Locational Marginal Pricing

When LMP is used

- The LMPs are computed for each node and market period in both the forward and balancing energy markets
- The forward energy market is typically for a period one day ahead, commonly called a day-ahead market
- The LMPs are utilized in the market settlement processes to determine generator payments and load charges by multiplying the amount of energy produced or consumed at a node by the LMP at that node.
- LMPs are used in ancillary service calculations and to price transmission and manage congestion and therefore the understanding of their calculation is crucial in North American power system economic analyses.

The short-run marginal cost at a node

- The short-run marginal cost at a node is equal to the change in optimal economic cost associated with supplying an additional increment of load at that node.
- This value is readily available from the Lagrangian multiplier associated with the nodal real power balance equation at the solution to the N-node Optimal Power Flow (OPF) problem:

$$\min \{ f(\boldsymbol{P}_{\mathrm{G}}, \boldsymbol{P}_{\mathrm{D}}) \}$$

where

$$f(\boldsymbol{P}_{\mathrm{G}}, \boldsymbol{P}_{\mathrm{D}}) = \sum_{i=1}^{N} C_{\mathrm{G},i}(P_{\mathrm{G},i}) - \sum_{i=1}^{N} C_{\mathrm{D},i}(P_{\mathrm{D},i})$$

This optimization is subject to:

$$\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{0}$$

$$\boldsymbol{h}(\boldsymbol{x}) \leq \boldsymbol{0}$$

The LMP for a node

Therefore, the LMP for node j is:

$$LMP_j = \lambda^*_{\mathrm{p},j} \tag{5}$$

where $\lambda^*_{\mathrm{p},j}$ is the Lagrangian multiplier at the optimal solution to (1) associated with the real power balance equation at node j. Obtaining the LMP from the OPF implicitly includes voltage, transmission and generation constraints that affect the economics of the delivery of energy in a transparent fashion.

A practical calculation of LMP

- In practical application the simplified DC system model is used in LMP calculations
- The LMP calculation is reflective of the method utilized in the PJM market, which is selected due to its prominence as the world's largest electricity market and its lengthy experience with LMPs
- The most notable influence of this method is the omission of the losses and the utilization of a DC system model
- Losses can be ignored without significant inaccuracy if they are small in value, as in the case of tightly meshed systems indicative of certain regions of North America
- Generation offers and load bids and not costs are utilized in the calculation

Few concepts of optimization theory

- The LMP at a given node can be decomposed into the marginal cost of generation based upon generator offers, congestion costs and marginal loss costs
- The LMP for node j can then be expressed as:

$$LMP_j = \lambda_s - \sum_{k=1}^{K} \mu_k \psi_{k,j}$$

- λ_s is the marginal price of generation at the reference node,
- K is the number of branches, and μ_k is the shadow price of the congestion of branch k.
- The shadow price, μ_k, is the change in cost due to an incremental
- relaxation of the constraint on branch k.
- The Injection Shift Factor (ISF) $\psi_{k,j}$ is the fraction of real power that flows on branch k due to a unit production of power at node j and consumption of power at the reference node under dc assumptions.
- The sum of products of μ_k and $\psi_{k,j}$ is the congestion cost

The OF

- The objective of the incremental optimization in the practical LMP calculation of an N-node system is:

$$\min \left\{ \sum_{i=1}^{N} O_{\mathrm{G},i}(\Delta P_{\mathrm{G},i}) - \sum_{i=1}^{N} O_{\mathrm{D},i}(\Delta P_{\mathrm{D},i}) \right\}$$

- where $\Delta P_{G,i}$ and $\Delta P_{D,i}$ are the changes in real power production and consumption from the operating point, and $O_{G,i}(\Delta P_{G,i})$ and $O_{D,i}(\Delta P_{D,i})$ are the offers and bids associated with the changes, respectively

The constrains

$$\sum_{i=1}^{N} \Delta P_{\mathrm{G},i} - \sum_{i=1}^{N} \Delta P_{\mathrm{D},i} = 0$$

$$\Delta P_{\mathrm{G},j}^{\min} \leq \Delta P_{\mathrm{G},j} \leq \Delta P_{\mathrm{G},j}^{\max} \quad \forall j \in (1, \ldots, N)$$

$$\Delta P_{\mathrm{D},j}^{\min} \leq \Delta P_{\mathrm{D},j} \leq \Delta P_{\mathrm{D},j}^{\max} \quad \forall j \in (1, \ldots, N)$$

$$- F_k^{\max} \leq \sum_{i=1}^{N} \{(\Delta P_{\mathrm{G},i} + P_{\mathrm{G},i})\psi_{k,i}\} -$$

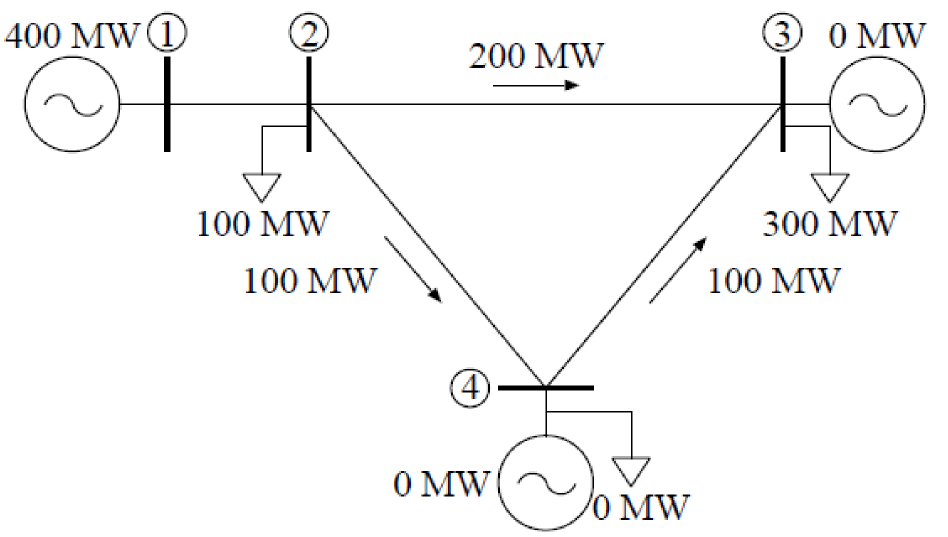
$$\sum_{i=1}^{N} \{(\Delta P_{\mathrm{D},i} + P_{\mathrm{D},i})\psi_{k,i}\} \leq F_k^{\max} \quad \forall k \in (1, \ldots, K)$$

- the superscript max and min refer to the maximum and minimum changes to real power production and consumption at each node, respectively.
- The limitations on branch flow must be obeyed:

A practical example

- The study system is comprised of four buses and four branches with equal reactances of 0.10 p.u.
- The corresponding ISFs for a unit of power produced at each bus with Bus 1 arbitrarily selected as the reference is shown in the following table:

Injection Bus	Branch Terminus Buses			
	1–2	2–3	2–4	4–3
1	0	0	0	0
2	−1	0	0	0
3	−1	−0.667	−0.333	−0.333
4	−1	−0.333	−0.667	0.333

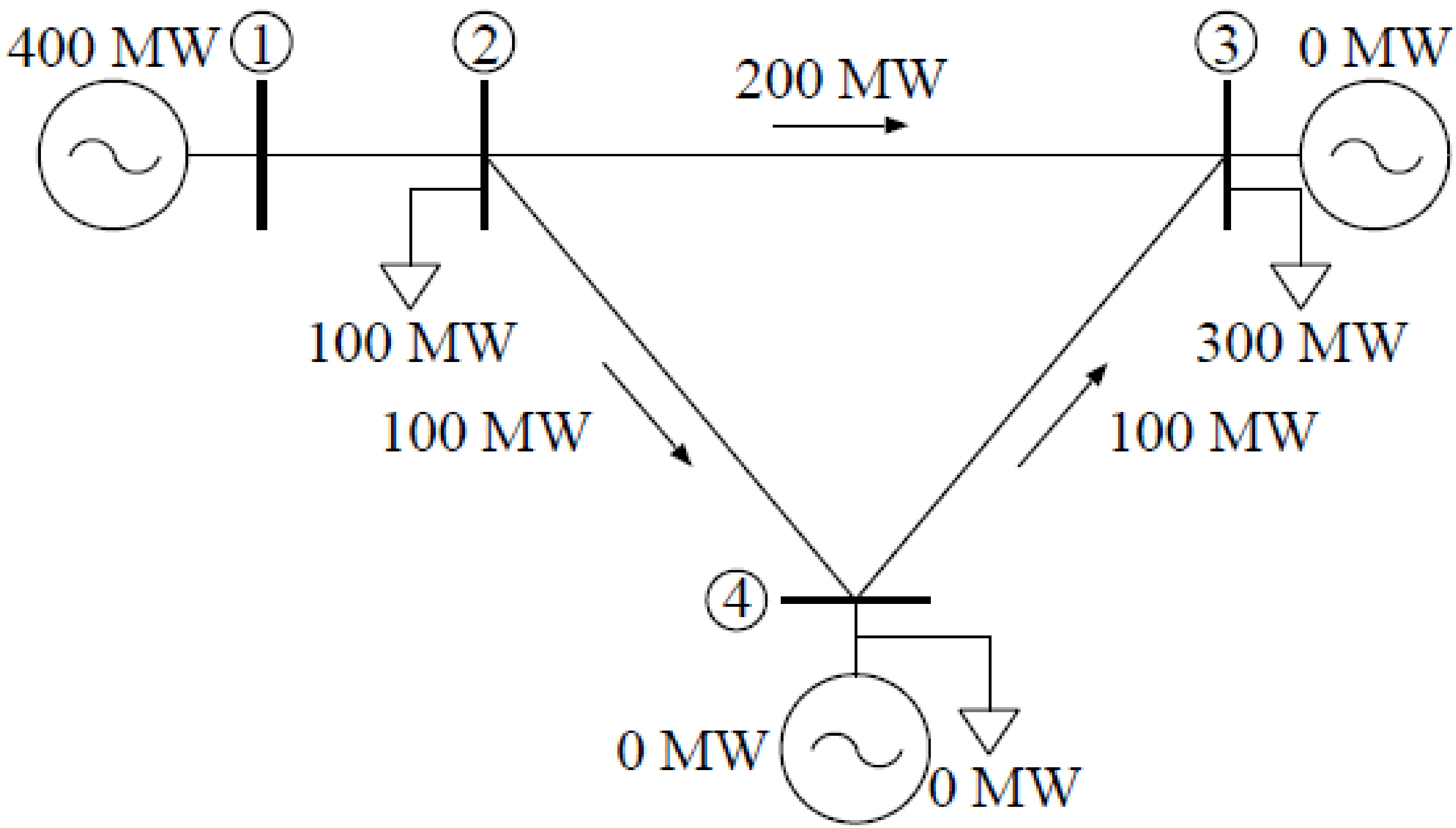


Generation and load data

Gen. Bus	c_1 (\$/MW)	$P^{\max}$ (MW)	$P^{\min}$ (MW)
1	20	500	0
3	25	200	0
4	30	200	0

Total load: 400 MW
The generator located in bus 1 is the most convenient generating unit

Unconstrained market

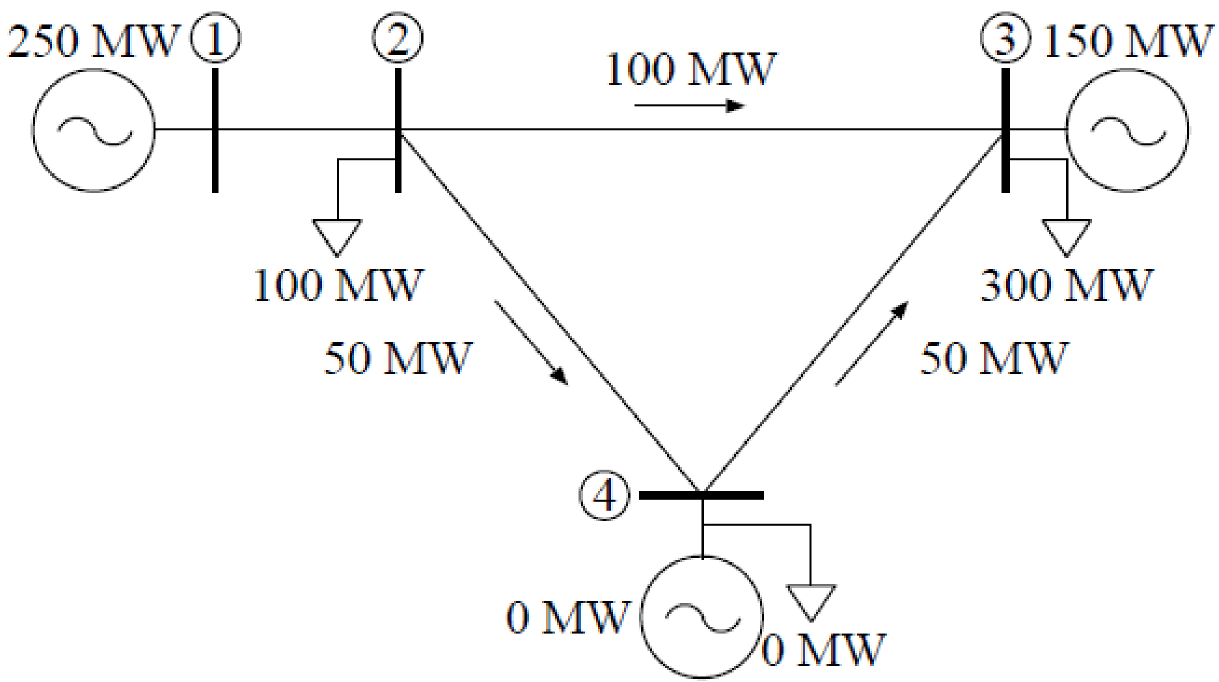


The LMP: **Unconstrained** case

Bus	LMP ($/MWh)	Generation (MW)	Load (MW)	Credit ($)	Charge ($)
1	20	400	–	8,000	0
2	20	–	100	0	2,000
3	20	0	300	0	6,000
4	20	0	0	0	0
Total				8,000	8,000

Constrained market

- The real power flow on branch 4–3 is limited to 50 MW and that the dispatch, load and resulting branch flows of the system are shown in the figure.
- The production cost for this dispatch is $8,750.



The LMPs

Bus	LMP ($/MWh)	Generation (MW)	Load (MW)	Credit ($)	Charge ($)
1	20	250	–	5,000	0
2	20	–	100	0	2,000
3	25	150	300	3,750	7,500
4	15	0	0	0	0
Total				8,750	9,500

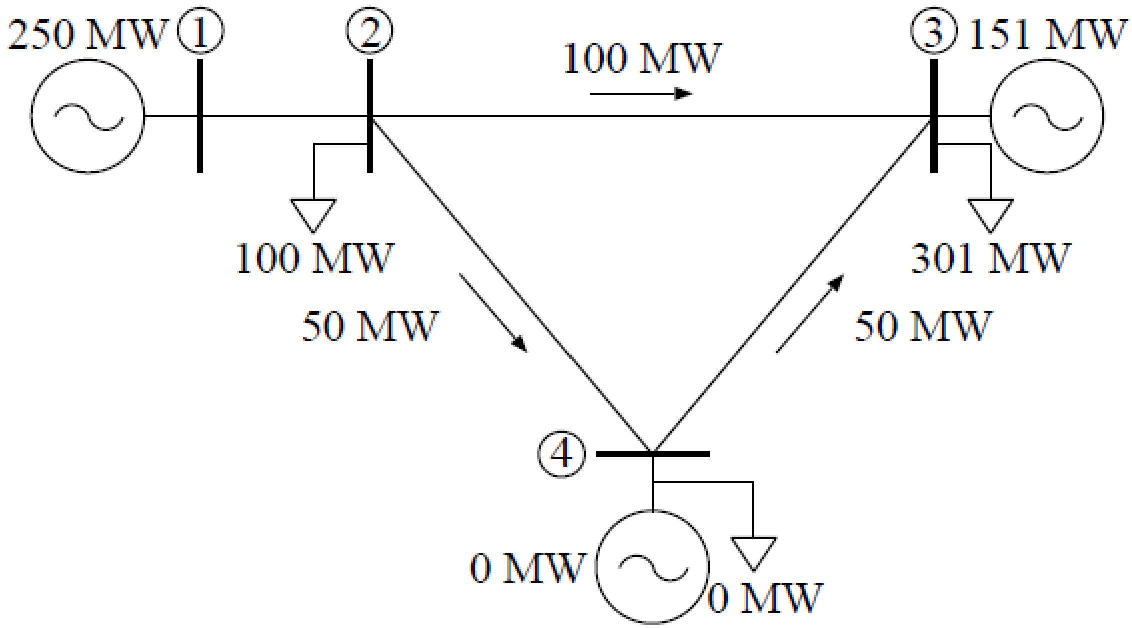
The LMP for buses1 and 2

- The LMP for Bus 1 and 2 are the same as in the unconstrained case as Generator 1 can supply one additional MW of power without violating the branch limit

The LMP for bus 3

- If the load at Bus 3 is increased to 301MW, Generator 1 cannot supply additional power since it would cause the power flow on branch 4–3 to increase past it's limit to 50.333 MW
- Instead, power must come from the next cheapest power source —Generator 3— whose increased power output to 151 MW does not affect the power flow on branch 4–3 if the load is concomitantly increased to 301 MW
- Therefore, the production cost increases to $8,775—an increase of $25, and hence the LMP at Bus 3 becomes $25

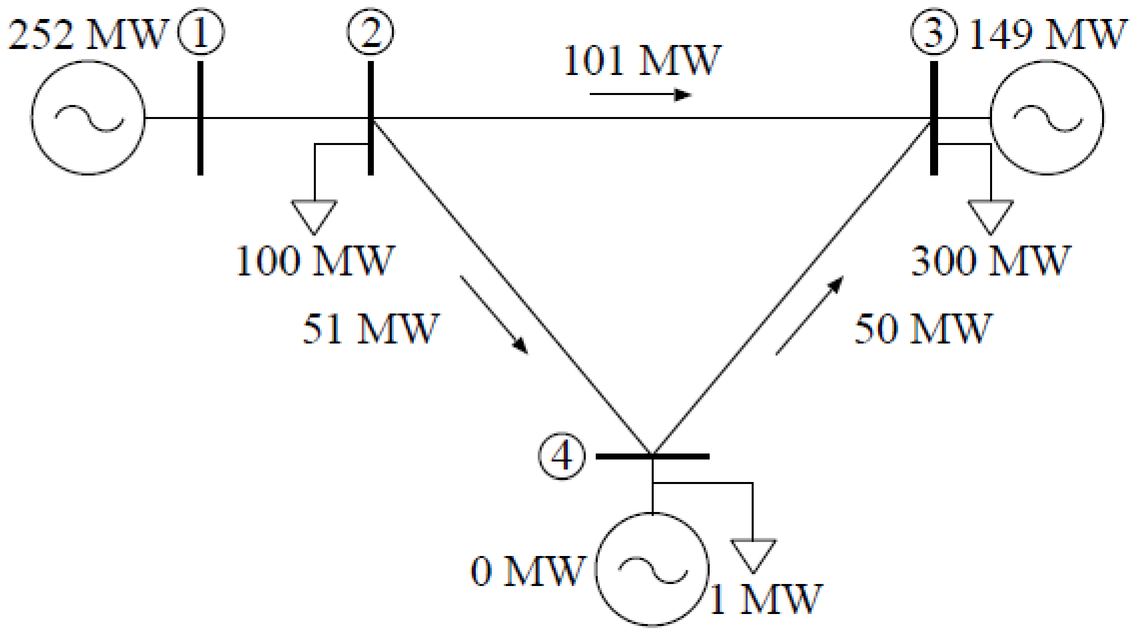
The LMP for bus 3



The LMP for bus 4

- In order to serve an additional MW of load at Bus 4 at the lowest production cost, Generator 1 increases its power output by 2 MW to 252 MW and Generator 3 decreases its power output by 1 MW to 149 MW
- The resulting power flow on branch 4–3 in this case is 50 MW and the net result is an increase in production cost by $15
- Therefore the LMP at Bus 4 is $15/MWh.
- Note that this value is less than the offer of any generator

The LMP for bus 4



Mathematical computation of LMP

- Solution of OP yields λ_s equal to \$ 20 and a non-zero μ_k for branch 4–3 of \$15
- Since the ISFs for Bus 1 and 2 onto branch 4–3 are zero, the LMP is equal to \$20/MWh
- For Bus 3, the ISF is equal to −0.333 so that the LMP becomes \$25/MWh
- In similar fashion, the LMP for Bus 4 is computed to be \$15/MWh
- The results are identical to the approach of increasing the demand by 1 MW at each bus

Location

LMP and the market signals

- Under the LMP methodology, power producers only get paid for deliverable energy, not on the gross power placed onto a power grid
- Location has become a major factor determining the profitability of a power plant
- There is
  - an economic disincentive to build power plants that can not actually deliver power to customers and
  - an incentive to build power lines able to relieve the congestion

The components of LMP in the Standard Market Design of FERC

- LMP=

  Clearing price +

  congestion charge +

  line loss charge

The clearing price is the same everywhere on a power system

Congestion charge and line loss charge are specific to each location

Types of locations in the Standard Market Design of FERC

- Node prices correspond directly to the price of energy at a specific piece of physical hardware (electrical bus where energy enters or leaves the transmission grid). Generators get paid the nodal price of the electricity bus where they deliver energy into the transmission grid

Types of locations in the Standard Market Design of FERC

- Zone prices are the average of all nodal within a limited geographical area
- Usually, the electric buyers pay the zone price for the power they receive

Types of locations in the Standard Market Design of FERC

- Hub prices are an average of selected nodal prices across several zones
- The hub price servers as the benchmark price for a power grid

Financial Transmission Right (FTR)

- FTRs are financial instruments
- Help customers to manage the price risk of having purchased or sold energy at a major hub and then being forced to pay a difference price when they deliver or receive energy at a specific node

Financial Transmission Right (FTR)

- FTRs are tradable contracts made between two parties
  - These parties take opposite sides of an obligation to pay or receive the difference in price between two nodes
  - If there is a congestion, one party will need to pay the other
  - This payment can go either way – either party can end up paying or received cash
- FTR option allows one party to pay an upfront fee (a premium) to avoid paying on a congestion charges
  - Buying an FTR option is like buying insurance against higher price due to congestion

Locational charging

- The requirement for location related charging are:
  - Locational signals to generators
  - Medium-term incentive to build network infrastructure
    - for base-case requirement
    - for variable requirements (capacity and redundancy)
  - Economy treatment of interconnection
  - Cost recovery and optimization of spend by the SO

Model for designation of electrical location

- Postage stamp
- Zonal
- Postage stamp with market splitting
- Nodal

Postage stamp

- All points are equivalent in terms of connection and use of system charging
- Advantages:
  - Simple
  - Suitable for immature electricity markets
- Disadvantages
  - No locational signals into the control area
  - Less efficient locational structure of generation and demand

Zonal model

- Zones are grouping of nodes
- Within a zone all producers and consumers are charged equally
- It is not too hard to understand
- Does not handle very well more local issues such as constraints

Postage stamp with market splitting

- The market spitting method offers a compromise between the purity of LMP and the liquidity offered by the postage stamp
- For the short-term pricing, the zones have the same prices unless there is an active constraint between them

Nodal

- It is an extension of the zonal model
- Each major bus has its own price
- The nodal pricing is the best mechanism from the point of view of an efficient market in which the market power and gaming problems are solved
- The market model is extremely complex and little understandable from the participants

Example

- A method of setting electricity rates where all end-use customers connected to the power grid pay the same price for electricity is known as what?

a) Postage stamp pricing
b) Nodal pricing
c) Locational marginal pricing
d) Zonal pricing

Correct answer: a

- The correct answer is a, postage stamp pricing. In this method, all customers pay the same price for electricity regardless of factors like distance from the generation plant or constraint issues.