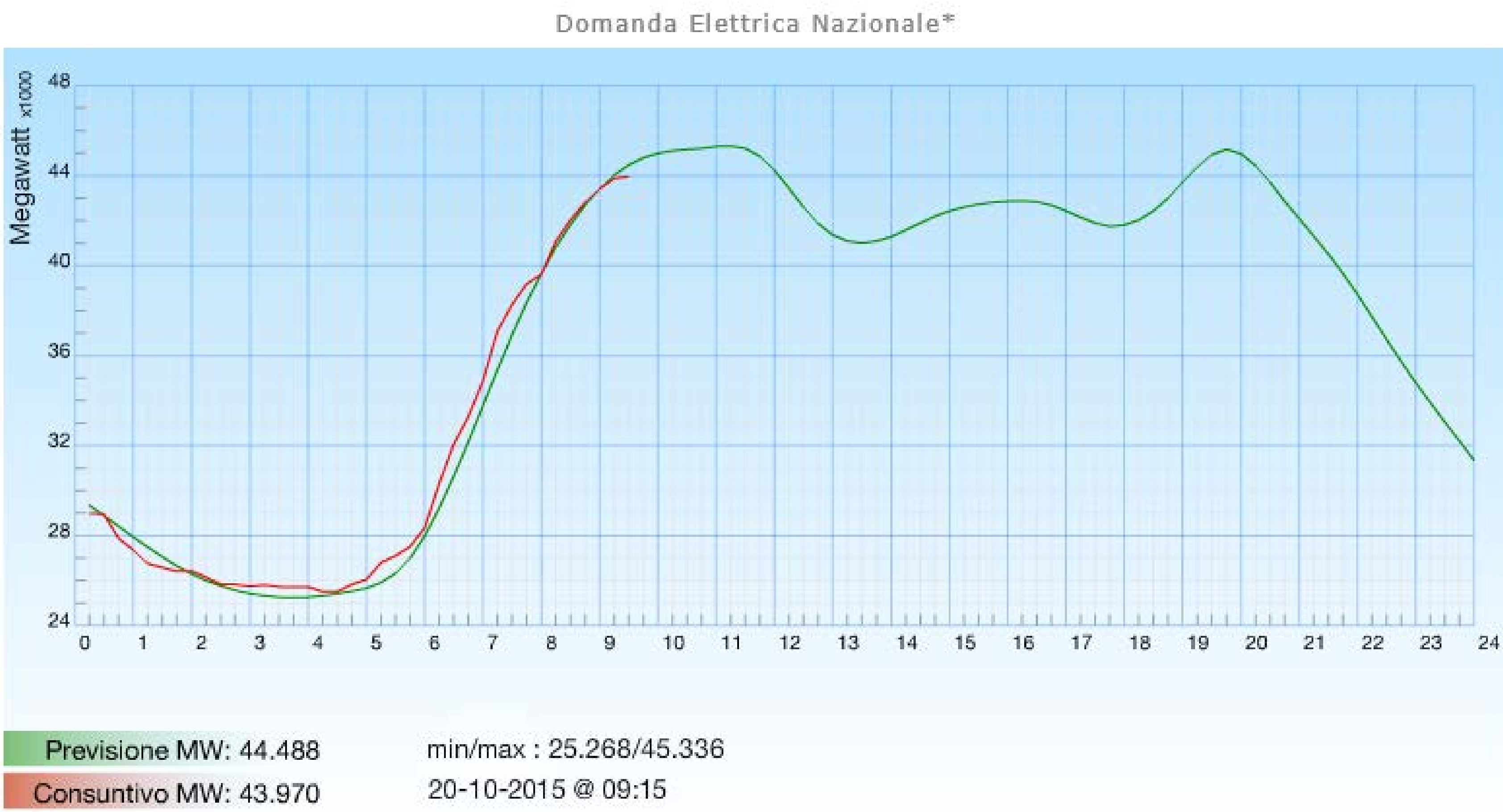
The Unit Commitment problem

Why we need to solve the UC problem

- We assume a generation company that holds many supply contracts
- Generally, the power curve is not constant
  - Therefore, it is necessary to choose the generating unit to put in service for each time interval



Why we need to solve the UC problem

- The generation company wants to maximize its profit therefore the generation production cost has to be minimized
- The UC problem consists in determining the generating units to keep in service for a given time interval minimizing the generation cost production
- It is necessary to determine beforehand the groups to keep in service for the next days (for example 1 week)
  - The start-up time could require many hours
- The generation company has a generation pattern with different generation units characterized by different
  - Start up and shut down costs
  - In service and out of service time
  - Generation costs
  - Technologies (GT, CCGT, Hydroelectric, …)
  - Fuels (nuclear, gas, coal, oil, …)
- Not all generating units are available in the UC problem due to the presence of maintenance programs that we assume known
- The problem:
  - For a given week, we want to choose the groups to keep in service
  - From the cumulative load demand, we know the fifteen minutes energy that has to be produced
  - The generation cost has to be minimized

The start-up cost

- Add or remove from the service a group generates a costs
  - The start-up cost and the shut-down cost
- These costs depend on the way a group is kept out of service
- Banking method: consists in keeping the boiler in conditions of temperature and pressure close to those operating
- Cooling method: is to let the boiler cool down after the decommissioning of the group

The banking method

$$Start - up\ cost = C_t \cdot t \cdot F + C_f$$

where:

- C_t is the hourly consumption fuel to keep the boiler temperature and pressure close to operating conditions
- t time (hours) from the shut-down
- F fuel cost
- C_f fixed cost (personnel and maintenance)

The cooling method

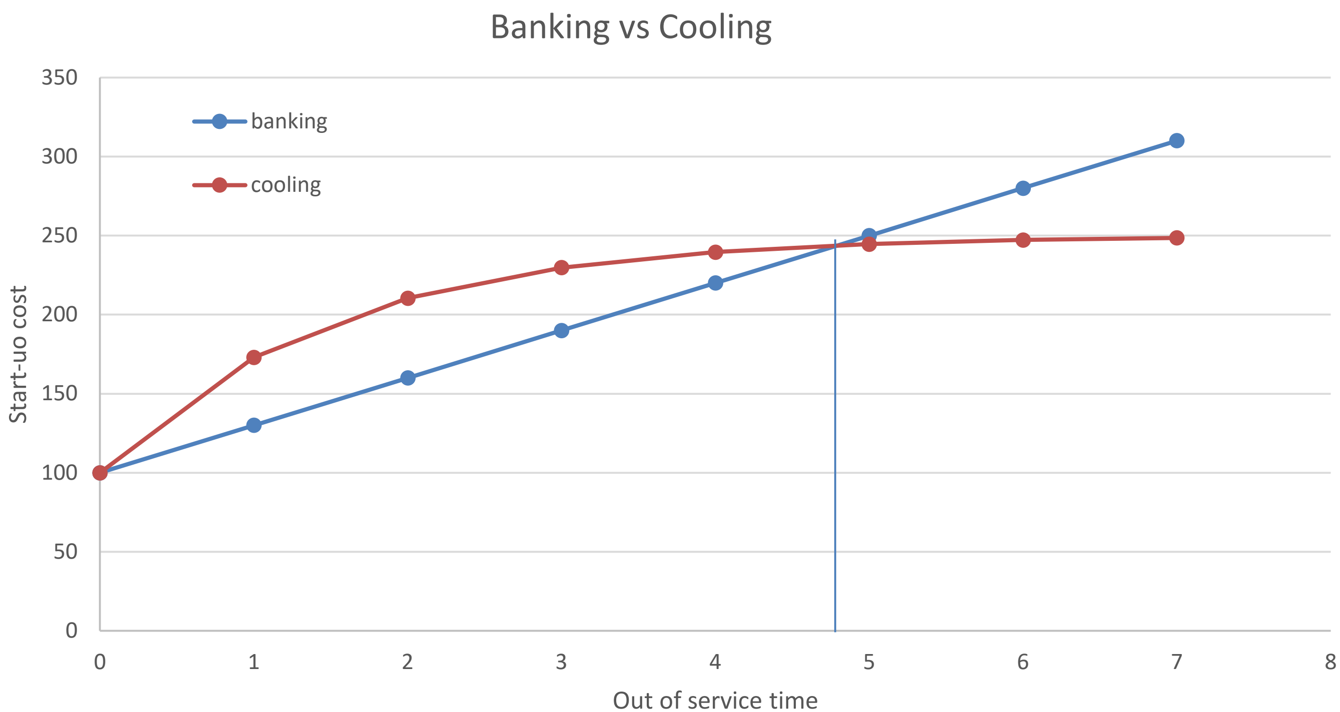
$$Start - up\ cost = C_c \cdot \left(1 - e^{-t/\alpha}\right) \cdot F + C_f$$

where:

- C_c consumption of fuel for cold start
- α thermal time constant of the unit

Comparison between cooling and banking

- The most convenient solution depends on out of service time



UC Mathematical model

$$\min_{Y,P} \sum_{j=1}^{N_I} \left(\sum_{i=1}^{N_T} a_{o,i} \cdot Y_{i,j} + a_{1,i} \cdot P_{i,j} + a_{2,i} \cdot P_{i,j}^2 + C_{su,i}(T_i) \right)$$

s.t.

$$\sum_{i=1}^{N_T} P_{i,j} = C_{tot,j} - C_{h,j} \qquad j = 1, \cdots, N_I$$

$$\sum_{j=2}^{N_0} \left(Y_{i,j} - Y_{i,j-1}\right)^2 \leq 2 \qquad i = 1, \cdots, N_T$$

$$Y_{i,j}\underline{P_i} \leq P_{i,j} \leq Y_{i,j}\overline{P_i} \qquad i = 1, \cdots, N_T \quad \mathrm{j} = 1, \cdots, N_I$$

$$\sum_{i=1}^{N_T} \left(Y_{i,j}\overline{P_i} - P_{i,j}\right) \geq R_j \qquad j = 1, \cdots, N_I$$

where:

$$T_i = \sum_{j=1}^{N_I} \left|1 - Y_{i,j}\right| \cdot \Delta t$$

T_i	out of service time of the unit i
Δt	time duration for the time interval j (i.e. 15 minutes)
N_I	total time interval number
$Y_{i,j}$	binary variable: 0 out of service, 1 in service
$P_{i,j}$	Power produced by ith unit in the jth time interval
$\underline{P_i}, \overline{P_i}$	minimum and maximum power of unit i
$a_{o,i}, a_{1,i}, a_{2,i}$	generation cost coefficients for unit i
$C_{su,i}(T_i)$	start-up cost of unit i
$C_{tot,j}$	total load for the time interval j
$C_{h,j}$	total hydroelectric power produced in the time interval j
R_j	minimum reserve requirement for the interval j

Hydro-thermal scheduling

- The UC is solved: we know the units in service for each time interval
- We want to compute the optimal hydro-thermal production minimising the total generation cost
  - It is necessary a coordination among all resources available

Daily scheduling activities

- Input
  - Energy Forecast (i.e. given by bilateral contracts)
  - Forecast of natural hydraulic contributions to the reservoir
  - Definition of operating reserve
- Output
  - Daily scheduling for each generating unit

Energy Forecast

- The generation company holds bilateral contracts
- For each customer a energy forecast for each time interval is done taking into account the customer characteristics

Forecast of natural hydraulic contributions to the reservoir

- Forecast of natural hydraulic contributions to each reservoir
- For particular situation, the forecast has to update continuously due to unexpected and heavy rainfall
  - Continuous forecast

Operating reserve definition

- It is necessary to guarantee a security operation of the system
- From the point of view of the generation company, the reserve guarantees that an out of service of a generating unit can be replaced from other units
  - Otherwise it is necessary to buy the energy in real time market
- A probabilistic procedure is used to define the reserve requirement
  - Depends on the characteristics of the generating unit pattern
  - Each generating unit is presented with tree possible state:
    - Out of service
    - In service at medium power output
    - In service at maximum power output

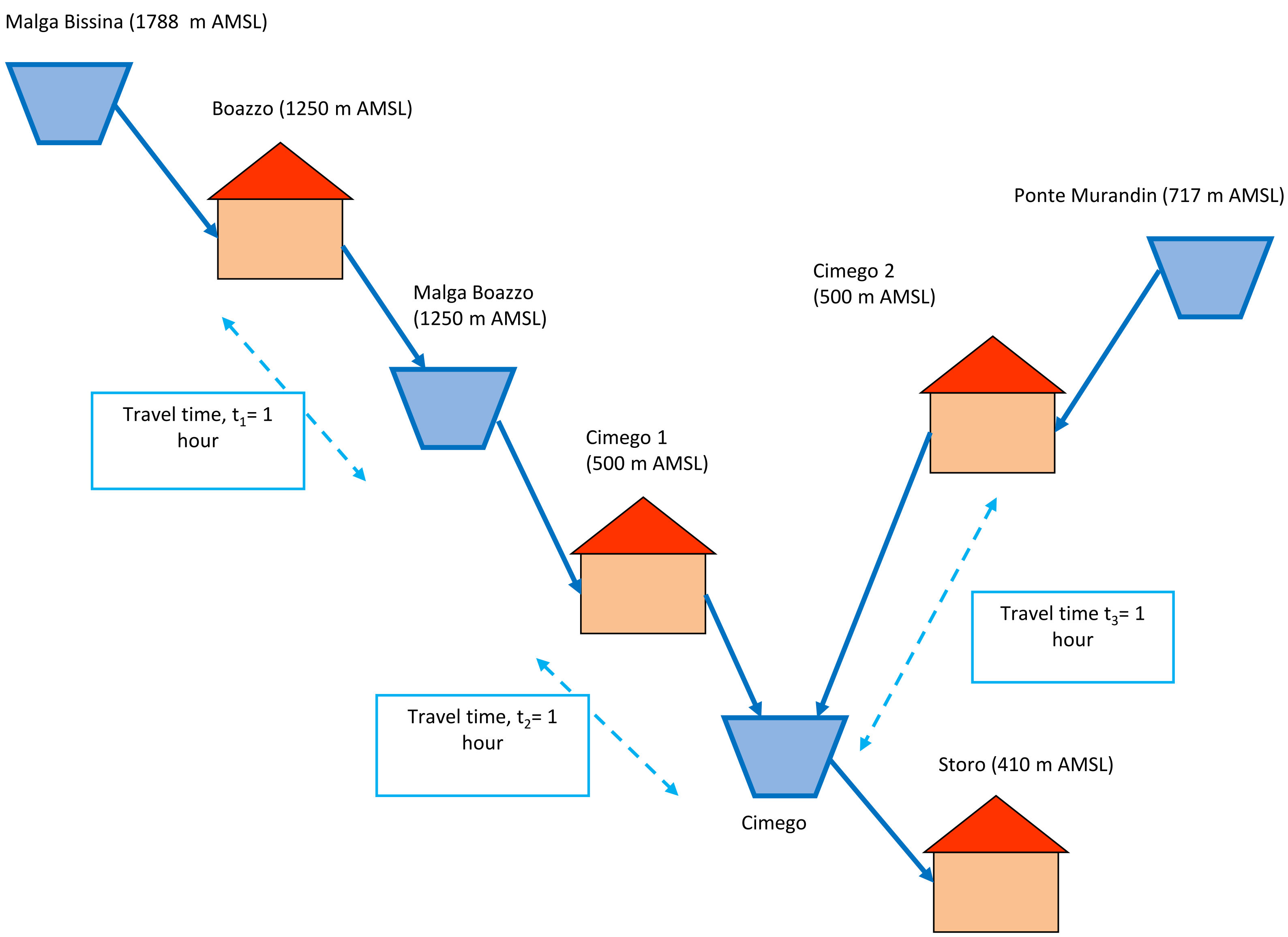
The problem

- The scheduling for each generating unit is computed for each fifteen minutes
- The objective: generation cost minimisation

The model

- Detailed description of the hydroelectric system
  - Hydro plant basin are take into acount: Hydro plants in a basin cannot be considered as independent producers in the market. The availability of energy in one of the plants depends on the ability to store water, the proper water inflows and the water delivered by upper hydro plants into the basin.
  - Moreover, the water delivered by upper hydro plants is accessible for production in the lower hydro plant after the travel time between the plants.
- Zonal representation of the transmission system
- A radial zonal model is adopted

Example: the Alto Chiese Basin and Relevant Hydro Plants



The model

- Zonal representation of the transmission system
- A radial zonal model is adopted

Procedure

- The fuel cost is minimised taking into account operational constraints and security:
  - Balance constraints for the reservoir belonging to the same basin
  - Daily constraints on the minimum and maximum level of the reservoir
  - Pump power plant constraints
  - Power plant (thermal and hydroelectric) technical constraints
  - Constraints on TTC among the transmission areas
- The scheduling given by the procedure depends strongly on the input given by the medium-term hydroelectric planning
- The thermoelectric units available are defined by the UC problem previously described

General consideration about the model

- From a mathematical point of view, the optimisation problem is a classical thermoelectric dispatch problem where the hydroelectric constraints are included
  - Big number of equations and variables
- The hydro-thermal scheduling problem is too big to solve.
- An alternative strategy solution is needed.
- A succession optimisation problem are solved where the hydro-problems and thermal-problems are separately solved
- The solution of the thermal problem gives the input for the hydro-electric problem
  - The Lagrange multiplier are used to link these two different problems

The thermal problem

- Assumption:
  - The hydroelectric production (Ph) is fixed and it is known
- The objective function CTO(P_h) is the cost production CTO(P_h) for a given hydroelectric production profile Ph
- The constraints are:
  - Balance constraint
  - Transmission constraints
  - Upper and lower bounds for the thermal power units

Thermal model

$$CTO(P_h) = min\left\{\sum_{i=1}^{N}\sum_{j\in T_T}\left(a_j \cdot P_{i,j}^2 + b_j \cdot P_{i,j} + c_j\right)\right\}$$

s.t.

$$\sum_{j\in T_T} P_{i,j} = \sum_{k=1}^{N_a} D_{a,i} - \sum_{j\in I_T} P_{i,j} \qquad i = 1, \cdots N$$

$$TR_{k,i} = \sum_{k=1}^{N_a} PTDF_{k,a} \cdot NI_{a,i} \qquad i = 1, \cdots, N \quad k = 1, \cdots, N_k$$

$$NI_{a,i} = D_{a,i} - \sum_{j\in(T_T\cup I_T)} P_{a,i,j} \qquad i = 1, \cdots, N \quad a = 1, \cdots, N_a$$

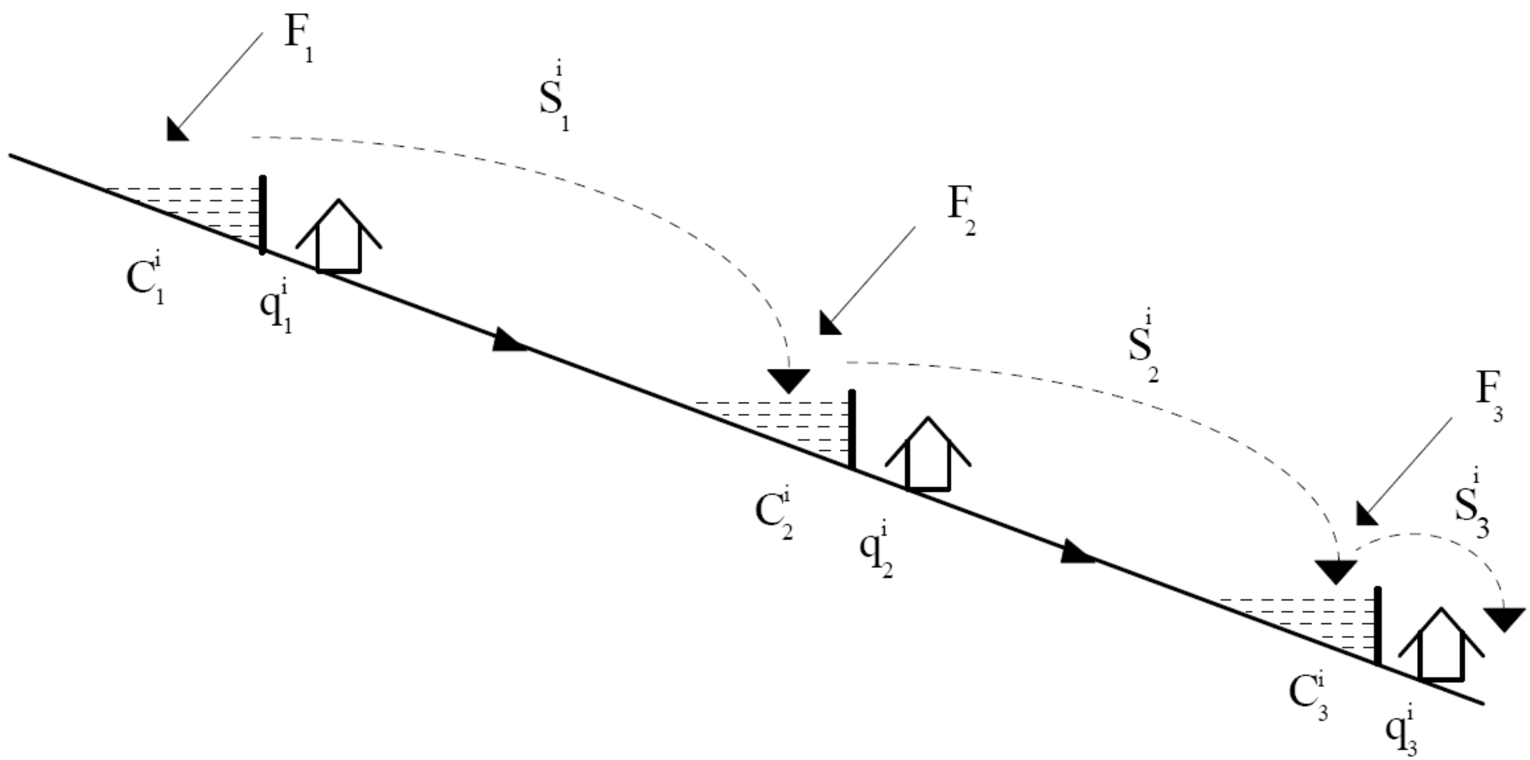
$$\underline{TR_k} \le TR_{k,i} \le \overline{TR_k} \qquad i = 1, \cdots, N \quad k = 1, \cdots, N_k$$

$$\underline{P_j} \le P_{i,j} \le \overline{P_j} \qquad i = 1, \cdots, N \quad j \in (T_T \cup I_T)$$

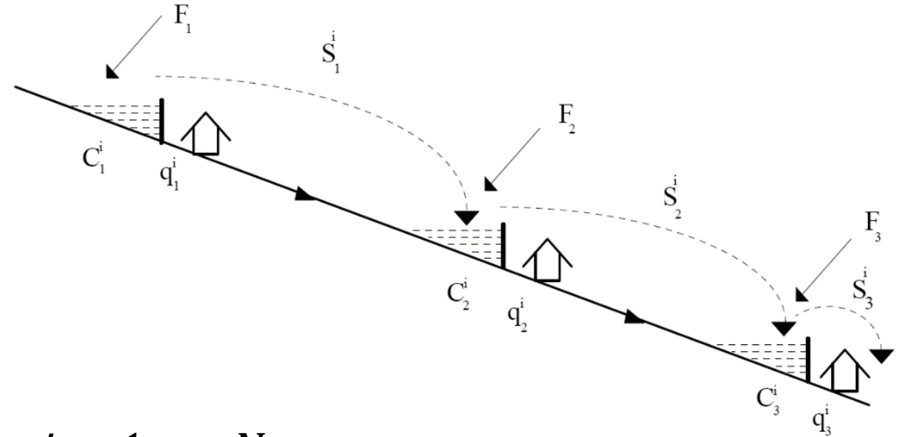
N	number of time periods
a_j, b_j, c_j	generation cost coefficients for thermal power plant j
T_T	set of thermoelectric power plants
I_T	set of hydroelectric power plants
$P_{i,j}$	power output for the jth power plant in the ith time interval
$D_{a,i}$	demand of area a in the time interval i
$TR_{k,i}$	power transit on the interface k in the time interval i
$PTDF_{k,a}$	power transmission and distribution factor between interface k and area a
$NI_{a,i}$	net injection in area a in the time interval i
$\underline{TR_k}, \overline{TR_k}$	minimum and maximum power transit on interface k
$\underline{P_j}, \overline{P_j}$	minimum and maximum power for the unit j

Hydroelectric constraints

- Each basin system in independent from other basin systems



Example of hydro constraints



$$C_1^i = C_1^{i-1} + F_1 - q_1^i - S_1^i \qquad i = 1, \cdots, N$$

$$C_2^i = C_2^{i-1} + F_2 + q_1^i + S_1^i - q_2^i - S_2^i \qquad i = 1, \cdots, N$$

$$C_3^i = C_3^{i-1} + F_3 + q_2^i + S_2^i - q_3^i - S_3^i \qquad i = 1, \cdots, N$$

$$0 \leq C_k^i \leq \overline{C_k} \qquad \begin{array}{l} i = 1, \cdots, N-1 \\ k = 1, \cdots, N_s \end{array}$$

$$\underline{C_k} \leq C_k^N \leq \overline{C_k} \qquad k = 1, \cdots, N_s$$

$$\underline{q_k^i} \leq q_k^i \leq \overline{q_k^i} \qquad \begin{array}{l} i = 1, \cdots, N \\ k = 1, \cdots, N_s \end{array}$$

This constraints takes into account the water availability defined by the medium-term planning

$\underline{q}_j^i, \overline{q}_j^i$	Valori minimo e massimo di q_j^i
N_s	Numero dei serbatoi
k	Indice dei serbatoi
F_k	Apporto naturale relativo al serbatoio k
S_k^i	Sfioro dal serbatoio k durante il periodo i
C_k^0	Volume iniziale di acqua nel serbatoio k
C_k^i	Volume di acqua nel serbatoio k alla fine del periodo i
C_k^{fin}	Minimo volume di acqua che deve rimanere nel serbatoio k alla fine della giornata
$\overline{C}_k$	Massimo valore per C_k^i
D_a^i	Domanda di carico dell'area α nel periodo i
D^i	$= \sum_a D_a^i$
Φ_{a1a2}^i	Transito di potenza attiva massimo ammissibile tra l'area $\alpha 1$ e l'area $\alpha 2$ durante il periodo i
a_j, b_j, c_j	Coefficienti della funzione (quadratica) costo di produzione dell'impianto termoelettrico j

Frank-Wolfe

Minimize $f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^{\mathrm{T}}E\mathbf{x} + \mathbf{h}^{\mathrm{T}}\mathbf{x}$

subject to $\mathbf{x} \epsilon \mathbf{P}$.

Step 1. Initialization. Let $k \leftarrow 0$ and let x_k be any point in $\mathbf{P}$.

Step 2. Convergence test. If $\nabla f(\mathbf{x}) = \frac{1}{2}(E + E^T)\mathbf{x} + \mathbf{h} = \mathbf{0}$ then Stop, we have found the minimum.

Step 3. Direction-finding subproblem. The approximation of the problem that is obtained by replacing the function f with its first-order Taylor expansion around x_k is found. Solve for $\bar{x}_k$:

Minimize $f(x_k) + \nabla^T f(x_k)\bar{x}_k$
Subject to $\bar{x}_k \epsilon \mathbf{P}$
(note that this is a Linear Program. x_k is fixed during Step 3, while the minimization takes place by varying $\bar{x}_k$ and is equivalent to minimization of $\nabla^T f(x_k)\bar{x}_k$).

Step 4. Step size determination. Find λ that minimizes $f(x_k + \lambda(\bar{x}_k - x_k))$ subject to $0 \leq \lambda \leq 1$. If $\nabla f(x_k)^T(\bar{x}_k - x_k) \geq 0$ then Stop, we have found the minimum in x_k.

Step 5. Update. Let $x_{k+1} \leftarrow x_k + \lambda(\bar{x}_k - x_k)$, let $k \leftarrow k + 1$ and go back to Step 2.