1 Introduction

The goal of this lab is to analyze a discrete system using Python's built-in functions and a function developed by Christopher Felton.

2 Equations

For this lab, we use the follwing equation.

$$y[k] = 2x[k] - 40x[k-1] + 10y[k-1] - 16y[k-2] \tag{1}$$

3 Methodology and Results

- Find H(z)

  We are going to use derivative in order to find the Laplace of the following equation:

$$y[k] = 2x[k] - 40x[k-1] + 10y[k-1] - 16y[k-2]$$

$$y[k] - 10y[k-1] + 16y[k-2] = 2x[k] - 40x[k-1]$$

$$Y(z) - 10z^{-1}Y(z) + 16z^{-2}Y(z) = 2X(z) - 40z^{-1}X(z)$$

$$Y(z)(1 - 10z^{-1} + 16z^{-2}) = X(z)(2 - 40z^{-1})$$

$$\frac{Y(z)}{X(z)} = \frac{(2 - 40z^{-1})}{(1 - 10z^{-1} + 16z^{-2})}$$

$$H(z) = \frac{(2 - 40z^{-1})}{(1 - 10z^{-1} + 16z^{-2})}$$

$$H(z) = \frac{(2 - 40z^{-1})}{(1 - 10z^{-1} + 16z^{-2})} \cdot \frac{z^2}{z^2}$$

$$H(z) = \frac{(2z^2 - 40z)}{(z^2 - 10z + 16)}$$

- Find h[k]

  Now, we are going to use partial fraction expansion in order to find h[k]:

$$H(z) = \frac{(2z^2 - 40z)}{(z^2 - 10z + 16)}$$

$$H(z) = \frac{(2z(z-20))}{((z-8)(z-2))}$$

$$\frac{H(z)}{z} = \frac{\frac{2z(z-20)}{z}}{((z-8)(z-2))}$$

$$\frac{H(z)}{z} = \frac{(2(z-20))}{((z-8)(z-2))}$$

$$\frac{(2(z-20))}{((z-8)(z-2))} = \frac{A}{(z-8)} + \frac{B}{(z-2)}$$

$$2(z-20) = A(z-2) + B(z-8)$$

$$put : z = 8$$

$$2(8-20) = A(8-2) + B(8-8)$$

$$-24 = 6A$$

$$A = \frac{-24}{6}$$

$$A = -4$$

$$put : z = 2$$

$$2(2-20) = A(2-2) + B(2-8)$$

$$-36 = -6B$$

$$B = \frac{36}{6}$$

$$B = 6$$

$$\frac{H(z)}{z} = \frac{(-4)}{(z-8)} + \frac{6}{(z-2)}$$

$$H(z) = \frac{(-4z)}{(z-8)} + \frac{6z}{(z-2)}$$

$$H(z) = \frac{(-4)}{(1-8z^{-1})} + \frac{6}{(1-2z^{-1})}$$

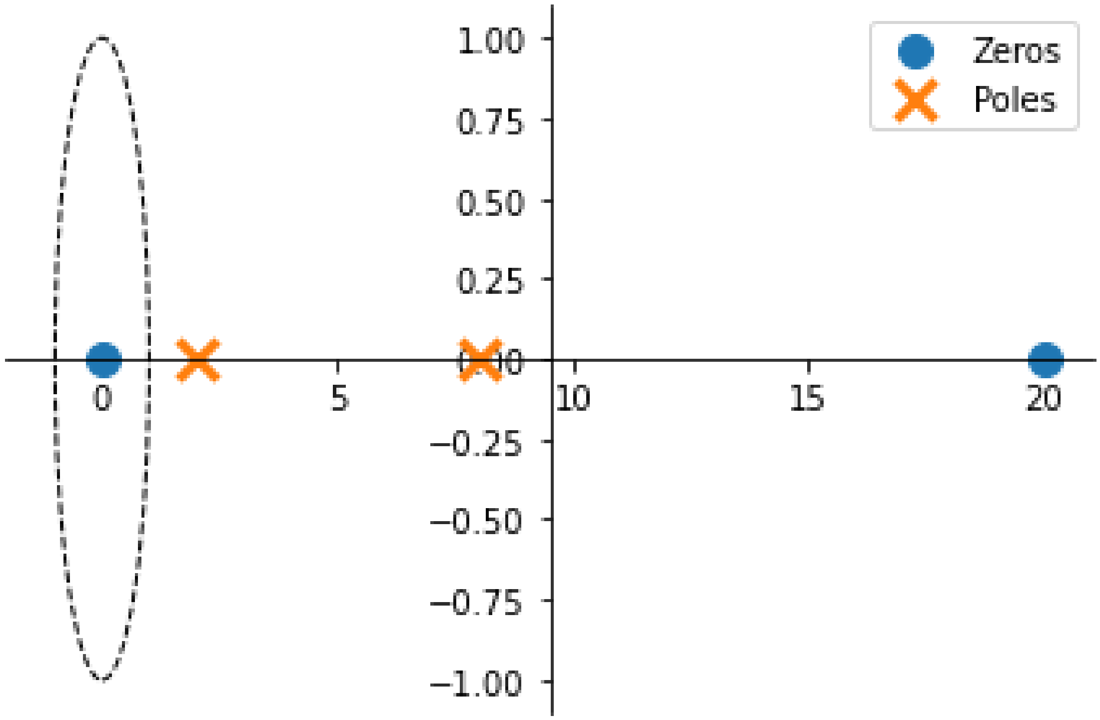
$$2 < |z| < 8$$

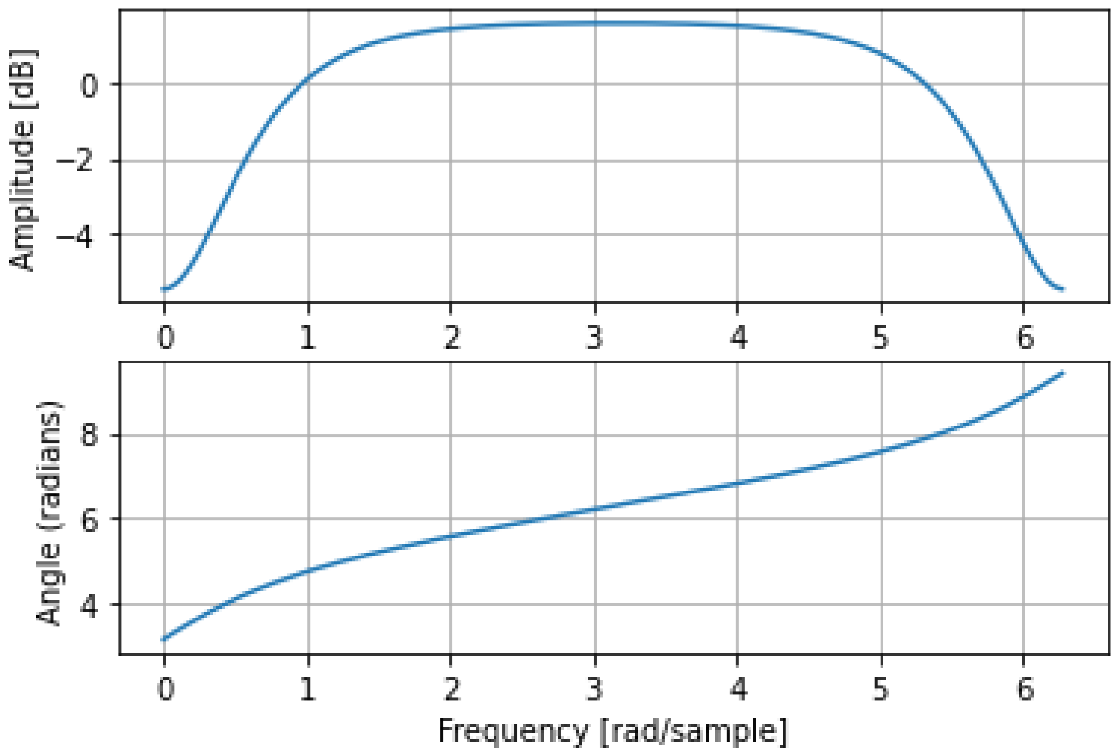
$$h(k) = -4(-8)^k u[-k-1] + 6(2)^k u[k]$$

- Python Code:

```python

    import numpy as np
    import matplotlib.pyplot as plt
    import scipy.signal as sig
    import zplane as zp

    #%% task 3

    num = [2, -40, 0]
    num1 = [2, -40]
    den = [1, -10, 16]

    r, p, k = sig.residue(num1, den)
    print("Residue: ", r)
    print("Poles: ",p)
    print("K: ",k)

    #%% task 4

    Z, P, K = zp.zplane(num, den)


    #%% task 5

    w, h = sig.freqz(num, den, whole=True)
    angles = np.unwrap(np.angle(h))

    plt.figure()
    plt.subplot(2, 1, 1)
    plt.plot(w, h)
    plt.xlabel("Frequency [rad/sample]")
    plt.ylabel("Amplitude [dB]")
    plt.grid()
    plt.subplot(2, 1, 2)
    plt.plot(w, angles)
    plt.grid()
    plt.xlabel("Frequency [rad/sample]")
    plt.ylabel('Angle (radians)')
    plt.show()
```

- The output plots of this code:





4 Questions

1. Looking at the plot generated in Task 4, is H(z) stable? Explain why or why not.

   H(z) is stable because the frequency response rises and then decays back down indicating that the function is stable.

5 Conclusion

In this lab, we learned the usage of how to analyze a discrete system using Python's.

6 Appendix

```
Reloaded modules: zplane
Residue:  [ 6. -4.]
Poles:  [2. 8.]
K:  []
```