in each status by age (right panel).



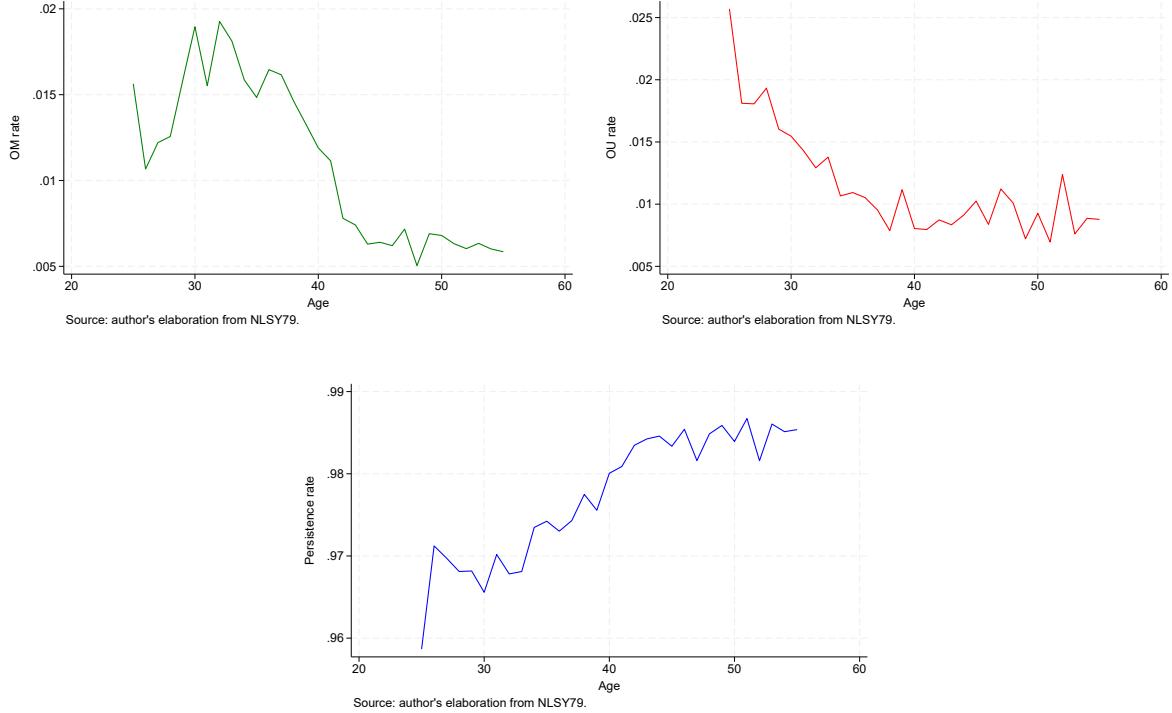
Note: This figure shows the share of undereducated workers by age (left panel) and the distribution of workers in each status: overeducation, matched or undereducation (right panel). Overeducation, undereducation and matched status are defined by comparing individual education to occupation requirements from O*NET. Series are unconditional cohort shares; see Section 2.1 for sample restrictions and measurement details. See Section 2.3 for details on the overeducation/undereducation/matched definitions.

2.7 Transitions from Overeducation

This section analyzes the transition of overeducated workers to alternative employment states. The findings, as depicted in Figure 4, reveal a clear lifecycle pattern in these transitions. The share of overeducated workers who successfully move to a matched job, one that aligns with their educational qualifications, reaches a peak at approximately age 32 and then declines rapidly. This pattern suggests that the early years of a career are a critical period for sorting and that the window for transitioning out of an overeducated role to a better-matched one narrows significantly with age. Additionally, transitions to unemployment are more frequent among younger workers. This is likely due to younger individuals being more willing to bear the costs of job search and turnover in an effort to find a better fit. As workers age and accumulate job-specific experience, they may become less inclined to risk unemployment, even if they are in a sub-optimal job.

These findings highlight two key mechanisms at play: a learning and sorting mechanism in the early career that helps correct initial mismatches, and a persistence mechanism later in life, where the costs of transitioning out of an overeducated job become prohibitive.

Figure 4: Monthly share of workers moving from overeducation to: matched (top left panel), unemployment (top right panel), remaining overeducated (bottom panel).



Note: This figure presents monthly transition rates from overeducation to (i) matched (OM rate), (ii) unemployment (OU rate), and (iii) remaining overeducated (Persistence rate). Rates are computed using the NLSY79 sample. Overeducation is defined as having attained a higher education level than the occupation's typical requirement. See Section 2.1 for sample restrictions and further details. See Section 2.3 for details on the overeducation/matched definition.

2.8 Overeducation Persistence

The core focus of this paper is the persistence of overeducation over a worker's career. As illustrated in Figure 4 (bottom panel), the monthly persistence rate of overeducation, the share of overeducated workers who remain in that status one month later, is not constant during the lifecycle. It is relatively low early in the career (approximately 96.5%) and increases to about 98.5% in later career stages.

This finding is crucial and highlights a significant insight: while overeducation may be a temporary “sorting” phase for some workers at the start of their careers, it becomes a much more permanent state as they age. This increased persistence suggests that the initial labor market frictions that lead to overeducation may be resolved early on, but once a worker settles into an overeducated job, the factors that drive persistence, such as the accumulation of non-transferable job-specific experience and the high costs of job searching, become more dominant.

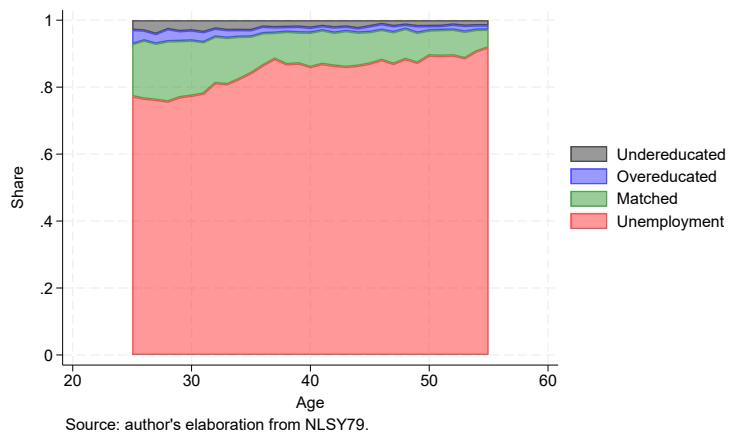
The detailed analysis of the components driving this persistence is complex and is

the subject of Section 4.3, where a formal decomposition is presented.

2.9 Transitions from Unemployment

This section examines the transitions of workers from unemployment to various employment statuses. The data (Figure 5) reveals a strong correlation between age and the likelihood of finding a new job. Specifically, younger, unemployed workers exhibit a higher degree of labor market fluidity, with approximately 20% successfully transitioning into a job each month at the beginning of their careers.

Figure 5: Monthly transition rates from unemployment by age.



Note: This figure presents monthly transition rates from unemployment to (i) matched (green area), (ii) overeducation (blue area), (iii) remaining unemployed (red area), and (iv) undereducation (black area). Rates are computed using the NLSY79 sample. Overeducation is defined as having attained a higher education level than the occupation's typical requirement. See Section 2.1 for sample restrictions and further details. See Section 2.3 for details on the overeducation/matched/undereducation definitions.

When these transitions occur, my findings indicate that the majority of the unemployed workers accept a matched job that aligns with their educational qualifications. However, a significant portion still ends up in either an overeducated or undereducated role. This highlights the ongoing nature of labor market sorting, even after a period of unemployment. The fact that a substantial share of unemployed workers finds a job that is not an ideal match suggests that short-term job-seeking goals, such as re-entering the workforce quickly, may sometimes take precedence over finding the perfect long-term fit.

2.10 Transitions from Matched

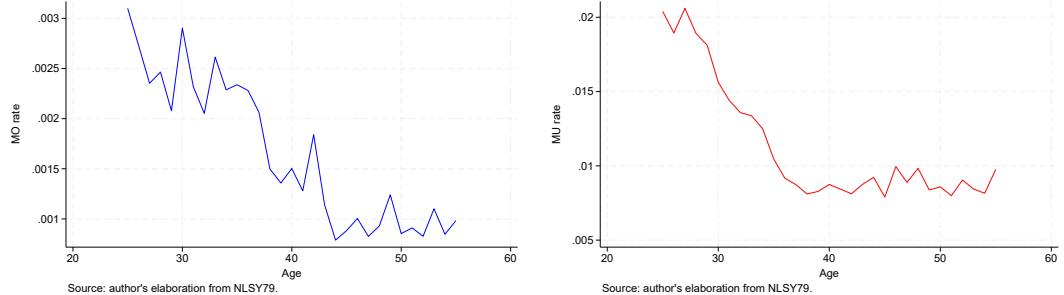
The career trajectories of workers who are in a matched job, one that is aligned with their educational qualifications, are more stable. My findings reveal that the movement

of these workers to an overeducated role is exceptionally rare. As illustrated in Figure 6, the transition rate from a matched job to an overeducated one is very low, fluctuating between 0.1% and 0.3%. This suggests that once a worker secures a well-fitting job, they are highly unlikely to regress into an overeducated state. This finding further emphasizes the “stickiness” of matched jobs and underscores the importance of the initial sorting phase in a worker’s career.

The transition from a matched job to unemployment is also examined, and I find that this happens more frequently among younger workers. This observation is consistent with the higher labor market fluidity of younger individuals, who may be more willing to change jobs or risk a period of unemployment in pursuit of better opportunities. This contrasts sharply with older workers, who, having likely found a stable job, show a much lower propensity for such transitions.

Overall, this analysis provides evidence that a matched job is a highly stable and desirable state, with a low probability of transition to either overeducation or unemployment, especially for workers later in their careers.

Figure 6: Monthly transition rates from matched to overeducated (left panel) and to unemployment (right panel).



Note: This figure shows monthly transition rates from matched employment to overeducation (left) and unemployment (right), by worker age. Rates are computed using the NLSY79 sample. Overeducation and matched statuses are defined by comparing individual education to occupation requirements from O*NET. See Section 2.1 for sample restrictions and further details. See Section 2.3 for details on the overeducation/matched definition.

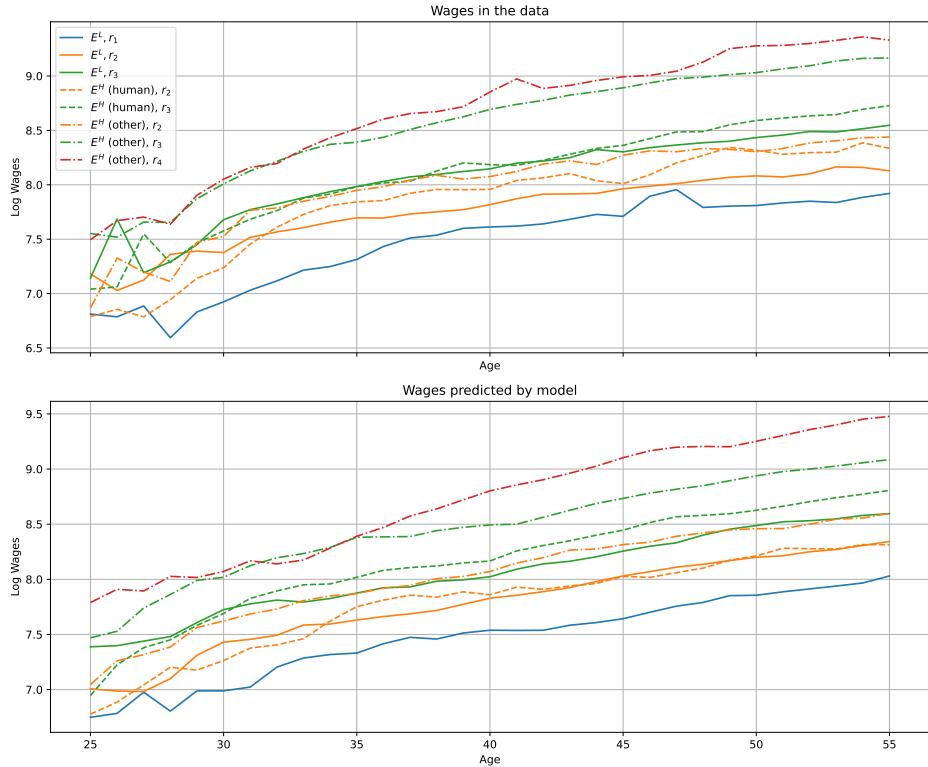
2.11 Age Profile of Wages by Complexity and Education

In this section the relationship between workers’ wages, job complexity, and educational level across the lifecycle is analyzed. The data are discussed to give a descriptive evidence and they will be used to calibrate the model in Section 4. Whether this is the result of sorting, firm productivity differences, or other mechanisms is not the focus of this paper, and will not be discussed here.

As depicted in Figure 7, the findings confirm a strong positive correlation between job complexity and wages. The complexity of a worker’s occupation is the primary

factor in explaining wage differences. Moreover, wages for all workers tend to follow a hump-shaped curve over their careers, rising in the early and middle years before plateauing or slightly declining later in life, as usual. Crucially, I find that workers in the most complex jobs consistently earn the highest wages, as expected.

Figure 7: Wage profile of monthly log wages, by complexity and education. Actual data (top panel) vs wages implied by the calibrated model (bottom panel).



Note: This figure shows the age profile of monthly log wages by job complexity (different colour) and education level (different line style). E^L corresponds to workers with a high school diploma or less, while $E^H(\text{human})$ corresponds to workers with a humanities college degree, and $E^H(\text{other})$ corresponds to workers with other types of college degrees. The top panel displays actual data from the NLSY79 sample, while the bottom panel presents wages implied by the calibrated model. Complexity levels are based on Baley et al. (2022), who compute complexity scores in 4 dimensions (verbal, mathematical, social, technical) for each occupation using O*NET skill descriptors. See Section 2.2 for details on the complexity classification. See Section 4 for details on the model calibration.

3 Theoretical Framework

3.1 The Environment

The economy consists of T overlapping generations. Time is discrete, and $t \in \{0, 1, 2, \dots, T\}$ is the age of the worker. Each risk-neutral worker is endowed with one indivisible unit of labor and maximizes expected lifetime utility, defined as the discounted sum of per-

period consumption with discount factor $\beta \in (0, 1)$. A continuum of risk-neutral firms, with positive measure, also populate the economy.

There are J different occupations, indexed by $j \in \{1, 2, \dots, J\}$. Occupations differ in their *complexity* r_j , which corresponds to the minimum skill requirement for that occupation. For each occupation, worker i has a skill level s_{ij} and an experience level $e_{ij} \in \{0, 1, 2, \dots, t\}$, defined as the number of past periods spent employed in occupation j . Experience is fully occupation-specific: if a worker moves to a different occupation, the accumulated experience does not transfer, and experience is zero in the new job. This assumption captures the idea that skills and knowledge acquired in one occupation may not be relevant or useful in another, especially when occupations are very different. While this is a simplification, as some skills may be transferable across occupations, it is useful for keeping the model tractable and highlights the role of job-specific experience in overeducation persistence. Nevertheless, introducing some degree of experience transferability would be an interesting extension for future research.

Output of a firm-worker match is given by

$$y(r_j, e_{ij}, s_{ij}, t) = \begin{cases} g(r_j, e_{ij}, t) & \text{if } s_{ij} \geq r_j \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where $g(\cdot)$ is increasing in both job complexity r_j and job-specific experience e_{ij} , and decreasing in age t to capture the possibility that productivity decrease due to aging (what is generally referred to as skill depreciation). So, if the skill requirement is satisfied ($s_{ij} \geq r_j$), the product of a match is $g(r_j, e_{ij}, t)$, whereas if the requirement is not satisfied, the match is not productive at all. It is important to highlight that, conditional on satisfying the skill requirement and given (r_j, e_{ij}, t) , workers are equally productive. This means that if $s_{ij} > r_j$, the worker has some skills ($s_{ij} - r_j$) that are not used in the job (overskilling), and these extra skills do not translate into higher productivity³.

3.2 Skill Level of the Worker

The skill level of a worker i in occupation j is given by

$$s_{ij} = s(a_{ij}, E_i)$$

³There is no doubt that, at the micro level, productivity variation may be relevant also within a complexity level. However, this is an approximation that helps to keep the model tractable. Moreover, there is some evidence that more educated workers are not more productive at a given job complexity level (Gautier et al. (2002)).

where $E_i \in \{E^L, E_f^H\}$ denotes the worker's education. Education is publicly observable and can be either low (E^L), corresponding to a high school diploma or less, or high (E_f^H), corresponding to a college degree in field f . Importantly, a college-educated worker in field f can benefit from higher education only in occupations that belong to the same field. In occupations associated with a different field ($f_i \neq f_j$), there is no distinction between E_f^H and E^L . The role of fields is further discussed in Section 3.3.

The second component of skill is the worker's innate ability in occupation j , denoted $a_{ij} \in \{a^L, a^H\}$. This can be also interpreted as the match-specific component of productivity between worker i and firms in occupation j . Ability is initially unobserved by both the worker and the firm (see Baley et al. (2022) for evidence on workers not having perfect information about their own ability). With probability α each period, the true value of a_{ij} becomes public knowledge; until then, ability remains unobserved and is denoted by $a_{ij} = a^0$.

Given the binary nature of both education and ability, there are four possible combinations, which correspond to four distinct skill levels, ordered as follows:

$$s(a^L, E^L) \leq s(a^L, E_f^H) \leq s(a^H, E^L) \leq s(a^H, E_f^H).$$

The lowest and highest skill levels are obvious. In principle, the two intermediate cases can be switched, depending on whether education or innate ability is more important. I chose this ordering for two reasons. First, there is evidence that lifetime earnings for high-ability high school graduates are higher than low-ability college graduates (Ichino et al. (2024)). Second, imposing $s(a^L, E_f^H) \geq s(a^H, E^L)$ would create a clear separation between the skill level of high and low educated individuals. This would leave little room for undereducation, which is instead a quite widespread condition (see Leuven and Oosterbeek (2011) and section 2.6).

3.3 Fields of study

Each occupation j is associated with a field of study f_j . A highly educated worker with a degree in field f (E_f^H) can exploit the benefits of higher education only in occupations linked to the same field, i.e. when $f_i = f_j$. In occupations associated with a different field ($f_i \neq f_j$), there is no distinction between workers with E_f^H and those with E^L .

For example, a worker holding a degree in philosophy cannot benefit from her education when applying for an engineering position; in that case, firms perceive no difference between her and an applicant without a college degree. The reverse also holds: a graduate in engineering does not gain an advantage when applying to a philosophy-related job.

This field-specificity implies that the value of higher education depends not only on the degree level, but also on the alignment between a worker's field of study and the field of the occupation.

3.4 Occupations and Job Complexity

Each occupation requires a minimum skill level r_j , which I refer to as the complexity of the job. Since there are four distinct skill levels, determined by education and innate ability, there are also four corresponding complexity levels: $r_1 \leq r_2 \leq r_3 \leq r_4$. The following matrix illustrates the four possible skill levels of the worker in a given occupation, along with the complexity levels where the minimum skill requirement is satisfied.

		Innate Ability	
		a^L	a^H
		$s(a^L, E_f^H)$ Req. satisfied: r_1, r_2	$s(a^H, E_f^H)$ Req. satisfied: r_1, r_2, r_3, r_4
Education	E^H	$s(a^L, E^L)$ Req. satisfied: r_1	$s(a^H, E^L)$ Req. satisfied: r_1, r_2, r_3
	E^L		

Note: This table summarizes the mapping between worker education (E^L, E_f^H), innate ability (a^L, a^H), and occupation complexity levels (r_1, r_2, r_3, r_4). It shows which education-ability combinations satisfy each complexity requirement and is intended to be a self-contained guide to the skill taxonomy used in the model. For example, a worker with low education and high ability (bottom-right entry) meets the skill requirements for occupations with complexity levels r_1, r_2 , and r_3 , whereas she will not be productive in jobs with complexity r_4 (the minimum requirement is high ability and high education). Remember that a given worker has a different ability level a_{ij} for each occupation j .

This implies that the set of occupations can be partitioned into four distinct complexity groups, each associated with a minimum skill level required for successful job performance.

Occupations with complexity r_1 are the least demanding and can be performed by all workers, since even the lowest education-ability combination suffices. By contrast, occupations with complexity r_4 are the most demanding and require both high education and high innate ability. Occupations characterized by complexity r_2 requires high education or high ability: for instance, a worker with $s(a^L, E_f^H)$ or $s(a^H, E^L)$ would meet the requirement. Finally, in jobs with complexity r_3 high ability is strictly necessary. Education per se is not required to meet the skill requirement, but it may serve

as a signal, as discussed in section 3.7.

Each group of occupations can include jobs tied to different educational fields. Hence, the match between a worker and an occupation depends not only on education and ability but also on the field-specific alignment, as previously described in Section 3.3.

3.5 Educational Choice

Since the purpose of this paper is to study overeducation persistence, I abstract from modeling explicitly the schooling decision process and assume that education is stochastically assigned before entering the labor market. More specifically, each individual must decide whether to pursue higher education in a given field f (denoted E_f^H), or to enter the labor market immediately with low education (E^L). The decision process is stochastic, but the probability of successfully completing higher education is increasing in the individual's average innate ability,

$$\bar{a}_i = \frac{1}{J} \sum_{j=1}^J a_{ij}.$$

This process implies that on average college-educated workers have higher innate ability than those with lower education. This is consistent with empirical evidence showing a positive correlation between innate ability and educational attainment (see Roth et al. (2015); Strenze (2007)). Moreover, it simplifies the analysis by avoiding the need to model the complex decision-making process involved in educational choices. At the same time, it captures important mechanisms that are relevant for the study of overeducation persistence, such as the signaling role of education and the sorting of higher-ability workers into higher educational levels.

As a result, individuals who attain higher education are, on average, more likely to possess high innate ability, provided their field of study aligns with the occupations they may enter. Formally,

$$P(a_{ij} = a^H \mid E_f^H) \equiv \bar{a}(E_f^H) > \bar{a}(E^L) \equiv P(a_{ij} = a^H \mid E^L) = P(a_{ij} = a^H \mid E_{f' \neq f}^H).$$

This implies that education plays a dual role in the model. First, it raises the worker's skill level through human capital accumulation (as described in Section 3.2). Second, it acts as a signal of higher expected innate ability. Crucially, the value of education is field-specific: a college degree in field f signals higher expected ability only in occupations whose associated field matches f . In occupations with $f_j \neq f_i$, the signal is uninformative, and workers with E_f^H are treated equivalently to those with E^L .

The reason why overeducation does not disappear completely in this framework is twofold. First, because of labor market frictions, workers may not find the right job immediately after entering the labor market, then the persistence channels of the model come into play. Second, the fact that individuals have incorrect beliefs about their own ability at the beginning of their career, may lead to incorrect educational choices.

3.6 The Labor Market

The type of a worker is summarized by $(E_i, e_{ij}, r_j, a_{ij}, t) = (\omega, t)$, that is, job-specific experience e_{ij} , complexity r_j and ability $a_{ij} \in \{a^0, a^L, a^H\}$ in the current (if employed) or most recent (if unemployed) occupation, education E_i and age t . Each vacancy must specify (ω, x, t) , where x is the value offered to the worker in lifetime utility. Search is directed, meaning that there is a continuum of submarkets indexed by (ω, x, t) . Workers decide optimally where to search based on their type, and firms decide how many vacancies to create and where to locate them. $\theta_t(\omega, x)$, the ratio between the vacancies and the individuals searching in a certain submarket, is the market tightness of the submarket (ω, x, t) . Workers and vacancies in the same submarket meet through a frictional process. Searching on-the-job is also possible: while employed workers are allowed to search for a job in a different occupation, but at a lower intensity than unemployed workers.

3.7 Educational Requirement

This section describes how firms determine the minimum educational requirement for each vacancy when a worker's innate ability is unknown (i.e., $a_{ij} = a^0$). As it will become clearer, education is treated both as a signalling/screening device (Spence (1978)) and as a worker's characteristic that can increase their skill level (human capital theory, Becker (1964)).

Since workers who satisfy the skill requirement $s_{ij} \geq r_j$ are equally productive, firms are only interested in maximizing the probability that a matched worker satisfies the requirement. Therefore, for a given occupation j , firms base their educational requirement E_j on the conditional probability $P(s_{ij} \geq r_j | E_i)$: the probability that a worker's skill level s_{ij} , which depends on education E_i and innate ability a_{ij} , is greater or equal than the minimum skill requirement r_j .

For jobs with the lowest complexity complexity r_1

$$P(s_{ij} \geq r_1 | E^L) = P(s_{ij} \geq r_1 | E^H) = 1$$

Thus, all workers satisfy the skill requirement regardless of their education and innate

ability. So, the minimum educational requirement is $E_j = E^L$.

On the other hand, for the most complex occupations r_4

$$P(s_{ij} \geq r_4 | E^L) = 0 \leq P(s_{ij} \geq r_4 | E_f^H) = P(a_{ij} = a^H | E_f^H) \equiv \bar{a}(E_f^H)$$

Therefore, the minimum educational requirement in this case is $E_j = E^H$.

For jobs with complexity r_3 , firms are not interested in education per se, but it can be a signal for high ability, more specifically

$$P(s_{ij} \geq r_3 | E^L) = P(a_{ij} = a^H | E^L) \equiv \bar{a}(E^L) < \bar{a}(E_f^H) \equiv P(a_{ij} = a^H | E_f^H) = P(s_{ij} \geq r_3 | E_f^H)$$

So in this case, firms may decide to open a vacancy for the same occupation, either directed at low-educated or high-educated workers, at potentially different conditions.

Similarly, for jobs with complexity $r_2 = s(E_f^H, a^L)$

$$P(s_{ij} \geq r_2 | E^L) = P(a_{ij} = a^H | E^L) \equiv \bar{a}(E^L) < 1 = P(s_{ij} \geq r_2 | E_f^H)$$

Also in this case, a vacancy may specify different educational requirements.

3.8 State, Stages and Contracts

The rest of the structure of the model is similar to Menzio et al. (2016), with some adaptations. I illustrate the main characteristics next.

At the beginning of each period, the aggregate state of the economy can be summarized by the tuple $\psi = (n, u, e, \gamma)$. The first component of ψ , $n(E, t)$ is the measure of non-participating workers. The second component $u(\omega, t)$ denotes the measure of unemployed workers. The third component $e(\omega, t)$ denotes the measure of employed workers. Finally, γ denotes the current realization of the stochastic process for the measure of newly born workers.

Every period is divided into five stages: entry-and-exit, separation, search, matching and production.

At the first stage, a non-participating worker of age t enters the labor market according to the process described in section 3.5, participating workers may exit the labor market permanently.

At the separation stage, an employed worker becomes unemployed with probability $d \in [\delta, 1]$, where d is a probability determined by the worker's employment contract and $\delta \in [0, 1]$ is the probability that the worker has to leave her job for exogenous reasons.

At the search stage, workers have the opportunity to search the labor market with a probability that depends on their employment status. In particular, if a worker is

unemployed at the beginning of the separation stage, she has the opportunity to search with probability $\lambda_u \in (0, 1]$. If a worker is employed in the separation stage and has not lost her job, she has the opportunity to search with probability $\lambda_e \in (0, 1]$. And if the worker lost her job during the separation stage, she cannot search in the current period. Whenever a worker has the opportunity to search, she optimally chooses which submarket to visit. Additionally, during the search stage, a firm decides the number of vacancies to create in each submarket. The cost of maintaining a vacancy for one period is $k > 0$.

At the matching stage, the vacancies and the workers who are searching in the same submarket come together through a frictional matching process. In particular, a worker searching in submarket (ω, x, t) meets a vacancy with probability $p(\theta_t(\omega, x))$, where $p : \mathbb{R}_+ \rightarrow [0, 1]$ is a twice-differentiable, strictly increasing and strictly concave function with boundary conditions $p(0) = 0$ and $p(\infty) = 1$. Similarly, a vacancy located in submarket (ω, x, t) meets a worker with probability $q(\theta_t(\omega, x))$, where $q : \mathbb{R}_+ \rightarrow [0, 1]$ is a twice-differentiable strictly decreasing function such that $q(\theta) = p(\theta)/\theta$, $q(0) = 1$ and $q(\infty) = 0$. When a vacancy and a worker of type (ω, t) meet in submarket (ω, x, t) , the firm offers to the worker an employment contract that is worth x in lifetime utility. If the worker rejects the offer, she returns to her previous employment position (i.e., unemployment or employment at another firm). If the worker accepts the offer, she leaves her previous employment position and enters a productive match with the firm.

At the production stage, an unemployed worker of type (ω, t) produces and consumes b units of output. A worker of type (ω, t) who is employed produces $y(r, e, s, t)$ units of output and consumes w , where w is the wage specified by the worker's employment contract. The worker and the firm observe their output with probability $\alpha \in (0, 1]$. At the end of the production stage, nature draws the measure of next period's entering cohort from the distribution $\Pi(\hat{\gamma}|\gamma)$. Throughout the paper, the hat indicates variables or functions in the next period.

As usual in this kind of models, I assume that employment contracts are complete in the sense that they can specify the wage paid by the firm to the worker, w , the probability that the worker and the firm break up at the separation stage, d , and the submarket where the worker should search while employed by the firm as a function of the history of the firm-worker match and of the aggregate economy. It follows that the firms always find it optimal to offer employment contracts that are bilaterally efficient, in the sense that these contracts maximize the sum of the worker's lifetime utility and the firm's lifetime profits from forming a match (see Menzio and Shi (2011) for more details).

3.9 Characterization of the Equilibrium

The value of unemployment for a worker of type $(\omega, t) = (E_i, e_{ij}, r_j, a_{ij}, t)$ is

$$U_t(\omega, \psi) = b + \beta \mathbb{E}_{\hat{\psi}|\psi} [U_{t+1}(\omega, \hat{\psi}) + \lambda_u R_{t+1}(\omega, \hat{\psi})]$$

$$R_{t+1}(\omega, \hat{\psi}) = \max \left\{ \max_x \{p(\theta_{t+1}(\omega, x, \hat{\psi}))[x - U_{t+1}(\omega, \hat{\psi})]\}, \max_{x,r} \{p(\theta_{t+1}(\omega_0, x, \hat{\psi}))[x - U_{t+1}(\omega, \hat{\psi})]\} \right\}$$

where $(\omega_0, t) = (E_i, e_{ij} = 0, r, a^0, t)$ is the type of the worker who accepts a job of complexity r she has never performed in the past. In the current period, the unemployed produces and enjoys b (home production, leisure, unemployment benefit). In the following period, with probability λ_u the worker can search into the labor market, where can choose whether to search for a job in the most recent occupation (first term of R_{t+1}) or a new career with no experience and unknown ability (second term of R_{t+1}); in both cases the optimal submarket is chosen by maximizing the probability of finding a job, $p(\theta)$ (decreasing in x), times the surplus deriving from the new employment condition, $x - U_{t+1}$.

The sum of the firm's lifetime profits and the worker's lifetime utility in a match with known $a \neq a^0$ is

$$V_t(\omega(a), \psi) = y(r_j, e_{ij}, s_{ij}, t) + \beta \mathbb{E}_{\hat{\psi}|\psi} \left[\max_{d \in [\delta, 1]} \{dU_{t+1}(\hat{\omega}(a), \hat{\psi}) + (1-d)[V_{t+1}(\hat{\omega}(a), \hat{\psi}) + \lambda_e S_{t+1}(\hat{\omega}(a), \hat{\psi})]\} \right]$$

$$S_{t+1}(\hat{\omega}(a), \hat{\psi}) = \max_{x,r} \{p(\theta_{t+1}(\omega_0, x, \hat{\psi}))[x - V_{t+1}(\hat{\omega}(a), \hat{\psi})]\}$$

where $\hat{\omega}(a) = (E_i, \hat{e}_{ij}, r_j, a, t)$ and $\hat{e}_{ij} = e_{ij} + 1$. In the current period, the employed produces y , which depends on whether the skill requirement is satisfied, $s_{ij} \geq r_j$, the experience of the worker in the current occupation, e_{ij} , the job complexity, r_j , and the age of the worker, t . In the following period, with probability d the match is destroyed and the worker moves to unemployment. Otherwise, the match continues, but the possibility of search on-the-job must be taken into account. In particular, an employed has the opportunity to search for a different job with probability λ_e . In this case, the worker optimally choose the submarket where to search, with no experience and unknown ability.

The sum of the firm's lifetime profits and the worker's lifetime utility in a match

with unknown ability $a = a^0$ is

$$\begin{aligned} V_t(\boldsymbol{\omega}(a^0), \psi) = & \alpha \mathbb{E}[V_t(\boldsymbol{\omega}(a), \psi) | E_i] + (1 - \alpha) \mathbb{E}[y(r_j, e_{ij}, s_{ij}, t) | E_i] + \\ & + (1 - \alpha) \beta \mathbb{E}_{\hat{\psi} | \psi} \max_{d \in [\delta, 1]} \{ d U_{t+1}(\hat{\boldsymbol{\omega}}(a^0), \hat{\psi}) + \\ & + (1 - d)(V_{t+1}(\hat{\boldsymbol{\omega}}(a^0), \hat{\psi}) + \lambda_e S_{t+1}(\hat{\boldsymbol{\omega}}(a^0), \hat{\psi})) \} \end{aligned}$$

In the current period, with probability α agents discover the true value of the innate ability of the worker in the current occupation, a_{ij} , otherwise they have an expectation of the output based on education. The following period is equivalent to the previous value function. Let's write explicitly the two expectation terms

$$\begin{aligned} \mathbb{E}[V_t(\boldsymbol{\omega}(a), \psi) | E_i] &= \bar{a}(E_i) V_t(\boldsymbol{\omega}(a^H), \psi) + (1 - \bar{a}(E_i)) V_t(\boldsymbol{\omega}(a^L), \psi) \\ \mathbb{E}[y(r, e, s, t) | E_i] &= \bar{a}(E_i) y(r, e, s(a^H), t) + (1 - \bar{a}(E_i)) y(r, e, s(a^L), t) \end{aligned}$$

where $\bar{a}_E(E_i) = P(a_{ij} = a^H | E_i)$ is the probability that a worker has high ability in a certain occupation, which depends on the educational level.

Market tightness should be such that

$$k \geq q(\theta_t(\boldsymbol{\omega}, x, \psi)) [V_t(\boldsymbol{\omega}, \psi) - x]$$

and $\theta_t(\boldsymbol{\omega}, x, \psi) \geq 0$ with complementary slackness. This condition ensures that firms behave optimally since, in submarkets with at least a vacancy, the cost of maintaining open a vacancy, k , must equal the expected benefit of an open vacancy (RHS), that is the probability of meeting a worker, $q(\theta)$, times the surplus extracted by the firm, $V - x$. If instead a submarket is empty ($\theta = 0$), it must be that the cost of the vacancy is greater than the expected benefit for the firm.

The unique equilibrium is block-recursive (BRE): a recursive equilibrium in which the agents' value and policy functions do not depend on the aggregate state of the economy ψ . The proof is reported in appendix A and follow Menzio et al. (2016) closely.

4 Calibration, Validation and Decomposition of Overeducation Persistence

4.1 Calibration

In this section, I describe the calibration strategy and results. First of all, I specify the functional form of the production function as

$$g(r_j, e_{ij}, t) = \rho_r((1 - \phi_1) + \phi_1 e_{ij}^{\phi_2}) - \phi_3 t$$

where ρ_r is the parameter that captures the complexity of the job (e.g., ρ_1 refers to complexity r_1), the term inside the parentheses is an increasing and concave function of the job-specific experience of the worker, and the last term captures skill depreciation. Moreover, following Menzio et al. (2016), the matching probability function is restricted to be of the form $p(\theta) = \min\{\theta^{\frac{1}{2}}, 1\}$. The parameters that need to be calibrated are the following. The discount factor β . The probabilities that the innate ability of a worker is high, conditional on their educational group: $\bar{a}(E^L)$, $\bar{a}(E_{hum}^H)$, $\bar{a}(E_{other}^H)$. These probabilities are crucial as they capture the signaling effect of education within the model. The search intensity of the unemployed, λ_u , and of the employed, λ_e . The unemployment flow value, b . The exogenous job destruction probability, δ . The learning probability, α . The flow cost of maintaining a vacancy open for the firm, k . Finally, all the 7 parameters of the production function $g(r_j, e_{ij}, t)$ (i.e., $\rho_1, \rho_2, \rho_3, \rho_4, \phi_1, \phi_2, \phi_3$). The calibration strategy is implemented as follows.

First, the parameters of the production function are calibrated using the age profile of wages described in section 2.11. More precisely, I assume that wages are a constant fraction of the match's productivity⁴. Then the distance between the average wages implied by the production function and the actual wages observed in the data is minimized. Figure 7 shows the monthly log wages of workers in the data compared to those predicted by the model. The calibrated model can replicate the lifecycle patterns for each education-complexity combination fairly accurately.

Second, the search intensity of the unemployed, λ_u is normalized to 1. The discount factor, β , is pinned down so that the annual real interest rate in the model, $\beta^{-12} - 1$, is equal to 4.04%, the average FED funds rate between 1982 and 2020. The beliefs when ability is unknown, $\bar{a}(E_i)$'s, are calibrated by using the average innate ability across all the occupations of each worker; the mean of this measure among all individuals in the same educational class E_i is used as the belief of being a^H when ability is not known (that is what $\bar{a}(E_i)$ captures). The exogenous separation rate, δ , has been set to 0.012,

⁴The constant fraction is allowed to be different across educational groups.

as calibrated by Baley et al. (2022) using the same data of this paper (NLSY79). The ratio between wages and the flow value of unemployment, w/b , which Hall and Milgrom (2008) estimated to be 0.71, is used to calibrate the flow value of unemployment, b .

The remaining parameters, α, λ_e, k , are those related to search and are calibrated by targeting some moments, as I describe in the following lines. First of all, I focus on workers younger than 35 years old. The reason is that young workers are more mobile across occupations and have a more pronounced learning process, making them more informative for the calibration of these parameters. Including older workers would make the calibration of the search parameters more difficult.

The monthly learning probability, α , is calibrated by targeting the ratio between the separation rate after the third and the eighteenth month of job-specific experience, $\log(haz_3/haz_{18})$, which Baley et al. (2022) estimated to be 1.37. To isolate the learning process, I restrict (only for the calibration of α) the attention to low-educated workers who are employed in jobs with complexity r_2 . The reason why I chose this group of people is that, in the model, when low-educated workers are employed in a job with complexity r_2 , the ability learning process is crucial in determining whether their match will continue or not⁵. Therefore, the separation rate of these kind of matches after different months of experience is very informative about the speed of the learning process of the model. To calibrate the search effort during employment, λ_e , I target the average transition from one occupation to another (EE rate). Intuitively, since in the model on-the-job search is possible only across different occupations, the rate at which workers change occupation is informative about the search effort while employed. Lastly, I use the average transition rate from unemployment to employment (UE rate) to calibrate the vacancy cost, k .

Table 4 displays the target and calibrated moments, which are quite close for the UE rate and the separation rate ratio. As far as transitions from one occupation to another are concerned, the model predicts a slightly lower EE rate; this may be due to the fact that, probably, people change occupation also for reasons that are outside of the model and do not affect the main conclusion of this paper (e.g., technological change and automation). Table 5 shows all the calibrated parameters of the model. The calibrated ability learning parameter (α) implies the monthly probability that agents discover the true ability of the worker in the current occupation is around 14%. This means that after 3 months, ability is unknown in the 63.6% of the matches; after 6 months, this share drop to approximately 40%, and after a year, 16% of the agents are involved in matches with unknown ability. Therefore, learning frictions play a non-negligible role in shaping the optimal decisions of the workers. The calibrated high-ability probability suggests

⁵If the ability of the worker turns out to be low, the match is revealed to be not productive and ends immediately.

there is a high degree of selection into education. Indeed, low-educated workers have 41.1% probability of being $a_{ij} = a^H$ in a certain occupation, whereas for humanities graduates this probability increases to 66.9% probability, followed by an even higher probability for non-humanities graduates, 73.4%. Finally, the production function calibrated parameters imply that jobs with complexity r_3 and r_4 are substantially more productive than less complex jobs, suggesting that the high-ability requirement is linked to more productive jobs. At the same time, the productivity in the most complex jobs (r_4) is only slightly higher than jobs with r_3 , indicating that there is not a strong incentive to move to a more complex occupation when employed in a r_3 complexity job.

Table 4: Model vs. Data moments.

Moment	Model	Data	Source
UE rate	0.2087	0.2099	NLSY79
EE rate	0.0048	0.0188	NLSY79
$\log(\text{haz}_3)/\log(\text{haz}_{18})$	1.3706	1.37	Baley et al. (2022)

Note: This table reports the calibration targets and the corresponding model-generated moments. “UE rate” denotes the monthly unemployment-to-employment transition rate; “EE rate” denotes the monthly employment-to-employment occupation-change rate; and $\log(\text{haz}_3)/\log(\text{haz}_{18})$ is the log ratio of separation hazards at 3 and 18 months of tenure. Data sources and sample restrictions are detailed in Section 2.1. Calibrated moments are generated using the theoretical transition probabilities of the model. For example, the UE rate is given by $\lambda_u p(\theta(\omega, t))$, whereas the EE rate is given by $(1 - d(\theta(\omega, t)))\lambda_e \max_{x,r} p(\theta(\omega_0, x, t))$.

Table 5: Calibrated parameters.

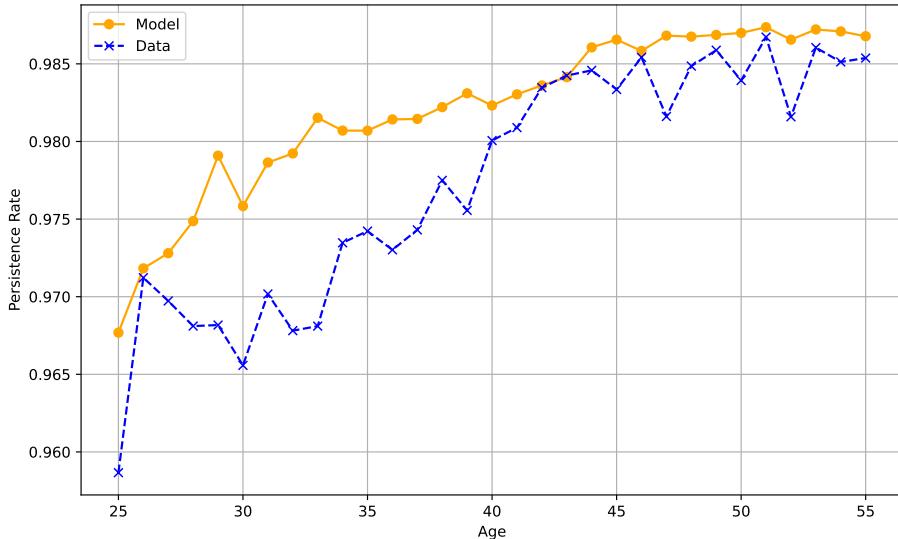
Parameter	Description	Value	Methodology
b	Unemployment flow value	5.3	Match $b/w = 0.71$, Hall and Milgrom (2008)
λ_u	Search intensity when unemployed	1	Normalized
λ_e	Search intensity while employed	0.8390	Match EE rate, calibrated
δ	Exogenous separation rate	0.012	Assigned, Baley et al. (2022)
α	Ability learning probability	0.1395	Match separation rate ratio, calibrated
k	Vacancy flow-cost	299.28	Match UE rate, calibrated
β	Discount factor	0.9967	Match average interest rate, assigned
ρ_1	Production function: complexity 1	10.938	Calibrated with age-profile of wages
ρ_2	Production function: complexity 2	11.377	Calibrated with age-profile of wages
ρ_3	Production function: complexity 3	11.88094	Calibrated with age-profile of wages
ρ_4	Production function: complexity 4	11.88104	Calibrated with age-profile of wages
ϕ_1	Production function	3.911	Calibrated with age-profile of wages
ϕ_2	Production function	0.03435	Calibrated with age-profile of wages
ϕ_3	Skill depreciation	0.00022	Calibrated with age-profile of wages
$\bar{a}(E^L)$	Probability of high ability: E^L	0.4112	NLSY79, assigned
$\bar{a}(E_{hum}^H)$	Probability of high ability: E_{hum}^H	0.669	NLSY79, assigned
$\bar{a}(E_{other}^H)$	Probability of high ability: E_{other}^H	0.734	NLSY79, assigned

Note: This table reports the calibrated parameters, their brief description, calibrated values, and the identification method or data source used. See Section 4 for details on the calibration procedure and targeted moments.

4.2 Validation: Overeducation Persistence in the Model

As a form of validation, Figure 8 provides a critical comparison, showing the monthly overeducation persistence rates from both the calibrated model and the NLSY79 data. The model successfully replicates the empirical pattern, capturing the initial, lower persistence rate early in the lifecycle and its subsequent increase with age. While the empirical fit of wages (Figure 7) was directly targeted, nothing directly related to the overeducation persistence has been used in the calibration strategy. So, the fact that the magnitude and the pattern of the persistence in the model is closely aligned with the data is not obvious and should be seen as an important result for the theory.

Figure 8: Overeducation monthly persistence: Model vs Data.



Note: This figure compares monthly overeducation persistence rates from the calibrated model and from the NLSY79 data. The solid line represents the empirical persistence (the same as in Figure 4). The dashed line is the persistence predicted by the model. Overeducation is defined as having attained a higher education level than the occupation's typical requirement (O*NET). Persistence is the share of overeducated workers who remain overeducated one month later. The persistence rate is not directly targeted in the calibration.

4.3 Decomposition of Overeducation Persistence: Methodology

This and the following section are dedicated to the key objective of this paper: decomposing the monthly persistence of overeducation into the main channels of the model. In the following lines, I will briefly recap the meaning of each channel and explain how I assign a certain channel to each overeducated worker in the dataset. First, the "tempo-

“temporary” channel is assigned when workers are searching for an alternative occupation, but they did not manage to find a new match due to search frictions. This is the case whenever the calibrated model predicts a positive matching probability⁶, given the worker’s type. The “apparent overeducation” channel is assigned to humanities graduates who are overeducated in jobs with complexity r_3 since, given their education type, they are in the best possible job (they do not satisfy the minimum requirement for r_4). If none of these two channels apply, the persistence is due to specialization and/or age. In this case, I will first assign a generic “age/experience” channel to the overeducated worker. Then, I will disentangle whether the persistence is mainly due to age or specialization. In particular, the minimum combination of age and experience reduction that would make the worker search for another occupation is computed. For example, if a 40-year-old worker with 10 years of experience would search for another occupation if she were 38 years old with 8 years of experience, then the age/experience reduction is 2 years of age and 2 years of experience.

4.4 Decomposition of Overeducation Persistence: Results

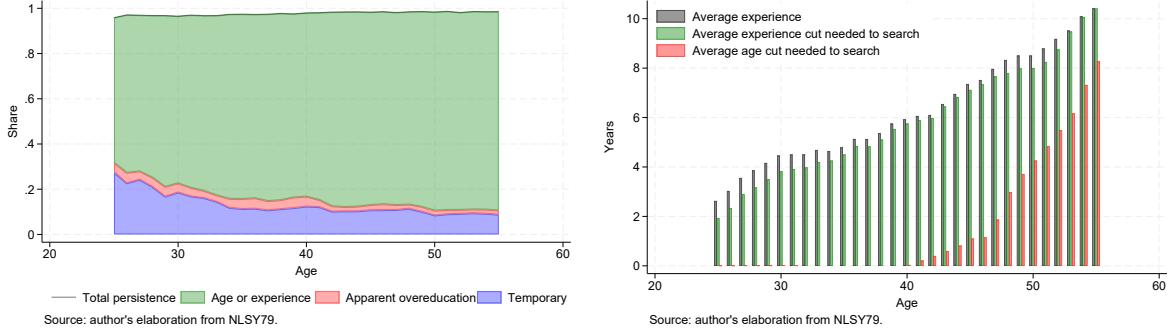
In this section, I present and discuss the results of decomposing the persistence of overeducation displayed in Figure 9. First, the persistence is partially temporary, and this channel becomes less relevant as workers get older, as expected. In particular, at the beginning of the working lifecycle, approximately 25 percentage points of the total monthly persistence are due to search frictions. This channel slowly decreases to around 10 percentage points and remains stable afterwards. The “apparent overeducation” channel plays a minor role, accounting for around 5 percentage points of the total persistence. This magnitude does not change significantly throughout the lifecycle.

The rest of the persistence in the first half of the lifecycle is almost entirely due to the job-specific experience, which leads overeducated workers to specialize in the current occupation. This means that overeducated workers accumulate experience that they do not want to lose by moving to a potentially more productive occupation. This mechanism is amplified by ability learning frictions, which add more uncertainty, the slower the learning process. Experience remains the major channel also in the second half of the lifecycle, but now age plays a more important role. In particular, after age 40, a mix between age and experience is the main driver of persistence.

Overall, the decomposition highlights a clear lifecycle pattern. Search frictions are mainly relevant at the beginning of careers, while specialization, reinforced later by age, drives long-run persistence.

⁶Actually, when the optimal θ is higher than a very low threshold (0.000001), which corresponds to a matching probability of 0.1%.

Figure 9: Decomposition of monthly overeducation persistence (NLSY79).



Note: This figure reports the decomposition of monthly overeducation persistence into the following channels: (i) **temporary** (search frictions), (ii) **apparent overeducation**, (iii) **specialization** (job-specific experience), and (iv) **age** (skill depreciation and a shorter time horizon). The left panel shows the lifecycle evolution of each channel's contribution, with age and specialization grouped together; the right panel isolates the joint roles of age and experience by showing the smallest reduction in age and experience needed to make it optimal for the worker to search for another occupation. Results are computed from the model calibrated to the NLSY79. See Section 4 for details on the calibration strategy.

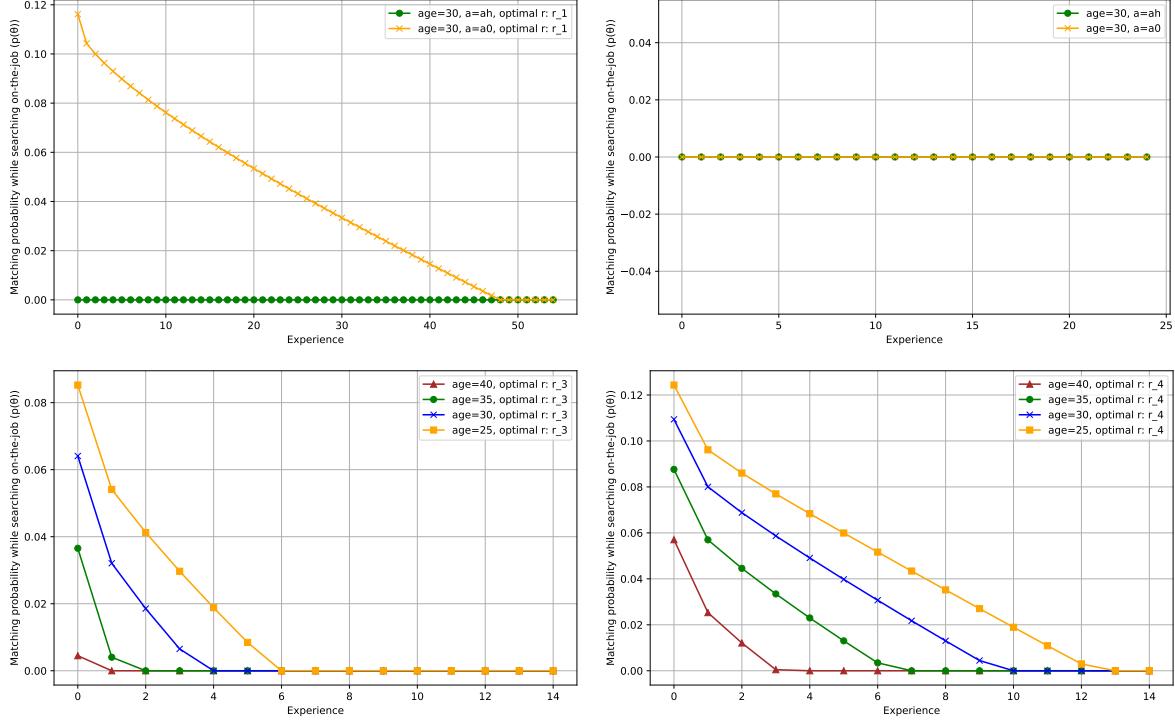
4.5 Learning Frictions for Low-educated Workers

Learning frictions can be potentially highly impactful on the workers' optimal choice. To see this, the top-left panel of Figure 10 illustrates the optimal choice of low-educated workers (E^L) employed in jobs with complexity r_2 . When ability is revealed to be high ($a_{ij} = a^H$), workers cease searching for alternative occupations: despite the possibility of moving to a more complex job (r_3), the risk associated with learning frictions makes it optimal to remain in the current match. Very different is the situation when the ability is unknown ($a_{ij} = a^0$). In this case, workers do search for an alternative occupation and the learning frictions are so heavy for them that they prefer to search for a new occupation with a lower complexity, r_1 , where they are sure to satisfy the skill requirement. In other words, the fact that their innate ability in the current occupation is unknown (and there is a high probability, around 60%, that $a_{ij} = a^L$) makes it optimal to search for jobs in the lowest complexity segment of the labor market and specialize in one of them. As they get more experienced, conditional on not having found another job due to search frictions, they have a higher opportunity cost of leaving their current occupation and, so, they search in submarkets where the employment contract value, x , is higher and the probability of matching, $p(\theta)$, is lower.

4.6 Learning Frictions for College Graduates

One may be concerned that a similar process also holds for college graduates employed in jobs with complexity r_3 when ability is unknown. However, Figure 10 (top-right

Figure 10: Matching probability while searching on the job for different worker types and job complexities.



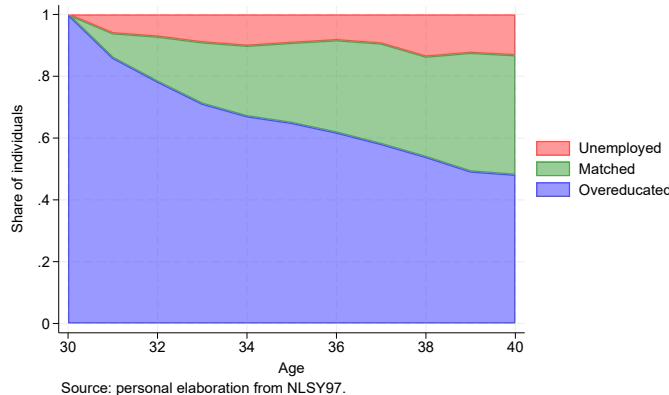
Note: Each panel reports the optimal on-the-job matching probability $p(\theta)$ on the y-axis and occupation-specific experience on the x-axis for a specific worker type and job complexity. The optimal r reported in the legend indicates the complexity level of the occupation where the worker is searching for a job while employed. Top-left: workers with no college degree (E^L) employed in jobs of complexity r_2 , aged 30, known high innate ability a^H vs unknown ability a^0 . Top-right: workers with a college degree (applies to both humanities and non-humanities) employed in jobs of complexity r_3 , aged 30, known high innate ability a^H vs unknown ability a^0 . Bottom-left: college graduates in humanities employed in jobs of complexity r_2 , different ages. Bottom-right: college graduates in non-humanities employed in jobs of complexity r_2 , different ages.

panel) shows that this is not the case. While employed in these kinds of jobs, college graduates do not search for alternative occupations, but they prefer to wait until they discover their ability level in the current occupation. The main difference with non-college graduates lies in the belief of being high-ability; this is much higher both for the humanities graduates (66.9%) and for non-humanities graduates (73.4%) compared to non-college graduates (41.1%). Therefore, the expected value of waiting and having the possibility of staying employed in a more complex occupation is high enough to outweigh the potential loss given by the possibility of being low-ability in the current occupation and restarting from zero in another occupation. Notice that in theory, non-humanities graduates could search for a more complex occupation in r_4 , but the potential gain is dominated by the job-specific experience loss, so that they prefer to stay in the current occupation.

4.7 College Graduates Optimal Choice in Lower Complexity Jobs

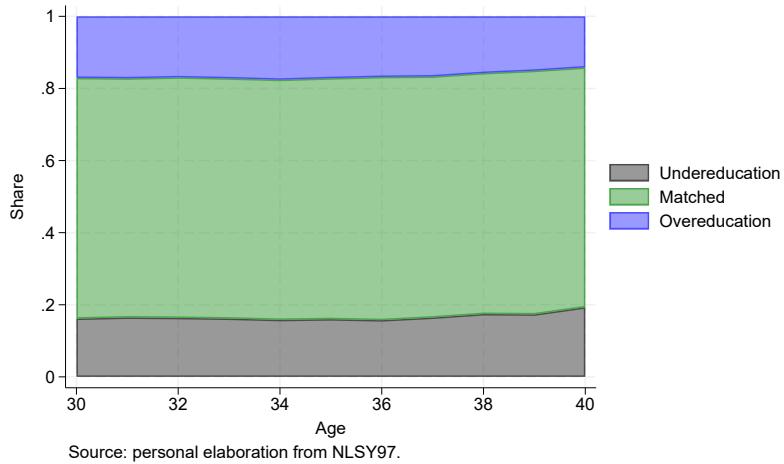
In this section, I discuss the optimal behaviour of college graduates in relatively low complexity jobs r_2 , the complexity level with the most overeducated workers. Figure 10 (bottom panels) shows their optimal matching probability, $p(\theta)$, when searching on-the-job. Both experience and age play an important role in determining the optimal choice of workers. In particular, for 30-year-old humanities graduates, it is optimal to search for an alternative occupation in complexity r_3 only within the first 6 months of employment in the current occupation. After this threshold, the "specialization" effect keeps workers attached to their current job (probably overeducated). The same pattern is even stronger for older workers, who have a shorter time horizon to enjoy the risky choice of moving to another occupation (if $a_{ij} = a^H$ in the new occupation) or to mitigate the loss of an eventually bad choice (if $a_{ij} = a^L$ in the new occupation). The same argument applies to non-humanities college graduates, but they search for a longer period, up to 12 months for 30-year-old workers, and with more intensity. This is the case because they have a higher probability of being high-ability in the new occupation, and they can search for more complex jobs (r_4) in which they are slightly more productive.

Figure 11: Evolution of the status of overeducated workers at age 30 (NLSY97).



Note: This figure is the analogue of Figure 1 for the younger cohort and follows all the individuals in the sample who are classified as *overeducated* at age 30 (NLSY97). The x-axis reports the age starting from 30; the y-axis shows the fraction of the original subsample (normalized to 100%) in each subsequent status. Series shown are: (i) **remaining overeducated** (blue area), that is the share still employed in occupations whose typical educational requirement (from O*NET) is lower than the worker's attainment; (ii) **transitioned to matched** (green area), that is the share employed in occupations whose typical requirement matches the worker's education; (iii) **transitioned to unemployment** (red area), that is the share currently unemployed. Overeducation is defined as having attained a higher education level than the occupation's typical requirement. See Section 2.1 for sample restrictions and further details. See Section 2.3 for details on the overeducation definition.

Figure 12: Share of workers in each status by age (NLSY97).



Note: This figure is the analogue of Figure 3 (right panel) for the younger cohort (NLSY97) and shows the share of workers in each status by age. See Section 2.1 for sample restrictions and further details. See Section 2.3 for details on the overeducation/matched/undereducation definitions.

5 Comparison with Different Cohort (NLSY97)

To assess the robustness of the results, I compare the baseline findings from the NLSY79 (older cohort) with those obtained using the NLSY97 (younger cohort). More specifically, the NLSY97 cohort refers to individuals born between 1980 and 1984.

Given that this newer cohort is very recent, individuals are around 40 years old at most, so only results about the first half of the lifecycle can be discussed. However, this is probably the most relevant part of the analysis, as it is more challenging to break down the potential inefficiencies due to overeducation as workers age. First, Figure 11 is the analogue of Figure 1 for the younger cohort. The pattern appears similar, although the persistence is slightly more pronounced. Indeed, after 5 years, more than 60% of overeducated workers at 30 are still experiencing the same condition (it was around 50% in NLSY79). After 10 years, almost 50% are still classified as overeducated (40% in NLSY79).

Second, the overeducation status in Figure 12 is very similar as well, showing both the share of overeducated and undereducated around 18% at 30 years old, followed by a little decrease (increase) in the share of overeducated (undereducated) before reaching 40 years old.

Finally, Figure 13 shows the decomposition of overeducation persistence. The key result is the same as in the baseline dataset: in the first half of the career, specialization is the most important driver leading to overeducation persistence. Here, the temporary channel is even less relevant, suggesting that overeducation is becoming more persistent.

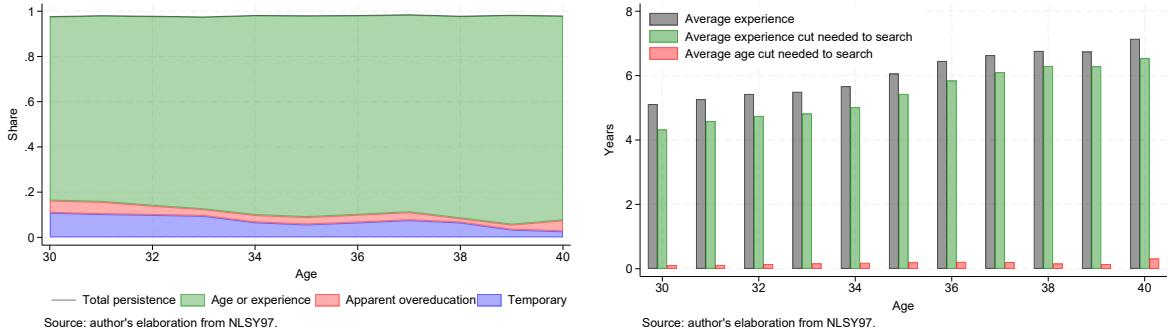


Figure 13: Decomposition of monthly overeducation persistence (NLSY97).

Note: This figure is the analogue of Figure 9 for the younger cohort (NLSY97) and reports the decomposition of monthly overeducation persistence into the following channels: (i) **temporary** (search frictions), (ii) **apparent overeducation**, (iii) **specialization** (job-specific experience), and (iv) **age** (skill depreciation and a shorter time horizon). The left panel shows the lifecycle evolution of each channel's contribution, with age and specialization grouped together; the right panel isolates the joint roles of age and experience by showing the smallest reduction in age and experience needed to make it optimal for the worker to search for another occupation. Results are computed from the model calibrated to the NLSY79 and applied to the NLSY97 cohort. See Section 4 for details on the calibration strategy.

Moreover, age and apparent overeducation also have a minor role in this case, at least in the first half of the lifecycle.

Overall, the comparison confirms that the key mechanisms identified in the model are robust across cohorts.

6 Conclusion

This paper has examined the persistence of overeducation over the lifecycle through the lens of a directed search model with heterogeneous workers and occupations. Using U.S. data from the NLSY79 and NLSY97, the model successfully replicates the empirical patterns of overeducation persistence and provides a structural decomposition of its underlying mechanisms.

Three main insights emerge. First, labor market frictions explain why many workers remain temporarily overeducated early in their careers, but their role fades with age. Second, specialization, driven by the accumulation of occupation-specific experience, emerges as the dominant force behind long-run persistence. Third, the interaction between specialization and age further amplifies this persistence in later stages of the lifecycle.

These findings highlight that overeducation is not merely a transient phenomenon but can result from structural mechanisms that lock workers into mismatched jobs. Policy implications follow directly from the mechanisms identified. First, because search

and especially learning frictions make early mismatches persistent as workers accumulate non-transferable job experience, policies that accelerate learning and improve early-career matching can materially reduce long-run overeducation. Second, incentives that lower the individual cost of moving to more complex occupations help undo specialization-driven traps. Finally, one may consider of reducing the extent of the mismatch in the first place when entering the labor market, for example through policies that aims at reducing search frictions and pushing individuals towards jobs more aligned with their education right after graduation. Overall, a lifecycle perspective is essential for understanding and addressing the persistence of overeducation.

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Appendix

A Proof of Block Recursive Equilibrium

The steps are similar to the proof of Theorem 1 in Menzio et al. (2016).

Let's proceed by backward induction. In the last period T , the value of unemployment is

$$U_T(\omega, \psi) = b$$

The joint value of the match is

$$V_T(\omega, \psi) = \begin{cases} y(r, e, s, t) & \text{if } a \neq a^0 \\ \mathbb{E}[y(r, e, s, t)|E] & \text{if } a = a^0 \end{cases}$$

Notice that both U_T and V_T do not depend on the state ψ , hence $U_T(\omega, \psi) = U_T(\omega)$ and $V_T(\omega, \psi) = V_T(\omega)$.

The market tightness must satisfy

$$\theta_T(\omega, x, \psi) = \begin{cases} q^{-1}\left(\frac{k}{V_T(\omega)-x}\right) & \text{if } x \leq V_T(\omega) - k \\ 0 & \text{otherwise} \end{cases}$$

Notice that $\theta_T(\omega, x, \psi) = \theta_T(\omega, x)$.

The problem of the unemployed is

$$R_T(\omega, \psi) = \max \left\{ \max_x \{p(\theta_T(\omega, x))[x - U_T(\omega)]\}, \max_{x,r} \{p(\theta_T(\omega_0, x))[x - U_{t+1}(\omega)]\} \right\}$$

Let's start from the second term, notice that $x = V_T(\omega) - \frac{k}{q(\theta_T(\omega, x))}$ whenever $x \leq V_T(\omega) - k$ and $\theta_T(\omega, x) = 0$ otherwise. So the second term can be rewritten as

$$\begin{aligned} & \max_{x,r} \{p(\theta_T(\omega_0, x'))[V_T(\omega_0) - \frac{k}{q(\theta_T(\omega_0, x))} - U_T(\omega)]\} = \\ &= \max_{x,r} \{-k\theta_T(\omega_0, x) + p(\theta_T(\omega_0, x))[V_T(\omega_0) - U_T(\omega)]\} \end{aligned}$$

Now, fixing r we can compute for each $\bar{r} \in \{r_1, r_2, r_3, r_4\}$

$$\begin{aligned} & \max_x \{-k\theta_T(\omega_0(\bar{r}), x) + p(\theta_T(\omega_0(\bar{r}), x))[V_T(\omega_0(\bar{r})) - U_T(\omega)]\} = \\ &= \max_\theta \{-k\theta + p(\theta)[V_T(\omega_0(\bar{r})) - U_T(\omega)]\} \end{aligned}$$

The maximizer of this problem gives the optimal choice for complexity \bar{r} , let's denote this maximizer as $\theta_{\bar{r}}^*$. Once we have computed this for each complexity level, we can compare the value function for each of the 4 maximizers to obtain the optimal choice for the second term (optimal new occupation), let's denote it as θ_{new}^* . Similarly, the first term can be rewritten as

$$\begin{aligned} & \max_x \{p(\theta_T(\omega, x))[V_T(\omega) - \frac{k}{q(\theta_T(\omega, x))} - U_T(\omega)]\} = \\ &= \max_x \{-k\theta_T(\omega, x) + p(\theta_T(\omega, x))[V_T(\omega) - U_T(\omega)]\} = \\ &= \max_\theta \{-k\theta + p(\theta)[V_T(\omega) - U_T(\omega)]\} \end{aligned}$$

Let's denote the solution of this problem as θ_{old}^* . The policy function of the unemployed is $\theta_T^u = \theta_{old}^*$ if the optimal first term is better than the optimal second term and $\theta_T^u = \theta_{old}^*$ otherwise. Notice that $\theta_T^u(\omega, \psi) = \theta_T^u(\omega)$ and $R_T(\omega, \psi) = R_T(\omega)$. Also, we denote as $r_T^u(\omega), e_T^u(\omega), a_T^u(\omega)$ the policy functions for the complexity, experience and ability associated with $\theta_T^u(\omega)$. It follows

$$x_T^u(\omega) = V_T(E, r_T^u, exp_T^u, a_T^u) - \frac{k}{q(\theta_T^u(\omega))}$$

A similar reasoning holds for the search problem of the employed

$$S_T(\omega, \psi) = \max_{x,r} \{p(\theta_T(\omega_0, x, \psi))[x - \mathbb{E}_{\hat{a}|a} V_T(\omega)]\}$$

Following the same steps for the second term of the unemployed problem above, it is possible to compute $\theta_T^e(\omega, \psi) = \theta_T^e(\omega)$, $S_T(\omega, \psi) = S_T(\omega)$, $r_T^e(\omega)$ and

$$x_T^e(\omega) = V_T(E, 0, r_T^e, a^0) - \frac{k}{q(\theta_T^e(\omega))}$$

The employment policy function d_T must solve

$$\max_{d \in [\delta, 1]} \{dU_T(\omega) + (1-d)[V_T(\omega) + \lambda_e S_T(\omega)]\}$$

Clearly $d_T = 1$ if $U_T(\omega) > [V_T(\omega) + \lambda_e S_T(\omega)]$ and $d_T = \delta$ otherwise. Also, $d_T(\omega, \psi) = d_T(\omega)$.

We can move to period $T-1$, where we have

$$\begin{aligned} U_{T-1}(\omega, \psi) &= b + \beta[U_T(\hat{\omega}) + \lambda_u R_T(\hat{\omega})] \\ V_{T-1}(\omega, \psi) &= y(r, e, s, t) + \beta\{d_T(\omega)U_T(\hat{\omega}) + (1-d_T(\omega))[\mathbb{E}_{\hat{a}|a}V_T(\hat{\omega}) + \lambda_e S_T(\hat{\omega})]\} \\ \theta_{T-1}(\omega, x, \psi) &= \begin{cases} q^{-1}\left(\frac{k}{V_{T-1}(\omega)-x}\right) & \text{if } x \leq V_{T-1}(\omega) - k \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Also in this case, the value functions do not depend on the state ψ . By repeating the same steps as above for all periods, we get the unique BRE.