Zonal (nodal) price computation

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Zonal market model

$$\max\ SW = \sum_{j=1}^{NL} p_{b,j}Q_{b,j} - \sum_{m=1}^{NG} p_{s,m}Q_{s,m} = \min(-SW) = min \sum_{m=1}^{NG} p_{s,m}Q_{s,m} - \sum_{j=1}^{NL} p_{b,j}Q_{b,j}$$

s.t.

$$\sum_{m=1}^{NG} Q_{s,m} - \sum_{j=1}^{NL} Q_{b,j} = 0$$

$$\underline{TR_k} \leq TR_k \leq \overline{TR_k}$$

$$TR_k = \sum_{n=1}^{NA} PTDF_{k,n} y_n$$

where

$$y_n = \sum_{i \in n} Q_{s,i} - \sum_{i \in n} Q_{b,i}$$

Lagrangian function

$$L = \sum_{m=1}^{NG} p_{s,m} Q_{s,m} - \sum_{j=1}^{NL} p_{b,j} Q_{b,j} -$$

$$+\lambda\left(\sum_{m=1}^{NG} Q_{s,m} - \sum_{j=1}^{NL} Q_{b,j}\right) -$$

$$+\sum_{k=1}^{NT} \underline{\mu_k}\left(TR_k - \underline{TR_k}\right) -$$

$$+\sum_{k=1}^{NT} \overline{\mu_k}\left(\overline{TR_k} - TR_k\right)$$

$$\underline{\mu_k} \geq 0$$

$$\overline{\mu_k} \geq 0$$

Marginal prince in the area k

- If the SW is maximised, the marginal price is the given by the variation of the OF due to a small withdrew energy in area k
- Therefore, the price is given by the derivative of the OF w.r.t. a withdrew of energy in area k, or, given by the derivative of the OF w.r.t an injection in area k where the sign is changed

$$p_k = \frac{\partial SW}{\partial Q_{b,k}} = -\frac{\partial SW}{\partial Q_{s,k}}$$

- But we want to minimize the –SW, therefore, the price is:

$$p_k = \frac{\partial(-SW)}{\partial Q_{s,k}}$$

Price in area k

- In the optimal solution point, the partial derivative of the OF=-SW w.r.t. the control variables are the following:

$$\frac{\partial L}{\partial Q_{s,k}} = \frac{\partial(-SW)}{\partial Q_{s,k}} - \lambda - \sum_{m=1}^{NT} \underline{\mu_m} \frac{\partial TR_m}{\partial Q_{s,k}} + \sum_{m=1}^{NT} \overline{\mu_m} \frac{\partial TR_m}{\partial Q_{s,k}} = 0$$

Therefore:

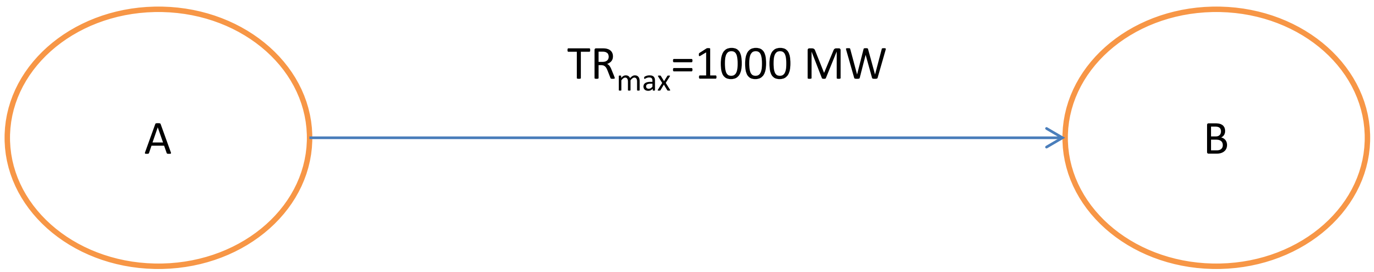
$$p_k - \lambda - \sum_{m=1}^{NT} \underline{\mu_m} \frac{\partial TR_m}{\partial Q_{s,k}} + \sum_{k=1}^{NT} \overline{\mu_m} \frac{\partial TR_m}{\partial Q_{s,k}} = 0$$

$$p_k = \lambda + \sum_{m=1}^{NT} \underline{\mu_m} \frac{\partial TR_m}{\partial Q_{s,k}} - \sum_{k=1}^{NT} \overline{\mu_m} \frac{\partial TR_m}{\partial Q_{s,k}}$$

Comments

- In case of no active transmission constraints, the Lagrangian multipliers associated to the non-equality constraints are equal to zero
  - The marginal price is equal to the Lagrangian multiplier associated to the equality constraint (balance constraint)
- Only one marginal price for the market
- Spatial price differentiation only if a transmission constraints is active, therefore in presence of transmission congestions

Example: price in area B



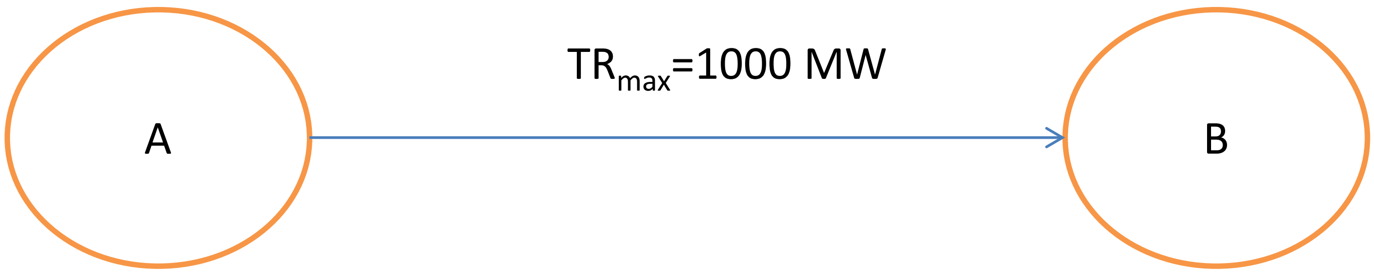
- We suppose the transit is equal to the maximum value (the transmission constraint is active)
- The power flows from area A to area B
- We assume that the area B is the slack bus of the system
  - $PTDF_a$=1
  - $PTDF_b$=0

$$TR = PTDF_a y_a + PTDF_b y_b = PTDF_a y_a$$

- The price in area B is given by the following equation:

$$p_b = \lambda - \overline{\mu}\frac{\partial TR}{\partial Q_{s,B}} = \lambda$$

Example: price in area A



- $\overline{\mu} > 0$
- $\frac{\partial TR}{\partial Q_{s,A}} > 0$ because an injection in area A increments the power on the interconnection line
- $p_a = \lambda - \overline{\mu}\frac{\partial TR}{\partial Q_{s,A}} < \lambda = p_b$

- The price in area A in lower than the price in area B
- The prince in exporting area are lower than the price in importing area

The impact of losses

$$\min \; -SW = \sum_{m=1}^{NG} p_{s,m} Q_{s,m} - \sum_{j=1}^{NL} p_{b,j} Q_{b,j}$$

s.t.

$$\sum_{m=1}^{NG} Q_{s,m} - \sum_{j=1}^{NL} Q_{b,j} - Pp\left(Q_{b,1}, Q_{b,2}, \dots, Q_{b,NL}, Q_{s,1}, Q_{s,2}, \dots, Q_{s,NG-1}\right) = 0$$

$$\underline{TR_k} \le TR_k \le \overline{TR_k}$$

$$TR_k = \sum_{n=1}^{NA} PTDF_{k,n} y_n$$

dove

$$y_n = \sum_{i \in n} Q_{s,i} - \sum_{i \in n} Q_{b,i}$$

$$L = \sum_{j=1}^{NL} p_{a,j} Q_{a,j} - \sum_{m=1}^{NG} p_{v,m} Q_{v,m} -$$

$$+\lambda \left(\sum_{m=1}^{NG} Q_{s,m} - \sum_{j=1}^{NL} Q_{b,j} \right.$$

$$\left. - Pp\left(Q_{b,1}, Q_{b,2}, \dots, Q_{b,NL}, Q_{s,1}, Q_{s,2}, \dots, Q_{s,NG-1}\right) \right) -$$

$$+ \sum_{k=1}^{NT} \underline{\mu_k} \left(TR_k - \underline{TR_k} \right) -$$

$$+ \sum_{k=1}^{NT} \overline{\mu_k} \left(\overline{TR_k} TR_k \right)$$

$$\underline{\mu_k} \geq 0$$

$$\overline{\mu_k} \geq 0$$

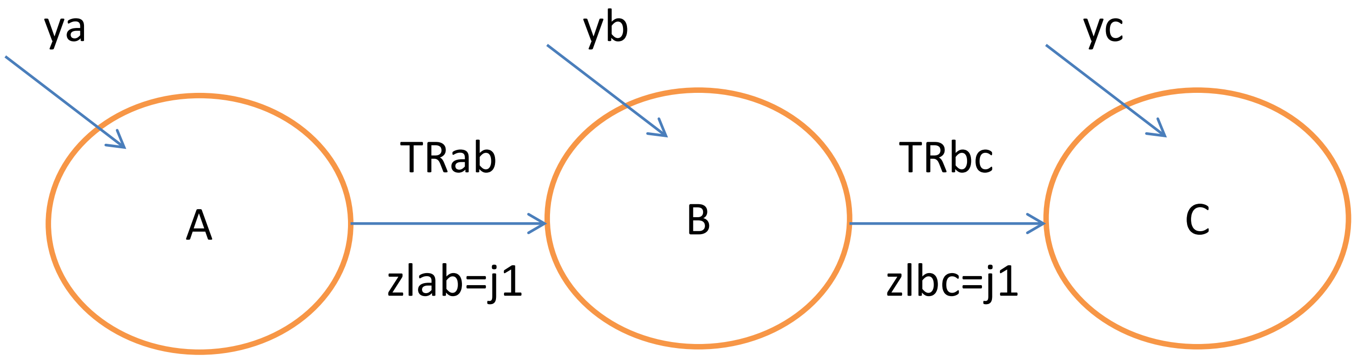
$$\frac{\partial L}{\partial Q_{s,k}} = \frac{\partial SC}{\partial Q_{s,k}} - \lambda\left(1 - \frac{\partial P_p}{\partial Q_{s,k}}\right) - \sum_{m=1}^{NT} \underline{\mu_m} \frac{\partial TR_m}{\partial Q_{s,k}} + \sum_{m=1}^{NT} \overline{\mu_m} \frac{\partial TR_m}{\partial Q_{s,k}}$$
$$= 0$$

Da cui

$$p_k = \lambda(1 - \beta) + \sum_{m=1}^{NT} \underline{\mu_m} \frac{\partial TR_m}{\partial Q_{s,k}} - \sum_{k=1}^{NT} \overline{\mu_m} \frac{\partial TR_m}{\partial Q_{s,k}}$$

PTDF computation

- PTDF: power transfer distribution factor



- We have to define a slack area (the final results does not depend on this choose)
- We adopt a DC power flow model
- $\delta c=0$

$$yl_{ab} = |1/zl_{ab}| = |-j1| = 1$$

$$TR_{ab} = yl_{ab}(\delta_a - \delta_b) = \delta_a - \delta_b$$

$$yl_{bc} = |1/zl_{bc}| = |-j1| = 1$$

$$TR_{bc} = yl_{bc}(\delta_b - \delta_c) = yl_{bc}(\delta_b) = \delta_b$$

y_a net injection in area a

y_b net injection in area b

y_c net injection in area c

Y=$B = \begin{bmatrix} yl_{ab} & -yl_{ab} \\ -yl_{ab} & yl_{ab} + yl_{bc} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ $\quad \delta = \begin{bmatrix} \delta_a \\ \delta_b \end{bmatrix}$ $\quad y = \begin{bmatrix} y_a \\ y_b \end{bmatrix}$

$$y_a = TR_{ab} = yl_{ab}(\delta_a - \delta_b) = yl_{ab}\delta_a - yl_{ab}\delta_b = \delta_a - \delta_b$$

$$y_b = TR_{bc} - TR_{ab} = yl_{bc}(\delta_b) - yl_{ab}(\delta_a - \delta_b) = -yl_{ab}\delta_a + (yl_{ab} + yl_{bc})\delta_b = -\delta_a + 2\delta_b$$

Adopting matrix:

$$y = B\delta$$

$$\delta = \begin{bmatrix} \delta_a \\ \delta_b \end{bmatrix} = B^{-1}y = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y_a \\ y_b \end{bmatrix} = \begin{bmatrix} 2y_{a+}y_b \\ y_{a+}y_b \end{bmatrix}$$

Now it is possible to compute the transits:

$$TR_{ab} = yl_{ab}(\delta_a - \delta_b) = yl_{ab}(2y_a + y_b - y_a - y_b) = yl_{ab}(y_a) = y_a$$

$$TR_{bc} = yl_{bc}(\delta_b) = yl_{bc}(y_a + y_b) = y_a + y_b$$

The derivative of the transits are the following:

$$PTDF_{ab,a} = \frac{\partial TR_{ab}}{\partial y_a} = 1$$

$$PTDF_{ab,b} = \frac{\partial TR_{ab}}{\partial y_b} = 0$$

$$PTDF_{bc,a} = \frac{\partial TR_{bc}}{\partial y_a} = 1$$

$$PTDF_{bc,b} = \frac{\partial TR_{bc}}{\partial y_b} = 1$$

It is possible to generalize the method

$$TR_{ij} = f(\delta_1, \delta_2, \dots, \delta_{N-1})$$

$$\frac{\partial TR_{ij}}{\partial y_k} = \frac{\partial TR_{ij}}{\partial \delta_1}\frac{\partial \delta_1}{\partial y_k} + \frac{\partial TR_{ij}}{\partial \delta_2}\frac{\partial \delta_2}{\partial y_k} + \cdots + \frac{\partial TR_{ij}}{\partial \delta_{N-1}}\frac{\partial \delta_{N-1}}{\partial y_k} = \sum_{j=1}^{N-1} \frac{\partial TR_{ij}}{\partial \delta_j}\frac{\partial \delta_j}{\partial y_k}$$

$\frac{\partial TR_{ij}}{\partial \delta_j}$ Very easy to compute

$\frac{\partial \delta_j}{\partial y_k}$ It is possible to use the sensitivities

$$y = B\delta$$
$$g(x,u) = y_{ass} - y = y_{ass} - B\delta = 0 = P - P_{calc}(\delta) = 0$$

$$x = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_{N-1} \end{bmatrix} \qquad u = \begin{bmatrix} P \\ P_2 \\ \vdots \\ P_{N-1} \end{bmatrix}$$

$$g(x_0 + \Delta x, u_0 + \Delta u) \cong g(x_0, u_0) + \frac{\partial g}{\partial x}\Delta x + \frac{\partial g}{\partial u}\Delta u \cong 0$$

But in the power flow equation solution :

$$g(x_0, u_0) = 0$$

Therefore:

$$g(x_0 + \Delta x, u_0 + \Delta u) \cong \frac{\partial g}{\partial x}\Delta x + \frac{\partial g}{\partial u}\Delta u = J_x \Delta x + J_u \Delta u \cong 0$$

$$\frac{\Delta x}{\Delta u} = -J_x^{-1} J_u$$

Example for a tree system busses

J_u Diagonal matrix of dimension (N-1)x(N-1)

$$J_u = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\partial g_a}{\partial P_a} & \frac{\partial g_a}{\partial P_b} \\ \frac{\partial g_b}{\partial P_a} & \frac{\partial g_b}{\partial P_b} \end{bmatrix}$$

$$\frac{\partial x}{\partial u} = -J_x^{-1} I = -J_x^{-1}$$

$$J_x = -B = \begin{bmatrix} -1 & +1 \\ +1 & -2 \end{bmatrix} = \begin{bmatrix} \frac{\partial g_a}{\partial \delta_a} & \frac{\partial g_a}{\partial \delta_b} \\ \frac{\partial g_b}{\partial \delta_a} & \frac{\partial g_b}{\partial \delta_b} \end{bmatrix}$$

$$\rightarrow J_x^{-1} = \begin{bmatrix} -2 & -1 \\ -1 & -1 \end{bmatrix} =$$

$$\frac{\partial x}{\partial u} = -J_x^{-1} J_u = -J_x^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\partial \delta_a}{\partial y_a} & \frac{\partial \delta_a}{\partial y_b} \\ \frac{\partial \delta_b}{\partial y_a} & \frac{\partial \delta_b}{\partial y_b} \end{bmatrix}$$

$$TR_{ab} = yl_{ab}(\delta_a - \delta_b)$$

$$TR_{bc} = yl_{bc}(\delta_b - \delta_c) = yl_{bc}(\delta_b)$$

$$\frac{\partial TR_{ab}}{\partial y_a} = \frac{\partial TR_{ab}}{\partial \delta_a}\frac{\partial \delta_a}{\partial y_a} + \frac{\partial TR_{ab}}{\partial \delta_b}\frac{\partial \delta_b}{\partial y_a} = (1)(2) + (-1)(1) = 1$$

$$\frac{\partial TR_{ab}}{\partial y_b} = \frac{\partial TR_{ab}}{\partial \delta_a}\frac{\partial \delta_a}{\partial y_b} + \frac{\partial TR_{ab}}{\partial \delta_b}\frac{\partial \delta_b}{\partial y_b} = (1)(1) + (-1)(1) = 0$$

$$\frac{\partial TR_{bc}}{\partial y_a} = \frac{\partial TR_{bc}}{\partial \delta_a}\frac{\partial \delta_a}{\partial y_a} + \frac{\partial TR_{bc}}{\partial \delta_b}\frac{\partial \delta_b}{\partial y_a} = (0)(2) + (1)(1) = 1$$

$$\frac{\partial TR_{bc}}{\partial y_b} = \frac{\partial TR_{bc}}{\partial \delta_b}\frac{\partial \delta_a}{\partial y_b} + \frac{\partial TR_{bc}}{\partial \delta_b}\frac{\partial \delta_b}{\partial y_b} = (0)(1) + (1)(1) = 1$$